



Article

Effects of Charged Martian Dust on Martian Atmosphere Remote Sensing

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Abstract: In this paper, the extinction property and optical depth of charged Martian dust at infrared band 3 THz–300 THz are studied using the Mie scattering theory. It is found that the extinction coefficients of Martian atmospheric dust and the dust optical depth (DOD) of the Martian atmosphere can be amplified significantly as the dust particles are charged. This extinction amplification has a peak, called amplification resonance, which shifts toward the upper left of the r - q parameter plane with increasing frequency. Here, r denotes the particle radius and q denotes the particle's total net charge. The amplification of the Martian DOD is more significant at high altitudes than at low altitudes because the particles at high altitudes are smaller. For example, at an altitude of 30–50 km, the dust optical depth at 30 THz can be increased by 60–200%. However, at 3 THz–10 THz, the DOD at the near surface altitude (0–10 km) can still be enhanced by ~80%. This implies that by treating the Martian dust as uncharged particles, the dust density constructed from the Martian DOD data might be overestimated. The estimation error of the dust density of the Martian atmosphere may be reduced by counting the enhancement of the DOD that is caused by charged dust.

Keywords: charged martian dust; Mars atmosphere Remote Sensing; optical depth; extinction coefficient; amplification resonance; extinction reduction



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1. Introduction

The Martian dust optical depth (DOD) is key data used in Mars climate simulations [1]. Many exploration missions have been launched to establish Mars global DOD data using the infrared channels onboard the Mars orbiter, such as the Thermal Emission Spectrometer (TES) of the Mars Odyssey Mission [2], the Emirates Mars Infrared Spectrometer (EMIRS) of the Emirates Mars Mission (EMM) [3], and the Mars Climate Sounder (MCS) [4]. From the DOD data, the vertical dust profiles of Martian atmosphere can be constructed [5,6] and fed into Mars climate models to predict the weather conditions on Mars, including temperature, wind speed, and dust density distribution [1]. These predictions are useful for future Mars missions (e.g., future long-term human missions to the surface of Mars). The dust density can also have impact on the opacity of the Martian atmosphere, which is important for the power management (e.g., the battery and solar panel management) of Mars landers and rovers [1,7–10].

The dust optical depth is related to the extinction properties of dust particles; therefore, the extinction property of Martian dust particles at a visible band (e.g., 488 nm and 647 nm [11]) and an infrared band (e.g., 5–50 μ m [12,13]) has been studied. To the best of our knowledge, most current studies on the extinction and optical depth of the particles treat the Martian dust as neutral particles (e.g., the work of refs. [14,15]). However, similarly to dust particles on Earth [16,17], dust particles in the Martian atmosphere can also be charged for many reasons, such as ultraviolet radiation from the Sun, or the collisions or friction between particles [18–20]. It is thought that the breakdown surface potential

is about 20–25 kV for a particle in the Martian atmosphere [21]. For this order of breakdown potential, the total net charge of a micro-sized dust particle can reach the order of $10^6 e$ [21–23], where e denotes the elementary charge, namely, $e = 1.602 \times 10^{-19}$ (unit in C, Coulomb). An experimental study on Mars analogue dust showed that the total net charge of a particle of radius $1.0 \mu\text{m}$ could reach $2 \times 10^5 e$ [24]. On the other hand, it has been found that the optical property of a charged particle, such as the extinction, absorption, and scattering of cross sections, can be changed [25–27]; therefore, the Mars DOD can also be changed if the dust particles are charged. That means the construction of dust density from the Mars DOD data (e.g., the TES data) might result in errors if one treats the atmospheric dust as neutral particles. In fact, this overestimation of dust density was already encountered. A model simulation of the Martian atmospheric dust distribution was performed using the vertical dust profile constructed from the TES DOD data [1]; it was found that the model prediction of dust density at altitudes above 15 km were overestimated, as compared with the vertical dust density distribution observed by NASA's Mars rover Phoenix. The Martian DOD data are often retrieved from infrared channels. For example, the TES, EMIRS, and MCS channels' wavelengths are $5 \mu\text{m}$ – $150 \mu\text{m}$, 6 – $40 \mu\text{m}$, and $\sim 22 \mu\text{m}$, respectively.

In this paper, the extinction coefficient and the dust optical depth of charged Martian dust at different altitudes are studied theoretically for the infrared band 3 THz – 300 THz , which covers the channels of TES, EMIRS, and MCS. The extinction coefficient of the charged Martian dust particle is calculated using the Mie scattering theory, where two different models of charged particles (the surface conductivity model and the Core-shell model [25,28]) are employed for the description of charged Martian dust. The calculated results indicate that the extinction of Martian dust particles and the Martian DOD can be amplified as the dust particles are charged, especially at high altitudes (e.g., above 20 km). The amplification of extinction and the DOD, as well as their dependence on altitude and frequency, are discussed. A preliminary attempt is made to correct the overestimated Martian dust profile, by counting in the enhancement of the DOD caused by charged Martian dust particles.

2. Model Description

2.1. Charged Martian Dust Particles

The wind on Mars can attain a velocity as high as 30 m/s [20]. As a result, the Martian surface is covered by regolith [20], and a huge amount of fine dust can be lifted from the surface and lofted to high altitudes within the Martian atmosphere. The low atmospheric pressure (about 5 – 10 mbar) and low humidity on Mars could make it easier for the dust to be charged due to rubbing or collisions [21]. The Martian dust can also get charged from exposure to cosmic radiations (e.g., ultraviolet (UV) rays [20]), as shown in Figure 1.

According to the theory of electrostatics, the surface potential V_S on a spherical particle's surface is given by Equation (1),

$$V_S = \frac{q}{4\pi r \epsilon_0} \quad (1)$$

where q (unit in Coulomb, [C]) denotes the total net charge of the charged particle, r , the particle radius, and ϵ_0 , the vacuum permittivity. The total charge q of the particle is limited by the breakdown potential ($V_{\text{breakdown}}$) of the particle. Some studies have pointed out that the breakdown potential $V_{\text{breakdown}}$ is about 20 kV [21,30]. Thus, the saturation charge of the charged particle is $q_{\text{max}} = 4\pi \epsilon_0 V_{\text{breakdown}}$, which is compared with the reported experimental results from refs. [22,24,31], as shown in Figure 2. Here, one can find that the experimental results are mostly lower than the saturation charge q_{max} predicted by (1).

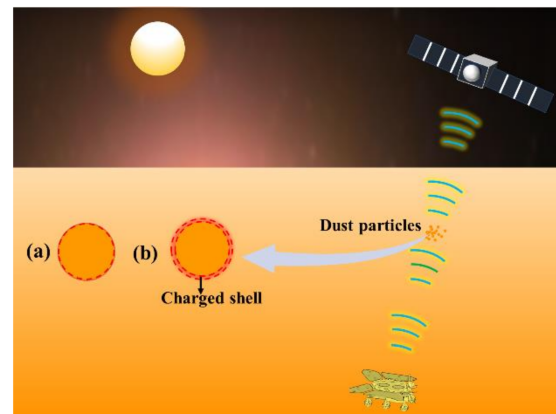


Figure 1. Schematic of the remote sensing of Martian atmospheric dust. (a) A dust particle with a charged shell that is much thinner than the particle size, which is assumed to form a surface current of zero-thickness [28]. (b) A dust particle with a charged shell that has a thickness comparable to the particle size, which can be described by a Core-Shell model [29].

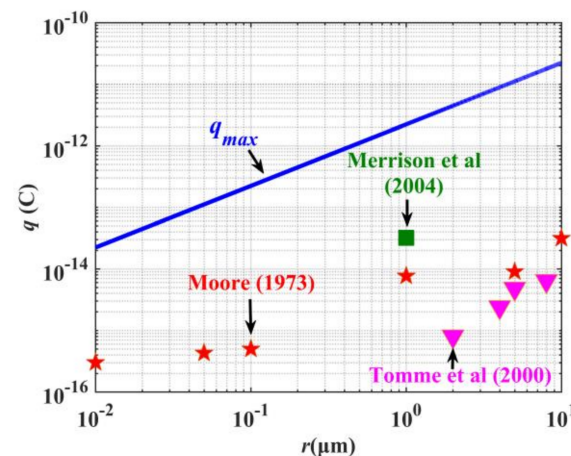


Figure 2. The dependence of the particle's saturation charge q_{max} on the particle radius. Here, it is assumed that the breakdown surface potential is 20 kV. The scatters are reported experimental data from refs. [22,24,31].

A study on the shapes of dust particles conducted by Ilan Koren et al. [32] showed that most aerosol particles were spherical. On the other hand, using the Lorenz–Mie theory and the T-matrix method, Yang et al. (2007) compared the optical properties of spherical and spheroidal dust particles. It was found that the effect of non-spherical shapes was large at visible wavelengths, but essentially negligible at infrared wavelengths [33]; therefore, the Martian atmospheric dust particles are assumed to be spherical in this paper.

It is well known that net charges of a solid are bound to a very small region in the vicinity of the solid surface, which can be treated as a charged shell of the solid. The thickness of the charged shell, denoted as t_s , is usually several nanometers (e.g., 1–3 nanometers) [34]. To study the extinction properties of charged Martian dust particles, there are two available theoretical models.

2.2. Surface Conductivity Model

The thickness of the charged shell is several orders of magnitude less than the particle radius, as shown by case (a) in Figure 1; therefore, when the particle is illuminated by a plane electromagnetic wave (EMW), it can be assumed that the net charges generate a surface current on the particle surface, the thickness of which is regarded as zero. This is the well-known Mie scattering theory for charged spheres in refs. [25,28] by Bohren and Huffman, which uses a phenomenological parameter, surface conductivity σ_s , to describe

the contribution of net charges to the extinction coefficient. This theory is widely used in many studies (e.g., the scattering of charged interstellar dust particles) [26]. Here, we call it the surface conductivity model in the rest of this paper.

The surface conductivity σ_s can be calculated using $\sigma_s = ie^2 n_s / m_e (\omega + i\gamma)$, which is a model that is widely used for research purposes [25]. Here, it is assumed the net charge carriers are electrons. m_e stands for the mass of electron ($m_e = 9.109 \times 10^{-31}$ kg), e is the elementary charge (i.e., 1.602×10^{-19} C), and γ is a constant defined as $\gamma = \gamma_c k_B T / \hbar$, where γ_c is a material-dependent constant [35], $k_B = 1.38 \times 10^{-23}$ J K⁻¹ is Boltzmann's constant, $\hbar = 1.046 \times 10^{-34}$ Js is Planck's constant (divided by the factor 2π), and T stands for the ambient temperature (in K).

Let n_s denote the number density (in 1/m²) of net charge carriers on the particle's surface; therefore, $n_s = q / 4\pi r^2 e$, where q denotes the total net charge of the charged particle. The refractive index of the particle's material is $m = \sqrt{\epsilon_r}$, where ϵ_r denotes the dielectric constant of the particle's material. As the particle is charged, an equivalent dielectric function of the charged particle can be obtained, which is $\epsilon_s = \epsilon_r + i \frac{2\sigma_s / r}{\epsilon_0 \omega}$ [25], where r denotes the particle radius and ϵ_0 denotes the vacuum permittivity. For a charged particle, an excitation frequency, ω_s , can be defined as $\omega_s = \omega_p / \sqrt{\epsilon_{real} + 2}$ for $\gamma \ll \omega$ (see ref. [36]), where ω_p is the surface plasma frequency [25], defined by $\omega_p = \sqrt{n_s e^2 / m_e \epsilon_0}$ and ϵ_{real} is the real part of the dielectric constant, the ϵ_r of the particle. Here, ω is the circular frequency of the incident electromagnetic wave (namely, $\omega = 2\pi f$), in which f stands for the frequency (in Hz) of the incident wave.

2.3. Core-Shell Model

To calculate the scattering of a particle that consisted of two different materials, Bohren and Huffman [28] also proposed a model of two concentric spheres, a large hollow sphere enclosing a small solid sphere, shown using case (b) in Figure 1. This model can also be employed to calculate the scattering of charged Martian atmospheric dust particles, by treating the hollow outer sphere as the charged region, and the inner sphere as the one free of net charge. Moreover, it is assumed that the particle has a charged shell with a finite thickness. We call it Core-shell model in this paper.

The refractive index of the core, free of net charges, is $m_1 = \sqrt{\epsilon_r}$. The dielectric constant of the outer sphere, denoted as ϵ_2 , can be calculated by $\epsilon_2 = \epsilon_r + i \frac{\sigma_{cond}}{\omega \epsilon_0}$. Here, σ_{cond} is the electric conductivity of the charged shell, which can be calculated by $\sigma_{cond} = \frac{ie^2 n_{cond}}{m_e (\omega + i\gamma)}$ [37]. Here, n_{cond} denotes the number density (in 1/m³) of net charges in the outer sphere, which can be calculated by $n_{cond} = 3q / 4\pi (r^3 - t_s^3) e$. The permittivity of the charged shell can be reorganized as $\epsilon_2 = \epsilon_r - \omega_p^2 / (\omega^2 + i\omega\gamma)$, where ω_p is the plasma frequency associated with the collective oscillation of the net charges, defined by $\omega_p = \sqrt{n_{cond} e^2 / (m_e \epsilon_0)}$. According to the Drude model of dielectric function, the excitation frequency of the net charges, denoted as ω_c , can be expressed as $\omega_c = \omega_p / \sqrt{-\text{Re}(\epsilon_2) + \epsilon_{real}}$ [38].

For the Core-shell model of the charged particle, an equivalent dielectric function, denoted as ϵ_{cs} , can also be defined, which is

$$\epsilon_{cs} = \frac{v_c}{v_c + v_s} \epsilon_r + \frac{v_s}{v_c + v_s} \epsilon_2, \quad (2)$$

in which v_c and v_s stand for the volume of the core and the charged shell, respectively.

By employing the two models, the scattering problem of a charged spherical particle can be solved using Mie's scattering theory, and the extinction coefficient of a charged sphere, denoted as Q_{ext} , can be expressed as the following form,

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \text{Re}(a_n + b_n) \quad (3)$$

where a_n and b_n are Mie scattering coefficients. One can find the expressions of Mie scattering coefficients a_n and b_n for both uncharged and charged spheres in ref. [28] by Bohren and Huffman. Let and denote the extinction coefficients of the charged and the uncharged particle, respectively. Thus, Q_{ext}^c/Q_{ext}^0 is called the extinction amplification factor of a charged particle.

3. Results and Discussions

The commonly reported sizes of dust particles in Martian atmosphere are in the range (0.01 μm , 10 μm) [6,23,38,39]. Under low and moderate dust loading conditions, which were observed by Viking, MPF, MGS/TES, MER/Mini-TES, and MER/Pancam, the effective particle radius lies in the range of 1.4–1.7 μm [8]. Under high dust loading conditions (e.g., the 2001 global storm), the observed effective dust particle radius is 1.8–2.5 μm [8]. For more diffuse conditions, the effective particle radius decreases from ~1.4–1.8 μm at the near-surface (<10 km), to ~1.0 μm at an altitude near 20 km, to 0.85 μm at an altitude of 20 km–30 km, and to ~0.5 μm at an altitude of 30 km–40 km, and it becomes less than 0.3 μm at an altitude above 40 km [8,40–44]. The infrared band is frequently used for Mars exploration missions, such as the Thermal Emission Spectrometer (TES, 5–150 μm) of the Mars Odyssey Mission [2], the Emirates Mars Infrared Spectrometer (EMIRS, 6–40 μm) of the Emirates Mars Mission (EMM), and Mars Climate Sounder (MCS, ~22 μm). In the following sections, the extinction coefficients of charged Martian atmospheric dust particles at these infrared bands and at different altitude levels, are calculated using the surface conductivity model and the Core-shell model.

3.1. Extinction Properties of Charged Martian Dust

Figure 3 shows the extinction amplification factors of charged Martian dust particles, denoted as Q_{ext}^c/Q_{ext}^0 , which are plotted as functions of the total net charge of the particle (denoted as q , unit in [C]). Here, four particle radii are selected for study (i.e., 0.5 μm , 1 μm , 1.5 μm and 2 μm [8,38,39]) which correspond to different altitude levels (i.e., altitude 30–40 km, near ~20 km and altitude near surface (<10 km), respectively). Considering the abovementioned sensors onboard Mars orbiters (i.e., TES, EMIRS, and MCS), four infrared frequencies/wavelengths of interest are considered, which are 3 THz/100 μm , 10 THz/30 μm , 30 THz/10 μm , and 300 THz/1 μm . The dielectric constants of particle materials for the four frequencies are $\epsilon_r = 4.3 + 1.05i$, $\epsilon_r = 4.6 + 2.73i$, $\epsilon_r = 5.3 + 3.16i$, and $\epsilon_r = 2.34 + 0.03i$, respectively, which are the experimental data from refs. [12,13,45].

As shown in Figure 3, for the considered frequencies and particle sizes, the surface conductivity model (Figure 3a,c,e,g) and Core-shell model (Figure 3b,d,f,h) give consistent results, and the amplification factors are basically greater than 1, which means the extinction coefficient of the Martian atmospheric dust particles will be enhanced when they are electrostatically charged. Particularly for the Martian dust particles at an altitude of 30 km–40 km and near 20 km, where the effective particle radii are 0.5 μm and 1 μm , respectively, the amplification factors at 3 THz have maximums with increasing q , which is called amplification resonance (as shown in Figure 3a–d). The peak value of the factor Q_{ext}^c/Q_{ext}^0 is over 4. The positions of amplification resonance on the q -axis only have a small difference between the two models. The amplification resonance peaks imply that at an altitude of 20–30 km, the extinction of Martian dust particles at the infrared frequency/wavelength 3 THz/100 μm could have a chance to be amplified by more than 400% when the particles are charged. The extinction factors at the infrared frequency/wavelength 10 THz/30 μm increase monotonously with increasing q and they reach their maximum as q gets saturated. The maximum of the factor Q_{ext}^c/Q_{ext}^0 is ~3 and ~1.6 for $r = 0.5 \mu\text{m}$ and 1.0 μm , respectively, as shown by the dashed lines in Figure 3a–d. With increasing frequency or particle size, the extinction amplification factor Q_{ext}^c/Q_{ext}^0 decreases. For example, the factor Q_{ext}^c/Q_{ext}^0 decreases to ~1.25 for $r = 0.5 \mu\text{m}$ and $f = 30 \text{ THz}$, even as q approaches q_{max} , as shown in Figure 3a,c.

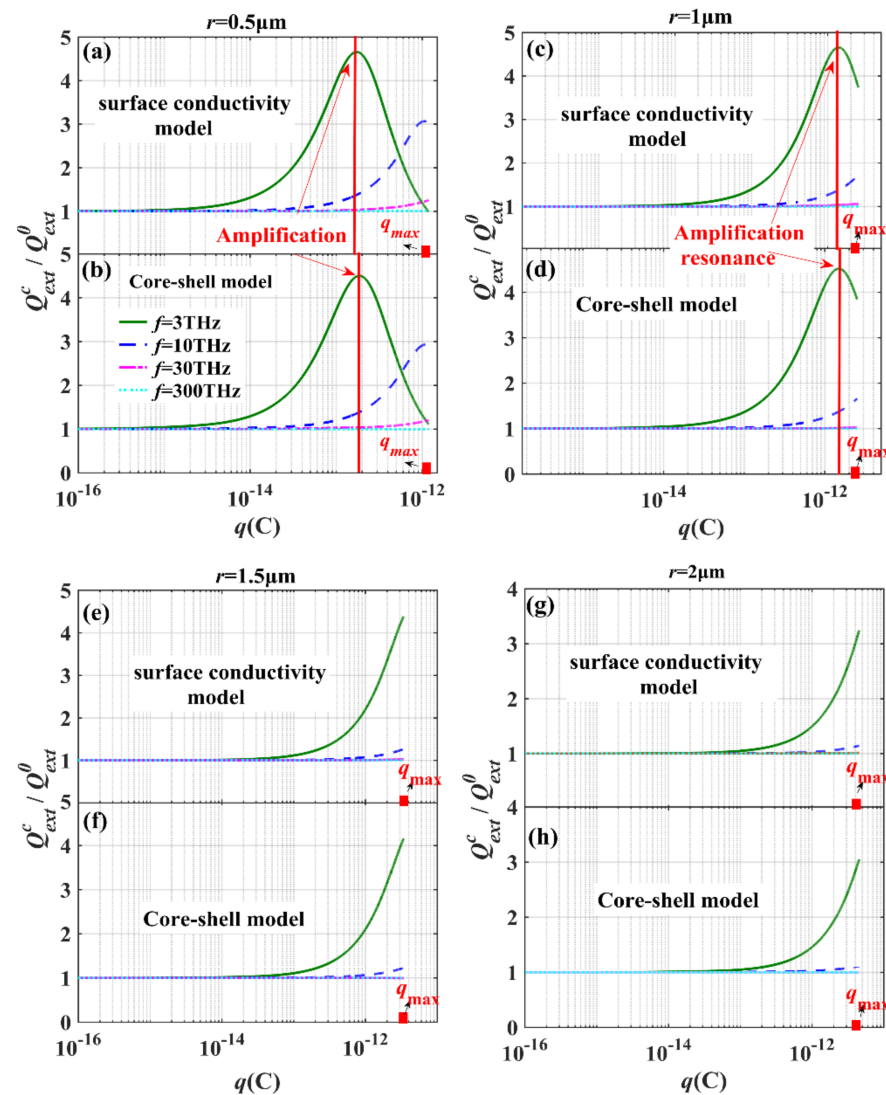


Figure 3. The extinction amplification factor (i.e., Q_{ext}^c / Q_{ext}^0) of charged Martian dust at infrared frequencies 3 THz, 10 THz, 30 THz, and 300 THz, and its dependence on q , the total net charge of the Martian dust particle. (a,c,e,g) are given by the surface conductivity model, while (b,d,f,h) are given by the Core-shell model for $r = 0.5 \mu\text{m}$, $1.0 \mu\text{m}$, $1.5 \mu\text{m}$ and $2.0 \mu\text{m}$, respectively.

One can get some implications from the results in Figure 3: the extinction amplifications of the charged Martian dust are not negligible at an altitude of 30–40 km for infrared frequencies/wavelengths 3–30 THz/10–100 μm , near an altitude of ~ 20 km for frequencies/wavelengths 3–10 THz/30–100 μm and at altitude < 10 km for the infrared frequency/wavelength near 3 THz/100 μm . One should notice that TES's and EIMRS's channels lie in these infrared bands. Theoretically, this implies the Martian dust particle density constructed from TES or EIMERS dust optical depth might be overestimated, especially at altitudes above 20 km, if one treats the dust as neutral particles.

To study the relationship between the extinction amplification factor of charged Martian dust particle (i.e., Q_{ext}^c / Q_{ext}^0) and the equivalent dielectric functions (i.e., ϵ_s and ϵ_{cs}), Figure 4 shows Q_{ext}^c / Q_{ext}^0 , ϵ_s and ϵ_{cs} as functions of the total net charge q of the particle for different infrared frequencies $f = 3$ THz, 10 THz, 30 THz, and 300 THz. Here, the particle radius is $r = 0.5 \mu\text{m}$. The left column, Figure 4a–d, are the results given by the surface conductivity model, whereas the right column, Figure 4a1–d1, are the results given by the Core-shell model. The right y-axis shows the values of the real and imaginary parts of ϵ_s and ϵ_{cs} . On the other hand, Q_{ext}^c / Q_{ext}^0 , ϵ_s and ϵ_{cs} are shown as functions of the particle

radius r in Figure 5, in which the particle's net charge is assumed to be saturated, namely, $q = q_{\max}(r)$.

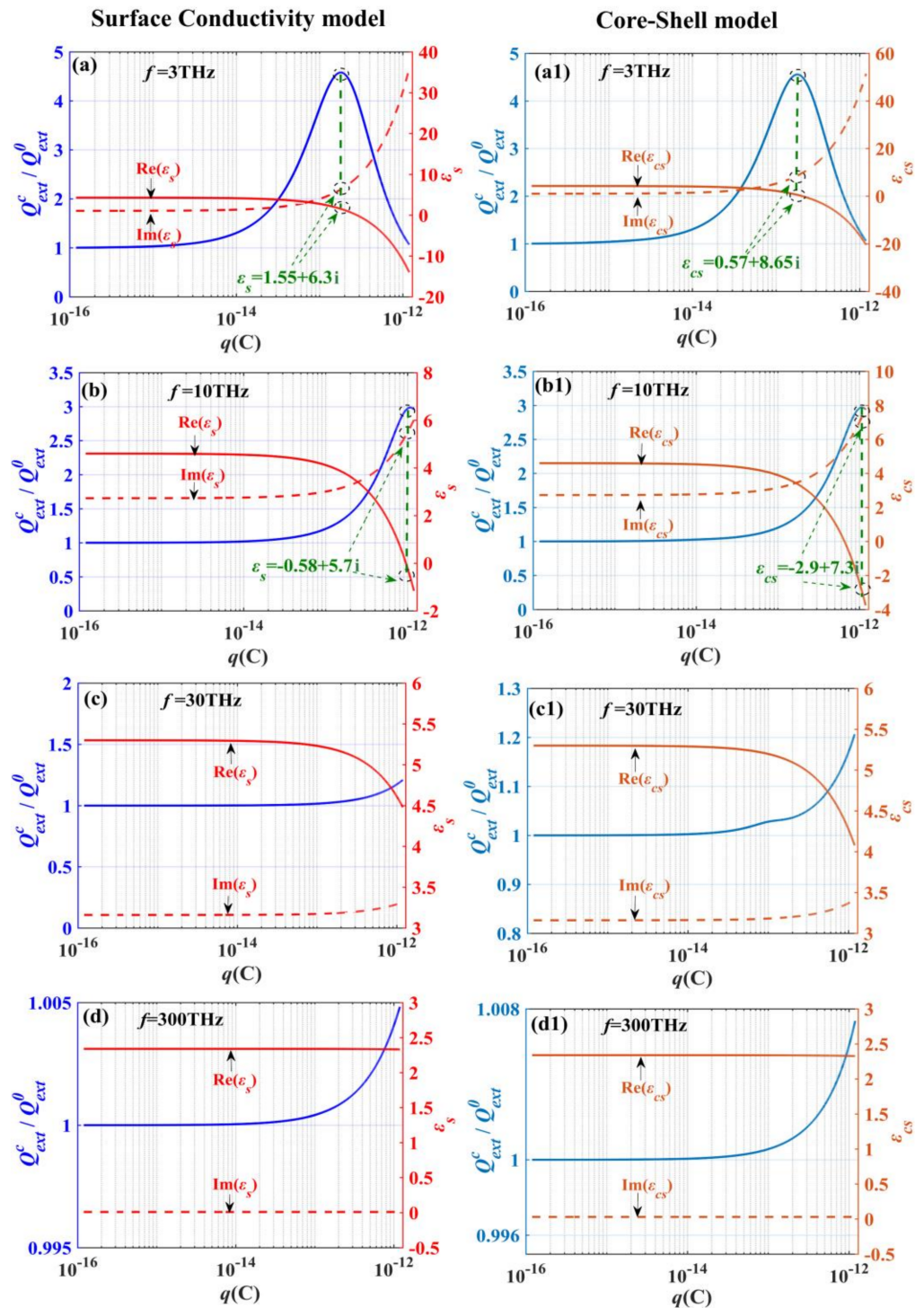


Figure 4. The extinction amplification factor $Q_{\text{ext}}^c / Q_{\text{ext}}^0$ and the equivalent dielectric functions (i.e., ϵ_s and ϵ_{cs}) given by the surface conductivity model (a–d) and the Core-shell model (a1–d1), respectively, which are plotted as functions of q , the total net charge of the particle. Here, the particle radius is $0.5 \mu\text{m}$.

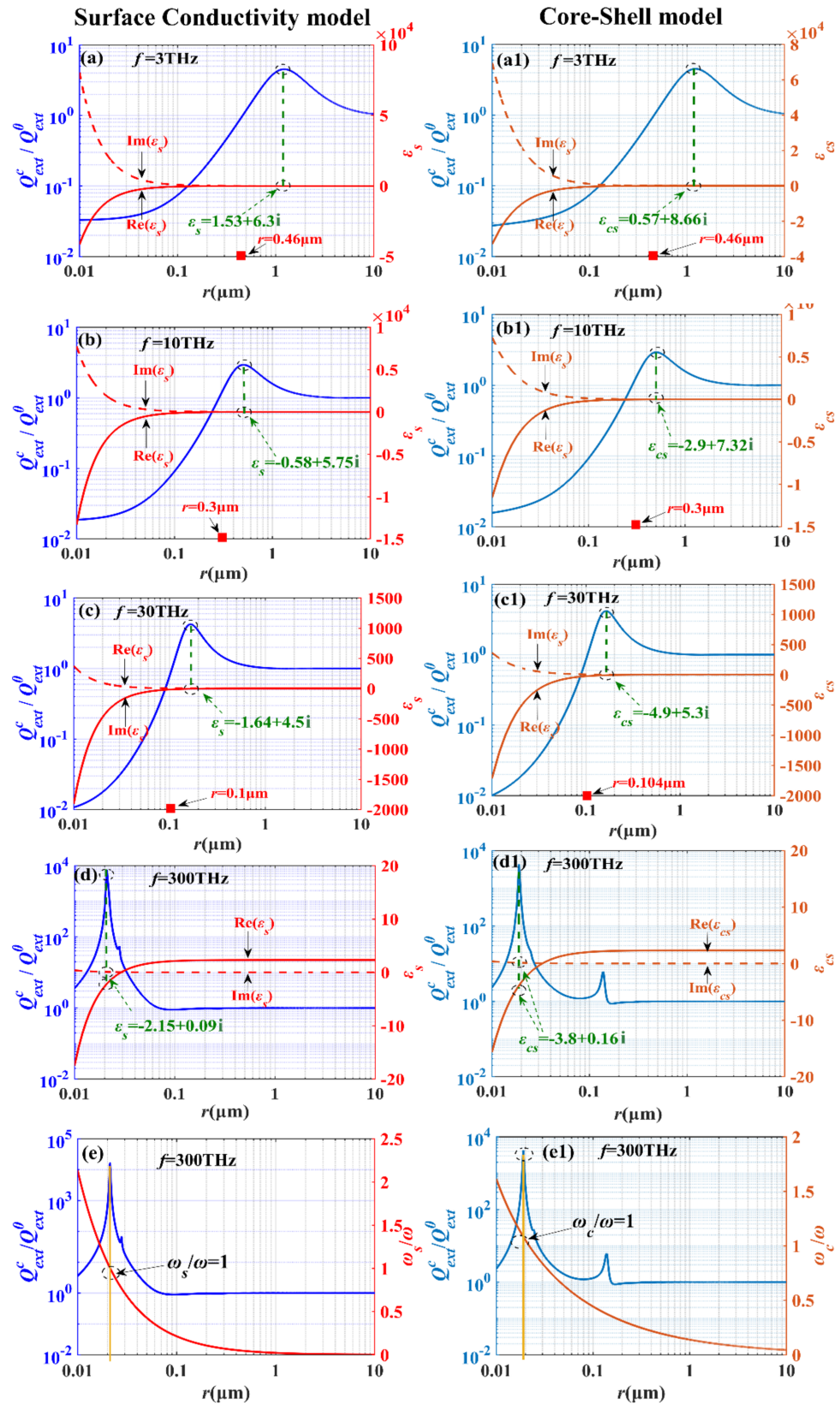


Figure 5. Extinction amplification factors Q_{ext}^c / Q_{ext}^0 and equivalent dielectric functions (i.e., ϵ_s and ϵ_{cs}), which are given by the surface conductivity model (a–d) and the Core-shell model (a1–d1), respectively, for frequencies $f = 3 \text{ THz}$, 10 THz , 30 THz and 300 THz . (e,e1) shows the normalized excitation frequencies (i.e., ω_s / ω and ω_c / ω) for $f = 300 \text{ THz}$ given by the two models, respectively. Here, for each fixed r , it is assumed that the particle's net charge gets saturated, namely by $q = q_{max}(r)$.

One can find from Figures 4 and 5 that the variation of the amplification factor, as a function of q and r , is accompanied by the variation of the equivalent dielectric functions $\epsilon_s(q, r)$ and $\epsilon_{cs}(q, r)$. By comparing Figure 4a with Figure 5a, one sees that at the positions of amplification resonance on the q -axis and r -axis, where Q_{ext}^c/Q_{ext}^0 takes the peak values, the equivalent dielectric function ϵ_s takes the same value for $f = 3$ THz. That is, at the infrared frequency $f = 3$ THz, the amplification resonance occurs as the dielectric function ϵ_s , as a function of q and r , which is equal to a particular value $1.55 + 6.3i$ (see Figures 4a and 5a). Similarly, the extinction amplification resonance for the frequency $f = 10$ THz occurs at $\epsilon_s(q, r) = -0.58 + 5.7i$, as shown in Figures 4b and 5b. It is the same for the Core-shell model's results, as shown in Figures 4a1,b1 and 5a1,b1. Table 1 lists the values of the equivalent dielectric functions ϵ_s and ϵ_{cs} , which produce the extinction amplification resonance of charged Martian dust particles at different infrared frequencies $f = 3$ THz, 10 THz, 30 THz and 300 THz.

Table 1. The values of the equivalent dielectric functions, ϵ_s and ϵ_{cs} , that produce the extinction amplification resonance of charged Martian dust particles for $f = 3$ THz, 10 THz, 30 THz, and 300 THz.

Frequency	Surface Conductivity Model		Core-Shell Model	
	Radius	ϵ_s	Radius	ϵ_{cs}
$f = 3$ THz	1.2 μm	$1.55 + 6.3i$	1.2 μm	$0.57 + 8.66i$
$f = 10$ THz	0.5 μm	$-0.58 + 5.75i$	0.5 μm	$-2.9 + 7.32i$
$f = 30$ THz	0.17 μm	$-1.64 + 4.5i$	0.165 μm	$-4.9 + 5.3i$
$f = 300$ THz	0.02 μm	$-2.15 + 0.09i$	0.019 μm	$-3.8 + 0.16i$

As reflected by Figures 4 and 5, the extinction amplification factors given by the two models, including the positions of extinction amplification on both the q -axis and r -axis, are consistent with each other at the infrared band 3 THz–300 THz. For larger Martian dust particles (e.g., $r = \sim 10$ μm), the extinction amplification factors are equal to 1, which means the charged particles' extinction coefficients are equal to the extinction coefficient of the uncharged particle, because the absorption of radiant flux by the surface net charges becomes much less than the absorption by the particle's bulk, with decreasing surface-to-bulk ratio. The extinction amplification resonance occurs on particles that are much smaller than the wavelength, as indicated by Figure 5a–d. For example, the position of amplification resonance is at $r = \sim 1.2$ μm for an infrared frequency/wavelength of 3 THz/100 μm , as displayed in Figure 5a,a1. Figure 5b,b1 indicates that the amplification peaks shift to $r = \sim 0.5$ μm for an infrared frequency/wavelength of 10 THz/30 μm . With the increase in frequency, the extinction amplification peak shifts toward the low end of r -axis, as indicated by Figure 5a–d,a1–d1. For $f = 300$ THz, a strong extinction amplification resonance occurs on small particles whose radii are about 20 nm. This strong extinction amplification of charged particles has been found in previous studies (e.g., refs. [46–50]).

It is well known that for particles much smaller than the wavelength of an incident wave, the particles' scattering fields can be approximated well by the Rayleigh scattering theory (i.e., dipole approximation) [28]; therefore, for a given fixed q of a particle much smaller than the wavelength, the results of the two models should approach the same limit case, the dipole approximation at the infrared band 3–300 THz. In conclusion, for the charged Martian dust particles at an altitude from the near-surface level to over 40 km, where $r = 0.3$ μm –2 μm , either the surface conductivity model or the Core-shell model can be employed to calculate their extinction coefficients at the infrared band 3 THz–300 THz, without resulting in significant difference. Figure 6 shows the difference between the two models for $f = 30$ THz and 300 THz, where $A = \frac{Q_{ext}^{c,SD} - Q_{ext}^{c,CS}}{Q_{ext}^{c,SD}} \times 100\%$. Here, $Q_{ext}^{c,SD}$ and $Q_{ext}^{c,CS}$ stand for the extinction coefficients given by the surface conductivity model and the Core-shell model, respectively.

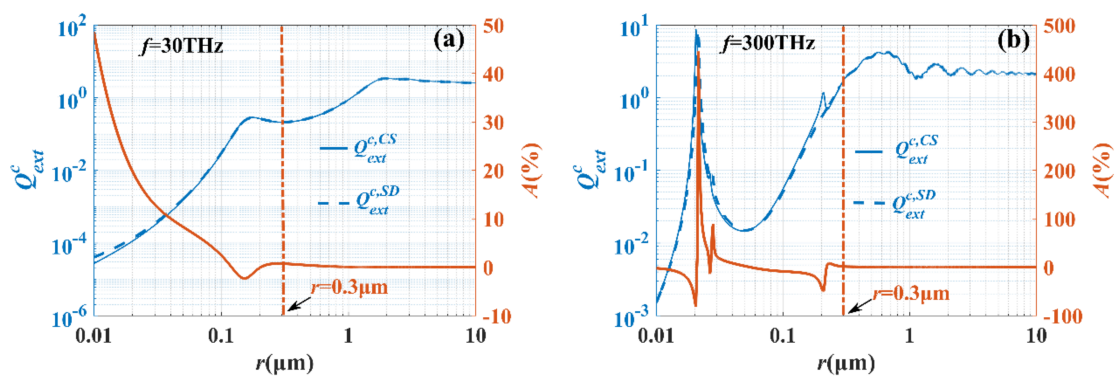


Figure 6. Difference between the surface conductivity model (dashed line) and the core-shell model (solid line) for (a) $f = 30$ THz and (b) $f = 300$ THz.

For a given fixed frequency, the excitation frequencies of the surface conductivity model and the Core-shell model (i.e., ω_s and ω_c) are functions of particle radius r and the particle's total net charge q . ω_s and ω_c for $f = 300$ THz are shown in Figure 5e,e1, which indicate that the extinction amplification resonances occur as the frequency of the incident wave (i.e., ω) approaches the extinction frequency ω_s or ω_c . That means the excitation frequency, ω_s or ω_c , can be considered as a characteristic of a charged particle.

As reflected in Figure 5a–d, to the left side of the extinction amplification resonance peak, the amplification factor Q_{ext}^c/Q_{ext}^0 can become less than 1 with the decrease of the particle radius. For example, Figure 5a,a1 shows that Q_{ext}^c/Q_{ext}^0 is less than 1 for $r < \sim 0.46 \mu\text{m}$ at frequency 3 THz. That means the extinction of the charged particle is reduced, compared with the uncharged particle, which is called extinction reduction here. With increasing frequency, the location of extinction reduction shifts toward the left end of the r -axis, as indicated by Figure 5 a–c. The location of extinction reduction shifts to $r < \sim 0.3 \mu\text{m}$ for $f = 10$ THz and to $r < \sim 0.1 \mu\text{m}$ for $f = 30$ THz.

Figure 7 shows the shift of the amplification resonance lines of charged particles on the r - q parameter plane. The thick solid lines, labelled as resonance lines, indicate the extinction amplification resonance positions for infrared frequencies 3 THz, 10 THz, 30 THz, and 300 THz. The dashed green line represents the saturation charge $q_{max}(r)$ of a particle of radius r . As already found in Figures 4 and 5, the resonance line shifts toward the upper-left of the r - q parameter plane as the frequency f gets higher. The intersection points of the resonance lines and the q_{max} -line correspond to the values of the particle radii listed in Table 1 (i.e., $1.22 \mu\text{m}$, $0.47 \mu\text{m}$, $0.16 \mu\text{m}$, and $0.02 \mu\text{m}$). These are the locations of extinction amplification peaks for $f = 3$ THz, 10 THz, 30 THz, and 300 THz, respectively, as shown in Figure 5a–d or Figure 5a1–d1. Taking the case of $f = 3$ THz as an example, one can see from Figure 7 that for a particle with $r \leq 1.22 \mu\text{m}$, the extinction amplification resonance can occur as the particle is charged. For $r > 1.22 \mu\text{m}$, however, the extinction amplification resonance cannot be reached, even if the particle's net charge gets saturated.

A shaded region is shown in Figure 7, which is between the q_{max} -line and the thin solid line. On the thin solid line, Q_{ext}^c/Q_{ext}^0 equals 1.0 for $f = 30$ THz, whereas for (r, q) , which lies in the shaded region, the factor Q_{ext}^c/Q_{ext}^0 is less than 1. One can call this region the *extinction reduction region* of 30 THz. For a given fixed frequency, one can obtain its extinction reduction region. With the increase of frequency, the lower boundary of the extinction reduction region also shifts toward the upper-left of the r - q parameter plane.

The scatters shown in Figure 7 (i.e., the solid stars and triangles) are the experimental data of the radii r and the total net charge q of charged Martian dust particles [22,24,31]. Some of them lie close to the extinction amplification resonance lines. One data point even lies in the shaded region, the extinction reduction region of 30 THz. On the other hand, the effective particle radii of Martian atmospheric dust particles is $0.3 \mu\text{m}$ – $1.8 \mu\text{m}$ at an altitude from near surface to over 40 km [8,40–43]; they cover the particle radii

labelled by the vertical dashed lines in Figure 7, which are the locations of extinction amplification resonance.

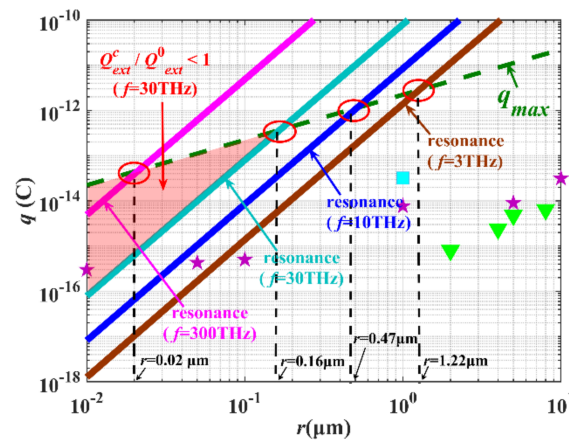


Figure 7. The shifts of the extinction amplification resonance and extinction reduction on the r – q parameter plane, as the frequency varies from 3 THz to 300 THz. Here, r denotes the radius of particle and q the particle’s total net charge. The thick solid lines (labelled with “resonance”) represent the positions of extinction amplification resonance. The shaded triangle region indicates positions, where the extinction reduction ($Q_{ext}^c / Q_{ext}^0 < 1$) occurs for $f = 30$ THz. The dashed line denotes the saturation charge $q_{max}(r)$ of a particle of radius r .

3.2. Effect of the Net Charge on the Martian Dust Optical Depth

The dust optical depth (DOD) of Martian atmosphere is used to construct the dust distributions on Mars. The dust distributions are used in the simulation of Mars’ climate, to predict the conditions for the entry, descent, and landing of Mars exploration landers [1]. As mentioned above, the extinction of charged Martian dust particles can be changed. Figure 8 shows that the Martian DOD at the infrared band 3 THz–300 THz can also be changed as the dust particles are charged. Here, τ^0 and τ^c denote the calculated DOD of uncharged and charged Martian atmospheric dust, respectively. The DOD at a given altitude is calculated for the vertical path from the atmosphere top to the given altitude. The vertical profile of Martian dust particle number density used for the calculation of DOD is $N \sim N_b \tau_s e^{-z/H}$ (see ref. [44]), where N_b is the number density at the surface. H stands for the scale height of the Martian atmosphere, which has an average value of 11.1 km and τ_s is the DOD of Martian atmosphere, which is the observed data of NASA’s Mars rovers Spirit and Opportunity. For $\tau_s = 6$, the particle density near the ground ($z = 0$) is ~ 140 (unit in $1/\text{cm}^3$) [44].

Compared with uncharged dust particles, charged Martian dust particles can produce larger dust optical depth. For infrared channels at 3 THz–10 THz, the observed DOD at the near surface (0–10 km) can be enhanced by $\sim 80\%$ (Figure 8b), as the dust particles are charged. At higher altitudes (e.g., 40–50 km), the enhancement of the DOD can increase to $\sim 200\%$ (Figure 8b), because the dust particles at high altitudes have smaller radii. With the increase in frequency (e.g., at 30–300 THz), the enhancement of the DOD caused by charged dust particle drops to $< 5\%$ at an altitude of 0–10 km (Figure 8c,d).

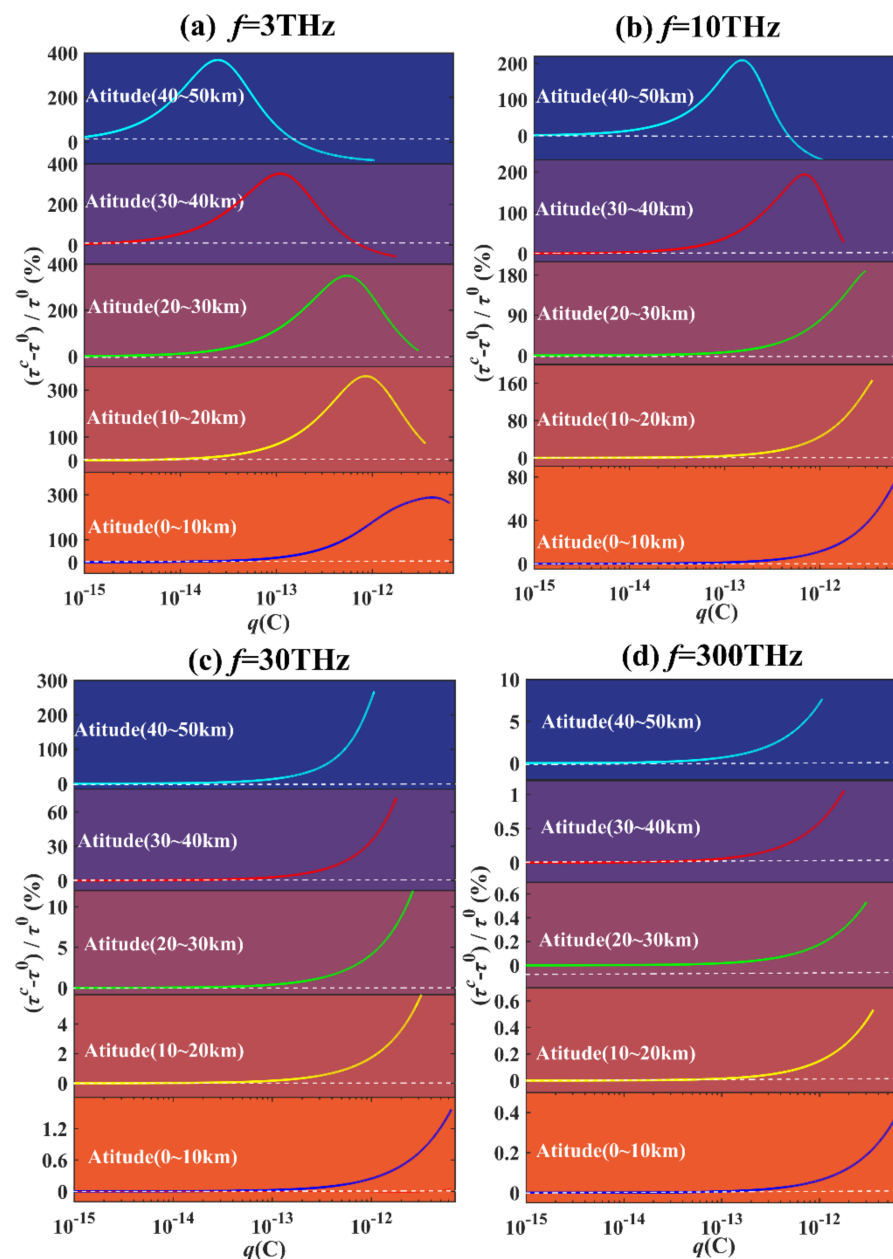


Figure 8. Difference between the DOD of charged Martian dust particles and uncharged dust particles for frequencies (a) $f = 3$ THz, (b) 10 THz, (c) 30 THz, and (d) 300 THz. Here, q is the total net charge of the charged Martian dust particle. The effective particle radii adopted in the calculation of DOD are $\sim 1.8 \mu\text{m}$ at an altitude of 0–10 km, $\sim 1 \mu\text{m}$ at an altitude of 10–20 km, $\sim 0.85 \mu\text{m}$ at an altitude of 20–30 km, $\sim 0.5 \mu\text{m}$ at an altitude of 30–40 km, and $\leq 0.3 \mu\text{m}$ at an altitude above 40 km (see ref. [8]).

Theoretically, by treating Martian dust as neutral particles, the dust particle density constructed from the observed DOD might be overestimated, due to the enhancement of the DOD by charged Martian dust particles. In fact, this overestimation of Martian dust particle density was encountered in research of the Martian atmosphere. A model simulation of Martian atmospheric dust distribution was performed using the vertical dust profile constructed from the TES dust optical depth data; it was found that the model prediction of dust density at altitudes above 15 km are higher than the dust density observed by NASA's Mars rover Phoenix [1]. The reason might be that the construction of dust distribution from the TES dust optical depth did not consider the enhancement of the DOD by charged particles. Here, by counting the enhancement of the DOD by charged

dust particles, the model prediction of the vertical dust density profile of the Martian atmosphere, denoted as N_0 (the line in blue, Figure 9), is corrected and compared with the observation data of Phoenix (i.e., the line in black), as shown in Figure 9. The corrected dust density, denoted as N_c (the line in red), is in better agreement with the observation data of Phoenix, especially at altitudes above 20 km. In our calculation, it is assumed that the dust optical depth was obtained by the TES channel of 10 THz/30 μm . The Martian dust surface potential is $V_s = 5$ kV [21], which is used to estimate the total net charge of the Martian dust particles. Given a fixed known DOD at an altitude, the correction factor can be determined, namely, $N_c/N_0 = Q_{ext}^0/Q_{ext}^c$. At higher altitudes, the overestimation is more significant. For example, N_0/N_c increases from 1.08 at an altitude of 20 km to ~ 1.4 at 34 km. Of course, further research is needed to understand how charged Martian dust particles affect the DOD.

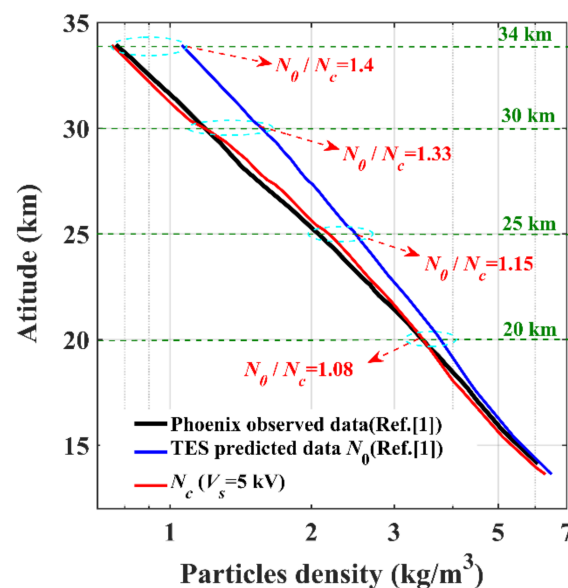


Figure 9. Vertical dust density profile of Martian atmosphere. Here, N_0 is the model prediction based on TES dust optical depth, given by ref. [1]. N_c denotes the corrected vertical profile of dust particle density, by counting in the contribution of the particles' net charges. The line in black is the observed data of Phoenix (see ref. [1]).

4. Conclusions

The extinction properties of charged Martian dust particles at different altitudes are studied by the Mie scattering theory, in which two models, the surface conductivity model and the Core-shell model, are adopted for the description of charged dust particles. It is found that the two models give consistent extinction data of charged Martian dust particles at an altitude from near surface (0–10 km) to above 40 km, for the infrared band 3 THz–300 THz, which covers the infrared channels of Mars orbiters (e.g., TES and MERIS); therefore, one can use either of the two models to study the extinction properties of charged Martian dust particles.

The extinction coefficients of Martian dust particles and the dust optical depth (DOD) data of infrared channels onboard Mars orbiters can be amplified as the dust particles are charged. The calculated results indicate that the extinction amplification usually occurs on particles that are much smaller than the infrared wavelength. With the increase in infrared frequency from 3 THz to 300 THz, the positions of the extinction amplification resonance of charged Martian dust particles shift toward the upper left of the r – q parameter plane. Here, r is the particle radius and q the particle's total net charge; therefore, the amplification of extinction and the DOD is more significant at high altitudes than at low altitudes. However, at 3 THz–10 THz, the amplification of the DOD at the near surface altitude (0–10 km) can still be significant.

That means the amplification of the DOD, which is caused by charged Martian dust, should be concerned when constructing the vertical dust profile from observed DOD data. By considering the enhancement of the DOD, the overestimated vertical dust profile constructed from DOD data (e.g., the TES data), can be corrected; however, the correction of vertical dust profiles constructed from DOD data needs further study, which should be of benefit to the model predictions of entry, descent, and landing conditions for future Mars missions.

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