

Article

Determination of Chimney Non-Verticality from TLS Data Using RANSAC Method

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Abstract: The continuous monitoring of tall industrial buildings is necessary to ensure safe operation. With technological advances in terrestrial laser scanning and other non-contact measurement methods, the methods and techniques for assessing the stability of tall industrial chimneys are evolving. This paper presents a method for determining the non-verticality and straightness of chimneys that offers significant advantages over existing methods. Narrow bands of scanned point clouds are processed at selected height intervals. Using the RANSAC method, points that do not belong to the chimney shell are filtered and the centre of the circle or ellipse is adjusted using the least squares method. The proposed method enables the efficient filtering of point clouds due to frequent obstructions on the chimney shell, the determination of the regularity of the chimney shell shape, a mathematical analysis of the chimney axis curvature, and an intuitive graphical representation of chimney non-verticality. The comparison of the results with other studies confirms the efficiency of the method.

Keywords: chimney; terrestrial laser scanning; non-verticality; RANSAC; least squares method; deformations



Citation: Pleterski, Ž.; Rak, G.; Kregar, K. Determination of Chimney Non-Verticality from TLS Data Using RANSAC Method. *Remote Sens.* **2024**, *16*, 4541. <https://doi.org/10.3390/rs16234541>

Academic Editor: Sara Gonizzi Barsanti

Received: 6 November 2024

Revised: 30 November 2024

Accepted: 2 December 2024

Published: 4 December 2024



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1. Introduction

Tall chimneys are high structures that are typically built near industrial facilities such as power plants, factories, refineries, and other facilities. They serve the crucial purpose of safely releasing gases and solid particles generated by combustion processes or industrial activities into the atmosphere. When planning such structures, attention must be paid not only to functionality but also to the selection of appropriate building materials, structural stability, and safety measures [1]. Chimneys must be constructed from materials capable of withstanding the high temperatures generated during combustion or other processes to prevent fire outbreaks or heat-related damage. Using materials that do not properly contract or expand can lead to structural damage or even collapse [2]. Moreover, external factors such as wind loads, solar radiation, earthquakes, and other environmental influences must also be included in the design and construction process to ensure the durability and resilience of the chimney.

In accordance with the safety measures for such structures, Slovenian legislation prescribes regular geotechnical monitoring to detect any damage and deformation of the structure. As part of the geodetic monitoring, non-verticality measurements can also be carried out to check compliance with the permissible values specified in the Eurocode standards. In the present case, Eurocode 3: Design of steel structures [3] is applicable. These are preventive safety measures aimed at ensuring regular maintenance and preventing possible serious damage to the structure that could jeopardise the operation of the facility or nearby residents. Determining the non-verticality of tall chimneys refers to the fact that tall chimneys are often not completely vertical, exhibiting inclinations or skewness. Various factors may contribute to this deviation, including elevated temperatures within the chimney, construction-related variables, or settlement of the structure [4,5]. As instrumentation has evolved, so has the methodology for such tasks. The simplest approach entails conducting

measurements using a total station (TS), where we determine the offset of the object surface from a plane defined by the collimation axis of the TS when the offset is measured with a tape measure [4]. Although this method offers high precision, it has several disadvantages, including the necessity for measurements in at least two orthogonal positions, the inaccessibility of measuring points on the object, and its inappropriateness in the case of irregular chimney shell shapes or obstacles near the object. Alternatives include methods that enable non-contact distance measurement, either by a modern TS with scanning capabilities or a terrestrial laser scanner (TLS), which allows us to obtain a point cloud of the chimney's surface. The literature on analysing the non-verticality of various industrial chimneys can be divided into three categories, namely the approximation of chimney cross-sections with geometric shapes (circle or ellipse), the approximation of the chimney by a geometric solid (frustum of a cone or cylinder), and alternative photogrammetric methods.

The approximation of a chimney with a circle or an ellipse is the most common approach in the literature. Gražulis et al. [5] presented a comparison of the determination of the centre of a chimney shell based on TLS measurements and terrestrial positioning system (TPS) measurements. In the TLS measurement, the point cloud was analysed at the same height levels as in TPS measurements, with the difference being that the centre point was determined using a narrow band of points at the specified height. The points were then projected onto horizontal planes at the height levels under consideration, from which the chimney centre was adjusted using the least squares method (LS method). In the traditional measurements, the calculation was carried out in a similar way, except that only eight points on the chimney shell were considered at each height level. In addition to the direct comparison of the TPS and TLS methods, the non-verticality was calculated from the differences in the coordinates of the calculated chimney centres. They found that the root mean square error (RMSE) was less than 2 mm in both cases, confirming the suitability of the TLS method for such tasks. A very similar analysis based on TLS data was also conducted by Muszynski and Milczarek, Barazzetti et al., Schneider, and Ötsch and Neuner [6–9]. They all analysed the non-verticality at different height levels by considering different width bands of the point cloud (2–5 cm), which were then projected onto a horizontal plane, to calculate the parameters of the fitted circle. Kaszowska et al. [10] investigated the tilt and tilt direction of the chimney by using a laser sensor designed for continuous tilt monitoring and TLS data. They found that geodetic methods are unsuitable in certain cases, as many chimneys or at least parts of the chimneys are not measurable due to nearby structural obstructions. When measurement is possible, they see the advantage of TLS in the ability to monitor the building geometry or compare it with the design, as discussed by Głowacki et al. [11]. They recognise the novelty in the use of laser sensors, as they allow for the continuous measurement of deviations in real-time, enabling the signalling of detected excessive deviations in real-time. In recent years, methods of using unmanned aerial vehicles (UAVs) for measurements of chimney non-verticality have also appeared in the literature. The main reason for this is that when using TLSs for tall structures, challenges arise with larger vertical angles, measuring longer lengths, and large angles of incidence. The problem manifests itself in a sparse point cloud and poor quality of the reflected signals, even when the measured lengths were shorter than the maximum lengths specified by the manufacturer. Harshit et al., Siwiec and Lenda, and Zrinjski et al. [12–14] determined non-verticality using the same approach, i.e., by fitting an ellipse or a circle. In practice, the UAV measurement method has not yet proven to be very suitable, as the merging of TLS and UAV point clouds is not optimal and the accuracy of the calculation from UAV clouds is lower.

The approximation of a chimney with a geometric solid is a procedure in which the entire chimney is treated as a single entity. Marjetič et al. [4], in their study of the gas turbine chimney of the Brestanica Thermal Power Plant (TEB), treated it as a frustum of a cone and assumed that the cone radius varies linearly with the height of the cone. They determined the non-verticality of the chimney based on the deviation of the chimney axis from the vertical. This was determined by the LS method adjustment using a scanned series

of points on the chimney shell recorded with the Leica Geosystems TCRP1201 + R1000 electronic TS. Kregar et al. [15] repeated the measurement of the TEB chimney, this time comparing the performance of the Leica TS30 electronic TS and the Riegl VZ-400 terrestrial laser scanner. In both cases, the chimneys were treated as cylinders using the LS method. Based on the results, they concluded that it is difficult to determine which approach is better, as the results are comparable. Practically the same methodology, namely cylinder fitting, was also used by Popović et al. [16].

Zheng et al. [17] introduced a method for determining the non-verticality of a chimney based on its silhouette. Their approach was to consider the chimney as a cylindrical structure and set up a geodetic network with 3–5 stations with known coordinates around it. At each station, they used the TOPCON Image Station to capture the left and right silhouettes of the chimney. They selected the most suitable photo, identified a projection point on the edge of the object, and determined its horizontal direction. At each station and at each elevation level, the centre and radius of the chimney were determined based on the intersection of sightlines using photogrammetry. The method assumes that the horizontal direction to the extreme points of the chimney silhouette represents a tangent to the chimney, with the average direction indicating the centre of the chimney. By intersecting the values from at least three stations, they calculated the coordinates of the chimney centre and the radius of the chimney shell centre. The authors argue that this method is more efficient, more accurate, and less time-consuming.

Primitive shape recognition is a common problem encountered in many fields of geometry-related computer science. Numerous methods have been proposed over the years, with the fundamental approaches including region-based methods, the Hough transform, and RANSAC [18–20]. Additionally, many modifications of these methods can be found in the literature, as thoroughly described by Schnabel et al. [21]. Point cloud segmentation plays a pivotal role in point cloud processing [20,22,23] and often serves as the initial step in extracting information from 3D point clouds. Segmentation algorithms group points based on specific criteria, usually connecting points that belong to the same surface. From this perspective, segmentation can be seen as the process of recognising simple surfaces in a point cloud [24]. In many applications, it is crucial to achieve high-quality and accurate results. This includes filtering out the noise, i.e., points that do not belong to the surface of the analysed object. Otherwise, model parameter estimation may include points that deviate from the model being estimated [25]. The RANSAC algorithm is particularly effective in the presence of noise and outliers [20,26] and was first introduced by Fischer and Bolles [27] in 1981. It is a remarkably simple yet powerful technique, which explains its widespread adoption. In addition to its simplicity, its ability to handle a high degree of data contamination while providing reliable parameter estimates is another key advantage [21,25]. Furthermore, the method is suitable for both 2D and 3D shape detection [20,21,23,28]. The utility and effectiveness of each model are demonstrated by its ability to correctly process multiple 2D and 3D geometric shapes [21], which makes it useful for a variety of tasks, as has been shown by several authors [15,21,28–30]. The RANSAC method, in combination with single height interval processing and fitting of circles, has been shown to be valuable for the non-verticality assessment of chimneys. Given that a TLS captures massive datasets with inherent noise, the proposed methodology simplifies the filtering-out of unwanted points, ensuring higher-quality results. With this approach, least squares (LS) adjustment can be performed on already filtered data. Additionally, circle or ellipse fitting represents simple geometric modelling of cross-sections, reflecting the true shape of a chimney cross-section. However, when analysing geometric 3D bodies, such as cylinders or cones, this simplification can lead to a loss of information about possible chimney twisting.

2. Materials and Methods

The proposed method is based on TLS data, analysed by filtering a point cloud using the RANSAC (RANDOM SAMPLE CONSENSUS) method and subsequent LS method

adjustment [27]. RANSAC is a method commonly used in computer vision and computer graphics to solve problems related to data containing noise or errors [31]. Its main advantage lies in the estimation of model parameters from a point cloud, even in the presence of significant noise in the data. The RANSAC method allows for the estimation of parameters for a mathematically describable shape or body that best fits the geometry of the scanned object [32].

The algorithm terminates when it identifies the largest number of points that best describe the selected model. This number must be sufficiently large to assert, with a certain probability that we choose ourselves (with a sufficient level of confidence), that at least one set of the random sample does not contain any outliers. Thus, the probability that the algorithm never selects only inliers is therefore [18]

$$1 - p = (1 - u^m)^N \quad (1)$$

where the variables are as follows:

- p —the confidence level (probability that only inliers will be randomly chosen in at least one of the repetitions);
- u —the expected percentage of inliers in a point cloud;
- m —the number of points needed to uniquely determine the parameters of a chosen mathematical model;
- N —the number of iterations needed to ensure (with a certain level of confidence) that at least one of the subsets contains only inliers.

The parameter values in our case were $u = 0.7$, $p = 0.95$, $t = 0.05$ m, and $m = 3$. N can be calculated from Equation (1) as

$$N = \frac{\log(1 - p)}{(1 - u^m)} \quad (2)$$

This approach allows us to analyse the chimney shell not as a single unit but in individual sections, which facilitates the assessment of both non-verticality and possible irregularities in the shape. In our study, we wanted to demonstrate the relevance and efficiency of the RANSAC method in the case of anomalies on the chimney shell. Additionally, we analysed whether each section of the chimney shell that intersects with the horizontal plane at different elevations actually has a circular shape or instead represents an ellipse.

2.1. Field Measurement and Utilised Instrumentation

A local geodetic micro-network was set up in the TEB area to monitor the movements and deformations of the TEB objects. In the first phase of the field survey, precise classical geodetic measurements of the network, including the reference points for the TLS, were carried out (Figure 1). This ensured that the positions of the reference points were determined with the highest accuracy (level of tenths of a millimetre). The local geodetic micro-network was measured using the Leica TS30 instrument. The TS30 provides an angular accuracy of $0.5''$ (tested according to ISO 17123-3:2001 [33]). The accuracy of the distance measurement with the Pinpoint EDM to Leica prism targets is $0.6 \text{ mm} + 1 \text{ ppm}$ (tested according to ISO 17123-4:2012 [34]) [35]. Horizontal directions, zenith distances and, slope distances between all visible points are observed, properly processed, and finally adjusted using the least squares method. The location of the GCP is shown in Figure 1, where the TS standpoints also correspond to the GCP for the TLS. The GCPs are determined in the project coordinate system using the 3D terrestrial geodetic network. The accuracy achieved for all points in the geodetic network is greater than or equal to 0.2 mm . Leica HDS $6''$ blue circular targets are cantered into the same tribraches, resulting in exactly the same coordinates with a constant height difference of 141.5 mm .

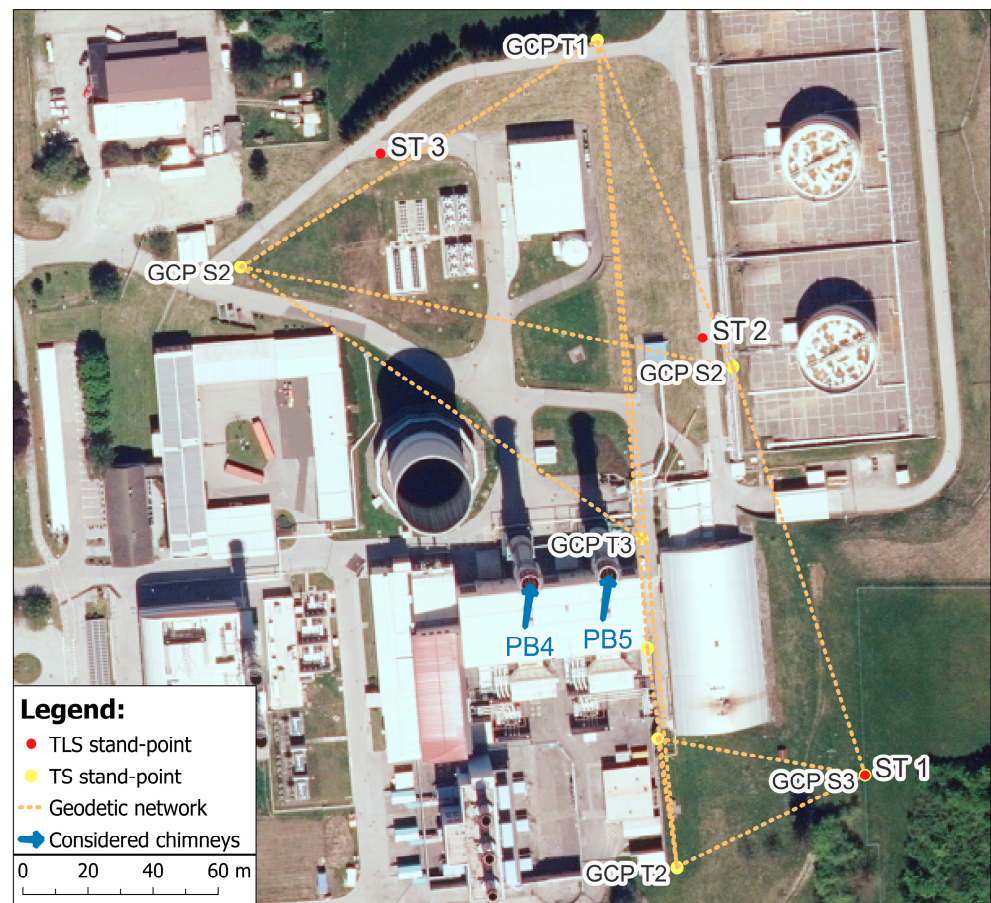


Figure 1. Layout of the local geodetic micro-network and display of TLS stations.

For the TLS, the instrument was temporarily stabilised on a tripod at each station. The stations were chosen so that 4–5 reference points were visible from each station, taking care to cover the geometry of the chimney shell as evenly as possible. The scanning was carried out with the Riegl VZ-400 instrument (Figure 2). The 3D terrestrial laser scanner RIEGL VZ-400 provides scan data acquisition with 5 mm accuracy and 3 mm repeatability, a measurement range of up to 600 m, and an efficient measurement rate of up to 122,000 measurements/s [36]. The instrument deflects the beam horizontally by rotating around the z-axis, resembling the operation of the TS. From this point of view, it is assumed that systematic errors manifest themselves in a similar way to a classical geodetic TS. As is common for laser scanners, the length measurements are taken directly on the surface of the scanned object without using reflectors. Therefore, a large number of points are measured every second, each of which has a unique position determined based on the horizontal angle, vertical angle, and slope distance. It should be noted that the coordinates of the points are measured in the coordinate system of the scanner. The final point positions were determined in the post-processing phase by registration based on tie points and georeferencing relative to known coordinates of reference points in the coordinate system of the geodetic network. TLS targets were scanned with a 1×1 mm grid on the target. The target centres were determined from the scanned point clouds of targets with procedures presented in [37,38], resulting in the coordinates and accuracies given in Table 1.

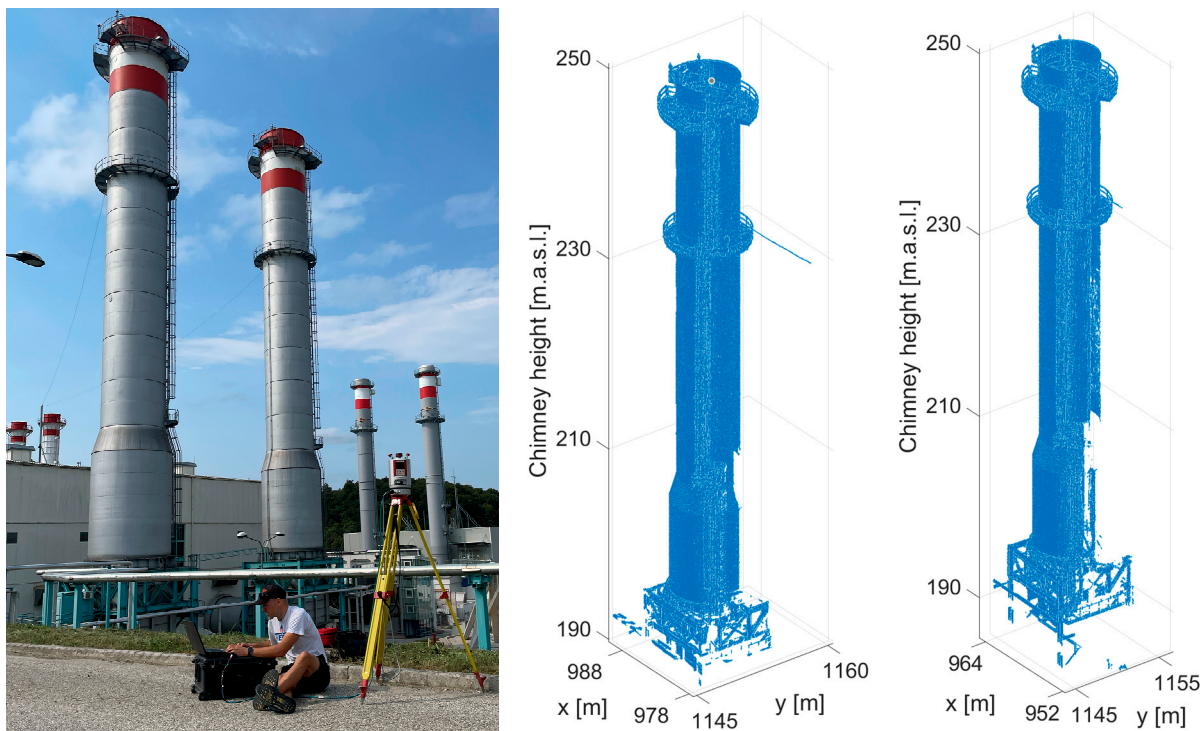


Figure 2. The scanning process of the chimneys under consideration (**left**) and point cloud of chimney shell PB4 (**middle**) and PB5 (**right**).

Table 1. Coordinates and accuracies of the target centres in the coordinate system of the scanner.

Station	Target	x [m]	y [m]	H [m]	σ_x [mm]	σ_y [mm]	σ_H [mm]
ST1	GCP S1	7.9662	−28.2271	1.8686	0.59	1.47	0.74
ST1	GCP S2	−33.5671	138.0895	−0.1333	2.17	2.69	3.55
ST1	GCP T1	−76.0140	240.3119	−1.0973	3.66	2.25	6.30
ST1	GCP T2	−44.4307	−49.3142	−13.1184	1.08	1.10	1.67
ST2	GCP S1	47.4958	−169.1779	5.5132	2.64	2.74	4.39
ST2	GCP S2	9.6583	−1.9979	2.2347	0.88	0.23	0.32
ST2	GCP S3	−141.1382	35.9880	−8.9799	2.84	2.23	3.65
ST2	GCP T1	−30.5027	101.1616	0.9655	1.71	2.68	2.64
ST2	GCP T3	−24.0025	−38.9064	−5.3311	0.84	1.04	1.15
ST3	GCP S2	108.0311	−71.1136	9.3690	2.45	2.17	3.24
ST3	GCP S3	−42.0870	−31.1561	−3.8964	1.32	1.11	1.31
ST3	GCP T1	69.4039	32.5766	5.9242	2.05	1.40	1.92
ST3	GCP T3	73.9037	−107.6498	2.1480	1.91	1.84	3.26

The adjustment for the determination of the transformation parameter returns also assessed the accuracies of the parameters (Table 2). The rotations are presented as rotation axis (unit vector) and angle.

Table 2. Accuracy parameters for the transformation of each setup.

Station	$\sigma_{rot.angle}$ [°]	σ_{vecx} [mm]	σ_{vecy} [mm]	σ_{vecz} [mm]	σ_{tx} [mm]	σ_{ty} [mm]	σ_{tz} [mm]
ST1	0.0138	0.024	0.019	0.005	1.343	1.389	2.176
ST2	0.0324	0.030	0.018	0.010	2.388	2.455	3.268
ST3	0.0319	0.020	0.016	0.011	2.402	2.094	2.981

To assess the influence of the uncertainty of the transformation parameters on the chimney inclination, we used the values $\sigma_{vec\ x}$ and $\sigma_{vec\ y}$ at a height of 65 m and obtained values of 1.99, 2.28, and 1.65 mm for the first, second, and third stations, respectively. Residuals of GCP after transformation into the local geodetic micro-network coordinate system are given in Table 3.

Table 3. Residuals of GCP after transformation into the local geodetic micro-network coordinate system.

Station	Target	Δ_x [mm]	Δ_y [mm]	Δ_H [mm]
ST1	GCP S1	0.04	1.67	0.72
ST1	GCP S2	3.08	1.23	2.96
ST1	GCP T1	2.92	0.71	1.49
ST1	GCP T2	0.14	1.68	0.16
ST2	GCP S1	9.61	6.25	6.56
ST2	GCP S2	2.00	2.84	10.75
ST2	GCP S3	8.69	4.59	3.26
ST2	GCP T1	10.59	0.61	7.06
ST2	GCP T3	3.21	3.60	6.03
ST3	GCP S2	4.29	1.33	6.78
ST3	GCP S3	6.36	0.73	7.07
ST3	GCP T1	3.48	0.01	5.83
ST3	GCP T3	0.86	0.36	2.05

Figure 2 shows the analysed chimneys (left) and the point clouds of the two chimneys obtained with the TLS (right). Even if the chimneys are analysed within a local coordinate system, the vertical component is based on the national height coordinate system. Therefore, the unit m.a.s.l. (metres above mean sea level) is used in all graphical representations.

The radius of the chimney shell at an altitude of 196 m.a.s.l. (bottom) is 3.66 m. Between altitudes of 206 m.a.s.l. and 210 m.a.s.l., the radius of the chimney shell decreases, evenly to 2.91 m. From an altitude of 210 m.a.s.l. to 250 m.a.s.l., the chimney shell no longer changes its shape. In the study, we looked at a 40 m long section of the chimney shell between 210 m.a.s.l and 250 m.a.s.l., for direct comparison with the results of Marjetič et al. [4], although the method presented allows the entire chimney shell to be analysed. All four chimneys were scanned from all three stations with the resolution set to 20 mm for the distance between the station and the chimney. Table 4 shows the number of points recorded for each scan.

Table 4. Number of captured points for each scan.

Station	Chimney	Nr. of Points	Station	Chimney	Nr. of Points	Station	Chimney	Nr. of Points
ST1	PB4	0.91 M	ST2	PB4	2.33 M	ST3	PB4	1.93 M
	PB5	0.86 M		PB5	2.54 M		PB5	1.74 M

2.2. Solution with a Circle

For each height interval, we determine the parameters of a circle by curve fitting. The equation of a circle in space has the following form:

$$(x_i - x_{s0})^2 + (y_i - y_{s0})^2 = r_0^2 \quad (3)$$

In Equation (3), the elements are x_i , y_i —coordinates of the point on the surface of the chimney shell; x_{s0} , y_{s0} —approximate coordinates of the centre of the circle; and r_0 —approximate radius of the circle in the plane with which we intersect the chimney shell.

From Equation (3), we can write a conditional equation F , which is of the following form:

$$F: (x_i - x_{s0})^2 + (y_i - y_{s0})^2 - r_0^2 = 0 \quad (4)$$

The functional model described by Equation (4) must be applied to each point measured on the surface of the chimney shell for each height band. Since Equation (4) represents a nonlinear system of equations, it must be linearised during the fitting process of the LS method by taking derivatives considering all observations and unknowns. The solution is determined using the general fitting model of the LS method.

$$\mathbf{A}\mathbf{v} + \mathbf{B}\Delta = \mathbf{f} \quad (5)$$

$$\Delta = \left(\mathbf{B}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{B} \right)^{-1} \cdot \left(\mathbf{B}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{f} \right) \quad (6)$$

$$\mathbf{Q}_{\Delta\Delta} = \left(\mathbf{B}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{B} \right)^{-1} \quad (7)$$

$$\hat{\mathbf{x}} = \mathbf{x}_0 + \Delta \quad (8)$$

$$\sigma_{xs} = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(1,1)}; \sigma_{ys} = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(2,2)}; \sigma_r = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(3,3)} \quad (9)$$

Or, in the case of an ellipse

$$\sigma_{xs} = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(1,1)}; \sigma_{ys} = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(2,2)}; \sigma_a = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(3,3)}; \sigma_b = \sigma_0 \sqrt{\Sigma_{\Delta\Delta}(4,4)} \quad (10)$$

where

$$\sigma_0 = \sqrt{\frac{\mathbf{v}^T \mathbf{P} \mathbf{v}}{r}} \quad (11)$$

In Equations (5)–(11), the elements are $\mathbf{A}$ —the matrix of partial derivatives concerning observations; $\mathbf{v}$ —the correction vector of observations; $\mathbf{B}$ —the matrix of partial derivatives for unknowns; Δ —the correction vector of unknowns; $\mathbf{f}$ —the vector of residuals; $\mathbf{Q}_{\Delta\Delta}$ —the matrix of cofactors of unknowns; $\hat{\mathbf{x}}$ —the adjusted values of the unknowns (parameters of circle/ellipse); r —the number of redundant observations; and $\mathbf{P}$ —the weight matrix.

The images below present the graphical results of the RANSAC method. Figure 3 (left) shows the input data for the RANSAC method at an elevation of 234 m.a.s.l. At certain height intervals of the chimney (such as the 234 m.a.s.l.), there were cases where obstacles such as a fence or platform (represented by a larger black circle) were incorporated into the circle-fitting process alongside the chimney's lateral surface (Figure 3 (left)). The same figure also shows a section with higher noise, most likely caused by multiple reflections of the laser beam from a metallic structure, such as a fence or platform. The reason why this only occurs at one station is related to the incident angle of the laser beam. Since the larger circle contains more points, an additional iteration of the RANSAC procedure was necessary, particularly for the data classified as outliers (red) in the first iteration (Figure 3 (right)). Figure 4 (left) shows the results of the second iteration of the RANSAC method, where red points indicate outliers and green points represent inliers. Figure 4 (right) shows the final result of the chimney shell analysis at a specific height. The red points indicate all the outliers, the green points represent all the inliers, and the black circle represents the fitted circle calculated using the LS method on the inliers.

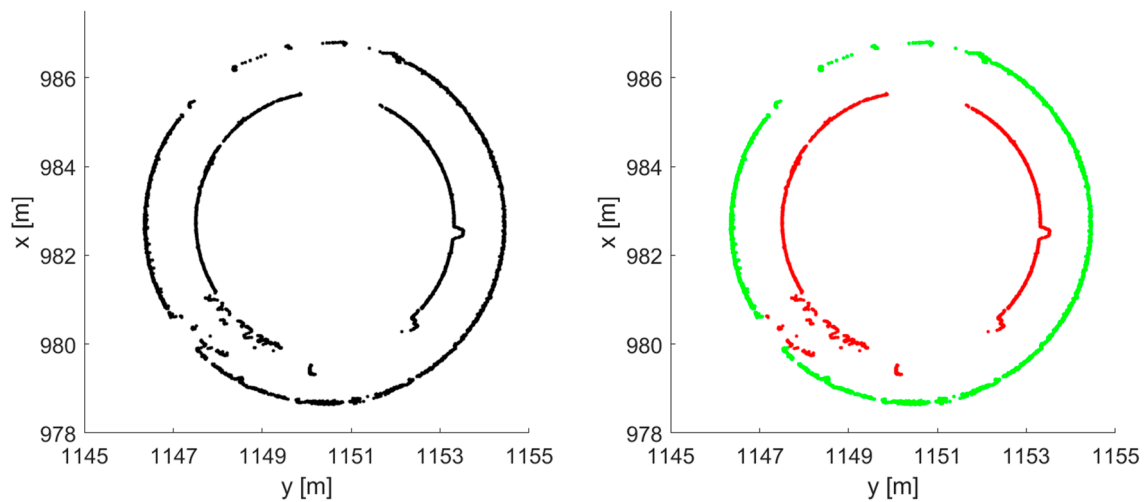


Figure 3. Input data (**left**) and result of the first iteration (**right**), with outliers shown in red and inliers in green.

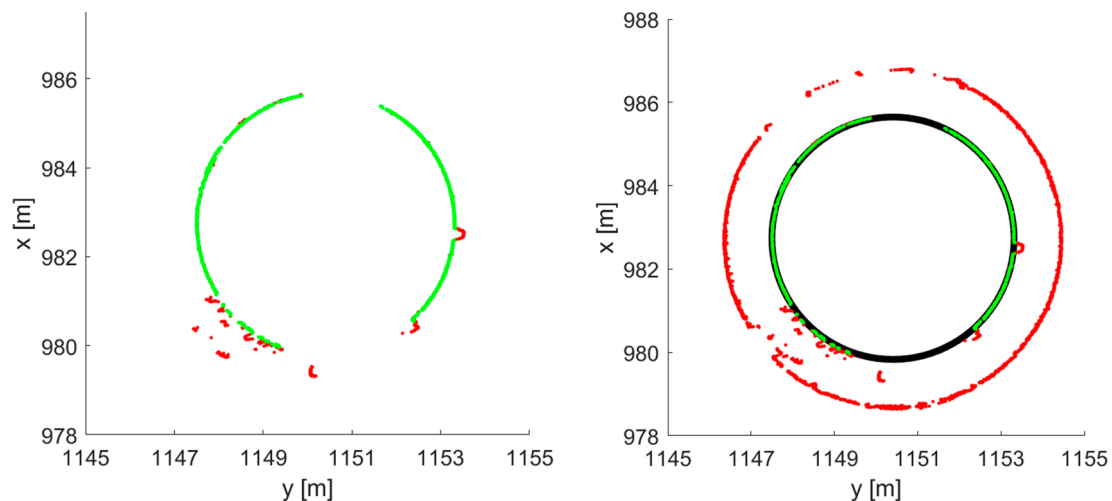


Figure 4. Result of the second iteration of the RANSAC method (**left**) and visual inspection of the results (**right**).

2.3. Solution with an Ellipse

In the second part of the study, we investigated whether it makes more sense to calculate with an ellipse or whether the use of the ellipse equation is also suitable if the chimney intersection point forms a circle. The crucial question was whether the results would remain correct if we carried out the calculations using the ellipse equation when the chimney is actually circular. The equation of an ellipse in space is expressed as follows:

$$\frac{(x_i - x_{s0})^2}{a^2} + \frac{(y_i - y_{s0})^2}{b^2} = 1 \quad (12)$$

In Equation (12), the elements are x_i , y_i —coordinates of the point on the surface of the chimney; x_{s0} , y_{s0} —approximate coordinates of the centre of the ellipse; and a , b —the major and minor semi-axes of the ellipse in the horizontal plane intersecting the surface of the chimney shell.

The functional relationship and the equations for the adjustment calculation remain unchanged. Only the partial derivatives in the matrices **A** and **B** are changed. To assess the flatness, we calculated the linear and numerical eccentricity. The linear eccentricity of the ellipse is a mathematical parameter that indicates the distance between the centre of the

ellipse and one of its foci. If a is the length of the major semi-axis of the ellipse and b is the length of the minor semi-axis of the ellipse, then the linear eccentricity is given by

$$e = \sqrt{a^2 - b^2} \quad (13)$$

The numerical eccentricity of an ellipse refers to the ratio between the distance between the foci of the ellipse and the length of its major semi-axis, expressed as

$$\varepsilon = \frac{e}{a} \quad (14)$$

If the numerical eccentricity is close to 0, this indicates that the foci of the ellipse are relatively close to the centre of the ellipse, which suggests that the ellipse is very close to a circle. If, on the other hand, the numerical eccentricity is close to 1, this indicates that the ellipse is very elongated. Due to the similarity between ellipses and circles, the plots closely resemble those shown in Figures 3 and 4.

2.4. Generating Height Bands

We conducted an analysis of the chimney shell in 1-metre height intervals, ranging from an altitude of 210 m.a.s.l. to 250 m.a.s.l. Each height interval represents a segment of the primary point cloud of the chimney shell, generated as a cross-section with the x-y plane. This was achieved by selecting points on each height plane, with the heights ranging from 210 m.a.s.l. to 250 m.a.s.l. in each metre. The height of each band was 1 cm, i.e., it consists of points corresponding to the height range $(n, n + 0.01)$ m. We opted for this approach to allow for a direct comparison with the results of Marjetič et al. [4]. An illustration of the height intervals is shown in Figure 5.

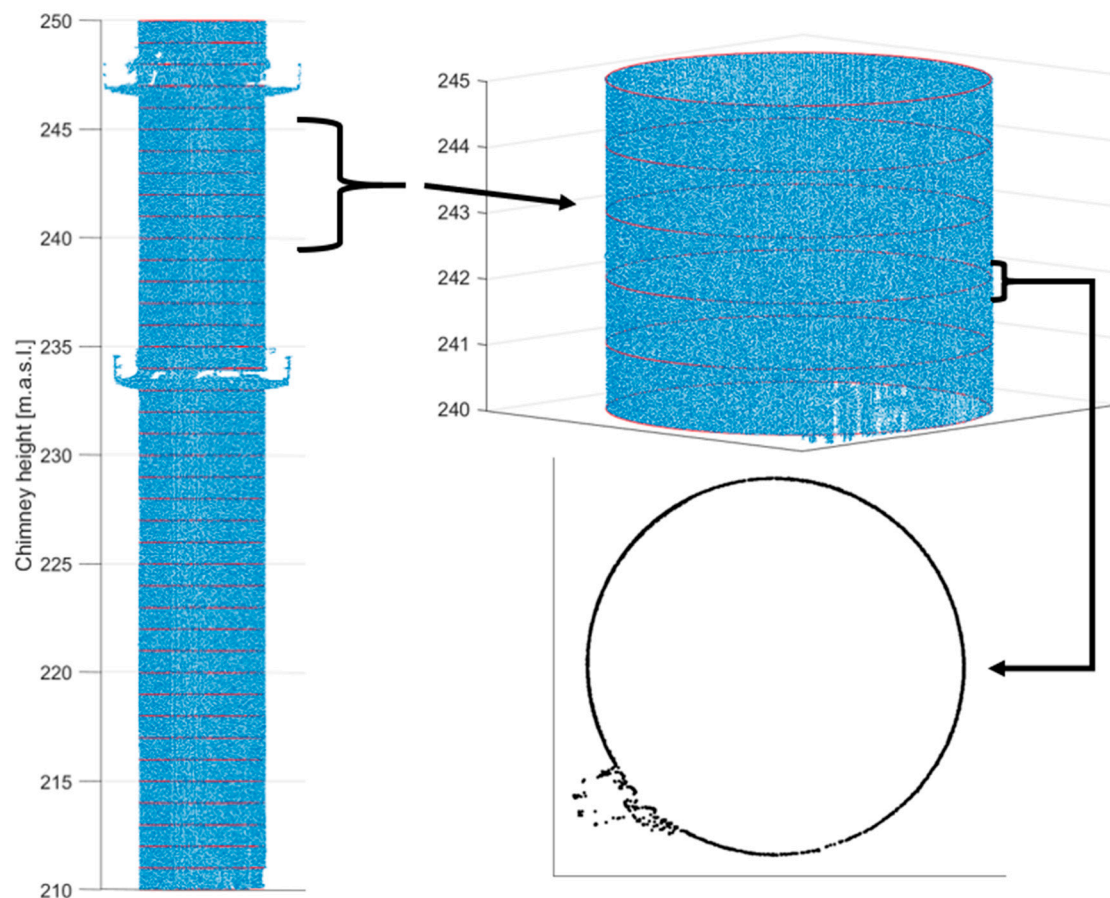


Figure 5. An illustration of height intervals.

Table 5 presents the results of the average number of points per height band and the average number of inliers for chimneys PB4 and PB5 considering both circular and elliptical fitting approaches.

Table 5. Results of the average number of points per height band and the average number of inliers.

Chimney	Solution with a Circle				Solution with an Ellipse			
	Avg. Num. of Pt. Per Height Band	St. Dev.	Avg. Number of Inliers	St. Dev.	Avg. Num. of Pt. Per Height Band	St. Dev.	Avg. Number of Inliers	St. Dev.
PB4	1808	152	1163	170	1821	119	1672	173
PB5	1265	139	760	130	1273	144	1149	140

Since the scanning stations were further away from the chimneys than the height of the chimney, we did not expect an extreme decrease in point cloud density at the top of the chimney. The scanning resolution was set according to the distance measured in the centre of the chimney. Therefore, the density is slightly lower in the upper part of the chimney and slightly higher in the lower part.

The effect is more pronounced on the sides of the chimney, where incidence angles are approaching perpendicularity. It might be useful to manually remove points at the edges. However, since the neighbouring station provides dense clouds in these areas, we rely on RANSAC to remove possibly erroneous points.

2.5. Calculation of Non-Verticality and Deviations

The RANSAC method selects the points for calculating the parameters at random, which means that the values obtained may vary slightly in subsequent iterations. To minimise possible errors and obtain the most accurate results possible, we repeated the calculations ten times at each computational level and averaged the results. Additionally, we examined the behaviour of the chimney at intermediate heights to determine whether it was tilted or otherwise deformed and whether the chimney axis could be described by mathematical curves. We then approximated various curves to determine which best fit the given data. We tested the fit of seven different curves, namely the following:

- Linear function ($Y = \beta_0 + \beta_1 \cdot X + \epsilon$);
- Exponential function ($Y = \beta_0 \cdot e^{\beta_1 \cdot X + \epsilon}$);
- Logarithmic function ($Y = \beta_0 + \beta_1 \cdot \ln(X) + \epsilon$);
- Inverse function ($Y = \beta_0 + \frac{\beta_1}{X} + \epsilon$);
- Polynomial function 2, 3, and 4 grades ($Y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$).

We translated the deviations to the coordinate origin (0, 0) and exchanged the coordinate axes in order to observe how the deviation along the x- or y-axis changes with increasing height. The optimum result is determined by minimising the sum of the squares of the point deviations from the curve. To assess the appropriateness, it is important to calculate the values of the sum of squares (SSE), total sum of squares (SST), and the coefficient of determination (R^2) [39].

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (15)$$

$$SST = \sum (y_i - \bar{y})^2 \sum_{i=1}^n (\hat{y}_i - \bar{y})^2 \quad (16)$$

$$R^2 = 1 - \frac{SSE}{SST} \quad (17)$$

where y_i —observed y value; $\bar{y}$ —sample mean of the observed y values; and $\hat{y}_i$ —estimated true mean.

The final analysis concerns the deviation of each point with respect to the deviation from the theoretical surface of a vertical cylinder. In Figure 6, the colour scale is generated such that each point is coloured according to its deviation from the theoretical vertical cylinder, calculated for each point using the following equation:

$$dist = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2} - r \quad (18)$$

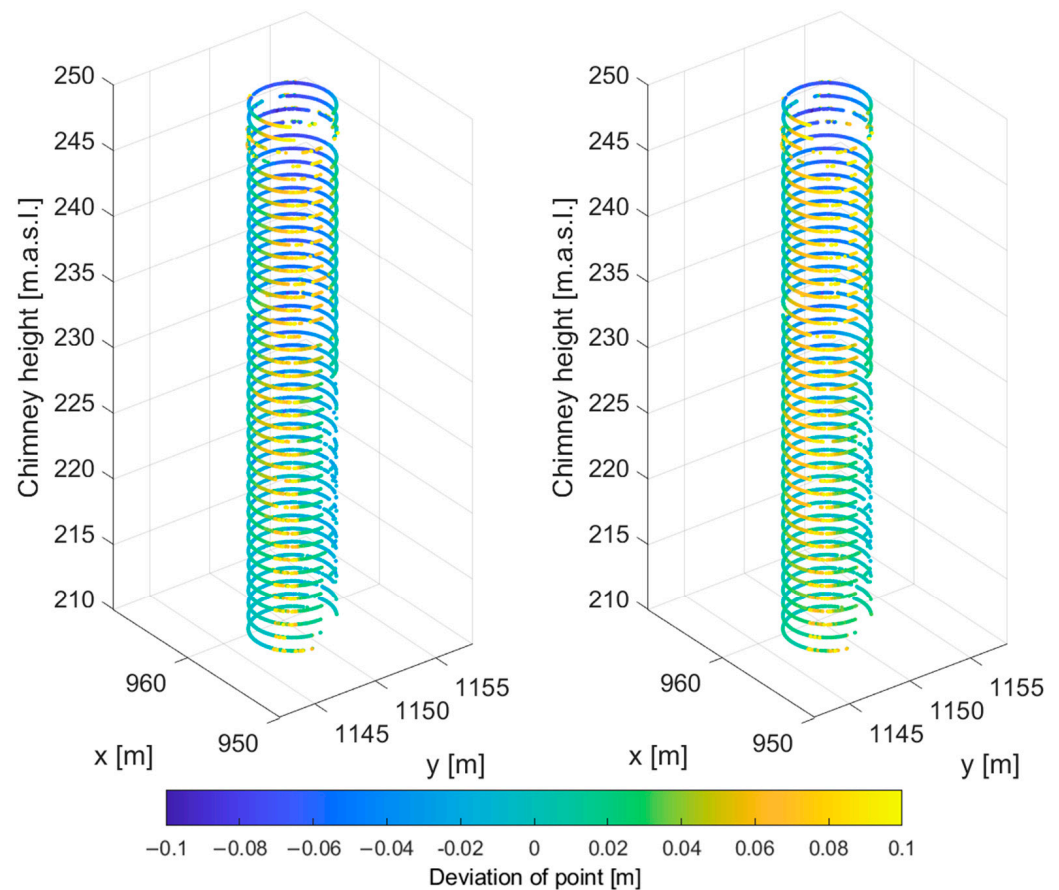


Figure 6. A visualisation of the deviation of points obtained through the RANSAC method from the theoretical cylinder—PB4—circle (**left**) and ellipse (**right**).

Alternatively, in the case of considering an ellipse

$$dist = \sqrt{\frac{(x_i - x_s)^2}{a^2} + \frac{(y_i - y_s)^2}{b^2}} - 1 \quad (19)$$

In Equations (18) and (19), the elements are x_i and y_i —coordinates of the point on the surface of the chimney; x_s and y_s —coordinates of the centre of the circle/ellipse; r —the radius of the circle; and a and b —major and minor semi-axes of the ellipse.

3. Results

The non-verticality of the chimney was calculated using the results at 210 m.a.s.l. At each calculation level, we determined the y - and x -coordinates of the chimney centre together with the corresponding radius or major and minor semi-axes of the ellipse. This allowed us to calculate the deviations from the reference values and juxtapose the results obtained with the known values from Marjetič et al. [4]. The results for the non-verticality

and the comparison are shown in Table 6, where only the values at the bottom of the chimney and the top of the chimney are included, allowing for a direct comparison with the results of the previous study [4]. At this point, we would like to emphasise that the comparison with an earlier study from 2011 (although the measurements were carried out using the same geodetic datum) does not represent a direct numerical comparison. Instead, it demonstrates that the proposed methodology provides reliable results that are consistent with established calculation methods.

Table 6. Comparison of scanning results between total station and scanner.

TPS [4]			
Results		Chimney—PB4	Chimney—PB5
Bottom of chimney	$y_s; \sigma_{y_s}$ [m]	1149.817; 0.003	1150.459; 0.002
	$x_s; \sigma_{x_s}$ [m]	958.815; 0.003	982.784; 0.002
	$r; \sigma_r$ [m]	2.908; 0.003	2.907; 0.002
Top of chimney	Δ_S [m]	0.079	0.083
Slope angle	ν [°]	159	202
TLS			
Solution with a Circle		Chimney—PB4	Chimney—PB5
Bottom of chimney	$y_s; \sigma_{y_s}$ [m]	1149.827; 0.008	1150.467; 0.007
	$x_s; \sigma_{x_s}$ [m]	958.819; 0.006	982.784; 0.005
	$r; \sigma_r$ [m]	2.910; 0.007	2.919; 0.006
Top of chimney	$y_s; \sigma_{y_s}$ [m]	1149.794; 0.004	1150.406; 0.004
	$x_s; \sigma_{x_s}$ [m]	958.760; 0.006	982.717; 0.006
	$r; \sigma_r$ [m]	2.896; 0.004	2.913; 0.003
Top of chimney offset	Δ_x [m]	−0.034	−0.061
	Δ_y [m]	−0.059	−0.067
	Δ_S [m]	0.068	0.090
Slope angle	ν [°]	244	218
Solution with an Ellipse		Chimney—PB4	Chimney—PB5
Bottom of chimney	$y_s; \sigma_{y_s}$ [m]	1149.826; 0.003	1150.470; 0.003
	$x_s; \sigma_{x_s}$ [m]	958.813; 0.005	982.763; 0.005
	$a; \sigma_a$ [m]	2.906; 0.004	2.906; 0.003
	$b; \sigma_b$ [m]	2.917; 0.007	2.932; 0.006
	ε	0.087	0.133
Top of chimney	$y_s; \sigma_{y_s}$ [m]	1149.786; 0.004	1150.408; 0.003
	$x_s; \sigma_{x_s}$ [m]	958.733; 0.007	982.717; 0.006
	$a; \sigma_a$ [m]	2.897; 0.004	2.903; 0.004
	$b; \sigma_b$ [m]	2.933; 0.009	2.925; 0.008
	ε	0.155	0.123
Top of chimney offset	Δ_x [m]	−0.039	−0.062
	Δ_y [m]	−0.080	−0.047
	Δ_S [m]	0.089	0.078
Slope angle	ν [°]	239	195

Based on the results, we can conclude that there are no significant differences between the two methods and the comparison with the values determined by Marjetič et al. [4]. It should be noted that the accuracy in determining the centre of the circle/ellipse and the corresponding radius or semi-axis of the ellipse is twice as bad in the case of the RANSAC method. The main reason for this is the fact that the rangefinder built into the Rigel-VZ400 scanner is of lower quality compared to the rangefinder in the TS, considering that both are non-contact measurements.

Based on the calculated coordinates of the ellipse or circle centre at all elevations, we analysed what happens at the intermediate elevations (Figure 7). Is the chimney tilt or otherwise deformed, and can the chimney axis be described by mathematical curves? This problem was analysed separately in the vertical x-H and y-H planes. For this purpose, the chimney was considered between the height levels of 210 m.a.s.l. and 250 m.a.s.l. at each height metre. In all cases, the height levels of 247 m.a.s.l. and 248 m.a.s.l. were omitted, as the number of points on the chimney surface at these two levels was too small due to the platform being at a height of about 246 m.a.s.l. and the scatter being too large, leading to inaccurate results. 3D displacements are shown in Figure 8.

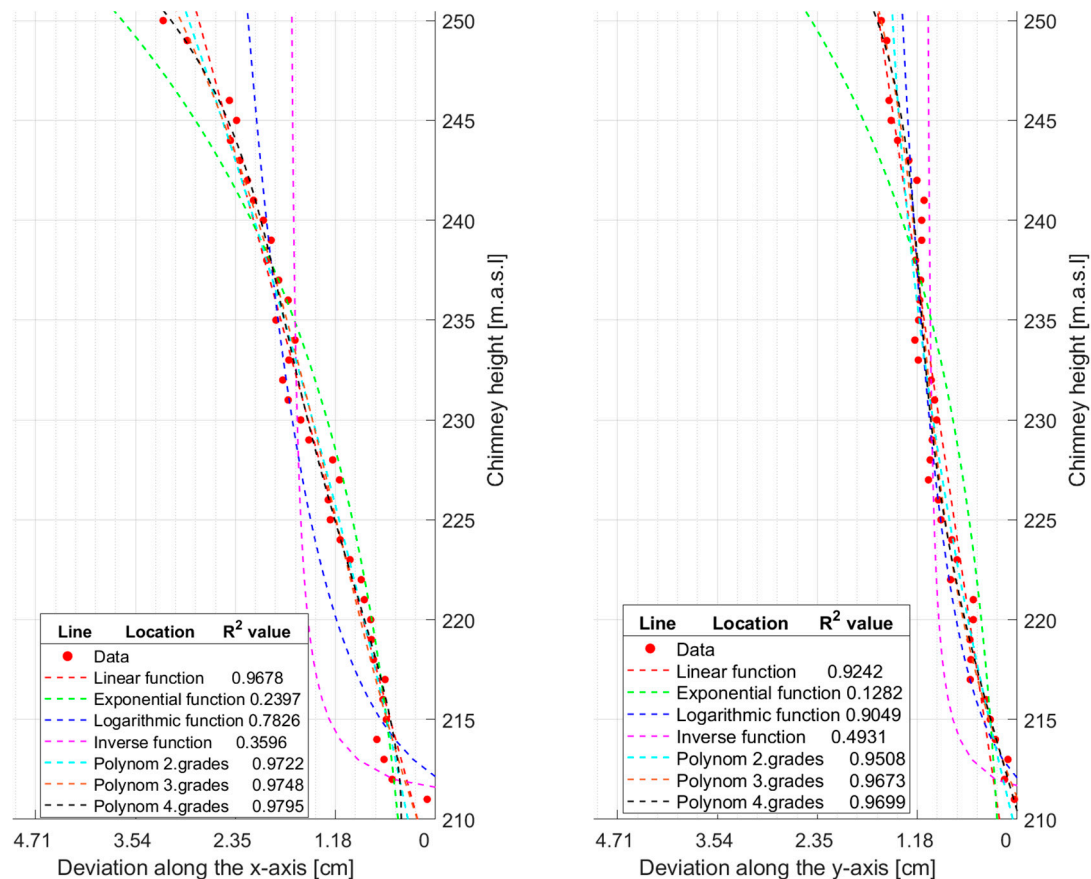


Figure 7. Approximation of curves at given offsets in xH plane—PB4—ellipse (right) and yH plane—PB4—ellipse (left).

The calculation of the chimney's tilt provides important information about the condition of the structure only if the allowable tilt values are known. The Eurocode 3 standards [3] prescribe the allowable horizontal deviation of the steel shell of a free-standing chimney, depending on the height above the foundation, with the following equation:

$$\Delta = \frac{H_C}{1000} \sqrt{1 + \frac{50}{H_C}} \quad (20)$$

The chimneys PB4 and PB5 are 65 m tall, which, according to the standard SIST EN 1993-3-2:2007 [40], means that the allowable deviation is 8.7 cm. The measured and calculated values for chimneys PB4 and PB5 (Table 6) do not exceed the allowable tilts. This indicates that the current tilt of the chimneys does not pose a risk of potential permanent deformation of the chimney structure.

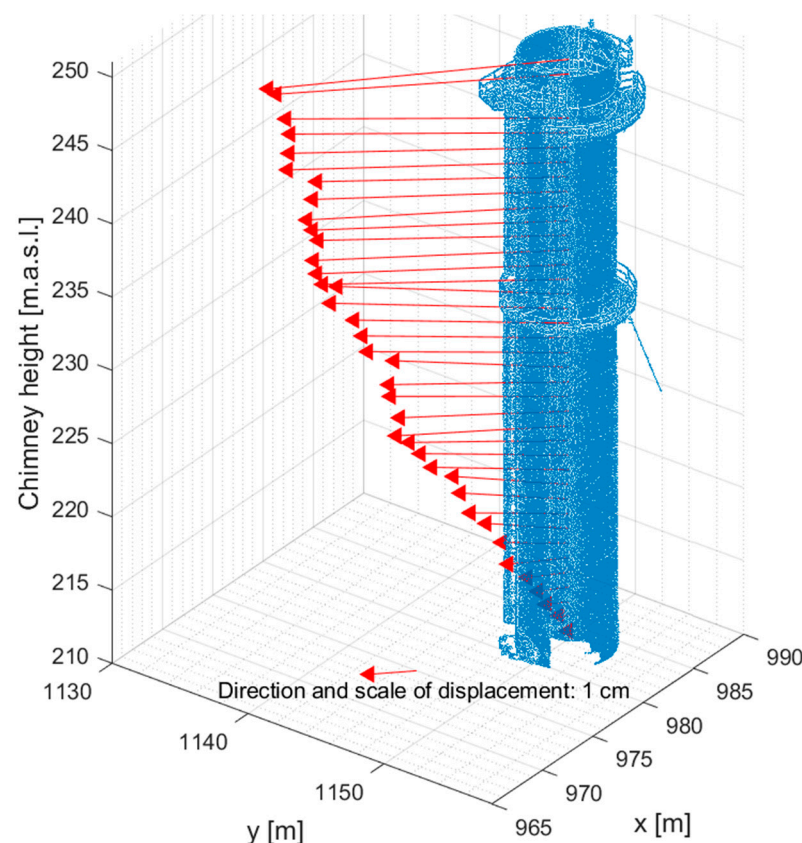


Figure 8. A 3D display of deviations from the vertical at every metre—solution with an ellipse.

4. Discussion

We have found that the application of the RANSAC method, together with the treatment of objects in height bands, is suitable or necessary to assess the non-verticality of chimneys and similar tall structures, especially when individual anomalies are present along the primary surface of the object. The RANSAC method offers notable advantages when dealing with height bands, including the ability to quickly filter out points that do not accurately represent the surface of the chimney, thereby mitigating the impact of grossly erroneous measurements on the results. Additionally, the chimney parameters are calculated using the LS method based on the filtered point cloud. Furthermore, when determining non-verticality based on height bands, there are no complications in the mathematical description of the shape, as the equations for circles, ellipses, rectangles, or squares are well established and easy to apply, even if the parameters vary with height—a challenge that arises when the object is treated as a whole.

The results suggest that it makes more sense to use the parameters of the ellipse when the cross-section of the chimney is circular, as this provides accurate results even if the cross-section is actually circular. However, if there are differences in the sizes of the semi-axes a and b , better results are obtained, allowing us to detect irregularities in the shape or to monitor whether the deviations from the prescribed shape are within acceptable limits. In addition, based on the calculated centre points of the height layers, we can approximate a curve through these points, facilitating the analysis of whether the chimney is leaning (linear function) or whether the structure is deformed (exponential or polynomial function).

From the graphical representations shown in Figure 7 (right), our analysis of chimney PB4's non-verticality (assuming a circular cross-section) reveals that there is no recognisable pattern of non-verticality or deviation from verticality in the x-H plane. However, examination of the y-H plane indicates a curvature of the chimney as deviations occur in various directions. It was found that third- and fourth-degree polynomial curves best fit

the measured data. The coefficient of determination values of 0.8244 and 0.8290 indicate a moderate fit of the curve to the data considering the expected scatter of the data.

For the PB5 chimney (assuming a circular cross-section), we observed that the deviations from the vertical axis of the chimney centre increase approximately proportionally. Moreover, the deviations in both considered planes (x-H plane and y-H plane) are approximately the same, which indicates an accurate calculation of the chimney inclination angle. The second-, third-, and fourth-degree polynomial curves fit the measured data best. The coefficients of determination values are 0.9658, 0.9672, and 0.9695, indicating a good fit of the curve to the data. This numerical confirmation is consistent with the graphical representation and shows that there are no notable differences in the degree of fit between the higher-order polynomials in this scenario.

For the PB4 chimney (assuming an elliptical cross-section), we observed that treating it as an ellipse leads to more interpretable results compared to treating it as a circle, as the trend of deviations is more consistent. It was found that second-, third-, and fourth-degree polynomial curves fit the measured data best. The coefficients of determination values are 0.9722, 0.9748, and 0.9795, respectively, indicating a strong fit of the curve to the data. This agreement with the graphical representation (Figure 7) emphasises the effectiveness of the selected polynomial curves in capturing the deviation trends.

In the case of chimney PB5 (assuming an elliptical cross-section), our results in the x-H plane show almost identical results compared to treating the chimney as a circle. However, there are differences in the y-H plane, particularly from 210 m.a.s.l. to 221 m.a.s.l., where the chimney appears to tilt in the opposite direction than when it was treated as a circle. This discrepancy can be attributed to the calculation of the ellipse centre at 210 m.a.s.l., where a slight difference in the x-coordinate of the ellipse centre results in an apparent deviation in the opposite direction. It was found that the third- and fourth-degree polynomial curves fit the measured data best, with coefficients of determination values of 0.9647 and 0.9685, respectively, indicating a good fit of the curve to the data.

The existing methods propose various approaches to calculate the non-verticality of the chimneys from point clouds, each of which has its limitations. Errors in the point measurement of chimneys are primarily due to the reflection of the laser beam, which depends on the angle of incidence and the material of the scanned object. Like all structures, tall objects such as chimneys require access to individual parts. In addition to the basic chimney structure, there are usually also obstacles on the chimney such as fences, stairs, platforms, etc. Points in the point cloud that do not represent the chimney surface are therefore gross errors that must be removed from the calculation process to ensure the quality of the chimney parameter calculations. In order to achieve high-quality results, appropriate filtering of the point cloud is required, where such points are excluded from the processing. However, this is often overlooked in the literature, with authors focusing mainly on objects without such anomalies. In cases with anomalies, their methods may provide insufficient results. Only Kregar et al. [15] reported on specific data filtering. Treating the chimney as a whole (geometric solid) does make sense if we assume that the object is not a rigid solid. However, external influences can cause not only changes in the position and orientation of the object but also deformations of its shape. Ötsch and Neuner [9] have already found that the chimney tilts differently at different times of the day and at different temperatures. Furthermore, due to its construction (whether made of bricks or individual metal modules), the chimney may not be straight from the bottom but may be twisted. While treating the object as a whole provides data on its non-verticality, it also loses certain information that can be critical to the treatment of such objects. Additional challenges arise with complex shapes for which the mathematical model is unknown.

Compared to previous methods, we found several advantages of looking at individual height bands in combination with the RANSAC method to determine the non-verticality of chimneys and other tall structures. By treating individual bands of point clouds representing cross-sections of the chimney shell and treating them at selected height intervals, we can calculate the parameters of the chimney shell centre and the corresponding radius or ellipse

semi-axes. These solutions apply to cases where the chimney shell is cylindrical. The main advantage of the RANSAC method when dealing with height bands is that it filters out points that do not represent the surface in the chimney shell in a single step, thus eliminating the influence of gross measurement errors on the results. Additionally, the parameters of the chimney shell are calculated using the LS method based on the filtered point cloud. In our research, we want to demonstrate that the RANSAC method is useful and necessary when anomalies are present on the chimney shell. Another important advantage relates to the mathematical background. The cross-section of the chimney shell is usually circular or elliptical, while other tall objects may have shapes such as rectangles, squares, etc., which are mathematically well-defined figures with respect to the band treatment. However, when treating an object as a whole, difficulties may arise in the mathematical description of such an object, especially for more complex structures (e.g., cooling towers, television towers, etc.). Using the results at all height levels, we can approximate a curve through the calculated centre points, allowing us to analyse whether the chimney shell is tilting (linear function) or whether the structure is possibly deformed (curve). To better visualise the deviation points from the theoretical cylinder and thus demonstrate the non-verticality of the chimney stack shell, each individual point can be coloured according to its distance from the vertical cylinder.

It is important to realise that this method (as well as others) only leads to accurate results if basic professional principles are adhered to. In the initial phase, it is crucial that the coordinates of the tie and reference points are determined very accurately. Poorly determined coordinates of these points lead to bad registration and georeferencing of the point cloud, which inevitably results in poor-quality data for further analysis. The chosen processing method must also ensure that the observed object (in our case, the chimney) is scanned from multiple positions. This ensures that a larger part of the circular arc is captured, reducing the potential for errors. Random points are thus selected over greater distances along the circular arc, which improves the accuracy of determining inliers and the parameters of the mathematical model. When analysing tall structures, two additional factors must be considered, namely that the density of the point cloud is likely to be lower in the upper sections of the chimney compared to the lower sections and when measuring the upper part of the chimney, a known phenomenon occurs where the laser points become elliptical, resulting in less accurate point coordinates. To mitigate these issues, the optimum TLS setups must be selected on-site during the measurement phase. It is recommended to position the TLS at a distance at least equal to the height of the object and if possible, on a higher structure close to the observed object. This ensures a better quality of measurements and reduces the likelihood of decreased point cloud density in the upper sections of the chimney or inaccuracies in the point coordinates.

5. Conclusions

TLSs are increasingly being recognised for their versatility and utility across a wide range of industries, including civil engineering, tunnelling, building information modelling (BIM), and reconstruction planning. These scanners offer a wealth of data points that provide a comprehensive insight into the object or construction site being surveyed. In addition, the measurement and processing, which also encompass registration and georeferencing, are generally fast and user-friendly. However, the subsequent analysis of the captured point clouds often requires more time and effort.

The method described here represents an innovative approach to assessing the non-verticality of chimneys. A notable advantage of this approach is that it treats the chimney as distinct segments and not as a single entity. This allows us to identify differences in the behaviour of the chimney at different elevations and to detect possible structural deformations. Moreover, the analytical methods are simpler when dealing with simple geometric shapes such as circles or ellipses as opposed to more complex solids such as truncated cones or cylinders. By comparing the results with the previous literature, we can confirm the suitability of the method and its ability to provide high-quality results.

Scenarios such as the calculation of the PB5 chimney non-verticality, where we opted for elliptical chimney cross-sections, are indeed a potential challenge. The complexity of determining the centre point at the intended height level leads to inaccuracies in the results. In such cases, it may be necessary to set the initial height for the calculation slightly higher or lower. Although this adjustment has no direct influence on the calculation of the non-verticality, it does have a significant effect on the accuracy of the results.

Author Contributions: Conceptualization, Ž.P., K.K. and G.R.; methodology, Ž.P. and K.K.; software, Ž.P.; validation, Ž.P., G.R. and K.K.; formal analysis, Ž.P.; investigation, Ž.P., G.R. and K.K.; data curation, Ž.P. and K.K.; writing—original draft preparation, Ž.P.; writing—review and editing, Ž.P., G.R. and K.K.; visualisation, Ž.P., G.R. and K.K.; supervision, G.R. and K.K.; project administration, G.R. and K.K.; funding acquisition, G.R. and K.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Slovenian Research Agency research core funding no. P2-0180 “Water Science and Technology, and Geotechnical Engineering: Tools and Methods for Process Analyses and Simulations” and P2-0227 “Geoinformation infrastructure and sustainable spatial development of Slovenia”.

Data Availability Statement: The authors confirm that access to all open-source data underpinning the findings of this study is available and is described in the study under Section 2. Access to the processed data depends on the use and is subject to individual agreements.

Conflicts of Interest: The authors declare no conflicts of interest.

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