remote sensing

Supplementary Data

In the main text, the equations of the results are shown, namely the row model-related equations. In order to further illustrate our research work, this additional text provides detailed mathematical physical ideas about the derivation for the row model. The material includes:

A) Derivation for the area fraction of the four components in the canopy closure;

B) Calculation of gap and vegetation probabilities.

A. Derivation for the Area Fraction of the Four Components in the Canopy Closure

Since the canopy closure consists of vegetation and gap, the area of gap in the canopy closure is soil. Therefore, the gap probability in the viewing direction is the soil fraction in the canopy closure in the viewing direction. According to mathematical probability, the soil fraction is:

$$S_z + S_g = P_o(\theta_o, x, h) \tag{A-1}$$

Then, the vegetation fraction of the canopy closure in the viewing direction is

$$S_c + S_i = 1 - P_o(\theta_o, x, h) \tag{A-2}$$

where $P_o(\theta_o, x, h)$ is the gap probability of canopy closure in the viewing direction; its specific expression can be seen Equation (B-23). Here, the area fraction of illuminated soil in the canopy closure (S_z) is the bidirectional gap probability of canopy closure ($P_{so}(\theta_s, \theta_o, x, h)$; its specific expression can be seen Equation (B-24)), from which we get:

$$S_z = P_{so}(\theta_s, \theta_o, x, h) \tag{A-3}$$

$$S_g = P_o(\theta_o, x, h) - P_{so}(\theta_s, \theta_o, x, h) \tag{A-4}$$

Equations (A-3) and (A-4) show the result of S_z and S_g in Equation (A-1). Next, we address S_c and S_i in Equation (A-2). Therefore, we introduce the expression of the area fraction of the illuminated vegetation in the continuous crops in the viewing direction derived by Verheof (1998, 2018) [1,2], i.e.,

$$S_c' = k' \int_0^h P_{so}(\theta_s, \theta_o, z) dz \tag{A-5}$$

where $\int_0^h P_{so}(\theta_s, \theta_o, z) dz$ is the bidirectional vegetation probability of continuous crops, and k' is the extinction coefficient of a continuous crop in viewing direction [3], and

$$k' = \frac{2}{\pi} L' \left\{ \left[ar\cos\left(\frac{-1}{\tan\theta_o \tan\theta_l} \right) - \frac{\pi}{2} \right] \cos\theta_l + \sin\left[ar\cos\left(\frac{-1}{\tan\theta_o \tan\theta_l} \right) \right] \tan\theta_o \sin\theta_l \right\} \tag{A-6}$$

where θ_l is the average leaf inclination angle, θ_o is viewing zenith angle, and L' is the differential leaf area index for continuous crops.

To apply Equation (A-5) to the canopy closure of row crops, we considered the row structure to modify L' to the differential leaf area index for canopy closure (L_{row}), i.e.,

$$L_{row} = (A_1 + A_2) L f(\theta_l) d\theta_l / A_1 h \tag{A-7}$$

where h is the canopy height, L is the leaf area index, and $f(\theta_l)$ is the leaf inclination distribution function (LADF). This study used an elliptic distribution function [4,5]. Considering Equations (A-6) and (A-7), the extinction coefficient of canopy closure in the viewing direction is

$$k=\frac{2}{\pi}L_{row}\left\{\left[ar\cos\left(\frac{-1}{\tan\theta_o\tan\theta_l}\right)-\frac{\pi}{2}\right]\cos\theta_l+\sin\left[ar\cos\left(\frac{-1}{\tan\theta_o\tan\theta_l}\right)\right]\tan\theta_o\sin\theta_l\right\} \quad \text{(A-8)}$$

When θ_o is replaced by solar zenith angle (θ_s), Equation (A-8) is the extinction coefficient of the canopy closure in the solar direction, i.e., K.

In the direction of the orthogonal row plane (*X*-axis), the canopy closure is separated by the soil of the between-row area, Therefore, the calculation of the bidirectional vegetation probability of row crops ($\int_0^h P_{so}(\theta_s,\theta_o,x,z)dz$) needs to take into account the integral boundary of the *X* axis, i.e., $\int_0^h P_{so}(\theta_s,\theta_o,z)dz$ need to be modified to $\int_0^h P_{so}(\theta_s,\theta_o,x,z)dz$ in order to be applicable to canopy closure; its specific expression can be seen Equation (B-32). Then, Equation (A-5) can be modified to the expression of the fraction of the illuminated vegetation area in the canopy closure, which is

$$S_c=k\int_0^h P_{so}(\theta_s,\theta_o,x,z)dz \quad \text{(A-9)}$$

where k is the extinction coefficient of canopy closure in the viewing direction. We substitute the expression of S_c into Equation (A-2), yielding:

$$S_i=1-P_o(\theta_o,x,h)-k\int_0^h P_{so}(\theta_s,\theta_o,x,z)dz \quad \text{(A-10)}$$

B. Calculation of Gap and Vegetation Probabilities

The gap probabilities are used as indicators to describe the transmittance in the canopy. We considered two issues when constructing a new approach to calculations of the gap probability. The first is the angle of light propagation (section B-1), and the other is the overlapping relationship between the two media (leaves and canopy closure) (see Section B-2).

B-1 The Angle of Light Propagation

In the angle of light propagation, we derived the inclined angle projected in the perpendicular plane (α) based on a triangular geometric relationship. The complex angle issue in three-dimensional space can be converted into the angle of the plane X–Z in Figure S1a, further simplifying the calculation. The inclined angle projected in the perpendicular plane is

$$\alpha = \arctan\left(\sin\varphi_{s/or}\tan|\theta|\right) \quad \text{(B-1)}$$

where $\varphi_{s/or}$ represents the azimuth angle between the solar or viewing direction and the row direction,; it is $|\varphi_s-\varphi_r|$ or $|\varphi_o-\varphi_r|$. Here φ_r is the row azimuth angle, and θ is the zenith angle, representing the general symbols of the viewing zenith angle (θ_o) and solar zenith angle (θ_s).

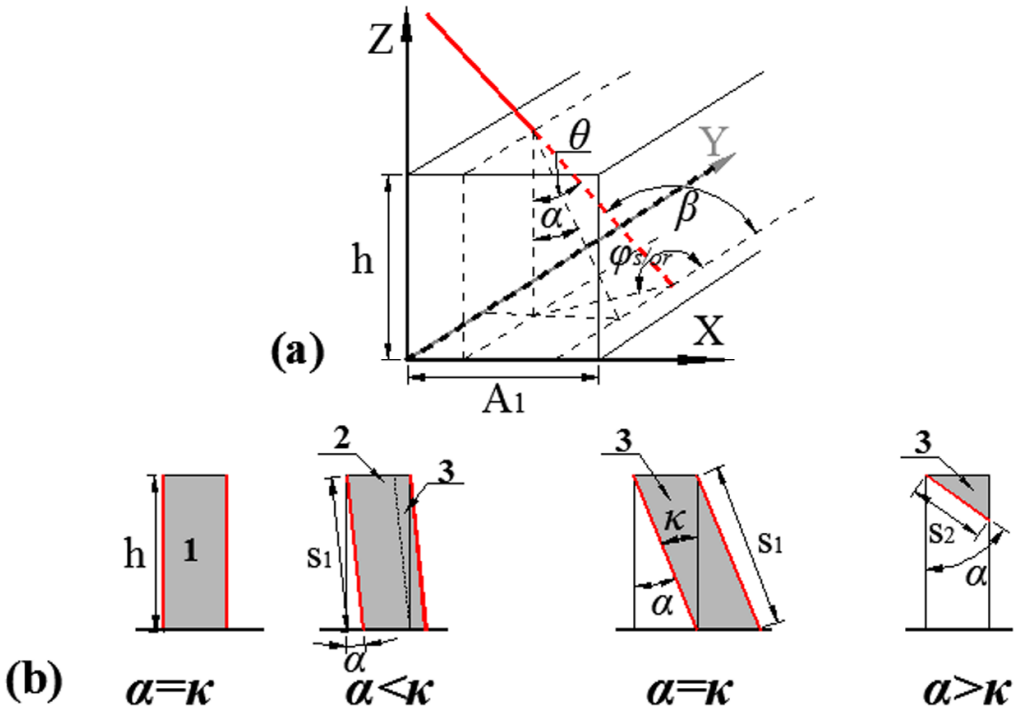


Figure S1. Geometric structure of row crops. (**a**) The angles in the three-dimensional space for row crops; and (**b**) the geometric relationship for canopy closure without overlap. The numbers in the figure are as follows: 1 = the square area, 2 = the parallelogram area, 3 = the triangle area. Here, α is the inclined angle projected in the perpendicular plane of the canopy closure, κ is the acritical angle, A_1 is row width, A_2 is the distance of between-row, h is canopy height, and s_1 and s_2 are path length.

B-2 The Overlapping Relationship

In this study, we assumed that the average gap probabilities in the first canopy closure would not be influenced by the overlap of the canopy closure, and that the average gap probabilities of the canopy closure after the first canopy closure would be influenced by the overlap of the previous canopy closure (Figure S2). Therefore, we divided the probabilities into categories to derive the average gap probability of canopy closure, i.e., (a) and (b).

(a) The average gap probabilities of canopy closure without overlap

To calculate the average gap probability of canopy closure without overlap, we defined a critical angle, i.e., κ, and $\kappa = \arctan(A_1/h)$. According to the geometric relationship in Figure S1b, the average gap probability of canopy closure without overlap includes four cases: α=0, $\alpha<\kappa$, $\alpha=\kappa$, and $\alpha>\kappa$. Taking the gap probabilities in the viewing direction as an example, we used k as the extinction coefficient (Equation A-8). For the gap probabilities in the solar direction, k needs to be replaced by K.

When α=0, the canopy closure is composed of a square (area 1 in Figure S1b). Based on the penetration function principle, i.e., the gap probability is $P=e^{-ks}$ [6], s is the path length. The average gap probability of a canopy closure without overlap is:

$$\overline{P_{n_o}} = e^{-kh} \quad \text{(B-2)}$$

When $\alpha<\kappa$, the canopy closure is composed of a triangle (area 3 in Figure S1b) and a parallelogram (area 2 in Figure S1b). Combining these geometric relations in mathematics, the sum gap probabilities in area 3 is:

$$P_{\Delta} = \int_{0}^{h\tan\alpha} e^{-k\frac{x}{\sin\alpha}} dx \tag{B-3}$$

Here, the integral in Equation (B-3) is calculated by the trapezoidal algorithm in the numerical calculation [7], that is, $\int_{a}^{b} f(x)dx = d\left[\frac{1}{2}f(x_0) + \sum_{i=1}^{N} f(x_i) + \frac{1}{2}f(x_{N+1})\right]$. Here, d is the step size in the numerical calculation. N is the number of samples for numerical integration. β is the azimuth of inclined angle (Figure S1a), and $\beta = \arcsin\left(\sin\varphi_{s/or}\sin|\theta|/\sin\alpha\right)$.

Since the integration is performed on the *X*-axis, we calculated the length for area 2 to be $A_1 - h\tan\alpha$. Therefore, the sum gap probability in area 2 is:

$$P_{\square} = \left(A_1 - h\tan\alpha\right)e^{-ks_1} \tag{B-4}$$

Here, s_1 is the path length in the canopy closure, and $s_1 = \frac{h}{\cos\alpha\sin\beta}$. Combining Equations (B-3) and (B-4), the average gap probability of canopy closure without overlap is:

$$\overline{P_{n_o}} = \frac{1}{A_1}\left(P_{\Delta} + P_{\square}\right) = \frac{1}{A_1}\left[\int_{0}^{h\tan\alpha} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx + \left(A_1 - h\tan\alpha\right)e^{-ks_1}\right] \tag{B-5}$$

When $\alpha = \kappa$, the canopy closure is composed of a triangle (area 3 in Figure S1b). Referring to a similar mathematical calculation principle, we can calculate the average gap probability of canopy closure without overlap as:

$$\overline{P_{n_o}} = \frac{1}{A_1}\int_{0}^{\frac{A_1}{\sin\varphi_r}} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx \tag{B-6}$$

When $\alpha > \kappa$, the canopy closure is composed of a triangle (area 3 in Figure S1b). According to Figure S1b, α changes will cause the path length in the canopy closure to change on the *Z* axis, i.e., $s_2 = \frac{z}{\cos\alpha}$. Therefore, the average gap probability of canopy closure without overlap is:

$$\overline{P_{n_o}} = P_{\Delta} = \int_{0}^{\frac{A_1}{\tan\alpha}} e^{-k\frac{z}{\cos\alpha}} dz \tag{B-7}$$

Finally, we summarized Equations (B-2), (B-5), (B-6), and (B-7); the general formula for the average gap probability of canopy closure without overlap is:

$$\overline{P_{n_o}} = \begin{cases} \frac{1}{A_1}\left\{\int_{0}^{h\tan\alpha} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx + \left[\left(A_1 - h\tan\alpha\right)\right]e^{-k\frac{h}{\cos\alpha\sin\beta}}\right\} & \kappa < \alpha \\ \frac{1}{A_1}\int_{0}^{\frac{A_1}{\tan\alpha}} e^{-k\frac{z}{\cos\alpha\sin\beta}} dz & \kappa \geq \alpha \end{cases} \tag{B-8}$$

(b) The average gap probability of canopy closures with overlap

The overlapping relationship between canopy closures is determined by the horizontal projected lengths of row height on the ground in the solar or viewing direction (Figure S2). Therefore, we defined l as the horizontal projected lengths of row height on the ground in the solar or viewing direction, and $l = h\tan\alpha$. After that, we could calculate the residual projection length in the solar

or viewing direction and the number of rows through projection, i.e., l_r and n Here, $l_r = l - n(A_1 + A_2)$ and $n = \lfloor l/(A_1 + A_2) \rfloor$.

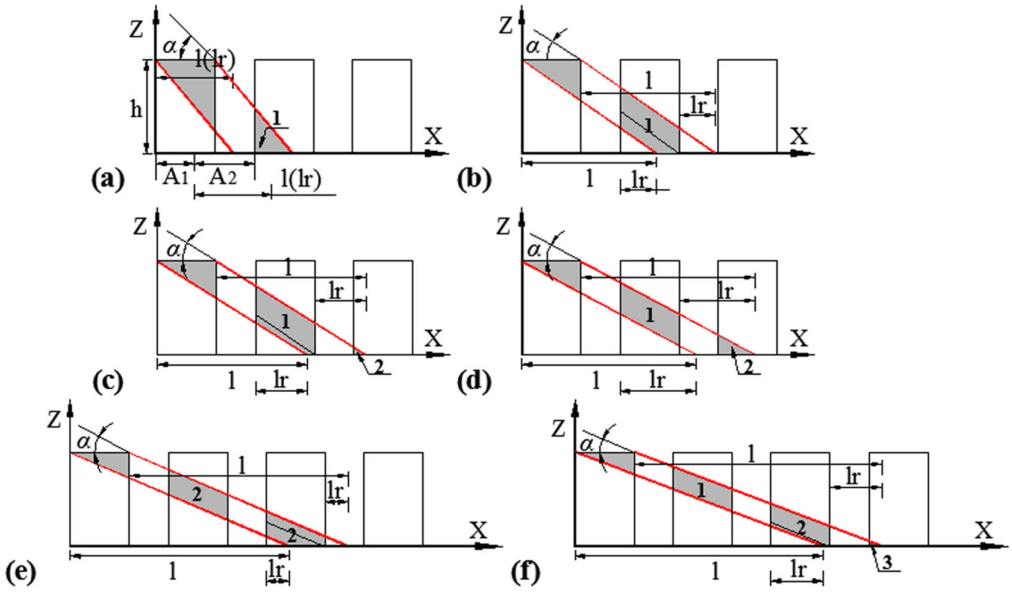


Figure S2. The sketch of the overlapping relationship between canopy closures as the changes in the viewing or solar direction ($A_1 \geq A_2$). (**a**) The case for $(A_2 < l_r \leq A_1 + A_2) \wedge (A_1 < l_r \leq A_1 + A_2) \wedge (n = 0)$, (**b**) the case for $(0 \leq l_r \leq A_2) \wedge (0 \leq l_r \leq A_2) \wedge (n = 1)$, (**c**) the case for $(0 \leq l_r \leq A_2) \wedge (A_2 \leq l_r \leq A_1 + A_2) \wedge (n = 1)$, (d) the case of $(A_2 < l_r \leq A_1 + A_2) \wedge (A_1 < l_r \leq A_1 + A_2) \wedge (n = 1)$, (**e**) the case for $(0 \leq l_r \leq A_2) \wedge (0 \leq l_r \leq A_2) \wedge (n = 2)$, (**f**) the case for $(0 \leq l_r \leq A_2) \wedge (A_2 \leq l_r \leq A_1 + A_2) \wedge (n = 2)$. Here l is horizontal projected lengths of row height on the ground in the solar or viewing direction, and l_r the residual projection length of l in the solar or viewing direction, n is the number of rows through projection, and $\wedge$ are the mathematical logic symbol for "and". Here the sketch only shows the case for $A_1 \geq A_2$, the case for $A_1 < A_2$ and the case for $A_1 \geq A_2$ slightly different but the principle is similar.

In Figure S2a, i.e., $(A_2 < l_r \leq A_1 + A_2) \wedge (A_1 < l_r \leq A_1 + A_2) \wedge (n = 0)$, area 1 is the closure of the first canopy with overlap. Similar to the calculation principle in Equation (B-7), we calculated the average gap probability in the first canopy closure with overlap:

$$\overline{P_1} = \frac{1}{A_1} \int_0^{l_r - A_2} e^{-k \frac{x}{\sin\alpha \sin\beta}} dx \tag{B-9}$$

In Figure S2b, i.e., $(0 \leq l_r \leq A_2) \wedge (0 \leq l_r \leq A_2) \wedge (n = 1)$, the first canopy closure with overlap consisted of a trapezoid and a parallelogram (area 1 in Figure S2b). Similarly to the calculation principle in Equation (B-5), we could calculate the average gap probability in the first canopy closure with overlap:

$$\overline{P_1} = \frac{1}{A_1} \left[\int_{lr}^{A_1} e^{-k \frac{x}{\sin\alpha \sin\beta}} dx + \left(h - \frac{2A_2}{\tan\alpha} \right) e^{-k \frac{A_1}{\sin\alpha \sin\beta}} \right] \tag{B-10}$$

Equations (B-9) and (B-10) are the overlap area with one canopy closure. We can therefore analyze the cases where the overlap area has two canopy closures, i.e., Figure S2c–d.

In Figure S2c, i.e., $\left(0 \le l_r \le A_2\right) \wedge \left(A_2 \le l_r \le A_1 + A_2\right) \wedge \left(n = 1\right)$, the first canopy closure with overlap consisted of a trapezoid and a parallelogram (area 1 in Figure S2c). We could calculate the average gap probability in the first canopy closure with overlap using:

$$\overline{P_1} = \frac{1}{A_1}\left[\int_{lr}^{A_1} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx + \left(h - \frac{2A_2}{\tan\alpha}\right)e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right] \tag{B-11}$$

The second canopy closure with overlap was a triangle area (area 2 in Figure S2c). The average gap probability was:

$$\overline{P_2} = \frac{1}{A_1}\int_0^{l_r - A_2} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx \tag{B-12}$$

In Figure S2d, i.e., $\left(A_2 < l_r \le A_1 + A_2\right) \wedge \left(A_1 < l_r \le A_1 + A_2\right) \wedge \left(n = 1\right)$, the first canopy closure with overlap consisted of a parallelogram (area 1 in Figure S2d). We could calculate the average gap probability as follows:

$$\overline{P_1} = \frac{1}{A_1}\left(h - \frac{A_1 + l_r}{\tan\alpha}\right)e^{-k\frac{A_1}{\sin\alpha\sin\beta}} \tag{B-13}$$

and the average gap probability in the second canopy closure with overlap was:

$$\overline{P_2} = \frac{1}{A_1}\int_0^{l_r - A_2} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx \tag{B-14}$$

Equations (B-11) to (B-14) are the overlap area with two canopy closures. Similarly, we could also calculate $\overline{P_1}$ and $\overline{P_2}$ in Figure S2e, i.e.,

$$\overline{P_1} = \frac{1}{A_1}\left[h - \frac{A_2 + \left(A_1 + A_2\right) + l_r}{\tan\alpha}\right]e^{-k\frac{A_1}{\sin\alpha\sin\beta}} \tag{B-15}$$

$$\overline{P_2} = \frac{1}{A_1}\left\{\int_{lr}^{A_1} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx + \left[h - \frac{2A_2 + \left(A_1 + A\right)_2}{\tan\alpha}\right]e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right\} \tag{B-16}$$

We could also calculate $\overline{P_1}$, $\overline{P_2}$ and $\overline{P_3}$ in Figure S2f, i.e.,

$$\overline{P_1} = \frac{1}{A_1}\left[h - \frac{A_2 + \left(A_1 + A_2\right) + l_r}{\tan\alpha}\right]e^{-k\frac{A_1}{\sin\alpha\sin\beta}} \tag{B-17}$$

$$\overline{P_2} = \frac{1}{A_1}\left\{\int_{lr}^{A_1} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx + \left[h - \frac{2A_2 + \left(A_1 + A_2\right)}{\tan\alpha}\right]e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right\} \tag{B-18}$$

$$\overline{P_3} = \frac{1}{A_1}\int_0^{l_r - A_2} e^{-k\frac{x}{\sin\alpha\sin\beta}}dx \tag{B-19}$$

We summarized the calculation rules of the above equations (Equations (B-9) to (B-19)), and extended it to the case of n canopy closures. Finally, we considered the case for $A_1 \ge A_2$ and $A_1 < A_2$,; we were

then able to obtain the following general equations for the three cases. When $\left(A_2 < l_r \le A_1 + A_2\right) \wedge \left(A_1 < l_r \le A_1 + A_2\right)$, the average gap probability of each canopy closure with overlap was:

$$\overline{P_n} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha} \right] e^{-k\frac{A_1}{\sin\alpha\sin\beta}} & n \ge 1 \\ \overline{P_{last}} = \dfrac{1}{A_1}\int_0^{l_r - A_2} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx & n \ge 0 \end{cases} \quad \text{(B-20)}$$

Here $\overline{P_n}$ represents $\overline{P_1}, \overline{P_2}, \overline{P_3}, \cdots, \overline{P_n}$. $\overline{P_{middle}}$ is the average gap probability in the middle canopy closure with overlap in the viewing (or solar) direction, and $\overline{P_{last}}$ is the average gap probability of the last canopy closure with overlap in the viewing (or solar) direction.

When $\left(0 \le l_r \le A_2\right) \wedge \left(0 \le l_r \le A_2\right)$, the average gap probability of each canopy closure with overlap is:

$$\overline{P_n} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha} \right] e^{-k\frac{A_1}{\sin\alpha\sin\beta}} & n \ge 2 \\ \overline{P_{last}} = \dfrac{1}{A_1}\left\{ \int_{lr}^{A_1} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx + \left[h - \dfrac{2A_2 + (n-1)\left(A_1 + A\right)_2}{\tan\alpha} \right] e^{-k\frac{A_1}{\sin\alpha\sin\beta}} \right\} & n \ge 1 \end{cases} \quad \text{(B-21)}$$

When $\left(0 \le l_r \le A_2\right) \wedge \left(A_2 \le l_r \le A_1 + A_2\right) \wedge \left(A_1 \ge A_2\right)$, the average gap probability of each canopy closure with overlap is:

$$\overline{P_n} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha} \right] e^{-k\frac{A_1}{\sin\alpha\sin\beta}} & n \ge 2 \\ \overline{P_{secend_to_last}} = \dfrac{1}{A_1}\left\{ \int_{lr}^{A_1} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx + \left[h - \dfrac{2A_2 + (n-1)\left(A_1 + A\right)_2}{\tan\alpha} \right] e^{-k\frac{A_1}{\sin\alpha\sin\beta}} \right\} & n \ge 1 \\ \overline{P_{last}} = \dfrac{1}{A_1}\int_0^{l_r - A_2} e^{-k\frac{x}{\sin\alpha\sin\beta}} dx & n \ge 1 \end{cases} \quad \text{(B-22)}$$

Here, $\overline{P_{secend_to_last}}$ is the average gap probability of the penultimate canopy with overlap in the viewing (or solar) direction.

B-3 Calculation of Gap and Vegetation Probabilities

Based on average gap probabilities of each canopy closure in section B-2, we considered the viewing and solar direction to calculate the gap and vegetation probabilities proposed in Section 2.2.1.

(a) Single directional gap probabilities

According to the principle of probability theory in mathematics, we know that the average gap probabilities in a certain viewing (or solar) direction should be the product of each canopy closure. Therefore, the gap probability of the canopy closure in the viewing (or solar) direction is:

$$P(\theta,x,h)=\begin{cases}\overline{P_{n_o}} & l\le A_2\\ \left(\overline{P_{n_o}}+\overline{P_{n_o}P_1}+\overline{P_{n_o}P_1P_2}+\cdots+\overline{P_{n_o}P_1P_2}\cdots\overline{P_{n-1}P_n}\right)\Big/n! & l>A_2\end{cases}$$
$$=\begin{cases}\overline{P_{n_o}} & l\le A_2\\ \overline{P_{n_o}}\left(1-\overline{P_1}-\overline{P_1P_2}-\cdots-\prod_{i=1}^{n}\overline{P_i}\right)\Big/n! & l>A_2\end{cases} \quad \text{(B-23)}$$

Here, $\overline{P_{n_o}}$ in (B-23) can refer to Eq. (B-8) and $\overline{P_1},\overline{P_2},\overline{P_3},\cdots,\overline{P_n}$ in (B-23) refers to Equations (B-20), (B-21), and (B-22). Equations (B-20) to (B-22) are the calculations for the viewing direction. For the solar direction, we replaced k in Equations (B-20), (B-21), and (B-22) with K. Equation (B-23) only considers one solar direction or the viewing direction. Therefore, Equation (B-23) is the single directional gap probabilities, i.e., the gap probabilities of canopy closure in the solar direction ($P_s(\theta_s,h)$), or the gap probabilities of the canopy closure in the viewing direction ($P_o(\theta_o,h)$).

(b) Bidirectional gap probability of canopy closure

When $P_s(\theta_s,h)$ and $P_o(\theta_o,h)$ are taken into account, the bidirectional gap probability of the canopy closure is $P_s(\theta_s,h)\cdot P_o(\theta_o,h)$. However, $P_s(\theta_s,h)$ and $P_o(\theta_o,h)$ are not system-independent in statistics. Therefore, a correlation coefficient function needs to be considered in modeling. This function is often called a "hotspot kernel function", and has been investigated by many modelers, including Kuusk (1985) [8] and Chen et al. (1997) [9]. When considering these factors, the bidirectional gap probability of the canopy closure is:

$$P_{so}(\theta_s,\theta_o,h)=P_s(\theta_s,h)\cdot P_o(\theta_o,h)+\left[P_s(\theta_s,h)-P_s(\theta_s,h)\cdot P_o(\theta_o,h)\right]\cdot F_\theta(\xi) \quad \text{(B-24)}$$

Here, $F_\theta(\xi)$ is a hotspot kernel function which is used to control the hotspot effect produced by multiangle observations. Chen et al. (1997) [10] gave the mathematical expression of a hotspot kernel function in the forest. We modified the parameters in their hotspot kernel function to use it with a canopy of row crops. The hotspot kernel function of the row crops is:

$$F_\theta(\xi)=C_1e^{-(\xi/\pi)C_2} \quad \text{(B-25)}$$

Here, C_1 and C_2 are hotspot factors. C_1 determines the magnitude of the hotspot, C_2 determines the width of the hotspot, and $C_2=\dfrac{kLh}{\cos\theta_s W_p}$, where W_p is the width of the leaves. ξ is the angle between the viewing direction and the sun direction, i.e.,

$$\xi=\arccos\left(\cos\theta_s\cos\theta_o+\sin\theta_s\sin\theta_o\cos\left|\varphi_s-\varphi_o\right|\right) \quad \text{(B-26)}$$

Here, φ_s is the solar azimuth angle and φ_o is viewing azimuth angle.

(c) Bidirectional vegetation probability of canopy closure

Canopy closures are composed of gaps and vegetation; the gap probability is $P=e^{-ks}$ [6], while the vegetation probability is $P_v=1-e^{-ks}$. We can calculate the vegetation probability of a canopy closure in the solar or viewing directions by analogy using Equations (B-8), (B-20), (B-21), and (B-22). Taking the viewing direction as an example, the average vegetation probability of canopy closure without overlap in the viewing direction is:

$$\overline{\int_0^h P_{n_o}(\theta_o,x,z)\,dz}=\begin{cases}\dfrac{1}{A_1}\left[\int_0^{h\tan\alpha}\left(1-e^{-k\frac{x}{\sin\alpha\sin\beta}}\right)dx+\left[(A_1-h\tan\alpha)\right]\left(1-e^{-k\frac{h}{\cos\alpha\sin\beta}}\right)\right] & \kappa<\alpha\\ \dfrac{1}{A_1}\int_0^{\frac{A_1}{\tan\alpha}}\left(1-e^{-k\frac{z}{\cos\alpha\sin\beta}}\right)dz & \kappa\ge\alpha\end{cases} \quad \text{(B-27)}$$

When $\left(A_2 < lr \le A_1 + A_2\right) \wedge \left(A_1 < lr \le A_1 + A_2\right)$, the average vegetation probability of each canopy closure with overlap in the viewing direction has the following general expression:

$$\overline{\int_0^h P_n(\theta_o, x, z)\,dz} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha}\right]\left(1 - e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right) & n \ge 1 \\ \overline{P_{last}} = \dfrac{1}{A_1}\displaystyle\int_0^{l_r - A_2}\left(1 - e^{-k\frac{x}{\sin\alpha\sin\beta}}\right)dx & n \ge 0 \end{cases} \tag{B-28}$$

When $\left(0 \le lr \le A_2\right) \wedge \left(0 \le lr \le A_2\right)$, the average vegetation probability of each canopy closure with overlap in the viewing direction has the following general expression:

$$\overline{\int_0^h P_n(\theta_o, x, z)dz} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha}\right]\left(1 - e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right) & n \ge 2 \\ \overline{P_{last}} = \dfrac{1}{A_1}\left\{\displaystyle\int_{lr}^{A_1}\left(1 - e^{-k\frac{x}{\sin\alpha\sin\beta}}\right)dx + \left[h - \dfrac{2A_2 + (n-1)\left(A_1 + A\right)_2}{\tan\alpha}\right]\left(1 - e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right)\right\} & n \ge 1 \end{cases} \tag{B-29}$$

When $\left(0 \le lr \le A_2\right) \wedge \left(A_2 \le lr \le A_1 + A_2\right) \wedge \left(A_1 \ge A_2\right)$, the average vegetation probability of each canopy closure with overlap in the viewing direction has the following general expression:

$$\overline{\int_0^h P_n(\theta_o, x, z)dz} = \begin{cases} \overline{P_{middle}} = \dfrac{1}{A_1}\left[h - \dfrac{A_2 + (\mathrm{n}-1)\left(A_1 + A_2\right) + l_r}{\tan\alpha}\right]\left(1 - e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right) & n \ge 2 \\ \overline{P_{secend_to_last}} = \dfrac{1}{A_1}\left\{\displaystyle\int_{lr}^{A_1}\left(1 - e^{-k\frac{x}{\sin\alpha\sin\beta}}\right)dx + \left[h - \dfrac{2A_2 + (n-1)\left(A_1 + A\right)_2}{\tan\alpha}\right]\left(1 - e^{-k\frac{A_1}{\sin\alpha\sin\beta}}\right)\right\} & n \ge 1 \\ \overline{P_{last}} = \dfrac{1}{A_1}\displaystyle\int_0^{l_r - A_2}\left(1 - e^{-k\frac{x}{\sin\alpha\sin\beta}}\right)dx & n \ge 1 \end{cases} \tag{B-30}$$

Equations (B-27) to (B-30) refer to the viewing direction. For the solar direction, we replaced k in Equations (B-27) to (B-30) with K. Then, the vegetation probability of canopy closure in the solar or viewing directions was:

$$\int_0^h P(\theta, z)dz = \begin{cases} \overline{\displaystyle\int_0^h P_{n_o}(\theta, x, z)dz} & l \le A_2 \\ \left\{\begin{array}{l} \overline{\displaystyle\int_0^h P_{n_o}(\theta, x, z)dz} + \left[1 - \overline{\displaystyle\int_0^h P_{n_o}(\theta, x, z)dz}\right]\overline{\displaystyle\int_0^h P_1(\theta, x, z)dz} \\ + \left[1 - \overline{\displaystyle\int_0^h P_{n_o}(\theta, x, z)dz}\right]\left[1 - \overline{\displaystyle\int_0^h P_1(\theta, x, z)dz}\right]\overline{\displaystyle\int_0^h P_2(\theta, x, z)dz} + \cdots \\ + \overline{\displaystyle\int_0^h P_i(\theta, x, z)dz}\prod_{i=1}^{n-1}\left[1 - \overline{\displaystyle\int_0^h P_{n_o}(\theta, x, z)dz}\right]\left[1 - \overline{\displaystyle\int_0^h P_{i-1}(\theta, x, z)dz}\right] \end{array}\right\} \Big/ n! & l > A_2 \end{cases} \tag{B-31}$$

The bidirectional vegetation probability of canopy closure is:

$$\begin{aligned} \int_0^h P_{so}(\theta_s, \theta_o, x, z)dz &= \int_0^h P_s(\theta_s, x, z)dz \int_0^h P_o(\theta_o, x, z)dz \\ &+ \left[\int_0^h P_s(\theta_s, x, z)dz - \int_0^h P_s(\theta_s, x, z)dz \int_0^h P_o(\theta_o, x, z)dz\right]\int_0^h C_1 e^{\frac{-(\xi/\pi)kLz}{\cos\theta_s W_p}}dz \end{aligned} \tag{B-32}$$

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