## Supplementary material

1. Details of the Mixed Sample Generation Process

A total of 20,000 mixed NDPI curves (NDPI ${ }_{\text {mix }}$ ) were simulated using Equation (13) for a three-endmember mixture. In Equation (13), $f_{\text {nonwheat1 }}$ and $f_{\text {nonwheat } 2}$ are the fractions of the two nonwheat endmembers and NDPI $I_{\text {nonwheat1 }}$ and NDPInonwheat2 ${ }^{2}$ are the NDPI curves of the two nonwheat endmembers. $f_{\text {wheat }}$ was generated 20,000 times with a random number generator from a uniform distribution ( $0-1$ ). fnonwheat1 and $f_{\text {nonwheat2 }}$ were generated by dividing ( $1-f_{\text {wheat }}$ ) randomly by a uniformly distributed number $c$ :

$$
\left\{\begin{array}{l}
f_{\text {nonwheat1 }}=c\left(1-f_{\text {wheat }}\right)  \tag{S1}\\
f_{\text {nonwheat2 }}=(1-c)\left(1-f_{\text {wheat }}\right)
\end{array}\right.
$$

NDPI $_{\text {wheat }}$ was randomly selected from the 10 winter wheat endmembers, and NDPInonwheat and NDPInonwheat2 were randomly selected from the 70 nonwheat endmembers. Thus, 20,000 mixed NDPI curves were simulated in total, among which 10,000 were positive samples ( $f_{\text {wheat }}>50 \%$ ) and the other 10,000 were negative samples ( $f_{\text {wheat }}<50 \%$ ).
2. Details of the Simulation of NDPI Curves with Cloud Contamination

In the cloud contamination simulation experiment, a random cloud distribution model was considered because such models cover most of cloudy scenarios:

$$
\begin{align*}
& 0<f_{\text {cloud }}<100 \%, 0<f_{\text {cloud key }}<f_{\text {key }}, 0<f_{\text {cloud nonkey }}<f_{\text {nonkey }} \\
& \text { subject to } f_{\text {cloud key }}+f_{\text {cloud nonkey }}=f_{\text {cloud }}, f_{\text {key }}+f_{\text {nonkey }}=1 \tag{S2}
\end{align*}
$$

where $f_{\text {cloud }}$ is the fraction of cloudy observations in the NDPI curves during the entire winter wheat growing season; $f_{\text {cloud key }}$ and $f_{\text {cloud nonkey }}$ are the fractions of cloud observations in the key and nonkey phenological stages, respectively; and $f_{\text {key }}$ and $f_{\text {nonkey }}$ are the fractions of the entire winter wheat growing season contributed by the key and nonkey stages, respectively. fcloud was generated from $0 \%$ to $100 \%$ in $10 \%$ increments, and $f_{\text {cloud key }}$ and $f_{\text {cloud nonkey }}$ were generated following the rules in Equation (S2), also in 10\% increments. In total, 2,390,953 cloudy NDPI curves were simulated, among which 1,062,923 curves are winter wheat and the other 1,328,030 are non winter wheat.

