

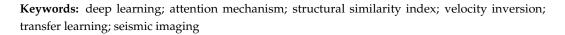


Article Deep Learning with Adaptive Attention for Seismic Velocity Inversion

Fangda Li^{1,2,3}, Zhenwei Guo^{1,2,3}, Xinpeng Pan^{1,2,3}, Jianxin Liu^{1,2,3}, Yanyi Wang^{1,2,3} and Dawei Gao^{1,2,3,*}

- Key Laboratory of Metallogenic Prediction of Nonferrous Metals and Geological Environment Monitoring, Ministry of Education, Changsha 410083, China
- ² Key Laboratory of Non-Ferrous Resources and Geological Hazard Detection, Changsha 410083, China
- ³ School of Geosciences and Info-Physics, Central South University, Changsha 410083, China
- Correspondence: daweigao@csu.edu.cn

Abstract: The subsurface velocity model is crucial for high-resolution seismic imaging. Although full-waveform inversion (FWI) is a high-accuracy velocity inversion method, it inevitably suffers from challenging problems, including human interference, strong nonuniqueness, and high computing costs. As an efficient and accurate nonlinear algorithm, deep learning (DL) has been used to estimate velocity models. However, conventional DL is insufficient to characterize detailed structures and retrieve complex velocity models. To address the aforementioned problems, we propose a hybrid network (AG-ResUnet) involving fully convolutional layers, attention mechanism, and residual unit to estimate velocity models from common source point (CSP) gathers. Specifically, the attention mechanism extracts the boundary information, which serves as a structural constraint in network training. We introduce the structural similarity index (*SSIM*) to the loss function, which minimizes the misfit between predicted velocity and ground truth. Compared with FWI and other networks, AG-ResUnet is more effective and efficient. Experiments on transfer learning and noisy data inversion demonstrate that AG-ResUnet makes a generalized and robust velocity prediction with rich structural details. The synthetic examples demonstrate that our method can improve seismic velocity inversion, contributing to guiding the imaging of geological structures.



1. Introduction

In seismic exploration, seismic imaging is one of the most commonly employed techniques for mapping the structure of the oil and gas reservoir, which contributes to further inferring reservoir characteristics (e.g., lithology, fluid- and fracture-property) [1–4]. A reliable macro-velocity model is a prerequisite for seismic imaging [5–7]. Velocity inversion methods mainly consist of stacking velocity analysis, migration velocity analysis, tomography, full-waveform inversion, and so on [8–11]. Full-waveform inversion (FWI) has become a promising method for estimating velocity models of complex structures [12–16].

FWI suffers from three main problems: strong nonlinearity, high computational cost, and inversion non-uniqueness [15]. Tarantola proposed a time-domain FWI based on generalized least squares, which addressed nonlinear inversion by linearization [17]. Considering the intrinsically nonlinear relationship between velocity and reflection coefficient, nonlinear optimization algorithms were used to establish the mapping between seismogram and velocity [18–20]. To improve the inversion efficiency, the frequency-domain FWI was developed [21,22]. Compared with the time domain, the frequency domain wavefield is decoupled. Therefore, multi-scale analysis was used in frequency-domain FWI [23,24]. The principle of this method is consecutively to use lower frequency information as the input to estimate the higher frequency part, finally yielding ideal results [25–27]. In order to address the non-uniqueness problem of inversion, Laplace-domain FWI was developed [28,29]. Because of the frequency insensitivity of the Laplace domain, it can obtain



Citation: Li, F.; Guo, Z.; Pan, X.; Liu, J.; Wang, Y.; Gao, D. Deep Learning with Adaptive Attention for Seismic Velocity Inversion. *Remote Sens.* **2022**, *14*, 3810. https://doi.org/10.3390/ rs14153810

Academic Editors: Ru-Shan Wu, Benfeng Wang and Jingrui Luo

Received: 28 June 2022 Accepted: 2 August 2022 Published: 7 August 2022

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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). the long-wavelength velocity model from a simple initial model [30,31]. Although previous studies indicate that FWI has the potential to image complex structures precisely, the objective function of FWI is strongly nonlinear, and it inevitably suffers from the aforementioned issues.

Therefore, velocity inversion needs an efficient and robust method to build an accurate model. Deep learning (DL) has been applied successfully in computer vision [32–35], demonstrating excellent nonlinear processability. Motivated by this idea, numerous studies have applied DL successfully in seismic denoising [36–38], seismic data reconstruction [39–41], seismic data interpretation and attribute analysis [42–44], and so on. In velocity inversion disciplines, DL has served as an alternative method to FWI with high precision, robustness, and efficiency [45–49].

The loss function, network architecture, and data type are thoroughly studied to improve the inversion resolution of complex stratigraphic boundaries. Local structures and details are mainly impacted by loss function [50,51]. Li et al. and Liu et al. proposed a novel loss function that combines structural similarity index (SSIM) and L1 norm, contributing to improving the inversion resolution [52,53]. The SSIM, suitable for the human visual system (HVS), evaluates the similarity of two images through lightness, contrast, and structure [54]. In terms of network architecture, Wu and Lin employed convolutional neural networks (CNN) with conditional random fields (CRF) to precisely predict the fault structure [55]. Li et al. proposed fully connected layers to learn spatially aligned feature maps, which improved the inversion accuracy of spatial locations of geological targets [52]. In selecting input data, common source point (CSP) gathers are always fed into the network for training the parameters of a network. However, they have weak spatial correspondence with velocity models. Nowadays, common-imaging point gathers (CIG) have been utilized for network training instead of seismograms, simplifying the feature extraction and showing excellent velocity model building [56,57]. Although the modern DL algorithms improve the inversion efficiency and effect, the experimental results show that it is insufficient to characterize stratigraphic boundaries and geological structures [49].

Therefore, this study focuses on improving the construction boundary extraction ability of modern DL algorithms. Here, we present a hybrid network (AG-ResUnet) consisting of Unet [58], residual units [59], and attention gates [60]. To precisely estimate the geological structure details in the velocity model, we introduce SSIM to the loss function and utilize SSIM to evaluate the inversion. Unlike other full reference image quality assessments (FR-IQAs) that focus on the value difference of each pixel, SSIM pays more attention to the local structural information in images, which is of great importance in reconstructing the structural details. In this study, we first train the hybrid network by a mix loss function, consisting of mean square error (MSE) and SSIM, to optimize each pixel and local patch velocity misfits simultaneously. Second, we compare the training effect of the mix loss function and MSE loss function and demonstrate that the mix loss function plays a vital role in retrieving local structures. To illustrate the strength of our method, we further compare predictions of AG-ResUnet and other methods (e.g., FWI and other conventional networks). Finally, we conduct the transfer learning and noisy data inversion experiments. The results demonstrate the generalization and robustness of our method. The synthetic examples indicate that our method can provide high-resolution recovery of the complex velocity model, thus significantly improving seismic imaging.

2. Methodology

2.1. Network Architecture

The input of network is CSP gathers calculated from a 2D reference velocity model, and the output is the predicted velocity model. The AG-ResUnet consists of the Unet frame, residual unit, and attention gate (AG) (Figure 1). In Unet architecture, high-resolution features from the encoder combine with the decoder output (red arrow in Figure 1), avoiding direct supervision and loss computation in high-level feature maps. Although multiple convolutional layers can establish strongly nonlinear mappings between seismogram and velocity, the degradation problem of deep networks still poses a significant challenge. To

tackle the degradation problem due to vanishing gradients, we introduce the residual skip connection (blue dotted line in Figure 1) between two non-adjacent layers in the encoder. By taking advantage of residual units, the network can continue learning new features with the vanishing of gradients.

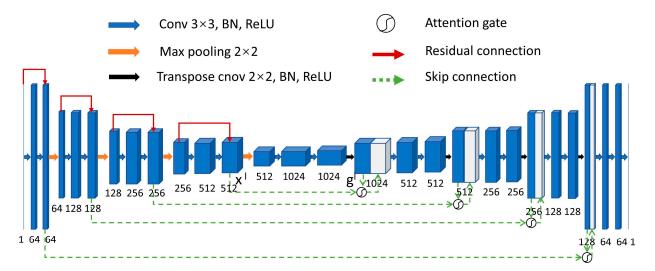


Figure 1. A block diagram of AG-ResUnet network architecture. Each box represents the output feature maps of a convolutional layer. The number at the bottom of each box denotes channels in the corresponding feature map. The down-sampling module consists of the repeated two convolutional layers with 3×3 kernel sizes (blue arrow), two batch normalization layers (BN), linear rectification function (ReLU), and the 2×2 max-pooling layer (orange arrow). Each up-sampling module replaces the max-pooling layer with a 2×2 transposed convolutional layer (black arrow).

AG (Figure 2), an attention mechanism, is employed in the medical segmentation, showing an excellent capability of boundary recognition [60]. Inspired by this idea, we utilize AGs in feature fusion, which adaptively focuses on the boundaries between geological targets. Its mathematical framework is as follows:

$$a^{l} = \sigma_{2} \left(\Psi^{T} \left(\sigma_{1} \left(W_{x}^{T} x^{l} + W_{g}^{T} g^{l} + b_{x} + b_{g} \right) \right) + b_{\Psi} \right)$$
(1)

$$= \alpha \cdot x$$
 (2)

where x^l is the low-level feature map at the *l* layer down-sampling, g^l is the *l* layer upsampling feature map, W_x^T , W_g^T , and Ψ^T are 1×1 convolutional layers, the superscript *T* means transpose, b_x , b_g , and b_{Ψ} are bias terms, σ_1 , σ_2 are ReLU and Sigmod activation functions, α is the gating coefficient, and \hat{x} is the feature map weighted by AG.

 $\hat{x} =$

We take the addition of feature maps of *x* and *g* input into the nonlinear function σ_1 to obtain the output feature map. Then the channels of the output feature map are adjusted to one by the convolutional layer Ψ . Finally, the feature map values are limited between zero and one by σ_2 , and multiply *x* to obtain the gating coefficient α . The α value is higher at geological boundaries.

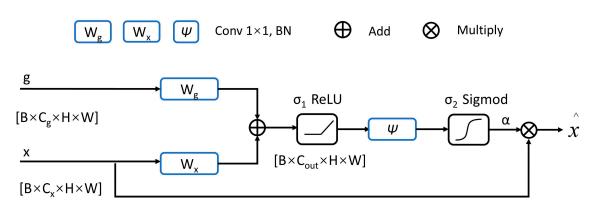


Figure 2. Attention gate work flow [60] The two input signals x^l and g^l of AG from the down-sampled and corresponding up-sampled feature maps, respectively, \hat{x} is the feature map weighted by AG.

2.2. Loss Function

Mean-square error (*MSE*) is the most common loss function for regression problems (e.g., velocity inversion),

$$L_{MSE}(y_i, y'_i) = \frac{1}{N} \sum_{i=1}^{N} (y_i - y'_i)^2$$
(3)

where *N* denotes the total number of pixels in one single image, y_i and y'_i represent velocity label and inversion velocity, respectively. L_{MSE} minimizes the only pixel-wise distance between output and target and ignores the texture structures. Consequently, the local structural information, critical for achieving high-resolution subsurface models, would not be retrieved using L_{MSE} only.

Local structures and details are vital factors to be focused on while recovering velocity models. To make the network optimization focus on local structural information, we introduce *SSIM* to the loss function. It describes the similarity of the two models from the perspective of local structure [54]. Evaluating model similarity with *SSIM* is known as full reference, meaning that a complete reference label (e.g., reference velocity models) is assumed to be known. The value of *SSIM* ranges from zero to one. The larger the value, the closer the predicted velocity model is to the ground truth. *SSIM* is defined as:

$$SSIM(y,y') = \frac{\left(2\mu_{y}\mu_{y'} + C_{1}\right)\left(2\sigma_{yy'} + C_{2}\right)}{\left(\mu_{y}^{2} + \mu_{y'}^{2} + C_{1}\right)\left(\sigma_{y}^{2} + \sigma_{y'}^{2} + C_{2}\right)}$$
(4)

where μ_y and $\mu_{y'}$ represent the mean values of y_i and y'_i , respectively, σ_y and $\sigma_{y'}$ are the corresponding standard deviation, $\sigma_{yy'}$ denotes the covariance of y_i and y'_i , and C_1 and C_2 represent constants to stabilize the division.

The combination of the *MSE* loss and the *SSIM* serves as a mix loss function (L_{Mix}) maintaining the advantages of both loss functions, which simultaneously optimize each pixel and local patch misfits,

$$L_{Mix}(y_i, y_i') = L_{MSE}(y_i, y_i') - L_{MSE}(y_i, y_i') \cdot SSIM(y_i, y_i')$$
(5)

where y_i and y'_i represent velocity label and inversion velocity, respectively.

2.3. Quantitative Metrics

Besides the loss values, three additional metrics are used to evaluate the performance of inversion, i.e., *SSIM*, peak signal-to-noise ratio (*PSNR*), and coefficient of determination (R^2). *SSIM* is vital for evaluating the inversion process since it considers the local structural information, while *PSNR* and R^2 only evaluate inversion by each pixel velocity.

The reconstruction quality of the velocity model is determined by *PSNR*, and the equation is as follows:

$$PSNR(y,y') = 20 \cdot log_{10}\left(\frac{Max(y)}{\sqrt{MSE(y,y')}}\right)$$
(6)

where y and y' represent the velocity label and the inversion velocity, respectively. The larger the *PSNR* value, the better the inversion.

 R^2 constantly evaluates the fit of the regression model, then gives the equation as follows:

$$R^{2}(y_{i}, y_{i}') = 1 - \frac{\sum_{i=1}^{N} (y_{i} - y_{i}')^{2}}{\sum_{i=1}^{N} (y_{i} - \overline{y_{i}})^{2}}$$
(7)

where $\overline{y_i}$ denotes mean values of the velocity label. The range of R^2 is generally zero to one. If the velocity inversion model and ground truth are the same, then the value of R^2 would be 1.

3. Experiments

3.1. Data Preparation

There are two kinds of datasets used in this study. One is the synthetic salt dome dataset, and the other is the Society of Exploration Geophysicists (SEG) [61] published salt dataset. The salt dome is the diapir structure typically associated with oil and gas. Imaging salt structures is necessary for studying reservoir structures. We refer to a strategy of random geological modeling [62,63] to build P-wave velocity (Vp) models. We first generate an initial model with 5~8 flat layers, and the Gaussian function is utilized to simulate the continuous fluctuation of the stratigraphic interface. The velocity of the first layer is randomly chosen from 1500~1600 m/s. The velocities of the remaining layers gradually increase, and the exact increment is randomly assigned in the range of 150–250 m/s. Finally, a salt dome structure with a fixed velocity of 4000 m/s is added to the flat model. The model (e.g., six models in Figure 3) has 200 samples in both the *x* and *z* directions, with a sampling interval of 10 m.

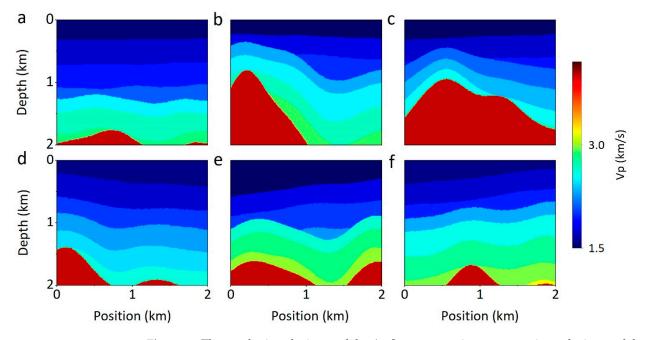


Figure 3. The synthetic velocity models. (**a**–**f**) represent six representative velocity models from synthetic dataset. The red geological targets represent salt domes; other layered structures are surrounding formations.

The SEG data is a 3D pseudo-real salt model built by SEG and the European Society of Geologists and Engineers (EAGE) based on geological data. We sliced it and made 500 independent 2D stratigraphic models (e.g., six models in Figure 4). The spatial size and grid interval are the same as those of the synthetic dataset. The velocity ranges from 1500 to 4482 m/s. Compared with the synthetic dataset, the salt structures of the SEG dataset are more irregular.

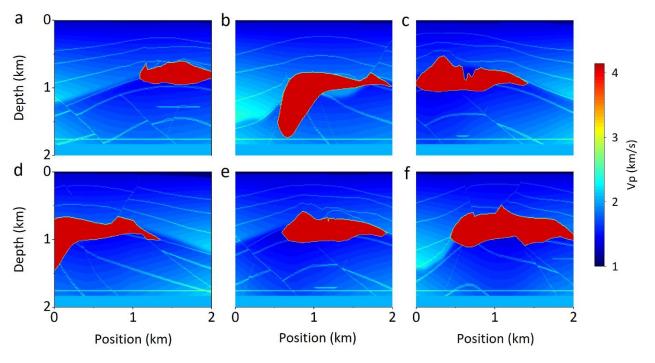


Figure 4. The SEG velocity models. (**a**–**f**) represent six representative velocity models from SEG dataset. The red geological targets represent salt, the blue represents background, and the white lines are stratigraphic boundaries.

CSP gathers, the input of the network, are generated through acoustic wave propagation (Equations (8) and (9)),

$$\frac{1}{v^2(x,z)}\frac{\partial^2 u(x,z,t)}{\partial t^2} = \left(\frac{\partial^2 u(x,z,t)}{\partial x^2} + \frac{\partial^2 u(x,z,t)}{\partial z^2}\right) + s(t) \tag{8}$$

$$s(t) = \left[1 - 2(\pi f_m t)^2\right] e^{-(\pi f_m t)^2}$$
(9)

where (x, z) is spatial coordinates and t is propagation time, v(x, z) denotes P-wave velocity, u(x, z, t) denotes the acoustic wavefield, s(t) is the Ricker wavelet, and f_m is the dominant frequency. We use the same parameters for both datasets (Table 1). According to the workflow mentioned above, we generated 3000 and 500 pairs of synthetic seismograms from the synthetic and SEG velocity models, respectively.

Table 1. Parameters of forward modeling. Forward modeling using finite difference method (FDM). All surface points are treated as receivers for each shot to simulate a full coverage seismic survey. Therefore, the number of receivers is 200 and the maximum offset is 2 km. The absorption boundary is perfectly matched layers (PMLs).

| Algorithm | Receiver Interval | Time Interval | Maximum Travel Time | Dominant Frequency | Source Points | Source Interval | Boundary Condition |
|-----------|----------------------|---------------|------------------------|-----------------------|---------------|-----------------|-----------------------|
| FDM | 10 m | 0.003 s | 3 s | 25 Hz | 6 | 28.5 m | PML |

3.2. Implementation Details

Before the training process, it is necessary to standardize the seismic records (Equation (10), which can improve training efficiency and prevent the vanishing gradient problem,

$$x' = \frac{x - mean(x)}{std(x)} \tag{10}$$

where *x* denotes raw seismogram, mean(x) and std(x) are mean value and standard deviation of *x*, respectively, and *x'* denotes standardized seismogram.

The neural network is implemented with PyTorch [64]. Seismograms of six sources corresponding to a synthetic velocity model are fed into the network. The training parameters are documented in Table 2. Here we conduct all the experiments in the same computer configuration. All network training in the study is performed on a GPU, model Tesla V100. Three thousand pairs of synthetic data are randomly split into two groups: training and test sets, in which there are 2700 and 300 pairs, respectively. The trained AG-ResUnet serves as an initial model to continue training on the SEG dataset in transfer learning. Five hundred pairs of SEG data are randomly split into 400 pairs of the training sets and 100 pairs of test sets. The transfer learning parameters are the same as in Table 2, except the epoch is 100.

Table 2. Parameters of training process in all networks.

| Optimizer | GPU | Learning Rate | Weight Decay | Batch Size | Epoch |
|-----------|------------|-------------------|-------------------|------------|-------|
| Adam | Tesla V100 | $1 	imes 10^{-3}$ | $1 	imes 10^{-4}$ | 8 | 200 |

4. Results

4.1. Optimized Performance of Loss Function

We first test the network optimization effect of the mix loss function from the synthetic dataset. Figure 5 exhibits the prediction results of AG-ResUnet trained with L_{MSE} and L_{Mix} , respectively. Notice that ground truth is included as the reference. It is clear that the third row of estimated models by L_{Mix} is close to the ground truth (Figure 5). For instance, the layered structure is clearly recovered in Figure 5b. Compared with L_{MSE} , inversion results of L_{Mix} have more accurate salt dome boundaries (Figure 5). The inversion results of the L_{Mix} are better than those of the L_{MSE} in terms of SSIM, PSNR, and R^2 .

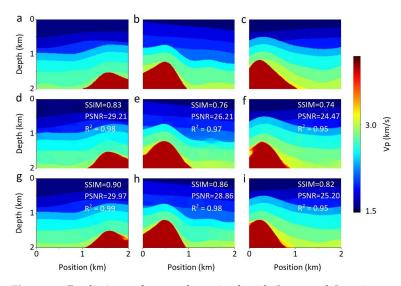


Figure 5. Predictions of networks trained with L_{MSE} and L_{Mix} in synthetic salt domes dataset. (**a–c**) Ground truth, (**d–f**) L_{MSE} results, and (**g–i**) L_{Mix} results. The red geological targets represent salt domes, and other layered structures surround formations. The quantitative metrics in the inversion image are calculated from the inversion result and the corresponding ground truth; the metrics in the following inversion image are calculated this way.

The metrics relationships between the L_{MSE} and L_{Mix} for the test set are shown in Figure 6. When the epoch is less than 25, the *PSNR* and R^2 values of L_{MSE} are slightly higher than those of L_{Mix} . As the training epoch increases, the test loss of L_{Mix} converging is lower than that of L_{MSE} , and *PSNR* and *SSIM* are significantly higher than those of L_{MSE} . The R^2 values of the two loss functions show the same change trend, both close to 1. From the test set evaluation metrics (Table 3), the loss value of L_{Mix} is an order of magnitude lower than that of L_{MSE} , while training times for both loss functions are almost the same. Tests based on synthetic data demonstrate that the L_{Mix} improves the inversion accuracy without introducing extra computation. The L_{Mix} is also used to train other networks in this study.

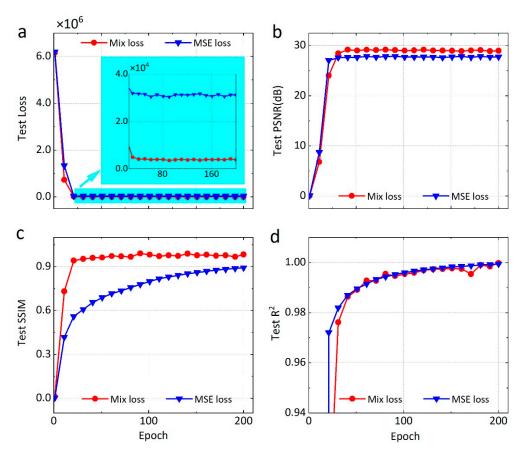


Figure 6. Comparison of quantitative metrics versus number of epoch between AG-ResUnet trained with L_{MSE} and L_{Mix} in synthetic salt domes dataset. (a) Test Loss, (b) test *PSNR*, (c) test *SSIM*, and (d) test R^2 . The red line represents the network's performance trained by the mix loss function, and the blue line represents the *MSE* loss function training result.

Table 3. Performance metrics statistics and cost time of different loss functions on synthetic test set.

| Dataset | Loss Function | Loss | PSNR | SSIM | <i>R</i> ² | Time |
|---------|------------------|----------|-------|--------|-----------------------|------------|
| Test | L _{MSE} | 29528.22 | 28.00 | 0.8636 | 0.9994 | 299 m 17 s |
| | L _{Mix} | 3577.75 | 29.31 | 0.9903 | 0.9999 | 308 m 13 s |

4.2. Qualitative Comparison

4.2.1. Comparison with Time-Domian FWI

Compared with AG-ResUnet and time-domain FWI, we analyzed performance between DL and the traditional inversion method. The forward parameters of FWI are assigned according to Table 1, and the conjugate gradient optimizer is employed. The inversion comparison based on the synthetic dataset is shown in Figure 7. Compared with the predictions and metrics of FWI, our approach has a more remarkable performance. The subsurface velocity models from AG-ResUnet (Figure 7j–l) are almost the same as the ground truth. Figure 7k,l can easily identify the salt dome outline. Although FWI can also retrieve the stratigraphic structure (Figure 7g–i), the resolution is low and structural boundaries are fuzzy.

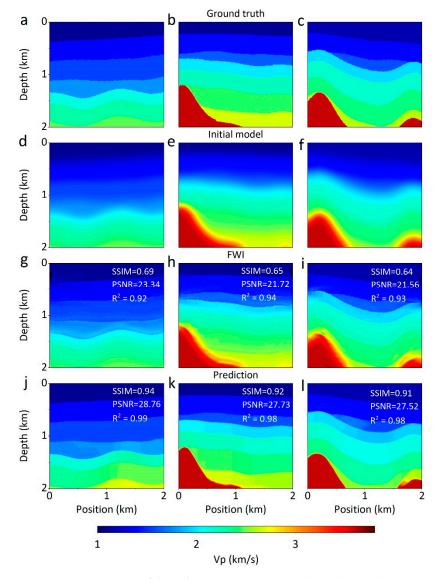


Figure 7. Comparisons of the velocity inversion in synthetic dataset. (**a**–**c**) Ground truth, (**d**–**f**) initial velocity model of FWI, (**g**–**i**) results of FWI, and (**j**–**l**) predictions of our method. The initial model is the true velocity model after Gaussian smoothing with mean 0 and variance 8.

To compare the efficiency of velocity inversion between FWI and AG-ResUnet, we illustrate the time consumed by FWI and AG-ResUnet for the training and inversion processes with the synthetic dataset (Table 4). Compared with FWI inversion time, the prediction time of the trained network is only 1.8 s, and the computational efficiency is much higher. To visualize the performance of our method, we plot the corresponding pseudo-logging data at the same position (Figure 8). It is clear that the velocity profiles of AG-ResUnet are nearly identical to the ground truth, but FWI velocity profiles exhibit slight fluctuations. What is more important is that AG-ResUnet is capable of precisely capturing the sudden velocity change (Figure 8).

| Dateset | Process | AG-ResUnet | FWI |
|-----------|-----------|--------------|--------------|
| Synthetic | Training | 308 min 13 s | N/A |
| | Inversion | 1.8 s | 272 min 54 s |

Table 4. Time consumed of FWI and AG-ResUnet for the training and inversion processes in synthetic dataset. N/A indicates that FWI had no training time.

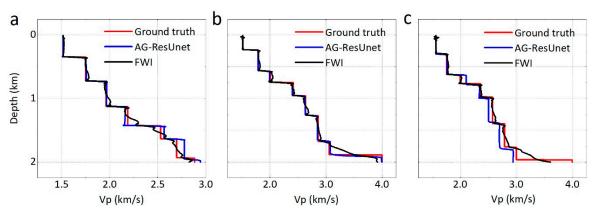


Figure 8. Vertical velocity comparison. (**a**–**c**) Pseudo-logging data of the ground truth (the models shown in Figure 7a–c) and corresponding predicted velocity at the position of 750 m. The red line represents the ground truth, the blue is the velocity variation of AG-ResUnet, and the black is FWI.

4.2.2. Comparison with Other Networks

To further demonstrate the superiority of AG-ResUnet for complex model inversion, we compared our approach with the other four DL algorithms, i.e., Unet, ResUnet, PSPnet, and DeepLab v3+, which have excellent ability in the segmentation field [65–68]. We apply the aforementioned network models to perform network training on synthetic datasets. We select predictions in the synthetic dataset to comprehensively compare the performance of algorithms (Figure 9). Generally, the results inverted by each network show relatively uniform and accurate velocity distribution. From the predictions and metrics, the velocity model predicted by AG-ResUnet is very close to the ground truth regarding the subsurface interface. Taking the salt dome structure as an example, AG-ResUnet successfully predicts the converging boundaries, while the predictions of other networks have significant errors.

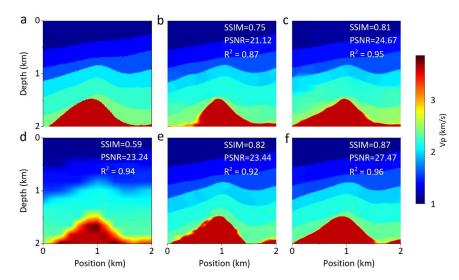


Figure 9. Comparison of inversion results in different networks. (a) Ground truth, (b) Unet, (c) ResUnet, (d) PSPnet, (e) DeepLab v3+, and (f) AG-ResUnet. The metrics in each result show the inversion effect in terms of quantification.

In order to comprehensively compare the trend of the metrics during the training process of each network, we show the metrics history of five networks from the test set (Figure 10). The metrics of each network share the same trend. The loss term decreases significantly within 25 epochs and then converges smoothly. Besides, AG-ResUnet has the lowest test loss, and *PSNR* and *SSIM* reached a high point. We summarize the performance metrics statistics and cost time of five networks with the synthetic test set (Table 5), indicating that AG-ResUnet is more capable of extracting boundary features than other methods. Although it has the longest training time, the time difference is within one order of magnitude, and performance is totally acceptable given the competing power. The inversion comparison of AG-ResUnet and ResUnet shows that the AG unit is of great importance for enhancing the capability of identifying the subsurface velocity boundaries.

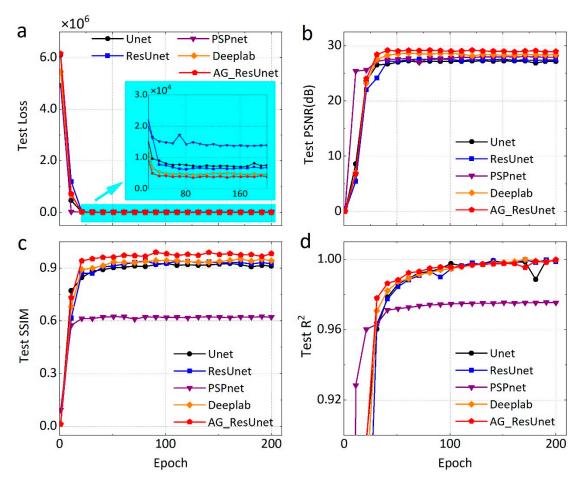


Figure 10. Quantitative metrics history of five networks trained in synthetic test dataset. (**a**) Test Loss, (**b**) test *PSNR*, (**c**) test *SSIM*, and (**d**) test *R*².

Table 5. Performance metrics statistics and cost time of five networks on synthetic test set. The bold numbers in this table indicate excellent performance.

| Dataset | Network | Loss | PSNR | SSIM | <i>R</i> ² | Time |
|---------|-------------|----------|-------|--------|-----------------------|--------------|
| | Unet | 7046.72 | 27.30 | 0.9265 | 0.9994 | 208 min 56 s |
| | ResUnet | 6045.29 | 28.05 | 0.9490 | 0.9998 | 295 min 26 s |
| Test | PSPnet | 13662.38 | 28.03 | 0.6249 | 0.9756 | 157 min 48 s |
| | Deeplab v3+ | 4290.33 | 28.76 | 0.9545 | 0.9996 | 247 min 38 s |
| | AG-ResUnet | 3577.75 | 29.31 | 0.9903 | 0.9999 | 308 min 13 s |

4.3. Network Generalization

In order to demonstrate the generalization of our method, we perform the transfer learning process with the SEG dataset and present the prediction results in Figure 11.

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The transfer learning results have accurately reconstructed the SEG salt body and clear subsurface interface. The inversion results and evaluation metrics on the SEG dataset demonstrate that transfer learning contributes to improving the trained network to identify new geological features.

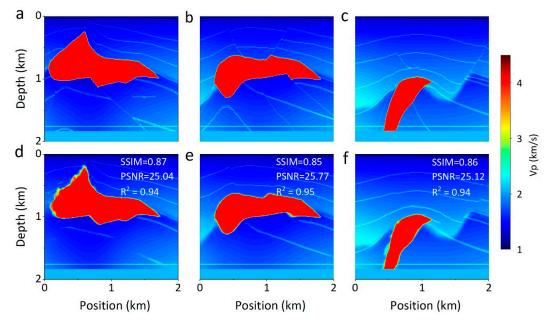


Figure 11. SEG salt body model inversion results. (**a**–**c**) SEG models, (**d**–**f**) Salt body predictions by AG-ResUnet. The ground truth models are included as the reference.

To further demonstrate the training effect of transfer learning on mini-batch datasets, we show the metrics trend for transfer learning from the SEG test set (Figure 12). The loss term decreases considerably within ten epochs and then converges gradually. Similarly, *PSNR*, *SSIM*, and R^2 rise substantially within ten epochs and stabilize around a high level. The training metrics of the mini-batch SEG dataset are close to the level of the synthetic dataset. In particular, *PSNR* is stable at around 31 dB, which has exceeded the training performance on the synthetic dataset. The results show that our method is generalizable, and the trained AG-ResUnet can be an initial model to predict other datasets.

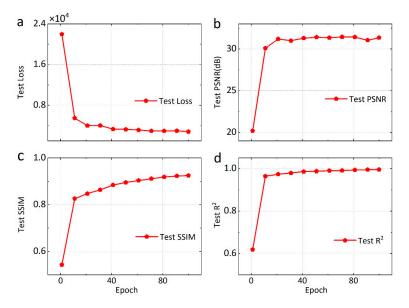


Figure 12. Quantitative metrics history of AG-ResUnet transfer learning in SEG test dataset. (a) Test Loss, (b) test *PSNR*, (c) test *SSIM*, and (d) test R^2 .

4.4. Network Stability

To analyze the stability of AG-ResUnet, we conduct additional experiments using both synthetic and SEG datasets under more realistic conditions. When the synthetic dataset is contaminated with a certain level of noise, the majority of predictions from AG-ResUnet (Figure 13) are close to the ground truth. However, overall predictions are slightly lower than those of clean data. When the noise level is low (e.g., SNR = 20 dB), it is clear in Figure 13d–f that the stratigraphic interfaces and the salt domes of inversion are accurate, with almost no artifacts. With the increase of noise (e.g., SNR \leq 10dB), the inversion results become poor, yet the velocity model can still be correctly recovered. Moreover, we use AG-ResUnet to invert the noise-contaminated SEG dataset, and the inversion results still show the aforementioned regularity (Figure 14). The results demonstrate that our method can yield a stable velocity model from noise-contaminated data.

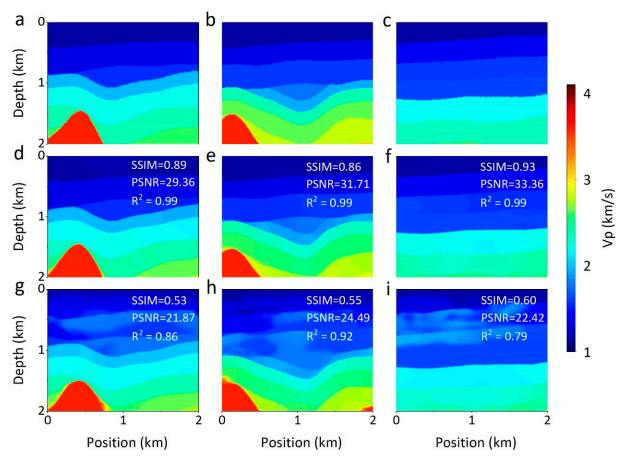


Figure 13. Sensitivity of our method to noise in synthetic dataset. (**a**–**c**) Ground truth, (**d**–**f**) predictions with 20 dB noisy seismic data, and (**g**–**i**) predictions with 10 dB noisy seismic data.

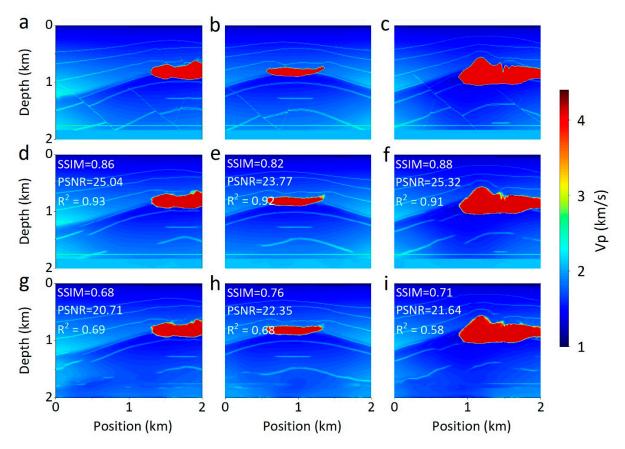


Figure 14. Sensitivity of our method to noise in SEG dataset. (**a**–**c**) SEG models, (**d**–**f**) predictions with 20 dB noisy seismic data, and (**g**–**i**) predictions with 10 dB noisy seismic data.

5. Discussion

We have demonstrated that our method can produce reliable results. The outstanding performance of AG-ResUnet benefits from the attention structure (i.e., AG). As illustrated by the workflow of AG in Section 2, this module fully considers the feature structure (e.g., geological targets and stratigraphic boundaries) extracted from the feature maps of the downsampling and upsampling process. We serve the feature structure extracted by AG as the training constraints. Therefore, our algorithm maintains a high boundary processing ability when dealing with different feature data. More importantly, the gating signal for each skip connection aggregates information from multiple scales, as shown in Figure 1, which increases the resolution of the reconstructed model and achieves better performance. By taking advantage of AG, our method is more robust and sensitive in extracting structural boundary information indicated by the inversion results of AG-ResUnet and other methods (Figures 7–9). Another reason for introducing AG into the network is that it is simple to calculate. Notice that the AG parameters can generally be updated by back-propagation without employing a sampling-based method used in the traditional attention mechanisms [69]. Comparing the quantitative metrics of ResUnet and AG-ResUnet in Table 5, it is clear that the training time remains the same after introducing AG. In other words, AGs can be easily integrated into traditional CNN architectures with minimal computational overhead, increasing the model sensitivity and accuracy for predicting velocity models.

Moreover, the characterization of structural details also depends on the loss function. The error sensitivity approach (e.g., *MSE* loss function, mean absolute error (MAE) loss function) computes the error between each pixel of the prediction and the ground truth, which is insufficient to extract relevant structural information between each element. Therefore, to further improve the resolution of inversion, we introduce the *SSIM* into the loss function, which has been widely used in geophysical research, e.g., seismic velocity inversion [56] and seismogram super-resolution reconstruction [70]. The *SSIM* term comprehensively

evaluates the structural similarity of the retrieved model so that the network optimization focuses on structural information. The inversion comparison in Figures 5 and 6 shows that the combination of *MSE* and *SSIM* terms contributes to estimating accurate velocity models and mitigating the problem of non-uniqueness in inversion.

By taking advantage of the aforementioned improvements, the transfer learning and stability experiment results demonstrate that AG-ResUnet has a strong generalization, robustness, and the potential of processing field data. Due to the difference in the geological structure of different areas, it is reasonable to apply transfer learning to field data processing [49,71,72]. Transfer learning, in which we use the previously trained network as an initial network for an updated velocity model due to the change in geological conditions, can reduce the training costs and be adapted to more general field conditions. Notice that transfer learning also requires sufficient data. However, field data are usually very limited, making it difficult for data-driven DL methods to work. Nowadays, the physics-guided networks, relying on prior physical information, show the potential to solve the velocity inversions in case of limited data [73–75]. Therefore, future work may pay more attention to using prior physical information to guide the optimization of the network.

6. Conclusions

This study proposes a novel and practical approach to directly estimate velocity models from seismic data using AG-ResUnet. We have demonstrated that our approach can produce a reliable result. Our method has two advantages in network architecture and loss function. The AG-ResUnet network introduces attention gates that can adaptively detect areas with significant velocity changes, equivalent to introducing additional structural constraints for processing. Furthermore, the mix loss function contributes to the inversion results approximating the ground truth. By taking advantage of these two improvements, our method enhances the ability of DL algorithms to characterize detailed structures and retrieve complex velocity models.

AG-ResUnet provides a reliable way to strengthen the complex structure extraction from CSP gathers, showing excellent transfer learning properties and strong generalization. Experimental results demonstrate that our approach yields a stable and robust estimation in the cases of noisy data. To summarize, AG-ResUnet has high accuracy, strong generalization, and robustness for velocity inversion. Our method contributes to imaging geologic reservoirs, CO_2 monitoring, and geotechnical engineering.

Author Contributions: Conceptualization, F.L. and D.G.; Formal analysis, F.L., X.P., J.L. and Y.W.; Funding acquisition, F.L., Z.G. and J.L.; Methodology, F.L., Z.G. and D.G.; Writing—original draft, F.L.; Writing—review & editing, F.L., Z.G. and D.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research is supported by the Natural Science Foundation of China (NSFC), 42074169; Major project of Hunan Province science and technology innovation(2020GK1021), and Graduate Independent Exploration and Innovation Project of Central South University, 2022ZZTS0556.

Data Availability Statement: The SEG salt model dataset can be obtained in SEG website (https://wiki.seg.org/wiki/Open_data (accessed on 10 December 2021). The synthetic dataset and network models associated with this research can be obtained by contacting the corresponding author.

Acknowledgments: We appreciate the editor for their suggestions to improve this paper. We also appreciate SEG for their open-access dataset, which is the base of transfer learning. The PyTorch libraries (https://pytorch.org (accessed on 15 March 2021) are used to train and build our deep-learning model.

Conflicts of Interest: The authors declare no conflict of interest.

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