

Orbit Predictions for Space Object Tracked by Ground-Based Optical and SLR Stations

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Abstract: In many cases, we have few optical observations over a short time span, and most of the information generated is too limited to compute a full orbit according to the angles-only method. This study aims to develop a mathematical model to determine the precise orbit from the optical observation data by the least squares method. We have used a set of the Global Navigational Satellite Systems, which are tracked by the Optical Satellite Tracking Station (OSTS) at the National Research Institute of Astronomy and Geophysics (NRIAG), Egypt, to access high-quality predictions for the orbits. We analyzed the orbit predictions from the observations of these satellites that are tracked from seven world stations using the laser ranging method, and the obtained results are compared with orbital elements produced using the Two-Line Element (TLE). The results showed that the orbital prediction accuracy differs for optical observations from laser observations because of the inaccuracy of the NORAD catalog information used; this is due to the difference between the time of observation and the epoch time of TLE.

Keywords: global navigational satellites; satellites laser ranging; orbit prediction; orbital propagation



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1. Introduction

Satellite Laser Ranging (SLR) is a technique for measuring the range of satellites, which represents the distance between the satellite and the tracking station. The SLR is considered to be one of the most accurate methods for tracking artificial earth satellites [1,2]. Galileo, the European Global Navigation Satellite System (GNSS), has 26 satellites in orbit. The International Laser Ranging Service introduces some SLR observations for Galileo satellites. They are equipped with a satellite laser ranging retroreflector consisting of 84 and 60 corner cube retroreflectors that allow the assessment of the orbit accuracy in space [3].

The laser tracking sensors transmit laser pulses and calculate the range by collecting the returned signals, where the angle data are generated by collecting the sunlight-illuminated satellites with about 2–5 arc second accuracy. The range measurement is collected by measuring the two-way time of flight with laser retroreflectors. The SLR is used in many applications, including space geodesy and geodynamics [4].

When the orbit is predicted using the North American Aerospace Defense Command (NORAD), the resultant pointing errors are usually in the order of tens to hundreds of arc seconds [5]. While we must have sufficiently accurate orbit predictions to achieve high efficiency of satellite laser tracking (better than 20 arc seconds) because of the narrow laser beam. Cordelli et al. [6] investigated the improvements to orbit predictions (OP) accuracy by combining the angle data from both the SLR and the Optical Ground Sites. Li et al. [7] have shown that using the Two-Line Elements to calculate positions every 10 min is sufficient to help achieve orbit determination (OD) convergence in the case where only the angular data over a short orbit arc are available. Lejba and Schillak [8] determined the positions and velocities of four SLR stations (Australian Yarragadee, North American Greenbelt, and two

European, Graz and Herstmonceux) from the orbits of LEO satellites. Where the orbital computations were performed on the basis of the observations of 20 SLR stations and then compared the final results with the LAGEOS data. Schillak et al. [9] used the results of the laser observations from the LARES satellite to determine the station coordinates, and they compared the results with the LAGEOS-1 and LAGEOS-2 satellites. The earth tides are generated by the gravitational effect of the Moon and the Sun, which cause variations in the mass distribution of the Earth. The estimated parameters Love and Shida numbers for second-degree tides are based on the analysis of the SLR data, where the values of these parameters are equal to 0.6140 ± 0.0005 for Shida and 0.0876 ± 0.0002 for Love (Jagoda and Rutkowska [10], Rutkowska and Jagoda [11]). Sośnica et al. [12] and Sośnica [13] studied ocean tide models based on the analysis of LAGEOS satellite altimetry data the results show that differences between LAGEOS orbits derived using modern gravity field models are rather small, while the more important role in the quality of resulting LAGEOS orbits result from ocean tides, station loading displacement corrections, and technical station errors. In this study, we combine angle and laser ranging data to investigate improvements in the accuracy of the orbit prediction using the OSTS station and some other laser ranging stations from the International Laser Ranging Service. Several results of Galileo orbits were demonstrated under a limited observation environment to verify the OP accuracy through the combination of angle and laser ranging data from several sites.

2. Sites Description

The optical characterization of satellites in different orbits is challenging because satellites in some orbits have low signal levels that typically require long exposure times although background stars will appear as streaks in the images, some satellites remain fixed in the field of view during the observation time, such as in GEO, and others appear as a movement trail, such as GNSS satellites, as shown in Figure 1. The observations are taken by selecting a field of view with a specific right ascension and declination, corresponding to a fixed solar phase angle (observatory–target–Sun), which remains outside of the Earth’s shadow. For image processing, we used APEX II software to make astrometry photometry, and identify the satellite in this image [14].

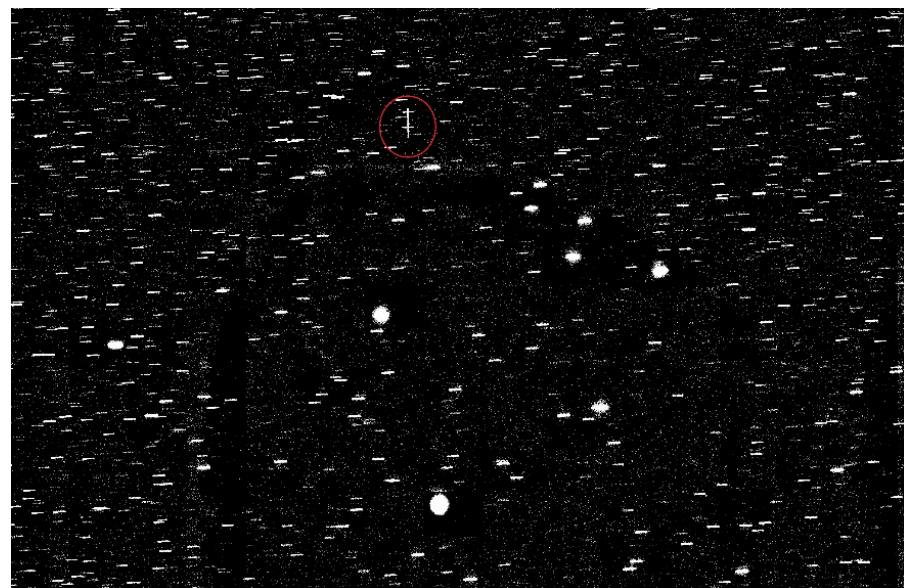


Figure 1. Examples of GEO and GNSS (inside red circle) satellites during observation from the OSTS station.

In this section, we will explain the design of the fundamentals of the electro-optical system [15] and SLR stations.

2.1. Optical Satellite Tracking Station

Electro-optical (EO) is used in this study from the Optical Satellite Tracking Station (OSTS) at Kottamia observatory, National Research Institute of Astronomy and Geophysics (NRIAG), Egypt, as shown in Figure 2, and more details are given in Table 1 [14,16]. The OSTS is a part of the International Scientific Optical Network (ISON) [17].



Figure 2. The OSTS station—NRIAG, Egypt.

Table 1. The fundamentals of the optical Satellite Tracking Station (OSTS).

Site Name	OSTS—NRIAG, Egypt
Latitude (deg)	29.933°N
Longitude (deg)	31.8823°E
Elevation (m)	470
Telescope Series	Celestron 11" Schmidt Astrograph
Telescope Aperture	280 mm
Telescope Focal Length	620 (mm)
Mount Type	CGE Pro Equatorial
CCD device	FLI MicroLine ML11002 Monochrome
Field of View (deg)	2.3 × 3.4

2.2. The Satellite Laser Ranging Stations

The Satellite Laser Ranging technique aims to measure the distance between the SLR station and the center of mass of the satellite equipped with a retroreflector. This is achieved by measuring the two-way time of flight of light pulses between the satellite and station. This requires correcting for the distance measured to the satellite for the effects of the decrease in the speed of light and the difference between the curved and straight ray paths (Jagoda et al. [18]). There are several factors that must be taken into consideration, such as the distance from the retroreflector to the mass center of the satellite, impact of satellite motion, Earth rotation, and relativistic effects (Jagoda et al. [18], Schillak [19], Schillak et al. [20]).

The laser observation equation for the i -th laser measurement of a distance to the satellite mass center can be written as (Jagoda et al. [18])

$$\frac{(\Delta t_i - \Delta c)C}{2} + \Delta a_i + \Delta m_c + \Delta r_{rc} + \Delta s_p + \Delta e_r - \Delta r_{bi} - \varepsilon_i - r_{c,i} = 0 \quad (1)$$

where Δt_i is the two-way time interval of flight of light pulses between the station and satellite for the i -th measurement, Δc is calibration correction, C is the speed of light, Δa_i and Δm_c are tropospheric delay for the i -th measurement (due to the twice passage of the light pulse through the atmosphere) and correction for the center of the satellite mass (25.1 cm for LAGEOS satellites), respectively, Δr_{bi} is the range bias of the observation, Δr_{rc} is the relativistic range correction, Δs_p and Δe_r are the impact of satellite motion and Earth rotation, respectively, ε_i is a random error for the i -th measurement, and $r_{c,i}$ is the distance calculated for the i -th measurement.

We have taken our data from different SLR stations, Mendeleevo, Moscow, Russia (1874), Zelenchukskaya Karachaevo, Cherkesi, Russia (1889), Changchun, Jilin, China (7237), Fangshan district Beijing, China (7249), Canberra Australian Capital Territory, Australia (7825), Potsdam Brandenburg, Germany (7841), and Matera Basilicata Italy (7941). The specifications contain the longitude, latitude, and height above mean sea level and these important parameters are given in Table 2.

Table 2. Laser satellite tracking stations (LSR).

Pad ID	1874	1889	7237	7249	7825	7841	7941
Latitude (deg)	56°N	43°N	43.79°N	39.61°N	35.31°S	52.38°N	40.65°N
Longitude (deg)	37°E	41°E	125.44°E	115.9°E	149.01°E	13.06°E	16.71°E
Elevation (m)	229	1155	274.9	82.300	805	123.5	536.9
Aperture (m)	0.25	0.25	0.60	0.60	1	0.44	1.50
Field of View (deg)	0.32	0.32	0.05	0.30	0.07	-	0.016
Magnitude (mag)	15	15	13	13	10	12	15
Transmit Efficiency	0.5	0.6	-	0.70	0.95	0.8	0.75
Receive Efficiency	0.5	0.5	-	0.70	0.95	0.4	0.87
Laser System Information							
Laser Type	ND: YAG						
Final Beam Diam (m)	0.03	0.03	0.06	0.01	1	0.15	0.010
Receiver System							
Wavelength (nm)	532	532	532	532	532	532	532
Field of View (")	20–120	-	20–420	-	12	10–60	60

3. Precise Orbit Determination

The Initial Orbit Determination (IOD) problem can be solved for the angles only by the Laplace, the Gauss, the Double-r, and the Gooding methods. These methods depend on choosing only three pairs of observations, with many observations wasted. For the angles-only method, we used either the topocentric right ascension, declination, and time of objects in the case of the optical system or azimuth and elevation in the case of SLR stations. From the angular measurements, we can form the vector of direction cosine with respect to a space-fixed reference frame. If we know the slant range and observation site's location, we could easily get an expression for the satellite's position vector [21–23].

To reduce the influence of measurement error and improve the accuracy of the initial orbit determination, the improved Laplace method utilizes all observations. However, the improved Laplace method cannot give the optimal IOD solution. In this work, we

used the least squares method to solve IOD problems and refine them by minimizing the measurement errors directly. The Laplace-LS IOD is the direct method to minimize the errors between the virtual and true observations [23].

Assuming we have n groups of noisy measurements of right ascensions (α) and declination (δ)

$$\tilde{\mathbf{y}}_j = (t_j, \tilde{\alpha}_j, \tilde{\delta}_j), \quad j = 1, 2, 3, \dots, n \quad (2)$$

The actual observation sequence with measurement noise is defined as

$$\tilde{\mathbf{Y}} = [\tilde{\alpha}_1, \tilde{\delta}_1, \tilde{\alpha}_2, \tilde{\delta}_2, \dots, \tilde{\alpha}_n, \tilde{\delta}_n]. \quad (3)$$

The theoretical observation sequence of the true orbit (without noise) is defined as

$$\bar{\mathbf{Y}} = [\bar{\alpha}_{t1}, \bar{\delta}_{t1}, \bar{\alpha}_{t2}, \bar{\delta}_{t2}, \dots, \bar{\alpha}_{tn}, \bar{\delta}_{tn}]. \quad (4)$$

Supposing the measurement errors follow a normal distribution with variance σ^2 , the observed ascensions and declinations have the following statistical characteristics

$$\tilde{\alpha}_j = N\left(\tilde{\alpha}_j, \frac{\sigma^2}{\cos^2 \tilde{\delta}_j}\right) \quad (5)$$

$$\tilde{\delta}_j = N(\tilde{\delta}_j, \sigma^2) \quad (6)$$

At any initial epoch t_0 and state x_0 , there is a theoretical observation sequence calculated by the dynamical model and the observation model.

$$\mathbf{Y}(x_0) = [\alpha_{t1}(x_0), \delta_{t1}(x_0), \dots, \alpha_{tn}(x_0), \delta_{tn}(x_0)]. \quad (7)$$

Now, we get the difference between the given orbit x_0 and the actual orbit from the weighted square sum of the errors between the theoretical observations and the actual observations.

$$\sum_{i=1}^m \left(\frac{(\alpha_{ti}(x_0) - \tilde{\alpha}_{ti})^2}{\frac{\sigma^2}{\cos^2 \tilde{\delta}_{ti}}} + \frac{(\delta_{ti}(x_0) - \tilde{\delta}_{ti})^2}{\sigma^2} \right). \quad (8)$$

The average angle error can be determined by the root mean square error (RMSE) of the errors (defined as the target function $J(x_0)$), which indicates the average angle error.

$$\min J(x_0) = \frac{1}{2\sqrt{m}} \sqrt{\sum_{i=1}^m (\alpha_{ti}(x_0) - \tilde{\alpha}_j)^2 \cdot \cos^2 \tilde{\delta} + (\delta_{ti}(x_0) - \tilde{\delta}_i)^2}. \quad (9)$$

After some process, we can estimate the Cartesian components (position and velocity) of the orbit state [23]. From state vectors, we can directly solve for the classical orbital elements, as follows in Appendix A.

4. Results and Discussions

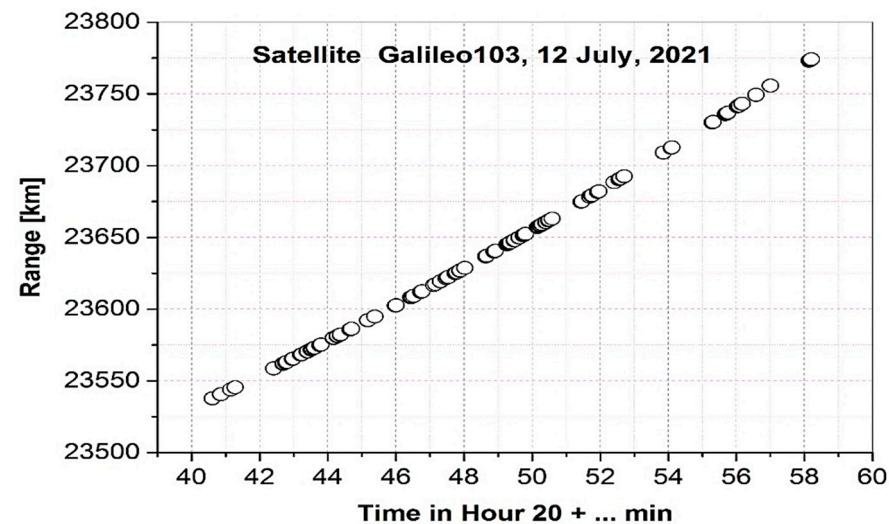
Now, we will determine the precise orbit using five space object targets in MEO that were selected (NORAD catalog ID 37139, 32393, 40315, 38857, and 37867). Table 3 shows the total number of observations from optical and SLR stations.

Table 3. The observations from optical and SLR stations.

NORAD ID	8 July 2021		9 July 2021		10 July 2021		11 July 2021	
	Optical	SLR	Optical	SLR	Optical	SLR	Optical	SLR
37139	-	-	OSTS	1874	-	-	-	-
32393	OSTS	-	-	-	-	7237	-	1889
40315	-	7249 7941	-	7249 1889	-	7825		OSTS
38857	-	7249	OSTS	-	-	-		7841
37867	OSTS	-	-	-	-	-	-	-

4.1. Orbital Element of Space Object Descriptions

The primary goal of Satellite Laser Ranging is the measurement of the time required for pulses emitted by a laser transmitter to travel to a satellite and return to the transmitting site, which is the range, as shown in Figure 3 for the satellite Galileo 103, as an example, which was observed on 12 July 2021.

**Figure 3.** The range of the satellite Galileo 103 observed on 12 July 2021.

The tracking object observed from different sites during the period from 8 to 11 July 2021 are given in Table 3. The description of the orbital element used for the real observation to demonstrate the OP accuracy and the proposed initial estimation is given in Table 4.

4.2. The Numerical Simulations for Optical and SLR Observation

In this subsection, we used the least-squares differential correction algorithm to determine the orbit of the satellites from the simulation of position and velocity measurements generated along the reference orbit. We developed a mathematical model using the MATLAB® package; it is applied to Optical and SLR observation parameters. The precise orbit determination is shown in Table 5. The table shows the orbital elements for the semi-major axis (a), inclination angle (i), and eccentricity (e) for the optical observation and the data available from NORAD and shows the difference between them; Δa , Δe , and Δi . The accuracy of the semi-major axis is about 600 to 2000 m, the inclination is about 10^{-3} , and the angle of eccentricity is about 10^{-4} .

Table 4. The description of space object targets in MEO.

NORAD ID	Object	Eccentricity	Perigee (km)	Apogee (km)	Semi-Major Axis (km)	Inclination Degree
37139	COSMOS 2464 (GLONASS 122)	0.0026558	19,062	19,198	25,508	64.044
32393	COSMOS 2434 (GLONASS107)	0.0001098	19,127	19,133	25,508	64.445
40315	COSMOS 2501 (GLONASS134)	0.0018922	19,082	19,178	25,508	63.755
38857	GALILEO-FM3 (Galileo103)	0.0003213	23,213	23,232	29,600	55.059
37867	COSMOS 2476 (GLONASS 128)	0.0020470	19,078	19,182	25,508	64.580

Table 5. Optical observation parameters.

Satellite Name	Time of Observer	Type of Data	Classical Orbital Elements					
			a (km)	Δa (km)	e	Δe	i (Degree)	Δi (Degree)
COSMOS 2476 (37867)	8 July 2021	TLE	25,508.24	0.69	0.0021	0.0002	64.575	0
		Obs	25,507.55		0.0023		64.575	
COSMOS 2434 (32393)	8 July 2021	TLE	25,510.57	0.98	0.0002	0.0001	64.455	0.002
		Obs	25,511.55		0.0001		64.457	
COSMOS 2464	9 July 2021	TLE	25,508.31	1.47	0.0026	0	64.054	0.001
		Obs	25,506.84		0.0026		64.053	
GALILEO-FM3 (38857)	9 July 2021	TLE	29,600.35	2.28	0.0004	0	55.054	0.003
		Obs	29,598.07		0.0004		55.051	
COSMOS 2501 (40315)	11 July 2021	TLE	25,508.24	1.28	0.0018	0	63.764	0
		Obs	25,509.52		0.0018		63.764	

On the other hand, the data given in Table 6 give the accuracy, as calculated from SLR observations in the semi-major axis ranging from 40 to 400 m. We have the observation of the object COSMOS 2464 on 9 July 2021 from optical and laser stations. The object COSMOS 2501 is tracking from two laser stations on the 8 and 9 of July 2021. The accuracy of the observation from the laser stations is nearly similar to the same results of different observation times. As we show, there is a clear difference in the results when comparing the two monitoring systems. This difference is because, in the case of the ground-based optical system, the space object appears as a trail and not as a point, so we take the start point and the end point for every shot; this leads to an error in angular measurements, and we are trying to fix that by fitting a line or polynomial to the trail to obtain more data input. This is because the OSTs has an equatorial mount, and the right ascension axis is fixed. While all SLR sensors are alt-azimuth mounts that have the rate of change on their axis.

Table 6. SLR observation parameters.

Stations ID	Time of Observer	Type of Data	Classical Orbital Elements					
			a (km)	Δa (km)	e	Δe	i (Degree)	Δi (Degree)
COSMOS 2501 (40315)	8 July 2021	TLE	25,508.25	0.03	0.0018	0	63.764	0
		SLR7249	25,508.22		0.0018		63.764	
		SLR7941	25,508.24		0.0018		63.764	
	9 July 2021	SLR7249	25,508.21	0.04	0.0018	0	63.764	0
		SLR1889	25,508.24	0.01	0.0018	0	63.764	0
	10 July 2021	SLR7825	25,508.21	0.04	0.0018	0	63.764	0
COSMOS 2464 (37139)	9 July 2021	TLE	25,508.31	0.05	0.0026	0	64.054	0
		SLR	25,508.26		0.0026		64.054	
COSMOS 2434 (32393)	10 July 2021	TLE	25,508.13	0.02	0.0002	0	64.455	0
		SLR.7237	25,508.11		0.0002		64.455	
	11 July 2021	SLR1889	25,508.13		0.0002	0	64.454	0
GALILEO-FM3 (38857)	8 July 2021	TLE	29,600.35	0.48	0.0004	0	55.054	0.03
		SLR7249	29,599.87		0.0004		55.051	
	11 July 2021	SLR7841	29,600.36		0.0004	0	55.054	0

The Figures 4–8 show the relationship between the position residuals and the time. The effects of perturbation on artificial satellites (Earth’s gravity, solar radiation pressure, and the third body) have been considered [9]. The model errors on relative position residuals of different examples are shown, where the green curve explains the variation in the x -axis after 3 days, the red curve shows the variation in the z -axis, and the yellow curve shows the variation in the y -axis. The results for different examples show the drift error for X residual position, Y residual, and Z residual are increases in the range of 10^{-4} km.

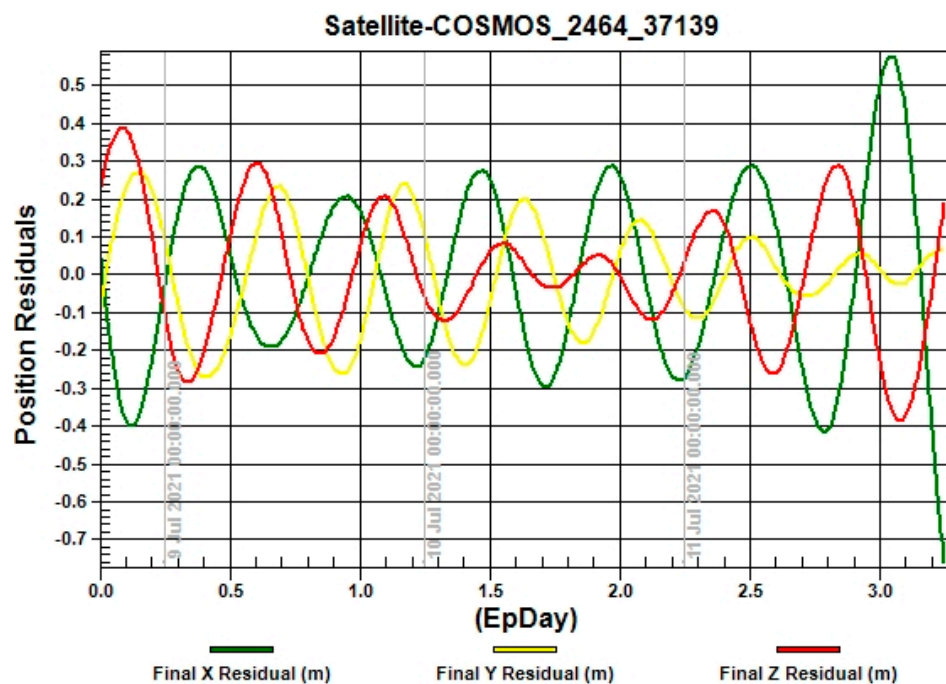


Figure 4. The relationship between the position residuals and the time for satellite “COSMOS-2464”.

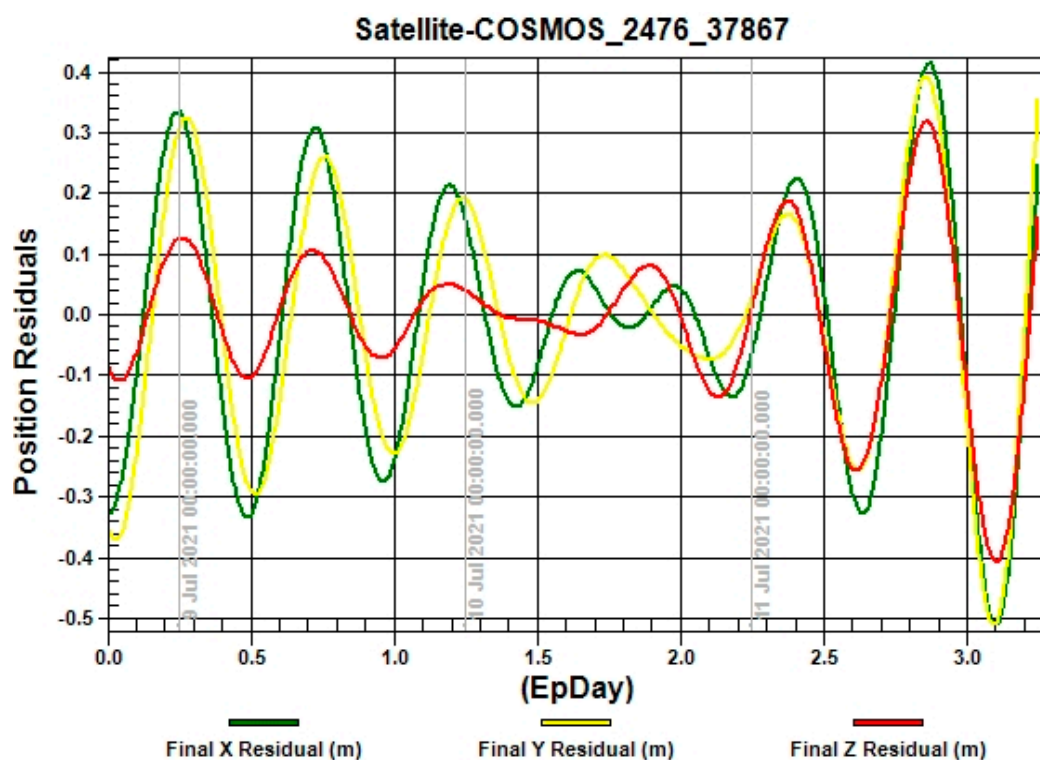


Figure 5. The relationship between the position residuals and the time for satellite “COSMOS-2476”.

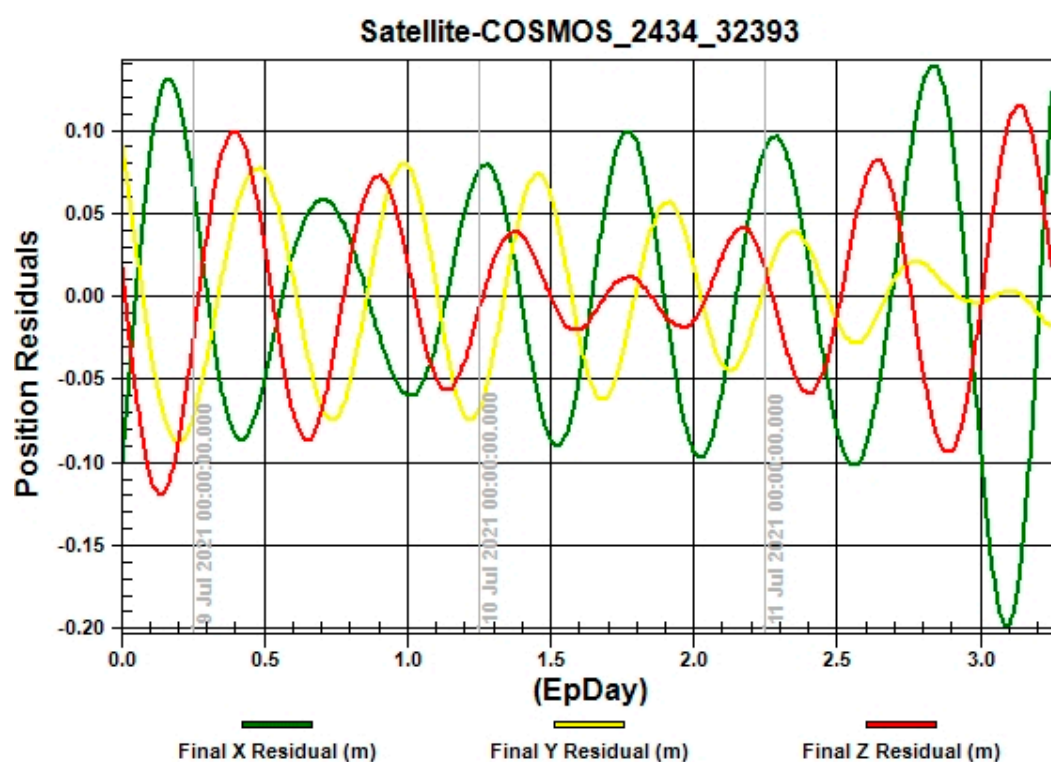


Figure 6. The relationship between the position residuals and the time for satellite “COSMOS-2434”.

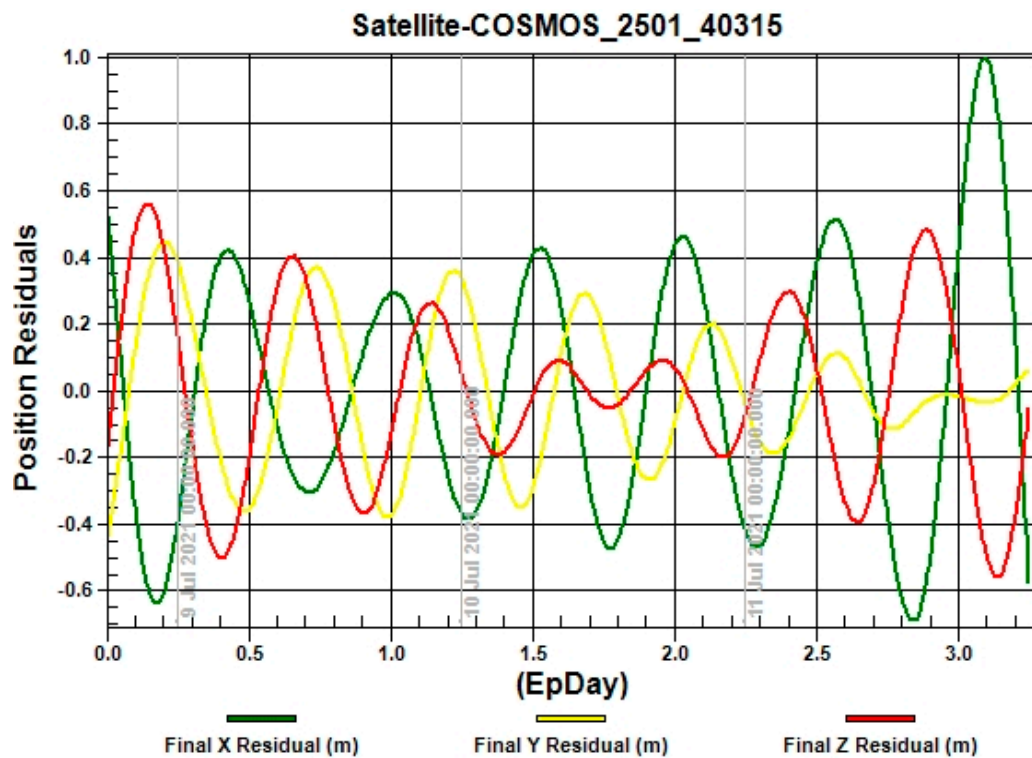


Figure 7. The relationship between the position residuals and the time for satellite “COSMOS-2501”.

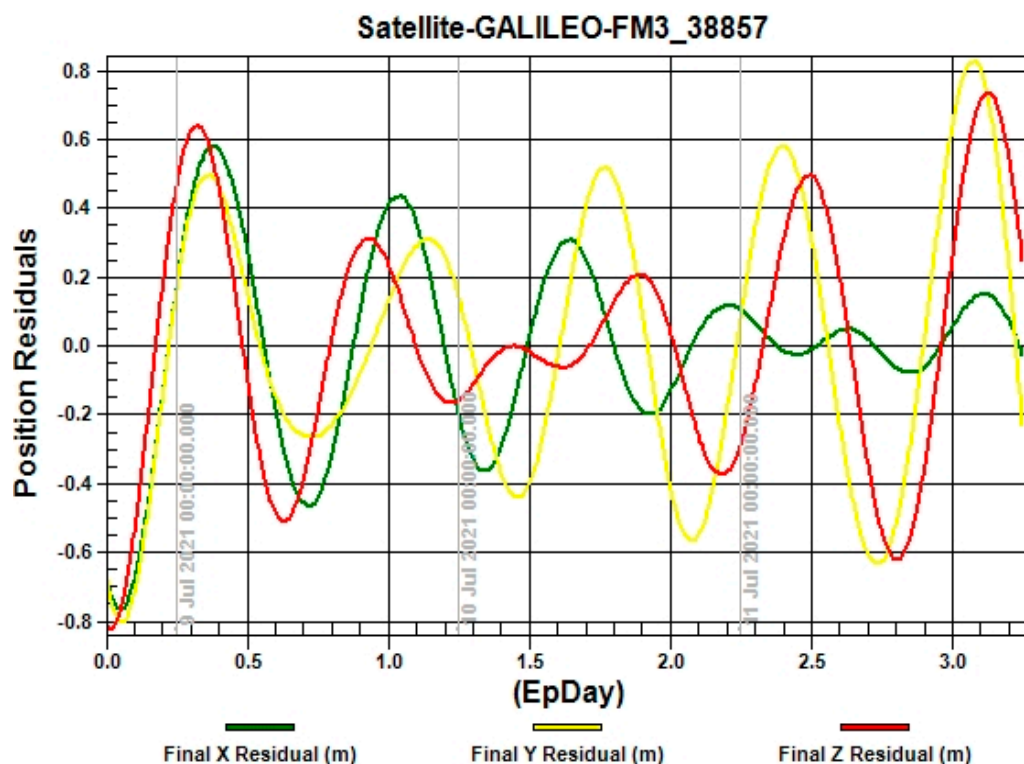


Figure 8. The relationship between the position residuals and the time for satellite “GALILEO-38857”.

5. Conclusions

In this paper, we developed a mathematical model to determine the precise orbit from optical and laser observation data. We applied our model to different Global Navigational Satellite Systems. The optical observation method is used by the Optical Satellite Tracking Station (OSTS) at NRIAG—Egypt, and the SLR observation methods are produced from

some stations of the International Laser Ranging Service for the Galileo satellites. We used the least squares method to refine the orbit derived from the IOD method; these methods depend on choosing only three pairs of observations. The cases using the angle and laser ranging data together (Cases 37139, 40315) satisfied the target OP accuracy (about 400 m at 19,000 km altitude). This is consequent to the error of OP accuracy coming from the inaccuracy of NORAD catalog information because of the difference between the time of observation and the epoch time of TLE. Compared with the TLE, our results showed that the accuracy of the calculations is better using SLR observations.

Author Contributions: A.M.A., S.K.T. and M.I.: common plan; A.M.A. and S.K.T.: the draft of the manuscript, and processed the original satellite data from OSTs; M.I., Z.L. and X.D.: provided the data from the SLR stations. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Cartesian Coordinates to Classical Orbital Elements (Elliptical Case)

We can compute orbital elements given the ECI position and velocity at time t for elliptical motion from the following equations. To compute the specific angular momentum and check for a degenerate orbit,

$$\vec{r} \times \vec{v} = \vec{h} = h_X \vec{u}_X + h_Y \vec{u}_Y + h_Z \vec{u}_Z$$

$$h = (h_X^2 + h_Y^2 + h_Z^2)^{1/2} \text{ and } (\vec{u}_X, \vec{u}_Y \text{ and } \vec{u}_Z) \text{ components of speed}$$

We compute the radius, r , and velocity, v ,

$$r = (X^2 + Y^2 + Z^2)^{1/2}$$

$$v = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2}$$

We compute the specific energy, ϵ , and verify elliptical motion,

$$\epsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

We compute the semi-major axis, a ,

$$a = -\frac{\mu}{2\epsilon}$$

We compute the eccentricity, e ,

$$e = \left(1 - \frac{h^2}{a\mu}\right)^{1/2}$$

We compute the inclination, i , ($0^\circ \rightarrow 180^\circ$),

$$\cos i = \frac{h_Z}{h}$$

We compute the right ascension of the ascending node, Ω , ($0^\circ \rightarrow 360^\circ$),

$$\Omega = \arctan\left(\frac{h_X}{-h_Y}\right)$$

We compute the argument of latitude, $\omega + \nu$, ($0^\circ \rightarrow 360^\circ$),

$$\omega + \nu = \arctan\left(\frac{\frac{Z}{\sin i}}{(X \cos \Omega + Y \sin \Omega)}\right)$$

We compute the true anomaly, ν , ($0^\circ \rightarrow 360^\circ$),

$$\cos \nu = \frac{a(1 - e^2) - r}{er}$$

If $\vec{r} \cdot \vec{v} > 0$ then $\nu < 180^\circ$. Or use

$$\nu = \arctan\left(\frac{\sqrt{\frac{p}{\mu}}(\dot{\vec{r}} \cdot \vec{r})}{p - r}\right), \text{ where } p = a(1 - e^2)$$

We compute the argument of periapse, ω ($0^\circ \rightarrow 360^\circ$),

$$\omega = (\omega + \nu) - \nu$$

We compute the eccentric anomaly, E , ($0^\circ \rightarrow 360^\circ$),

$$\tan \frac{E}{2} = \left[\frac{(1 - e)}{(1 + e)}\right]^{1/2} \tan \frac{\nu}{2}$$

where E is in the same half-plane as ν .

This equation will yield the correct quadrant for ν .

We compute the time of periapse passage, T (note that E must be in radians),

$$T = t - \frac{1}{n}(E - e \sin E), \quad n = \sqrt{\frac{\mu}{a^3}}$$

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