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Multi-Resolution STAP for Enhanced Ultra-Low-Altitude Target Detection

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Abstract: In this paper, an ultra-low-altitude target (ULAT) detection approach, referred to as the multi-resolution space-time adaptive processing (STAP), is proposed to enhance the target detection performance in a missile-borne radar system. In this respect, the whole base band is divided into a series of equal-width and center-frequency-diverse sub-bands with the frequency diversity technique, which enhances the multipath-target coupled (MTC) effect with the decreased range resolution. Hence, it is feasible to exploit the multipath signal power to improve the output signal-to-clutter-plus-noise ratio (SCNR) performance of sub-band STAP. In this regard, the mechanism of the MTC effect is analyzed numerically for the efficient sub-band STAP. However, such SCNR improvement is achieved at the cost of target tracking performance loss. Hence, the full-band STAP is further applied for multipath-target separation based on the target range-Doppler locations detected by the joint multiple sub-bands $\Sigma\Delta$ -STAP, which also alleviates the dynamic target attenuation and the corresponding target Doppler history corruption within the long coherent processing interval (CPI). On this basis, the SCNR performance is further improved by applying coherent accumulation among sub-CPIs, in which the clutter suppression performance degradation and coherent accumulation loss of STAP are alleviated within the sub-CPIs. Numerical and measured results corroborate the effectiveness of ULAT detection with the considered multi-resolution STAP.

Keywords: frequency diversity; multi-resolution; multipath signal; space-time adaptive processing (STAP); ultra-low-altitude target



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1. Introduction

For a missile-borne radar, it is challenging to detect an ultra-low-altitude target (ULAT), due to the strong clutter environment and the specular multipath interference adjacent to the target, which leads to target attenuation and target tracking performance loss [1,2]. Moreover, within the long coherent processing interval (CPI), the coherent accumulation loss is induced by the target range walking (RW) and dynamic target-glint (TG) issues, while the clutter suppression suffers from the nonhomogeneous clutter sample problem induced by the RW and nonstationary electromagnetic scattering of clutter [3]. Hence, it requires an enhanced ULAT detection algorithm in the missile-borne radar system.

Space-time adaptive processing (STAP) algorithms have been studied widely in [4–7], and the STAP weight vector essentially consists of a whitening operation followed by a spatial-temporal 2D match filter. However, the adjacent multipath interference is strongly coupled with the ULAT in the joint range-Doppler (R-D) domain, which inevitably corrupts the target detection performance of STAP based approaches. In specific, when the multipath interference is mistakenly contained in the training samples for clutter covariance matrix (CCM) estimation, the spatial-temporal two-dimensional (2D) main lobe of the STAP filter will be corrupted, which is referred to as the target pollution problem. In this regard, a joint magnitude and phase constrained STAP approach was proposed in [6], which has advantages in the anti-interference match filtering with the main lobe shape-preserving

technique. However, even though the multipath induced target pollution is circumvented, the multipath-target coupled (MTC) effect causes the dynamic TG problem in the narrow-band radar system, leading to the target attenuation and target tracking performance loss. To tackle this issue, the wide-band radar system can be applied instead, in which the improved range resolution alleviates the MTC effect and the TG problem, correspondingly. However, the problems arise with the wide-band radar system, such as the target splitting and the RW of both the target and clutter [8], which also degrade the target detection performance.

In order to avoid the target attenuation from the TG problem, it is important to decouple the multipath interference from the target. Spatial filtering based methods are proposed for the multipath interference separation [9–11]. However, the accuracy is far from enough for the robust separation purpose. Moreover, the temporal coherence differences induced by the sea surface roughness are also utilized for the multipath interference separation from target [12]. However, such differences are less reliable because of the complex and stochastic roughness of sea surface. Based on the received signal model in the multiple-input multiple-output radar system, a direction of arrival estimation algorithm is developed for resolution enhancement by optimizing the multipath reflected coefficient properties for the transmitter and receiver [13].

In this paper, a multi-resolution joint processing approach is proposed with the STAP-based radar system utilizing different bandwidths, which is referred to as the multi-resolution STAP. Instead of regarding the multipath signal as a destructive interference to be separated, we consider the multipath signal as a constructive signal component, whose power is supposed to contribute to the target output power, since it is strongly coupled with the ULAT signal. To improve the signal-to-clutter-plus-noise ratio (SCNR) performance with the multipath signal power, the sub-band segmentation is introduced to enhance the MTC effect with the frequency diversity technique [14–16], following the magnitude-phase MTC effect mechanism analysis derived for the efficient SCNR-enhanced sub-band STAP. Furthermore, with the joint complementary detection among independent sub-band STAP outputs, the probability of detection (P_d) can be improved on the basis of every single involved independent sub-band STAP output. However, the reduced range resolution in the strong MTC effect leads to target tracking performance loss in the missile-borne radar system. To tackle this issue, the full-band STAP is applied for multipath-target separation, based on the target R-D locations detected by the joint multiple sub-bands STAP. In this stage, the dynamic TG problem is removed by multipath-target decoupling, which circumvents the possible TG-induced target attenuation and corresponding Doppler history corruption within the long CPI. On this basis, the SCNR performance is further improved by applying coherent accumulation among sub-CPIs, in which the clutter suppression performance degradation and coherent accumulation loss of STAP are alleviated within the sub-CPIs. The novelties and contributions of this paper are summarized as follows.

- i We not only circumvent the target attenuation induced by the multipath signal, but also exploit the multipath signal power to enhance the ULAT detection performance. In specific, the mechanism of the MTC effect is derived for the fast searching of SCNR-enhanced sub-band distribution. Furthermore, the consistent CCM estimation scheme is proposed in the sub-band STAP to reduce computational burden, and the clutter training sample selection scheme is also proposed to satisfy the sample demand.
- ii By dividing the whole base band into multiple independent sub-bands, it is feasible to improve the target detection performance with the joint complementary detection among multiple sub-band STAP outputs.
- iii On the basis of the SCNR-enhanced sub-band STAP detection results, the dynamic TG-induced target attenuation is further alleviated by multipath-target decoupling with the full-band STAP, which constrains the target Doppler history and provides the foundation for further SCNR improvement within the long-CPI.

The remainder of this paper is organized as follows. Section 2 establishes the signal model. The proposed multi-resolution STAP is presented in Section 3. Section 4 shows numerical and experiment results, and the conclusion is drawn in Section 5.

2. Signal Model

2.1. Geometric Configuration and Multipath-Target Signal Model

Without loss of generality, Figure 1 shows the geometric configuration of a missile-borne radar system. The radar seeker is employed in the forward-looking mode, as a common practice. Note that the beam is steered to a low-grazing angle area to alleviate the geometry-induced range dependence of clutter in a forward-looking mode [17], and the platform's flight stability is also improved with the stabilization algorithm [18]. The velocity of radar is v_a , which is along the X-axis. The radar seeker is assumed to be a sum-and-difference ($\Sigma\Delta$) channel based radar, which has been widely applied in the missile-borne radar system. Assume that K pulses are transmitted during a CPI, and the pulse repetition frequency (PRF) is defined as f_r . Note that the O-XY plane is the sea, and the Z-axis is perpendicular to the sea.

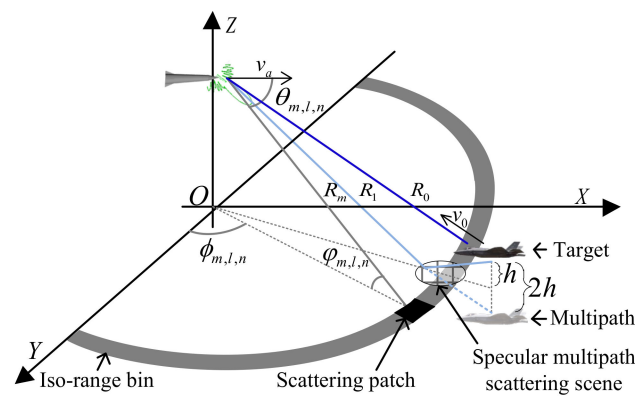


Figure 1. Geometric configuration of missile-borne radar.

The slant range of the target corresponding to the k -th pulse is denoted by $R_{0,k}$, i.e.,

$$R_{0,k} = R_0 - v_a \cos(\theta_0)T(k-1) - v_0T(k-1), \quad (1)$$

where R_0 and v_0 denote the initial slant range and initial radical velocity of the target, respectively, θ_0 is the conic angle with respect to v_a , and $T = 1/f_r$ is the pulse repetitive time (PRT). Here the second-order or even higher-order terms of the slant range are ignored within the limited CPI. The multipath echo signal follows the classical four-path model [1,2], which is a range-extended target signal regarded as the summation of echoes reflected from many scattering patches in the specular multipath scattering scene. However, for the sake of simplicity, we consider the multipath signal as a point-target-like signal with a specific slant range $R_{1,k}$; that is

$$R_{1,k} = \sqrt{R_{0,k}^2 + (2h)^2 + 4R_{0,k}h \sin(\varphi_0)}, \quad (2)$$

where φ_0 denotes the grazing angle of the target. The transmitted LFM signal is written as

$$\Psi(k, \tau) = \psi(\tau)e^{j2\pi f_c t} = \text{rect}\left(\frac{\tau}{T_p}\right)e^{j\pi\gamma\tau^2}e^{j2\pi f_c t}, \quad (3)$$

where $\psi(\tau)$ is the base band waveform of the transmitted LFM signal, $\text{rect}(x) = \begin{cases} 1, & |x| \leq 1/2 \\ 0, & |x| > 1/2 \end{cases}$ is the window function, T_p is the pulse width, f_c is the carrier frequency, γ is the chirp ratio, τ is the fast-time, $t = (k-1)T + \tau$ is the total time. By ignoring the Doppler frequency

modulation on the fast-time dimension, the base band signal of the target is derived after digital down conversion in the k -th pulse, which is expressed as

$$x_T(k, \tau) = \xi_0 \text{rect}\left(\frac{\tau - 2R_{0,k}/c}{T_p}\right) e^{j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} e^{-j\frac{4\pi f_c R_{0,k}}{c}}, \quad (4)$$

where ξ_0 is the coefficient, and c is the light speed. Substitute $R_{1,k}$ for $R_{0,k}$ in (4), we can derive the multipath signal as

$$x_M(k, \tau) = \xi_1 \text{rect}\left(\frac{\tau - 2R_{1,k}/c}{T_p}\right) e^{j\pi\gamma(\tau - \frac{2R_{1,k}}{c})^2} e^{-j\frac{4\pi f_c R_{1,k}}{c}}, \quad (5)$$

where ξ_1 denotes the synthetic coefficient of the multipath echo signal. After implementing fast Fourier transform with respect to the fast-time dimension, the target signal is further expressed in the pulse-range frequency domain as

$$x_T(k, f) = \xi_0 \psi(f) e^{-j\frac{4\pi(f_c + f)R_{0,k}}{c}}, \quad (6)$$

where $\psi(f)$ is the spectrum of $\psi(\tau)$, and it is distributed within the whole transmitted bandwidth $B = \gamma T_p$, while the spectral leakage is outside the transmitted bandwidth. For a particular case, $B = 80$ MHz, the center frequency $f_{c,0} = 0$ MHz, the sampling frequency $f_s = 192$ MHz, the $\psi(f)$ is shown in Figure 2. Moreover, the frequency diversity based sub-band segmentation is demonstrated in Figure 2, in which the whole base band is divided into equal-width and non-overlapped sub-bands with different colors.

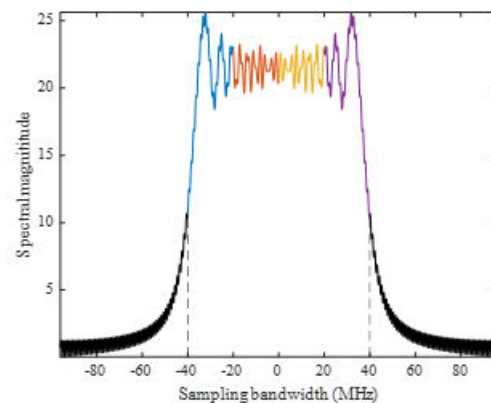


Figure 2. Demonstration of frequency diversity-based sub-band segmentation.

In fact, as the general case, the equal-width and overlapped sub-band segmentation scheme is carried out with the sliding window technique for the proposed sub-band STAP, specified by the sub-band width B_{sub} and the sub-band center frequency interval Δf . It is also seen from the Figure 2 that, even though there are some envelope differences between sub-band spectrum segments, we can still derive the center-frequency-modulation (CFM) based sub-band signals for the MTC effect mechanism analysis, according to the time-frequency linearly coupled LFM signal. Taking the target signal as an example, the zero-center-frequency sub-band signal is derived as the reference sub-band signal; that is

$$\begin{aligned} x_{T,sub,0}(k, \tau) &= \xi_0 \psi_{sub}\left(\tau - \frac{2R_{0,k}}{c}\right) e^{-j\frac{4\pi f_c R_{0,k}}{c}} \\ &= \xi_0 \text{rect}\left(\frac{\gamma(\tau - 2R_{0,k}/c)}{B_{sub}}\right) e^{j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} e^{-j\frac{4\pi f_c R_{0,k}}{c}} \end{aligned} \quad (7)$$

where $\psi_{sub}(\tau)$ is the sub-band waveform with the bandwidth B_{sub} . Then, we can further derive the i -th sub-band signal via CFM, i.e.,

$$x_{T,sub,i}(k, \tau) = e^{j2\pi\Delta f_i(\tau - \frac{2R_{0,k}}{c})} x_{T,sub,0}(k, \tau), \quad (8)$$

where Δf_i denotes the center frequency for $x_{T,sub,i}(k, \tau)$, and the corresponding sub-band waveform is expressed as

$$\psi_{sub,i}(\tau) = e^{j2\pi\Delta f_i\tau} \psi_{sub}(\tau). \quad (9)$$

Implementing range compression to $x_{T,sub,i}(k, \tau)$ with the matched filter coefficients $\psi_{sub,i}(\tau)$, it yields

$$\begin{aligned} & \bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\ &= \xi_0 e^{-j\frac{4\pi f_c R_{0,k}}{c}} e^{j2\pi\Delta f_i(\tau - \frac{2R_{0,k}}{c})} \left(\frac{B_{sub}}{\gamma} - \left(\tau - \frac{2R_{0,k}}{c} \right) \right) \\ & \sin c \left[\gamma \left(\tau - \frac{2R_{0,k}}{c} \right) \left(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma} \right) \right] \end{aligned} \quad (10)$$

where the $\bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub})$ denotes the range-compressed complex envelope of $x_{T,sub,i}(k, \tau)$. The derivation of the sub-band range-compressed envelope can refer to the Appendix A, while we define the range-compressed multipath signal as $\bar{x}_{M,sub,i}(k, \tau, \Delta f_i, B_{sub})$ in the same way, i.e.,

$$\begin{aligned} & \bar{x}_{M,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\ &= \xi_1 e^{-j\frac{4\pi f_c R_{1,k}}{c}} e^{j2\pi\Delta f_i(\tau - \frac{2R_{1,k}}{c})} \left(\frac{B_{sub}}{\gamma} - \left(\tau - \frac{2R_{1,k}}{c} \right) \right) \\ & \sin c \left[\gamma \left(\tau - \frac{2R_{1,k}}{c} \right) \left(\tau - \frac{2R_{1,k}}{c} - \frac{B_{sub}}{\gamma} \right) \right] \end{aligned} \quad (11)$$

It is observed from (10) and (11) that the range-compressed complex envelope of the sub-band signal depends on the Δf_i and B_{sub} . On the one hand, the range-compressed gain and resolution of the envelope decrease with the decreased sub-band width. On the other hand, the different center frequencies of sub-band distribution produce different impacts of phase modulation terms on the complex envelope. In general, it can be concluded that the MTC effect is generated from the joint magnitude-phase coupling of range-compressed envelope superposition, which depends on the sub-band segmentation.

2.2. Space-Time Signal Model

Considering that the clutter signal can be regarded as the summation of echoes reflected from many scattering patches in the scene, the range-compressed clutter echo corresponding to the i -th sub-band is modeled as

$$\begin{aligned} & \bar{x}_{C,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\ &= \sum_{m=1}^M \bar{x}_{C,m,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\ &= \sum_{m=1}^M \xi_m e^{-j\frac{4\pi f_c R_{m,k}}{c}} e^{j2\pi\Delta f_i(\tau - \frac{2R_{m,k}}{c})} \left(\frac{B_{sub}}{\gamma} - \left(\tau - \frac{2R_{m,k}}{c} \right) \right) \\ & \sin c \left[\gamma \left(\tau - \frac{2R_{m,k}}{c} \right) \left(\tau - \frac{2R_{m,k}}{c} - \frac{B_{sub}}{\gamma} \right) \right] \end{aligned} \quad (12)$$

$$R_{m,k} = R_m - v_a \cos(\theta_m) T(k-1), \quad (13)$$

where ξ_m is the coefficient of the scattering patch, R_m is the initial slant range, θ_m is the conic angle with respect to v_a . Note that (10)–(12) are all the range-compressed received echo signals of a single channel in the joint slow-time and fast-time domain, and the multi-

channel echo signal in the $\Sigma\Delta$ -channel based radar system is obtained with the weighting of antenna patterns, which is expressed as

$$\begin{aligned} \mathbf{x}(k, \tau, \Delta f_i, B_{sub}) &= \bar{\mathbf{x}}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) + \bar{\mathbf{x}}_{M,sub,i}(k, \tau, \Delta f_i, B_{sub}) + \\ &\quad \bar{\mathbf{x}}_{C,sub,i}(k, \tau, \Delta f_i, B_{sub}) + \mathbf{n} \\ &= \mathbf{s}_s(\phi_0, \varphi_0) \bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) + \\ &\quad \mathbf{s}_s(\phi_0, \varphi_M) \bar{x}_{M,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\ &\quad + \sum_{m=1}^M \mathbf{s}_s(\phi_m, \varphi_m) \bar{x}_{C,m,sub,i}(k, \tau, \Delta f_i, B_{sub}) + \mathbf{n} \end{aligned} \quad (14)$$

$$\mathbf{s}_s(\phi, \varphi) = [P_\Sigma(\phi, \varphi), P_{\Delta,\phi}(\phi, \varphi), P_{\Delta,\varphi}(\phi, \varphi)]^T, \quad (15)$$

where the superscript $(\cdot)^T$ denotes the transpose operator, $\mathbf{s}_s(\phi, \varphi)$ denotes the spatial steering vector consisting of complex weighting coefficients $P_\Sigma(\phi, \varphi)$, $P_{\Delta,\phi}(\phi, \varphi)$ and $P_{\Delta,\varphi}(\phi, \varphi)$ from the sum, azimuth-difference and elevation-difference antenna patterns, respectively, ϕ_m and φ_m denote the corresponding azimuth and grazing angles for clutter, respectively, the ϕ_0 denotes the azimuth angle for both target and multipath signals, the φ_1 denotes the grazing angle for the multipath signal, and the $\mathbf{n}$ denotes the additive white Gaussian noise (AWGN). Note that the multi-channels echo signal $\mathbf{x}(k, \tau, \Delta f_i, B_{sub}) \in \mathbb{C}^{3 \times 1}$ is a sample of $\Sigma\Delta$ -channels for a given fast-time and slow-time instance. By stacking echoes corresponding to all K pulses into a vector, it yields

$$\mathbf{x}(\tau, \Delta f_i, B_{sub}) = [\mathbf{x}^T(1, \tau, \Delta f_i, B_{sub}), \mathbf{x}^T(2, \tau, \Delta f_i, B_{sub}), \dots, \mathbf{x}^T(K, \tau, \Delta f_i, B_{sub})]^T, \quad (16)$$

where $\mathbf{x}(\tau, \Delta f_i, B_{sub}) \in \mathbb{C}^{3K \times 1}$ denotes the space-time 2D data corresponding to a particular fast-time time instance for the i -th sub-band signal, which includes the possible target, multipath, clutter, and AWGN.

3. Multi-Resolution STAP Based ULAT Detection

3.1. SCNR-Enhanced Sub-Band STAP

3.1.1. Mechanism Analysis of MTC Effect

It is observed from (10) that there is a delay-dependent phase modulation term $e^{j2\pi\Delta f_i(\tau - \frac{2R_{0k}}{c})}$ for the range-compressed complex envelope of a point target, which is the key to the SCNR-enhanced sub-band STAP. With the sub-band distribution searching with respect to the Δf_i and B_{sub} , it is feasible to achieve the in-phase magnitude-phase superposition of complex envelopes for two adjacent point targets, producing a stronger output power with the so-called SCNR-enhanced sub-band distribution. Motivated by this concept, the power of the adjacent specular multipath signal can contribute to the target output power on the magnitude-phase superposed complex envelope, by exploiting the strong MTC effect with a properly reduced B_{sub} . However, the SCNR-enhanced sub-band distribution is unknown because of the dynamic and stochastic scattering coefficients, especially for the environment-coupled ξ_1 . Hence, it will lead to a heavy computational burden for the sample matrix inverse (SMI) [19,20] based sub-band STAP with the searching of Δf_i and B_{sub} .

For the purpose of enhanced ULAT detection, we introduce the concept of maximum possible output power in the joint magnitude-phase MTC effect, which represents the target peak envelope power achievable with the contribution of the multipath signal power. Here, for the sub-band distribution specified by a pair of Δf_i and B_{sub} , the signal-to-interference-ratio (SIR) is defined as

$$\text{SIR} = \frac{\max_{k,\tau} \left(|\bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub})|^2 \right)}{\max_{k,\tau} \left(|\bar{x}_{M,sub,i}(k, \tau, \Delta f_i, B_{sub})|^2 \right)}, \quad (17)$$

where the $\max_{k,\tau}(\cdot)$ denotes the maximum value operator within the value range of k and τ . According to (10) and (11), the condition of the in-phase envelope superposition will be reached when the delay-dependent phase modulation term compensates the phase difference between the multipath and target envelopes at $R_{1,k}$ and $R_{0,k}$; that is

$$\angle\left(\xi_0 e^{-j\frac{4\pi f_c R_{0,k}}{c}}\right) = \angle\left(\xi_1 e^{-j\frac{4\pi f_c R_{1,k}}{c}} e^{j2\pi\Delta f_i\left(\frac{2R_{0,k}}{c} - \frac{2R_{1,k}}{c}\right)}\right), \quad (18)$$

where $\angle(\cdot)$ denotes the phase angle calculation operator. Under this condition, the joint magnitude-phase superposed complex envelope reduces to the magnitude superposed envelope. The range-compressed gain and resolution of the envelope decrease with a smaller sub-band width, which decreases the peak magnitudes of multipath-target envelopes, and extends the envelopes between $R_{1,k}$ and $R_{0,k}$. Hence, the searching of B_{sub} is required for the SCNR-enhanced sub-band distribution.

In general, we can derive the mechanism of joint magnitude-phase MTC effect in theory, which are categorized as follows.

- i The Δf_i of the SCNR-enhanced sub-band distribution keeps constant with different B_{sub} , because the maximum possible output power is produced under the in-phase envelope superposition condition, which corresponds to a specific Δf_i .
- ii The SCNR performance trend of the sub-band STAP is stable and periodic with respect to the Δf_i of the sub-band distribution, because of the periodicity of the Δf_i -dependent phase coupling in the envelope superposition.

Based on this mechanism analysis, the searching of SCNR-enhanced sub-band distribution can be applied more efficiently by being simplified into two steps, which will be detailed in the next part. Moreover, the mechanism analysis of the MTC effect will be validated with numerical and measured results in Section 4.

3.1.2. Joint Multiple Sub-Bands $\Sigma\Delta$ -STAP

In this part, the $\Sigma\Delta$ -STAP [21–23] is applied within different sub-bands based on the aforementioned mechanism analysis, which is referred to as the joint multiple sub-bands $\Sigma\Delta$ -STAP. The statistical-methodology-based CCM estimation of STAP is expressed as

$$\hat{\mathbf{R}}_{CN,i} = \frac{1}{\|\Omega\|_0} \sum_{\tau \in \Omega} \mathbf{x}(\tau, \Delta f_i, B_{sub}) \mathbf{x}(\tau, \Delta f_i, B_{sub})^H, \quad (19)$$

where Ω denotes the sampling range of secondary data surrounding with the cell under test (CUT), the superscript $(\cdot)^H$ denotes the conjugate transpose operator. According to the well-known Reed–Mallett–Brennan rule for statistical-methodology-based STAP, the output SCNR loss is less than 3 dB, where the number of independent and identically distributed (I.I.D) training samples is more than twice of the system's degrees-of-freedom (DOFs) [20]. However, as seen from (12), the dependence of adjacent training samples increases dramatically with the decreased range resolution, which corrupts the I.I.D characteristic of the training samples and cause the CCM estimation errors due to insufficient training samples. To tackle such issue, we propose a training sample selection scheme for the sub-band STAP, which is categorized into two cases: (1) selecting training samples with a constant interval scaled by the ratio of the full-band width to sub-band width. In this way, the same training sample number, of twice the system DOFs, is required, which represents the same computation complexity involved in the CCM estimation as the full-band STAP condition. (2) Selecting adjacent training samples (e.g., the full-band STAP condition), while expanding the sampling range with the ratio of the full-band width to sub-band width. In this way, with more scaled training sample demand (twice the system DOFs), the more computation complexity will be involved in the CCM estimation. In practice, we can choose different cases in the training sample selection scheme according to the performance requirement. However, in either case, the sampling range will be inevitably expanded proportionally. Under this condition, even though the sample demand of inde-

pendent distribution characteristic is satisfied by the proposed training sample selection scheme, the demand for the identical distribution characteristic will be more difficult to be met simultaneously in the nonhomogeneous clutter condition. The nonhomogeneous clutter suppression methods are not detailed herein [24–28], since we mainly focus on the SCNR-enhanced sub-band STAP in this part.

We further derive the weight vector of the sub-band $\Sigma\Delta$ -STAP as

$$\mathbf{w}_{n,\phi,\varphi} = \mu \hat{\mathbf{R}}_{CN,i}^{-1} \mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi), \quad n = 0, 1, \dots, N-1, \quad \phi, \varphi \in \Theta, \quad (20)$$

$$\mu = \frac{1}{\mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi) \hat{\mathbf{R}}_{CN,i}^{-1} \mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi)}, \quad (21)$$

$$\mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi) = \mathbf{s}_t(f_{d,n}) \otimes \mathbf{s}_s(\phi, \varphi), \quad (22)$$

where $\mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi) \in \mathbb{C}^{3K \times 1}$ denotes the spatial-temporal 2D steering vector, and μ is the normalized factor, $\mathbf{s}_t(f_{d,n})$ is the temporal steering vector corresponding to the normalized Doppler sampling frequency $f_{d,n}$, i.e.,

$$\mathbf{s}_t(f_{d,n}) = [1, e^{j2\pi f_{d,n}}, \dots, e^{j2\pi f_{d,n}(K-1)}]^T, \quad (23)$$

and Θ is the variable range of the matched angles ϕ and φ . As seen from (15), the spatial steering vector $\mathbf{s}_s(\phi, \varphi)$ is consisted of $P_\Sigma(\phi, \varphi)$, $P_{\Delta,\phi}(\phi, \varphi)$ and $P_{\Delta,\varphi}(\phi, \varphi)$; however, it is difficult to obtain the complex weighting coefficients of antenna patterns in practice. Alternatively, according to the characteristic of $\Sigma\Delta$ -antenna patterns, we apply a simplified spatial steering vector corresponding to the main lobe direction in the $\Sigma\Delta$ -STAP based radar system; that is

$$\mathbf{s}_s(\phi, \varphi) = [1, 0, 0]^T. \quad (24)$$

According to the aforementioned mechanism analysis of the MTC effect, the SCNR-enhanced sub-band STAP is applied with the searching of sub-band distributions. The equal-width and overlapped sub-band segmentation will be introduced within the whole transmitted bandwidth, which will lead to a considerable computational burden with the redundant CCM estimation in the searching of Δf_i and B_{sub} . As observed from (12), the phase modulation term $e^{-j\frac{4\pi f_c R_{m,k}}{c}}$ produces the Doppler frequency $f_{d,m} = 2v_a \cos(\theta_m) f_c / c$ for the scattering patch, which determines the phase coherence of CCM. Meanwhile, it is obvious that the delay-dependent phase modulation term $e^{j2\pi \Delta f_i (\tau - \frac{2R_{m,k}}{c})}$ is negligible comparing with $e^{-j\frac{4\pi f_c R_{m,k}}{c}}$ for a sample of pulses at a specific fast-time instance τ , since the Δf_i is small enough than the f_c , while the time-delay coupling terms are both dependent on the same $R_{m,k}$. Hence, by following the concept of the negligible phase term involved in the frequency diversity-based approaches [16], we can consider that the Doppler frequency of clutter signal is not coupled with the center frequency of sub-band distribution, which gives facilities for the consistent CCM estimation scheme for different sub-band distributions with a specific sub-band width, and reduces the computational complexity of the SCNR-enhanced sub-band STAP, correspondingly. Moreover, it is also seen from (12) that the sub-band width only has an impact on the range resolution instead of the phase modulation term. Hence, it is feasible to utilize high-resolution training samples in the full-band radar system for the consistent CCM estimation of different sub-band widths, which can circumvent the aforementioned sampling range expanding in the sub-band STAP and can be referred to as the third training sample selection scheme, correspondingly. However, it will lead to a considerable echo data storage demand by applying signal processing in full-band and sub-band radar systems for CCM estimation and match filtering, respectively, which challenges the performance limited missile-borne radar seeker.

As for the practical implementation of the SCNR-enhanced sub-band STAP, the effective target detection instead of the SCNR performance evaluation is applied in the

searching of the sub-band distributions. In the first step, the sub-band STAP is applied with different center frequencies of sub-band distribution, corresponding to a presumed B_{sub} . For the sub-band STAP outputs approaching to the in-phase superposition-based maximum possible output power, the target detection performance is improved with the enhanced SCNR performance. In the second step, for the successful detection case in the former step, we further apply the sub-band STAP with different B_{sub} , corresponding to the invariant successfully-detected center frequencies of sub-band distribution. Otherwise, we repeat the first step with another presumed B_{sub} . As a result, there may be several successful detections achieved in the searching of sub-band distributions, including the false alarms and the target detected at the correct R-D location. For the multiple sub-band STAP outputs in different sub-band distributions, the joint complementary detection is utilized to further improve the P_d , referred to as the joint multiple sub-bands STAP based on the independent STAP outputs in different sub-band distributions; that is

$$P_{d,syn} = 1 - P_{f,syn} = 1 - \prod_{i=1}^I (1 - P_{d,i}), \quad (25)$$

where $P_{d,i}$ is the P_d for the i -th sub-band STAP output involved, $P_{d,syn}$ is the synthetic P_d of the joint multiple sub-bands STAP based on the joint complementary detection, $P_{f,syn}$ denotes the probability of simultaneous failed detection for all sub-band STAP outputs involved. It indicates that the target detection performance of the joint multiple sub-bands STAP is improved on the basis of every single involved independent sub-band STAP output.

3.2. Full-Band STAP Based Multipath-Target Separation

With the SCNR-enhanced sub-band STAP, the target detection performance can be improved with the enhanced MTC effect. However, such performance improvement is achieved at the cost of the target tracking performance loss in the MTC effect, which is a key problem for the target tracking precision demanded missile-borne radar system. To tackle this issue, we further introduce the full-band STAP for multipath-target separation, based on the R-D location of the target detected in the joint multiple sub-bands STAP outputs. Hence, the 2D parameter searching of the matched Doppler frequency and CUT is alleviated for the full-band STAP, which can be referred to as the R-D specified full-band $\Sigma\Delta$ -STAP.

In addition to the target tracking, the full-band STAP also shows the advantage for nonhomogeneous clutter suppression with the decreased sampling range demand. It should be noted that the nonhomogeneous clutter could be induced by the factors arising with the long CPI, including the RW and the nonstationary electromagnetic scattering of clutter, leading to the clutter suppression performance degradation. Moreover, the target RW also causes the coherent accumulation loss within the long CPI, and the RW of the target and clutter cannot be mitigated simultaneously with Keystone transform [29], because of their conflict Doppler ambiguity factors. However, it is important to improve the output SCNR performance within the long CPI. Such contradiction between the long-CPI demanded performance and the accompanying problems calls for a robust long-CPI clutter suppression method, correspondingly.

By applying the R-D specified full-band $\Sigma\Delta$ -STAP within the sub-CPIs, the pulse-Doppler (PD) based long-CPI coherent accumulation is further carried out to improve the SCNR performance. In this regard, the whole CPI is divided into equal-length and overlapped sub-CPIs, in which the RW and nonstationary electromagnetic scattering issues are alleviated for the robust adaptive clutter suppression and coherent accumulation loss mitigation, simultaneously. These sub-CPIs are obtained with the sliding window processing, as shown in Figure 3, in which the constant sliding window interval is supposed to be 1 PRT for the sake of simplicity. We suppose that the sub-CPI contains K_{sub} pulses; thus, there are $K - K_{sub} + 1$ sub-CPIs within the whole CPI. The echo snapshot within the

sub-CPI is modeled by extracting corresponding rows of $\mathbf{x}(\tau, 0, B)$, according to the rules of the sliding window processing, which is denoted by $\mathbf{x}_j(\tau, 0, B)$ in the j -th sub-CPI. For the further long-CPI coherent accumulation, it is important to constrain the target Doppler history of clutter-suppressed STAP outputs among sub-CPIs, including the aspects of linear phase response constrained STAP filters design and the mitigation of dynamic TG-induced target attenuation, which are summarized as follows.

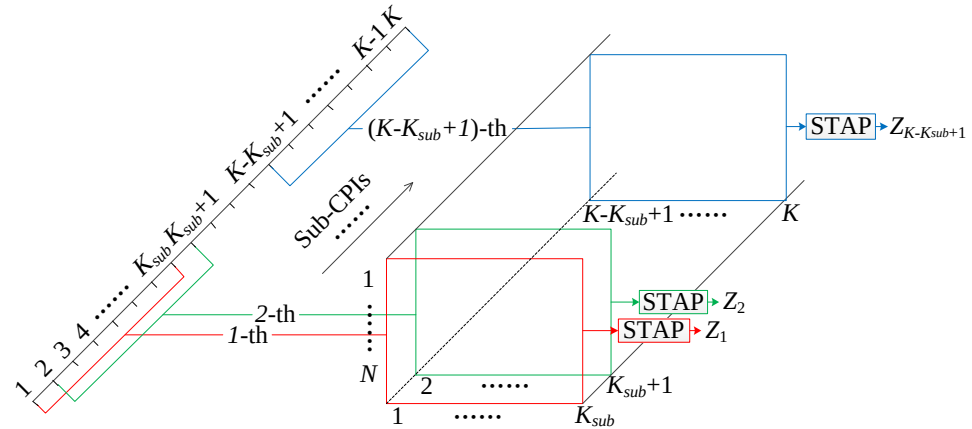


Figure 3. Diagrammatic sketch of sliding window processing.

The inconsistent sub-CPI second-order CCM estimations in the calculation of STAP weight vectors will lead to the nonlinear phase response, which might cause the target Doppler history corruption of STAP outputs among sub-CPIs and the SCNR performance degradation of the following long-CPI coherent accumulation. To tackle this issue, we adopt a sample selection [30] and covariance smoothing [31] approaches with the whole secondary data, by which the linear phase response of STAP is guaranteed with the consistent CCM estimation among sub-CPIs; that is

$$\mathbf{R}_{sub} = \frac{1}{J_s} \sum_j \frac{1}{\|\Omega_j\|_0} \sum_{\tau \in \Omega_j} \mathbf{x}_j(\tau) \mathbf{x}_j(\tau)^H, \quad (26)$$

where J_s denotes the covariance matrix smoothing times among sub-CPIs, and Ω_j denotes the selection range of training samples in the j -th sub-CPI, obtained by the statistical analysis of secondary data. Note that the J_s can be decreased to reduce the computational complexity in practice, because the statistical homogeneity of the selected training samples have already been improved for the covariance matrix smoothing. By updating the CCM with selected stationary training samples in the covariance matrix smoothing, the aforementioned long-CPI induced clutter suppression performance degradation is avoided.

The $\mathbf{s}_t(f_{d,n})$ is designed to be consistent in the STAP filters among sub-CPIs, in which the mismatched Doppler frequency difference between $f_{d,n}$ and the target Doppler frequency $f_{d,0} = 2(v_a \cos(\theta_0) + v_0)f_c/c$ has no impact on the target Doppler history of STAP outputs. In specific, considering the case that the subspaces of the target and clutter are orthogonal, we can obtain $\hat{\mathbf{R}}_{CN,0} \mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi) = 0$ and the STAP output within the j -th sub-CPI is expressed as

$$\begin{aligned} Z_j &= \mathbf{w}_{n,\phi,\varphi}^H \mathbf{x}_j(\tau, 0, B) = \frac{\mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi) \hat{\mathbf{R}}_{CN,0}^{-1} \mathbf{x}_j(\tau, 0, B)}{\mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi) \hat{\mathbf{R}}_{CN,0}^{-1} \mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi)} \\ &\approx \frac{\mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi) \mathbf{x}_j(\tau, 0, B)}{\mathbf{s}_{s,t}^H(f_{d,n}, \phi, \varphi) \mathbf{s}_{s,t}(f_{d,n}, \phi, \varphi)} \end{aligned} \quad (27)$$

Substituting (5)–(7) into (10), while simplifying the target signal in $\mathbf{x}_j(\tau, 0, B)$ into a steering vector based formation, we can further derive

$$Z_j \approx \frac{\mathbf{s}_t(f_{d,n})^H \mathbf{s}_t(f_{d,0})}{\|\mathbf{s}_t(f_{d,n})\|_2^2} \xi_0 G_p e^{-j \frac{4\pi f_c R_{0,j}}{c}} = \frac{\xi_0 G_p e^{-j \frac{4\pi f_c R_{0,j}}{c}}}{K_{sub}} \sum_{k=0}^{K_{sub}-1} e^{-j 2\pi \Delta f_d k}, \quad (28)$$

where G_p denotes the range-compressed gain. It is observed that the target Doppler history of clutter-suppressed STAP outputs is constrained among sub-CPIs with the linear phase response of sliding window processing. Thus, the long-CPI coherent accumulation can be carried out for further SCNR improvement.

In practice, the ξ_1 is inevitably dynamic and stochastic, since it is synthesized complicatedly by the echoes reflected from many scattering patches in the specular multipath scattering scene. Hence, it is infeasible to satisfy the condition of in-phase envelope superposition in (1) within the long CPI, because of the limited delay-dependent phase modulation term with respect to the dynamic multipath-target phase difference. This will cause the dynamic TG-induced target attenuation problem and the corresponding target Doppler history corruption in the MTC effect. By decoupling the multipath from the target, the full-band STAP alleviates the dynamic TG-induced target attenuation, which constrains the target Doppler history and gives facilities for the further long-CPI coherent accumulation.

In specific, the computational complexity of the full-band STAP based multipath-target separation is analyzed. As for the R-D specified full-band $\Sigma\Delta$ -STAP stage, the R-D location of the target detected in the joint multiple sub-bands STAP outputs is utilized as the coarse prior knowledge, which can be utilized to reduce high computational complexity induced by the searching operation of full-band STAP. The CCM can be estimated with at least $6K_{sub}$ training samples in the selected sub-CPI, and the corresponding computational complexity is $O(6K_{sub}(3K_{sub})^2 + (3K_{sub})^3 + (K - K_{sub} + 1)((3K_{sub})^2 + N_r 3K_{sub}))$, where N_r is the number of guard cells involved in the sub-CPI. As for the following PD based long-CPI coherent accumulation, the corresponding computational complexity is $O(N_r * (K - K_{sub} + 1) * \log(K - K_{sub} + 1))$. The flowchart of the proposed multi-resolution STAP is shown in Figure 4.

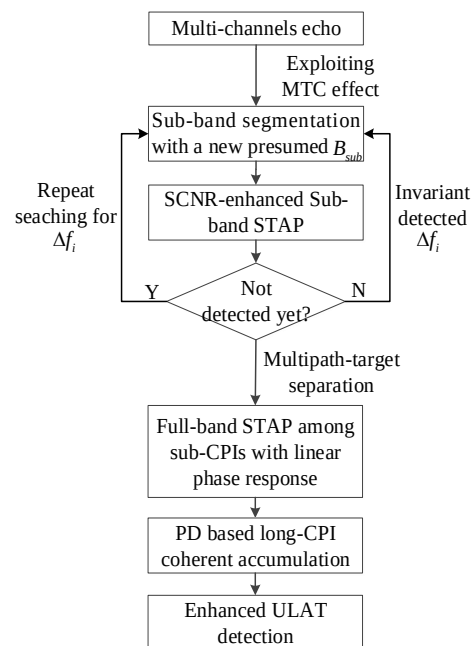


Figure 4. Flowchart of the proposed multi-resolution STAP.

4. Performance Analysis

In this section, numerical and experimental examples are provided to validate the aforementioned mechanism analysis and assess the performance of the proposed multi-resolution STAP, respectively, where strong clutter, specular multipath interference, and AWGN are in the forward-looking airborne radar seeker verification system. Four-core Intel(R) Core(TM) CPU of 3.6 GHz with 32 GB random access memory and MATLAB R2015b software are utilized to perform numerical and experimental analyses.

4.1. Mechanism Validation of MTC Effect

In this section, the mechanism of the joint magnitude-phase MTC effect is validated with numerical and measured results, which provides the foundation for the efficient SCNR-enhanced sub-band STAP.

4.1.1. Numerical Analysis

According to the CFM based sub-band signal model in (7) and (8), the numerical analysis is carried out for the MTC effect with a particular case. In this case, two adjacent point targets are simulated as the multipath and target in the scene, respectively, and both the ζ_0 and ζ_1 are set to the real number for the sake of simplicity. The range bin index difference between the multipath and target is 5, and the SIR is assigned to 1.6 dB; the transmitted waveform simulation parameters are listed in Table 1.

Table 1. Parameters of Waveform.

Parameter	Value	Parameter	Value
Carrier frequency	14.5 GHz	Waveform type	LFM
Full-band width	80 MHz	Sub-band width	20 MHz
Sampling frequency	192 MHz	Pulse width	1 μ s

The simulation results of the magnitude-phase MTC effect are shown in Figure 5. It is observed from Figure 5a that the maximum output power degrades dramatically after range-compressed envelope superposition with the sub-band center frequency $\Delta f_{i,out} = -3$ MHz, because the out-phase superposition is present between multipath and target. Meanwhile, it is also observed from Figure 5b that the maximum output power has been enhanced significantly after envelope superposition with the sub-band center frequency $\Delta f_{i,in} = 15$ MHz, which corresponds to the in-phase superposition based MTC condition. To further interpret the mechanism behind these results, the phase angles are numerically evaluated at $R_{0,k}$ as shown in the Table 2. It is seen that the phase angles of the target complex envelope keep constant with different center frequencies of sub-band distribution, while the phase angles of the multipath envelope are almost in-phase and opposite-phase with those of the target envelope, corresponding to the sub-band center frequencies $\Delta f_{i,out}$ and $\Delta f_{i,in}$, respectively. According to (7) and (8), we derive the theoretical phase angle differences of the multipath envelope at $R_{0,k}$, corresponding to $\Delta f_{i,out}$ and $\Delta f_{i,in}$; that is

$$\Delta\beta = \text{angle} \left(e^{j2\pi|\Delta f_{i,in} - \Delta f_{i,out}| \times \left| \frac{2R_{0,k}}{c} - \frac{2R_{1,k}}{c} \right|} \right). \quad (29)$$

Substituting the parameters of the range bin index difference, $\Delta f_{i,out}$ and $\Delta f_{i,in}$ into (12), we derive $\Delta\beta = 168.75$ deg, which coincides with the numerical results listed in Table 2.

In conclusion, the simulation results in Figure 5 and the sub-band range-compressed envelope derivation in (10), (11) have been validated with numerical results. The power of the adjacent specular multipath signal can be utilized to enhance the target output power of the superposed envelope, which provides the foundation for the SCNR-enhanced sub-band STAP. Moreover, it can be seen in Figure 5b that, even though the SIR value is only 1.6 dB in this case, the location of the maximum output power is still close to the target.

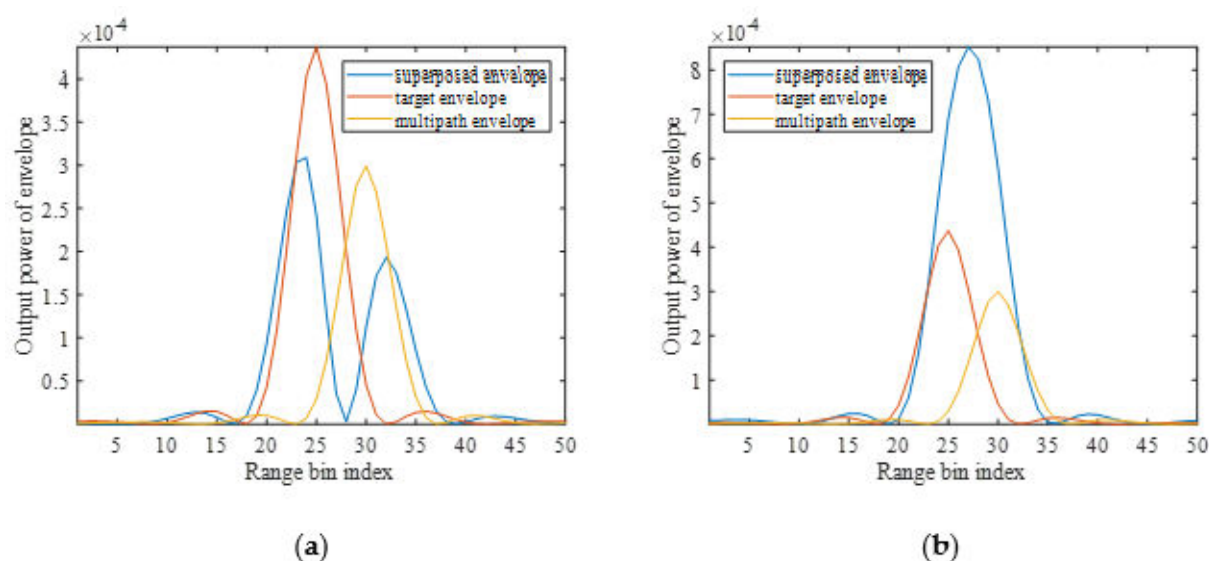


Figure 5. Simulation of the magnitude-phase superposed envelope in the MTC effect. (a) Out-of-phase superposed envelope with the center frequency of -3 MHz. (b) In-phase superposed envelope with the center frequency of 15 MHz.

Table 2. Phase Angle Analysis at Target Location.

Center Frequency of Sub-Band	Target	Multipath
-3 MHz	-7.50 deg	165.08 deg
15 MHz	-7.50 deg	-3.67 deg

4.1.2. Experimental Analysis

In this part, the measured data based sub-band STAP is carried out to further validate the mechanism of the joint magnitude-phase MTC effect. The measured data of the sea clutter, as well as a cooperative ULAT, are collected by an airborne radar seeker with $\Sigma\Delta$ -channels in forward-looking mode, and the $\Sigma\Delta$ three-channels are synthesized by the analogue beamforming technique, including the azimuth and elevation difference channels. The parameters of the radar system are listed in Table 3, and the waveform parameters coincide with those in Table 1. In this example, the output SCNR performance is evaluated for the SCNR-enhanced sub-band STAP with the searching of sub-band distributions.

Table 3. Parameters of Radar System.

Parameter	Value	Parameter	Value
Platform height	1.247 km	Platform velocity	80 m/s
PRF	20 KHz	Beam elevation direction	9.96 deg
CPI	6.4 ms	Beam azimuth direction	84.5 deg
Guard cells	50	Full-band training samples	200

Figure 6a shows the full-band R-D spectrum of measured data in the sum channel with the R-D algorithm [32], in which the range bin index axis includes the guard cells zone and the sampling range of STAP. It is seen that the main lobe clutter is very strong, while the geometry-induced range-dependent sidelobe clutter also exists—weak enough to be neglected in this case. Figure 6b shows the output R-D spectrum of the full-band STAP; it is seen that the main lobe is suppressed sufficiently and the coherent accumulation of the multipath and target signals are achieved with the match filtering of STAP, while the sidelobe clutter is too weak to compete with the detected target. Figure 6c shows the partial enlargement view of Figure 6b; it is seen that the adjacent specular multipath signal is very strong, while the range-extended phenomenon occurs in the multipath signal, in

practice, because the four-path model based multipath signal is actually the summation of echoes reflected from many scattering patches in the specular multipath scattering scene. It is obvious that the multipath-target decoupling is achieved with the full-band STAP, which represents the superior target tracking performance in the missile-borne radar system, correspondingly. In comparison, the results of the SCNR-enhanced sub-band STAP are shown in Figure 6d–f, corresponding to those in Figure 6a–c, respectively, with the sub-band width of $B_{sub} = 20\text{MHz}$. According to the training sample selection scheme mentioned in Section 3, it is seen that the clutter suppression performance is still robust enough, as shown in Figure 6e, compared with the R-D spectrum of the sub-band echo signal in Figure 6d. As seen from the partial enlargement view in Figure 6f, the multipath signal power has contributed to the magnitude-phase superposed target output power with the enhanced MTC effect, which is achieved at the cost of the target tracking performance loss.

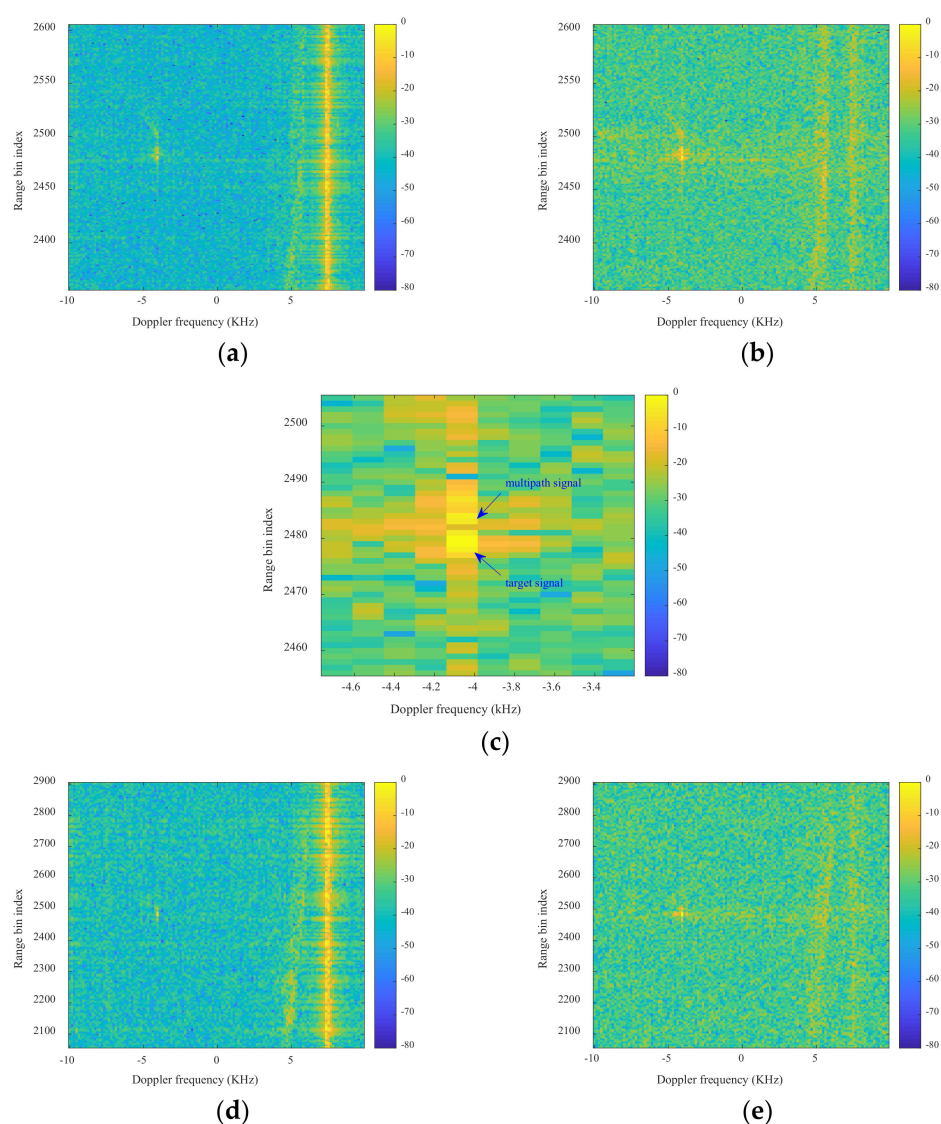


Figure 6. Cont.

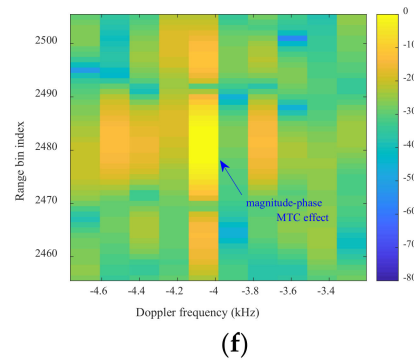


Figure 6. Experiment results of the sub-band STAP. (a) R-D spectrum of measured data in the sum channel with full-band. (b) Output R-D spectrum of the full-band STAP. (c) Partial enlargement view of (b). (d) R-D spectrum of measured data in the sum channel with sub-band. (e) Output R-D spectrum of the sub-band STAP. (f) Partial enlargement view of (e).

Figure 7a shows the SCNR performance of the sub-band STAP with respect to different sub-band distributions, corresponding to different Δf_i and B_{sub} for the 1-th measured data frame, in which the full-band STAP performance is also demonstrated as the reference, referred to as the case with $B_{sub} = 80\text{MHz}$. It is obvious that the SCNR-enhanced sub-band STAP achieves the better performance than the full-band STAP with the SCNR performance improvement of 5.3 dB, which is the performance evaluation corresponding to the measured results in the Figure 6. Meanwhile, it is also seen that the mechanisms summarized in the Section 3.1 have been validated with the output SCNR performance curves based on the measured results, which can be further categorized into two aspects: (1) the trend of SCNR performance keeps stable with respect to the center frequencies of sub-band distribution, which gives the facilities for the fast searching of SCNR-enhanced sub-band distribution. In specific, the coarse searching is first applied with a larger sub-band center frequency interval, and then the refined searching can be further applied around the successfully-detected center frequencies with a smaller sub-band center frequency interval; (2) the center frequency of the SCNR-enhanced sub-band distribution almost keeps identical with most sub-band widths, which is also conducive to the fast searching of SCNR-enhanced sub-band distributions with the invariant successfully-detected center frequencies. In addition, the same SCNR performance evaluation of sub-band STAP is also carried out based on the 2-th measured data frame, which also follows the aforementioned mechanism summarization.

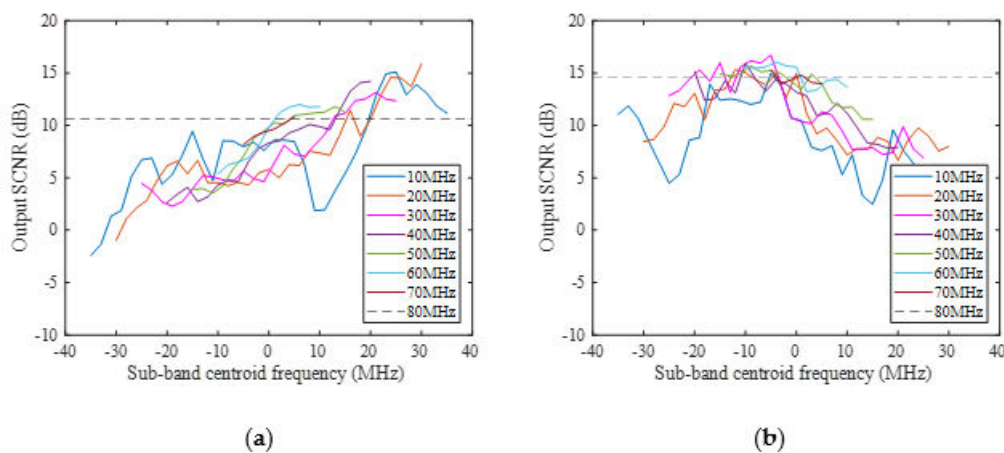


Figure 7. SCNR performance of sub-band STAP with respect to different sub-band distributions (a) 1-th measured data frame based result. (b) 2-th measured data frame based result.

4.2. Experiment Results of Joint Multi-Resolution STAP

In this section, the R-D specified full-band $\Sigma\Delta$ -STAP is applied for multipath-target separation based on the target R-D locations detected by the joint multiple sub-bands $\Sigma\Delta$ -STAP, followed by the further SCNR performance improvement with long-CPI target coherent accumulation. In specific, the parameters of sliding window processing are listed in Table 4, and the other parameters coincide with those in Tables 1 and 3.

Table 4. Parameters of Sliding Window Processing.

Parameter	Value	Parameter	Value
CPI	25.6 ms	Beam azimuth direction	84.5 deg
Guard cells	50	Full-band training samples	200

Figure 8 shows the output SCNR performances of sub-band STAP with respect to center frequencies of sub-band distribution within different sub-CPIs, corresponding to $B_{sub} = 30$ MHz, where the color bar indicates the output SCNR values. It is obvious that the dynamic TG effect is present among sub-CPIs, according to the significant variations of SCNR performances. As for the sub-CPIs with poor SCNR performances, the in-phase superposition based MTC condition cannot even be met, because of the limited delay-dependent phase modulation term under the condition of the stochastic ξ_1 in (1), which follow the summarization in Section 3.2. To tackle the dynamic TG-induced target attenuation issue and the corresponding Doppler history corruption, it is necessary to decouple the multipath from target with the increased bandwidth, which constrains the target Doppler history of clutter-suppressed STAP outputs among sub-CPIs.

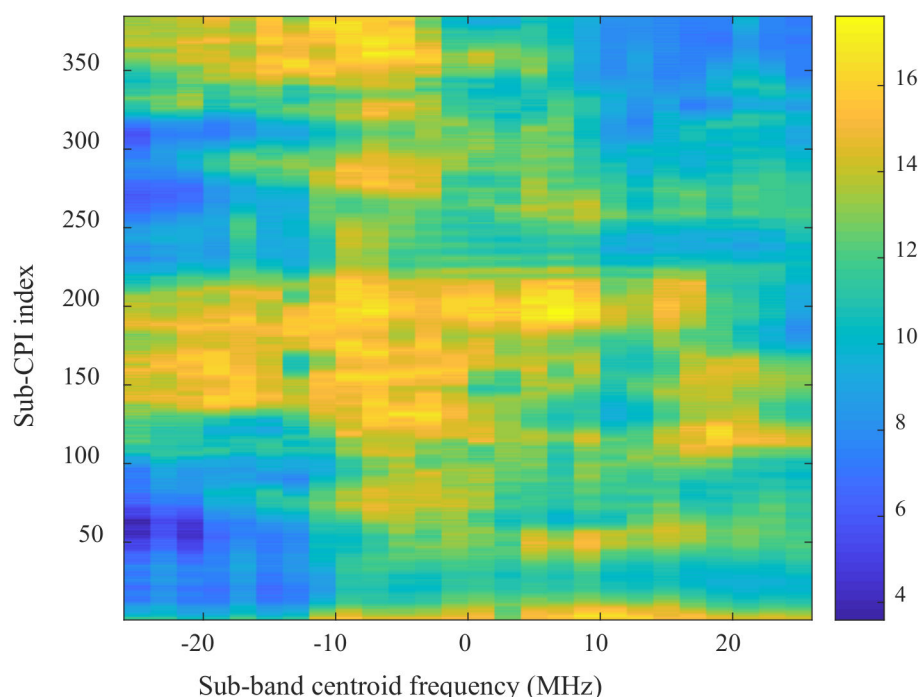


Figure 8. SCNR performance of the sub-band STAP with respect to sub-band distributions within different sub-CPIs.

As shown in Figure 9a, the R-D specified full-band $\Sigma\Delta$ -STAP outputs will be stacked into the slow-time dimension in the sequence of sub-CPI orders, which contributes to the recognized trajectories of the target and a multipath by robust clutter suppression and multipath-target separation. Figure 9b shows the RW correction result of clutter suppressed outputs in Figure 9a by Keystone transform. It is seen that the RW of trajectories are

mitigated for both the target and multipath. Figure 9c shows the further target coherent accumulation result based on the clutter suppressed outputs in Figure 9b. It is seen that the target spectral peak is accumulated in the R-D domain with the presence of residual clutter, while the adjacent multipath spectral peak also exists individually.

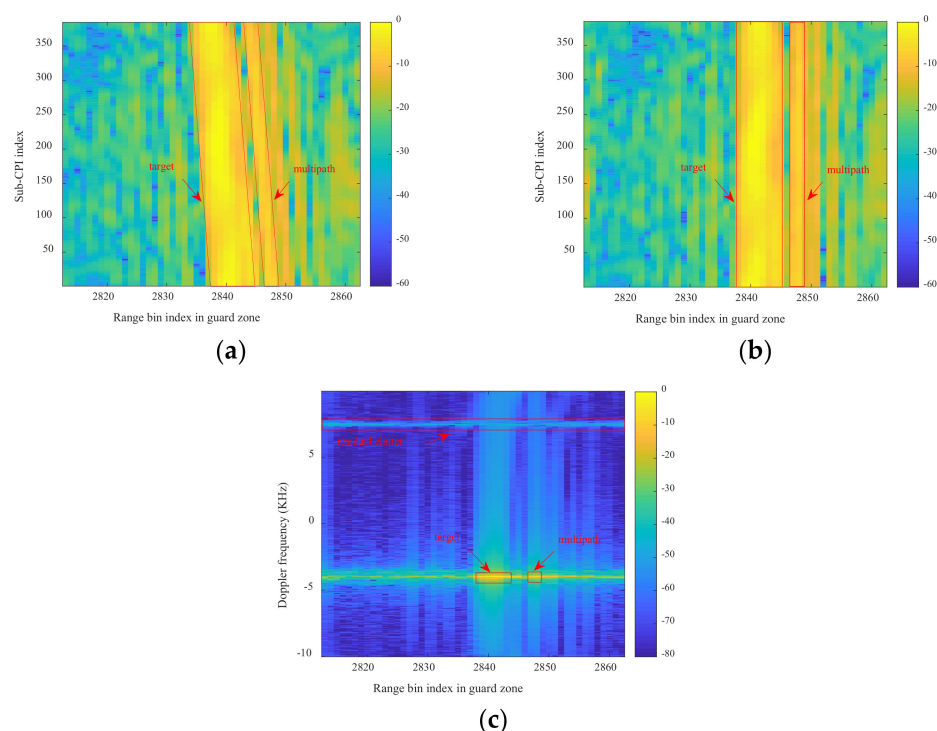


Figure 9. Experiment results of joint multi-resolution STAP. (a) R-D specified full-band $\Sigma\Delta$ -STAP outputs. (b) RW correction result. (c) Further coherent accumulation result.

As the metric of target detection performance of STAP, Table 5 shows the comparison of output SCNRs. It is seen that the proposed method achieves significant SCNR improvement after target coherent accumulation in the full-band radar system. In comparison, it is seen that the output SCNR performance degrades dramatically with the conventional full-band STAP within the long-CPI, because of the nonhomogeneous clutter suppression performance degradation and the coherent accumulation loss induced by target RW. Moreover, the so-called mono-resolution STAP is introduced for comparison by applying the SCNR-enhanced sub-band STAP within sub-CPIs, followed by the same target coherent accumulation, which requires the repeat searching of SCNR-enhanced sub-band distributions within different sub-CPIs because of the dynamic TG effect. It is seen that the output SCNR performance is poorer than the proposed method, even with a much higher STAP output SCNR performance contributed by the multipath signal power. Hence, it verifies that the aforementioned analysis that the dynamic TG-induced target attenuation and Doppler history corruption is present with the MTC effect, which leads to the SCNR performance degradation.

Table 5. SCNR Performance Analysis.

Method	STAP Output SCNR	Output SCNR of Further Coherent Accumulation	Computational Consuming Time
Proposed multi-resolution STAP	8.25 dB	40.04 dB	4.34 s
Mono-resolution STAP	20.35 dB	35.77 dB	Null
Conventional full-band STAP	4.75 dB	Null	3.37 s

5. Conclusions

ULAT detection is a challenging problem in missile-borne radar systems, due to the strong clutter and MTC effect with the presence of adjacent multipath interferences. To tackle this issue, an enhanced ULAT detection approach is proposed with the frequency diversity technique, referred to as the multi-resolution STAP, where the multipath signal power is exploited for SCNR performance improvement. Specifically, by deriving the mechanism of the MTC effect, it is feasible to improve target detection performance with the SCNR-enhanced sub-band STAP, accordingly. However, it is achieved at the cost of target tracking performance loss. To tackle this issue, the R-D specified full-band STAP is further applied for multipath-target separation based on the target R-D locations detected by the joint multiple sub-bands $\Sigma\Delta$ -STAP. In addition, the dynamic TG-induced target attenuation and the corresponding target Doppler history corruption are also mitigated with multipath-target separation, which gives facilities for further SCNR performance improvement with long-CPI coherent accumulation. The computer simulations and experimental results have demonstrated the effectiveness of the proposed method in ULAT detection enhancement.

Possible future studies should focus on the experimental measured data-based ULAT detection and the range-dependent clutter compensation in the $\Sigma\Delta$ -STAP-based radar system.

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Appendix A. Derivations of Sub-Band Range-Compressed Envelope for Target

In this appendix section, the sub-band range-compressed envelope in (10) is derived in detail, by implementing temporal convolution with $\psi_{sub,i}^*(-\tau)$ to the $x_{T,sub,i}(k, \tau)$, where $(\cdot)^*$ denotes the conjugate operator; that is

$$x_{T,sub,i}(k, \tau) = \zeta_0 \text{rect}\left(\frac{\gamma(\tau - 2R_{0,k}/c)}{B_{sub}}\right) e^{j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} e^{j2\pi\Delta f_i(\tau - \frac{2R_{0,k}}{c})} e^{-j\frac{4\pi f_c R_{0,k}}{c}}, \quad (A1)$$

$$\psi_{sub,i}(\tau) = \text{rect}\left(\frac{\gamma\tau}{B_{sub}}\right) e^{j\pi\gamma\tau^2} e^{j2\pi\Delta f_i\tau}. \quad (A2)$$

After temporal convolution between (13) and (14), it yields

$$\begin{aligned} \bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) &= \int_{-\infty}^{+\infty} \psi_{sub,i}^*(t) x_{T,sub,i}(k, t) dt \\ &= \zeta_0 e^{-j\frac{4\pi f_c R_{0,k}}{c}} \int_{-\infty}^{+\infty} \text{rect}\left(\frac{\gamma t}{B_{sub}}\right) e^{-j\pi\gamma t^2} e^{-j2\pi\Delta f_i t} \\ &\quad \text{rect}\left(\frac{\gamma(t + \tau - 2R_{0,k}/c)}{B_{sub}}\right) e^{j\pi\gamma(t + \tau - \frac{2R_{0,k}}{c})^2} e^{j2\pi\Delta f_i(t + \tau - \frac{2R_{0,k}}{c})} dt \end{aligned} \quad (A3)$$

It is further expressed as

$$\bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) = \zeta_0 e^{-j\frac{4\pi f_c R_{0,k}}{c}} e^{j2\pi\Delta f_i(\tau - \frac{2R_{0,k}}{c})} e^{j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} \int_{-\frac{B_{sub}}{2\gamma}}^{\frac{B_{sub}}{2\gamma} - \tau + \frac{2R_{0,k}}{c}} e^{j2\pi\gamma(\tau - \frac{2R_{0,k}}{c})t} dt, \quad (A4)$$

where the t denotes a temporary variable, representing the fast time dimension. Applying evaluation of the integral in (16) with the Euler formula, it yields

$$\begin{aligned}
 & \int_{-\frac{B_{sub}}{2\gamma}}^{\frac{B_{sub}}{2\gamma} - \tau + \frac{2R_{0,k}}{c}} e^{j2\pi\gamma(\tau - \frac{2R_{0,k}}{c})t} dt \\
 &= \frac{e^{-j2\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} e^{j\pi B_{sub}(\tau - \frac{2R_{0,k}}{c})} - e^{-j\pi B_{sub}(\tau - \frac{2R_{0,k}}{c})}}{j2\pi\gamma\left(\tau - \frac{2R_{0,k}}{c}\right)} \\
 &= \frac{\left(e^{-j\pi\gamma(\tau - \frac{2R_{0,k}}{c})(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma})} - e^{j\pi\gamma(\tau - \frac{2R_{0,k}}{c})(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma})}\right) e^{-j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2}}{j2\pi\gamma\left(\tau - \frac{2R_{0,k}}{c}\right)} \quad (A5) \\
 &= e^{-j\pi\gamma(\tau - \frac{2R_{0,k}}{c})^2} \left(\frac{B_{sub}}{\gamma} - \tau + \frac{2R_{0,k}}{c}\right) \frac{\sin\left(\pi\gamma\left(\tau - \frac{2R_{0,k}}{c}\right)\left(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma}\right)\right)}{\pi\gamma\left(\tau - \frac{2R_{0,k}}{c}\right)\left(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma}\right)},
 \end{aligned}$$

Substituting (17) into (16), it yields

$$\begin{aligned}
 & \bar{x}_{T,sub,i}(k, \tau, \Delta f_i, B_{sub}) \\
 &= \xi_0 e^{-j\frac{4\pi f_c R_{0,k}}{c}} e^{j2\pi\Delta f_i(\tau - \frac{2R_{0,k}}{c})} \left(\frac{B_{sub}}{\gamma} - \left(\tau - \frac{2R_{0,k}}{c}\right)\right) \\
 & \sin c\left[\gamma\left(\tau - \frac{2R_{0,k}}{c}\right)\left(\tau - \frac{2R_{0,k}}{c} - \frac{B_{sub}}{\gamma}\right)\right]. \quad (A6)
 \end{aligned}$$

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