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Towards Equitable Support in First-Year Engineering Mathematics

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Abstract

Advancing Sustainable Development Goals 4 and 10 in higher education requires understanding students' diverse academic backgrounds and how they relate to performance in early mathematics courses that are critical for academic progression. This study addresses this need by linking cohort-level admission profiles with individual course-specific predicted probabilities of failure. Anonymized admission data from 193 first-year engineering students at a regional university in southern Chile were analyzed using 7 academic and school-background characteristics. Gower dissimilarity and partitioning around medoids identified 3 partially overlapping profiles (average silhouette width = 0.435) that summarized cohort heterogeneity and were associated with subsequent Algebra and Calculus trajectories (Cramér's $V = 0.285$). Separate ridge-penalized logistic models, evaluated through 100 stratified training–validation splits, achieved mean areas under the receiver operating characteristic curve (AUC) of 0.832 and 0.735, with Brier scores of 0.185 and 0.221 for Algebra and Calculus, respectively, indicating that the same admission information was more predictive of failure in Algebra. Internal validation showed that the predicted probabilities of failure spanned too narrow a range in both courses. Their use to determine which students are offered support requires validation in new cohorts. Individual probabilities nevertheless varied substantially within profiles and overlapped across them. Admission profiles can therefore inform cohort-level support planning, while course-specific probabilities provide preliminary signals for individual review without assigning fixed labels. These complementary signals may inform early, differentiated, and non-stigmatizing academic support, whose effects on equitable student progression and retention remain to be evaluated.

Keywords: sustainable higher education; students at risk; educational analytics; admission profiles; Gower distance; partitioning around medoids; ridge logistic regression; early prediction; engineering education

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Academic Editors: Patrícia Albergaria-Almeida and Marta Abelha

Received: 25 October 2026
Revised: 4 November 2026
Accepted: 5 November 2026
Published: 7 November 2026

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1. Introduction

Higher education can contribute to social mobility, human-capital formation, and the sustainable development of territories. These benefits depend not only on access but also on students' progression and completion [1,2]. In Latin America and the Caribbean, the expansion of university enrollment has broadened access opportunities without eliminating

inequalities associated with social background, prior academic preparation, and persistence through graduation [1]. Within this context, Sustainable Development Goal 4 calls for inclusive, equitable, and quality education, while Sustainable Development Goal 10 seeks to reduce inequalities [2]. For higher education institutions (HEIs), these commitments extend beyond simply providing access; they also involve recognizing the different conditions under which students begin their studies and supporting their academic progression [3]. Equality emphasizes access to the same opportunities, whereas equity focuses on fairness and recognizes that students who begin under heterogeneous conditions may require different forms or intensities of support to achieve common learning objectives [3]. Quantitative evidence may inform institutional decisions aligned with these goals, but it must be interpreted in relation to structural conditions and does not by itself demonstrate that educational outcomes have become more equitable or sustainable [4,5].

The first year is a critical stage in educational trajectories because early academic discontinuation can affect subsequent persistence and degree completion [6,7]. Institutional dropout must, however, be distinguished from departure from the tertiary education system. Students who do not continue at their original institution may transfer to another institution or enter a new cohort and therefore remain in tertiary education. Chile's Servicio de Información de Educación Superior (Higher Education Information Service, SIES) addresses this distinction through the "non-enrollment rate," which measures departure from the tertiary system while excluding students who continue their studies elsewhere [8]. According to the SIES report for the 2024 cohort, this non-enrollment rate was 7.6% at Chilean universities, compared with 8.5% in 2020 and 8.4% in the preceding year [8]. The decline remained below one percentage point over four years, indicating that departure from tertiary education continues to represent a persistent institutional challenge. These patterns are shaped by multiple factors, including academic preparation, socioeducational conditions, expectations, and the personal, organizational, academic, and social demands of the initial university experience [6,7,9]. The beginning of the university trajectory is consequently an important stage for identifying emerging difficulties and offering timely support [10].

The transition from school mathematics to university mathematics involves cognitive, didactic, and sociocultural changes that extend beyond mastery of previously acquired knowledge [11]. These changes are particularly relevant in introductory science, technology, engineering, and mathematics (STEM) courses in the basic sciences, which require high levels of abstraction and the development of competencies associated with logical–mathematical reasoning [12]. Early mathematics courses such as Elementary Algebra, Precalculus, and Differential Calculus may therefore be especially demanding. Students must engage with new forms of reasoning and heightened academic expectations while drawing on diverse prior educational experiences [13].

Evidence from STEM education shows that academic preparation before university admission is closely associated with performance in introductory courses [14]. In engineering programs, initial mathematical preparation has also been associated with overall academic performance and the timely completion of studies [15]. Difficulties in early mathematics courses may consequently affect both immediate performance and progression through the degree. This issue is particularly relevant for groups historically underrepresented in higher education, for whom introductory courses may become critical junctures affecting persistence in STEM programs [16]. These associations should be interpreted as reflecting interactions among prior educational opportunities, current academic demands, and institutional support rather than as fixed deficits of individual students [4,5].

Within early engineering mathematics, Elementary Algebra and Differential Calculus represent distinct knowledge domains in international curricular frameworks [17]. Ele-

mentary Algebra emphasizes symbolic relationships, generalization, and equation solving, whereas Differential Calculus focuses on variation, rates of change, and local behavior. Therefore, combining both courses into a single measure could conceal differences in the admission characteristics associated with passing or failing each course.

The present study aims to examine how students' admission profiles relate to their academic performance in first-year Calculus and Algebra, with a view to informing the early identification of students who may benefit from additional academic support. Outcomes are analyzed separately for each course, while both are considered components of students' early mathematics trajectory. This course-specific perspective is relevant because early academic experiences can influence persistence in STEM programs [18], and first-year support can contribute to retention and graduation in engineering [19,20].

1.1. Admission Heterogeneity and Profiles

Students do not enter university under homogeneous conditions. Admission route, the type and academic track of the secondary school attended, school achievement, and admission scores express distinct dimensions of prior educational experience. In Chile, research has documented relationships between the secondary-school curricular track and access to and persistence in STEM pathways [21], as well as between university-selection instruments and first-year academic performance [22]. These variables neither operate in isolation nor determine university outcomes on their own; rather, their combination configures differentiated admission profiles. In this study, an *admission profile* is defined as a multidimensional configuration of student characteristics measured before university, constructed through unsupervised methods without using subsequent academic outcomes. Identifying such configurations is more informative than comparing overall means. Recent studies illustrate the value of this approach. Calderón Ortiz et al. [23] used clustering to identify multidimensional university-access profiles in a regional Mexican context and linked their findings to educational equity and Sustainable Development Goal 4. In Chile, Muñoz-Muñoz et al. [24] identified socioeducational profiles associated with first-year dropout in engineering and framed the phenomenon as a problem of institutional sustainability. Denaro et al. [25] used multi-institutional cluster analysis to identify configurations of opportunity gaps in introductory STEM courses. Collectively, these studies show that clustering can reveal heterogeneity relevant to understanding access, performance, and persistence.

These profiles must be interpreted cautiously. Differences between groups should not automatically be attributed to individual deficits because they may also reflect inequalities in prior educational opportunities and in the institutional conditions students encounter. Critical equity approaches in STEM recommend contextualizing quantitative comparisons and examining the mechanisms that generate differentiated outcomes [4]. Similarly, researchers have proposed replacing interpretations centered exclusively on "achievement gaps" with analyses that consider the educational opportunities available to different groups [5].

1.2. Identifying Admission Profiles for Early Support

Educational data mining provides tools for identifying patterns in academic records and converting them into evidence that can support decision-making [26,27]. When admission data combine quantitative and categorical variables, Gower distance can represent them within a common dissimilarity matrix [28]. On this matrix, partitioning around medoids (PAM) can construct groups represented by actual observations [29]. Constructing profiles exclusively from admission information allows subsequent course outcomes to be examined as external evidence of how initial configurations relate to academic trajectories, without using those outcomes to define the groups themselves.

Such profiles should be interpreted as contextual summaries of recurring configurations, not as rigid student types, rankings of ability, or predetermined academic trajectories [4,5]. The same caution applies to estimated probabilities of failure, which should not be understood as intrinsic or permanent student attributes. Academic performance reflects interactions among prior educational opportunities, current learning conditions, curricular demands, teaching and assessment practices, and institutional support [6,7,9]. Consequently, neither a profile nor an estimated probability determines what a student can achieve.

Profile analysis can be complemented by supervised models that assess whether initial information discriminates between students who pass and those who fail. Reviews of academic prediction recognize the value of pre-admission information and emphasize evaluating models with data not used for estimation [30,31]. Ridge-penalized logistic regression is appropriate when an interpretable model is required and coefficient instability among related predictors must be reduced [32]. These approaches address connected but distinct levels of the educational problem. Profiles make cohort heterogeneity interpretable, whereas course-specific models retain individual variation and allow admission information to be related separately to each early mathematics outcome. Learning analytics, however, should not be reduced to producing indicators. It should maintain an explicit connection to learning processes [33]. Prediction has institutional value only when its outputs are understandable, timely, and actionable for student support [34–36]. Their joint interpretation can therefore connect differentiated cohort-level planning with individual follow-up without converting profiles or probabilities into fixed student labels.

Ethical frameworks for early-warning systems (EWS) emphasize that predictive accuracy alone is insufficient to justify their use. Patterson et al. [37] highlight the importance of anticipation, inclusion, reflexivity, and responsiveness in the responsible development of early-alert systems. Similarly, Khalil et al. [38] warn that learning analytics may reinforce existing educational inequalities, while Alfredo et al. [39] emphasize stakeholder participation and human oversight. These considerations are particularly relevant when admission characteristics may reflect unequal prior educational opportunities. Thus, any future use of admission profiles and predicted probabilities to guide early support would require transparent communication, student and staff consultation, individual review, and evaluation of how access to support and its benefits are distributed across student groups. These principles inform the potential application of our findings, as the present study does not implement or evaluate an EWS.

Research on mathematics support provides a pedagogical basis for connecting early identification with educational responses. Robinson and Croft [40] describe diagnostic testing linked to follow-up support for first-year engineering students. Ambrose et al. in Chapters 1 and 5 in [41] explain how assessing prior knowledge, setting explicit learning goals, and providing practice with informative feedback can guide instruction. Mayerhofer et al. [42] reported short-term improvements in aspects of motivation and learning skills after mathematics bridging, with several favorable developments weakening during the subsequent semester. Their findings encourage paying attention to continuity of support. Atif et al. [36] documented teachers' concerns about false alerts and their use of student records to interpret warning scores. Together, these contributions inform support approaches that combine diagnostic evidence, continued learning opportunities, and professional review.

1.3. Open Questions

Previous research has established associations between admission characteristics and subsequent university performance [14,15,22]. These associations describe general re-

lationships, but they do not show how multiple entry characteristics combine within a heterogeneous cohort or how individual probabilities may differ among students with similar background configurations. Clustering has been used to identify multidimensional patterns related to educational access, performance, and persistence [23–25], while predictive studies have evaluated the capacity of pre-admission information to anticipate academic outcomes [30,31]. Some studies have combined both approaches to examine undergraduate performance [43]. Less attention has been given to connecting admission profiles constructed from mixed data with out-of-sample individual predictions for separate early engineering mathematics courses.

This study addresses this gap by using admission profiles to make cohort heterogeneity interpretable and course-specific models to preserve individual variation within those profiles. Connecting these outputs can reveal how different initial configurations are associated with subsequent mathematics trajectories and identify students whose estimated probabilities differ from the broader pattern of their profile. Elementary Algebra (hereafter, Algebra) and Differential Calculus (hereafter, Calculus) are analyzed separately because admission characteristics may have different predictive value in each course. By connecting admission profiles with course-specific probabilities, the resulting evidence can inform whether early support should be provided across the cohort, differentiated by profile or course, or prioritized for individual follow-up.

Accordingly, the study was guided by two sequential research questions as follows:

RQ1. What admission profiles can be identified from the mixed characteristics available for the 193 students, and how are these profiles associated with their trajectories in Elementary Algebra and Differential Calculus?

RQ2. To what extent can the same admission characteristics support out-of-sample estimation of individual probabilities of failure in each course, and how do these probabilities vary within the identified profiles?

The analyses directly examine associations between admission profiles and subsequent course trajectories and evaluate the predictive performance of the individual models. They do not assess the effectiveness of any support strategy or demonstrate improvements in equity or institutional sustainability. These are broader implications of the study. The results may inform timely and differentiated support, but the design and evaluation of such interventions remain subjects for future research [34–36].

The remainder of the article follows this sequence: Section 2 presents the cohort, variables, and analytical framework, with detailed statistical formulations reported in the Supplementary Material, Section S1. Section 3 moves from the overall cohort to admission profiles, mathematics trajectories, course-specific predictive performance, and individual probabilities within profiles. Section 4 interprets these connected levels of information and their institutional implications. Finally, Section 5 summarizes the contribution and scope of the study.

2. Materials and Methods

This quantitative, observational study connects cohort-level characterization with individual, course-specific prediction in a single analytical workflow. Figure 1 distinguishes the data inputs, analytical methods, outputs, and levels of analysis. It also indicates where RQ1 and RQ2 are addressed and how their results are connected. The statistical analyses were performed using R (version 4.4.1) [44].

As summarized in Figure 1, Sections 2.1–2.4 address RQ1 at the cohort level. Section 2.1 describes the study context, participants, and ethical safeguards. Section 2.2 then defines the admission characteristics and mathematics outcomes. Section 2.3 describes the construction and characterization of the admission profiles. Section 2.4 relates those profiles to

subsequent Algebra and Calculus trajectories. Finally, Sections 2.5 and 2.6 address RQ2 at the individual level by developing course-specific predictive models and then interpreting the resulting probabilities within the independently constructed admission profiles. Details on the mathematical formulations, computational procedures, and extended diagnostic analyses are reported in the Supplementary Material, Section S1.

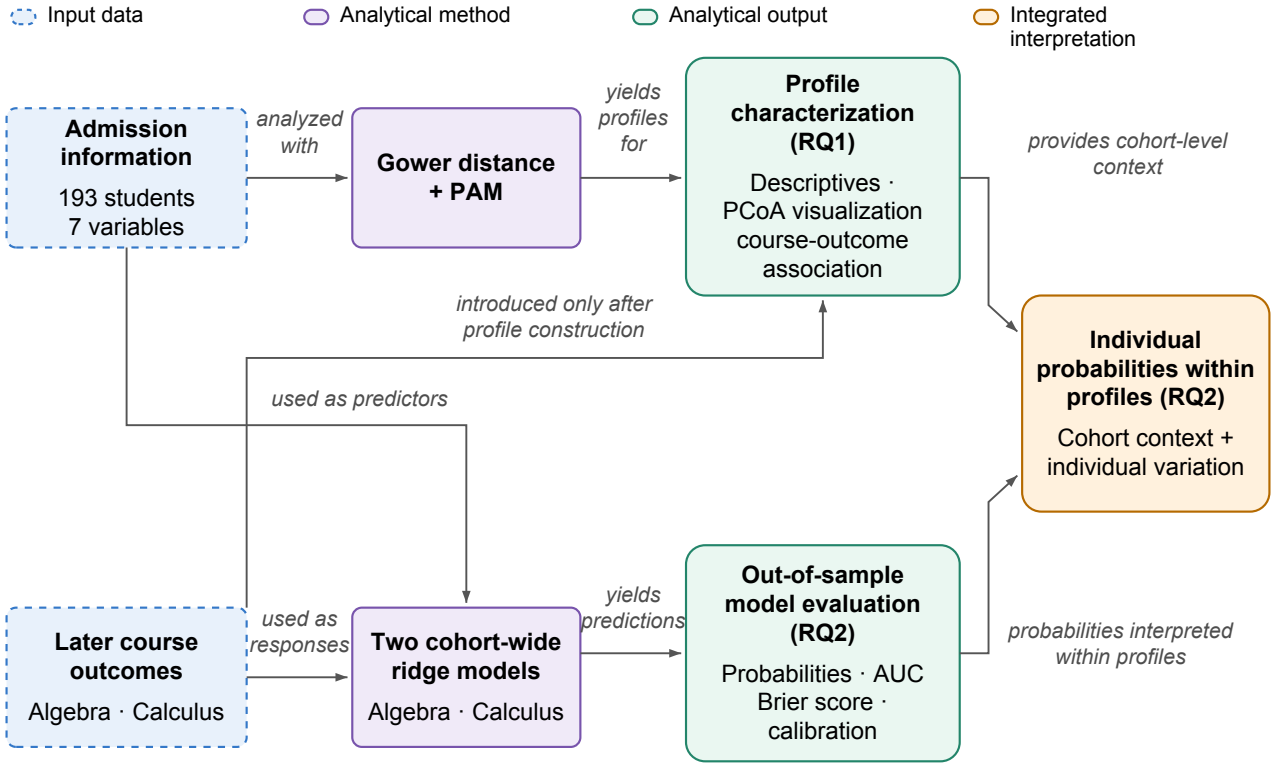


Figure 1. Analytical workflow connecting admission-profile characterization (RQ1) with course-specific prediction and the interpretation of individual probabilities within profiles (RQ2). Algebra and Calculus outcomes enter the profiling branch only after profile construction and therefore do not determine profile membership.

2.1. Context, Participants, and Ethics

The analysis used anonymized administrative data from first-year students at Universidad Católica de Temuco (UCT), a regional university located in Temuco, La Araucanía, southern Chile. UCT was established to serve La Araucanía and the country's southern macroregion [45]. This setting has a distinctive sociocultural context. According to the 2024 Census, 34.5% of the regional population identified as belonging to an Indigenous people [46]. A 2022 institutional account reported that approximately 80% of students entering UCT were the first generation in their families to enter higher education and that more than 30% of enrolled students identified with Indigenous peoples [47]. These figures characterize the regional and institutional setting and are not assumed to describe the study participants.

The anonymized data contained no direct student identifiers and were stored in private institutional repositories accessible only to authorized personnel. The study was approved by the Ethics Committee of Universidad Católica de Temuco under protocol CEIUCT08212003/25. Any action informed by the results would follow institutional confidentiality and ethical procedures, involve human review, and use estimated probabilities only to offer support, never to restrict students' opportunities. The admission characteristics and subsequent mathematics outcomes used in the analytical workflow are defined in Section 2.2.

2.2. Admission Characteristics and Mathematics Outcomes

Admission profiles were constructed exclusively from 7 characteristics recorded before university course performance. This ensured that Algebra and Calculus outcomes did not define the profiles and allowed the profiles to be related to subsequent performance without circularity. The same admission characteristics were later used as predictors in the separate course-specific models.

NEM is the official Chilean acronym for *Notas de Enseñanza Media* and refers to the overall grade average obtained throughout secondary education, transformed to the score scale used for university admission. The ranking score measures a student's academic position relative to previous cohorts from the same school. The mathematics admission score records performance on the mathematics component of the Higher Education Access Test (*Prueba de Acceso a la Educación Superior*, PAES), whereas the weighted admission score combines the selection factors according to the weights established for entry into the engineering program. Admission route distinguished regular admission from inclusive-access pathways. Secondary-school track distinguished humanities–science from technical–vocational education. School administrative sector distinguished municipal, private subsidized, private non-subsidized, and local education service schools. Table 1 summarizes the variables and their official measurement ranges [48].

Table 1. Admission characteristics used to construct the profiles and as predictors in the course-specific models.

Characteristic	Type	Measurement	Range or Categories
NEM score	Quantitative	University admission score	100–1000 points
Ranking score	Quantitative	University admission score	100–1000 points
Mathematics admission score	Quantitative	PAES Mathematics score	100–1000 points
Weighted admission score	Quantitative	Engineering-program admission score	100–1000 points
Admission route	Categorical	University entry pathway	Regular admission; inclusive-access pathway
Secondary-school track	Categorical	Secondary-school curricular track	Humanities–science; technical–vocational
School administrative sector	Categorical	Secondary-school administrative sector	Municipal; private subsidized; private non-subsidized; local education service

The academic outcomes were the final grades and pass–fail status in Algebra and Calculus. Grades followed the institutional scale ranging from 1.0 to 7.0, with 4.0 being the minimum passing grade. The binary outcomes were combined to define 4 possible mathematics trajectories: (a) passing both courses; (b) failing Algebra only; (c) failing Calculus only; and (d) failing both courses. These categories summarize students' recorded pass–fail outcomes in the two courses and are distinct from the probabilities estimated by the predictive models. For each course-specific model, failure was coded as 1 and passing as 0. Keeping the outcomes separate allowed the analysis to determine whether admission characteristics provided different predictive information for Algebra and Calculus.

The seven admission characteristics therefore provide the common input to both analytical branches: cohort-level profile construction in Section 2.3 and individual course-specific prediction in Section 2.5. The mathematics outcomes enter only after profile construction, as described in Section 2.4.

2.3. Cohort-Level Characterization

The seven mixed-type admission variables were represented using Gower dissimilarity, and partitioning around medoids (PAM) was used to identify interpretable configurations of students [28,29]. Gower dissimilarity was selected because it allowed the quantitative and categorical admission characteristics to contribute to a common dissimilarity measure while preserving their respective measurement structures [28]. PAM complemented this choice because its medoids correspond to observed admission records rather than synthetic centroids, facilitating descriptions based on configurations that actually occurred in the cohort [29]. The resulting profiles were treated as empirical summaries of cohort heterogeneity rather than as natural or definitive categories of students.

Solutions with two to six profiles were compared using average silhouette width, and a leave-one-variable-out analysis examined the robustness of the selected solution. Average silhouette width was used as a comparative measure of cohesion and separation, not as evidence that the profiles represented discrete student types [49]. For the sensitivity analysis, each admission variable was omitted in turn, the Gower and PAM procedures were repeated, and the resulting profile solution was compared with the complete-variable partition; complete procedures are provided in the Supplementary Material, Section S1.1.

Principal coordinates analysis (PCoA) was used only to visualize the distance structure and did not determine profile membership [50]. The selected profile solution, its descriptive characterization, the PCoA visualization, and the sensitivity results are reported in Section 3.2. The mathematical definitions, algorithms, and complete sensitivity procedures are reported in the Supplementary Material, Section S1.1, while the extended interpretation of the PCoA dimensions is provided in the Supplementary Material, Section S4. Once these profiles had been constructed without using university course performance, Algebra and Calculus outcomes were introduced as external variables to characterize subsequent mathematics trajectories, as described in Section 2.4.

2.4. Profiles and Mathematics Trajectories

After the profiles had been constructed exclusively from admission information, Algebra and Calculus outcomes were introduced as external characterization variables. Their association with the four mathematics trajectories was evaluated using a chi-square test with Monte Carlo estimation, Cramér's V, and standardized residuals [51].

The analysis of the four Algebra–Calculus trajectories across admission profiles may involve small numbers of students in certain profile-trajectory combinations, potentially limiting statistical power to detect weaker associations [52]. Although Monte Carlo estimation reduces reliance on the asymptotic chi-square approximation when expected frequencies are small, it does not overcome these limitations [51].

In light of these considerations, the association analysis is intended to identify overall patterns between admission profiles and mathematics trajectories, without drawing definitive conclusions about individual students or sparsely represented combinations. The results of this association analysis are reported in Section 3.3. Complete statistical details are provided in the Supplementary Material, Section S1.3. Because this analysis concerns relationships between cohort-level profiles and outcomes rather than individual prediction, the course-specific predictive analysis is described separately in Section 2.5.

2.5. Individual Course-Specific Prediction

Two ridge-penalized logistic models were fitted, one for failure in Algebra and one for failure in Calculus, using the same seven original admission variables [53,54]. Ridge regularization was selected to accommodate correlation among the academic predictors and stabilize coefficient estimation [53,54]. Predictive performance was evaluated

through 100 stratified 70/30 training–validation splits, with stratification based on the four joint Algebra–Calculus trajectories. Each split contained 135 training observations and 58 validation observations and was shared by both models. Within each training set, missing mathematics admission scores were imputed using the median, quantitative predictors were standardized using their means and standard deviations, and categorical predictors were dummy encoded. These transformations were then applied to the validation set. For each course, the regularization parameter was selected within the training set using stratified 10-fold cross-validation with binomial deviance as the evaluation criterion and the one-standard-error rule. The held-out validation observations were excluded from preprocessing, parameter selection, and coefficient estimation. Discrimination, probabilistic error, and classification performance were assessed on held-out observations [55,56]. Calibration was assessed using the held-out predictions from each split by estimating the calibration-in-the-large intercept and calibration slope, summarized by their medians and quartiles. The calibration curves used each student's mean out-of-sample probability and a single observed outcome per student and course. Mean predicted probabilities were also compared with observed proportions of failure within each profile. The complete procedure is presented in the Supplementary Material, Section S1.7. After internal evaluation, final models were refitted using all 193 students. Neither Gower distances nor PAM membership was included as a ridge predictor. Profiles summarize cohort-level configurations, whereas ridge estimates student-level and course-specific probabilities from the original admission characteristics. The course-specific predictive results are reported in Section 3.4. The resulting out-of-sample probabilities are subsequently related to the independently constructed admission profiles in Section 2.6. The complete model formulation, preprocessing, tuning, validation, and metric definitions are reported in the Supplementary Material, Section S1.4.

2.6. Linking Individual Predictions to Admission Profiles

To connect the two analytical levels, each student's mean out-of-sample predicted probability was assigned to the admission profile identified independently through Gower and PAM. For each student and course, this mean was calculated using only the validation splits in which that student was held out. The analysis summarizes the distribution and within-profile variation in these probabilities separately for Algebra and Calculus. This step does not create profile-specific models; it uses profiles to contextualize individual estimates and to show why profile membership alone is insufficient for prioritizing support. The results of this cross-level comparison are reported in Section 3.5, and additional details concerning the generation of the out-of-sample probabilities are provided in Supplementary Section S1.5. Together, Sections 2.3–2.6 complete the analytical sequence summarized in Figure 1, moving from cohort-level characterization to mathematics trajectories, individual course-specific prediction, and finally the interpretation of individual probabilities within the admission profiles.

3. Results

The results follow the analytical sequence summarized in Figure 1. Section 3.1 first describes overall Algebra and Calculus outcomes in the analytical sample. Section 3.2 reports the cohort-level admission-profile characterization and overlap, and Section 3.3 examines the subsequent mathematics trajectories across those profiles. Section 3.4 then reports the separate individual predictive results for Algebra and Calculus. Finally, Section 3.5 presents the distribution of individual out-of-sample probabilities within the independently constructed admission profiles.

3.1. Analytical Sample and Overall Mathematics Outcomes

These overall results provide the baseline against which the profile-specific patterns reported in the following subsections are interpreted. The sample comprised 193 first-year engineering students. Mean grades in Algebra and Calculus were similar. In Algebra, the mean grade was 3.70, the median was 3.40, and the standard deviation was 1.20. In Calculus, the mean was 3.74, the median was 4.00, and the standard deviation was 1.18. Although the means were close, the Calculus median reached the minimum passing grade, whereas the Algebra median remained below this threshold.

Four mathematics trajectories were distinguished: students who passed both courses, students who failed Algebra but passed Calculus (Algebra-only failure), students who passed Algebra but failed Calculus (Calculus-only failure), and students who failed both courses. Seventy students passed both courses, representing 36.3% of the sample. Seventy-nine students (40.9%) failed both Algebra and Calculus, 27 students (14.0%) had Algebra-only failure, and 17 students (8.8%) had Calculus-only failure. Thus, the outcomes included both concurrent and course-specific patterns of failure.

3.2. Cohort-Level Admission Profiles: Characteristics and Overlap

The PAM solution with $k = 3$ identified three admission profiles. The average silhouette width was 0.435 for $k = 3$, compared with 0.404 for $k = 2$, 0.347 for $k = 4$, 0.334 for $k = 5$, and 0.285 for $k = 6$. The three-profile solution therefore provided the highest average silhouette width among the solutions considered, although the difference from $k = 2$ was modest. The profiles are consequently interpreted as a useful summary of cohort heterogeneity rather than as sharply separated or natural student categories. To avoid rigid or potentially stigmatizing labels, the groups are reported as Profile 1, Profile 2, and Profile 3.

Table 2 summarizes the quantitative admission characteristics used in profile construction, whereas Table 3 reports subsequent Algebra and Calculus performance. The mean-based summaries are complemented below by the silhouette analysis, sensitivity analysis, PCoA visualization, and trajectory distributions, which provide information about overlap and within-profile heterogeneity.

Table 2 shows the profile sizes and mean quantitative admission characteristics. Profile 1 comprised 90 students and had the highest mean mathematics admission and weighted admission scores. Its mean NEM and ranking scores were also high. Profile 2 comprised 75 students and had lower mathematics and weighted admission scores than Profile 1 but slightly higher mean NEM and ranking scores. Profile 3 comprised 28 students and had the lowest mean values for all four quantitative admission indicators.

Table 2. Profile size and mean quantitative admission characteristics.

Profile	*n*	Mathematics Score	NEM	Ranking	Weighted Score
Profile 1	90	686	756	773	650
Profile 2	75	631	767	787	609
Profile 3	28	603	645	665	588

The categorical admission characteristics also contributed to the multivariate configuration. In particular, Profile 3 had a greater representation of students from the technical–vocational secondary-school track. The sensitivity analysis further indicated that the school-background variables were important for the stability of the profile solution.

Table 3 reports the subsequent university performance descriptively and separately from the admission characteristics. Profile 1 had the highest mean grades in both Algebra and Calculus and the highest Algebra passing rate. Profile 2 had lower mean grades but

the highest Calculus passing rate, while its Algebra passing rate was lower. Profile 3 had the lowest mean grades and passing rates in both courses.

Table 3. Subsequent Algebra and Calculus performance by admission profile.

Profile	Algebra Grade	Calculus Grade	Passed Algebra (%)	Passed Calculus (%)
Profile 1	4.02	4.00	56.7	53.3
Profile 2	3.60	3.67	44.0	57.3
Profile 3	2.97	3.09	10.7	21.4

The leave-one-variable-out sensitivity analysis supported the three-profile characterization for five of the seven admission variables. Removing NEM, ranking, mathematics admission score, weighted admission score, or admission route retained $k = 3$ as the optimal solution, with adjusted Rand index (ARI) values between 0.945 and 1.000 relative to the complete-variable partition. In contrast, removing secondary-school track changed the optimum to $k = 2$ and reduced the ARI to 0.655, while removing school administrative sector changed the optimum to $k = 2$ and reduced the ARI to 0.210. These results indicate that the profile configuration was particularly sensitive to the two school-background variables. Complete results are reported in the Supplementary Material, Section S3.

PCoA Visualization

Figure 2 shows the two-dimensional PCoA projection of the Gower distance matrix. Each point represents a student, and colors indicate the profiles obtained using PAM. Proximity between points indicates greater similarity in the admission variables used to calculate the distance.

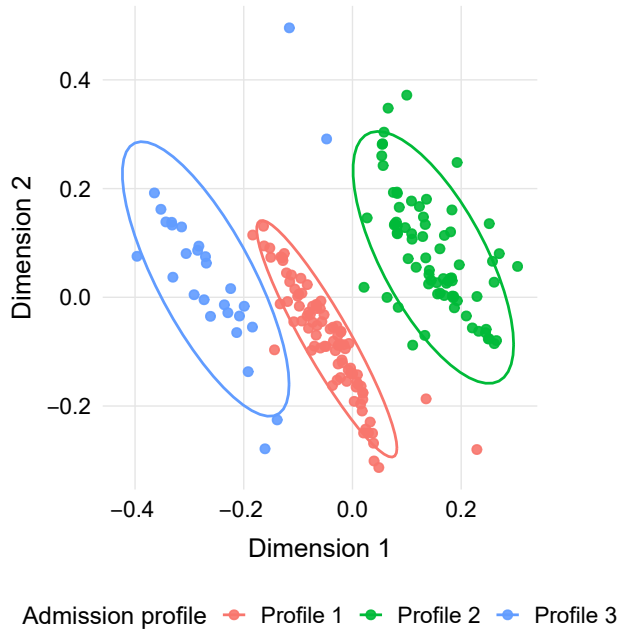


Figure 2. Two-dimensional principal coordinates analysis projection of the Gower distance matrix by admission profile. Each point represents a student. Colors indicate the profiles obtained using PAM with $k = 3$. Ellipses summarize the spread of the students' projected coordinates within each admission profile.

The two-dimensional projection shows partial overlap among the three profiles rather than three completely separated regions, consistent with the average silhouette width of 0.435. PCoA did not determine profile membership; it was used only to display the distance structure represented by the full Gower matrix.

The first two PCoA dimensions represented 55.04% of the variation captured by the dissimilarity matrix: 31.30% for the first dimension and 23.74% for the second. The third dimension represented an additional 12.47%, yielding 67.51% cumulatively across the first three dimensions. Consequently, the apparent separation or overlap in Figure 2 should not be interpreted as a complete representation of the multivariate distance structure. The Supplementary Material, Section S4, reports the associations between the original admission characteristics and the first PCoA dimensions.

3.3. Algebra and Calculus Trajectories Across Profiles

As defined in Section 2.2, the categories in this analysis represent observed course outcomes rather than predicted probabilities: Algebra-only failure denotes failure in Algebra with passing in Calculus, Calculus-only failure denotes passing in Algebra with failure in Calculus, and failure in both denotes concurrent failure. These outcomes were introduced only after the admission profiles had been constructed.

The distribution of mathematics trajectories differed across profiles (see Table 4). In Profile 1, 46.7% of students passed both courses and 36.7% failed both courses. In Profile 2, 33.3% passed both courses and 32.0% failed both courses; Algebra-only failure reached 24.0%, compared with 6.7% in Profile 1 and 10.7% in Profile 3. This was the most distinctive course-specific outcome pattern in Profile 2. In Profile 3, 78.6% of students failed both courses, whereas only 10.7% passed both.

The 36.7% concurrent failure in Profile 1 is also noteworthy because it occurred despite this profile having the strongest average admission indicators and mean university grades. Outcomes therefore varied substantially within profiles, so profile membership did not uniquely determine subsequent course performance.

Table 4. Distribution of Algebra and Calculus trajectories by admission profile.

Profile	Passed Both	Algebra-Only Failure	Calculus-Only Failure	Failed Both
Profile 1	42 (46.7%)	6 (6.7%)	9 (10.0%)	33 (36.7%)
Profile 2	25 (33.3%)	18 (24.0%)	8 (10.7%)	24 (32.0%)
Profile 3	3 (10.7%)	3 (10.7%)	0 (0.0%)	22 (78.6%)

Association Analysis

The association analysis evaluated whether the distribution of these four trajectories differed across the independently constructed admission profiles. The association was statistically significant but modest in magnitude. The chi-square statistic was $\chi^2 = 31.398$, with $p < 0.001$. Because some expected frequencies were below 5, the p-value was estimated using a Monte Carlo simulation with 10,000 replicates. Cramér's V was 0.285, indicating that the profiles were associated with the trajectories but did not provide a deterministic classification of course outcomes.

Table 5 identifies the profile–trajectory combinations contributing most strongly to the association. Profile 1 had an overrepresentation of students who passed both courses (standardized residual 2.81) and an underrepresentation of Algebra-only failure (residual −2.74). Profile 2 had an overrepresentation of Algebra-only failure (residual 3.20) and an underrepresentation of failure in both courses (residual −2.01). Profile 3 had a marked

overrepresentation of failure in both courses (residual 4.38) and an underrepresentation of students who passed both courses (residual −3.04).

Table 5. Standardized residuals for the association between admission profile and mathematics trajectory.

Profile	Passed Both	Algebra-Only Failure	Calculus-Only Failure	Failed Both
Profile 1	2.81	−2.74	0.55	−1.13
Profile 2	−0.68	3.20	0.73	−2.01
Profile 3	−3.04	−0.54	−1.78	4.38

3.4. Course-Specific Ridge Models and Predictive Performance

Following the model specification and validation procedure described in Section 2.5, separate ridge logistic models were estimated for failure in Algebra and Calculus using the same seven admission characteristics defined in Section 2.2. After internal evaluation, each model was refitted using all 193 students to obtain the final course-specific equations reported below. The complete model formulation, predictor preprocessing, regularization procedure, tuning, and validation strategy are provided in the Supplementary Material, Section S1.4, while the complete penalized coefficient table and predictor coding are reported in the Supplementary Material, Section S2.

The four quantitative predictors were standardized before model estimation. Let $z_{\mathrm{NEM},i}$, $z_{\mathrm{Rank},i}$, $z_{\mathrm{Math},i}$, and $z_{\mathrm{Weighted},i}$ denote the standardized NEM, ranking, mathematics admission, and weighted admission scores for student i. The categorical terms are represented by indicator variables for inclusive-access admission ($I_{\mathrm{IA},i}$), technical–vocational secondary education ($I_{\mathrm{TV},i}$), municipal school ($I_{\mathrm{MUN},i}$), private non-subsidized school ($I_{\mathrm{PNS},i}$), and local education service school ($I_{\mathrm{LES},i}$). The corresponding reference categories are regular admission, humanities–science secondary education, and private subsidized schools; full coding details are given in the Supplementary Material, Section S2.

For Algebra, the fitted linear predictor was

$$\begin{aligned}\hat{\eta}_{iA} = {} & 0.212 - 0.244 z_{\mathrm{NEM},i} - 0.224 z_{\mathrm{Rank},i} - 0.270 z_{\mathrm{Math},i} - 0.334 z_{\mathrm{Weighted},i} \\ & + 0.056 I_{\mathrm{IA},i} + 0.125 I_{\mathrm{TV},i} - 0.012 I_{\mathrm{MUN},i} + 0.005 I_{\mathrm{PNS},i} + 0.025 I_{\mathrm{LES},i}.\end{aligned} \tag{1}$$

For Calculus, the fitted linear predictor was

$$\begin{aligned}\hat{\eta}_{iC} = {} & -0.005 - 0.158 z_{\mathrm{NEM},i} - 0.147 z_{\mathrm{Rank},i} - 0.252 z_{\mathrm{Math},i} - 0.227 z_{\mathrm{Weighted},i} \\ & + 0.117 I_{\mathrm{IA},i} + 0.129 I_{\mathrm{TV},i} - 0.100 I_{\mathrm{MUN},i} + 0.007 I_{\mathrm{PNS},i} - 0.015 I_{\mathrm{LES},i}.\end{aligned} \tag{2}$$

For either course, the estimated individual probability of failure was obtained from the corresponding linear predictor through

$$\hat{p}_{ij} = \frac{\exp\left(\hat{\eta}_{ij}\right)}{1 + \exp\left(\hat{\eta}_{ij}\right)}, \qquad j \in \{A, C\}, \tag{3}$$

where A and C denote Algebra and Calculus, respectively. Equations (1)–(3) correspond to the final models refitted on the complete analytical sample; the predictive-performance results below were obtained separately from held-out observations according to the validation procedure in Section 2.5.

Table 6 summarizes out-of-sample performance across the 100 stratified splits. The Algebra model achieved a mean area under the receiver operating characteristic curve (AUC) of 0.832, with a 95% empirical stability interval from 0.754 to 0.909, indicating good

discrimination between students who passed and failed. The Calculus model achieved a mean AUC of 0.735, with an interval from 0.642 to 0.831, corresponding to more moderate discrimination and greater variability across splits.

Table 6. Out-of-sample predictive performance across 100 stratified splits. Reported values are means across repetitions; the 95% empirical stability interval (ESI) for AUC comprises the 2.5th and 97.5th percentiles across repetitions.

Course	Mean AUC (95% ESI)	Brier Score	Sensitivity	Specificity	Balanced Accuracy
Algebra	0.832 (0.754–0.909)	0.185	0.813	0.632	0.723
Calculus	0.735 (0.642–0.831)	0.221	0.651	0.663	0.657

The lower Brier score for Algebra indicates that its estimated probabilities had less error than those for Calculus. At a 0.50 classification threshold, the Algebra model correctly identified 81.3% of students who failed and 63.2% of those who passed, yielding a balanced accuracy of 72.3%. Performance for Calculus was more even but lower, with 65.1% sensitivity, 66.3% specificity, and 65.7% balanced accuracy. The higher AUC, lower Brier score, and higher balanced accuracy for Algebra indicate that the same admission characteristics provided stronger predictive information for Algebra failure than for Calculus failure.

Figure 3 presents the distribution of individual out-of-sample failure probabilities. As specified in Section 2.5, each student's mean probability was calculated only from repetitions in which that student belonged to the validation set. In Algebra, the mean probability was 0.448 among students who passed and 0.630 among those who failed. The corresponding probabilities in Calculus were 0.455 and 0.534. The greater separation between observed outcome groups in Algebra is consistent with the difference in discriminatory performance reported in Table 6.

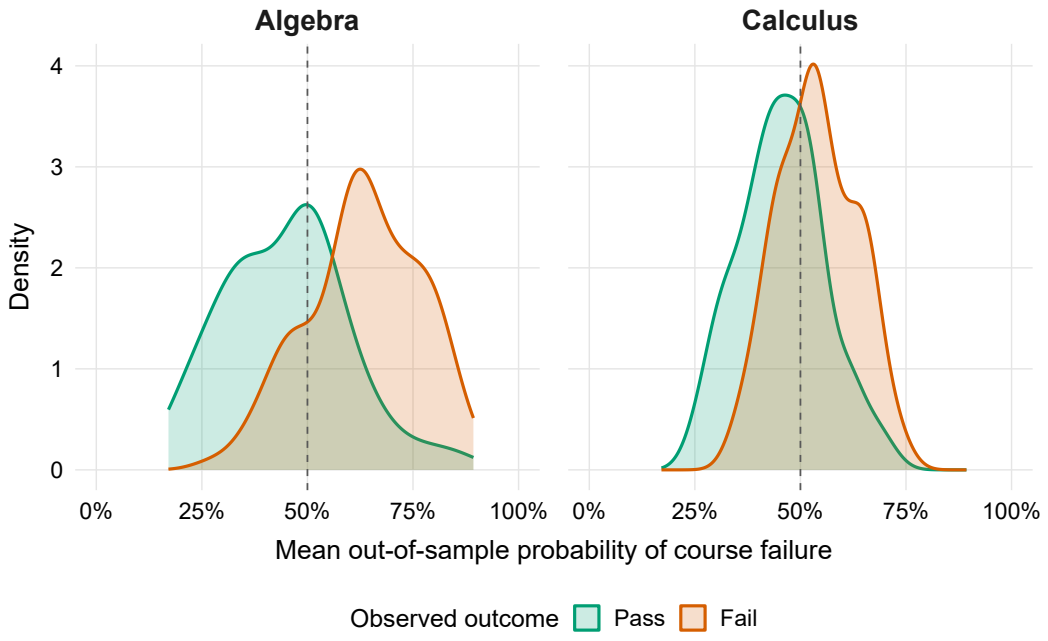


Figure 3. Distribution of the mean out-of-sample probability of failure by observed outcome. Each individual probability was calculated exclusively from repetitions in which the student belonged to the 30% validation set. The vertical line indicates the 0.50 classification threshold.

The fitted coefficients in Equations (1) and (2) show that all four quantitative admission characteristics had negative penalized coefficients in both courses. Conditional on the other predictors, higher values of these admission measures were therefore associated with lower estimated probabilities of failure. In Algebra, the weighted admission score had the largest coefficient magnitude among the quantitative predictors, with a penalized odds ratio of 0.716 per one-standard-deviation increase, corresponding to an approximate 28.4% reduction in the estimated odds of failure. In Calculus, the mathematics admission score had the largest coefficient magnitude, with a penalized odds ratio of 0.778, corresponding to an approximate 22.2% reduction.

The categorical coefficients were comparatively small after regularization. These estimates represent conditional predictive contributions within the ridge models and should not be interpreted as independent causal effects. The complete coefficient estimates and corresponding penalized odds ratios are reported in the Supplementary Material, Section S2.

Across the 100 validation splits, the median calibration-in-the-large intercept was 0.004 for Algebra and 0.012 for Calculus. The median calibration slopes were 2.28 and 2.38, respectively. Although the intercepts were close to zero, slopes above one indicate that the predictions reflected smaller differences in risk than those observed. The estimated curves suggest overestimation in the lower probability range and underestimation across much of the upper range. Supplementary Table S1 reports the medians and quartiles across splits, and Supplementary Figure S1 shows the course-specific calibration curves based on student-level mean out-of-sample probabilities.

3.5. Individual Predicted Probabilities Within Admission Profiles

Following the cross-level analysis described in Section 2.6, the mean out-of-sample probabilities generated by the global ridge models were examined within the three admission profiles identified independently through Gower dissimilarity and PAM. No additional model was fitted within the profiles and profile membership was not included as a ridge predictor. For each course, the analysis therefore relates the individual probabilities produced by the global model to the cohort-level admission profiles.

Table 7 compares the failure rate with the mean out-of-sample predicted probability within each profile. In Algebra, mean predicted failure probabilities increased from 51.2% in Profile 1 and 54.9% in Profile 2 to 67.3% in Profile 3. The corresponding failure rates were 43.3%, 56.0%, and 89.3%, respectively. In Calculus, mean predicted probabilities were 47.5%, 49.3%, and 57.9% across Profiles 1–3, compared with failure rates of 46.7%, 42.7%, and 78.6%.

Table 7. Failure rates and mean out-of-sample predicted probabilities from the global ridge models by admission profile. Differences are calculated as the mean predicted probability minus the observed failure rate and are expressed in percentage points (pp).

Course	Profile	*n*	Observed Failure (%)	Mean Predicted Failure (%)	Difference (pp)
Algebra	Profile 1	90	43.3	51.2	+7.9
	Profile 2	75	56.0	54.9	−1.1
	Profile 3	28	89.3	67.3	−22.0
Calculus	Profile 1	90	46.7	47.5	+0.8
	Profile 2	75	42.7	49.3	+6.6
	Profile 3	28	78.6	57.9	−20.7

The global models therefore assigned the highest mean probability of failure to Profile 3 in both courses. However, the magnitude of the probabilities did not reproduce the profile-

level failure rates exactly. In particular, the mean predicted probabilities for Profile 3 were lower than the failure rates by 22.0 percentage points in Algebra and 20.7 percentage points in Calculus. These values are descriptive predicted–observed discrepancies at the aggregate profile level and should not be interpreted as formal calibration estimates or as measures of individual discrimination. Of the 28 students in Profile 3, 25 failed Algebra and 22 failed Calculus. The small subgroup size limits the precision of the observed proportions, and the magnitude of the predicted–observed discrepancies may differ in other cohorts. These discrepancies alone do not establish systematic underestimation beyond the cohort studied. Supplementary Figure S2 presents the corresponding comparison for all three admission profiles. The profile-level ordering was stable across the repeated validation procedure described in Section 2.5. Profile 3 had the highest mean predicted probability in 97% of Algebra repetitions and 98% of Calculus repetitions. The complete repetition-level comparison of profile rankings based on mean predicted failure probabilities and on observed failure rates is reported in the Supplementary Material, Section S1.6.

The profile-level means in Table 7, however, do not describe the variation in individual predicted probabilities within each profile. To examine this variation, Table 8 summarizes the student-level mean out-of-sample probabilities within Profiles 1–3 separately for Algebra and Calculus, while Figure 4 displays their distributions and overlap.

Table 8. Distribution of individual mean out-of-sample predicted probabilities of failure within admission profiles.

Course	Profile	*n*	Mean	Median	SD	Q1–Q3	Min–Max
Algebra	Profile 1	90	0.512	0.516	0.169	0.371–0.627	0.171–0.862
	Profile 2	75	0.549	0.544	0.153	0.446–0.648	0.250–0.881
	Profile 3	28	0.673	0.696	0.147	0.603–0.780	0.287–0.893
Calculus	Profile 1	90	0.475	0.476	0.104	0.396–0.541	0.254–0.707
	Profile 2	75	0.493	0.479	0.094	0.433–0.542	0.306–0.723
	Profile 3	28	0.579	0.585	0.096	0.530–0.651	0.319–0.744

Individual predicted probabilities exhibited within-profile variation in both courses (Table 8). Dispersion was greater in Algebra, with standard deviations ranging from 0.147 to 0.169, compared with 0.094 to 0.104 in Calculus. Profiles 1 and 2 showed substantial overlap in their interquartile ranges, particularly in Calculus, where their medians were similar (0.476 and 0.479, respectively). Profile 3 had higher median probabilities in both Algebra (0.696) and Calculus (0.585), although its interquartile range still overlapped with those of the other profiles. Moreover, its individual probabilities ranged from 0.287 to 0.893 in Algebra and from 0.319 to 0.744 in Calculus. Thus, assignment to the same admission profile did not imply a uniform predicted probability of failure: the global ridge models retained substantial individual variation within profiles, while predicted probabilities also overlapped across profiles.

Figure 4 complements the distributional summaries in Table 8 by showing the extent of overlap in student-level probabilities across admission profiles. Profiles 1 and 2 overlapped substantially in both courses, while Profile 3 was generally shifted toward higher estimated probabilities but still overlapped with the other profiles. Some students in Profiles 1 and 2 received higher estimates than some students in Profile 3, and students who passed and failed also showed overlapping probability distributions within each profile. These patterns reinforce that profile membership provides cohort-level context but cannot substitute for individual course-specific probability estimates.

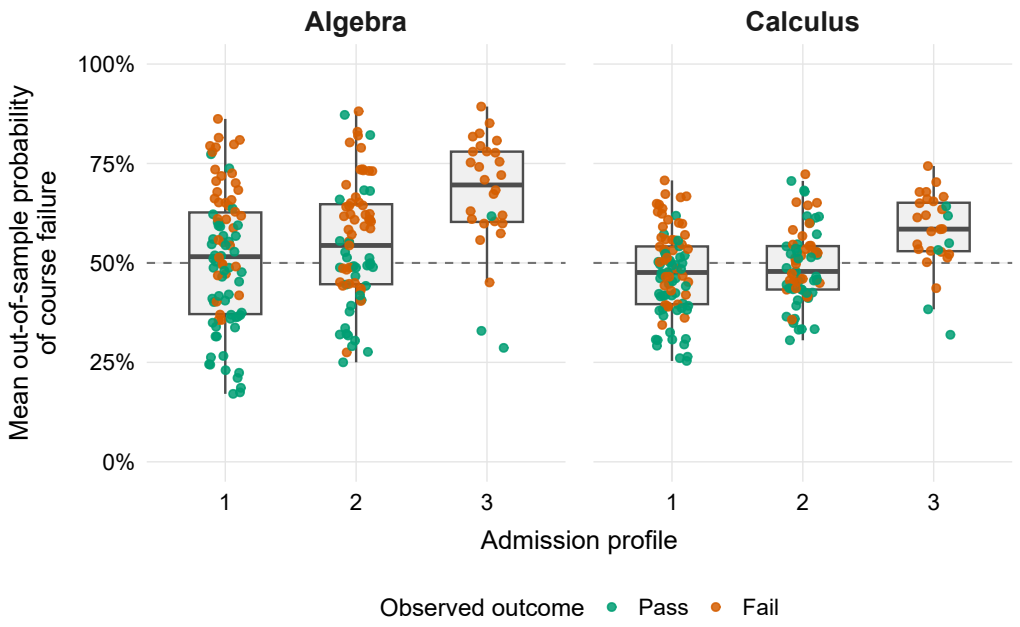


Figure 4. Distribution of individual mean out-of-sample predicted probabilities of failure within admission profiles for Algebra and Calculus. Each point represents one student and is based only on repetitions in which that student belonged to the validation set. Boxes summarize the within-profile distributions. The dashed horizontal line indicates a predicted probability of failure of 0.5 (50%).

4. Discussion

Section 4.1 examines the contribution of connecting cohort and individual analyses. Sections 4.2 and 4.3 discuss profile uncertainty and possible explanations for the course-specific patterns, respectively. Section 4.4 translates these interpretations into support options for prospective evaluation, and then Section 4.5 specifies conditions for equitable use. Finally, Section 4.6 identifies the limits of the evidence and priorities for further research.

4.1. Principal Findings and Analytical Contribution

The analytical contribution of these results lies in identifying what is lost when either profiles or individual probabilities are considered alone. The within-profile distributions in Section 3.5, particularly Figure 4, show why a group average cannot represent every student's estimated probability. Conversely, admission profiles situate individual estimates within recurring educational backgrounds. Their joint interpretation can therefore inform different questions: what provision might be relevant across a cohort, and which students might warrant an individual conversation? This is an interpretive contribution rather than evidence that clustering improved prediction, since the models used the original admission variables without profile membership (Sections 2.5 and 2.6).

These findings are consistent with research identifying prior academic characteristics as relevant predictors of success in higher education and emphasizing the value of early detection for planning timely interventions [30,31,43]. Here, restricting predictors to admission information makes the estimates potentially available before course difficulties become apparent. The associated trade-off is that early availability of admission information provides little knowledge about the learning processes that produce those difficulties.

4.2. Admission Profiles as Contextual, Non-Deterministic Summaries

The sensitivity findings in Section 3.2 make the representation of school background consequential for interpretation. Removing secondary-school track or administrative sector altered the partition, whereas omitting individual admission scores largely preserved it. As detailed in the Supplementary Material, Section S3, this procedure tests dependence on the selected variables, not reproducibility in another cohort. The contrast suggests that the profiles capture configurations of schooling as well as differences in academic scores. It does not establish that school sector or track causes failure. The limited change after removing individual scores may also reflect information shared among correlated academic measures rather than their educational irrelevance.

The modest advantage of the selected partition and the partial overlap reported in Section 3.2 further argue against fixed categories of mathematical preparedness. Combined with the outcome variation in Section 3.3, these findings support interpreting the admission profiles as cohort-level summaries of recurring configurations rather than fixed student types, while using profile descriptions to initiate enquiry into students' needs. They cannot identify whether a particular student needs conceptual instruction, help with study organization, or support with circumstances outside the classroom. Those distinctions require diagnostic tasks and the student's own account.

The reproducibility of the profiles requires evaluation in independent cohorts. Given their sensitivity to school-background characteristics, changes in cohort composition could alter the number and composition of the profiles. Evaluation should repeat the selection of the number of profiles and examine both the similarity of their admission characteristics and their associations with Algebra and Calculus trajectories. Until then, the profile labels and support priorities identified in this sample should not be transferred automatically to other cohorts.

4.3. Course-Specific Prediction in Algebra and Calculus

Table 6 shows that, using the same seven admission characteristics and training–validation splits, the models achieved a mean AUC of 0.832 for Algebra and 0.735 for Calculus. Thus, in this cohort, admission information provided better discrimination between students who subsequently failed and those who passed Algebra than Calculus. This descriptive difference motivates an educational interpretation, but it does not identify its cause or establish that Algebra is inherently more predictable in other contexts.

One plausible explanation is a closer alignment between the prior preparation represented by the admission variables and the prerequisites emphasized in Algebra. If success in the local Algebra course depends substantially on previously acquired fluency in manipulating expressions and solving equations, admission variables may capture a greater proportion of the preparation relevant to that outcome. Calculus also requires algebraic fluency, but additionally involves interpreting functions, limits, and rates of change through symbolic, graphical, and numerical representations, consistent with the curricular domains described by SEFI [17] and research emphasizing the importance of connecting multiple representations in university mathematics [57]. Differences in how students develop and coordinate these competencies during the course may be less fully captured by aggregate admission scores. This is a possible explanation for the lower AUC in Calculus, not a demonstrated difference in the competencies of the students analyzed.

Course structure and assessment practices offer complementary hypotheses, given the importance of aligning learning objectives, teaching activities, and assessment [58]. Differences in the sequencing and pace of prerequisite review, opportunities for practice and feedback, or the timing and weighting of assessments could influence the extent to which prior preparation is reflected in final pass–fail outcomes. Similarly, the relative

emphasis on routine symbolic procedures, conceptual justification, and application tasks could affect the relationship between admission variables and assessed performance. The present analysis did not examine these course characteristics or students' mastery of specific prerequisites and therefore cannot distinguish among these explanations. Testing these hypotheses in future research would require comparing course syllabi, assessment tasks, and marking criteria with early diagnostic results and student work. Together with the pattern of Algebra-only failure in Profile 2 (see Section 3.3), the observed difference supports course-specific diagnostic assessment rather than reliance on a single admission-based measure of mathematical preparedness.

4.4. Implications for Prospective Early Support

Building on the approaches discussed by Robinson and Croft [40], Ambrose et al. [41], Mayerhofer et al. [42], and Atif et al. [36] for diagnostic assessment, targeted learning support, mathematics bridging, and professionally reviewed early alerts, respectively (see Section 1), the trajectories (Table 4) suggest differentiated priorities for support. Failure occurred in every profile, including 36.7% concurrent failure in Profile 1, supporting broadly accessible diagnostic assessment and mathematics support rather than restricting provision to students in profiles with less favorable admission characteristics [40]. At the same time, the higher prevalence of Algebra-only failure in Profile 2 suggests a potential emphasis on Algebra-focused support guided by diagnostic evidence [41], whereas the 78.6% rate of concurrent failure in Profile 3 suggests evaluating coordinated bridging and continued tutoring across both courses [42].

The finding for Profile 1 supports offering early diagnostic assessments also to students with favorable admission characteristics. Tasks should align with course-specific prerequisites, such as manipulating expressions and solving equations in Algebra, and interpreting functions and their representations in Calculus. Reviewing these tasks alongside admission information and in consultation with the student could guide support. This proposal requires prospective evaluation, as the study did not include early diagnostic assessments.

The individual probability results add a second level of information to these profile-level patterns. Although Profile 3 showed the highest mean and median predicted probabilities of failure in both courses, the distributions in Table 8 and Figure 4 also showed substantial variation within profiles and overlap across them. Consequently, profile membership may help institutions anticipate broad support needs, but it should not determine which individual students are invited to or prioritized for specific interventions. Individual course-specific probabilities can instead provide an additional signal for review, to be considered alongside diagnostic evidence and student consultation [36].

On this basis, Table 9 translates the main empirical patterns into prospective support options. The table distinguishes what was found in the present study from how those findings could inform support design and from the outcomes that would need to be evaluated in a subsequent cohort. The proposed responses are therefore implications for prospective testing rather than interventions whose effectiveness was established here.

These prospective responses are consistent with previous mathematics-support research without implying that their effectiveness was tested here. Robinson and Croft [40] and Ambrose et al. in Chapters 1 and 5 in [41] support the use of diagnostic information to connect prior knowledge with targeted practice and feedback, which is consistent with broadly accessible diagnostics followed by tutoring focused on demonstrated needs. Mayerhofer et al. [42] further suggest that mathematics bridging is better considered as part of continued support rather than as a stand-alone response. In the present study, these approaches are linked specifically to the cohort-wide occurrence of failure, the Algebra-only pattern in Profile 2, and the concurrent-failure pattern in Profile 3; the cited literature

provides a rationale for prospective testing rather than evidence that these responses would be effective in this cohort.

Table 9. Support options suggested by the study's findings. All responses and evaluation measures are proposals for prospective evaluation.

Finding in This Cohort	Proposed Support Response	Evaluation Measures
Failure occurred in every profile.	Open diagnostics and access to tutoring.	Participation and support uptake across profiles.
Algebra-only failure was overrepresented in Profile 2.	Algebra tutoring guided by diagnostic tasks.	Progress on assessed concepts and Algebra outcomes.
Failure in both courses was overrepresented in Profile 3.	Bridging and continued tutoring for needs spanning both courses.	Sustained participation and outcomes in each course.
Probabilities varied within profiles and overlapped across profiles.	Individual review and an agreed support offer.	Uptake and unmet needs among students not invited.

These support options were not evaluated in the present study and should therefore be treated as proposals for prospective testing. The 0.50 threshold used in the predictive analysis served only as a classification convention and should not be interpreted as an institutional decision rule. Criteria for individual follow-up would instead need to consider the course-specific evidence available for each student, institutional support capacity, and the consequences of false-positive and false-negative alerts. Access to support should remain broadly available under the safeguards discussed in Section 4.5.

4.5. Ethical and Equitable Use of Predictive Analytics

Variables such as school administrative sector, secondary-school track, and admission route may reflect structural inequalities. Because these characteristics contribute to both the admission profiles and the individual probability estimates, their prospective use should be examined in terms of how support opportunities are distributed across student groups rather than interpreted as fixed attributes of individual students. Profile membership and estimated probabilities should therefore be used to broaden opportunities for support, not to restrict eligibility or label students. Transparent communication of their purpose and uncertainty, together with opportunities to correct inaccurate records and continued access to support regardless of profile or estimated probability, would be essential safeguards. Early-warning systems should likewise be embedded within institutional support processes rather than reduced to automatic classification [35,36].

Any prospective implementation should therefore be evaluated not only in terms of predictive performance but also in terms of how support opportunities, participation, and subsequent benefits are distributed across student groups. Where sample sizes permit, these patterns should be examined by admission route, secondary-school track, and school administrative sector. Such analyses would provide prospective evidence about equitable implementation rather than demonstrate that the present models are already fair. Moreover, because the available data did not include age, gender, territorial origin, first-generation status, Indigenous identification, or direct socioeconomic indicators (Section 2.1), equity cannot be assessed across those dimensions in the present cohort. The connection to sustainability is therefore prospective and institutional: a support system informed by these results could only be considered sustainable and equitable if access, participation, educational benefit, and differential effects are evaluated over time. The present study

establishes patterns in mathematics outcomes and internal predictive performance, not improvements in equity, retention, or institutional sustainability.

4.6. Limitations and Future Research

The study has several limitations. It analyzed a single institutional cohort of 193 students, which limits the generalizability of both the admission profiles and the predictive results. Repeated 70/30 evaluation provides evidence of internal predictive stability, but it repeatedly draws training and validation observations from the same cohort and therefore does not substitute for temporal or external validation; accordingly, the reported percentiles were not interpreted as confidence intervals. The evaluation described in Section 2.5 and Supplementary Material, Section S1.4, therefore supports claims about internal predictive performance only. Similarly, the leave-one-variable-out sensitivity analysis evaluates one aspect of profile robustness but does not establish that the three-profile structure will recur in another cohort or institution. Future research should therefore examine both the transportability of the predictive models and the reproducibility of comparable admission configurations and mathematics trajectories in independent cohorts.

Although relevant admission characteristics were included, process variables such as attendance, motivation, employment demands, socioemotional well-being, and participation in institutional support were not available. The absence of learning-platform activity, study habits, and students' own accounts also limits the ability to examine factors that may contribute to the course outcomes beyond the information available at admission. These variables should not simply be added to the present models, because they become available after university entry and would therefore address a different prediction point. Future research could instead evaluate a sequential approach in which the admission-based probabilities provide an initial estimate and are subsequently updated with early-course evidence such as diagnostic results, attendance, or learning activity. Such an approach would require separate prospective validation to determine whether the additional information improves course-specific prediction and the identification of support needs.

Profile 3 contained only 28 students. Its profile-specific trajectory rates and predicted–observed discrepancies should therefore be interpreted descriptively and require confirmation in independent cohorts.

Calibration was assessed through internal validation in a single cohort (Supplementary Material, Section S1.7). If invitations to academic support depended on exceeding a fixed probability threshold, the underestimation observed in Profile 3 could leave some students who need support without an invitation. This study did not evaluate such a decision rule. Calibration would therefore need to be assessed in independent cohorts before these probabilities are used to determine which students are offered support. Separately, the support options proposed in Table 9 require prospective evaluation to determine whether they improve participation, learning, or course outcomes. Such evaluation should use an appropriate comparison design so that effects associated with the support can be distinguished from differences in uptake, selection into intervention, or variation between cohorts.

5. Conclusions

This study linked two complementary levels of evidence for examining early mathematics difficulties in first-year engineering: admission profiles constructed from pre-university characteristics and individual course-specific predicted probabilities of failure in Algebra and Calculus. The profiles summarized recurring configurations within the entering cohort, whereas the individual course-specific probabilities retained substantial variation within those profiles and overlapped across them (Section 3.5, Figure 4). Their joint interpretation

therefore preserves the distinction between group-level patterns and individual estimates rather than treating profile membership as a proxy for student-specific probability.

The predictive results also showed that the relationship between admission characteristics and subsequent mathematics outcomes was course-specific. The ridge model for Algebra achieved stronger discrimination and lower probabilistic error than the corresponding model for Calculus, with an AUC of 0.832 and a Brier score of 0.185 for Algebra, compared with an AUC of 0.735 and a Brier score of 0.221 for Calculus. This difference supports modeling the two courses separately and cautions against treating early mathematics difficulty as a single undifferentiated outcome. Internal calibration assessment showed that the predicted probabilities spanned too narrow a range. Their use to guide individual decisions requires validation in independent cohorts.

From an institutional perspective, the results support differentiated rather than profile-determined support. Difficulties were present across all profiles, including 36.7% concurrent failure in Profile 1, while Algebra-only failure was overrepresented in Profile 2 (24.0%) and concurrent failure reached 78.6% in Profile 3. At the same time, individual predicted probabilities varied substantially within profiles and overlapped across them. These findings suggest that profiles may help anticipate broad patterns of need across the cohort, whereas student-level course-specific probabilities provide complementary information for individual review. Support decisions should therefore combine these signals with diagnostic evidence rather than rely on profile membership or a fixed probability threshold alone.

These findings should be interpreted within the limits of a single institutional cohort and internal validation. Independent validation is needed before the course-specific models are used operationally, and the proposed support framework should be evaluated prospectively for its effects on participation, mathematics outcomes, and equitable access to support. The contribution of the present study therefore lies not in demonstrating improved equity or sustainability, but in providing an interpretable framework that connects admission heterogeneity, individual course-specific probabilities, and prospective support decisions while preserving the distinction between cohort-level context and student-level evidence.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/su182111374/s1. Figure S1: Course-specific calibration in the 193 students. The curve represents the frequency of failure estimated using logistic regression with a natural spline (three degrees of freedom). The circles show the observed proportions of failure by quintile of predicted probability. The diagonal represents agreement between the predicted probability and the observed frequency. Figure S2: Predicted and observed failure by admission profile. Each point represents one profile within a course. Points above the diagonal indicate that the observed proportion of failure exceeded the mean predicted probability. Table S1: Course-specific calibration across the 100 validation splits. Values are medians, with the first and third quartiles in brackets. Table S2: Estimates from the final ridge-penalized logistic models refitted on the complete analytical sample. Penalized odds ratios were calculated as $\exp(\hat{\beta})$. For quantitative variables, coefficients and odds ratios correspond to a one-standard-deviation increase after standardization. Table S3: Sensitivity of the admission-profile solution to removal of one admission characteristic at a time. Table S4: Percentage of the PCoA representation associated with the first five dimensions. Table S5: Correlations between quantitative admission characteristics and PCoA dimensions. Table S6: Eta-squared associations between categorical admission characteristics and PCoA dimensions. References [28,29,32,49–51,53–56] are cited in the Supplementary Materials.

Author Contributions: Conceptualization, methodology, validation, investigation, and writing—original draft preparation, H.J.G., C.T.-U., I.H., O.V., P.Z. and E.V.-M. (equal contributions); formal analysis, H.J.G. (lead), with contributions from C.T.-U., I.H., O.V., P.Z. and E.V.-M.; writing—review and editing, C.T.-U. (lead), with contributions from H.J.G., I.H., O.V., P.Z. and E.V.-M. All authors agree to be accountable for their contributions and to ensure that questions concerning the accuracy

or integrity of the work are appropriately investigated and resolved. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially supported by the Departamento de Ciencias Matemáticas y Físicas at Universidad Católica de Temuco through project FINGPEP202504. The authors acknowledge support from Eindhoven University of Technology for open-access publication.

Institutional Review Board Statement: The study was reviewed and approved by the institutional Ethics Committee of Universidad Católica de Temuco (protocol CEIUCT08212003/25).

Informed Consent Statement: Written informed consent was obtained from all students whose data were included in the study.

Data Availability Statement: The student-level data analyzed in this study are not publicly available due to student privacy and institutional confidentiality requirements. The data are held by Universidad Católica de Temuco, and access is restricted to authorized personnel.

Acknowledgments: During the preparation of this manuscript, the authors used ChatGPT (GPT-5.6), Gemini 3 Flash, NotebookLM (web version), Claude Opus 4.6, and Grok 4 to assist with grammatical correction and language refinement. The authors reviewed and edited all suggested revisions and take full responsibility for the content of this publication.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study, in the collection, analysis or interpretation of data, in the writing of the manuscript, or in the decision to publish the results.

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