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Article

Predicting the Trend of Taiwan's Electronic Paper Industry by an Effective Combined Grey Model

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Abstract: Electronic paper (e-paper) is a major sector of Taiwan's Optoelectronic industry. It has paid much attention on the development of flexible displays. Even though the market is booming, the future is still unclear for business development. No research has yet forecasted the future market size of the e-paper industry. In addition, proposing an appropriate forecasting model to understand the trend of this industry plays a crucial role for market players and government's authorities in formulating correct strategies. Therefore, in this paper, the future market size of Taiwan's e-paper industry is predicted by an effective combined grey model. Two combinations of DGM(2,1) and Verhulst model with Fourier series and Markov chain, namely FM-Verhulst and FMDGM(2,1), were presented. Based on the annual data of Taiwan's e-paper industry, the results show that the forecasting performances of two FM-Verhulst and FMDGM(2,1) models are highly accurate compared with other grey models. Precision is 96.36% and 97.77%, respectively. However, for long-term prediction, the FMDGM(2,1) model obtains the best performance in all proposed grey models. With obtained forecasting results in Taiwan's e-paper industry by the FMDGM(2,1) model, it can be pointed out that the future market size of Taiwan's e-paper would slowly increase in the next few years.

Keywords: forecasting; electronic paper; grey model; Fourier series; Markov chain

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1. Introduction

Electronic paper is a portable, reusable storage and display medium, typically thin and flexible. It is, literally, the electronic substitution for printed pages. Typically, it reproduces mainly static text, usually monochrome, with the high flexibility of a whole screen, so it may even be folded or rolled like traditional paper [1]. As per the discussion of Genuth [2], e-paper offers several advantages, such as being made of flexible material, requiring ultra-low power consumption, low-cost manufacture and, most importantly, easy and convenient to read. Thus, prospective e-paper is just around the corner, with the promise of holding libraries on a chip and replacing most printed newspaper before the end of the next decades. As a result, business for electronic paper display is noteworthy in the coming years.

In fact, e-paper has entered the market with the appearance of e-reader products. Along with the success of Kindle e-books, the e-paper application market has grown rapidly since 2008 with more and more information products such as e-book readers, information displays, signboards, electronic tags, digital clocks, and so on. Data from the Display Search determined that the value of e-paper display modules will grow from $431 million in 2009 to $6 billion in 2015 and close to $10 billion in 2018 [3]. This means that e-paper application products will be a huge market with a lucrative business to explore. At the present, some main market players are entering this lucrative market, including Taiwan display maker Prime View International (PVI), AU Optronics Corp (AUO), Chung Hwa Picture Tube, Japan's Bridgestone, and Korea's LG Display Company. Among these companies, PVI is the largest provider, which supplies more than 90% of global e-paper modules for the readers of Amazon and Sony [4].

Presently, along with existing application on the e-paper-based products such as e-readers, billboards, tags, watches, mobile phone, and flexible e-paper (Compared with the traditional electronic paper, flexible electrophoretic display components fabricated on plastic substrates rather than a glass substrate can be bent repeatedly, so that it is closer to real paper [5]) devices, there are many concept applications for e-paper which utilize the bistable characteristics of e-paper displays, such as radio frequency identification (RFID), small display, smart card, and others [6,7]. However, with the current stage of the technology, e-paper is still driven mainly by e-readers and some information products, due to several limitations of e-paper features, such as non-full color, less brightness, low refresh rate, significant weight, small panel size, *etc.* For the reasons of e-paper being commercialized, it does not yet fully satisfy the performance desired for existing and emerging applications [5].

With the future direction for display technologies, flexible displays have become the mainstream of the global displays market, especially in e-paper technologies [7–9]. Therefore, after the successful commercialization of monochrome electronic paper, the development of next generation flexible, video, and color e-paper products has propelled [10]. Over the past few years, the global display manufacturers have invested a large amount of money in the research and development of flexible display technology. Some technological research institutes have been established in various countries, such as the Flexible Display Center at Arizona State University, the United States Display Consortium R&D projects for Flexible Display, the Technology Research Association for Advanced Display Materials (TRADIM) in Japan, the Industrial Technology Research Institute (ITRI) in Taiwan, Samsung and LGD's Display Center, the Electronic and Telecommunication Research Institute (ETRI), Korea Electronic Institute (KETI) in South Korea, *etc.*

In Taiwan, e-paper is a major sector of the optoelectronic industry, which has paid attention to the development of the e-paper industry in recent years. After the first flexible e-paper, which was fabricated by Taiwan's E-ink Holding in 2000 [11], following the technical tendency in the global display industry, the e-paper-based flexible display has also been promoted by several big manufacturers. In 2009, one of a dominating market players, Taiwan's AUO, developed a 6-inch flexible e-paper display, which can be repeatedly bent, making it even closer to the functionality of real paper. This flexible e-paper display is portable and unbreakable, significantly increasing the product's cushion [12]. Moreover, along with the effort of manufacturers, the support of government in the technological development of e-paper has a critical role in strengthening the competitive advantages of Taiwan's e-paper industry. Recently, ITRI, a government technological research center, has fabricated a novel e-paper named i2R e-paper. The i2R e-paper is rewriteable and environmentally-friendly thermal printable e-paper with high-resolution capability (300 dpi). For these technical advances, the novel applications of e-paper has been foreseen to be used for e-banner, e-cards, e-signage, e-badges, and e-tickets with several electrical addressing and refreshing mechanisms [13]. As mentioned above, with the holding of key technologies for e-paper and the experience of manufacturing consumer electronics over a long history, Taiwan's e-paper has more competitive advantage to be a leader in the global e-paper sector as well as entering more e-paper-based products into mass production [14]. Despite these advantages, with current technologies in the display industry, the e-paper-based product market remains in an early stage of development. For the long term, in order to improve e-paper performance and expand its applications, technological breakthroughs *via* R&D investment are indispensable. This market would provide huge growth potential and gives more opportunities, as well as challenges, for Taiwan's local manufacturers and other rivals from South Korea and Japan [4].

Due to the high investment capital and rapid change property of high-tech industries, most policymakers and stakeholders have intentionally chosen a simple and quick way to find out possible market trends and changing demands in the short- and medium-term [15]. As a result, using a suitable forecast method to accurately predict the market size for Taiwan's e-paper industry is necessary. It plays a vital role to reduce the risk from the uncertain environment and offer some critical information to draft the correct strategies, as well as plan the manufacturing schedules.

As shown in literature [16], from a long-term perspective, a reliable forecast in the technology field can help maximize gain and minimize loss for organizations. The forecasting in the technology field has therefore been recognized as an effective way in order to anticipate and understand the potential direction, rate, and effects of technological change [17–19]. Some forecasting techniques, such as time-series cross-sectional regression (TSCS), artificial neural network (ANN), hybrid grey model, exponential smoothing, Grey-Markov, genetic algorithms, and grey models have been suggested for the high-tech industry [20–23]. However, to our knowledge, none of the previously-published papers proposes an effective forecasting model to understand the trend of the e-paper industry. Thus, this work forecasts the future market size of Taiwan's e-paper industry by employing an effective combined grey model.

Compared with other forecasting approaches, Grey system theory has been widely applied in various fields for two decades, due to small sample size, simple calculation and high accuracy in short-term forecasting. Grey system theory has been used as an effective tool for understanding the uncertain environment under the limitation of information [24]. As a core of the grey system theory, the GM(1,1) model successfully estimated the future trend in many fields, such as energy [25,26], economics [27], and engineering [28]. Despite this, previous studies also found some defects of the grey model, and its performance still can be improved [29]. Therefore, in order to enhance the forecasting capability of the grey model and extend the scope of application of Grey system theory, the literature has provided numerous solutions. Some new types of grey model, such as GM(2,1), DGM(2,1), and Verhulst model, were modified based on GM(1,1) grey system theory [30,31]. Although the DGM(2,1) forecasting model is considered when the data sequence has fluctuating characteristics [30], some research pointed out that its model performance was not high and was difficult to apply directly to actual estimation [32]. From this, some works on improving the prediction of the DGM(2,1) model have been discussed [33–35]. On the other hand, some Grey forecasting models combining with other approaches, such as Grey-Fourier [36], Grey-Markov [26,37], Grey-Fuzzy or Grey-Taguchi [38,39], were developed to improve the forecasting precision.

The Fourier Markov Grey forecasting model (FMGM(1,1)), which integrated GM(1,1), Fourier series and Markov chain, is one type of a combined grey model with high accuracy prediction. FMGM(1,1) has been adopted to forecast the development of typhoon rainfall systems and a steel price system; simulation results show that FMGM(1,1) has obtained high precision and it can effectively predict the system developing under an instable environment [29]. To take long-term operation into consideration, FMGM(1,1) has been successfully applied to predict the turning time of stock markets [40]. Recently, MFGM(1,1) has also presented excellent predictive accuracy in the case of forecasts with short-term historical and randomly fluctuating data [41]. From these obtained results, the MFGM(1,1) model predicted accurately under an instable environment, randomly fluctuating data and a limited data sample. However, the integrating of Fourier series and Markov chain into DGM(2,1) and Verhulst models has not yet been proposed. Thus, in this paper, the combination of DGM(2,1) and Verhulst models with Fourier series and Markov chain, namely FM-Verhulst and FMDGM(2,1), in the same way in [29,40] will be investigated based on the annual data of Taiwan's e-paper industry. The precision of these combined models are compared with traditional grey models and FMGM(1,1) to evaluate the capability of forecasting. Finally the best model is selected to predict the trend of Taiwan's e-paper industry.

Following this introduction, the rest of this paper is structured as follows. In Section 2, mathematical functions of proposed models are briefly illustrated. Empirical results analyzing Taiwan's e-paper industry are discussed in Section 3. The final section presents the remarkable conclusions of this study.

2. Methodology

Grey system theory was developed by Professor Deng in 1989 and focused on the relation between the analytical model construction and for circumstances such as no certainty, multi-data input, discrete data, and insufficient data through predicting and decision-making. Regarding the grey predicting sector, several types of grey models can be supported in this system, such as GM(1,1), GM (m,n), and the Verhulst model. The literature review of these models will be presented in this section [42].

2.1. Basic Grey Forecasting Model GM(1,1)

GM(1,1) is understood as a one variable, first-order grey model and is the most popular and widely applied in the previous research in the forecasting field. Based on Grey system theory [24], the mathematical function of GM(1,1) is described as:

Step 1: Suppose an original series with n entries is:

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(k), \ldots x^{(0)}(n) \right\} \tag{1}$$

where, $x^{(0)}(k)$ is the value at time $k\ (k = 1, 2, \ldots, n)$.

Step 2: From the original series $x^{(0)}$, a new series $x^{(1)}$ can be generated by a one time accumulated generating operation (1-AGO), which is

$$x^{(1)} = \left\{ x^{(1)}(1), x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(k), \ldots, x^{(1)}(n) \right\} \tag{2}$$

where, $x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j)$.

Step 3: A first-order differential equation with one variable is expressed as following:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{3}$$

where, a is the developing coefficient and b is the grey input coefficient. These two coefficients can be determined by the least squares method as the following:

$$[a, b]^T = \left(B^T B \right)^{-1} B^T Y \tag{4}$$

where,

$$B = \begin{bmatrix} -\left(x^{(1)}(1) + x^{(1)}(2) \right)/2 & 1 \\ -\left(x^{(1)}(2) + x^{(1)}(3) \right)/2 & 1 \\ \vdots & \vdots \\ -\left(x^{(1)}(n-1) + x^{(1)}(n) \right)/2 & 1 \end{bmatrix} \text{ and}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n) \right]^T$$

Solve Equation (4), the time response function of the GM (1,1) is given by:

$$\hat{x}^{(1)}(k) = \left[x^{(0)}(1) - \frac{b}{a} \right] e^{-a(k-1)} + \frac{b}{a} \qquad (k = 2, 3, \ldots, n) \tag{5}$$

Based on the operation of one time inverse accumulated generating operation (1-IAGO), the predicted series $\hat{x}^{(0)}$ can be obtained as following:

$$\hat{x}^{(0)} = \left\{ \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \hat{x}^{(0)}(3), \ldots, \hat{x}^{(0)}(k), \ldots, \hat{x}^{(0)}(n) \right\} \tag{6}$$

where, $\begin{cases} \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \qquad (k = 2, 3, \ldots, n) \end{cases}$.

2.2. Direct Grey Forecasting Model DGM(2,1)

DGM(2,1) is a version of the grey forecasting models in which the original series in Equation (1) and the new sequence $x^{(1)}$ are used to construct the whitenization equation of DGM(2,1) by setting up a second-order differential equation. DGM(2,1) is a single sequence, second-order, linear dynamic model and its mathematical formula is presented as following [24,30,31]:

$$\frac{d^2x^{(1)}}{dt^2}+a\frac{dx^{(1)}}{dt}=b \tag{7}$$

where,

$$\hat{P}=\left[\hat{a},\hat{b}\right]^T=\left(B^TB\right)^{-1}B^TY$$

$$B=\begin{bmatrix}-x^{(0)}(2) & 1\\ -x^{(0)}(3) & 1\\ \cdots & \cdots\\ -x^{(0)}(n) & 1\end{bmatrix} \text{ and}$$

$$Y=\begin{bmatrix}x^{(0)}(2)-x^{(0)}(1) & 1\\ x^{(0)}(3)-x^{(0)}(2) & 1\\ \cdots & \cdots\\ x^{(0)}(n)-x^{(0)}(n-1) & 1\end{bmatrix}$$

According to Epuation (7), time response function can be given by:

$$\hat{x}^1(k+1)=\left(\frac{\hat{b}}{\hat{a}^2}-\frac{x^{(0)}(1)}{\hat{a}}\right)e^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}}(k+1)+(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}})\frac{1+\hat{a}}{\hat{a}} \tag{8}$$

where, $\begin{cases}\hat{x}^{(0)}(1)=\hat{x}^{(1)}(1)\\ \hat{x}^{(0)}(k)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \qquad (k=1,2,\ldots,n)\end{cases}$.

2.3. Grey Verhulst Model

The Grey Verhulst model was the biological growth model proposed by German biotech mathematician Verhulst in 1837. This model is usually used to predict the trends for a fixed value of reached saturation and stability. Similar to the GM(1,1) model, from the original series $x^{(0)}$ in Equation (1), the Grey Verhulst model can be defined as following [43]:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b(x^{(1)})^2 \tag{9}$$

where,

$$\hat{P}=[\hat{a},\hat{b}]^T=(B^TB)^{-1}B^TY$$

$$B=\begin{bmatrix}-\frac{1}{2}(x^{(0)}(1)+x^{(0)}(2)) & \left[-\frac{1}{2}(x^{(0)}(1)+x^{(0)}(2))\right]^2\\ -\frac{1}{2}(x^{(0)}(2)+x^{(0)}(3)) & \left[-\frac{1}{2}(x^{(0)}(2)+x^{(0)}(3))\right]^2\\ \vdots & \vdots\\ -\frac{1}{2}(x^{(0)}(n-1)+x^{(0)}(n)) & \left[-\frac{1}{2}(x^{(0)}(n-1)+x^{(0)}(n))\right]^2\end{bmatrix} \text{ and}$$

$$Y=\left[x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\right]^T$$

The solution of $x^{(0)}(t)$ at time k is:

$$\hat{x}^{(1)}(k+1)=\frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1)+(\hat{a}-\hat{b}x^{(0)}(1))e^{\hat{a}k}} \tag{10}$$

where, $\begin{cases}\hat{x}^{(0)}(1)=\hat{x}^{(1)}(1)\\ \hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \qquad (k=1,2,\ldots,n)\end{cases}.$

2.4. Fourier Residual Modified Model (FGM)

The Fourier correction approach is one of the residual correction approaches; it is used to transform the residual into frequency spectra and then select the low-frequency term. In grey prediction, Fourier series are common to increase the precision of grey models when the data is significantly fluctuating and its efficiency has been proven in several studies [22,36,44].

Let x and $\hat{x}$ be the actual series of n entries and the predicted series obtained from grey models respectively. Based on the predicted series $\hat{x}$, a residual series named ε is defined as:

$$\varepsilon=\{\varepsilon(2),\varepsilon(3),\varepsilon(4),\ldots,\varepsilon(k),\ldots,\varepsilon(n)\} \tag{11}$$

where, $\varepsilon(k)=x(k)-\hat{x}(k) \quad with \quad (k=2,3,\ldots,n)$.

Expressed in Fourier series, error residual $\varepsilon(k)$ is rewritten as:

$$\varepsilon(k)=\frac{1}{2}a_0+\sum_{i=1}^{F}\left[a_i\, cos\left(\frac{2\pi i}{n-1}k\right)+b_i\, sin\left(\frac{2\pi i}{n-1}k\right)\right]\ (k=1,2,\ldots,n) \tag{12}$$

where, $F=\left[(n-1)/2-1\right]$ is the minimum deployment frequency of Fourier series and the only select integer number [45].

P and C matrixes can be defined as following:

$$P=\begin{bmatrix}\frac{1}{2} & cos\left(\frac{2\pi\times 1}{n-1}\times 2\right) & sin\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \cdots & cos\left(\frac{2\pi\times F}{n-1}\times 2\right) & sin\left(\frac{2\pi\times F}{n-1}\times 2\right)\\ \frac{1}{2} & cos\left(\frac{2\pi\times 1}{n-1}\times 3\right) & sin\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \cdots & cos\left(\frac{2\pi\times F}{n-1}\times 3\right) & sin\left(\frac{2\pi\times F}{n-1}\times 3\right)\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \frac{1}{2} & cos\left(\frac{2\pi\times 1}{n-1}\times n\right) & sin\left(\frac{2\pi\times 1}{n-1}\times n\right) & \cdots & cos\left(\frac{2\pi\times F}{n-1}\times n\right) & sin\left(\frac{2\pi\times F}{n-1}\times n\right)\end{bmatrix} \text{ and}$$

$$C=\left[a_0,a_1,b_1,a_2,b_2,\ldots,a_F,b_F\right]^T$$

Then, the residual series is rewritten as:

$$\varepsilon=P\cdot C \tag{13}$$

The parameters $a_0,a_1,b_1,a_2,b_2,\ldots,a_F,b_F$ in Equation (12) are obtained by using the ordinary least squares method (OLS) which results in the equation of:

$$C=\left(P^TP\right)^{-1}P^T\left[\varepsilon\right]^T \tag{14}$$

Once the parameters are calculated, the modified residual series $\hat{\varepsilon}$ is then achieved based on the following expression:

$$\hat{\varepsilon}(k)=\frac{1}{2}a_0+\sum_{i=1}^{F}\left[a_i\,cos\left(\frac{2\pi i}{n-1}k\right)+b_i\,sin\left(\frac{2\pi i}{n-1}k\right)\right] \tag{15}$$

From the predicted series $\hat{x}$ and $\hat{\varepsilon}$, the Fourier modified series $\tilde{x}$ of series $\hat{x}$ is determined by:

$$\tilde{x}=\{\tilde{x}(1),\tilde{x}(2),\tilde{x}(3),\ldots,\tilde{x}(k),\ldots,\tilde{x}(n)\} \tag{16}$$

where, $\begin{cases}\tilde{x}(1)=\hat{x}(1)\\ \tilde{x}(k)=\hat{x}(k)+\hat{\varepsilon}(k) \quad (k=2,3,\ldots,n)\end{cases}$.

2.5. Grey Markov Model (MGM)

The idea of the application of the Markov state transition matrices is to enhance the capability of the grey model when random fluctuations occur in the sampling data. This algorithm uses the transition matrices to divide the state and to calculate the probability, so the future evolution of the conditional probability relies on the current state of the system but not on its history. Markov state transition matrices play the remedial role in overcoming the limitation of the grey forecasting model and are discussed in recent works. These literatures in [46–48] indicated that Markov state matrices have clearly increased the accuracy of grey forecasting models.

In this study, we use the predicted values from the grey models to conduct Markov state transition matrices first. The S states are then defined for each time step according to the distribution of the grey predicted series, and each state is an interval whose width is a fixed portion between the maximum and the minimum of the whole residual errors. With those states, the state transition between two adjacent time steps can be defined. The state transition probability from state i to state j after m steps is achieved as:

$$P_{ij}=\frac{M_{ij}(m)}{M_i}\,(i,j=1,2\ldots m) \tag{17}$$

where, $M_{ij}(m)$ is denoted the number from state i to state j after m steps and M_i is the number of state S_i. So, the m-step of transition matrix is given by:

$$P=\begin{bmatrix} p_{11}(m) & p_{12}(m) & \ldots & p_{1j}(m)\\ p_{21}(m) & p_{22}(m) & \ldots & p_{2j}(m)\\ & & \ldots & \\ p_{i1}(m) & p_{i2}(m) & \ldots & p_{ij}(m)\end{bmatrix} i,j=\overline{1,n} \tag{18}$$

$P(m)$ reflects the transition rule between different states and is the foundation of the forecasting model. We can predict the future trend of the systems by studying the stochastic transition matrix $P(m)$. If $P(1)$ has more than two lines whose probability values are the same or close to each other, it is difficult to precisely decide the next direction of the system. It is necessary for learning and checking the matrix $P(1)$ or $P(m)$ $m\geq 3$. At the same time, the transition of the system can be decided by checking $P(1)$ or $P(m)$ $m\geq 2$.

After determining the future transition state of a system, the possibilities of a certain state for the next step are obtained by the probability in r vectors denoted as $\{a_i(T),i=1,2\ldots,r\}$ at time step T. The centers of r-state as $\{v_i,(i=1,2\ldots,r)\}$ are defined first. The predicted values for the next step are then obtained as follows:

$$\hat{x}'^{(0)}(T+1)=\tilde{x}^{(0)}(T+1)+\sum_{i}^{r}a_i(T)v_i \tag{19}$$

where, a_i is the corresponding weigh for the state i.

2.6. Fourier Markov Grey Modified Model (FMGM)

The Fourier Markov Grey Modified Model (FMGM) is a forecasting model that has been developed by a combination of GM(1,1), Fourier theory, and Markov chain [49]. This model has been successfully applied, with high accuracy, compared to the traditional GM(1,1) model [29,40,41]. However, the combination of Fourier series and Markow chain with DGM(2,1) and Verhulst method have not yet been researched until now. Thus, in this paper, these models will be propsed, and their forecasting peformance aslo investigated, based on the trend analysis of the Taiwan e-paper industry. Firstly, the residual errors from the predicted series of three traditional grey models consisting of GM(1,1), DGM(2,1), and Verhuslt model are adopted. After that, a Fourier-corrected approach and Markov chain are integrated into these grey models in the same way as [29,40,41,49,50]. Finally, the performance of the proposed models is compared with traditional grey models and FMGM(1,1) to collect the best model which is capable of analyzing the trend of the Taiwan e-paper industry. The overall procedure to obtain the modified model is introduced as below:

Step 1: From the original series $x^{(0)}$ in Equation (1), the predicted series of $\hat{x}^{(0)}$ for GM(1,1), DGM(2,1) and Verhulst models are obtained by Equations (6), (8), and (10), respectively, and then residual error sequences r is determined as follows:

$$\hat{x}_g^{(0)}(k)=\left\{x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\right\} \tag{20}$$

$$r_g^{(0)}(k)=\left\{r^{(0)}(2),r^{(0)}(3),\ldots,r^{(0)}(n)\right\} \tag{21}$$

where, $r^{(0)}(k)=x^{(0)}(k)-\hat{x}^{(0)}(k)$ ($k=2,3,\ldots n$).

Step 2: Calculate the Fourier series by Equation (22) as following:

$$r_f(k)=\frac{1}{2}a_0+\sum_{i=1}^{F}\left[a_i\,cos\left(\frac{2\pi i}{n-1}k\right)+b_i\,sin\left(\frac{2\pi i}{n-1}k\right)\right]\quad k=2,3,n \tag{22}$$

From Equations (12)–(16), the parameters of Fourier series are calculated and the corresponding predicted series are obtained as follows:

$$\hat{y}_f^{(0)}(k)=\hat{x}_g^{(0)}(k)+r_f(k)\quad k=2,3,n \tag{23}$$

Step 3: Divide states and calculate the transition probability of each state by Markov chain:

Let $\hat{y}_{fm}(k)$ is a Markov chain. The values of $\hat{y}_{fm}(k)$ are distributed in the region of $\hat{y}_f^{(0)}(k)$, that may be divided into some interval according to concrete circumstance. The state partition is an important step in the Markov chain. However, there is no standard rule to divide these state intervals. In general, state partition is identified depending on the historical data and research subject [37]. The width of the interval is equal to a fixed portion between the maximum and the minimum of the whole residual errors.

When S states and probability matrix P of each state in the Markov chain are determined from Equations (17) and (18), we can define the tendency of transition probability, which may help to decide the future state transition of a system.

Step 4: Obtain predicted values:

When the future state transition is determined, the predicted values can be calculated similarly by Equation (19) and demonstrated as below

$$\hat{y}_{fm}^{(0)} = y_f^{(0)}(k) + \sum_{i}^{r} a_i(T)v_i \quad k = 2,3,n \tag{24}$$

where, a_i is the corresponding weigh for the state i.

2.7. Evaluating Performance of the Prediction Models

For evaluating the performance of proposed prediction models, three criteria are applied for comparing the accuracy of each model based on the actual value $x^{(0)}(k)$ and predicted value $\hat{x}^{(0)}(k)$. Three criteria consisting of relative percentage error, the mean absolute percentage error, and the prediction accuracy will be used in this work. Their calculation is introduced as follows:

Relative percentage error (*RPE*) measures the size of error as a percentage of actual value. This index reflects the difference between the actual value and the forecasted value. The formula of *RPE* is expressed as following:

$$RPE = \frac{\left| x^{(0)}(k) - \hat{x}^{(0)}(k) \right|}{x^{(0)}(k)} \times 100\% \tag{25}$$

Mean of average percentage error (*MAPE*) is one error measurement which is popularly applied in forecasting. *MAPE* denotes the average relative size of the predicted error. *MAPE* is defined as following:

$$MAPE = \left(\frac{1}{n} \sum_{k=1}^{n} \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{\hat{x}^{(0)}(k)} \right) \times 100\% \tag{26}$$

Prediction accuracy or precision rate (ρ) measures the level of the closeness of the statement of forecast quantity and the actual value. This index is calculated by the following function:

$$\rho = 1 - MAPE \tag{27}$$

When *MAPE* is close to 0, the forecasting model is taking place at high accuracy, and has given good performance and *vice versa*. Based on the values of *MAPE* and ρ of each model, the level of forecasting accuracy can be classified into four grades, as shown in Table 1 [51]:

Table 1. Classifying of the level of forecasting accuracy.

Grade Level	MAPE	Accuracy (ρ) ρ = 1 − MAPE
Excellent	<0.01	>0.95
Good	<0.05	>0.90
Qualified	<0.10	>0.85
Unqualified	≥0.10	≤0.85

In addition, to assess the forecasting capability of the grey model, the developing coefficient a of the grey model is used as a criterion [52]. The range of a is listed in Table 2 as:

Table 2. Forecasting capability of Grey Model.

Range of Developing Coefficient α		Forecasting Capability
1	$-\alpha \leq 0.3$	The model can be used for medium and long-term forecasting
2	$0.3 < -\alpha \leq 0.5$	The model is suitable for short-term forecasting
3	$0.5 < -\alpha \leq 0.8$	The model is carefully employed in short-term forecasting
4	$0.8 < -\alpha \leq 1$	The model should be modified with residual
5	$-\alpha > 1$	The model is not suitable for forecasting

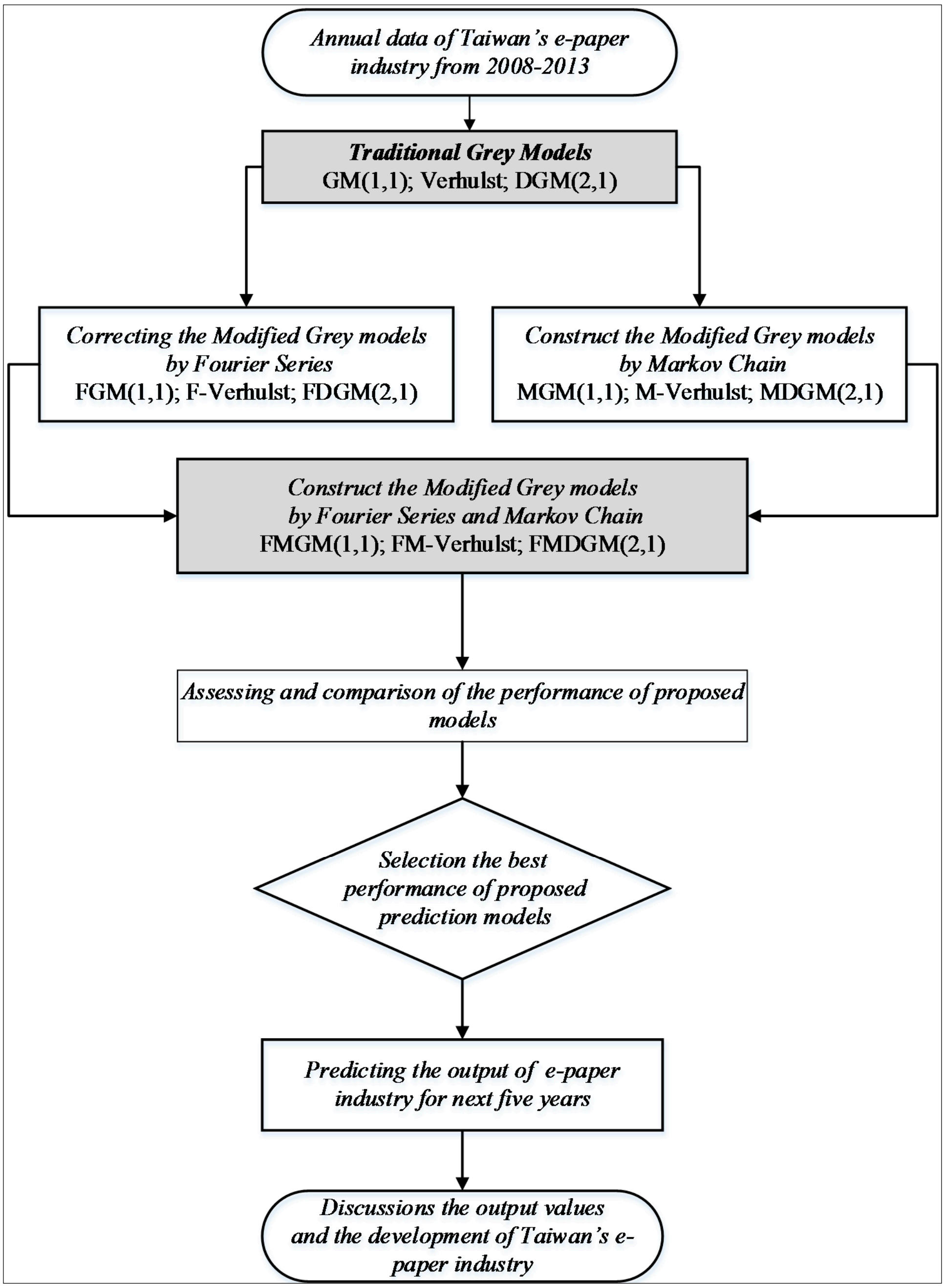


Figure 1. The flowchart of this work.

To clearly assess the performance of some different combined grey models, as well as to realize the trend of Taiwan's e-paper industry, this study uses the production values of e-paper from Taiwan Photonics Industry and Technology Development Association (PIDA) as the input data to construct the forecasting models. The flowchart of this work is shown in Figure 1.

3. Results and Discussion

In this section, the comparison of prediction capability of proposed models is conducted. The prediction model with the best performance is employed to calculate the output of Taiwan's e-paper industry in the next five years. Some discussions about the development of this industry are also presented.

As discussion of Liu and Dang [52], the developing coefficient can be considered as a criterion to assess the predictive capability of a forecasting model. Based on the data of the Taiwan e-paper industry, the developing coefficient a and input coefficient b in the model of GM(1,1), Verhulst and DGM(2,1) are shown in Table 3.

Table 3. Coefficient *a* and *b* of three models; GM(1,1), DGM(2,1), and Verhulst.

	GM(1,1)	**DGM(2,1)**	***Verhulst***
a	−0.19537	0.18369	−0.92298
b	12,195.76	8763.24	0.7029×10^{-5}

As shown in Table 3, the coefficients of GM(1,1) and DGM(2,1) fall into the first range of Table 2. This means that two proposed models can be utilized to the medium- or long-term forecasting. However, the Verhulst model is not appropriate for forecasting, so the residual error of this model must be corrected.

From the above-obtained results, the predicted results of Taiwan's e-paper industry by using nine combined grey models are shown in Tables 4 and 5. The status of predicted errors of different models is also shown in Figure 2.

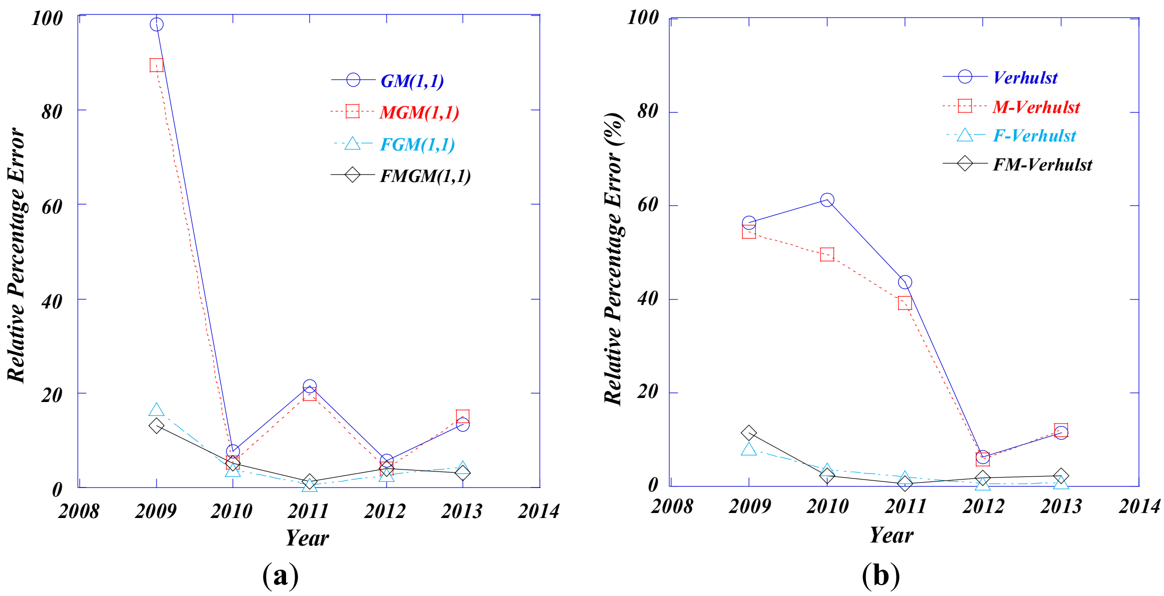
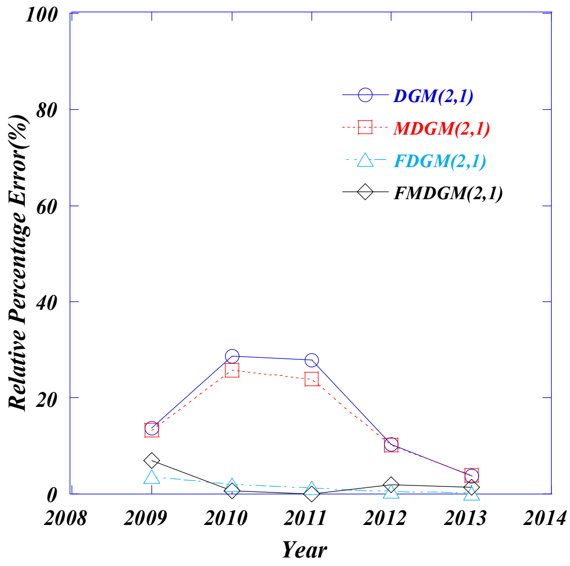


Figure 2. *Cont.*



(c)

Figure 2. (**a**) Status of the relative error of GM(1,1) and different modified models; (**b**) Status of the relative error of Verhulst and different modified models; (**c**) Status of the relative error of DGM(2,1) and different modified models.

Table 4. The predicted results of Taiwan's e-paper industry by Traditional Grey models and Markov Grey models.

Year	Actual Values (NTD Million)	*Traditional Grey Models*					
		GM(1,1)		Verhulst		DGM(2,1)	
		Predicted	RPE (%)	Predicted	RPE (%)	Predicted	RPE (%)
2008	2090	2090		2090		2090	
2009	6994	13,864.34	98.23	3046.10	56.45	6034.48	13.72
2010	18,268	16,855.67	7.73	7066.44	61.32	13,027.28	28.69
2011	26,130	20,492.41	21.58	14,714.57	43.69	18,846.61	27.87
2012	26,430	24,913.81	5.74	24,759.73	6.32	23,689.39	10.37
2013	26,705	30,289.15	13.42	29,766.25	11.46	27,719.50	3.80
MAPE (%)			29.34		35.85		16.89
Precision ρ (%)			70.6		64.15		83.11
Predicting level		Unqualified		Unqualified		Unqualified	
Year	Actual Values (NTD million)	*Markov Grey Models(MGM)*					
		MGM(1,1)		M-Verhulst		MDGM(2,1)	
		Predicted	RPE (%)	Predicted	RPE (%)	Predicted	RPE (%)
2008	2090	2090		2090		2090	
2009	6994	13,252.30	89.48	3186.29	54.44	6066.24	13.27
2010	18,268	17,300.63	5.29	9206.62	49.60	13,559.04	25.78
2011	26,130	20,937.43	19.87	15,854.76	39.32	19,878.37	23.93
2012	26,430	25,358.83	4.05	24,899.92	5.79	23,721.15	10.25
2013	26,705	30,734.03	15.08	29,906.43	11.99	27,751.26	3.92
MAPE (%)			26.75		32.23		15.43
Precision ρ (%)			73.25		67.77		84.57
Predicting level		Unqualified		Unqualified		Unqualified	

Table 5. The predicted results of Taiwan's e-paper industry by different Fourier residual modified models and Fourier Markov Grey modified models.

Year	Actual Values (NTD Million)	***Fourier Residual Modified Models(FGM)***					
		FGM(1,1)		**F-Verhulst**		**FDGM(2,1)**	
		Predicted	**RPE (%)**	**Predicted**	**RPE (%)**	**Predicted**	**RPE (%)**
2008	2090	2090		2090		2090	
2009	6994	8155.12	16.60	6431.62	8.04	6738.79	3.65
2010	18,268	17,559.72	3.80	18,930.99	3.62	18,625.82	1.96
2011	26,130	26,114.90	0.50	25,619.65	1.95	25,806.25	1.24
2012	26,430	27,162.71	2.70	26,592.78	0.62	26,596.02	0.63
2013	26,705	25,534.55	4.38	26,951.96	0.92	26,760.12	0.21
MAPE (%)			5.53		3.03		1.54
Precision ρ (%)			94.47		96.97		98.46
Predicting level		Good		Excellent		Excellent	
Year	**Actual Values (NTD million)**	***Fourier Markov Grey Modified Models(FMGM)***					
		FMGM(1,1)		**FM-Verhulst**		**FMDGM(2,1)**	
		Predicted	**RPE (%)**	**Predicted**	**RPE (%)**	**Predicted**	**RPE (%)**
2008	2090			2090		2090	
2009	6994	7915.23	13.17	6191.73	11.47	6498.90	7.07
2010	18,268	17,319.84	5.19	18,691.10	2.31	18,385.93	0.64
2011	26,130	26,452.52	1.23	25,957.27	0.60	26,143.87	0.05
2012	26,430	27,500.33	4.05	26,930.41	1.89	26,933.64	1.90
2013	26,705	25,872.17	3.12	27,289.58	2.19	27,097.74	1.47
MAPE (%)			5.35		3.71		2.23
Precision ρ (%)			94.65		96.26		97.77
Predicting level		Excellent		Excellent		Excellent	

For assessing the prediction capacity of the proposed models, some evaluative criteria are computed and integrated into Tables 4 and 5 and Figure 3. The results show that the MAPE values of three traditional grey models consisting of GM(1,1), DGM(2,1), and Verhulst fall from 29.34% to 16.89%. Therefore, these models do not pass the level of prediction and cannot apply to the e-paper industry directly, while all proposed combined grey models have more precision than three traditional grey models. Obviously, predicted errors of the combined models are significantly reduced, as shown in Figure 2. As shown in Table 4, the prediction rates of three MGM models including MGM(1,1) , M-Verhulst, and MDGM(2,1) are 73.25%, 67.77%, and 84.57%, respectively. Thus, three MGM models have better prediction performance than GM(1,1) (70.6%), DGM(2,1) (64.15%), and Verhulst (83.11%). Nevertheless, the highest precision rate of the MGM(1,1) model still places at 84.57%, identifying that MGM models do not reach the qualified predicting level. On the contrary, as shown in Table 5, the value of accuracy rates range from 94.47% to 98.49%, FGM models clearly achieved excellent prediction performance compared with MGM models. Furthermore, FGM models and the FMGM models have approximately achieved the accuracy rate. Thus, it is difficult to identify which are better models. Fortunately, as the notes of [41], the Fourier correction approach is to increase prediction capability from the considered input data set and does not change the local characteristics of grey model prediction. Thus, it can be concluded that FMGM models have obtained high performance compared with other prediction models.

Consideration of the effectiveness of different FMGM models, the results indicated that the accuracy of FMGM(1,1) (94.65%) has less than that of FM-Verhulst (96.26%) and FMDGM(2,1) (97.77%), as in Table 5. Furthermore, FMDGM (2,1) gets the best performance in all proposed models, with the precision rate exceeding 97% . In addition, the forecasting accuracy of FMDGM(2,1) reflects Level 1 in Table 2. Thus FMDGM(2,1) can be used in middle- or long-term forecasting [45].

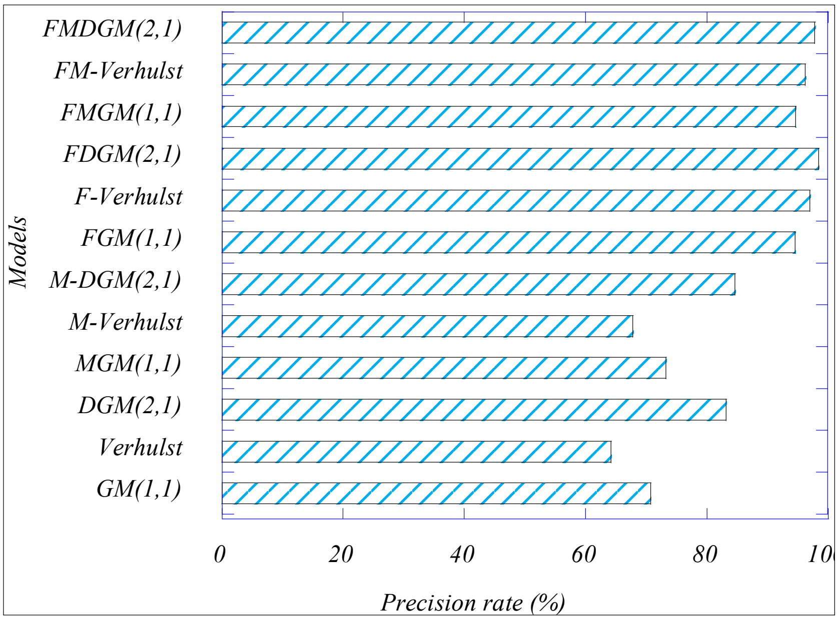


Figure 3. The accuracy of the different grey models.

As noted above, the FMDGM(2,1) model can be used successfully to analyze and explain the future trend of the e-paper industry. To act in line with the goal of this article, the FMDGM(2,1) model is suggested to estimate the output of this industry for the next five years.

According to the predicted series of $\hat{y}^{(0)}_{f(DGM2,1)}(k)$, which was created by the FDGM(2,1) model, two adjacent intervals are established based on the residual series and actual series, two states are divided as $v_1 = [-0.037, 0.01]$, $v_2 = [0.01, 0.028]$. Then, the number of historical data in each interval can be observed as: *M1 = 2*; *M2 = 4*. The transition probability matrix of state $P^{(m)}$ is also calculated by using Equations (17) and (18) and is shown in one-step as following:

$$P(1) = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 4/4 \end{bmatrix} \tag{28}$$

Finally, we use the values of *P(1)* to judge the state of next step. According to this, the predicted values of Taiwan's e-paper industry in the five next year most likely transfer to state 2. Therefore, the output of the sample can be forecasted by using Equation (23). The predicted results are illustrated in Table 6 and the forecasting curve of the e-paper industry is represented in Figure 4.

As shown, Taiwan's e-paper industry saw rapid development in the periods from 2008 to 2011. The production values reach NTD 26,130 million by 2011, rising at an annual growth rate of 1150% from NTD 2090 million in 2008. Unfortunately, this index was not changed for the period from 2011 to 2013, and it was only 2.2%. Furthermore, Taiwan's e-paper production values are predicted a slow increase in the next few years. However, the gap in the growth rate remains in a stable state. This value will reach to NTD 36,729.07 million in the next year and grow to NTD 40,024.76 million in 2018. This phenomenon can be evidenced by the rise of tablets, such as the iPad and other thin, light tablets entering the e-reader market in the past four years have caused the dramatic decrease of the growth of e-paper production. Additionally, the e-paper industry has been faced the bottleneck due to some limitations of e-paper technical features, such as non-full color, less brightness, low refresh rate, significant weight, and small panel size, all of which have significant effects to the development of the e-paper industry.

Table 6. The forecasted output values of Taiwan's e-paper industry for the five next years.

Year	Output of E-Paper Industry (Unit: NTD Million)
2014	32,954.83
2015	36,562.72
2016	36,729.07
2017	39,911.26
2018	40,024.76

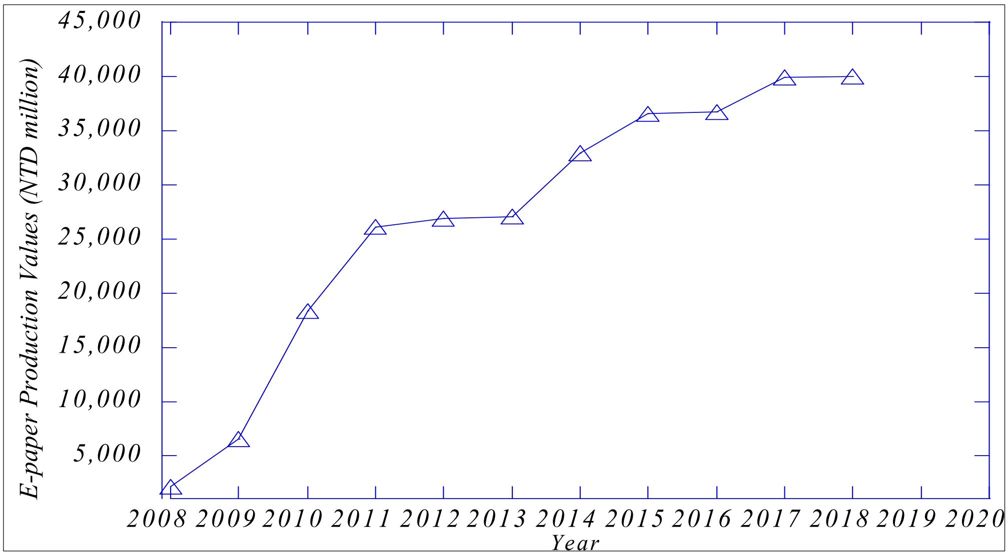


Figure 4. The forecasting curve of e-paper industry.

4. Conclusions

This paper predicted the future market size of Taiwan's e-paper industry by applying several grey models. Two combinations of DGM(2,1) and Verhulst models with Fourier series and Markov chain, namely FM-Verhulst and FMDGM(2,1), were presented. Based on the annual data of Taiwan's e-paper industry, the forecasting performance of the combined grey models and several traditional grey models were investigated. The results show that three models of GM(1,1), DGM(2,1), and Verhulst and three Markov grey models of MGM(1,), MDGM(2,1), and M-Verhulst cannot predict accurately the future trend of Taiwan's e-paper industry, while three Fourier Grey models of FGM(1,1), FDGM(2,1), and F-Verhulst achieved excellent prediction performance of Taiwan's e-paper industry. In addition, compared with FMGM(1,1), two FM-Verhulst and FMDGM(2,1) models have better forecasting accuracy. However, for long-term prediction, the results showed that the FMDGM(2,1) model obtained the best performance in all proposed grey models in this article. With obtained forecasting results of

Taiwan's e-paper industry by FMDGM(2,1) model, it can be claimed that the future market size of Taiwan's e-paper would slowly increase in the next few years. The value of e-paper production will reach NTD 36,729.07 million in the next year and grow to NTD 40,024.76 million in 2018.

Author Contributions

In this paper, Ying-Fang Huang contributed to collect the data and pointed out analysis ideas, while Chia-Nan Wang contributed to design the theoretical verifications. Hoang-Sa Dang analyzed data and prepared for the manuscript. Shun-Te Lai is involved in results discussion. All authors have both read and approved the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

Nomenclature

$x^{(0)}$	Original series
$x^{(1)}$	First-order generated sequence by AGO
a,b	Coefficients of the grey different equation
$k = 1,2,\ldots,n$	Discrete time
$\hat{x}$, $\tilde{x}$, $\hat{x}'$, y, $\hat{y}$	Forecasted series
$\varepsilon(k), r(k)$	Residual error sequence of k point
n	Length of sequence in model
F	Minimum deployment frequency of Fourier series
$a_0, a_1, b_1, a_2, b_2, \ldots, a_F, b_F$	Parameters of Fourier series
S	Transition state of Markov chain
M	State space of the Markov chain
P	Transition probability matrix
m	Step of transition
a_i	Corresponding weigh for the state i.
Greek symbols	
ρ	Accuracy rate
Subscripts	
i, j, m, n, f, r, g	Indices
Superscripts	
i, j	Indices
T	Transpose of matrix

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