Sustainable Mobility: Using a Global Energy Model to Inform Decision Makers

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1. Mathematical Formulation of Model

This model is formulated as a Linear Program (LP). The objective is to minimize the cost to society based on the available supply and required demand to meet global sustainability targets. Mathematically describing the relationships between energy sources and conversion of these sources to meet the demands from the various sectors involves the introduction of numerous inputs and constraints, as presented below.

1.1. Input

In this section, we introduce the sets and input parameters such as the type of energy supply, the type of energy conversion plants, and the demand from different transportation modes. The parameters capture costs associated with different investments and storage, estimation of future emissions, the amount of energy produced by different sources in different regions, etc.

1.1.1. Sets First we describe the *Energy* sets of all energy options for both primary energy sources and secondary energy carriers.

E = all energy sources and carriers.
$E^I \subset E$ = all energy sources that be converted (both primary and secondary).
$E^{ICG} \subset E^I$ = energy sources that can be used for co-generation of heat and electricity.
$E^{IP} \subset E^I$ = primary energy sources.
$E^{IPT} \subset E^{IP}$ = primary energy sources that can be traded between regions.
$E^{IPF} \subset E^{IP}$ = primary energy sources that are fossil fuels.
$E^O \subset E$ = all energy carriers (i.e. converted from primary energy sources).
$E^{OT} \subset E^O$ = energy carriers that can be traded between regions.
$E^{OCG} \subset E^O$ = energy carriers that can be produced from co-generation.
$E^{OM} \subset E^O$ = energy carriers that can be used in the transportation sector.
$E^{OS} \subset E^{OM}$ = synthetic and gaseous fuels used in the transportation sector.
$E^{OR} \subset E^{OM}$ = energy carriers for the road transportation sector.
$E^{ORL} \subset E^{OR}$ = synthetic fuels and petroleum-based fuels for road transport.

Next, we describe the *Transportation Mode* sets:

M = all transportation modes
$M^P \subset M$ = passenger transportation modes
$M^V \subset M$ = passenger cars and road-based freight
$M^L \subset M$ = heavy road-based transportation modes
$M^F \subset M$ = all freight transportation modes
$M^B \subset M$ = sea-based freight transportation modes

The *Energy Conversion Plant* sets are as follows:

F = energy conversion plant types.
$F^{CC} \subset F$ = energy conversion plant types that can capture carbon.
$F^{CG} \subset F$ = energy conversion plant types that can be used for co-generation of electricity and heat.

The sets for *Engine Technologies* are below:

P = all powertrain technology types.
P^H = all hybrids and plug-in hybrids.
P^B = all internal combustion engines and fuel cells.

The following are the sets for *Regions and Time periods*:

R = all regions.
T^T = all time periods.
T = all time periods excluding the historical time periods.

1.1.2. Parameters The parameters and their units are described below.

$\alpha^c_{e_i,e_o,f}$ = Life time of energy conversion plants of type $f \in F$ that convert energy in $e_i \in E^I$ to energy out $e_o \in E^O$. [yr]
$\alpha^i_{e_o}$ = Life time of infrastructure for $e_o \in E^{OS}$. [yr]
$\alpha^v_{e_o,p,m}$ = Life time of vehicle powertrains of type $p \in P$ using transportation mode $m \in M^V$ and transportation fuel $e_o \in E^{OM}$. [yr]
$b^c_{e_i,e_o,f,r}$ = Capacity in energy conversion plants of type $f \in F$ in region $r \in R$ that convert energy from $e_i \in E^I$ to $e_o \in E^O$, in the model's initial year. [TW]
$\tilde{b}^p_r$ = Small "kick-start" value when a new primary energy source is introduced in region $r \in R$. $[\frac{\text{EJ}}{\text{decade}}]$
$\tilde{b}^c_r$ = Small "kick-start" value when a new energy conversion technology is introduced in region $r \in R$. $[\frac{\text{TW}}{\text{decade}}]$
$\tilde{b}^i_r$ = Small "kick-start" value when new infrastructure is introduced in region $r \in R$. $[\frac{\text{TW}}{\text{decade}}]$
$\tilde{b}^v_r$ = Small "kick-start" value when a new powertrain technology is introduced in region $r \in R$. $[\frac{\text{Gvehicles}}{\text{decade}}]$
$\beta^c_{e_i,e_o,f,t}$ = Capacity factor (the share of maximum capacity that is used per year) for energy conversion plants of type $f \in F$ converting $e_i \in E^I$ to $e_o \in E^O$ in every time period $t \in T$. [-]
$\beta^i_{e_o}$ = Capacity factor for infrastructure for fuels $e_o \in E^{OS}$. [-]
$\gamma_{e_o,p,m,t}$ = A time $t \in T$ dependent relative powertrain efficiency factor that relates the powertrain $p \in P$ efficiency to conventional ICEV for transportation modes $m \in M$ and fuel $e_o \in E^{OM}$. [-]
$c^p_{e_i,r}$ = Extraction cost for primary energy sources $e_i \in E^{IPT}$ in regions $r \in R$. $[\frac{\text{GUSD}}{\text{EJ}}]$
$c^c_{e_i,e_o,f,t}$ = Investment cost, adjusted if assuming different investment and discount rates, for energy conversion plants of type $f \in F$ that convert $e_i \in E^I$ to $e_o \in E^O$ in time period $t \in T$. $[\frac{\text{GUSD}}{\text{MtC}}]$
$c^i_{e_o}$ = Investment cost, adjusted if assuming different investment and discount rates,

for infrastructure distributing energy $e_o \in E^{OS}$. $[\frac{\text{GUSD}}{\text{MtC}}]$
$c^v_{e_o,p,t}$ = Investment cost, adjusted if assuming different investment and discount rates, for vehicles in transportation mode $m \in M^V$ with powertrain $p \in P$ using energy $e_o \in E^{OR}$ in time period $t \in T$. $[\frac{\text{GUSD}}{\text{MtC}}]$
$c^{pt}_{e_i}$ = Cost for transportation of primary energy sources $e_i \in E^{IPT}$ when trading between regions. $[\frac{\text{GUSD}}{\text{EJ}}]$
$c^{st}_{e_o}$ = Cost for transportation of energy carriers $e_o \in E^{OT}$ when trading between regions. $[\frac{\text{GUSD}}{\text{EJ}}]$
$c^{\tilde{pt}}_{e_i}$ = Additional, distance depending, cost for transportation of primary energy sources $e_i \in E^{IPT}$ when trading between regions. $[\frac{\text{GUSD}}{\text{EJ}}]$
$c^{\tilde{st}}_{e_o}$ = Additional, distance depending, cost for transportation of energy carriers $e_o \in E^{OT}$ when trading between regions. $[\frac{\text{GUSD}}{\text{EJ}}]$
c^f = Cost for carbon storage from fossil CCS. $[\frac{\text{GUSD}}{\text{MtC}}]$
c^b = Cost for carbon storage from bioenergy CCS. $[\frac{\text{GUSD}}{\text{MtC}}]$
$c^{\tilde{b}}$ = Additional transportation cost applied to bioenergy CCS. $[\frac{\text{GUSD}}{\text{EJ}}]$
$c^t_{r,t}$ = Cost for emitting CO_2 (carbon tax) in region $r \in R$ and time period $t \in T$. $[\frac{\text{GUSD}}{\text{MtC}}]$
$d^e_{r,t}$ = Electricity demand for region $r \in R$ and time period $t \in T$. [EJ/yr]
$d^h_{r,t}$ = Heat demand for region $r \in R$ and time period $t \in T$. [EJ/yr]
$d^t_{r,m,t}$ = Energy demand for each transportation mode $m \in M$ in every region $r \in R$ and time period $t \in T$. [EJ/yr]
δ_{r_1,r_2} = Table of rough distances between regions $r_1 \in R$ and $r_2 \in R$ [km]
ε^b_t = Estimation of future natural CO_2 sinks in biota time periods $t \in T$. [MtC/yr]
ε^l_t = Estimation of future emissions from land use changes for periods $t \in T$. [MtC/yr]
ε^h_t = Historical fossil emissions for time periods $t \in T$. [MtC/yr]
ζ = Constant used in carbon cycle model. [-]
$\eta_{e_i,e_o,f,t}$ = Energy conversion efficiency when energy is converted from $e_i \in E^I$ to $e_o \in E^O$ in plants of type $f \in F$ in time period $t \in T$. [-]
η^{ccb} = Carbon capture efficiency in bioenergy CCS plants. [-]
η^{ccf} = Carbon capture efficiency in fossil CCS plants. [-]
$\eta^{cg}_{e_i,e_o,f}$ = The share of energy output that is heat when converting $e_i \in E^{ICG}$ to $e_o \in E^{OCG}$ in co-generation plants of type $f \in F^{CG}$. [-]
θ_{e_i} = Carbon dioxide emission factors for $e_i \in E^{IPT}$ $[\frac{\text{MtC}}{\text{EJ}}]$
θ^s = Carbon dioxide emission factor from the use of CTL/GTL. $[\frac{\text{MtC}}{\text{EJ}}]$
$\iota^p_{r,t}$ = Growth limit on extraction of primary energy sources for every region $r \in R$ and time period $t \in T$. $[\frac{\text{EJ}}{\text{decade}}]$
$\iota^b_{r,t}$ = Growth limit on biomass plantation for regions $r \in R$ and periods $t \in T$. $[\frac{\text{EJ}}{\text{decade}}]$
$\iota^c_{r,e_i,e_o,f,t}$ = Growth limit on energy conversion plants of type $f \in F$ that convert $e_i \in E^I$ to $e_o \in E^O$ for every region $r \in R$ and period $t \in T$. $[\frac{\text{TW}}{\text{decade}}]$
$\iota^i_{r,t}$ = Growth limit on infrastructure for regions $r \in R$ and periods $t \in T$. $[\frac{\text{TW}}{\text{decade}}]$
$\iota^{df}_{r,t}$ = Absolute number in order to reach zero when phasing out a transportation fuel for every region $r \in R$ and time period $t \in T$. $[\frac{\text{EJ}}{\text{decade}}]$
$\iota^{dc}_{r,t}$ = Absolute number in order to reach zero when phasing out a conversion technology for every $r \in R$ and time period $t \in T$. $[\frac{\text{EJ}}{\text{decade}}]$
ι^l = Max growth limit, in relative terms, on primary energy extraction, energy conversion capacity, infrastructure capacity, and vehicle stock. [-]
κ = Global growth limit on carbon storage capacity. $[\frac{\text{MtC}}{\text{decade}}]$
μ = Number of seconds per year expressed in millions. [Ms]
ξ^{im} = Max fraction of electricity produced from intermittent energy sources. [-]
ξ^{cge} = Max fraction of electricity produced in co-generation plants. [-]

ξ^{cgh} = Max fraction of heat produced in co-generation plants. [-]
ξ^{cc} = Max fraction of heat produced in CCS plants. [-]
ξ^{dc} = Min fraction of previous time step's energy conversion. [-]
ξ^{df} = Min fraction of previous time step's transportation fuels. [-]
$\xi^{om}_{e_i,e_o}$ = Operation and maintenance cost as fraction of investment cost for energy conversion plants that converts $e_i \in E^I$ to $e_o \in E^O$. [-]
$\xi^{e}_{e_i}$ = Electricity requirements if applying CCS when converting $e_i \in E^I$ to electricity (fraction on energy converted). [-]
ξ^{b}_{m} = Share of heavy road-based transportation modes $m \in M^L$ that can be of powertrain type BEV. [-]
ξ^{p}_{m} = Share of heavy road-based transportation modes $m \in M^L$ that can be of powertrain type PHEV. [-]
ξ_{m} = Fraction of time that a PHEV operates in battery mode for vehicles in transportation modes $m \in M^V$. [-]
$n^{v}_{r,m,t}$ = Number of vehicles for transportation mode $m \in M^V$ in every region $r \in R$ and time period $t \in T$. $[\frac{\text{Gvehicles}}{\text{yr}}]$
ϕ_{t_1,t_2} = Parameterized Impulse Response Function giving the contribution to the CO_2 concentration in year $t_1 \in T$ as a result of emissions from year $t_2 \in T$. $[\frac{\text{MtC}}{\text{yr}}]$
$\sigma_{e_i,e_o,f}$ = Indicates if energy conversion is allowed at a conversion plant $f \in F$ from a primary energy source $e_i \in E^I$ to an energy carrier $e_o \in E^O$. [-]
ψ = Pre-industrial atmospheric CO_2 concentration. [ppm]
r^d = Discount rate. [-]
u^{cc} = Max storage capacity for global captured carbon. [MtC]
$u^{y^p}_{e_i,r,t}$ = Annual maximum supply potential (non-fossil sources) of primary energy source $e_i \in E^{IP}$ in every region $r \in R$ and time period $t \in T$. [EJ/yr]
$\tilde{u}^{y^p}_{e_i,r}$ = Total primary energy supply potential (fossil sources) of energy source $e_i \in E^{IP}$ in every region $r \in R$. [EJ]

1.1.3. Variables First, we introduce the following continuous variables associated with ***Costs*** and expressed in units [GUSD/yr] each region $r \in R$ and time period $t \in T$.

- $x^p_{r,t} \in \mathbb{R}^+$ is the total cost for extracting primary energy sources.
- $x^c_{r,t} \in \mathbb{R}^+$ is the total cost, aggregated over all investments (e.g. energy conversion capacity, infrastructure, and vehicles).
- $x^{om}_{r,t} \in \mathbb{R}^+$ is the total cost for operation and maintenance.
- $x^{tp}_{r,t} \in \mathbb{R}^+$ is the total transportation cost for importing primary energy sources.
- $x^{ts}_{r,t} \in \mathbb{R}^+$ is the total transportation cost for importing energy carriers.
- $x^{cc}_{r,t} \in \mathbb{R}^+$ is the total cost for storing carbon.
- $x^{b}_{r,t} \in \mathbb{R}^+$ is the total additional transportation cost if applying CCS on bio-energy
- $x^{t}_{r,t} \in \mathbb{R}^+$ is the total cost if applying carbon taxes to the model.

Second, we introduce the continuous variables associated with ***Energy*** in units [EJ/yr] for every time period $t \in T$, dividing these into: main energy flow, energy conversion, import and export, and transportation sector. The variables associated with the ***Main Energy Flow*** are as follows:

- $y^p_{r,e_i,t} \in \mathbb{R}^+$ is the amount of primary energy $e_i \in E^{IP}$ extracted in each region $r \in R$.
- $y^{pf}_{r,e_i,t} \in \mathbb{R}^+$ is the amount of primary energy $e_i \in E^{IP}$ used in a region $r \in R$ after import/export has taken place.
- $y^{l}_{r,e_i,t} \in \mathbb{R}^+$ is the amount of energy $e_i \in E^I \setminus E^{IP}$ that will be converted a second time in each region $r \in R$.
- $y^{s}_{r,e_o,t} \in \mathbb{R}^+$ is the amount of energy carriers $e_o \in E^O$ produced in each region $r \in R$.
- $y^{sf}_{r,e_o,t} \in \mathbb{R}^+$ The amount of energy carriers $e_o \in E^O$ that meets the demand after import/export in each region $r \in R$.

- $y^{cg}_{r,t} \in \mathbb{R}^+$ is the amount of heat produced using co-generation technologies in each region $r \in R$.
- $y^{cce}_{r,t} \in \mathbb{R}^+$ is the additional electricity, added to the regional demand, if the model choose to use CCS in heat generation plants in each region $r \in R$.
- $y^{csp}_{t} \in \mathbb{R}^+$ is the total amount of energy from CSP.

The continuous variable associated with ***Energy Conversion*** is $w^{c}_{r,e_i,e_o,f,t} \in \mathbb{R}^+$, which is the amount of energy converted from $e_i \in E^I$ to $e_o \in E^O$ by plants of type $f \in F$ in each region $r \in R$ and time period $t \in T$ and is in units [EJ/yr] .

The continuous variables associated with ***Import and Export*** are in units of [EJ/yr] for every time period $t \in T$ and are as follows:

- $y^{pet}_{r,e_i,t} \in \mathbb{R}^+$ is the amount of primary energy $e_i \in E^{IPT}$ exported from a region $r \in R$.
- $y^{set}_{r,e_o,t} \in \mathbb{R}^+$ is the amount of energy carriers $e_o \in E^{OT}$ exported from a region $r \in R$.
- $y^{pit}_{r,e_i,t} \in \mathbb{R}^+$ is the amount of primary energy $e_i \in E^{IPT}$ imported to a region $r \in R$.
- $y^{sit}_{r,e_o,t} \in \mathbb{R}^+$ is the amount of energy carriers $e_o \in E^{OT}$ imported to a region $r \in R$.
- $y^{pt}_{r_1,r_2,e_i,t} \in \mathbb{R}^+$ is the amount of primary energy $e_i \in E^{IPT}$ imported from region $r_1 \in R$ to a region $r_2 \in R$.
- $y^{st}_{r_1,r_2,e_o,t} \in \mathbb{R}^+$ is the amount of energy carriers $e_o \in E^{OT}$ imported from region $r_1 \in R$ to a region $r_2 \in R$.

The next couple of continuous variables associated with the ***Transportation Sector*** are in units of [EJ/yr] for every region $r \in R$ and time period $t \in T$ and are as follows: $y^{sm}_{r,e_o,p,m,t} \in \mathbb{R}^+$ is the amount of transportation fuels $e_o \in E^{OM}$ required for powertrain of type $p \in P$ for the transportation mode $m \in M$ and $y^{ev}_{r,t} \in \mathbb{R}^+$ is the amount of electricity used in the transportation sector.

The following continuous variables associated with ***Investment and Capital*** are for every region $r \in R$ and time period $t \in T$:

- $z^{c}_{r,e_i,e_o,f,t} \in \mathbb{R}^+$ in units $[\frac{\text{TW}}{\text{decade}}]$ is new investments in energy conversion capacity for converting $e_i \in E^I$ to $e_o \in E^O$ in plant type $f \in F$.
- $z^{i}_{r,e_o,t} \in \mathbb{R}^+$ in units $[\frac{\text{TW}}{\text{decade}}]$ is new investments in infrastructure capacity for distributing energy carrier $e_o \in E^{OM}$.
- $z^{v}_{r,e_o,p,m,t} \in \mathbb{R}^+$ in units $[\frac{\text{Gvehicles}}{\text{decade}}]$ is new investments in vehicles used in transportation mode $m \in M^V$ with powertrain $p \in P$ running on fuel $e_o \in E^{OM}$.
- $v^{c}_{r,e_i,e_o,f,t} \in \mathbb{R}^+$ in units [TW] is the aggregated capacity in energy conversion plants for converting $e_i \in E^I$ to $e_o \in E^O$ in plant type $f \in F$.
- $v^{i}_{r,e_o,t} \in \mathbb{R}^+$ in units [TW] is the aggregated infrastructure capacity for energy carriers $e_o \in E^{OS}$.
- $v^{v}_{r,e_o,p,m,t} \in \mathbb{R}^+$ in units [Gvehicles] is the aggregated number of vehicles with powertrain $p \in P$ using fuel $e_o \in E^{OM}$ in transportation mode $m \in M^V$.

The next few continuous variables are associated with the ***Carbon Emitted*** and can take on both positive values and negative values when using the strategy of negative emissions (e.g., to reach really low CO_2 targets, the model might want to use CCS on bioenergy): $s^{e}_{r,t} \in \mathbb{R}$ are the annual emissions in each region $r \in R$ and time period $t \in T^T$ in units [MtC/yr]; $s^{g}_{t} \in \mathbb{R}$ are the global annual emissions aggregated over all regions in units [MtC/yr]; and s are the total global emissions aggregated over all time periods and regions in units [MtC].

The following continuous variables are associated with the ***Carbon Captured*** and are in units in units [MtC/yr] for every region $r \in R$ and time period $t \in T^T$: $s^{f}_{r,t} \in \mathbb{R}^+$ is the annual amount of carbon captured from fossil fuels; $s^{b}_{r,t} \in \mathbb{R}^+$ is the annual amount of carbon captured from biomass; and $s^{t}_{r,t} \in \mathbb{R}^+$ is the annual amount of carbon captured from fossil fuels and biomass.

The last two variables are associated with the ***CO_2 Concentration***: $s^{c}_{t_1,t_2} \in \mathbb{R}$ is the carbon contribution generated by the carbon cycle module in time period $t_1 \in T^T$ from time period $t_2 \in T^T$ in units of [MtC/yr]; and $s^{ac}_{t} \in \mathbb{R}^+$ is the atmospheric CO_2 concentration, from the carbon cycle module, in every time period $t \in T^T$ in units of [ppm].

1.1.4. Objective Function The objective of this model is to minimize total global energy system cost: the expression in the denominator is the discount factor.

$$\text{Minimize} \sum_{r \in R} \sum_{t \in T} \frac{\tilde{t}\left(x^p_{rt} + x^c_{rt} + x^{om}_{rt} + x^{tp}_{rt} + x^{ts}_{rt} + x^{cc}_{rt} + x^b_{rt} + x^t_{rt}\right)}{(1 + r^d)^{t - t_0}} \;\; \forall \;\; t_0 \in \{1990\}$$

1.1.5. Constraints We have divided the constraints into logical groups around the following: cost, primary energy flow, secondary energy flow, transport balances and limitations, investments and depreciations, and emissions.

Costs

The first set of constraints define the cost decision variables for simplification of notation in the objective function:

- Total cost for extracting primary energy sources for each region and time period:

$$x^p_{rt} = \sum_{e_i \in E^{IPT}} c^p_{e_i r} y^p_{r,e_i,t} \;\; \forall \;\; r \in R, t \in T \tag{1}$$

- Total cost, aggregated over all investments done (energy conversion capacity, infrastructure, and vehicles) for each region and time period:

$$x^c_{rt} = \sum_{e_i \in E^I} \sum_{e_o \in E^O} \sum_{f \in F} c^c_{e_i e_o f t} z^c_{r e_i e_o f t} + c^i_{e_o} z^i_{r e_o t} + c^v_{e_o p t} z^v_{r e_o p m t} \;\; \forall \;\; r \in R, t \in T \tag{2}$$

- Total cost for operation and maintenance for each region and time period:

$$x^{om}_{rt} = \sum_{e_i \in E^I} \sum_{e_o \in E^O} \sum_{f \in F} [\xi^{om}_{e_i e_o} c^c_{e_i e_o f t} \eta_{e_i e_o f t} / \mu] w^c_{r e_i e_o f t} \;\; \forall \;\; r \in R, t \in T \tag{3}$$

- Total transportation cost for importing primary energy sources in each region and time period:

$$x^{pt}_{r_1 t} = \sum_{e_i \in E^{IPT}} \sum_{r_2 \in R \setminus \{r_1\}} [c^{pt}_{e_i} + \delta_{r_1 r_2} c^{\tilde{pt}}_{e_i}] y^{pt}_{r_1 r_2 e_i t} \;\; \forall \;\; r_1 \in R, t \in T \tag{4}$$

- Total transportation cost for importing energy carriers in each region and time period:

$$x^{st}_{r_1 t} = \sum_{e_o \in E^{OT}} \sum_{r_2 \in R \setminus \{r_1\}} [c^{st}_{e_o} + \delta_{r_1 r_2} c^{\tilde{st}}_{e_o}] y^{pt}_{r_1 r_2 e_o t} \;\; \forall \;\; r_1 \in R, t \in T \tag{5}$$

- Total cost for storing carbon (including transportation of CO_2) in every region and time period:

$$x^{cc}_{rt} = c^f s^f_{rt} + c^b s^b_{rt} \;\; \forall \;\; r \in R, t \in T$$

- Total additional transportation cost for carbon capture and storage from bioenergy conversion plants in every region and time period:

$$x^b_{rt} = \sum_{e_o \in E^O} \sum_{f \in F^C C} c^{\tilde{b}} w^c_{r e_i e_o f t} \;\; \forall \;\; r \in R, t \in T \tag{6}$$

- Total cost for each region and time period if applying carbon taxes to the model:

$$x^t_{rt} = c^t_{rt} s^e_{rt} \;\; \forall \;\; r \in R, t \in T \tag{7}$$

Primary Energy Flow

We first describe the constraints around ***Extraction and Trade***.

- Upper bound on the amount of non-fossil primary energy sources extracted in each region in every time period:

$$y^p_{re_it} \leq u^{y^p}_{re_it} \;\; \forall \;\; r \in R, e_i \in E^{IP}, t \in T \tag{8}$$

- Upper bound on the amount of fossil primary energy sources extracted in each region over all time periods:

$$\sum_{t\in T} \tilde{t} y^p_{re_it} \leq \tilde{u}^{y^p}_{re_i} \;\; \forall \;\; r \in R, e_i \in E^{IP} \tag{9}$$

- The primary energy used in a region must be equal to the primary energy extracted in the region plus the primary energy that has been imported to the region minus the primary energy that has been exported from the region:

$$y^{pf}_{re_it} = y^p_{re_it} + y^{pit}_{re_it} - y^{pet}_{re_it} \;\; \forall \;\; r \in R, e_i \in E^{IPT}, t \in T \tag{10}$$

- The use of primary energy sources, that can not be traded, must be equal to the primary energy extracted in the region in every time period:

$$y^{pf}_{re_it} = y^p_{re_it} \;\; \forall \;\; r \in R, e_i \in E^{IP} \setminus E^{IPT}, t \in T \tag{11}$$

- The import of primary energy sources to one region must equal the sum of all exports from other regions to this specific region:

$$y^{pit}_{r_ie_it} = \sum_{r_e \in R\setminus\{r_i\}} y^{pt}_{r_ir_ee_it} \;\; \forall \;\; r_i \in R, e_i \in E^{IPT}, t \in T \tag{12}$$

- The export of primary energy sources from one region must equal the sum of all imports to other regions from this specific region:

$$y^{pet}_{r_ee_it} = \sum_{r_i \in R\setminus\{r_e\}} y^{pt}_{r_ir_ee_it} \;\; \forall \;\; r_e \in R, e_i \in E^{IPT}, t \in T \tag{13}$$

The couple of constraints around ***Energy Conversion Balancing Primary Energy Sources*** are itemized below:

- The primary energy used in a region must be equal to the amount of primary energy converted to energy carriers in energy conversion plants, in each region and every time period:

$$y^{pf}_{re_it} = \sum_{f\in F} \sum_{e_o \in E^O} w^c_{re_ie_oft} \;\; \forall \;\; r \in R, e_i \in E^{IP}, t \in T \tag{14}$$

- Here we define the global use of CSP. In the scenarios where we assume that CSP will not be a large scale available technology, we set the variable y^{csp}_t to zero.

$$y^{csp}_t = \sum_{r\in R} y^p_{re_it} \;\; \forall \;\; e_i \in \{\text{solar\_CSP}\}, t \in T \tag{15}$$

Secondary Energy Flow
We start by introducing the constraints involving ***Energy conversion to energy carriers and trade***.

- For energy carriers that can not be traded between regions, the amount of energy carriers used in a region must be equal to the amount of primary energy sources converted in the region's energy conversion plants times the conversion efficiency in each region and every time period:

$$y^s_{re_ot} = \sum_{f\in F} \sum_{e_i \in E^I} \eta_{e_ie_oft} w^c_{re_ie_oft} \;\; \forall \;\; r \in R, e_o \in E^O \setminus E^{OT}, t \in T \tag{16}$$

• For energy carriers that can be traded between regions, the amount of energy carriers used in a region equals the amount of primary energy sources converted in the region's energy conversion plants times the conversion efficiency, plus the energy carriers that have been imported to the region minus the energy carriers that have been exported from the region, for each region and every time period:

$$y^{s}_{re_ot} = \sum_{f\in F}\sum_{e_i\in E^I} \eta_{e_ie_oft} w^{c}_{re_ie_oft} + y^{sit}_{re_ot} - y^{set}_{re_ot} \;\; \forall \;\; r\in R, e_o \in E^{OT}, t\in T \tag{17}$$

• The import of energy carriers to one region must equal the sum of all exports from other regions to the specified region:

$$y^{sit}_{r_ie_ot} = \sum_{r_e\in R\setminus\{r_i\}} y^{st}_{r_ir_ee_ot} \;\; \forall \;\; r_i\in R, e_o \in E^{OT}, t\in T \tag{18}$$

• The export of energy carriers from one region must equal the sum of all imports to other regions from the specified region:

$$y^{set}_{r_ee_ot} = \sum_{r_i\in R\setminus\{r_e\}} y^{st}_{r_ir_ee_ot} \;\; \forall \;\; r_e\in R, e_o \in E^{OT}, t\in T \tag{19}$$

• The amount of heat produced in co-generation with electricity equals the exogenously given share of heat in such conversion plants times the total amount of energy produced in these plants, for each region and every time period:

$$y^{cg}_{rt} = \sum_{e_i\in E^{ICG}}\sum_{e_o\in E^{OCG}}\sum_{f\in F^{CG}} \eta^{cg}_{e_ie_of} w^{c}_{re_ie_oft} \;\; \forall \;\; r\in R, t\in T \tag{20}$$

• These constraints specify the balance (i.e. transfer) of energy carriers produced, imported, and exported to the next model node where energy carriers meet the demand. The amount of energy carriers meeting each region's demand must be less than or equal to the amount of energy carriers produced and traded for each energy carrier except heat, and for each region and every time period:

$$y^{sf}_{re_ot} \leq y^{s}_{re_ot} \;\; \forall \;\; r\in R, e_o \in E^{O}\setminus\{\text{heat}\}, t\in T \tag{21}$$

• The energy carrier heat is balanced (i.e. transferred) to the next model node where energy carriers meet the demand. The amount of heat meeting the demand in a region must be less than or equal to the amount of heat produced in conventional heat generation plants plus the heat produced in co-generation plants, for each region and every time period:

$$y^{sf}_{re_ot} \leq y^{s}_{re_ot} + y^{cg}_{rt} \;\; \forall \;\; r\in R, e_o \in \{\text{heat}\}, t\in T \tag{22}$$

The next group of Secondary Energy Flow Constraints involve the ***Conversion of Energy Carriers to Other Energy Carriers***:

• Some energy carriers (e.g., electricity and hydrogen) can be converted to other energy carriers (e.g., heat). There must be an equal or larger amount of an energy carrier produced in the first energy conversion process than what can be used in this second conversion process, for each of the energy carriers allowed to be converted a second time and for every region and period:

$$y^{l}_{re_it} \geq \sum_{f\in F}\sum_{e_o\in E^O} w^{c}_{re_ie_oft} \;\; \forall \;\; r\in R, e_i \in E^{I}\setminus E^{IP}, t\in T \tag{23}$$

• The amount of energy carriers converted again must be less than or equal to the difference between the energy carriers produced, including import and export, and the energy carriers meeting the demand for each of the energy carriers allowed to be converted a second time, each region and every time period:

$$y^{l}_{re_it} \leq y^{s}_{re_it} - y^{sf}_{re_it} \;\; \forall \;\; r \in R, e_i \in E^{I} \setminus E^{IP}, t \in T \tag{24}$$

The following group of Secondary Energy Flow Constraints involve ***Balancing Heat and Electricity demand***:

• The total amount of heat generated in a region must be greater than or equal to the exogenously given heat demand in each region and every time period:

$$y^{sf}_{re_ot} \geq d^{h}_{rt} \;\; \forall \;\; r \in R, e_o \in \{\text{heat}\}, t \in T \tag{25}$$

• The net amount of electricity generated in a region must be greater than or equal to the exogenously given electricity demand plus the additional electricity needed if the model chooses to apply CCS on heat generation plants, as well as if the model chooses to use electricity in the transportation sector (e.g., for PHEVs and BEVs) in each region and time period:

$$y^{sf}_{re_ot} \geq d^{e}_{rt} + y^{cce}_{rt} + y^{ev}_{rt} \;\; \forall \;\; r \in R, e_o \in \{\text{elec}\}, t \in T \tag{26}$$

We group the Secondary Energy Flow Constraints on ***Balancing Demand on Fuels for Transport*** as follows:

• For non-electric vehicles, the amount of transportation fuels produced (except electricity) equals the fuel required for all transportation modes and all powertrain types for each region and time step:

$$y^{sf}_{re_ot} = \sum_{m \in M} \sum_{p \in P} y^{sm}_{re_opmt} \;\; \forall \;\; r \in R, e_o \in E^{OM} \setminus \{\text{elec}\}, t \in T \tag{27}$$

• The total amount of electricity used in a region's transportation sector equals the electricity required for all transportation modes (e.g. passenger rail, freight rail, high speed rail) and all powertrain types (e.g. PHEVs and BEVs) for each region and time step:

$$y^{ev}_{rt} = \sum_{m \in M} \sum_{p \in P} y^{sm}_{re_opmt} \;\; \forall \;\; r \in R, e_o \in \{\text{elec}\}, t \in T \tag{28}$$

• The exogenously given energy demand for the different transportation modes must be equal to the amount of available fuels (for all fuels and powertrain options) times a relative powertrain efficiency factor in each region and time period:

$$d^{t}_{rmt} = \sum_{e_o \in E^{OM}} \sum_{p \in P} \gamma_{e_opmt} y^{sm}_{re_opmt} \;\; \forall \;\; r \in R, m \in M, t \in T \tag{29}$$

The next group of Secondary Energy Flow Constraints involve ***Limitations on Co-generation and Intermittent Energy***:

• The amount of electricity coming from co-generation plants is limited to a certain share of the electricity demand for all regions and time periods.

$$\sum_{e_i \in E^{ICG}} \sum_{f \in F^{CG}} \eta_{e_ie_oft} w^{c}_{re_ie_oft} \leq \xi^{cge} d^{e}_{rt} \;\; \forall \;\; r \in R, e_o \in \{\text{elec}\}, t \in T \tag{30}$$

• The amount of heat coming from co-generation plants is limited to a certain share of the heat demand for all regions and time periods.

$$\sum_{e_i \in E^{ICG}} \sum_{f \in F^{CG}} \sum_{e_o \in E^{OCG}} \eta^{cg}_{e_ie_of} w^{c}_{re_ie_oft} \leq \xi^{cgh} d^{h}_{rt} \;\; \forall \;\; r \in R, t \in T \tag{31}$$

• Electricity from intermittent energy sources (e.g., wind and solar) without electricity storage, such as hydrogen production, is maximized to a certain share of total electricity production.

$$w^c_{re_ie_oft} + w^c_{r\tilde{e_i}e_oft} \leq \xi^{im} y^{sf}_{re_ot} \;\; \forall \;\; r \in R, e_i \in \{\text{wind}\}, \tilde{e_i} \in \{\text{solar}\}, e_o \in \{\text{elec}\}, f \in \{0\}, t \in T$$

We group the Secondary Energy Flow Constraints on ***Limitations on Technology Growth and Depreciation*** as follows:

• This limits the relative growth on energy conversion capacity for each combination of primary energy source and energy carrier, type of conversion plant, region, and time period.

$$v^c_{re_ie_oft_2} \leq e^{\tilde{t}\ln(1+\iota^l)} v^c_{re_ie_oft_1} + \tilde{b}^c_r \;\; \forall \;\; r \in R, e_i \in E^I, e_o \in E^O, f \in F, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{32}$$

• This limits the growth on energy conversion capacity, in absolute number, for each energy carrier, region, and time period.

$$v^c_{re_ie_oft_2} \leq v^c_{re_ie_oft_1} + \frac{\iota^c_{re_ie_oft_1}}{\beta^c_{e_ie_oft_1}} \;\; \forall \;\; r \in R, e_i \in E^I, e_o \in E^O, f \in F, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{33}$$

• This limits the relative growth on infrastructure capacity for each energy carrier, region, and time period.

$$v^i_{re_ot_2} \leq e^{\tilde{t}\ln(1+\iota^l)} v^i_{re_ot_1} + \tilde{b}^i_r \;\; \forall \;\; r \in R, e_o \in E^O, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{34}$$

• This limits the growth on infrastructure capacity, in absolute number, for each energy carrier, region, and time period.

$$v^i_{re_ot_2} \leq v^i_{re_ot_1} + \frac{\iota^i_{rt_1}}{\beta^i_{e_o}} \;\; \forall \;\; r \in R, e_o \in E^O, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{35}$$

• This limits the relative growth on vehicles for each combination of powertrain, fuels, transportation mode, region, and time period.

$$v^v_{re_opmt_2} \leq e^{\tilde{t}\ln(1+\iota^l)} v^v_{re_opmt_1} + \tilde{b}^v_r \;\; \forall \;\; r \in R, e_o \in E^O, p \in P, m \in M, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{36}$$

• This limits the relative growth on extraction of primary energy sources for each source, region, and time period.

$$y^p_{re_it_2} \leq e^{\tilde{t}\ln(1+\iota^l)} y^p_{re_it_1} + \tilde{b}^p_r \;\; \forall \;\; r \in R, e_i \in E^I, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{37}$$

• This limits the growth on extraction of fossil primary energy sources for each source, region, and time period.

$$y^p_{re_it_2} \leq y^p_{re_it_1} + \iota^p_{rt_1} \;\; \forall \;\; r \in R, e_i \in E^{IPF}, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{38}$$

• This constraint limits the growth on extraction of biomass for each region and time period.

$$y^p_{re_it_2} \leq y^p_{re_it_1} + \iota^b_{rt_1} \;\; \forall \;\; r \in R, e_i \in \{\text{bio}\}, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{39}$$

• This limits the decay to capture energy system inertia when phasing out an energy conversion plant for each primary energy source converted to any energy carrier, in all types of conversion plants, for each region and time period.

$$w^c_{re_ie_oft_2} \leq \xi^{dc} w^c_{re_ie_oft_1} - \iota^{dc}_{rt_1} \;\; \forall \;\; r \in R, e_i \in E^I, e_o \in E^O, f \in F, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{40}$$

• This constraint limits the decay when phasing out a transportation fuel technology for each transportation mode, powertrain technology, region, and time period.

$$y^{sm}_{re_opmt_2} \leq \xi^{df} y^{sm}_{re_opmt_2} - \iota^{df}_{rt_1} \ \ \forall \ r \in R, e_o \in E^{OM}, p \in P, m \in M, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \tag{41}$$

Transport Balances and Limitations

The first grouping is to ***Balance the Number of Cars and Trucks to the Different Fuels.***

• The amount of transportation fuels must be less than or equal to the stock of vehicles for region, time period, and road based fuel and powertrain combination, except for PHEVs, times a conversion and scaling factor. This factor consists of the exogenously given energy demand for transport divided with the exogenously given total amount of vehicles and the relative powertrain efficiency factor:

$$y^{sm}_{re_opmt} \leq \frac{d^t_{rmt}}{\gamma_{e_opmt} n^v_{rmt}} v^v_{re_opmt} \ \ \forall \ r \in R, e_o \in E^{OR}, p \in P \setminus \{\text{phev}\}, m \in M^V, t \in T \tag{42}$$

• The amount of liquid fuels for PHEVs must be less than or equal to the stock of PHEVs times a conversion and scaling factor. This factor consists of the exogenously given energy demand for transport multiplied with the non-electric share of energy needed for PHEVs, divided with by the exogenously given total amount of vehicles and the relative powertrain efficiency factor, for each road based fuel, region, and time step:

$$y^{sm}_{re_opmt} \leq \frac{(1-\xi_m) d^t_{rmt}}{\gamma_{e_opmt} n^v_{rmt}} v^v_{re_opmt} \ \ \forall \ r \in R, e_o \in E^{ORL}, p \in \{\text{phev}\}, m \in M^V, t \in T \tag{43}$$

The next constraint associated with Transport Balances and Limitations is around ***Non-electric Fuels for PHEVs***:

• This constraint balances the right amount of liquid fuels to complement the electricity in PHEVs:

$$\sum_{e_o \in E^{OM}} \frac{\gamma_{e_opmt}}{1-\xi_m} y^{sm}_{re_opmt} = \frac{\gamma_{\tilde{e}^opmt}}{\xi_m} y^{sm}_{r\tilde{e}^opmt} \ \ \forall \ r \in R, \tilde{e}_o \in \{\text{elec}\}, p \in \{\text{phev}\}, m \in M^V, t \in T \tag{44}$$

The next couple of constraints associated with Transport Balances and Limitations are on ***Limitations on Heavy Electric Vehicles***.

• The powertrain technologies HEVs and PHEVs are assumed to play a limited role in future transportation modes. Trucks and Buses are therefore limited to a certain share of energy demand for these modes.

$$\sum_{e_o \in E^{ORL}} \frac{\gamma_{e_opmt}}{1-\xi_m} y^{sm}_{re_opmt} \leq \xi^p_m d^t_{rmt} \ \ \forall \ r \in R, m \in M^L, p \in P^H, t \in T \tag{45}$$

• The BEV powertrain technology is assumed to play a limited role in future transportation modes. Trucks and Buses are therefore limited to a certain share of energy demand for these modes:

$$\gamma_{e_opmt} y^{sm}_{re_opmt} \leq \xi^b_m d^t_{rmt} \ \ \forall \ r \in R, e_o \in \{\text{elec}\}, p \in \{\text{BEV}\}, m \in M^L, t \in T \tag{46}$$

Investment and Depreciation

The first grouping of constraints is around ***Energy Conversion Plants, Infrastructure, and Vehicles***:

• The initial capacity (for 1990) in energy conversion plants is equal to the exogenously given capacity plus new investments made in 1990.

$$v^c_{re_ie_oft} = b^c_{re_ie_of} + \tilde{t} z^c_{re_ie_oft} \ \ \forall \ r \in R, e_i \in E^I, e_o \in E^O, f \in F, t \in \{1990\} \tag{47}$$

• Aggregated capacity in energy conversion plants equals new investments plus the aggregated capacity from the previous time period multiplied by a depreciation factor based on a plant's assumed life time.

$$v^c_{re_ie_oft_2} = \tilde{t}z^c_{re_ie_oft_2} + e^{\tilde{t}\ln(1-1/\alpha^c_{e_ie_of})}v^c_{re_ie_oft_1} \;\; \forall \; r \in R, e_i \in E^I, e_o \in E^O, f \in F, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \quad (48)$$

• Aggregated capacity in fuel infrastructure equals new investments plus the aggregated capacity from the previous time period multiplied by a depreciation factor.

$$v^i_{re_ot_2} = \tilde{t}z^i_{re_ot_2} + e^{\tilde{t}\ln(1-1/\alpha^i_{e_o})}v^i_{re_ot_1} \;\; \forall \; r \in R, e_o \in E^{OS}, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \quad (49)$$

• The aggregated number of vehicles, with a certain powertrain technology, equals new investments plus the aggregated stock of vehicles from the previous time period multiplied by a depreciation factor.

$$v^v_{re_opmt_2} = \tilde{t}z^v_{re_opmt_2} + e^{\tilde{t}\ln(1-1/\alpha^v_{e_opm})}v^v_{re_opmt_1} \;\; \forall \; r \in R, e_o \in E^{OM}, m \in M^V, t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \quad (50)$$

The second grouping of constraints on Investment and Depreciation is around ***Balancing Investments to Energy Flow***:

• The energy flow in energy conversion plants times its conversion efficiency must be less than or equal to the aggregated capacity in energy conversion plants (converted from TW into EJ/yr).

$$\eta_{e_ie_oft}w^c_{re_ie_oft} \leq \beta^c_{e_ie_oft}\mu v^c_{re_ie_oft} \;\; \forall \; r \in R, e_i \in E^I, e_o \in E^O, f \in F, t \in T \quad (51)$$

• The amount of fuels used must be less than or equal to the aggregated capacity in fuel infrastructure (converted from TW into EJ/yr).

$$y^{sf}_{re_ot} \leq \beta^i_{e_o}\mu v^i_{re_ot} \;\; \forall \; r \in R, e_o \in E^{OS}, t \in T \quad (52)$$

The next grouping of constraints on Investment and Depreciation is around ***CCS and Limitations***:

• This limits the total amount of captured carbon from fossil fuels in each region and time period. When using "less than or equal" the regions have the possibility to invest in CCS-technology but wait a decade or so before using it.

$$s^f_{rt} \leq \sum_{e_i \in E^{IPF}} \sum_{e_o \in E^O} \sum_{f \in F^{CC}} \theta_{e_i}\eta^{ccf}w^c_{re_ie_oft} \;\; \forall \; r \in R, t \in T \quad (53)$$

• This limits the total amount of captured carbon from bio-energy in each region and time period.

$$s^b_{rt} \leq \sum_{e_o \in E^O} \sum_{f \in F^{CC}} \theta_{e_i}\eta^{ccb}w^c_{re_ie_oft} \;\; \forall \; r \in R, e_i \in \{\text{bio}\}, t \in T \quad (54)$$

• This sets the total amount of carbon captured in each region and every time period. In the scenarios where we assume that CCS will not be a large scale available technology, we set this variable to zero.

$$s^t_{rt} = s^f_{rt} + s^b_{rt} \;\; \forall \; r \in R, t \in T \quad (55)$$

• This sets the global limitation on carbon storage expansion for every time period.

$$\sum_{r \in R} s^t_{rt_2} \leq \sum_{r \in R} s^t_{rt_1} + \tilde{t}\kappa \;\; \forall \; t_1 \in T, t_2 \in T : t_2 = t_1 + \tilde{t} \quad (56)$$

• This constraint sets the global limitation on carbon storage.

$$\sum_{r\in R}\sum_{t\in T}\tilde{t}s^{t}_{rt} \leq u^{cc} \tag{57}$$

• This specifies the maximum heat demand that can be fulfilled from CCS facilities for every region and time period.

$$\sum_{e_i\in E^{IPT}}\sum_{f\in F^{CC}} w^{c}_{re_ie_oft} \leq \xi^{cc} d^{h}_{rt} \ \ \forall\ \ r\in R, e_o \in \{\text{heat}\}, t\in T \tag{58}$$

• This specifies the additional electricity needed if the model choose to use CCS in heat generation plants for each region and time period. Note that we assume that negligible additional heat is needed when applying CCS.

$$y^{cce}_{rt} = \sum_{e_i\in E^{IPT}} \xi^{e}_{e_i} w^{c}_{re_ie_oft} \ \ \forall\ \ r\in R, e_o\in\{\text{heat}\}, f\in\{\text{CCS}\}, t\in T \tag{59}$$

The next constraint on Investment and Depreciation is around ***Allowed Energy Conversion Paths***:

• This constraint whether or not energy conversion from a primary energy source to an energy carrier is allowed for each primary energy source, energy carrier, and type of energy conversion plant. When $\sigma_{e_ie_of} = 1$, then the constraint guarantees that it is not allowed, as the constraint is then active.

$$\sum_{r\in R}\sum_{t\in T}\sigma_{e_ie_of} w^{c}_{re_ie_oft} = 0 \ \ \forall\ \ e_i\in E^{I}, e_o\in E^{O}, f\in F \tag{60}$$

Emission Calculations

• This constrains the carbon dioxide emissions in each region and time period. The emissions are based on the amount of primary energy sources used in a region after import and export. For tradable secondary energy carriers that contain carbon in the fuel (i.e., CTL and GTL), the emissions are separated between the producing and using region. Captured emissions from applying CCS are subtracted from the total emissions.

$$s^{e}_{rt} = \sum_{e_i\in E^{IPF}} \theta_{e_i} y^{pf}_{re_it} - s^{t}_{rt} - \theta^{s} y^{set}_{re_ot} + \theta^{s} y^{sit}_{re_ot} \ \ \forall\ \ r\in R, e_o\in\{\text{CTL/GTL}\}, t\in T \tag{61}$$

• This sets the global carbon dioxide emissions for each time period. When analyzing different CO_2 reduction targets, we vary the upper bounds on variable s^{g}_{t}.

$$s^{g}_{t} = \sum_{r\in R} s^{e}_{rt} \ \ \forall\ \ t\in T \tag{62}$$

• This sets historical global emissions for each time period.

$$\varepsilon^{h}_{t} = \sum_{r\in R} s^{e}_{rt} \ \ \forall\ \ t\in T^{T}\setminus T \tag{63}$$

• This sets the aggregated global carbon dioxide emissions.

$$s = \sum_{r\in R}\sum_{t\in T}\tilde{t}s^{e}_{rt} \tag{64}$$

The next couple of constraints involve the ***Carbon Cycle Model***:

- The carbon contribution to the atmospheric CO_2 concentration from one time period to another time period depends on the emissions from fossil fuels, as well as estimated future emissions from land use changes and estimated future biota sinks, times a factor given by the impulse response function.

$$s^{c}_{t_1 t_2} = \phi_{t_1 t_2} \left(\sum_{r \in R} s^{e}_{r t_2} + \varepsilon^{l}_{t_2} - \varepsilon^{b}_{t_2} \right) \quad \forall \; t_1, t_2 \in T^T \tag{65}$$

- This sets the atmospheric CO_2 concentration.

$$s^{ac}_{t_1} = \psi + \zeta \sum_{t_2 \in T^T} s^{c}_{t_1 t_2} \quad \forall \; t_1 \in T^T \tag{66}$$

Note that the mathematical formulation presented in this paper can be modified with different constraints based on the desired scenarios. For example, we could include upper bounds on the use of nuclear energy, assuming no expansion beyond current global capacity or limit the amount of hydrogen used in aviation. The model described in this paper is not meant to be all inclusive but is meant to lay the foundation upon which any modeler can expand based on their specific questions.