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Performance Evaluation and Requirement Analysis for Chronometric Leveling with High-Accuracy Optical Clocks

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Abstract: The high-precision unification of global height datum has long been a hot issue in the field of geodesy. The chronometric leveling method originates from the gravitational redshift effect of general relativity, which may provide a new solution for the unification of global height datum. The height difference between the two locations could be measured via the frequency comparison of high-precision optical clocks. We build the error model for chronometric leveling, mainly including the measurement systematic error of two optical clocks, frequency statistical error of two optical clocks, and transmission path error of optical fiber when using optical fiber as carrier. Then, we put forward the schemes to achieve chronometric leveling. The performance of the height measurement using different schemes is evaluated. Additionally, considering the requirements of geodesy, four typical height measurement accuracy factors for chronometric leveling are proposed. Meanwhile, the corresponding accuracy requirements for optical clocks and frequency transmission techniques are also given. It has important guiding significance for the unification of the global height datum and related geoscience fields using high-accuracy chronometric leveling in the future.

Keywords: relativistic geodesy; global height datum unification; chronometric leveling; performance evaluation; requirement analysis; optical clocks; frequency transmission technique; optical fiber frequency transmission

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1. Introduction

The unification of the global height datum is of great significance for international engineering constructions, geophysical and geodynamic research, and global environmental change monitoring. One of the main tasks of the International Association of Geodesy (IAG) is to establish a global height datum unification with accuracy up to 1 cm. At present, there are four main methods for the unification of the height datum: geodetic leveling, oceanographic approach, the geodetic boundary value problem (GBVP), and the Global Navigation Satellite Systems (GNSS)/geoid method [1–4]. The geodetic leveling method combines spirit leveling and gravimetry to connect the regional height datum, which can achieve sub-millimeter accuracy in a short distance. However, it has the problems of error accumulation and inability to conduct cross sea survey, and is not suitable for the connection of long-distance height datum [5]. The oceanographic approach uses the mean dynamic topology to determine the potential difference or vertical deviation between regional height datum. The accuracy of this method depends on the accuracy of the marine terrain model [6]. Because of the poor accuracy of the marine terrain model at the tide gauge station representing the starting point of the height datum, the high-precision unification of height datum should not be achieved by the oceanographic approach. The implementation of the GBVP method requires global ground gravity data, which are hard to obtain in practice. The GNSS/geoid method unifies the height datum via the global gravity field model and GNSS, whose accuracy can reach several centimeters to several decimeters. However, in the area with sparse observation data, the error can reach $\pm$1m in extreme cases [5,7,8]. In summary, the existing methods cannot achieve the unification of global height datum with an accuracy of 1 cm, which is of great significance to develop new methods to solve this problem.

Chronometric leveling is a method to determine the height difference based on the gravitational redshift effect of general relativity, which is called "relativistic geodesy" [9,10]. The gravitational redshift effect indicated that a clock at higher altitude ticks faster than one at a lower altitude [11]. When the fractional uncertainty of clocks reaches 1×10^{-18}, a height difference change of about 1 cm on the surface can be detected according to the results of frequency comparison. In the recent past, fractional uncertainty of optical clocks has entered low 10^{-18}, which can preliminarily satisfy the needs of centimeter-scale height measurements [12]. At present, the fractional uncertainty of five kinds of optical clocks has reached the order of 10^{-18} [13–19]. Especially, the optical clock developed by the National Institute of standards and Metrology (NIST) has the highest accuracy. The clock systematic fractional uncertainty has reached incredible 9.4×10^{-19}, corresponding to a height difference of 9 mm, approximately. In 2022, the gravitational redshift effect is tested in 10^{-21}, and the variation of sub-millimeter height differences can be detected [20,21]. It will be possible to realize the unification of global height datum with an accuracy of 1 cm by chronometric leveling in the future.

Researchers have made much progress using static and transportable optical clocks to adopt chronometric leveling. Benefitting from well-conditioned laboratories, the static optical clock first realized the decimeter-level and centimeter-level chronometric leveling experiments. In 2010, two static Al^+ optical clocks were connected by a 75 m long optical fiber [22]. The systematic fractional uncertainties of the two optical clocks were 8.6×10^{-18} and 2.3×10^{-17}, respectively. The height measurement result was 37 ± 15 cm, which was highly consistent with the known value of 33 cm. Takano et al. (2016) used a 15 km long urban telecommunication optical fiber to connect Sr optical clocks in two laboratories, and realized a height measurement with an uncertainty of about 5 cm under the condition of a height difference of about 148 m [23]. The systematic fractional uncertainties of the two optical clocks were 4.8×10^{-18} and 8.3×10^{-18}, respectively. Remote clock comparison reached an uncertainty of 5×10^{-17} via 1415 km of telecom fiber between Paris and Braunschweig, with negligible contributions from the frequency transfer technique [24]. Recently, the longest experimental distance of clock comparison has reached 2220 km via optical fiber, which is enough to support intercontinental height datum connection [25]. Outdoor chronometric leveling based on transportable optical clocks has also achieved certain progress. In 2018, a static optical clock and a transportable Sr clock were connected by a 150 km optical fiber in the mountainous area on the boundary of France and Italy, realizing chronometric leveling in a relatively adverse environment [26]. The height difference between the measurement results and the classical geodetic method was just about 20 cm. In 2020, a pair of high-precision transportable Sr optical clocks were connected by an optical fiber in Tokyo Skytree, successfully realizing the chronometric leveling with a vertical height difference of 450 m and an uncertainty of about 5 cm [27]. We may conclude that the optical fiber frequency transmission (OFFT) technique makes a clock comparison feasible. Combining optical clocks and the frequency transmission techniques, the unification of global height datum may be achieved with unprecedented accuracy in the future. However, the performance evaluation and requirement analysis of chronometric leveling have not been established so far.

This paper aims to analyze the error sources for chronometric leveling, evaluate the performance of different schemes, and put forward the requirements for realizing the unification of the global height datum with different accuracy via chronometric leveling. First, we build the accuracy model for chronometric leveling. The height uncertainty of chronometric leveling depends on three parts of error when using optical fiber as carrier: measurement systematic error of the optical clock, frequency statistical error of the optical clock, and transmission path error of the optical fiber. Then, we describe a recent height measurement experiment via a pair of optical clocks. The experiment illustrated the potential for chronometric leveling. Then, we put forward a variety of schemes to realize chronometric leveling based on different optical clocks and OFFT systems, which may serve the height measurement or height datum unification with different accuracy. In addition, considering the requirements of geodesy, four typical accuracy factors of chronometric leveling are proposed, and the accuracy requirements of optical clocks and frequency transmission techniques to achieve these factors are also given. It has important guiding significance for using optical clocks and frequency transmission techniques to achieve chronometric leveling and realize the unification of the global height datum in the future.

2. Principle of Chronometric Leveling

According to the gravitational redshift effect from general relativity, clocks tick faster for higher latitude, which can establish the relationship between the height and frequency [9, 11]. This method is now known as chronometric leveling [28]. The essence of chronometric leveling is the geopotential difference observation. Because the conversion process between geopotential and height is relatively mature, chronometric leveling can also be regarded as a height measurement method. Its principle is shown in Figure 1. If locations 1 and 2 each have a clock, with frequencies of f_1 and f_2, the frequency difference between the clocks can be determined by frequency transmission techniques such as optical fiber or satellite. The gravitational potential difference and height difference between the two points can be obtained as follows [3,9,29].

$$\Delta h = h_2 - h_1 = \frac{W_1 - W_2}{g} = \frac{c^2}{g}\frac{f_2 - f_1}{f_1} \approx 9.1 \times 10^{17}\frac{f_2 - f_1}{f_1} \tag{1}$$

where h_1 and h_2 are the orthometric height of locations 1 and 2, W_1 and W_2 are the gravitational potential of locations 1 and 2, f_1 and f_2 are the frequency of optical clocks 1 and 2, c is the speed of light in vacuum, known as c = 299,792,458 m/s, and g is the gravitational acceleration of locations 1 and 2, assumed $g = g_1 = g_2 = 9.8$ m/s^2. If the frequency ratio of optical clocks changes by 1.0×10^{-18}, the corresponding height difference is about 9.1 mm, which has the potential to achieve the unification of the global height datum with state-of-the-art accuracy.

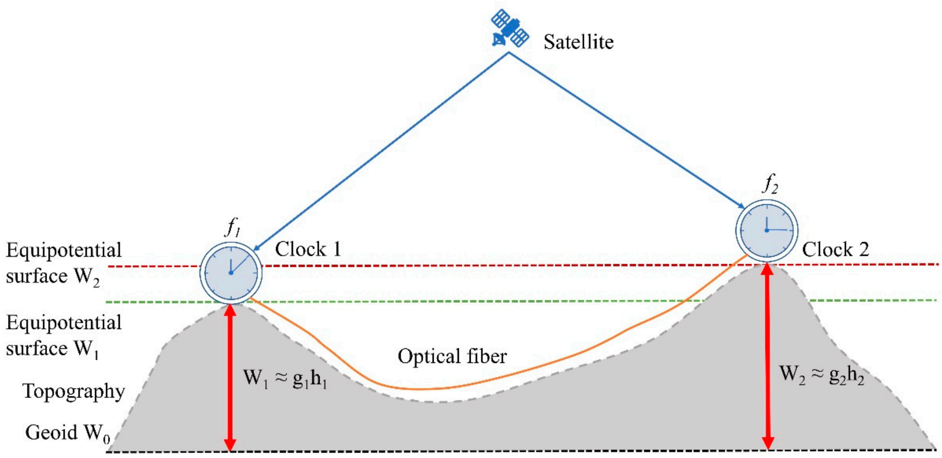

Figure 1. Schematic of chronometric leveling.

3. Error Sources of Chronometric Leveling

Chronometric leveling inevitably contains different kinds of error items, mainly including three parts: the measurement systematic error of the optical clock, the frequency statistical error of the optical clock, and the transmission path error during the process of clock comparison. Different error terms jointly determine the accuracy of the height measurement. According to the law of error propagation, we may conclude that the uncertainty of height measurement can be expressed as

$$\hat{\sigma} = \sqrt{\sigma_{sys}^2 + \sigma_{sta}^2 + \sigma_{path}^2} \quad (2)$$

where $\hat{\sigma}$ is the uncertainty of height measurement, $\sigma_{sys} = \sqrt{\sigma_{sys-clock1}^2 + \sigma_{sys-clock2}^2}$ is the total measurement systematic error from the two optical clocks used, $\sigma_{sta} = \sqrt{\sigma_{sta-clock1}^2 + \sigma_{sta2-clock2}^2}$ is the frequency statistical error from the two optical clocks used, and σ_{path} is the path error due to the frequency transmission technique.

Different frequency transmission techniques differ in their sources of error. According to carriers, existing frequency transmission techniques can be divided into three main categories: satellite-based frequency transmission techniques, very long baseline interferometry (VLBI) frequency transmission techniques, and optical fiber-based frequency transmission techniques [30]. Satellite-based frequency transmission techniques mainly include two-way satellite time and frequency transfer (TWSTFT) and Global Navigation Satellite System precision point positioning (GNSS-PPP), etc., with fractional uncertainty up to the 10^{-16} order of magnitude [31]. The fractional uncertainty of VLBI frequency transmission can also reach the 10^{-16} order of magnitude [32]. These methods can achieve long-distance frequency transmission at low cost, but they cannot meet the needs based on centimeter-level chronometric leveling. OFFT, with advantages of environmental stability and low signal loss, is by far one of the most accurate methods for frequency transmission. Available experiments show that the fractional uncertainty of frequency transmission of OFFT could be lower than 10^{-19} [33–35]. Implementing chronometric leveling via OFFT could be the most mature and reliable method at present.

The detailed items for each error source are shown in Figure 2 when OFFT is used for clock comparison. The measurement systematic error of the optical clock mainly includes eleven items, including blackbody radiation (BBR), second-order Doppler shift, and so on, which should be evaluated before application [16,18,36]. In addition, the frequency statistical error of the optical clock may be caused by outliers, drift, and noise characteristic of the output frequency [37]. The error term caused by optical fiber transmission mainly includes the Doppler time delay, environment error, and random error.

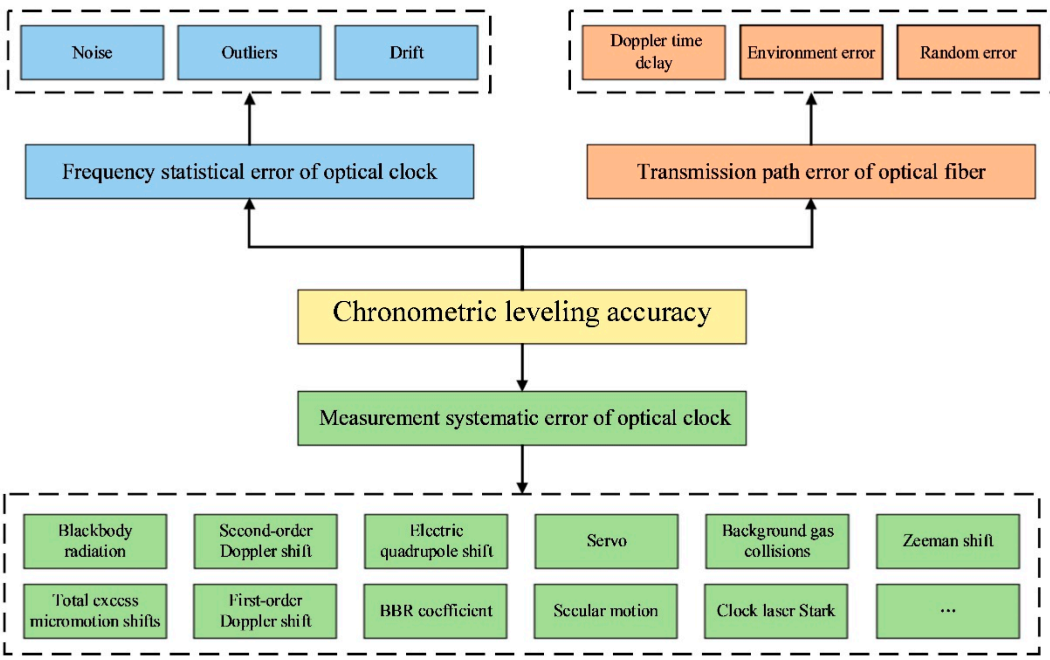

Figure 2. The main sources of error terms in chronometric leveling via OFFT.

3.1. Measurement Systematic Error of Optical Clock

The optical clock inevitably contains some systematic error terms, which will cause the offset of the output frequency and have a certain degree of uncertainty [30]. The main offset term can be detected and corrected by optical techniques, so the remaining uncertainty will determine the accuracy of the height measurement. The systematic uncertainty σ_{sys1} of a single optical clock can be simply expressed as

$$\sigma_{sys1} = \sqrt{\sigma_{BBR}^2 + \sigma_{D2}^2 + \sigma_{Others}^2} \tag{3}$$

where σ_{BBR} is the uncertainty caused by BBR, σ_{D2} is the uncertainty of the second-order Doppler shift, σ_{Others} is the uncertainty of the sum of other effects, including the Zeeman shift, electric quadrupole shift, etc. Other measurement systematic error items include servo, background gas collisions, Zeeman shift, etc. [38]. These error items mainly depend on the design and manufacture of the optical clock, which should be evaluated before practical application [39].

3.2. Frequency Statistical Error of Optical Clock

An optical clock is a complex system composed of a variety of electrical and optical devices. Due to the influence of environmental factors such as temperature, electric field, humidity, and magnetic field, the output frequency of an optical clock is inevitable a variable value. The frequency model mainly includes a fixed frequency and a small disturbance term, which can be expressed as [37]

$$f(t) = f_0 + r(t) \tag{4}$$

where f_0 is the fixed frequency, $r(t)$ is the disturbance term of the optical clock varying with time, known as $r(t) \ll f_0$. Meanwhile, the output frequency may have outliers and a frequency drift. The outliers can be eliminated by the rheinda rule and the frequency drift can be corrected by linear fitting and other methods [40]. In summary, the error caused by the output frequency may be dominated by the noise characteristic of the disturbance term.

The noise model of an atomic clock could be described by five kinds of independent energy spectrum noises, and the total noise model is the linear superposition of five kinds of noise. The basic model has been widely accepted internationally and can be expressed as [37,41]

$$f_i(t) = \sum_{\alpha=-2}^{2} z_\alpha(t) \tag{5}$$

which represents five kinds of independent noises, which are random walk frequency modulation noise (RWFM), flicker frequency modulation noise (FFM), white frequency modulation noise (WFM), flicker phase modulation noise (FPM), and white phase modulation noise (WPM). Since noise models of an atomic clock are time-varying, the standard deviation of frequency could be a divergence value. The Allan variance is usually used to calculate the uncertainty of the optical clock frequency [41,42].

The terms of the frequency statistical error mainly include outliers, drift, frequency noise, and so on, which will affect the accuracy of the height measurement [40,41]. Among them, the outliers can be eliminated by the rheinda rule, the data gap can be linearly interpolated, and the drift can be estimated and corrected by the logarithmic fitting method [43]. Hence, the frequency noise characteristic of optical clocks may dominate the frequency statistical error.

3.3. Transmission Path Error of Optical Fiber

The magnitude of the fiber path error has an important impact on the realization of high-precision height measurements. In practice, differences in accuracy may be caused by factors such as the specification and distance of optical fibers. The evaluation factors of optical fiber frequency transmission systems are usually 1-s stability, the system noise floor, and the time to reach the system noise floor [33,44]. The higher the 1-s stability of an optical fiber frequency transmission system, the better its short-term stability. The system noise floor is the theoretical limit of the optical fiber frequency transmission system, which is related to the material, length and laser wavelength. Generally, the system noise floor is measured with a short distance optical fiber several meters long after the construction. The better the optical fiber frequency transmission system, the lower the system noise floor. In addition, the time required to reach the system noise floor is also a significant factor. According to the existing experimental results, the model between the speed of accuracy improvement and time is usually τ^{-1} or $\tau^{-3/2}$, where τ is the averaging time, that is, the slope of the time Allan deviation graph is usually -1 or $-3/2$ [45–47].

4. High-Accuracy Clocks and Test

Chronometric leveling with centimeter level accuracy was achieved based on the two transportable $^{40}Ca^{+}$ ion optical clocks developed by Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences (APM). The chronometric leveling is essentially the frequency comparison of two optical clocks. The main hardware includes the time-frequency transmission link and a pair of optical clocks. The flow diagram is shown in Figure 3 [36]. Optical clock 1 was fixed on the experimental platform as a reference and optical clock 2 was placed in the horizontal position (−0.035 m), low position (−0.792 m), and high position (1.072 m) in turn. Finally, optical clock 2 returns to the horizontal position. A total of four periods of observation were carried out. The observation periods 1~3 lasted for eleven days, respectively. Then, period 4 on the horizontal position was carried out again at an interval of one month to verify the stability of the instrument, which lasted for five days. Due to the influence of the magnetic field around the experimental site, the observation time of a single day was only about 6 h. During the measurement process, no adjustment was made to the optical clock except for the height of the instrument. Frequency transmission was carried out through a 10 m long optical fiber. The actual height was measured directly via the classical geodetic method with an uncertainty of about 1 cm.

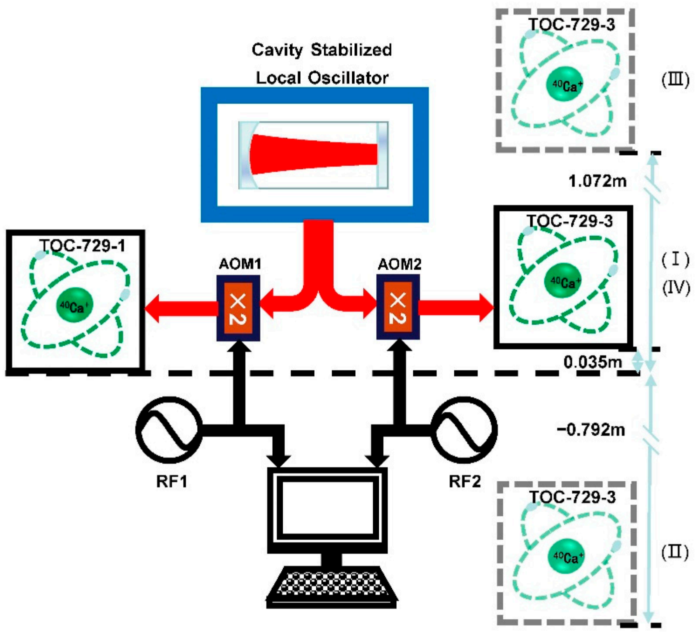

Figure 3. Flow diagram of using APM clocks for chronometric leveling [36].

The difference between the result of the height measurement with the optical clock and the geometric method was only a few centimeters. The average values of the three positions of the height measurement with the optical clock are −0.05 m, −0.83 m, and 1.02 m, respectively. The difference from the true value measured by the classical geodetic method is 0.015 m, 0.038 m, and 0.052 m, respectively, indicating that the difference between the observed results and the true value could be reduced to cm level after multiple observations.

The uncertainty of the final elevation measurement of the experiment was about 21 cm. The two optical clocks used in this experiment are $^{40}Ca^{+}$ ion optical clocks developed by APM, and their systematic errors are both 1.3×10^{-17}, the corresponding height uncertainty about 0.1192 m. Using two optical clocks, the uncertainty of the final elevation measurement needs to be multiplied by $\sqrt{2}$, and the systematic error of the final optical clock measurement was about 0.1686 m. Limited by the stability and observation time of the clock laser, the frequency statistical error of the transportable $^{40}Ca^{+}$ ion optical clock of APM is $4.8 \times 10^{-15}/\sqrt{\tau}$. The longer the averaging time, the smaller the frequency statistical error. Hence, the frequency statistical error in this experiment was 1.0×10^{-17}, the corresponding height uncertainty about 0.090 m. The optical fiber with a time-frequency transmission link of 10 m was supplemented by noise elimination technology to eliminate the impact of path noise on the measurement. The magnitude of the fiber path noise was less than 1.0×10^{-18}, the corresponding height uncertainty less than 9 mm. Compared with the error of the optical clocks, the impact of the transmission path on the experiment might be ignored. According to the covariance propagation law, we combine the above three error sources and the final height uncertainty was approximately 21 cm.

5. Performance Evaluation and Requirement Analysis

5.1. Clocks and OFFT Systems

The measurement systematic error of the optical clock depends on the overall design of the optical clock and the operating environment, which may dominate the accuracy of the height measurement. We assume four typical optical clocks with accuracy ranging from 10^{-17} to 10^{-19}, which are named Clock 1~4. The systematic error sources and values of Clock 1~4 are shown in Figure 4 and Table 1. Both Clock 1 and 2 are $^{40}Ca^{+}$ optical clocks developed by APM [18,36]. The former is a miniaturized transportable optical clock, and the systematic error reaches 1.3×10^{-17}. The latter could only be operated in the laboratory with larger volume, but the measurement systematic error reached 3×10^{-18}. Clock 3 is the most accurate optical clock so far developed by NIST [16]. Clock 4 is an ultra-high precision optical clock that we assume may be realized in the future. The measurement systematic error reaches 3×10^{-19}, the corresponding height error approximately 0.0028 m. If chronometric leveling is achieved by using a pair of Clock 1~4, the total measurement systematic error can be ranging from 0.1686 m to 0.0040 m.

Table 1. Measurement systematic error items of existing and potential optical clocks.

Number	Source	Clock 1 ($\times 10^{-17}$)	Clock 2 ($\times 10^{-18}$)	Clock 3 ($\times 10^{-19}$)	Clock 4 ($\times 10^{-19}$)
1	Blackbody radiation	0.92	2.7	4.2	—
2	Second-order Doppler shift	0.02	0.9	—	—
3	Electric quadrupole shift	0.92	0.4	—	—
4	Servo	0.14	0.4	—	—
5	Background gas collisions	0.12	—	2.4	—
6	Zeeman shift	0.07	—	3.7	—
7	Total excess micromotion shifts	—	0.2	5.9	—
8	First-order Doppler shift	—	0.3	2.2	—
9	BBR coefficient	—	0.3	—	—
10	Secular motion	—	—	2.9	—
11	Clock laser Stark	—	—	2	—
	Total	1.3	3	9.4	3
	Height uncertainty by one clock	0.1192 m	0.0275 m	0.0086 m	0.0028 m
	Height uncertainty by a pair of clocks	0.1686 m	0.0389 m	0.0122 m	0.0040 m

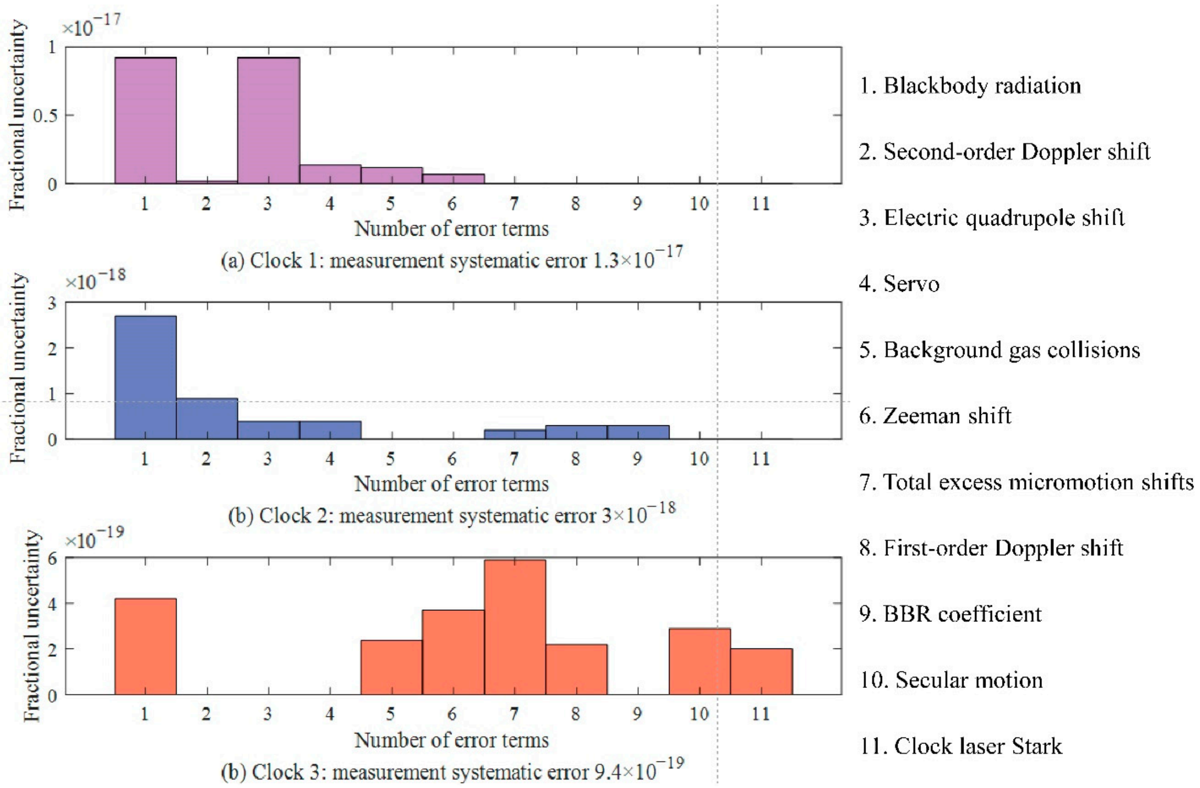

Figure 4. Measurement systematic error items of existing optical clocks.

The frequency statistical error comes from the dynamic change of the output frequency of the optical clock, which is closely related to the design of the optical clock and the clock laser used. In the following, we assume that the sampling frequency is 1 s and the output frequency conforms to the normal distribution. Currently, the Allan deviation is equal to the uncertainty value in the measurement. We select three existing optical clocks with different short-term stability, and look forward to the short-term stability that optical clocks can achieve in the future. Since the systematic measurement error of the optical clock has a certain correlation with its frequency statistical error, the selected optical clock is consistent with that described above. The specific relationship between the frequency statistical error and averaging time is shown in Figure 5 [16,18,36]. The lowest and highest stabilities in a second reach 4.8×10^{-15} and 1.0×10^{-16}, respectively, and with the increase of the averaging time, the accuracy improves continuously. When the averaging time is set as 10^6 s, the corresponding fractional uncertainty and the corresponding height error are shown in Table 2. Clock 1 has the largest statistical error, corresponding to a height error of 0.0440 m, while Clock 4 has the lowest statistical error, corresponding to a height error of about 0.0009 m. However, in practice, two optical clocks are required to realize the chronometric leveling, and the total statistical error will be the superposition of two independent error terms. The final statistical error and height error need to be multiplied by $\sqrt{2}$. The height error using two Clock 1 will reach 0.0624 m, and the same as Clock 4 will reach 0.0013 m. The frequency statistical error using a pair of Clock 2 or 3 are 0.0385 m and 0.0156 m, respectively.

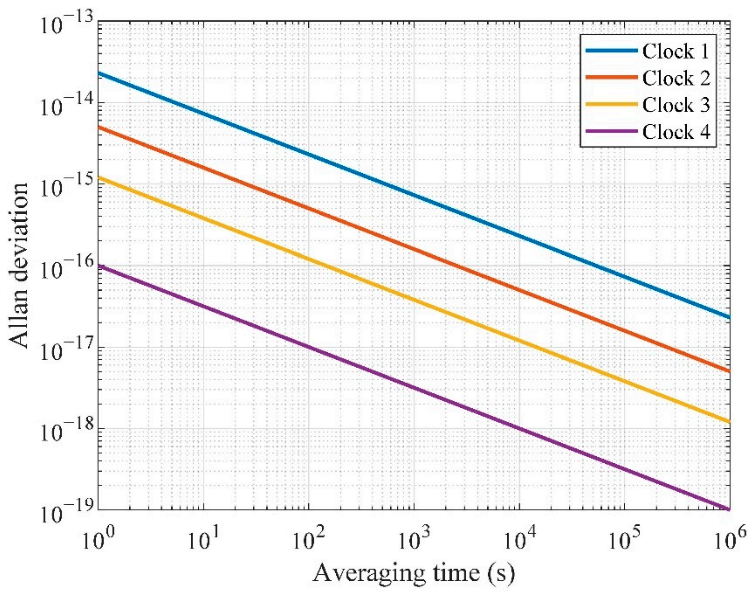

Figure 5. Frequency statistical error of different optical clocks.

Table 2. Frequency statistical error parameters of existing and future optical clocks.

Clock	Frequency Statistical Error Model	Frequency Statistical Error at 10^6 s	Frequency Statistical Error of a Pair of Clocks at 10^6 s	Height Uncertainty by a Single Clock (m)	Height Uncertainty by a Pair of Clocks (m)
1	$4.8 \times 10^{-15}/\sqrt{\tau}$	4.8×10^{-18}	6.8×10^{-18}	0.0440	0.0624
2	$3.0 \times 10^{-15}/\sqrt{\tau}$	3.0×10^{-18}	4.2×10^{-18}	0.0275	0.0385
3	$1.2 \times 10^{-15}/\sqrt{\tau}$	1.2×10^{-18}	1.7×10^{-18}	0.0110	0.0156
4	$1.0 \times 10^{-16}/\sqrt{\tau}$	1.0×10^{-19}	1.4×10^{-19}	0.0009	0.0013

The magnitude of transmission path error has an important impact on the realization of high-precision height measurement. We choose four proven optical fiber frequency transmission systems with accuracy ranging from 10^{-18} to 10^{-21}, which are named as Systems A~D, respectively [35,47–49]. The error models between accuracy and averaging time of different optical fiber frequency transmission systems are shown in Figure 6. By analyzing Figure 6, we may conclude that the uncertainty of different optical fiber systems has a linear relationship with time. When the average time is 2×10^4 s, the system noise floor may be achieved. The error model transmission distance and error model, system noise floor, and corresponding height uncertainty are shown in Table 3. All of Systems A~D could complete hundreds of kilometers of frequency transmission with uncertainty of 2×10^{-18} to 2.1×10^{-21}, with the corresponding height uncertainty ranging from 0.0183 m to negligible 0.00002 m. Optical fiber frequency transmission systems with different accuracy serve different optical clock comparisons with different accuracy.

Table 3. Transmission path error and parameters of existing OFFT systems.

System	Transmission Path Error Model	System Noise Floor	Distance (km)	Height Uncertainty (m)
A	$1 \times 10^{-14}/\tau$	2×10^{-18}	480	0.0183
B	$5 \times 10^{-15}/\tau^{3/2}$	4×10^{-19}	920	0.0037
C	$5.4 \times 10^{-16}/\tau$	1.7×10^{-20}	680	0.0002
D	$7 \times 10^{-18}/\tau$	2×10^{-21}	100	0.00002

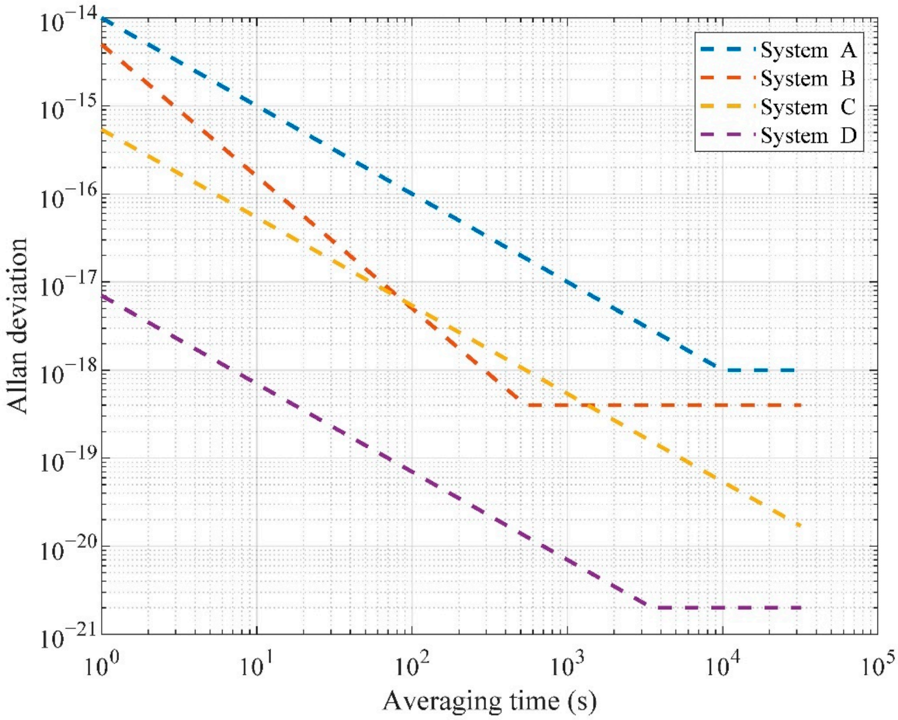

Figure 6. Accuracy of different OFFT systems.

5.2. Performance Evaluation

Four optical clocks and four optical fiber frequency transmission systems are combined into 16 chronometric leveling schemes, respectively, which can realize height measurements with different accuracy. The detail combination of clocks and optical fiber frequency transmission systems are shown in Table 4. Especially, each scheme needs two optical clocks and a set of OFFT system to achieve the height measurement.

Table 4. Scheme elements of using different clocks and OFFT systems.

Schemes	Clock	OFFT System
1	1	A
2		B
3		C
4		D
5	2	A
6		B
7		C
8		D
9	3	A
10		B
11		C
12		D
13	4	A
14		B
15		C
16		D

Based on the above 16 schemes, we could conclude that the specific accuracy results are those shown in Table 5 and Figure 7. The results of Table 5 and Figure 7 could be understood as follows: the first column is the serial number of our theoretical schemes.

Each scheme includes a pair of optical clocks and the corresponding optical fiber frequency transmission system, which corresponds to Table 4. By changing the optical clocks and optical fiber frequency transmission systems, the measurement with different accuracy can be realized finally. The second column and the third column are the systematic errors of the optical clock used in the scheme and their corresponding errors, and their sources and uncertainties are detailed in Table 1. The fourth column is the statistical error of the output frequency of the optical clock. The mathematical model and the preset accuracy are described in detail in Table 2. The fifth column is the transmission error of optical fiber transmission systems A~D, which is from the last column of Table 3. The sixth column is the total error obtained by the errors from the third column to the fifth column according to the covariance propagation law. For instance, scheme 1 in Table 5 includes two clock 1 and OFFT system A. The systematic error of two clock 1 is 0.1686 m (Table 1), the statistical error is 0.0624 m (Table 2), and the transmission path error is 0.0183 m (Table 3). According to the covariance propagation law, the uncertainty of scheme 1 may achieve 0.1807 m. In addition, we draw the following conclusions by analyzing Table 5 and Figure 7:

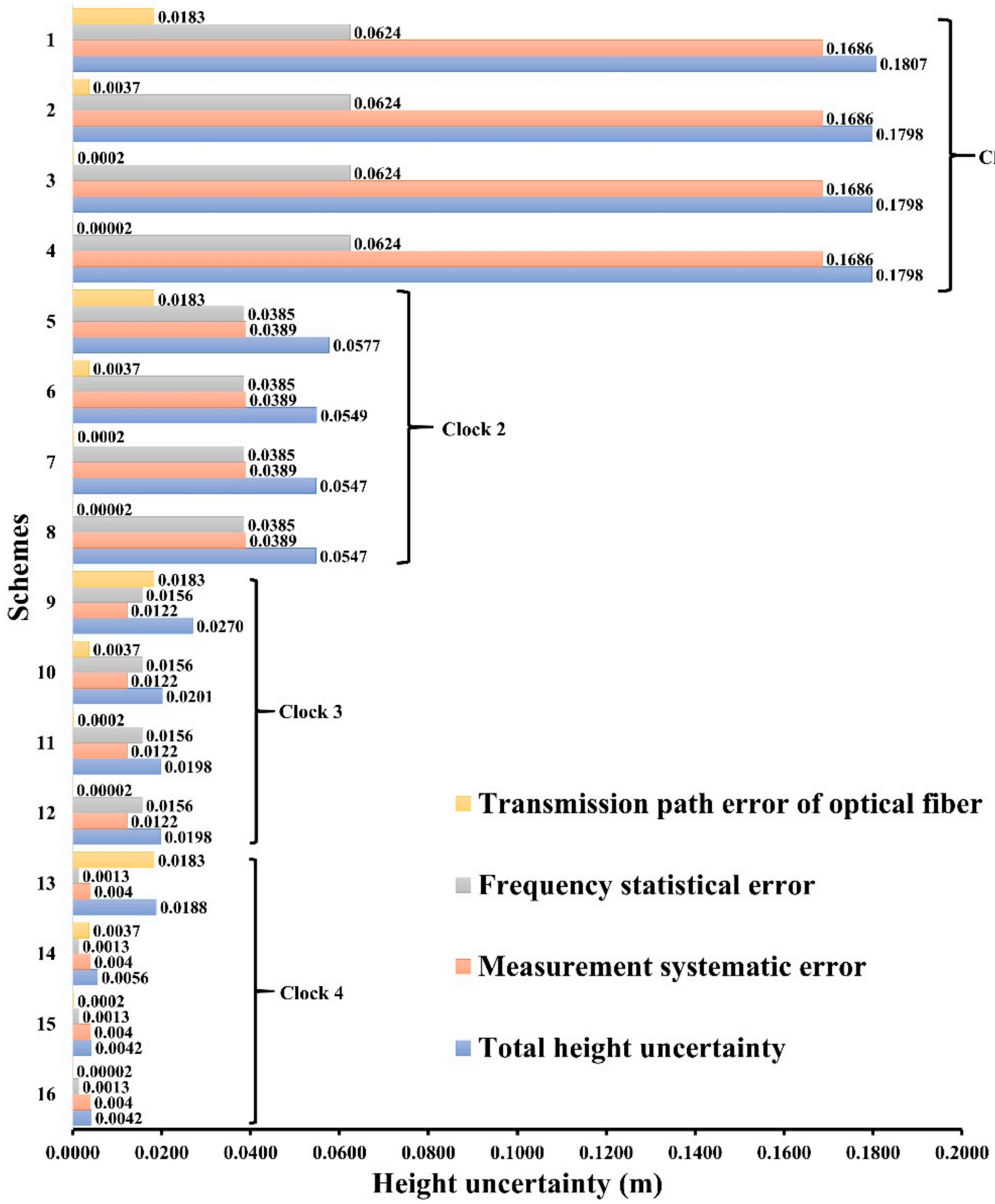

Figure 7. Uncertainty of height measurement for different schemes.

Table 5. Height uncertainty under different schemes of chronometric leveling.

Schemes	Clock	Measurement Systematic Error (m)	Frequency Statistical Error (m)	Transmission Path Error (m)	Height Uncertainty (m)
1	1	0.1686	0.0624	0.0183	0.1807
2				0.0037	0.1798
3				0.0002	0.1798
4				0.00002	0.1798
5	2	0.0389	0.0385	0.0183	0.0577
6				0.0037	0.0549
7				0.0002	0.0547
8				0.00002	0.0547
9	3	0.0122	0.0156	0.0183	0.0270
10				0.0037	0.0201
11				0.0002	0.0198
12				0.00002	0.0198
13	4	0.0040	0.0013	0.0183	0.0188
14				0.0037	0.0056
15				0.0002	0.0042
16				0.00002	0.0042

(1) The theoretical analysis results in this paper are consistent with the actual measurements. Schemes 1~4 adopt a pair of transportable $^{40}Ca^{+}$ optical clocks developed by APM and optical fiber frequency transmission systems with different accuracy levels. Since the error order of the four optical fiber transmission schemes is at least one order of magnitude lower than that of the optical clock, the uncertainty contribution of a height measurement mainly comes from the measurement systematic error and frequency statistical error of the optical clocks. The theoretical results show that the height uncertainty is about 18 cm. In 2022, the experiment has been completed in the laboratory [36]. Two transportable $^{40}Ca^{+}$ optical clocks were connected by a 10 m long optical fiber. One of the optical clocks was fixed, and the other optical clock was located at three different heights for the experiments. The uncertainty of the height measurement was about 21 cm. By comparing the theoretical and practical results, the difference between them is approximately 3 cm.

We believe that the reasons mainly include the following four points:

(a) The longer averaging time setting reduces the frequency statistical error and makes the theoretical error smaller. We assume that the averaging time is 10^6 s, the frequency statistical error reaches 6.8×10^{-18}, and the corresponding height uncertainty is 0.0624 m. However, in the actual experiment, considering that the measurement systematic error of a single clock is 1.3×10^{-17}, the corresponding height uncertainty is about 0.1193 m. Even if the averaging time is extended indefinitely until the statistical error is negligible, the uncertainty of a height measurement remains at the decimeter level. In practice, the frequency statistical error reached 1×10^{-17}, the corresponding height uncertainty about 0.0920 m. Compared with the frequency statistical error of the actual experiment, the theoretical statistical error is reduced by about 0.0296 m.

(b) The rounding error of the measurement systematic error of optical clocks makes the theoretical error smaller. The measurement systematic error of the two optical clocks in this paper is 1.8385×10^{-17}. However, the original article also contained some extremely small systematic shifts that we have not listed; after rounding, the actual measurement systematic error was given as 1.9×10^{-17}. Compared with the actual measurement systematic error, the theoretical measurement systematic error is reduced by 0.0056 m, approximately.

(c) The difference between the theoretical gravity value and the actual gravity value makes the theoretical result smaller. We assume that the gravity value is $g = 9.8\ m/s^2$, while the gravity value in the measured experiment is about $g = 9.793461\ m/s^2$. According to Equation (1), the gravity value is larger than the actual gravity, which directly leads to the smaller uncertainty of the height measurement. If the gravity value in the practical experiment is replaced by the value we assume, then the uncertainty of the final height measurement can be reduced by 0.0002 m.

(d) The Influence of optical fiber frequency transmission path error. The path error due to the 10 m optical fiber frequency transmission using noise cancellation technique is not worth considering. Strictly, it might still have a small impact on the height measurement uncertainty.

(2) Constructing an optical fiber frequency transmission system, whose transmission path error is one order of magnitude lower than the measurement systematic error of the connected optical clocks, could well implement chronometric leveling. Taking schemes 1~4 in Figure 7 and Table 5 as an example, they own two totally identical optical clocks. The difference among the four schemes lies in the different accuracy of the selected optical fiber transmission systems. The largest path error caused by the optical fiber transmission systems is 0.0183 m in scheme 1. For scheme 1 which can only achieve the height uncertainty about 18 cm, the variance contribution rate from the optical fiber transmission system is no more than 1%. If the optical fiber transmission path error is further reduced to 0.0037 m, or even 0.00002 m, the uncertainty of the height measurement will not be decreased by more than 0.0001 m. The fractional uncertainty of the optical fiber transmission system is 2×10^{-18} in scheme 1, while the fractional uncertainty of the total measurement systematic error of the two Clock 1 is 1.8385×10^{-17}. We may draw the conclusion that the optical clock comparison could be completed well when the transmission path error of the optical fiber is less than one order of magnitude of the measurement systematic error. The same conclusion could be drawn from the other 12 experimental schemes.

(3) Using the ultra-high precision optical clocks and OFFT technique, the height measurement uncertainty may reach less than 1 cm, and the unification of a high-precision height datum might be realized. The final height measurement uncertainty can reach 0.0198 m theoretically when a pair of Clock 3 combinate with OFFT system C. After the Clock 4 with ultra-high precision is realized in the future, it can be combined with the existing optical fiber frequency transmission system D, and the uncertainty of the height measurement may reach 0.0042 m. Moreover, it has the potential to directly carry out direct observations spanning hundreds or even thousands of kilometers. The unification of global height datum may be achieved with the precision of 1 cm.

5.3. Requirement Analysis

Fixed requirements for the accuracy of height measurement are usually needed in the field of geodesy. The target accuracy factors are set as 5 cm, 1 cm, 0.5 cm, and 0.1 cm, respectively, to explore the extent to which the optical clocks and frequency transmission techniques can be specifically applied to the unification of the height datum and other geoscience fields. The total measurement systematic error and frequency statistical error of two optical clocks have a certain correlation with the optical clock itself. The former can only rely on the progress of quantum science, while the latter can be reduced relatively easily by prolonging the observation time. With reference to the actual measurement results of Liu et al. (2022), we set $\sigma_{sys} = 2\sigma_{sta}$. Meanwhile, the frequency transmission technology is still developing rapidly, and more high-precision frequency transmission means may appear in the future [50,51]. Here, according to the conclusion obtained above, we assume that the frequency transmission path error σ_{path} is less than 1 order of magnitude of σ_{sys} i.e., $\sigma_{path} = 0.1\sigma_{sys}$.

On the premise of the above assumptions, the σ_{sys}, σ_{sta} and σ_{path} of the target accuracy are solved according to Equations (1) and (2). Results are shown in Table 6. It is worth noting that the systematic error and statistical error here refer to the total error of the two optical clocks required for chronometric leveling. The systematic error and statistical error for a single optical clock should be divided by $\sqrt{2}$. According to the results of Table 6, when σ_{sys}, σ_{sta} and σ_{path} reach 4.86×10^{-18}, 2.43×10^{-18} and 4.86×10^{-19}, respectively, the height measurement with an uncertainty of 5 cm could be realized. This target accuracy has been preliminarily realized [23,27]. When σ_{sys}, σ_{sta} and σ_{path} are further reduced to 9.71×10^{-19}, 4.86×10^{-19} and 9.71×10^{-20}, respectively, which may meet the current requirements of IAG for the unification of the height datum with 1 cm uncertainty [52].

Moreover, with the progress of optical clock and frequency transmission technology, chronometric leveling has great potential to realize the unification of the global height datum with state-of-the-art accuracy of 0.5 cm or even 0.1 cm. Table 6 shows that when σ_{sys}, σ_{sta} and σ_{path} reach 9.71×10^{-20}, 4.86×10^{-20}, 9.71×10^{-21}, respectively, the height measurement with the accuracy of 0.1 cm may be realized. If the frequency transmission technique can realize intercontinental optical clock comparison, a global high-precision height datum unification might be achieved by using chronometric leveling. At present, the gravitational redshift effect has been tested at the order of 10^{-21} in the laboratory, and the optical fiber frequency transmission technique could basically meet the demand for clock comparison [20,21,33,53]. In the future, the above four accuracy factors are possible to be achieved with the development of quantum science and technology.

Table 6. Requirements for measurement systematic error σ_{sys} and frequency statistical error σ_{sta} of a pair of optical clocks and transmission path error σ_{path} under different target accuracy.

Target Accuracy (cm)	Measurement Systematic Error σ_{sys}	Frequency Statistical Error σ_{sta}	Transmission Path Error σ_{path}
5	4.86×10^{-18}	2.43×10^{-18}	4.86×10^{-19}
1	9.71×10^{-19}	4.86×10^{-19}	9.71×10^{-20}
0.5	4.86×10^{-19}	2.43×10^{-19}	4.86×10^{-20}
0.1	9.71×10^{-20}	4.86×10^{-20}	9.71×10^{-21}

6. Conclusions

Chronometric leveling is expected to provide a new solution for achieving high-precision height datum unification. Hence, in this paper, we put forward the schemes for different target accuracy by chronometric leveling. The error sources for realizing the chronometric leveling height measurement are deeply analyzed when using optical fiber as the carrier. The results of the theoretical analysis show that:

(1) The measurement systematic error of the optical clocks may depend on the performance of the instrument and operating environment and the measurement systematic error of the current highest precision optical clock can reach 8.6 mm. The systematic error is the systematic frequency shift caused by the electric field, magnetic field, temperature, and so on, which could be evaluated and corrected to a certain accuracy through quantum technical means. The measurement systematic error mainly depends on the performance of the optical clock used. The corresponding height difference of the existing optical clock with the highest accuracy could reach 8.6 mm. With the progress of quantum technology and laser science, more optical clocks with higher accuracy may appear to serve the chronometric leveling.

(2) The most important influence of the frequency statistical error of optical clocks is the continuous observation time and noise characteristics of optical clocks. By analogy with atomic clocks, the frequency statistical error of the optical clock will include outliers, data gap, and possible drift, which will be improved through the data preprocessing method. The biggest influence factor may be the noise characteristics caused by the temperature, vibration, and so on. The frequency noise characteristic models of existing optical clocks are all expressed as WFM. The accuracy can be improved by one order of magnitude when the averaging time is extended by two orders of magnitude. However, considering the observation data of the optical clock published at present, the continuous observation time is relatively short. We cannot exclude that there will be drift, RWFM noise, and other noises that may affect the accuracy during long-term observation.

(3) Using the high-precision optical clocks combined with OFFT systems is expected to achieve the unification of high-precision height datum with a height measurement

uncertainty of 1 cm. Combining different optical clocks and OFFT systems could achieve height measurements with different accuracy. In practice, the frequency transmission accuracy should be ensured more than 1 order of magnitude higher than the accuracy of the connected optical clock, to make the variance contribution rate of the path error about 1%. It can basically realize the chronometric leveling of the order corresponding to the error of the optical clock. The theoretical analysis results show that the measurement systematic error of the next-generation optical clock measurements may reach 3×10^{-19} with stable output frequency. The optical frequency statistical error of the optical clock is expected to reach 1×10^{-19}. Combined with the high-precision optical fiber frequency transmission technology, the height measurement with an uncertainty of about 4.2 mm may be realized. The problem of error accumulation and cross sea transmission might be overcome. It is expected to achieve the unification of the global height datum with the accuracy of 1 cm proposed by IAG.

(4) Assuming that the precision of optical clock and frequency transmission technique can reach the given precision, the unification of the height datum with an uncertainty of 1 cm or even 1 mm may be achieved. When the fractional frequency uncertainty of the total measurement systematic error and output statistical error of the two optical clocks reaches 9.71×10^{-19} and 4.86×10^{-19}, respectively, and the fractional frequency uncertainty of the adopted frequency transmission technique is 1 order of magnitude better than the systematic error of the optical clock measurement, the height measurement with the accuracy of 1 cm may be realized. If the frequency transmission technology can realize intercontinental frequency transmission, it is possible to unify the global height datum with the accuracy of 1 cm. With the improvement of the accuracy of optical clock and frequency transmission technology, the accuracy of unification of the height datum via chronometric leveling might be higher.

To our knowledge, this is the first detailed quantitative study of height uncertainty for chronometric leveling. Meanwhile, considering the requirements of geodetic field for height measurement, the requirements for optical clocks and frequency transmission accuracy are put forward, which have important guiding significance for the practical application of optical clocks and frequency transmission techniques in the future. Frequency transmission technique and quantum technology are developing rapidly. In the future, it is expected to achieve centimeter-level or even millimeter-level height difference measurements within a few hours, and the time resolution may exceed any existing observation techniques [29,54]. Transportable optical clocks, static optical clocks, and a variety of frequency transmission technologies may be combined to form optical clock networks, which are expected to achieve the establishment of new quantum benchmarks, and long-term observation of gravity potential difference, and height difference [23,27,55]. It may be used to establish a geoid, surface vertical displacement monitoring, verification of satellite gravity, gravity field determination, deep space navigation, and other geoscience fields [56–62].

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