

Article

Supervirtual Refraction Interferometry in the Radon Domain

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Abstract: Accurate picking of seismic first arrivals is very important for first arrival travel time tomography, but the first arrivals appearing at far offsets are often more difficult to pick accurately due to the low signal-to-noise ratio (SNR). The conventional supervirtual refraction interferometry (SVI) method can improve the SNR of first arrivals to a certain extent; however, it is not suitable for seismic data that interfered by strong noise. In order to better process the first arrivals at far offsets with serious noise interference, we propose a modified method, in which SVI implemented in the Radon domain (RDSVI) due to the cross-correlation in the Radon domain have a better effect. According to the kinematic characteristics of first arrival refractions, SVI is performed in the linear Radon domain. Both synthetic data and field data demonstrate the proposed method can enhance the effective signal and attenuate the strong noise simultaneously, so as to significantly improve the SNR of the first arrival data. Meanwhile, the RDSVI method is tested on the first arrival data with missing traces, which proves that this method can overcome the influence of abnormal traces and is suitable for the reconstruction of sparsely sampled seismic data.

Keywords: supervirtual refraction interferometry; signal-to-noise ratio; cross-correlation; Radon transform



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1. Introduction

As first arrivals are usually used to provide information on surface velocity information for static correction and tomography [1], as well as image the shallow structure at near surface [2] and deep structure of the Earth [3–6], the accurate picking of seismic first arrivals is very important in seismic exploration. According to Snell's law, first arrival refractions usually appeared at the far offset section in the seismic record. Due to the long propagation distance, spherical divergence, formation attenuation and scattering, they always show the characteristics of weak energy, strong noise interference and low signal-to-noise ratio (SNR), which make it difficult to perform accurate travel time picks; thus, they cannot provide high-quality first arrival data for subsequent processing, such as static correction and travel time tomography.

Seismic interferometry (SI) is an efficient remedy to enhance the seismic data. Cross-correlation is a common algorithm in the interferometry [7,8]. To improve the SNR of first arrivals, cross-correlation was applied to the first arrival data, along with the stacking in the source domain [9]. However, this can only obtain the virtual first arrivals that excited at a certain underground point as the virtual source, whose position does not match the actual source. Meanwhile, the exploration aperture is reduced, because the cross-correlation process shortens the ray path of the first arrivals. To reset the virtual source to the surface, the virtual first arrival was convolved with the original first arrival data [10], successfully resetting the virtual source to the actual source position. This method is referred to as supervirtual refraction interferometry (SVI), and the principle was elaborated in detail soon afterward [11,12]. It is a two-step strategy: the original first arrivals are first cross-correlated and stacked in the source domain to generate the virtual first arrival; then, the first arrivals are retrieved by the convolution of the virtual first arrival with the original

first arrival data. After applying SVI, the SNR of first arrivals can be improved due to the enhancement of head wave energy and suppression of random noise. Therefore, the quality of first arrival picking will be improved and allow a further investigation of the subsurface velocity structure.

The proposal of SVI provides help to improve the SNR of first arrivals, and this method has continued to develop in recent years. Al-Hagan et al. [13] extended SVI and proposed an iterative supervirtual refraction interferometry (ISVI) method. In this method, the retrieved first arrivals are reused as input in the SVI procedure for further SNR enhancement through iteration. SVI was also applied to 3D seismic data as another extension [14–16]. To ensure the consistency of stacking number and to balance the energy of retrieved first arrival traces, the virtual first arrival was cross-correlated with the original first arrival data and stacked with the convolution result [17], realized the balance of energy between the traces of retrieved first arrivals. The advantages and disadvantages of 2D, 3D, and iterative SVI methods were summarized, respectively [18]. Xu et al. [19] proposed an extrapolated supervirtual refraction interferometry (ESVI) method, which increased the maximum offset and pick number of first arrivals, as well as expanded the exploration aperture. SVI was also widely used in practical case investigations. This method was applied to seismic data collected at the western fault region of Saudi Arabia and obtained the reconstructed first arrivals [20–22]. SVI was also implemented in OBS data [23], realized the increase of source-receiver offsets, and provided reliable first arrival picking. Edigbue et al. [24] performed semiautomatic 3D SVI in western Saudi Arabia. SVI was used in residual static correction [25,26] and seismic signal enhancement [27–32] as well.

The core of enhancing the effective signal in SVI is stacking in the source domain during cross-correlation [33–38]. The random seismic can be attenuated through stacking. Theoretically, the SNR of the retrieved first arrival can be improved by a factor of $\sqrt{N}$, where N is the number of sources involved in stacking. However, since the seismic noise will also involve in the cross-correlation process, part of the noise energy will be enhanced at the same time. For the case that the raw seismic data containing strong noise, conventional cross-correlation even may cause unexpected distraction, so the improvement of the SNR will be very limited. To improve the effect of cross-correlation on effective signal enhancement, some modified methods have been proposed, such as performed cross-correlation in the plane wave domain [39] to retrieve the reflections in specific ray directions with an improvement of the imaging effect, and implemented cross-correlation in the parabolic Radon domain [40] to achieve the reconstruction of seismic data with missing traces and suppression of interference artifacts. Aiming at the problem that the conventional SVI method is not ideal in processing the first arrival data interfered by strong noise, we propose a method that performs SVI in the Radon domain (RDSVI). Since the kinematics of first arrivals at far offsets are usually linear, together with the excellent noise immunity of the linear Radon transform, we decide to apply SVI in the linear Radon domain. Different from the conventional cross-correlation in the temporal domain that all data will involve in operation, on account that just the ray parameters in the range of the effective signal energy cluster need to be cross-correlated in the Radon domain, seismic noise are largely avoided participating in calculations, which can result in a greatly attenuate of random noise in the reconstructed first arrivals. The processing results of both synthetic data and field data demonstrate that the proposed method can significantly improve the SNR of first arrivals data with strong noise.

2. Methods

2.1. Review of Conventional SVI

We first briefly review the theory of the two-step conventional SVI method. Based on the far-field approximation [33], the virtual refraction can be obtained by the cross-correlation of two traces in one seismic shot and then integrating along the shot domain:

$$\text{Im}[g(\mathbf{A}|\mathbf{B})^v] \approx k \int g(\mathbf{B}|\mathbf{S})^* g(\mathbf{A}|\mathbf{S}) d^2\mathbf{S} \quad (1)$$

where Im denotes the imaginary part of the Green's function, k means the average wavenumber. $g(\mathbf{A}|\mathbf{B})^v$ represents the retrieved virtual refraction that excited at receiver position $\mathbf{B}$, received at receiver position $\mathbf{A}$. $g(\mathbf{A}|\mathbf{S})$ and $g(\mathbf{B}|\mathbf{S})$ represent original refractions that excited at source position $\mathbf{S}$, received at receiver position $\mathbf{A}$ and $\mathbf{B}$, respectively. $*$ denotes complex conjugation. When integrating along the shot domain, the retrieved virtual refractions of each shot will be stacked, and the amplitude will be enhanced due to the constructive interference, whereas the phase of the seismic noise is arbitrary, the destructive interference will eliminate the noise signal during the superposition. Under the combination of constructive interference and destructive interference, the effective signal is enhanced, while the noise signal is eliminated simultaneously, thus achieving the suppression of the seismic noise by integrating along the shot domain.

As Figure 1a shows, cross-correlation can eliminate the common ray path of refractions, to reconstruct a virtual wavefield that is stimulated somewhere underground while received at the surface. The solid line and dotted line represent the positive and negative time components of the virtual refraction wavefield, respectively. $\otimes$ denotes cross-correlation. Since the sources that generate refracted waves are all located in the stationary phase zone, they can contribute to enhancing the effective signal energy by integrating along the source domain. If there are enough postcritical sources, SVI might provide good noise suppression.

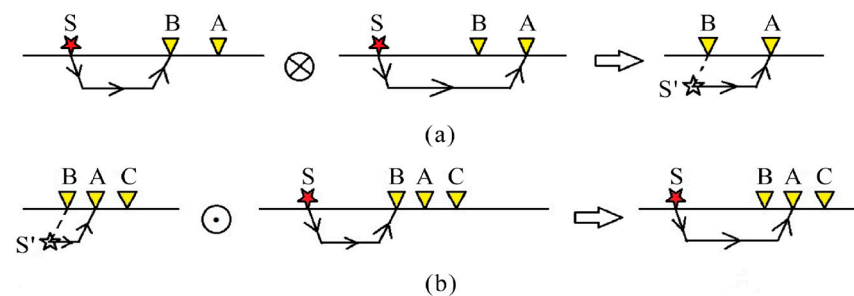


Figure 1. Sketch of SVI: (a) Virtual refraction interference, $\otimes$ denotes cross-correlation. (b) Convolution between the virtual refraction and original data, $\odot$ denotes convolution.

Then the retrieved virtual refraction is convolved with the original seismic data under the assumption of far-field approximation:

$$g(\mathbf{A}|\mathbf{S})^{svic} \approx 2ik \int g(\mathbf{A}|\mathbf{B})^v g(\mathbf{B}|\mathbf{S}) d^2\mathbf{B} \quad (2)$$

where $g(\mathbf{A}|\mathbf{S})^{svic}$ represents the convoluted refraction. Convolution extends the ray path of refraction, and reset the virtual source to the actual source position at the surface, as presented in Figure 1b. $\odot$ denotes convolution. The SNR of the reconstructed refraction is further improved by integrating along the receiver domain.

Windowing around the first arrivals before SVI is an important step to suppress artifacts. This procedure can avoid other complex wavefields to participate in the operation that may affect the reconstruction of first arrivals, as well as reduce the amount of data involved in the calculation. Previous experience has shown the smaller the window, the less artifacts appear in the retrieved traces [21].

Although the conventional SVI can enhance the refraction signal to a certain extent, meanwhile, the seismic noise will also involve in the calculation and may cause an unexpected energy enhancement, even interference artifacts may occur. If the noise interference in the original record is serious, the effect of conventional SVI on improving the SNR will be greatly reduced. In addition, despite the exploration aperture is recovered, with the shortening of the source-virtual source distance when stacking in the receiver domain, the number of receivers participating in stacking will continuing decrease, which will result in the inconsistent stacking numbers between traces at different offsets, therefore the SNR improvement of traces at near offsets will be worse than traces at far offsets.

2.2. RDSVI

To solve the problem that the conventional SVI method is not suitable for the first arrival refraction data interfered by strong noise, we implement cross-correlation in the linear Radon domain according to the kinematics of first arrivals at far offsets is linear.

The forward and inverse linear Radon transform in the frequency domain are expressed as:

$$u(p) = \int_{-\infty}^{+\infty} g(x) \exp(j\omega px) dx \quad (3)$$

$$g(x) = \int_{-\infty}^{+\infty} u(p) \exp(-j\omega px) dp \quad (4)$$

where ω denotes angle frequency, $g(x)$ and $u(p)$ represent seismic data in the temporal domain and Radon domain, respectively. The item $g(\mathbf{B}|\mathbf{S})$ in Equation (1) can be rewritten according to Equation (4) as:

$$g(\mathbf{B}|\mathbf{S}) = \int u_{\mathbf{B}}(p_{\mathbf{B}}) \exp(-j\omega p_{\mathbf{B}}x) dp_{\mathbf{B}} \quad (5)$$

and its complex conjugation is:

$$g(\mathbf{B}|\mathbf{S})^* = \int u_{\mathbf{B}}(p_{\mathbf{B}})^* \exp(j\omega p_{\mathbf{B}}x) dp_{\mathbf{B}} \quad (6)$$

Substituting Equation (6) into Equation (1) will get:

$$g(\mathbf{A}|\mathbf{B})^v \approx \iint u_{\mathbf{B}}(p_{\mathbf{B}})^* \exp(j\omega p_{\mathbf{B}}x) g(\mathbf{A}|\mathbf{S}) dp_{\mathbf{B}} dx \quad (7)$$

Since the item $\int g(\mathbf{A}|\mathbf{S}) \exp(j\omega p_{\mathbf{B}}x) dx$ equals to $u_{\mathbf{A}}(p_{\mathbf{B}})$, Equation (7) can be rewritten as:

$$g(\mathbf{A}|\mathbf{B})^v \approx \int u_{\mathbf{B}}(p_{\mathbf{B}})^* u_{\mathbf{A}}(p_{\mathbf{B}}) dp_{\mathbf{B}} \quad (8)$$

The ray parameter $p_{\mathbf{B}}$ is defined by the position of receiver $\mathbf{B}$. Equation (8) shows that cross-correlation can be implemented in the linear Radon domain.

The concrete steps that perform cross-correlation in the Radon domain are: (1) sort the refraction data from common shot gather to common receiver gather; (2) transform the common receiver gather into the linear Radon domain; (3) cross-correlate the Radon domain data with the same ray parameter; (4) stacking the cross-correlated traces across ray parameters. The refraction signal will be transformed into energy clusters in the linear Radon domain, yet the noise energy will not converge. Compared with the cross-correlation in the temporal and spatial domain, noise energy can be largely avoided involve in the operation in the linear Radon domain. Taking this benefit, the SNR of the retrieved virtual refraction can be significantly improved.

Due to the limitation of seismic acquisition space, the superposition during the Radon transform can only be performed in the limited temporal domain, which will be affected by the smooth effect. As well as the least squares algorithm may produce artifacts in the Radon domain, the energy convergence in the Radon model will be influenced. To obtain a Radon model with more concentrated effective signal energy, we apply the sparse Radon transform based on iterative model shrinkage (SRTIS) [41,42]. In each iteration of the SRTIS, the Radon model updates can be expressed as:

$$\mathbf{m}^k = T_{\alpha} \left\{ \mathbf{m}_{k-1} + 2tF^{-1} \left[\left(\mathbf{L}^T \mathbf{L} \right)^{-1} \mathbf{L}^T (F(\mathbf{d}) - \mathbf{L}F[\mathbf{m}_{k-1}]) \right] \right\} \quad (9)$$

where $\mathbf{d}$ is the seismic data in the t - x domain, $\mathbf{m}^k$ represents the time-domain Radon model estimation in the k th iteration; $\mathbf{L}$ is the Radon transform operator; F and F^{-1} denote the forward and inverse Fourier transform, respectively; t is an appropriate step size controlling

the convergence speed to some extent, and it is typically selected from the bound of 0–1 to ensure the convergence of the SRTIS. T_α is the shrinkage operator defined by:

$$T_\alpha\{\mathbf{m}, k\}_{ij} = \left(|m_{ij}| - \alpha \frac{K-k}{K} \tilde{m}_{ij} \right)_+ \text{sgn}(m_{ij}) \quad (10)$$

where α is a scale, in general $0 < \alpha < 1$; K is the predefined maximum iteration number; $\mathbf{m} = \{m_{ij}\}$, $\tilde{\mathbf{m}} = \{\tilde{m}_{ij}\}$, $\tilde{\mathbf{m}} = \tilde{\mathbf{m}}(\max(|\mathbf{m}|)/\max(|\tilde{\mathbf{m}}|))$, $\tilde{\mathbf{m}}$ is the output of the 2D averaging filter of $|\mathbf{m}|$, and $(x)_+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$. The energy cluster of the effective signal in the Radon domain will be more converged after SRTIS, thus helping to obtain a better cross-correlated result.

To solve the problem of poor reconstruction of refracted waves at near offset traces in convolution, the retrieved virtual refraction is cross-correlated with original seismic data:

$$g(\mathbf{A}|\mathbf{S})^{svix} \approx 2ik \int g(\mathbf{B}|\mathbf{A})^{v*} g(\mathbf{B}|\mathbf{S}) d^2\mathbf{A} \quad (11)$$

where $g(\mathbf{A}|\mathbf{S})^{svix}$ represents the cross-correlated refraction.

To further improve the SNR, this step is also implemented in the linear Radon domain:

$$g(\mathbf{A}|\mathbf{S})^{svix} \approx \int u_{\mathbf{B}|\mathbf{A}}(p_{\mathbf{B}})^{v*} u_{\mathbf{B}|\mathbf{S}}(p_{\mathbf{B}}) dp_{\mathbf{B}} \quad (12)$$

Cross-correlation will eliminate the common ray path of the refractions. Contrary to convolution, with the shortening of the source–virtual source distance, the number of receivers participating in stacking will continue to increase; therefore, the SNR improvement effect of near offset traces will be better than traces at far offset.

Both convolution and cross-correlation processing can enhance the refraction signal, but the energy distribution of two retrieved results is unbalanced due to the different superposition times, respectively. To ensure the consistency of superposition in the receiver domain and equalize the energy of the reconstructed refractions, the two results are summed:

$$g(\mathbf{A}|\mathbf{S})^{svi} = g(\mathbf{A}|\mathbf{S})^{svic} + g(\mathbf{A}|\mathbf{S})^{svix} \quad (13)$$

In this way, the number of superpositions is equal to the number of receivers, solving the problem that the energy of reconstructed refraction is affected by the change of offset.

Here we use a simple noise-contaminated refraction data to demonstrate the contribution of SRTIS on the convergence of effective signal during the linear Radon transform. Figure 2a is the input noisy refraction after windowing. Figure 2b presents the normal Radon model. Due to the limitations such as smoothing effects, many unexpected interferences will occur in the Radon model, resulting in the insufficient convergence of effective signal. As a contrast, the effective signal energy is more concentrated after SRTIS, with noise energy attenuated at the same time, as shown in Figure 2c. When performing the cross-correlation in the Radon domain after SRTIS, only the ray parameters within the range of the effective signal energy need to be cross-correlated, which can provide great help to suppress the interference artifacts.

The same input in Figure 2a is then reconstructed through RDSVI to explain how the intermediate result varies with offsets. The results of each step in the RDSVI method are shown in Figure 3. It shows in Figure 3a that the virtual refraction energy is enhanced after stacking in the source domain, while the travel time is shorter than original data, because the cross-correlation eliminates the common ray path of refraction from source to receivers. The convolution between virtual refraction and original data will lead to a stacking number reduction on retrieved first arrival refraction at near side traces compared to far side traces; thus, the stacking energy of near side traces is insufficient, as presented in Figure 3b. Contrary to Figure 3b, Figure 3c shows that the cross-correlation will cause a

stacking number reduction to the retrieved first arrival refraction at far side traces compared to near side traces, and the stacking energy of far side traces is insufficient. The combine of the two reconstructed results makes up for the inconsistency of stacking numbers at different offsets. The final reconstructed result is shown in Figure 3d, in which we could see the energy of all traces is continuous and enhanced.

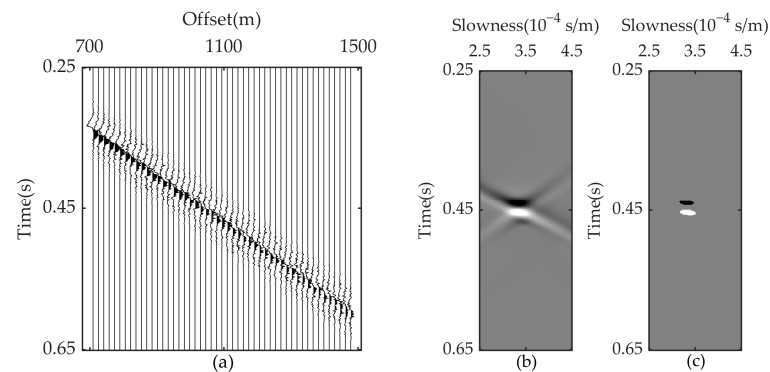


Figure 2. Input noisy refraction and the corresponding linear Radon transform results: (a) Input noisy refraction. (b) Normal linear Radon model. (c) Same as (b) after SRTIS.

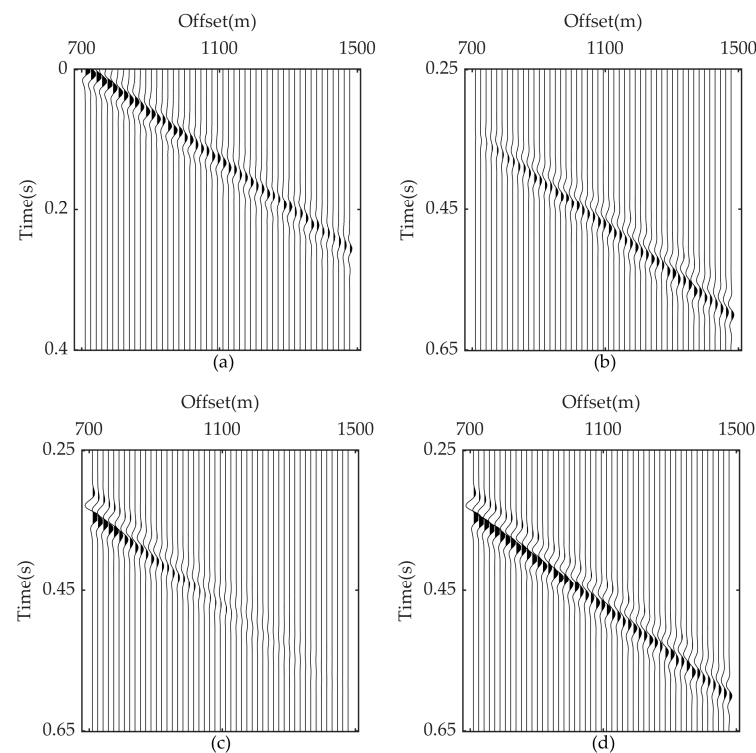


Figure 3. Results of each step in RDSVI: (a) Retrieved virtual refraction. (b) Convolution between virtual refraction and original data. (c) Cross-correlation between virtual refraction and original data. (d) Stacking of (b,c).

The flowchart of RDSVI is as follows (Algorithm 1):

Algorithm 1: RDSVI

- (1) **Input:** Original common shot gathers g^s ;
- (2) Windowing around the first arrivals;
- (3) Convert to common receiver gathers: $g^s \xrightarrow{\text{sort}} g^r$;
- (4) Radon transform and SRTIS: $u^{ri}(p) = \mathbf{RS}[g^r]$;
- (5) Cross-correlation in the Radon domain: $g^v = u_{\mathbf{B}}^{ri}(p_{\mathbf{B}}) \otimes u_{\mathbf{A}}^{ri}(p_{\mathbf{B}})$;
- (6) Convolution: $g^{svic} = g^v \odot g^s$;
- (7) Radon transform and SRTIS of original data: $u^{si}(p) = \mathbf{RS}[g^s]$;
- (8) Radon transform and SRTIS of virtual refraction: $u^{vi}(p) = \mathbf{RS}[g^v]$;
- (9) Cross-correlation in the Radon domain: $g^{svix} = u_{\mathbf{C}}^{vi}(p_{\mathbf{C}}) \otimes u_{\mathbf{C}}^{si}(p_{\mathbf{C}})$;
- (10) Stacking: $g^{svi} = g^{svic} + g^{svix}$;
- (11) **Output:** reconstructed first arrival g^{svi} .

3. Numerical Results**3.1. Synthetic Data Example**

To demonstrate the effectiveness of our method, a velocity model (Figure 4) is used to generate the synthetic seismic record. The 101 sources (red stars) are placed uniformly on the surface with a distribution range of 0–2000 m at 20 m shot intervals. The 251 receivers (yellow triangles) evenly distributed along the surface from 0 m to 5000 m with a spacing of 20 m. Using a 2D finite-difference forward modeling program, the synthetic noise-free common shot gathers that contain all wavefields are generated. One shot with the source located at the start of the survey is shown in Figure 5a. It can be seen that the amplitude of the first arrivals at far offsets is weaker compared to that at near offsets due to the increase in wave propagation distance. Strong random Gaussian white noise is then added to the original common shot gathers, as Figure 5b presents. Since the noise seriously reduces the SNR of seismic data, the first arrivals are basically invisible now, especially at the far offset section after 3000 m, where the first arrivals are completely masked by strong noise, which sets a big obstacle for accurate first arrival picking. Windowing is applied around the first arrivals before the reconstruction. The severe noise interference in the original records makes it difficult to determine the time window; for convenience in the time window selection, we adopted a preprocessing step for the noise-contaminated common shot gathers, including band-pass filtering and f-x deconvolution. The preprocessing step can singularize the refraction event, especially at far offsets, which can help us to determine the time window. However, because the preprocessing may cause damage to the effective signal, this step only provides the basis for the selection of the time window, and the selected time window will be applied to the original noisy data in Figure 5b. The seismic data after preprocessing is presented in Figure 5c, and the zoom view of the part from 2000 m to 5000 m (red square) that contains the far offset first arrival refractions is shown in Figure 5d. To ensure all the first arrival signals are included, we choose a large time window (red rhombus) with a time length of 300 ms, such that some later arrivals after the target first arrival refractions are also included.

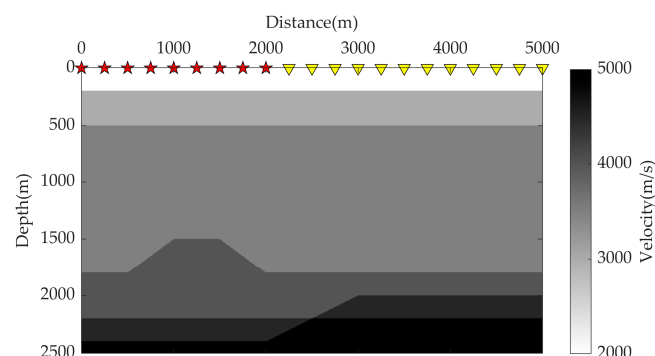


Figure 4. Velocity model. Red stars denote sources, and yellow triangles represent receivers.

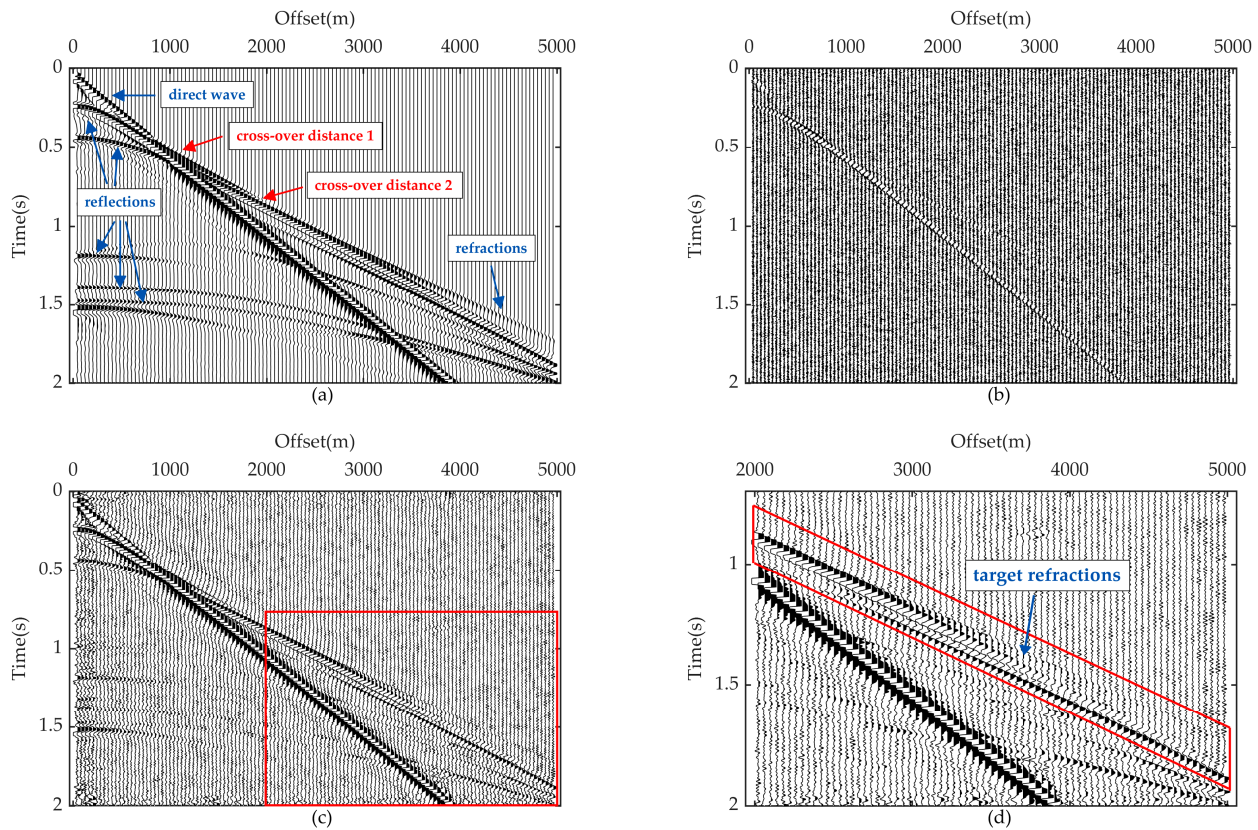


Figure 5. Synthetic common shot gathers: (a) Noise-free record. (b) Same as (a) after adding strong random noise to mask the first arrivals. (c) Same as (b) after preprocessing. (d) Zoom view of the far offset section in (c), where red rhombus denotes the selected time window.

Figure 6 shows the original first arrival records and the reconstructed results of the conventional SVI and RDSVI, respectively. It can be found from the original noise-contaminated data in Figure 6a that the weak first arrival signals are almost masked by strong noise. We reconstructed the first arrivals with conventional SVI and RDSVI, and the results are shown in Figure 6b,c, respectively. The reconstruction of conventional SVI is not satisfactory; only the first arrivals between 2000 and 3000 m are enhanced to a certain extent, but there remains a lot of noise, and interference artifacts exist in the reconstructed traces. While the first arrivals after 3000 m are still unrecognizable, which does not change much compared with the original data, this indicates that, due to the strong noise susceptibility of the conventional SVI, it is not suitable for processing the first arrivals data with strong noise. In contrast, the interference of noise is eliminated during the reconstruction through RDSVI, and only a small amount of noise remains. The first arrivals at all offsets are retrieved well with an obvious phase, as Figure 6c presented. The first arrival event in the reconstructed result matches the noise-free data in Figure 6d; meanwhile, the amplitude is enhanced, which proves that this method is suitable for reconstructing the first arrivals with strong noise. To further clarify the enhancing effect of the two methods, we calculated the SNR of each reconstructed result with the help of the corresponding reconstructed results of the noise-free data [43]:

$$SNR = 10 \log \frac{\|\mathbf{s}\|_2^2}{\|\mathbf{s} - \mathbf{s}_{est}\|_2^2} \quad (14)$$

where $\mathbf{s}$ and $\mathbf{s}_{est}$ represent the reconstructed result of noise-free data and the reconstructed result of noise-contaminated data that are to be estimated, respectively. Through calculation, the SNR of the conventional SVI result is only 1.7 dB, which does not meet our demand.

While the SNR of the RDSVI result reaches up to 13.41 dB, which is a great improvement compared with the conventional method.

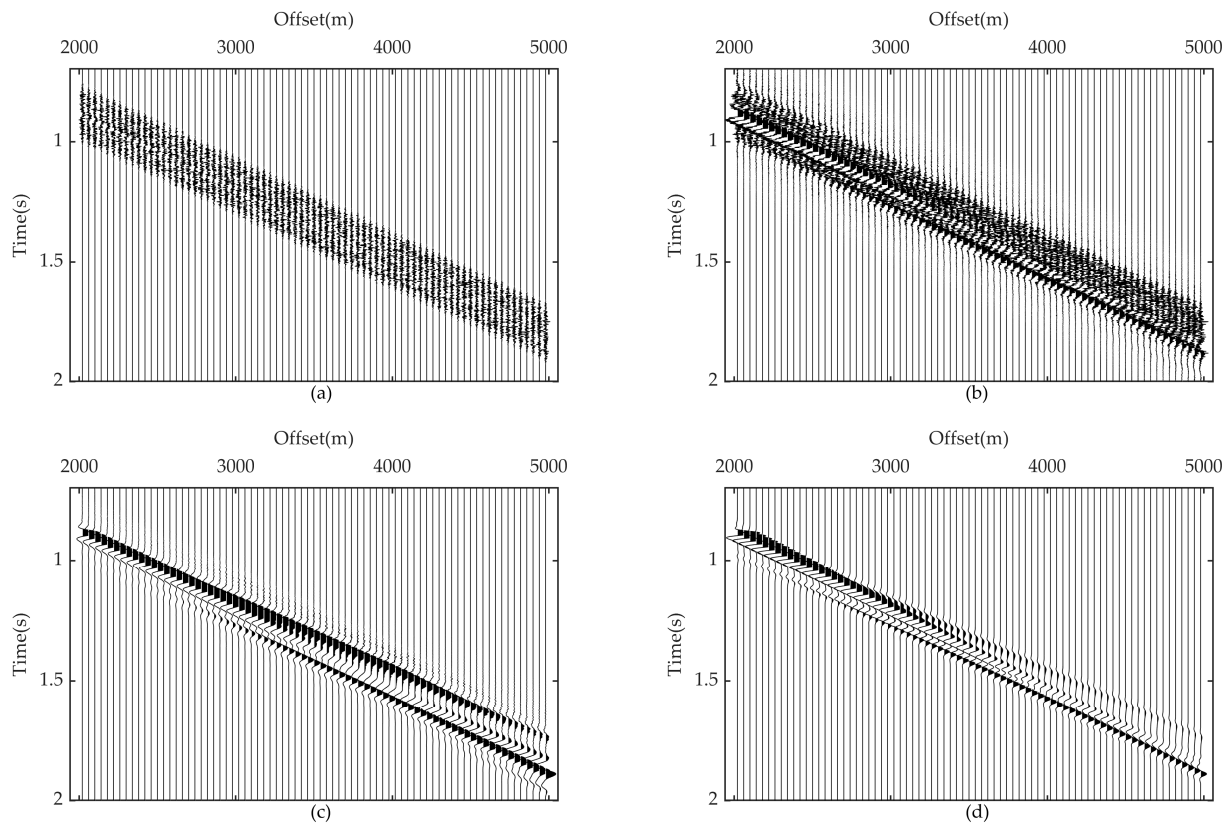


Figure 6. Original first arrival data and reconstructed results: (a) Noise-contaminated data. (b) Reconstructed result by conventional SVI. (c) Reconstructed result by RDSVI. (d) Noise-free data.

To further investigate the changes of amplitude and phase in the reconstructed results, we display the first trace and the last trace before and after reconstruction for waveform comparison, as shown in Figure 7. Figure 7a–d present the waveforms of the first trace of the four data in Figure 6, respectively. It can be seen that both retrieved traces recover the amplitude well, but due to the poor suppression of conventional SVI, the phase distortion of the first arrival signal is serious, as Figure 7b shows. In contrast, based on the excellent noise immunity of RDSVI, the phase of the retrieved signal is clear and smooth, as presented in Figure 7c. Figure 7e–h show the waveforms of the last trace of the corresponding four data. On account of the energy of the first arrival at far offset being weak, coupled with the interference by strong noise, the waveform of the original noisy data is so disordered that the first arrival signal is completely unrecognizable, let alone being picked accurately, as Figure 7e shows. The retrieved trace of conventional SVI in Figure 7f is basically no different from the original trace, the waveform is still disordered, and no effective signal can be identified. On the contrary, there are only some minor phase distortions in the retrieved trace of RDSVI, most of the noise is attenuated, and the effective signal is clear and easy to recognize. The amplitude of the first arrival signal is also enhanced compared with the noise-free trace. Both the reconstructed result and the waveform demonstrate that the RDSVI method can enhance the first arrivals well and significantly improve the SNR when processing the seismic data that interfered by strong noise.

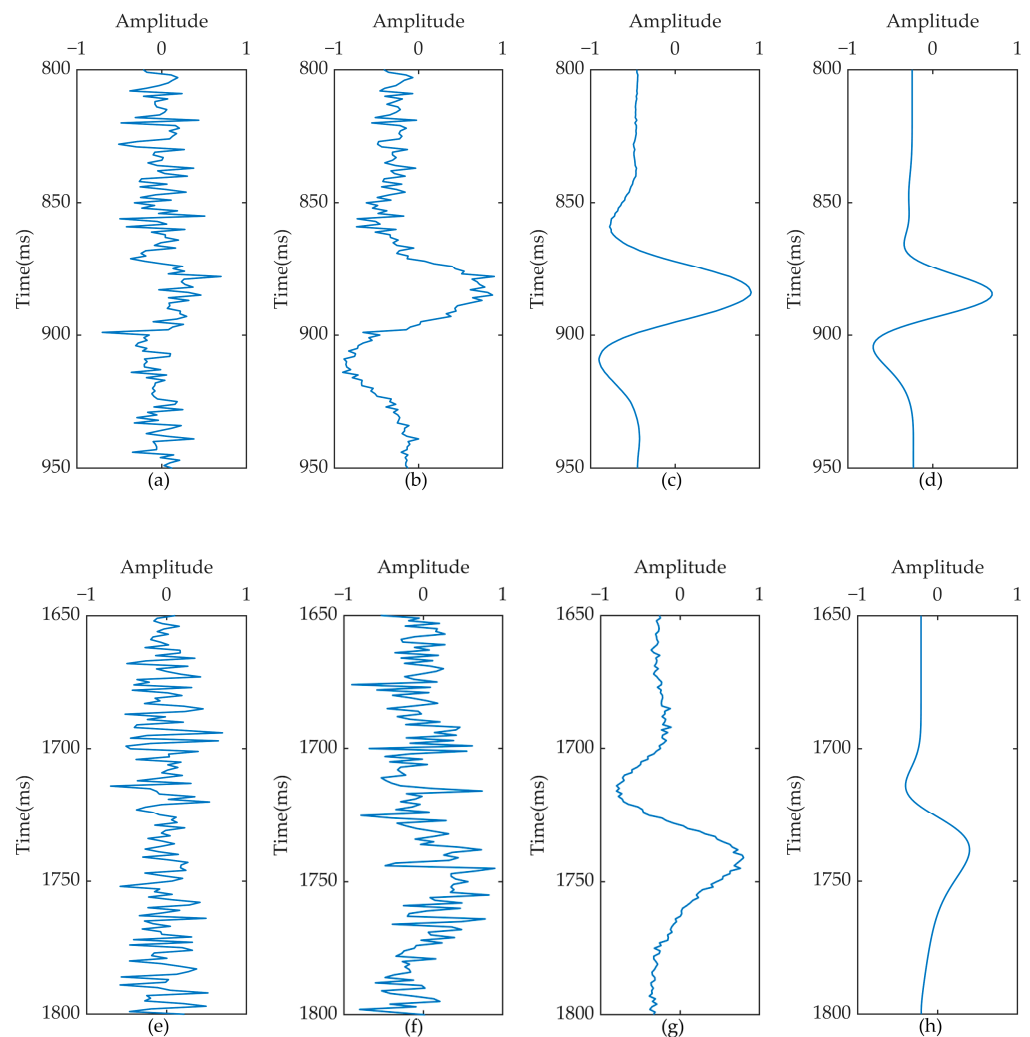


Figure 7. Waveform comparison of single trace before and after reconstruction: (a–d) The first trace of original noise-contaminated data, reconstructed data by conventional SVI, reconstructed data by RDSVI, and original noise-free data, respectively. (e–h) The last trace of the four data, same as (a–d).

To verify the help of the reconstructed data on the first arrival picking, we performed automatic travel time picking on the first arrival data before and after reconstruction. Since the SNR of the original noisy data is too low, the first arrivals cannot be picked properly; here, we use the picks of noise-free data as the accurate result. Figure 8a shows the travel time picks versus offset for the noise-free original shot gather (black circles), the conventional SVI shot gather (blue crosses), and the RDSVI shot gather (red dots). It can be seen that there are obvious errors in the picking result of conventional SVI. Since the retrieved first arrivals at far offsets are still not well identified, resulting in missing picks at some offsets, statistics show that the missing rate reaches 46.3%, whereas the picking results of RDSVI at all offsets are in good agreement with the accurate result. Figure 8b presents the travel time difference between the reconstructed result and the accurate result, from which we could find that the time difference of the conventional SVI method is more perturbed, and the maximum time difference reaches up to 40 ms. Such an unstable picking result will greatly affect the subsequent static correction or first arrival tomography. In contrast, the time difference of the RDSVI method is more stable and the perturbation is smaller. The statistical analysis of the time difference results shows that the time difference of conventional SVI is very scattered, and all of them are more than 4 ms, which cannot meet the accuracy requirements of the first arrival picking, whereas all the time difference of RDSVI is less than 4 ms, and most of the picking results are within 2 ms, as shown

in Figure 8c. The first arrival picking result demonstrates that RDSVI can reconstruct and enhance the first arrivals in the case of poor quality of the original seismic records, providing high-quality data for accurate travel time picks.

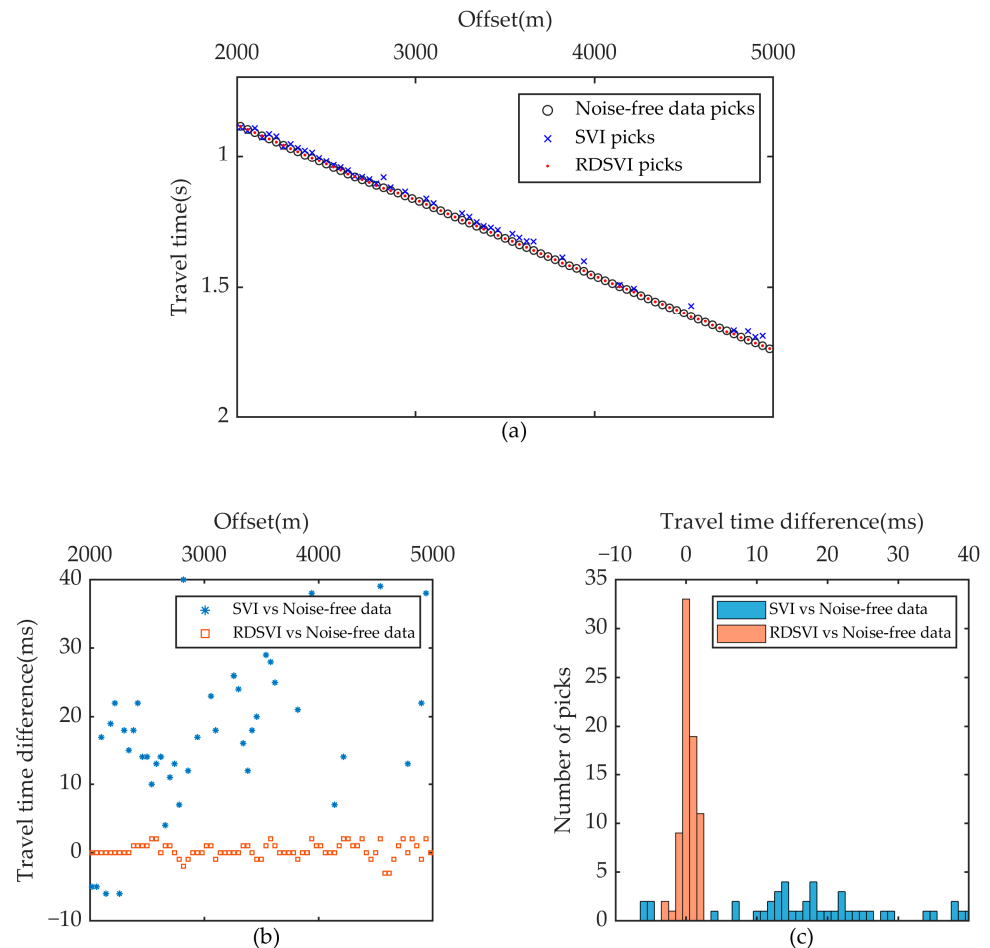


Figure 8. Comparison of automatic first arrival picking results: (a) Travel time picks against offset. (b) Difference in travel time picks between the original noise-free data and two reconstructed results. (c) Corresponding histogram of (b).

3.2. Field Data Example

A shallow, first arrival refraction dataset collected in Gansu Province, China, is processed to verify the application of the proposed method in the actual situation. There are 19 shots stimulated, received by 71 receivers on the surface. Figure 9a shows one shot of this dataset. It can be seen that there exists strong noise interference in the first 30 traces, with the consistency of the first arrivals destroyed at the last 20 traces, making it difficult to pick accurately. The raw seismic records are windowed first to obtain the first arrival data for the following process, as shown in Figure 9b. The conventional SVI and RDSVI are both used to reconstruct the first arrivals, and the results are presented in Figure 9c,d, respectively. It can be seen from Figure 9c that the amplitude of first arrivals was enhanced through conventional SVI; however, the continuity of the event is still not recovered, and new interference artifacts are introduced into the retrieved traces at the same time, which may affect the subsequent first arrival picking, whereas the RDSVI method attenuates the noise at the first 30 traces well, and the continuity of the last 20 traces is recovered in the meantime. The amplitude and phase of first arrivals at all offsets are clear and easy to recognize, as Figure 9d presents.

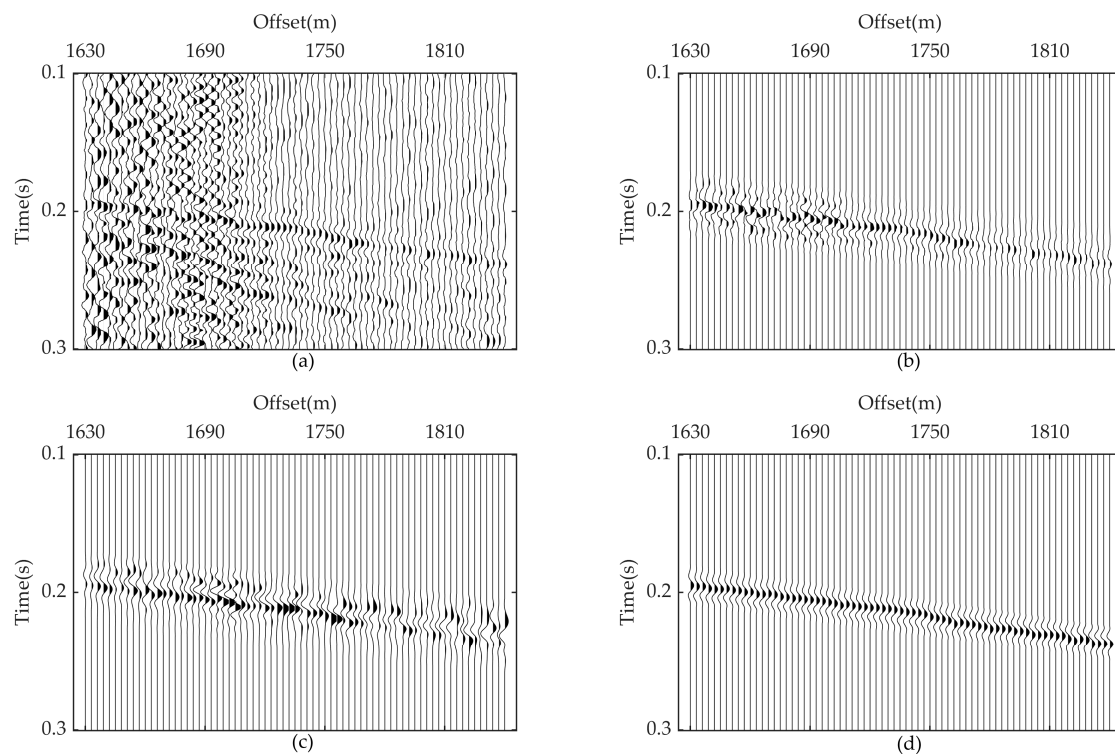


Figure 9. Raw field data and reconstructed results: (a) Raw field data. (b) Muting surrounding the first arrivals. (c) Reconstructed result by conventional SVI. (d) Reconstructed result by RDSVI.

The automatic travel time picking was applied to the field data before and after reconstruction to further validate the picking accuracy. Both picked results are displayed in Figure 10. The fluctuation of results picked in raw data (black circles) is severe at the first 30 traces mainly due to the noise interference, and since the first arrivals at the last 20 traces are hard to recognize, some picked results are missing in this part. For the results picked in conventional SVI data (blue crosses), because of the distraction of interference artifacts, there are many abnormal values in the picking results, and the continuity is even worse than the raw data picks. On the contrary, the results picked in RDSVI data (red dots) are stable and match the retrieved event in Figure 9d well, mainly because the noise were attenuated, together with the recovery of the consistency of first arrivals at far offsets. Both successfully reconstructed data and reliable first arrival's picking results of field data example demonstrate the effectiveness of RDSVI.

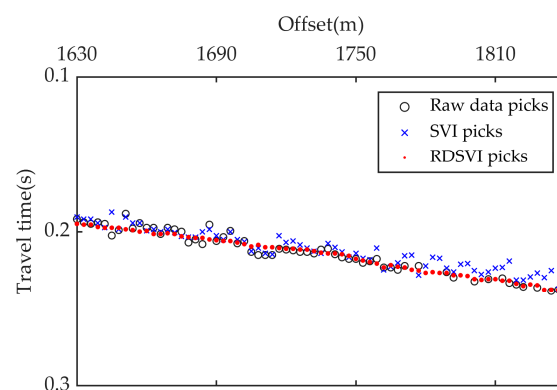


Figure 10. Automatic first arrival picking results.

The long propagation distance of first arrivals will result in some seismic information missing in traces at different offsets. Specifically, low-frequency components are missing at the near offset traces, whereas high-frequency components are missing at the far offset traces in the original data. The missing seismic information can be compensated during the convolution and cross-correlation step in the RDSVI method. The retrieved traces at near offsets will contain low-frequency components of original traces at far offsets; meanwhile, the retrieved traces at far offset will contain high-frequency components of original traces at near offsets, which can lead to a frequency spectrum change. After compensation, the main frequency of retrieved traces at near and far offsets are pulled down and up, respectively. The first and last trace before and after reconstruction are extracted for the frequency spectrum analysis to see the change in the main frequency, as shown in Figure 11.

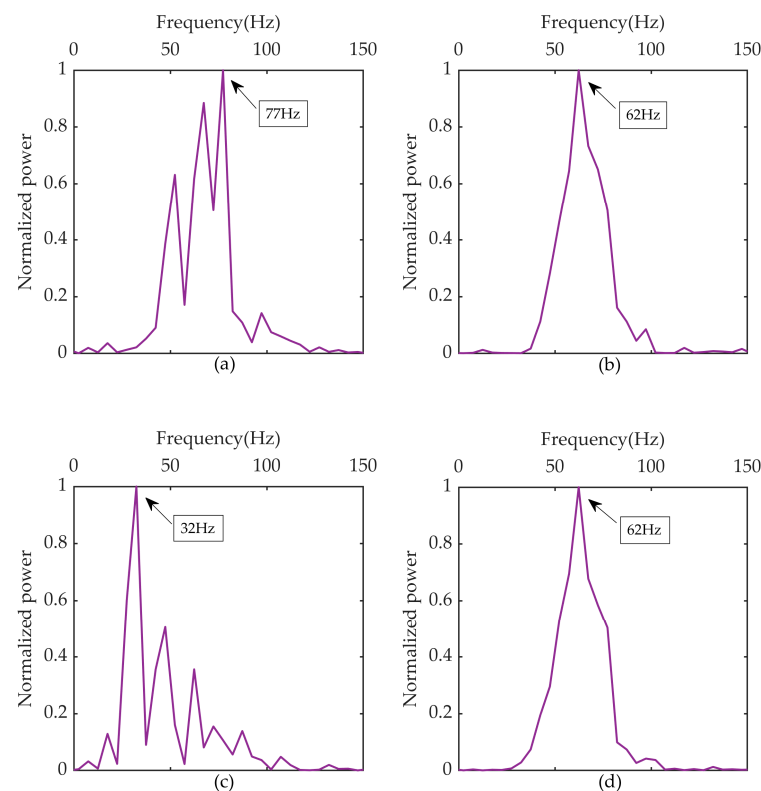


Figure 11. Frequency spectrum analysis: (a) The first raw trace. (b) The first retrieved trace. (c) The last raw trace. (d) The last retrieved trace.

4. Discussion

In practical seismic data acquisition, due to the factors such as dead or severely corrupted traces, surface obstacles, acquisition aperture, or budgetary constraints, the collected datasets are usually irregular along the spatial direction or sparsely sampled, resulting in missing seismic traces. The conventional SVI method cannot recover the information of missing traces, which is a serious impact on the accurate first arrival picking. To verify whether the RDSVI method can retrieve the continuity of the first arrivals, as well as reconstruct the data with a higher SNR in the case of processing the seismic data with missing traces, we tested and compared the reconstructed results of the RDSVI method on first arrival data with different trace missing ratios.

We performed random trace missing processing on the original noisy first arrivals data (Figure 6a) and simulated the cases of seismic data with 10%, 30%, and 50% traces missing, respectively. The RDSVI method is used for first arrival reconstruction. Figure 12 displays the three input missing data and the corresponding reconstructed results, respectively. It can be seen that, in the case of 10% missing, RDSVI reconstructs the first arrival well, signif-

icantly improving the SNR and overcoming the influence of traces missing simultaneously. The amplitudes at the missing traces present a good match with the reconstructed results of the non-missing original data (Figure 6d). In the case of 30% missing, this method also has a good effect on improving the SNR, but the ability of noise attenuation is not that good as before, and the phase distortion at far offsets is a little obvious. As to 50% traces missing in the original data, the kinematic characteristics of the reconstructed first arrivals can still be retrieved well; however, the attenuation of noise is significantly reduced, especially at far offsets, as well as the phase distortion becomes severe, which may affect the precious of the subsequent first arrival picking.

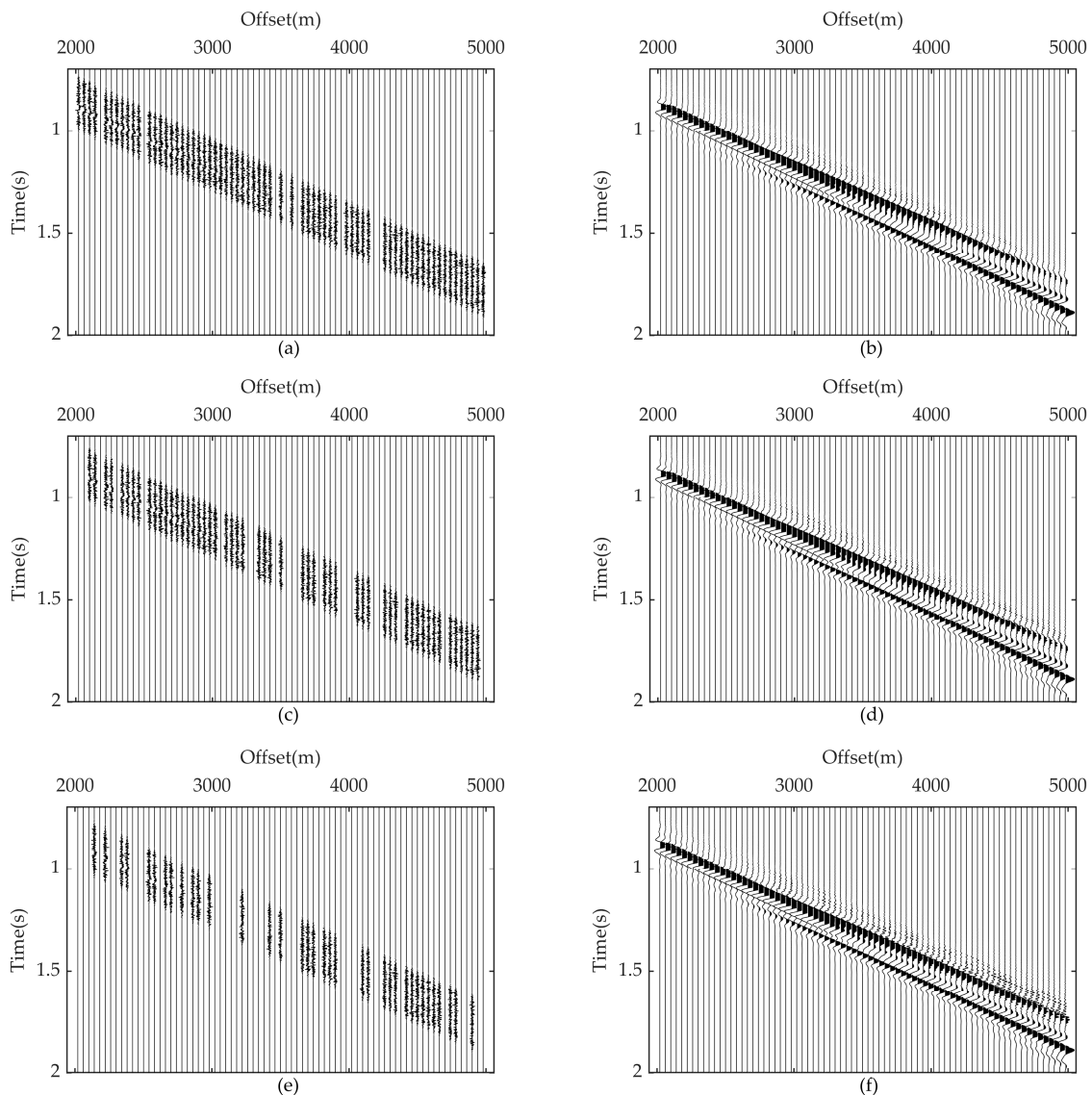


Figure 12. Different trace missing data and corresponding reconstructed results: (a,b) 10% missing data and reconstructed result, (c,d) 30% missing data and reconstructed result, and (e,f) 50% missing data and reconstructed result.

To sum up, based on the characteristic of the Radon transform that integrates along the linear path, in the face of seismic data with missing traces, the RDSVI method can significantly improve the SNR of the first arrivals while interpolating the seismic data, which is more in line with the actual data processing situation. Compared with the conventional interpolation method that requires many input parameters, this method is completely data-driven and does not need manual intervention. In general, this method

can accomplish the task of first arrival reconstruction with the original missing data with a missing rate no more than 30%. However, in the case of the data with a large scale of trace missing, coupled with the interference of strong noise, the effect of this method on the reconstruction and interpolation will be greatly reduced due to the insufficient effective signal information provided and the weak energy of the original first arrival data. Combined with the analysis results above, we recommend using this method when the deletion ratio of original data is less than 10%. In practical seismic data acquisition, the proportion of abnormal traces is usually under 5%, so this method is suitable for the reconstruction and interpolation of first arrivals in most cases.

5. Conclusions

Aiming at the problem that conventional SVI has a limited ability to improve the SNR of the first arrival data interfered with by strong noise, based on the excellent noise immunity of the Radon transform, we proposed the RDSVI method that implemented SVI in the Radon domain. Since the cross-correlation in the Radon domain is more effective on noise suppression than in the temporal domain, and with the help of SRTIS to improve the convergence of effective signal energy in the Radon domain, this method can better attenuate the strong noise and significantly improve the SNR of the first arrival data. Both synthetic data and field data show that RDSVI can obtain a better reconstructed result with stronger amplitude and clearer phase of first arrival signals compared to conventional SVI. The travel time picking result of RDSVI are accurate and stable, and the offsets of pickable first arrivals are also increased after reconstruction. Meanwhile, when facing the data with missing traces, this method can also provide help for seismic data interpolation. These results verify the effectiveness of this method in processing the first arrival data with strong noise. Based on the feature of the integral path of the linear Radon transform, not only limited to refractions, but also for other straight-line seismic events or approximately straight-line shape, this method can be used for signal reconstruction and enhancement.

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