Sustainability **2015**, *7*, 13085-13103; doi:10.3390/su71013085

OPEN ACCESS

sustainability

ISSN 2071-1050

www.mdpi.com/journal/sustainability

Article

A Mathematical Programming Approach to the Optimal Sustainable Product Mix for the Process Industry

Noha M. Galal * and Ahmed F. Abdul Moneim

Department of Industrial and Management Engineering, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport (AASTMT), AbuKir Campus, Alexandria 1029, Egypt; E-Mail: mail@ahmedfarouk.org

* Author to whom correspondence should be addressed; E-Mail: noha.galal@staff.aast.edu; Tel.: +20-3-562-2366 (ext. 1368); Fax: +20-3-561-0950.

Academic Editor: Tin-Chih Toly Chen

Received: 1 July 2015 / Accepted: 18 September 2015 / Published: 25 September 2015

Abstract: The increasing concerns about the environment and the depletion of natural resources are the main drivers for the growing interest in sustainability. Manufacturing operations are frequently considered to have an adverse effect on the environment. Hence, the sustainable operation of manufacturing facilities is a vital practice to ensure sustainability. The aim of this paper is to find the optimum product mix of a manufacturing facility to maximize its sustainability. A mixed integer non-linear programming model is developed to specify the product mix in order to maximize a proposed sustainability index (SI) of a manufacturing facility. The sustainability index comprises the economic, environmental and social pillars of sustainability in a weighted form using the analytic hierarchy process (AHP). The model results allow the identification of the prospective improvements of manufacturing sustainability.

Keywords: mathematical modeling; optimization; sustainable manufacturing

1. Introduction

Manufacturing is a vital requirement for a nation's development and its social and economic welfare. The manufacturing processes transform a set of inputs to outputs having a certain utility to the customer. Yet, the conversion process entails the generation of emissions and wastes and, to some extent, an inefficient use of available resources. For these reasons, academics and practitioners, driven in part by governmental legislation and customer awareness, have made efforts to minimize the adverse effects of manufacturing operations. A set of approaches and principles have gained importance as "green manufacturing", "environmental conscious manufacturing" and, more recently, "sustainable manufacturing". Sustainable manufacturing is defined by the United States Department of Commerce as: "the creation of a manufactured product with processes that have minimal negative impact on the environment, conserve energy and natural resources, are safe for employees and communities, and are economically sound [1]". The main terminology and underlying concepts of sustainability are reviewed in [2]. Numerous research efforts were dedicated to structure the field of sustainable manufacturing and to identify tools for integrating sustainability aspects alongside established traditional concepts of manufacturing [3–5]. Essential to the implementation of sustainability concepts in manufacturing is the existence of sustainability assessment tools. A review of tools applied in manufacturing is given in [6,7].

Increasing global competition and customers requiring a high variety of products cause the determination of the product mix to be an essential planning issue for profitability and customer satisfaction. The purpose of this paper is to determine the sustainable product mix and to set major operational parameters to maximize a proposed manufacturing sustainability index.

The remainder of this paper is structured as follows. The next section presents a literature review of the product mix problem considering the sustainability dimensions. Section 3 gives a detailed description of the proposed mathematical model for the sustainable product mix problem and presents a numerical instance to illustrate the implementation of the proposed model. The solution of the numerical illustration is presented in Section 4 together with the analysis of the obtained results and how these can be used in setting benchmarks for manufacturing sustainability. Finally, conclusions are drawn in Section 5, and possible areas for future research are suggested.

2. Literature Review

Recently, environmental stewardship has become one of the targets to aim at when determining the product mix. The problem of considering the optimum green product mix has been tackled in the literature in a number of research works [8–11]. Letmathe and Balakrishnan [8] have incorporated environmental constraints with other traditional production planning constraints. In their work, two mathematical models have been proposed; the first identified the optimal product mix, and the second selected additionally the type of operation to be used in production. The objective was to maximize profit under carbon emission trading policy. Tsai *et al*. [10] modeled a green product mix problem with capacity expansion features. Again, the objective was to maximize profit, considering the following cost elements: machine cost, direct labor cost, direct material cost, environmental pollution and product level cost. Activity-based costing and the theory of constraints have been integrated in [11]. This integration facilitates addressing accurately the cost of products and incorporating operational constraints [11]. Another approach to consider environmental aspects in the traditional product mix problem is using stochastic multi-objective programming to arrive at a sustainable production plan maximizing the expected return while minimizing the pollution penalties, subject to a set of environmental constraints [9].

It can be concluded that the authors have tended to monetize the environmental impact through carbon trading and taxing [8–11]. Yet, carbon trading policies are still not enforced in all countries. Furthermore, there might be scale differences in the production costs and the estimated environmental costs. This could lead to underestimating the value of resulting emissions. Hence, there is a need to have an objective function capable of fairly balancing between economic benefit and the resulting environmental and social burden or benefit. It may be also observed that the majority of research efforts are towards the prevention or reduction of environmental pollution [12]. Even when sustainable technologies are addressed, they considered technologies satisfying environmental constraints. Thus, there is a lack of considering the totality of sustainability dimensions, economic, environmental and social, in determining the product mix.

A number of sustainability assessment tools exist for applications at company and shop floor levels. The Organization for Economic Co-operation and Development (OECD) has developed the OECD sustainable toolkit [13]. The toolkit provides a total of 18 indicators categorized according to the input, operations and output of a manufacturing process. It is observed that the indicators are mainly environment oriented. The advantage of the toolkit is that it includes a set of measurable indicators with easy to access data. Yet, the lack of direct consideration of social and economic aspects is obvious.

Chen *et al*. [6] conducted a literature review to assess a set of twelve sustainability tools used at the factory level. The investigated tools were evaluated against four criteria: rapid assessment, application at the factory level, generic applicability and holistic view of sustainability. They concluded that the existing tools fail to satisfy all four criteria simultaneously, and hence, no tool efficiently aids facility planners in developing sustainable factories. Similarly, Joung *et al*. [7] conducted a review on indicators for sustainable manufacturing encompassing a set of 11 indicators. They presented a classification scheme of the NIST covering five dimensions of sustainability: environmental stewardship, economic growth, social well-being, technological advancement and performance management.

Samuel and Hashim [14] applied the framework developed by the Lowell Center for Sustainable Production (LASP) and the Global Reporting Initiative (GRI) to assess the sustainability of petrochemical industry in Malaysia. Chen *et al*. developed a sustainability measure for small and medium-sized enterprises (SMEs) in [15]. The developed tool relies on a database and a survey of 133 questions to cover all three dimensions of sustainability. Results of the different indicators are weighted and normalized and aggregated to a single score. The tool could help decision makers in identifying potential areas of improvement.

Applying the different sustainability measures in decision making is critical to ensuring implementing and improving sustainable practices on system design and at process levels [16]. Al-Sharrah *et al*. develop a multi-objective mixed integer linear programing model of design [17]. The proposed model considered three objectives: environmental, economic and safety. The model delivered an optimal network structure and the quantity to be produced at each plant. More recently, Vimal *et al*. [18] applied the sustainability measures for deploying sustainability at the process level. A comprehensive review of sustainability measures has been presented, and a graph theoretic approach has been applied to determine the interrelationship of the different sustainability measures for the shielded metal arc welding process.

In a previous work [19], a sustainability index (SI) has been proposed to assess the sustainability of a manufacturing unit. The proposed SI has been applied in the same manner proposed in [7], to assess the sustainability of the manufacturing operations and creating benchmarks and goals for improvements. The objective of the current work is to use the developed sustainability index, in the planning phase, specifically for defining the product mix of a manufacturing facility. Changes were necessary to adjust the proposed SI to fit the application in the planning phase. The adjusted index is used as an objective function, and a set of environmental, social and typical production constraints have been considered. The resulting mathematical model is a mixed integer non-linear programming model that identifies the product mix and other operating parameters of a manufacturing unit to maximize its sustainability.

The contribution of this paper is to determine the sustainable product mix capturing the totality of sustainability aspects (economic, environmental and social). This work contributes to planning sustainable manufacturing operations; determining the variety and quantities to be produced and setting up major operational parameters as overtime and training budget, which are vital determinants of sustainability of operations. The product mix problem has been previously addressed with environmental considerations [8–11]. The current work differs from previous work in that it offers a mathematical model considering the totality of aspects of sustainability, not only environmental, to help systematically plan and improve sustainable manufacturing activities.

3. Mathematical Model Development

3.1. Problem Definition

Consider a manufacturing system, such as that described in Figure 1, producing a variety of products. The system transforms a set of natural resources with the aid of labor, technology and financial resources to outputs. These outputs are the required products to be sold in the market in addition to scrap resulting from inefficiencies in the transformation process and some recyclable material. The operation of this manufacturing system has economic, environmental, as well as social impacts on its surrounding environment. Thus, to ensure the sustainability of the manufacturing system and to mitigate any resulting adverse effects, a conscious planning process is necessary. The classical target of maximizing the economic benefit of the manufacturing operation is no longer the sole and vital objective in the current era with the increased customer awareness and stringent environmental legislation. Thus, the manufacturing system faces a planning problem of what product mix to offer, *i.e.*, the variety and quantity of each type to produce so as to maximize manufacturing sustainability. Quantity and variety decisions are vital planning decisions, since they determine the amount of resources consumed, as well as the amount of output generated. Besides planning the product mix, further operational decisions have to be made that also shape the efficiency of the manufacturing operation. These decisions are the amount of renewable energy to use, quantities to be scraped or recycled, the amount of investments made in training personnel and the amount of overtime to use. These decisions heavily affect the economic, environmental and social aspects of the facility and, thus, determine the manufacturing sustainability. The target is to arrive at decisions concerning the product mix and the aforementioned operating parameters, so as to maximize system sustainability, taking into consideration the limited availability of labor time, energy, budget, material and to satisfy the forecasted demand.

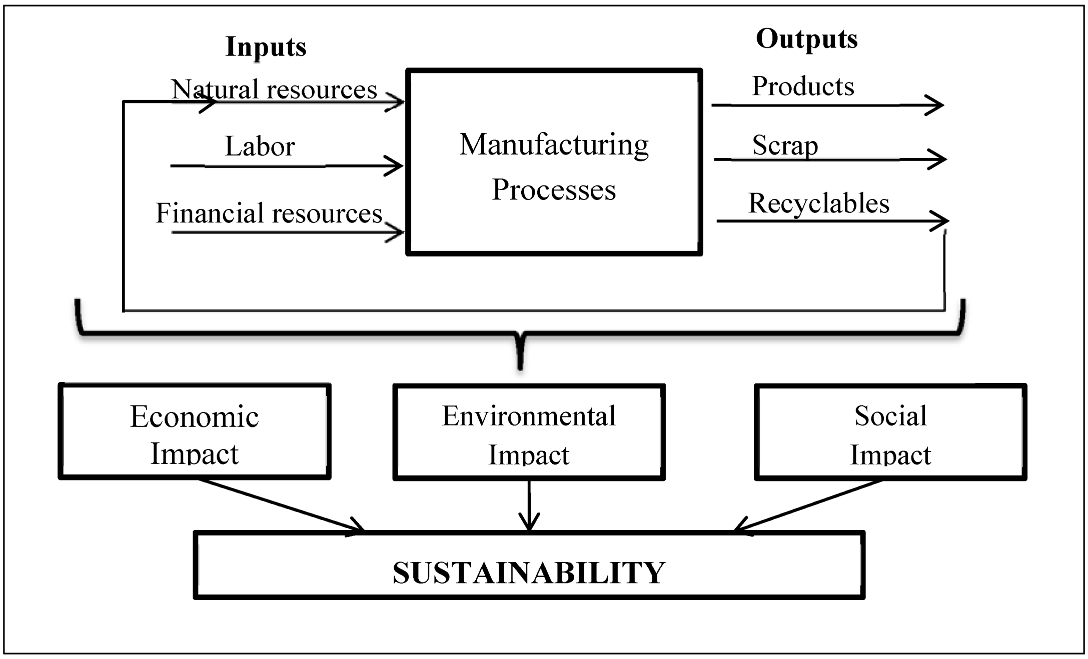

Figure 1. Elements of a manufacturing system affecting its sustainability.

3.2. Mathematical Model

The aim of this paper is to devise a mathematical model to help decision makers in setting up their product mix and other vital operating parameters, so as to maximize manufacturing sustainability. Thus, the objective function should reflect sustainability, while the constraints reflect the limited available resources. Since the sustainability consists of three pillars, economic, environmental and social, the objective function has to include these elements. An integration of these incommensurable aspects is challenging. In a previous work [19], the authors have suggested a sustainability index (SI) to integrate all three pillars into a single score having a value between zero and one, with one indicating complete sustainability. The sustainability measure expressed by the objective function has a hierarchal structure as proposed in [19] and depicted in Figure 2. It addresses the three pillars of sustainability: environmental, economic and social. Each of the pillars is evaluated by a set of sub-indicators, as shown in Figure 2. Thus, the first hierarchy consists of three elements. The environmental indicator is further divided into two further hierarchies, while the economic and social consist only of one further hierarchal level. All of the indicators and sub-indicators are so devised such that they have a value ranging from zero to one. This formulation of the objective function differs from previous approaches found in the literature, where the incommensurability problem has been approached through monetizing carbon emissions. Furthermore, the proposed model deals with all three aspects of sustainability, unlike previous approaches, which dealt mainly with economic and environmental aspects. The suggested formulation also integrates some operational decisions with the product mix decision. Assuming that the manufacturing facility has a predetermined capacity, the optimal product mix, overtime needed, amounts scrapped and recycled and the allocated training budget are sought. A mixed integer non-linear programming model is proposed for determining the optimum sustainable product mix. The measure of sustainability used is SI, as described by the objective function Elements Equations (1) to (15).

3.2.1. Model Assumptions

The following assumptions have been made:

- Only one type of emissions is considered, which is CO_2.
- Demand is deterministic and constant.
- Exact amounts of raw material required for producing each product type and CO_2 emissions resulting from the production are known.

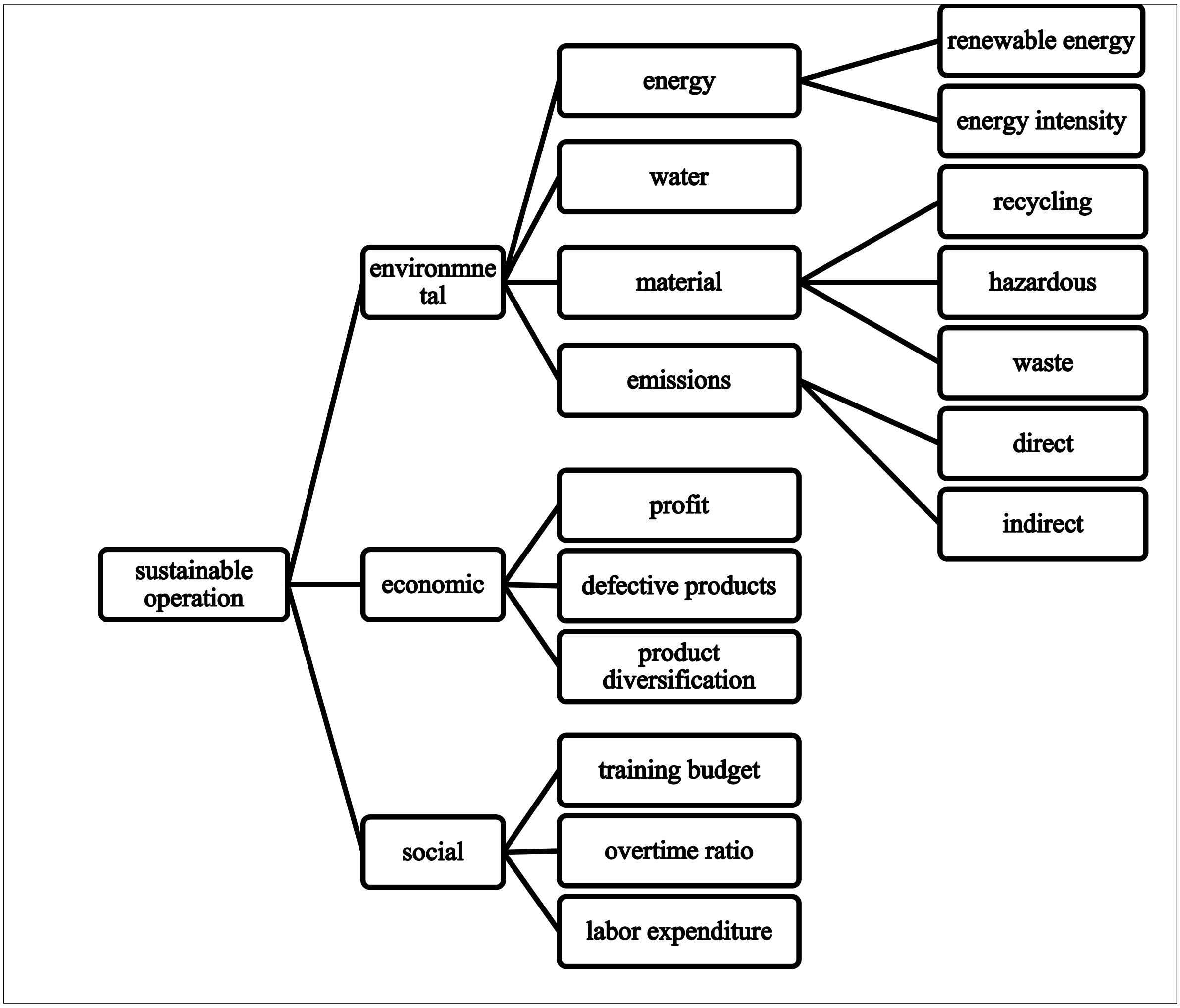

Figure 2. Hierarchal structure of the sustainability measure used in the objective function.

3.2.2. Model Nomenclature

Indices:

i	Elements of the first hierarchy level of environmental indicators
j	Elements of the second hierarchy level in environmental indicators
k	Product type, $k = 1, \ldots, N$
l	Elements of the first hierarchy level in economic and social indicators
p	Input type
m	Hazardous material type

Parameters

MH_k	Man-hours/unit weight of product *k*
N	Number of products
β_k	Ratio of recyclable products
δ_{mk}	Amount of hazardous material *m* in product *k* (kg)
RT	Available regular time (h)
W_k	Amount of water consumed per unit weight of product *k* (m^3/kg)
WW_k	Amount of waste water per unit weight of product *k* (m^3/kg)
QC_k	Amount of CO_2 generated in the production of the unit weight of product *k*
c_{p1}	Amount of emissions resulting from one kWh of electricity generated from conventional generation (kg CO_2/kWh)
c_{p2}	Amount of emissions resulting from the transportation of the unit weight per unit distance (kg CO_2/tkm)
λ_k	Percentage of defects of product *k*
D_k	Demand for product *k* (kg)
e_k	Amount of energy consumed in producing a unit weight of product *k* (kWh/kg)
E_{min}	Minimum allowable percentage of renewable energy used (%)
E_{max}	Maximum percentage of renewable energy used (%)
B	Available working capital (Egyptian Pound (EGP); 1EGP=0.13 USD)
Ce	Cost of 1 kWh of electricity via renewable resources (EGP/kWh)
Cc	Price of electricity purchased from the grid (EGP/kWh)
q_{pk}	Quantity of input type *p* in product *k* (kg/kg)
c_p	Unit cost of input type *p* (EGP/kg)
p_k	Selling price of the unit weight of product *k* (EGP/kg)
d_k	Transportation distance of product *k* (km)
Q	The maximum possible number of diversified products in the considered industry
M	Total manpower
f_k	Product fraction
E_l	Labor rate for regular time (EGP/worker hour)
E_o	Labor rate for over time (EGP/worker hour)
H_m	Maximum permissible amount of hazardous material of type *m* to include (kg)
Ov_{max}	Maximum allowed overtime expressed as a percentage of regular time (%)
Bt_{min}	Minimum training budget (EGP)
W_{1ij}	Weight of sub-indicator *i* of the *j*-th element in first hierarchal level of the environmental indicators
W_{2l}	Weight of the *l*-th element of economic indicators
W_{3l}	Weight of the *l*-th element of social indicators

Decision Variables:

Bt	Training budget (EGP)
x_k	Amount produced from product *k* (kg)

e_r	Renewable energy used expressed as the percentage of total energy necessary to produce a unit weight of product (%)
r_k	Amount of product *k* to be recycled (kg)
s_k	Amount of product *k* to be scrapped (kg)
Ov	Amount of overtime needed (h)

3.2.3. Objective Function

The objective function for the proposed model is the sustainability index, which is to be maximized. It is expressed by a series of dimensionless ratios all having a value between zero and one, as will be described next. The use of the ratio facilitates the aggregation of the different elements of the objective function to a score as described by Equation (19).

$$I_{111} = \frac{e_r \sum_k x_k e_k}{\sum_k e_k x_k} \tag{1}$$

$$I_{112} = 1 - \frac{\sum_k e_k x_k [C_e e_r + C_c(1-e_r)]}{\sum_k \sum_p c_p q_{pk} x_k + \sum_k e_k x_k [C_e e_r + C_c(1-e_r)] + RT \times M \times E_l + E_o(\max(\sum_k MH_k x_k - RT \times M, 0))} \tag{2}$$

$$I_{123} = 1 - \frac{\sum_k WW_k x_k}{\sum_k W_k x_k} \tag{3}$$

$$I_{132} = \frac{\sum_k r_k}{\sum_k \sum_p q_{pk} x_k} \tag{4}$$

$$I_{133} = 1 - \frac{\sum_k \sum_m \delta_{mk} x_k}{\sum_k \sum_p q_{kp} x_k} \tag{5}$$

$$I_{134} = 1 - \frac{\sum_k s_k}{\sum_k \sum_p q_{kp} x_k} \tag{6}$$

$$I_{141} = 1 - \frac{\sum_k QC_k x_k}{\sum_k QC_k x_k + c_{p1} \sum_k x_k e_k (1 - e_r) + c_{p2} \sum_k d_k x_k} \tag{7}$$

$$I_{142} = 1 - \frac{c_{p1} \sum_k x_k E(1 - e_r) + c_{p2} \sum_k d_k x_k}{\sum_k QC_k x_k + c_{p1} \sum_k x_k e_k (1 - e_r) + c_{p2} \sum_k d_k x_k} \tag{8}$$

$$I_{21} = \frac{\sum_k x_k p_k - \sum_k \sum_p c_p q_{pk} x_k + \sum_k e_k x_k [C_e e_r + C_c(1-e_r)] + RTME_l + E_o(\max(\sum_k MH_k x_k - RTM, 0)) + Bt}{\sum_k x_k p_k} \tag{9}$$

$$I_{22} = 1 - \frac{\sum_k \lambda_k x_k}{\sum_k x_k} \tag{10}$$

$$f_k = \frac{x_k}{\sum_k x_k} \qquad \forall k, x_k \neq 0 \tag{11}$$

$$I_{23} = \frac{\sum_{k=1}^{N} f_k \, ln(f_k)}{ln(1/Q)} \tag{12}$$

$$I_{32} = \frac{Bt}{\sum_k \sum_p c_p q_{pk} x_k + \sum_k e_k x_k [C_e e_r + C_c (1 - e_r)] + RTME_l + E_o(\max(\sum_k MH_k x_k - RTM, 0)) + Bt} \quad (13)$$

$$I_{33} = 1 - \frac{\max(\sum_k MH_k x_k - RTM, 0)}{RTM} \quad (14)$$

$$I_{34} = \frac{RTME_l + E_o(\max(\sum_k MH_k x_k - RTM, 0))}{\sum_k \sum_p c_p q_{pk} x_k + RTME_l + E_o(\max(\sum_k MH_k x_k - RTM, 0)) + \sum_k e_k x_k [C_e e_r + C_c (1 - e_r)] + Bt} \quad (15)$$

Equations (1) through (8) address the environmental dimension of sustainability, represented by four fields: energy, material, water and emissions. The energy field is described by two sub-indicators, renewable energy Equation (1) and energy intensity Equation (2). In Equation (1), the use of renewable energy is assessed by the ratio of renewable energy used to the total energy consumed in production. The renewable energy used is expressed as a fraction e_r of the total energy used in production ($\sum_k e_k x_k$). Energy intensity Equation (2) is the ratio of the cost of total energy consumed to the cost of total inputs, including raw material, energy and labor cost (regular and overtime). The cost of energy comprises both conventional ($\sum_k e_k x_k [C_c(1 - e_r)]$) and renewable energy ($\sum_k e_k x_k [C_e e_r]$) costs. The material cost term ($\sum_k \sum_p c_p q_{pk} x_k$)in the denominator of Equation (2) is the sum of costs of all input types *p* for all products *k*. As for labor costs, they include both regular costs ($RT \times M \times E_l$) and overtime costs if needed ($E_o(\max(\sum_k MH_k x_k - RT \times M, 0)$). The inclusion of labor costs is only necessary when the production time requirement exceeds the available regular time. This is guaranteed by the use of the max function. Since the increase in this ratio will negatively affect the sustainability of the system, the ratio is subtracted from one, to ensure that an increase of the indicator value towards one increases the sustainability. The ratio of wasted water to total amount of water input to production is given in Equation (3). The third field of the environmental impact addresses the material usage. Three types of material are monitored: recycled material, hazardous material and waste. Each is compared to the total material input ($\sum_k \sum_p q_{pk} x_k$), as per Equations (4) to (6). Hazardous material and waste ratios are subtracted from one to ensure that the increase in value of the indicator improves sustainability. Emissions are the last element to describe the environmental pillar. Two types of emissions are considered, direct Equation (7) and indirect emissions Equation (8). Direct emissions originate from CO_2 emissions resulting from the manufacturing operations inside the plant ($\sum_k QC_k x_k$). Indirect emissions, on the other hand, are caused by the energy generation ($c_{p1} \sum_k x_k e_k (1 - e_r)$) and transportation ($c_{p2} \sum_k d_k x_k$), which proceed outside the plant, yet are necessary to accomplish the manufacturing operations.

The economic performance is measured via Equation (9) through Equation (12). The first sub-indicator in this group is the profit fraction, which compares the profit to the sales revenue. Profit is calculated as the difference between sales revenue and total costs comprised of material ($\sum_k x_k p_k$), labor ($\sum_k e_k x_k [C_e e_r + C_c(1 - e_r)]$), energy ($\sum_k e_k x_k [C_e e_r + C_c(1 - e_r)]$) and training budget, *Bt* Equation (9). The economic dimension is also affected by the quality of the resulting output. Thus, the ratio of defective units to total output is expressed by Equation (10). Again, this ratio is subtracted from one to ensure its increase improves sustainability. The last sub-indicator in this category is the product diversification ratio. Customer demand is usually characterized by the high variety requirement. The diversification in the product mix, a marketing concept, is expressed by Equation (12). The assessment is based on the entropy measure of product diversification [20]. The calculation of Equation (12) relies on determining the product fraction, f_k, given in Equation (11). To avoid ln (zero) from occurring in Equation (12), the calculation of f_k is restricted to non-zero values of x_k. A more sustainable manufacturing system is capable of producing a variety of products with the same resources.

The social dimension of sustainability is measured by the impact of the manufacturing operations on the workers inside the plant (Equations (13) to (15)). Three sub-indicators are used in this respect: training budget ratio, overtime ratio and labor intensity. A more sustainable manufacturing operation is guaranteed if the organization spends more on training and developing the skills of its operators Equation (13), ensures that the workers do not suffer from extended working hours Equation (14) and gives the workers a reasonable monetary reward Equation (15). Thus, the training budget is compared to total input costs in Equation (13). The overtime ratio compares the overtime used, if any, $(\max(\sum_k MH_k x_k - RT \times M, 0))$ to the regular time $(RT \times M)$. Extended working hours decrease sustainability; therefore, the ratio is subtracted from one. Finally, the labor expenditure encompassing wages, incentives and insurance is compared to the total expenditure Equation (15).

Since each indicator/sub-indicator may have different relative importance depending on the organization strategy and industry type, weights are assigned to each indicator and sub-indicator, to reflect the decision makers' preferences. The weight calculation is to be accomplished via the analytic hierarchy process (AHP) [21], as in [19]. AHP is based on the concept of the pairwise comparison of the indicators/sub-indicators to arrive at a weight for each element of the objective function. With reference to Figure 2, a set of seven series of pairwise comparisons has to be established in order to arrive at a relative weight for each of the 15 elements included in the objective function. First, the three main elements, economic, environmental and social, are compared to each other. Then, the importance of the elements of each of the main three dimensions is relatively assessed. The remaining set of pairwise comparisons is performed to determine the relative importance of each of the sub-indicators of energy, material and emissions. In order to obtain the global weight of each sub-criterion, the weights obtained from the pairwise comparison of the sub-indicators are multiplied by their respective parent indicator weight [21]. For example, to get the global weight of the "recycling" indicator, its weight has to be multiplied by both the "material" and "environment" indicators. The AHP allows for including the subjective evaluation of experts. Through incorporating a consistency check, AHP reduces bias in decision makers' subjective evaluations. The inconsistency ratio is calculated by the ratio CI/RI, where CI is the consistency index and RI is the random index, the value of which depends on the number of criteria being compared. RI signifies the consistency index when the entries of the comparison matrix **A** are completely random. The pairwise comparison is considered consistent if the ratio CI/RI is less than 0.1.

Following the hierarchy structure (Figure 2), all sub-indicators under each of the three pillars are multiplied by their respective weights and aggregated as indicated by Equations (16) to (18). The result is three indicator values representing the environmental, economic and social dimensions, respectively.

$$I_{EN} = \sum_{i=1}^{4} \sum_{j=1}^{n_i} w_{1ij} I_{1ij} \tag{16}$$

$$I_{EC} = \sum_{l=1}^{3} w_{2l} I_{2l} \tag{17}$$

$$I_{SC} = \sum_{l=1}^{4} w_{3l} I_{3l} \tag{18}$$

The three components are thought of as a three-dimensional vector and are added accordingly to obtain the resultant ($\sqrt{I_{EN}^2 + I_{EC}^2 + I_{SC}^2}$) (Figure 3). In order to arrive at a value representing a relative sustainability measure, the resultant vector is divided by the maximum theoretical sustainability value, by substituting all indicators or sub-indicators by a value of one and using the current weight structure. The maximum theoretical sustainability is expressed in the denominator of Equation (19).

$$\text{Maximize } SI = \frac{\sqrt{I_{EN}^2 + I_{EC}^2 + I_{SC}^2}}{\sqrt{\left(\sum_{i=1}^{4}\sum_{j=1}^{n_i} w_{1ij}\right)^2 + \left(\sum_{i=1}^{3} w_{2i}\right)^2 + \left(\sum_{i=1}^{4} w_{3i}\right)^2}} \tag{19}$$

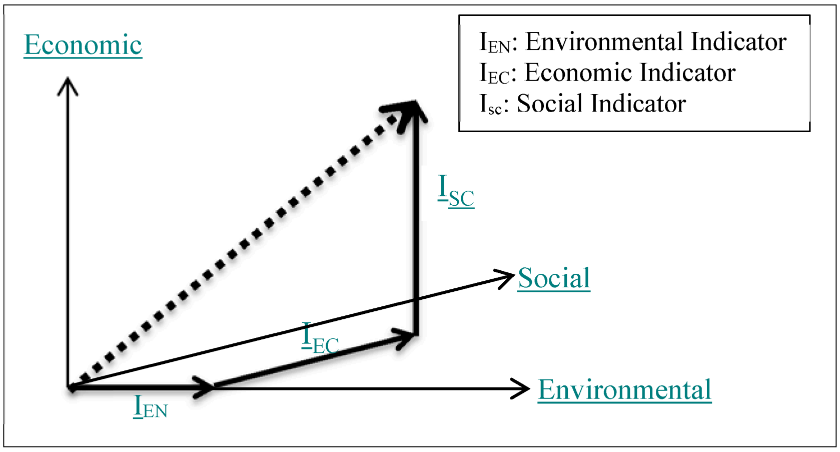

Figure 3. Three-dimensional presentation of the sustainability measure.

Equation (19) is the objective function of the proposed model, which is intended to be maximized. The value of the objective function is normalized. Thus, the maximum possible value is one, indicating the complete sustainability of the manufacturing operation.

3.2.4. Constraints

Six types of constraint sets are considered in the proposed model, as described next.

Time and labor availability constraints: Labor time availability is expressed by constraints Equation (20) and Equation (21). Overtime exists, when the required man-hours for production exceed the available regular time; otherwise, the overtime is set to zero Equation (20). Constraint Equation (21) indicates that the amount of overtime ($\sum_k X_k \; MH_k - RT \times M$) should not exceed the maximum allowable overtime ratio expressed as a percentage of regular time ($Ov_{max} \times RT \times M$). Overtime used has to be limited by the maximum allowable overtime indicated by labor unions or some target value set by the organization policy Equation (21).

$$Ov = \max(\sum_k MH_k x_k - RTM, 0) \tag{20}$$

$$Ov \leq Ov_{max} \times RT \times M \tag{21}$$

Percentage of renewable energy constraint: The use of renewable energy sources improves sustainability. The percentage of energy generated from renewable resources to the total energy used should be between the minimum and maximum target values set by the authority or the organization. This constraint addresses the gradual implementation of the use of renewable energy resources and, thus, is suitable for initiating sustainability measure implementation in small and medium-sized companies in emerging economies.

$$E_{min} \leq e_r \leq E_{max} \tag{22}$$

Demand constraint: The amount of production of each product should be less than or equal to the maximum demand of that product as in Equation (23).

$$x_k \leq D_k \qquad \forall k \tag{23}$$

Material-related constraints: This constraint set includes three categories of constraints. The first Equation (24) ensures that the amount recycled cannot exceed the maximum technically feasible recyclable amount.

$$r_k \leq \beta_k \lambda_k x_k \qquad \forall k \tag{24}$$

The second set states that the amount of hazardous material used should not exceed the maximum allowable hazardous material content permissible as indicated by the environmental authorities.

$$\sum_k \sum_m \delta_{mk} x_k \leq H_m \qquad \forall k, m \tag{25}$$

Finally, the material balance constraint Equation (26) ensures that defective products are either scrapped or recycled.

$$\lambda_k x_k = s_k + r_k \qquad \forall \text{k} \tag{26}$$

Budget constraint: Total production cost consisting of the sum of labor, material, and energy costs together with the budget invested in training the personnel should not exceed the available working capital Equation (27). Furthermore, the training budget should be greater than a minimum budget value set by the authority or by the organization according to its mission and vision Equation (28).

$$\sum_k \sum_p c_p q_{pk} x_k + RT \times M \times E_l + Ov \times RT \times M \times E_o + \sum_k e_k x_k [C_e e_r + C_c(1 - e_r)] + Bt \leq B \tag{27}$$

$$Bt \geq Bt_{min} \tag{28}$$

Variable bounds: Finally, the limit on the sub-indicator value to be between zero and one and the non-negativity constraints are expressed in Equations (29) and (30), respectively:

$$0 \leq I_{1ij}, I_{2l}, I_{3l} \leq 1 \qquad \forall i, j, l \tag{29}$$

$$Bt, X_k, er, X_{kr}, x_{ks} \geq 0 \tag{30}$$

3.3. Numerical Illustration

The model has been implemented using a hypothetical example in a process industry. Determining the product mix for an industrial facility is a vital decision that determines its profit. Decision makers need to offer a number of products satisfying customer demand and maximizing their profit. Yet, when considering the environmental and social impact of the product mix, a tradeoff may be necessary. Along with setting the product mix, other operating decisions may affect sustainability, such as how much overtime is necessary to allocate and how to deal with defective products. In view of the emerging environmental legislation and customer pressure, the company is considering planning its product mix taking all three aspects of sustainability into consideration. Nevertheless, the three dimensions of sustainability are not of equal importance to the decision maker. Since the environmental regulations are still emerging, it is given a lower importance than the economic aspects, which remain the main driver. Thus, the three dimensions are given different relative weights to reflect the case being considered. It has been assumed that a medium-sized company is planning its product mix of three products (k = 1, 2, 3). Given are the annual demand, energy consumption, man-hour requirements, input material cost and quantity, cost values, amount of water wasted and emission values, as shown in Table 1. The company needs to determine how much of each type to produce, the amount of overtime, if necessary, to allocate and the amount to recycle or to dispose of in order to maximize its sustainability.

Table 1. Input data.

Parameter	Value	Units	Parameter	Value	Units	Parameter	Value	Units
	0.02288			358.30			100	
MH_k	0.00763	h/kg	p_k	139.30	EGP	d_k	200	km
	0.00572			114.00			400	
	0.0014×10^{-3}			9.518×10^{-6}			1000	
δ_{mk}	0.0007×10^{-3}	kg	QC_k	4.759×10^{-5}	kg CO_2/kg	D_k	1000	ton
	0.0014×10^{-3}			1.904×10^{-5}			1000	
	0.2473			0.1 0.2 0.1			0.300	
W_k	0.4946	m^3	q_{pk}	0.2 0.3 0.8	kg/kg	c_p	0.233	EGP/kg
	0.4122			0.7 0.5 0.5;			0.637	
	0.0393			25.5×10^{-3}		E_o	36.75	EGP/h
WW_k	0.0196	m^3	e_k	8.5×10^{-3}	kWh/kg			
	0.0131			6.4×10^{-3}		E_l	24.5	EGP/h
B	8,629,140	EGP	Ce	1.2	EGP/kWh	E_{min}	2	%
RT	2400	h	-	-	-	E_{max}	7	%
Ov_{max}	30	%	c_{p2}	3.041×10^{-3}	kg CO_2/kgkm	H_m	20	kg
λ_k	0.07	%	c_{p1}	6.648×10^{-4}	kg CO_2/kWh	Q	12	-
-	-	-	Cc	0.75	EGP/kWh	M	50	worker
β_k	0.36	%	Bt_{min}	$0.0025 \times B$	EGP	N	3	-

The input data are based on the following assumptions: a one shift operation for 300 days a year; the overtime rate is 1.5-times the regular labor rate; and scrap and recyclable percentages are constant for all three products. In order to identify the relative weight of the sustainability indicator, the AHP [21] has been applied. To arrive at the weight given in Table 2, the following steps were necessary. First, a questionnaire is given to an expert. It consists of a set of seven pairwise comparisons to cover all of the indicators and sub-indicators. The expert is asked to rate the relative importance of the indicator or sub-indicator using a scale of numerical values ranging from one to nine. The value of one means that both indicators being compared are of the same importance, while nine signifies that the indicator is absolutely more important than the other. From the questionnaire, a comparison matrix, **A**, is developed. In this numerical illustration, a set of seven comparison matrices is necessary. An example of a comparison matrix is given in Table 3. The weight of each criterion is calculated by normalizing all of the entries of the comparison matrix, **A**, *i.e.*, each element is divided by the sum of its respective column, and then averaging the row entries. The relative weight is given in the last column of Table 3. To check for the consistency of the results, a consistency ratio is calculated. This ratio is obtained by multiplying the comparison matrix **A** by the weight vector and dividing the result by the weight vector, then averaging the elements of the resulting vector to obtain a value x. The resulting numeral value is subtracted from the number of indicators being compared, m, and divided by the number of indicators under comparison less one, as in Equation (33).

$$CI = \frac{x - m}{m - 1} \tag{33}$$

Table 2. Weights for the sustainability indicators in the objective function.

Environmental Indicators		**Economic Indicators**	
Indicator	**Weight**	**Indicator**	**Weight**
I111	0.013	I21	0.583
I112	0.004	I22	0.141
I123	0.053	I23	0.056
I132	0.002	**Social Indicators**	
I133	0.007	**Indicator**	**Weight**
I134	0.001	I32	0.015
I141	0.035	I33	0.062
I142	0.004	I34	0.006

Table 3. Comparison matrix for the first hierarchal level of the sustainability index.

Indicator	Environmental	Economic	Social	Weight
Environmental	1	1/7	2	0.13738
Economic	7	1	8	0.77984
Social	1/2	1/8	1	0.08277

Finally this *CI* value is divided by *RI*; if the result is less than 0.1, the results are consistent and accepted; and otherwise, the questionnaire is repeated.

To arrive at the weight value for sub-indicators at a higher hierarchical level as the renewable energy (Figure 2), it is necessary to get the values of the relative weight for it and for the energy and environment indicators, as described above. The product of the weight of all three indicators is calculated to form a global indicator value. The weight structure elicited from the decision maker is given in Table 2.

4. Results and Discussion

The proposed model (a mixed integer non-linear programming model) has been solved using Lingo 14, and a global optimum solution has been obtained. Table 4 summarizes the model results. The optimum sustainability achievable for the manufacturing system under study is 0.5083 (50.83%). The results call for a product mix consisting of 13,246.48 and 2,078.47 kg of Products 1 and 2, respectively. No overtime is needed, and a renewable energy generation of 0.7% of the energy consumed in production is recommended. The percentage of recycled and scrapped material for the two products is 2.1% and 4.9%, respectively. The training budget is set at 21,572.85 EGP annually, representing 0.25% of the total budget. Thus, the proposed model succeeds in determining the product mix taking the three aspects of sustainability into consideration. Furthermore, it helps in setting up other operational parameters as determining the amount of renewable energy to use, the training budget to allocate to worker development and assigning defective products to the different end of life options (recycling or scrapping). This is achieved taking the preference of the decision makers as to the relative importance of the different sustainability indicators into consideration. In the above-described illustration, the economic dimension is still given the highest importance. This is the case where the sustainability concepts are still introduced or when strict environmental regulations are still missing.

Table 4. Summary of the results.

Variable	Value	Units
Bt	21,572.85	(EGP)
x_k	13,246.48; 2078.47; 0	(kg)
Ov	0	(h)
z_0	0	-
e_r	0.007	(%)
r_k	278.18; 43.65; 0	(kg)
s_k	649.1; 101.8; 0	(kg)
SI	0.5083	-

The obtained results may be further used as guidance for enhancing the sustainability of the manufacturing operation. The value of each indicator is given in Table 5. The indicators have been devised to take on a minimum value of zero and a maximum value of one, with a larger value indicating an improvement in performance. Thus, these indicator values can be used to guide the decision maker in prioritizing his effort for improving sustainability. Figure 4 depicts the prospective improvement in each of the indicators. According to the current weight structure of the objective function, there are four indicators that have a wide margin of improvement to increase sustainability. The increase of renewable energy usage (I111), recycling (I132), reduction of indirect emissions (I142) and the increase in operator training and development (I32) are the highest four prospective fields of improvement. In the current solution, these indicators are barely satisfied; thus, there is large room for improvement. Three out of these four indicators relate to the environmental pillar, while one belongs to the social pillar. Next is increasing product diversification (I23) in the economic pillar. Hence, through the analysis of the results, it is possible to establish guidelines for improving manufacturing sustainability.

Table 5. Summary of indicator values.

Environmental Indicators		Economic Indicators	
Indicator	**Value**	**Indicator**	**Value**
I111	0.007	I21	0.41
I112	0.998	I22	0.93
I123	0.870	I23	0.160
I132	0.021	**Social Indicators**	
I133	0.9999	**Indicator**	**Value**
I134	0.9951	I32	0.007
I141	0.999	I33	1
I142	0.000	I34	0.990

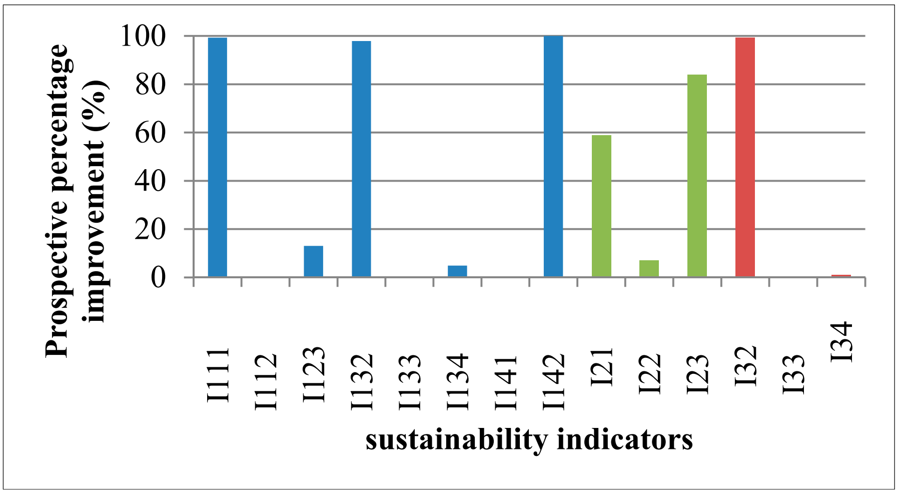

Figure 4. Prospective maximum percentage improvement of economic, environmental and social indicators.

In order to gain insight into the effect of including sustainability in the decision making process, the model has been resolved for another scenario considering only the economic dimension, as is the case with the traditional product mix problem. In the proposed model, the economic dimension is presented by three indicators. The weight of these economic indicators is assumed to be equal, while the weights of all of the environmental and social indicators are set to zero. The new results for Scenario 2 are given in Table 6. When comparing the results from the first scenario (Table 4) with the second scenario (Table 6), a number of differences can be identified. First, all three types of products are produced, and not only two, as in the first scenario. This can be justified by the objective function, which targets increasing the economic benefit without any environmental or social restrictions. Second, no recycling is recommended; again, this is due to the lack of environmental restrictions. Third, less of the renewable energy usage is suggested. Finally, there is a decrease by 8.04% of the original sustainability index value. Thus, these differences in the results emphasize the contribution achieved by incorporating all sustainability dimensions in the decision making process. Considering only one dimension (the economic) does not satisfy the sustainability targets.

Table 6. Summary of results when only the economic aspect is considered.

Variable	Value	Units
Bt	21,572.85	(EGP)
x_k	9347.1; 6108.5; 1884.3	(kg)
Ov	0	(h)
z_0	0	
e_r	0.002	(%)
r_k	0; 0; 0	(kg)
s_k	654.3; 427.6; 131.9	(kg)
SI	0.46742	-

Hence, the proposed model determines the product mix, taking all three pillars of sustainability into consideration. Previous studies have considered the environmental aspects in addition to the traditional economic aspects. The proposed model presents an efficient tool for setting up major production parameters for sustainable manufacturing. The nature of the objective function (SI) being dimensionless facilitates benchmarking performance among different facilities. Furthermore, analysis of the results helps decision makers with identifying areas for improving sustainability. Using AHP facilitates the incorporation of decision makers' preferences about the relative importance of the different sustainability dimensions. It can further easily model facilities at different implementation levels of sustainability concepts through changing the weight structure.

5. Conclusions

This work has introduced a formulation for a sustainable product mix problem in the process industry. The contribution of the proposed formulation is its ability to addresses all three pillars of sustainability at the planning level. The main advantage of the proposed measure of sustainability is the minimal amount of effort required in data collection. It mainly relies on data usually collected in all plants for cost analysis and quality control. This fact makes the model applicable in facilities introducing sustainability concepts. It thus contributes to encouraging the implementation of sustainable practices in manufacturing, especially in emerging economies, where there is still a lack of sustainability awareness and related legislation. The inclusion of AHP to give the relative weight of importance to the terms of the objective function also makes the model suitable for its intended usage in SMEs of emerging economies. Different weight structures may reflect different stages of implementing sustainability in manufacturing. At the introductory level, the economic dimension prevails over the decision making process with the lack of environmental legislation. The introduction of environmental legislation by local authorities or foreign authorities, in the case of export, will enforce the higher relative importance of the environmental dimensions. The same concept is applicable to the social dimension, which is a very important dimension for SMEs in emerging economies, which are usually labor-intensive industries. The consideration of the proposed quantitative measures is a step forward towards the quantification of the social dimension, which is usually hard to quantify. The model arrives at the optimum product mix for maximum sustainability under the current work environment and the predefined weight structure. Yet, the results allow also for identifying the prospective measures for improving sustainability. Through results analysis, a plan for increasing sustainability can be well defined. Furthermore, the model can be used to optimize the product mix for sustainability at different stages of introducing sustainability measures through changing the weight structure. The current research can be extended by integrating the model with life cycle assessment results, incorporating stochastic demand and considering capacity expansions. It is also applicable to other forms of industries with minor changes in the definition of decision variables and model parameters.

Acknowledgments

The authors acknowledge the time and effort of the two anonymous referees whose constructive comments have helped improve the quality of this manuscript.

Author Contributions

Both authors have contributed to the formulation of the proposed mathematical model and discussed the results. Noha M. Galal has solved the model using Lingo software. Ahmed F. Abdul Moneim has thoroughly revised the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

1. How Does Commerce Define Sustainable Manufacturing? Available online: http://www.trade.gov/competitiveness/sustainablemanufacturing/how_doc_defines_SM.asp (accessed on 25 December 2014).
2. Glavič, P.; Lukman, R. Review of sustainability terms and their definitions. *J. Clean. Prod.* **2007**, *15*, 1875–1885.
3. Jovane, F.; Yoshikawa, H.; Alting, L.; Boër, C.R.; Westkamper, E.; Williams, D.; Tseng, M.; Seliger, G.; Paci, A.M. The incoming global technological and industrial revolution towards competitive sustainable manufacturing. *CIRP Ann. Manuf. Technol.* **2008**, *57*, 641–659.
4. Rosen, M.A.; Kishawy, H.A. Sustainable manufacturing and design: Concepts, practices and needs. *Sustainability* **2012**, *4*, 154–174.
5. Bi, Z. Revisiting system paradigms from the viewpoint of manufacturing sustainability. *Sustainability* **2011**, *3*, 1323–1340.
6. Chen, D.; Schudeleit, T.; Posselt, G.; Thiede, S. A state-of-the-art review and evaluation of tools for factory sustainability assessment. *Procedia CIRP* **2013**, *9*, 85–90.
7. Joung, C.B.; Carrell, J.; Sarkar, P.; Feng, S.C. Categorization of indicators for sustainable manufacturing. *Ecol. Indic.* **2013**, *24*, 148–157.
8. Letmathe, P.; Balakrishnan, N. Environmental considerations on the optimal product mix. *Eur. J. Oper. Res.* **2005**, *167*, 398–412.
9. Rădulescu, M.; Rădulescu, S.; Rădulescu, C.Z. Sustainable production technologies which take into account environmental constraints. *Eur. J. Oper. Res.* **2009**, *193*, 730–740.

10. Tsai, W.-H.; Lin, W.-R.; Fan, Y.-W.; Lee, P.-L.; Lin, S.-J.; Hsu, J.-L. Applying a mathematical programming approach for a green product mix decision. *Int. J. Prod. Res.* **2012**, *50*, 1171–1184.
11. Tsai, W.-H.; Chen, H.-C.; Leu, J.-D.; Chang, Y.-C.; Lin, T.W. A product-mix decision model using green manufacturing technologies under activity-based costing. *J. Clean. Prod.* **2013**, *57*, 178–187.
12. Yuan, C.; Zhai, Q.; Dornfeld, D. A three dimensional system approach for environmentally sustainable manufacturing. *CIRP Ann. Manuf. Technol.* **2012**, *61*, 39–42.
13. OECD Sustainable Manufacturing Toolkit. Avalable online: http://www.oecd.org/sti/inno/48101937.pdf (accessed on 25 December 2014).
14. Samuel, V.B.; Agamuthu, P.; Hashim, M.A. Indicators for assessment of sustainable production: A case study of the petrochemical industry in Malaysia. *Ecol. Indic.* **2013**, *24*, 392–402.
15. Chen, D.; Thiede, S.; Schudeleit, T.; Herrmann, C. A holistic and rapid sustainability assessment tool for manufacturing SMEs. *CIRP Ann. Manuf. Technol.* **2014**, *63*, 437–440.
16. Jayal, A.D.; Badurdeen, F.; Dillon, O.W.; Jawahir, I.S. Sustainable manufacturing: modeling and optimization challenges at the product, process and system levels. *CIRP J. Manuf. Sci. Technol.* **2010**, *2*, 144–152.
17. Al-Sharrah, G.; Elkamel, A.; Almanssoor, A. Sustainability indicators for decision-making and optimisation in the process industry: The case of the petrochemical industry. *Chem. Eng. Sci.* **2010**, *65*, 1452–1461.
18. Vimal, K.E.K.; Vinodh, S.; Raja, A. Modelling, assessment and deployment of strategies for ensuring sustainable shielded metal arc welding process—A case study. *J. Clean. Prod.* **2015**, *93*, 364–377.
19. Sustainable Manufacturing Indicators. Available online: http://gccbs2013.aast.edu/newgcc/images/pdf/sustainable%20manufacturing%20indicators.pdf (accessed on 25 December 2014).
20. Park, K.; Jang, S.S. Effect of diversification on firm performance: Application of the entropy measure. *Int. J. Hosp. Manag.* **2012**, *31*, 218–228.
21. Saaty, T.L. Decision making with the analytic hierarchy process. *Int. J. Serv. Sci.* **2008**, *1*, 83–98.