

Article

Stable Benefit Allocation for Economically Sustainable Coordination of Distributed Energy Resources: A WRT-Inspired Fidelity-Controlled Reduction Framework

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Abstract

Local energy sharing can support more sustainable distribution systems by coordinating rooftop photovoltaic generation, behind-the-meter storage, and flexible demand among prosumers. Its expansion to larger energy communities, however, requires benefit-allocation mechanisms that remain computationally tractable while maintaining physically meaningful and sufficiently stable incentives for heterogeneous participants. This paper develops a fidelity-controlled WRT-inspired player-reduction framework for scalable cooperative energy sharing in distribution networks. Participants are characterized by physical-economic signatures incorporating temporal net demand, operational flexibility, stand-alone economics, marginal contribution, and network sensitivity. Candidate partitions generated through hierarchical entropy weighting and density-based grouping are screened according to feature-space compactness, member-wise game consistency, and aggregation-interaction consistency. Reduced coalitions are restored to their original participants before network-constrained physical evaluation, and the reduced-space nucleolus is subsequently recovered at the participant level. A split-coalition correction with independent out-of-sample validation is further introduced to improve allocation stability. For the primary $N = 10$ benchmark, the game consistent reduced formulation (Case 3) retains 6.07% of the original coalition space, limits the normalized L_1 allocation error to 3.67%, and achieves a $5.40\times$ speedup over the full-space nucleolus. The complete stability-corrected framework (Case 4) further reduces the out-of-sample violation probability from 0.900% to 0.433% while retaining a $1.73\times$ speedup. Across five independently generated $N = 10$ instances, Case 4 achieves a median normalized L_1 allocation error of 8.74% and a median speedup of $1.50\times$. At $N = 30$, the reduced representation contains 16,382 proper coalitions, corresponding to an approximately 6.55×10^4 -fold reduction in coalition space. By enabling scalable and stability-aware benefit allocation while retaining participant heterogeneity and network operating constraints, the framework provides computational and economic enabling conditions for sustained participation in prosumer communities with distributed renewable energy resources. These outcomes concern the scalability and economic viability of cooperative participation.



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1. Introduction

1.1. Background and Motivation

As rooftop photovoltaic (PV) generation, behind-the-meter energy storage, and other flexible resources continue to expand in distribution systems, an increasing number of consumers are able to participate actively in local energy management. Through energy sharing, surplus renewable generation and flexible demand can be coordinated among neighboring prosumers, which may reduce grid purchases and improve the local utilization of distributed energy resources [1–3]. However, the value of such cooperation depends on more than energy balance. When participants are connected at different locations of a distribution feeder, their transactions may have different effects on nodal voltages and branch power flows [4,5]. Meanwhile, differences in net-demand profiles and storage capability also affect the flexibility that each participant can contribute. These physical characteristics make the value of cooperation dependent on both participant composition and network operating conditions.

From a sustainability perspective, the viability of such energy-sharing communities depends not only on the efficient use of distributed renewable resources but also on their ability to maintain economically acceptable participation as the number and heterogeneity of prosumers increase. A benefit-allocation mechanism that becomes computationally impractical or produces unstable participant incentives can therefore limit the long-term scalability of cooperative energy sharing. In this context, computational scalability and coalition stability are treated in this study as enabling conditions for economically sustainable prosumer participation.

When prosumers cooperate, the operating savings obtained at the system level must eventually be distributed among individual participants. Sustained cooperation requires sufficient collective benefit and an allocation that remains acceptable to individual participants. Coalitional-game models have been used for this purpose in local energy communities, where the payoff received by each participant is linked to its role in collective operation [6,7]. The problem becomes more involved when participants differ considerably in demand, renewable generation, storage flexibility, and electrical location. For example, two prosumers with similar energy consumption may contribute differently to a coalition if the PV profiles, available storage, or network impacts are different [8]. The resulting allocation should reflect the cooperative value created under the actual operating conditions of the system.

The difficulty becomes more pronounced as the number of participating prosumers increases. In a coalitional game, evaluating an allocation such as the nucleolus requires the values of a rapidly growing number of possible coalitions. For the network-constrained energy-sharing problem considered here, each coalition value is obtained by solving an optimization problem. The calculation accounts for storage schedules, grid transactions, voltage limits, and branch-flow constraints. Thus, increasing the participant population enlarges not only the coalition space but also the number of repeated physical scheduling problems that must be solved. This computational burden can quickly make direct full-space evaluation impractical. Prosumer clustering has therefore been investigated as a way of reducing the effective number of players in cooperative energy management [9]. The central difficulty is that a computationally smaller game remains useful only when the reduction retains the cooperative behavior needed for the subsequent allocation.

1.2. Literature Review

In the existing literature, cooperation among prosumers has been studied through local energy sharing, transactive energy, and coalitional game formulations. Coalitional game models in [10,11] coordinate renewable generation and distributed storage so that partici-

pants can obtain savings beyond individual operation. The low voltage community framework in [12] further incorporates power flow constraints and allocates community costs using the Shapley value and a Nash-based scheme. Shared storage is considered in [13,14], where different market and access arrangements are designed to distribute storage benefits among community members. Coalition formation has also been introduced into peer-to-peer trading. The formulations in [15,16] allow prosumers to form trading groups according to the economic gains obtained through cooperation, while related models in [17,18] consider priority-based trading and alternative profit distribution mechanisms. Other studies have extended game-based coordination to social preferences [19], community microgrids [20], and contribution-based renewable energy sharing [21]. These works demonstrate that participant heterogeneity and resource ownership can influence collective operation and the economic outcomes received by individual members.

The allocation of cooperative gains has also received considerable attention, since different solution concepts emphasize different properties of the resulting payoff. The storage market design in [13] examines cooperative and noncooperative arrangements, while [22] compares the Shapley value, the core, the nucleolus, and other allocation rules together with participant exit conditions. A broader discussion of community energy systems in [23] identifies cost allocation as an important factor in maintaining viable cooperation. Coalition stability is treated more explicitly in [24], where possible participant deviations are considered during coalition formation. Stable bilateral contracts are obtained through a coalitional assignment game in [25], and a stable coalition partition for networked microgrids is developed in [26]. Nucleolus-based allocation has also been used in cooperative energy management to limit coalition dissatisfaction [6]. Its computation requires information from a rapidly expanding set of coalitions. This difficulty becomes more pronounced when each coalition value must be obtained from an underlying energy management problem rather than from a direct analytical expression.

Several methods have been proposed to reduce the computational burden associated with cooperative allocation. Ref. [9] constructs marginal contribution profiles for prosumers and reduces the number of effective players before nucleolus estimation. Stratified random sampling is then used to assess the estimation accuracy, allowing the reported problems to increase from fewer than 15 participants to more than 100. Approximation has also been studied for other cooperative allocation rules. The Shapley value methods in [27] use sampling and prosumer clustering to support computation for communities containing up to 200 participants. Another approach is reported in [25], where a distributed negotiation mechanism addresses scalability while retaining stable bilateral contracts. Ref. [28] subdivides the original cooperative game using participant characteristics and reports substantial reductions in computation time. Its results also show that the choice of subdivision criteria affects the properties retained from the original game. These studies make clear that player reduction can extend the practical scale of cooperative energy management, while the information retained during aggregation determines the quality of the reduced representation.

Distribution network constraints have also been incorporated into local energy coordination because the physical consequences of a transaction depend on the network in which it takes place. The cooperative framework in [12] includes power flow constraints when scheduling community resources. More explicit network treatment is found in [29,30], where peer-to-peer market outcomes are coordinated with distribution system operating limits. A coalition graph game in [31] incorporates local voltage management and examines the stability of the resulting coalition structure. Network feasibility is also considered in the cooperative negawatt framework in [16], which evaluates voltage profiles and power losses associated with energy trading. Other studies consider multiple sharing regions [32]

or coordinated planning with bilateral transactions between distribution networks and microgrids [33]. Together, these studies show that electrical location, network limits, and participant flexibility can affect the physical consequences of cooperation. These factors become especially relevant when several original participants are represented collectively by a smaller number of players.

These scalable approaches differ substantially in their allocation targets, computational mechanisms, stability objectives, and information requirements. To make these distinctions explicit, Section 6.7 provides a quantitative comparison between representative existing approaches and the proposed framework using their reported computational scale and the corresponding results obtained in this study.

1.3. Research Gap and Contribution

However, several issues remain when player reduction is introduced into cooperative energy sharing with network constraints. First, the quality of a reduced game depends on which prosumers are grouped together. Existing scalable methods use marginal contribution profiles or participant characteristics to reduce the number of effective players [9,28]. These criteria can substantially reduce the coalition space, but similar participants may still behave differently when they join different coalitions. The contributions can also change when several members are aggregated because storage flexibility, net demand, and network location interact with the other participants in the coalition. A grouping result based on a single similarity measure may therefore provide a computationally efficient reduced game without fully preserving the cooperative relationships of the original system. Second, the physical meaning of coalition values needs to be retained after reduction. Network constrained energy sharing models explicitly account for voltage limits, branch flows, and other distribution system operating conditions [12,29–33]. When several original prosumers are represented by one reduced player, using an averaged or representative participant may alter the operating conditions associated with that coalition. The resulting coalition value can then differ from the value obtained when the original participants are evaluated under the same physical model. Third, coalition stability has been examined through participant deviations, stable bilateral contracts, and stable coalition partitions [24–26]. However, an additional issue arises when the allocation problem is solved after player reduction. A coalition may contain only part of a reduced group and is therefore not explicitly represented in the reduced coalition space. Although such coalitions are excluded from the reduced game, they can still arise after the group payoff is distributed back to individual prosumers. These participants may have incentives to deviate even when the allocation satisfies the stability conditions of all coalitions represented in the reduced game. The reduced space solution alone may therefore be insufficient to assess stability after participant-level payoff recovery.

To address these limitations, this paper develops a fidelity-controlled reduction framework to support scalable and sustainable cooperative energy sharing in distribution networks. The proposed method reduces the number of effective players while preserving the cooperative relationships and physical characteristics of the original system. By improving the scalability and stability of benefit allocation, the framework is intended to facilitate economically sustainable participation in larger prosumer energy communities. It further examines participant-level coalition stability after the reduced allocation is mapped back to individual prosumers. The main contributions of this paper are summarized as follows:

1. A fidelity-controlled WRT-inspired reduction framework is developed for large-scale network-constrained cooperative energy-sharing games. Each participant is characterized by a physical–economic signature incorporating temporal net demand, operational flexibility, stand-alone economics, cooperative marginal contribution, and network sensitivity. Hierarchical entropy weighting and density-based grouping are

combined with three complementary admissibility criteria that constrain feature-space dispersion, member-wise contextual game inconsistency, and aggregation-induced interaction distortion. Among the admissible partitions, the smallest reduced-player representation is selected, enabling participant reduction to be governed by cooperative-game fidelity rather than feature similarity alone.

2. A restore-before-evaluate reduced-game construction and participant-level recovery mechanism is established to connect the reduced game with the original physical system. Instead of replacing each participant group with an averaged representative, every reduced coalition is restored to its constituent participants and evaluated using the original network-constrained operating model before its characteristic value is assigned. The nucleolus is then determined in the reduced coalition space and mapped back to individual participants through contribution-guided intra-group shares. This construction preserves the grand-coalition value and retains the original physical interpretation of reduced-coalition values while avoiding exhaustive nucleolus evaluation in the full participant space.
3. A stability-certified de-clustering procedure is proposed to address split coalitions that are not explicitly represented in the reduced game. A two-stage lexicographic correction first minimizes the maximum positive excess over sampled split coalitions and then minimizes the deviation from the contribution-guided initial allocation while preserving each reduced-group payoff. The resulting allocation exactly preserves the excess of representable coalitions and maintains budget balance and individual rationality. Independent out-of-sample split-coalition testing, together with a Hoeffding-based probabilistic certificate, is further introduced to quantify stability beyond the coalitions used for correction.

2. Overall Framework of the Proposed WRT-Inspired Cooperative Allocation Method

2.1. WRT-Inspired Framework Architecture and Information Flow

The proposed method is organized around the transformation of the original participant-level cooperative game into a reduced game and the subsequent recovery of individual payoffs. As illustrated in Figure 1, the framework operates across the original participant space $\mathcal{N}$ and the reduced game space $\mathcal{G}$, while retaining the original physical operating model for coalition valuation. The reduction is therefore applied to the cooperative player representation rather than to the underlying network-constrained operating problem. The resulting information flow follows a whole–reduction–restoration structure, followed by participant-level payoff recovery and stability treatment.

The framework starts from the original participant space $\mathcal{N}$. Participant load, PV generation, energy storage characteristics, and distribution network topology are incorporated into the network constrained coalition operating model. The resulting coalition operating cost $J(S)$ is converted into the cooperative value $v(S)$, thereby defining the full-dimensional cooperative game. Information from this original game is used to construct the physics–economic signature of each participant, including its temporal net demand, operational flexibility, stand-alone economic performance, marginal contribution, and electrical network sensitivity. Hierarchical entropy weighting and DBSCAN generate candidate partitions, which are subsequently screened using feature radius, member-wise game consistency, and aggregation interaction consistency. The admissible partition with the minimum reduced player dimension defines a complete partition $\mathcal{C} = \{C_1, \dots, C_G\}$ of the original participant set $\mathcal{N}$ and the corresponding reduced-player index set $\mathcal{G} = \{1, \dots, G\}$. No original participant is discarded during this reduction: each $i \in \mathcal{N}$ belongs to exactly one cluster C_g , while participants not assigned to a multi-member density cluster are retained

as singleton clusters. Thus, the transition from $\mathcal{N}$ to $\mathcal{G}$ aggregates the cooperative game representation of the original participants rather than selecting or removing participants.

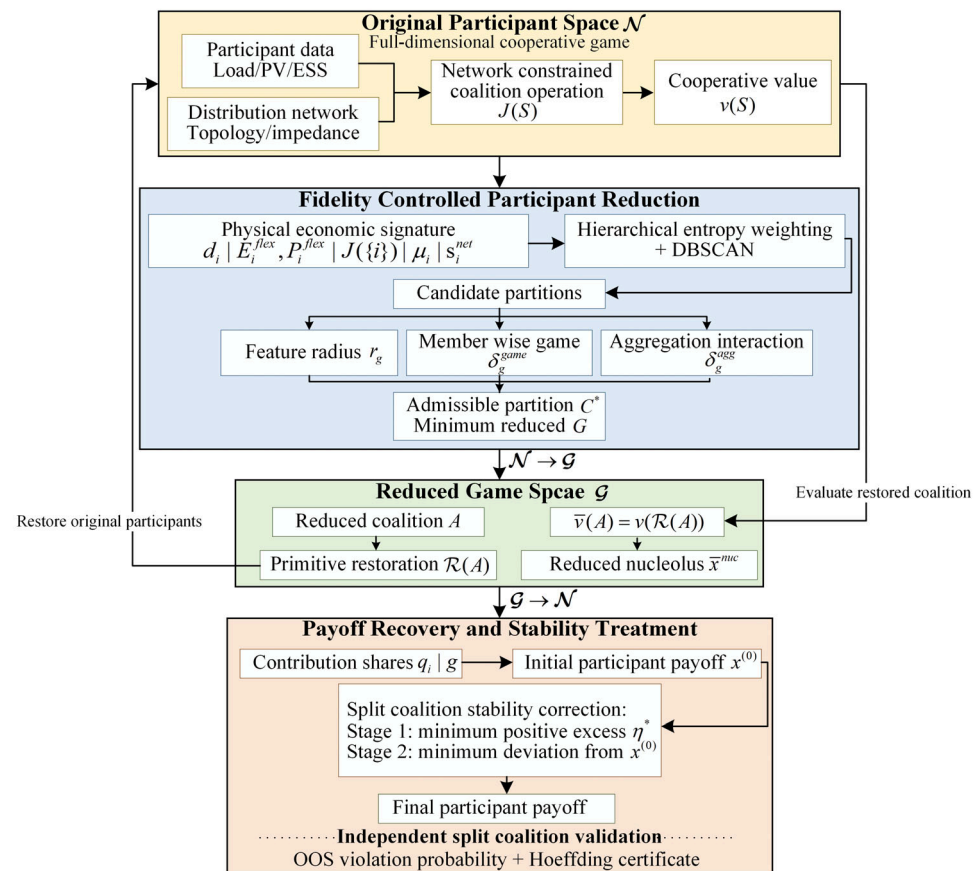


Figure 1. Architecture of the proposed WRT-inspired cooperative allocation method.

Moreover, the transition to the reduced game does not replace the participants within a cluster by an averaged or equivalent prosumer. For each reduced coalition $A \subseteq \mathcal{G}$, the primitive restoration operator $\mathcal{R}(A) = \bigcup_{g \in A} C_g$ recovers all original participants represented by the selected reduced players. The restored coalition is then returned to the original network constrained operating model for physical evaluation. Its resulting original game value is assigned to the reduced coalition through $\bar{v}(A) = v(\mathcal{R}(A))$. Hence, the reduced game compresses the combinatorial player space while preserving the original participant and network primitives used to evaluate coalition values. The reduced nucleolus $\bar{x}^{muc}$ is subsequently determined from the restored reduced characteristic function.

The cluster level nucleolus must finally be recovered in the original participant space. The retained contribution shares $q_{i|g}$ are first used to obtain an initial participant payoff $x^{(0)}$ while preserving the payoff allocated to each reduced cluster. Because the reduced game explicitly represents only coalitions formed by complete clusters, this initial inverse mapping does not directly control coalitions that split one or more clusters. A two-stage stability correction is therefore applied. The first stage minimizes the largest positive excess over the sampled split coalitions, and the second stage minimizes the deviation from the contribution-guided initial allocation while keeping the cluster totals unchanged. The resulting participant payoff is finally assessed using independently sampled split coalitions through the out-of-sample violation probability and the associated Hoeffding certificate.

2.2. Whole–Reduction–Restoration Functional Modules

The full-dimensional cooperative game provides the reference characteristic function from which both allocation and reduction information are derived. Each coalition value is determined from the original network-constrained physical scheduling problem. This preserves the dependence of cooperation benefits on participant flexibility and electrical operating limits. At the same time, the exponential number of coalitions in the original participant space creates the computational motivation for reduction.

The participant representation provides the information basis for identifying candidates that may be aggregated. Physical proximity in the feature space is evaluated using temporal net demand, operational flexibility, stand-alone economic performance, marginal contribution, and network sensitivity. The representation captures both the operating characteristics of a participant and its relevance to cooperative value formation.

Candidate partitions obtained from the weighted feature space are not accepted directly. Participants that appear similar in physical or economic characteristics may behave differently when embedded in different coalition contexts. The proposed reduction therefore introduces member-wise game consistency and aggregation interaction consistency in addition to feature compactness. The first criterion evaluates whether participants within the same cluster exhibit comparable contextual marginal contribution patterns, whereas the second evaluates whether representing them jointly as one reduced player introduces excessive nonadditive interaction. The final partition is the greatest compression that satisfies all prescribed fidelity requirements. For the selected partition, the reduced characteristic function is constructed through primitive restoration rather than representative participant approximation. The nucleolus is then solved over the reduced player set, thereby reducing the number of coalition evaluations required by the allocation problem. The reduced nucleolus determines cluster-level payoffs and therefore requires an inverse mapping to recover participant-level allocation. Contribution shares provide the initial within-cluster distribution, while the subsequent stability correction addresses split coalitions that are absent from the reduced game. This preserves the reduced cluster allocation while improving participant-level coalition stability.

2.3. Overall Computational Workflow

The proposed method is implemented sequentially from original-game characterization to participant reduction, reduced-game allocation, payoff recovery, and independent stability validation. The complete computational procedure is summarized in Algorithm 1. Candidate partitions are evaluated independently over the prescribed DBSCAN configuration set, and only partitions satisfying all three fidelity requirements are retained for final selection.

Once the admissible partition is fixed, reduced coalition values are evaluated by restoring the corresponding original participants and reusing the original network-constrained operating model. The full-space nucleolus is not required by Algorithm 1 and is used only as a small-scale benchmark in Section 6. Moreover, the validation coalitions are generated only after the final participant allocation has been obtained and therefore do not feed back into participant grouping, partition selection, or stability correction.

Algorithm 1 Overall workflow of the proposed WRT-inspired cooperative allocation method

Require: Participant/network data; $\mathcal{K}_{DB}; \tau_\phi, \tau_m, \tau_a$; sampling settings.

Ensure $C^*, \bar{x}^{nuc}, x^{SC}$, and OOS stability metrics.

1. Construct the original cooperative game using (1)–(15). ◇ Full-dimensional game
 2. Construct participant signatures ξ_i using (16)–(20)
 3. Obtain ϕ_i and $d_{i,j}$ using (21)–(29). ◇ Participant representation
 4. **for** each $\kappa \in \mathcal{K}_{DB}$ **do**
 5. Generate $\mathcal{C}^{(\kappa)}$ using (30)–(32)
 6. Evaluate $r_g^{(\kappa)}, \delta_g^{game,(\kappa)}$ and $\delta_g^{agg,(\kappa)}$ using (33)–(39).
 7. **if** all clusters satisfy (40) **then**
 8. Retain $\mathcal{C}^{(\kappa)}$.
 9. **end if**
 10. **end for**
 11. Select C^* according to (41)–(42). ◇ Fidelity-controlled reduction
 12. **for** each $A \subseteq \mathcal{G}$ **do**
 13. Restore $\mathcal{R}(A)$ and evaluate $\bar{v}(A) = v(\mathcal{R}(A))$ using (44).
 14. **end for** ◇ Primitive restoration
 15. Determine $\bar{x}^{nuc}$ using (45)–(47).
 16. Construct $q_{i|g}$ and recover $x^{(0)}$ using (43) and (48).
 17. Generate $\mathfrak{S}_{tr}$ and solve (50)–(51) for x^{SC} . ◇ Payoff recovery
 18. Generate an independent validation set after x^{SC} is fixed.
 19. Evaluate $\hat{p}_{viol}(\tau)$ and Hoeffding certificate $\diamond$ Independent validation (55)–(57).
 20. **return** $C^*, \bar{x}^{nuc}, x^{SC}$, and OOS stability metrics.
-

3. Full-Dimensional Cooperative Energy-Sharing Game

This section establishes the original network constrained cooperative energy sharing game before any participant reduction is introduced. The physical operating model is first formulated for an arbitrary coalition $S \subseteq \mathcal{N}$. The resulting operating costs are then converted into coalition savings, which define the characteristic function used for nucleolus allocation. The associated coalition enumeration burden motivates the reduced formulation developed in Section 4.

3.1. Coalition Operating Model

Let $\mathcal{N} = \{1, \dots, N\}$ and $\mathcal{T} = \{1, \dots, T\}$ denote the sets of prosumers and scheduling periods, respectively, where $N = |\mathcal{N}|$ and $T = |\mathcal{T}|$. Indices $i, j \in \mathcal{N}$ and $t \in \mathcal{T}$ denote prosumers and scheduling periods, respectively. The net demand of each prosumer provides the basic energy imbalance entering the coalition scheduling model and is defined as Equation (1).

$$d_{i,t} = P_{i,t}^L - P_{i,t}^{PV} \quad (1)$$

where $d_{i,t}$ denotes the net power demand of prosumer i at time t ; and $P_{i,t}^L, P_{i,t}^{PV}$ represent the local load demand and PV generation of prosumer i at time t respectively. When $d_{i,t} > 0$, it indicates that the local load demand exceeds the PV generation, while $d_{i,t} \leq 0$ indicates that the PV generation can cover the local demand.

For an arbitrary coalition $S \subseteq \mathcal{N}$, coordinated operation is performed over the scheduling horizon to minimize its total grid transaction and storage degradation costs. The resulting coalition operating cost $J(S)$ is expressed by Equation (2).

$$J(S) = \min_{\mathbf{y}_S} \sum_{t \in \mathcal{T}} \left[\lambda_t^b P_{S,t}^{gbuy} - \lambda_t^s P_{S,t}^{gsell} + \sum_{i \in S} c_i^{\deg} (P_{i,t}^{ch} + P_{i,t}^{dis}) \right] \Delta t \quad (2)$$

where λ_t^b and λ_t^s denote the grid buying and selling price respectively; $P_{S,t}^{gbuy}$ and $P_{S,t}^{gsell}$ represent the grid import and export power of coalition S at time t respectively; $c_i^{\deg}$ is the coefficient of battery degradation cost; and $P_{i,t}^{ch}, P_{i,t}^{dis}$ are charging and discharging power respectively; and $\mathbf{y}_S$ collects the operational decision variables of coalition S .

In the present formulation, the load demand $P_{i,t}^L$ and PV generation $P_{i,t}^{PV}$ are treated as exogenous operating profiles and are not directly controlled by the distribution utility. The coordinated operating decisions concern the charging and discharging schedules of the storage devices owned by participating prosumers. Therefore, practical implementation requires the participating prosumers to provide the necessary operating information and permit coordinated scheduling of their storage resources. Such coordination may be carried out by an authorized community operator or aggregator and does not require the distribution utility to have direct device-level control of the PV systems or behind-the-meter (BTM) storage. The distribution network model is used to ensure that the resulting schedules remain within the prescribed voltage and branch flow limits. Detailed communication and control architectures are outside the scope of this study.

The minimization in Equation (2) is subject to the participant-level storage constraints and distribution network operating constraints in Equations (3)–(10). To ensure overall supply–demand balance within the coalition:

$$P_{S,t}^{gbuy} - P_{S,t}^{gsell} = \sum_{i \in S} (d_{i,t} + P_{i,t}^{ch} - P_{i,t}^{dis}) \quad (3)$$

Storage dynamics and the limit of storage energy and power:

$$E_{i,t+1} = E_{i,t} + \eta_i^{ch} P_{i,t}^{ch} \Delta t - \frac{P_{i,t}^{dis} \Delta t}{\eta_i^{dis}} \quad (4)$$

$$\begin{aligned} \underline{E}_i &\leq E_{i,t} \leq \bar{E}_i, \\ 0 &\leq P_{i,t}^{ch} \leq \bar{P}_i^{ch}, \quad 0 \leq P_{i,t}^{dis} \leq \bar{P}_i^{dis} \end{aligned} \quad (5)$$

where $\underline{E}_i$ and $\bar{E}_i$ are the minimum and maximum stored energy of prosumer i respectively; $E_{i,t}$ denote the stored energy state of prosumer i at time t ; η_i^{ch} and η_i^{dis} are the charging and discharging efficiency of prosumer i respectively; and $\bar{P}_i^{ch}, \bar{P}_i^{dis}$ denote the maximum charging and discharging power of prosumer i respectively.

Prohibition of simultaneous charging and discharging of the same energy storage device during the same time period:

$$0 \leq P_{i,t}^{ch} \leq u_{i,t} \bar{P}_i^{ch}, \quad 0 \leq P_{i,t}^{dis} \leq (1 - u_{i,t}) \bar{P}_i^{dis} \quad (6)$$

where $u_{i,t} \in \{0,1\}$ is a binary variable for charging state.

To prevent the optimizer from artificially emptying the energy storage at the endpoint, define $E_{i,T+1} = E_{i,1}$. The participant-level schedules are next mapped to their corresponding

distribution network buses. Let b_i denote the connection bus of prosumer i . The resulting active and reactive power withdrawals at each bus are obtained as

$$\begin{aligned} p_{n,t} &= \sum_{i \in S, b_i=n} (d_{i,t} + P_{i,t}^{ch} - P_{i,t}^{dis}), \\ q_{n,t} &= \sum_{i \in S, b_i=n} q_{i,t}^{net} \end{aligned} \quad (7)$$

where b_i denotes the connection bus of prosumer i ; $p_{n,t}$ and $q_{n,t}$ are the nodal active and reactive power withdrawals at distribution network bus n at time t respectively; and $q_{i,t}^{net}$ is the net reactive demand of prosumer i at time t .

A linearized radial distribution power flow representation, commonly adopted in distribution-level flexibility and energy trading models [3,4], is used to incorporate network operating constraints. Under the positive withdrawal convention, the squared nodal voltage is expressed as Equation (8). The branch power flow relation is expressed in Equation (9), and the network operating limits are given in Equation (10).

$$u_t = u_0 \mathbf{1} - R p_t - X q_t \quad (8)$$

$$f_t = H p_t \quad (9)$$

$$\underline{u} \leq u_t \leq \bar{u}, \quad -\bar{f} \leq f_t \leq \bar{f} \quad (10)$$

where u_t is the squared voltage vector at time t , and $\underline{u}, \bar{u}$ denote the voltage limits; u_0 denotes the square of slack bus voltage; $\mathbf{1}$ denotes an all-ones vector of compatible dimension; p_t and q_t are the nodal active and reactive withdrawal vectors at time t respectively; R and X represent the sensitivity matrix between active/reactive power and voltage respectively; f_t denotes the active branch-flow vector; and $\bar{f}$ is the vector of active branch-flow limits. Reactive-power effects are retained in the voltage relation, whereas the branch-loading constraint adopts a linear active-power flow proxy to preserve computational tractability. H represents the sensitivity matrix between node power withdrawal and branch power flow.

3.2. Conventional Cooperative-Game Allocation

The coalition operating costs are converted into a transferable utility savings game by measuring the reduction in total stand-alone operating cost obtained through cooperation:

$$v(S) = \sum_{i \in S} J(\{i\}) - J(S), \quad v(\emptyset) = 0, \quad v(\{i\}) = 0 \quad (11)$$

where $v(S)$ is the coalition collaboration benefits/cost savings. Under this definition, $v(\{i\}) = 0$ because a singleton coalition has no cooperative savings.

A feasible allocation of the grand coalition benefit must satisfy efficiency and individual rationality:

$$\sum_{i \in \mathcal{N}} x_i = v(\mathcal{N}), \quad x_i \geq v(\{i\}) = 0 \quad (12)$$

where x_i is the payoff allocated to prosumer i . The set of payoff vectors satisfying (12) defines the imputation set $\mathcal{I}(v)$. To measure the incentive of any coalition to deviate from a candidate allocation $\mathbf{x}$, its excess is defined as Equation (13).

$$e(S, \mathbf{x}) = v(S) - \sum_{i \in S} x_i \quad (13)$$

where $\mathbf{x}$ denotes the payoff vector of prosumers, and $e(S, \mathbf{x})$ denotes the excess or dissatisfaction of coalition. When $e(S, \mathbf{x}) > 0$, it means the coalition can gain more benefits by withdrawing on its own.

A payoff belongs to the core if no proper coalition has a positive excess $e(S, \mathbf{x}) \leq 0$, $\forall S \subsetneq \mathcal{N}$.

A larger positive excess indicates greater dissatisfaction because coalition S can obtain a higher payoff by acting independently. The nucleolus has been widely adopted in cooperative energy management to obtain a unique payoff allocation by lexicographically minimizing coalition dissatisfaction [6,9]. For the full-dimensional energy sharing game considered here, it is expressed as

$$x^{nuc} = \arg \text{lex min sort}_{x \in \mathcal{I}(v)} \{e(S, x) : \emptyset \neq S \subsetneq \mathcal{N}\} \quad (14)$$

where $\mathcal{I}(v)$ denotes the imputation set, and x^{nuc} is the full-game nucleolus.

3.3. Computational Scalability Bottleneck

The computational burden of nucleolus-based energy allocation grows rapidly with the number of participants because coalition values must be evaluated over an exponentially expanding coalition space, which has also been identified as a key scalability limitation in prosumer cooperative energy management [9]. Exact nucleolus evaluation requires the characteristic values of all relevant coalitions in the original participant space. The number of nonempty proper coalitions grows exponentially with N , given by Equation (15).

$$N_{coal}^{full} = 2^N - 2 \quad (15)$$

where N_{coal}^{full} denotes the number of full-game proper coalitions.

Each coalition value additionally requires the solution of the network-constrained operating problem in Equations (2)–(10). The computational burden therefore arises not only from the number of coalition constraints in the nucleolus problem, but also from the repeated physical optimization required to construct the characteristic function. This motivates reducing the effective player dimension while retaining the original physical model for coalition evaluation.

4. Proposed WRT-Inspired Physics–Economic Game Reduction

The reduction is constructed in two layers. Participant characteristics are first used to generate candidate partitions with reduced dimensionality. Because similarity in physical or economic features does not necessarily imply similarity in cooperative behavior, these candidates are subsequently screened using game-theoretic consistency criteria. Only partitions satisfying the prescribed feature and game fidelity requirements are eligible for reduced game construction.

4.1. Participant Physical–Economic Characterization

Participant aggregation should distinguish prosumers according to the factors that determine both their operational role and their contribution to cooperation. Four participant-specific components are first considered before the electrical network effect is introduced in Section 4.2: temporal net demand, storage flexibility, stand-alone operating cost, and cooperative marginal contribution. The net demand trajectory of prosumer i can be expressed as $d_i = [d_{i,1}, d_{i,2}, \dots, d_{i,T}]^T$. Its available energy and bidirectional power flexibility are characterized by Equation (16).

$$E_i^{flex} = \bar{E}_i - \underline{E}_i, P_i^{flex} = \bar{P}_i^{ch} + \bar{P}_i^{dis} \quad (16)$$

where E_i^{flex}, P_i^{flex} are the available flexible energy and bidirectional power-flexibility span of prosumer i respectively.

The stand-alone operating cost $J(\{i\})$ captures the economic position of the participant before cooperation. In addition, its leave-one-out marginal contribution is defined

as $\mu_i = v(\mathcal{N}) - v(\mathcal{N} \setminus \{i\})$, which measures the change in grand coalition savings when participant i is removed.

4.2. Network-Sensitivity Embedding

Participants with similar demand and flexibility may nevertheless have different effects on the distribution network when connected at different electrical locations. Network sensitivity is therefore incorporated into the participant representation so that aggregation does not ignore differences in voltage and branch-flow impacts.

$$M_{\ell n} = \begin{cases} 1, & \ell \in \mathcal{P}(0, n), \\ 0, & \text{otherwise} \end{cases}, \mathbf{M} = [M_{\ell n}] \quad (17)$$

where $\mathbf{M} = [M_{\ell n}]$ denotes the path matrix of bus to branch; and $\ell \in \mathcal{P}(0, n)$ represents the branch ℓ belongs to the radial path from slack bus to bus n .

Consistent with the topology-based linearized radial network representations used in recent distribution system studies [3,4], the sensitivity matrices in this work are constructed directly from the feeder path structure and branch impedances. The derivation of the topology-based network-sensitivity matrices in Equation (18) is provided in Appendix A.

$$\begin{aligned} \mathbf{R} &= 2\mathbf{M}^T \text{diag}(\mathbf{r}) \mathbf{M}, \\ \mathbf{X} &= 2\mathbf{M}^T \text{diag}(\mathbf{x}^{line}) \mathbf{M}, \\ \mathbf{H} &= \mathbf{M} \end{aligned} \quad (18)$$

where $\mathbf{r}, \mathbf{x}^{line}$ are the resistance and reactance vectors of branch respectively.

The participant electrical network fingerprint s_i^{net} is shown in Equation (19):

$$s_i^{net} = [\mathbf{R}_{:,b_i}^T, \mathbf{X}_{:,b_i}^T, \mathbf{H}_{:,b_i}^T]^T \quad (19)$$

The raw physical-economic signature ξ_i includes the operation, economics, cooperative marginal contribution, and electrical-network impact, as shown in Equation (20):

$$\xi_i = [d_i^T, E_i^{flex}, P_i^{flex}, J(\{i\}), \mu_i, (s_i^{net})^T]^T \quad (20)$$

4.3. Entropy-Weighted Participant Grouping

The participant signature is divided into five semantically distinct feature blocks. Let $\mathcal{B} = \{dem, flex, eco, game, net\}$ denote the block set corresponding to net-demand trajectory, operational flexibility, stand-alone economics, cooperative-game contribution, and network sensitivity, respectively. For each $b \in \mathcal{B}$, $\mathcal{K}_b$ denotes the coordinate-index set associated with block b , and $K_b = |\mathcal{K}_b|$ is its dimension. The blocks form a disjoint partition of the K feature coordinates.

To address the issue of inconsistent units and numerical scales among features, the feature normalization $z_{i,k}$ is introduced, as shown in Equation (21):

$$z_{i,k} = \begin{cases} \frac{\xi_{i,k} - \xi_k^{\min}}{\xi_k^{\max} - \xi_k^{\min}}, & \xi_k^{\max} > \xi_k^{\min}, \\ 0, & \xi_k^{\max} = \xi_k^{\min} \end{cases} \quad (21)$$

where $\xi_k^{\max}, \xi_k^{\min}$ are the maximum and minimum values of feature k .

The feature information proportion $\pi_{i,k}$ is shown as Equation (22):

$$\pi_{i,k} = \frac{z_{i,k} + \varepsilon\pi}{\sum_{j \in \mathcal{N}} (z_{j,k} + \varepsilon\pi)} \quad (22)$$

where $\varepsilon_\pi > 0$ is a small dimensionless numerical regularizer used only to avoid the logarithm of zero in the entropy calculation.

Entropy-based weighting has been used in recent energy system studies to assign larger weights to features carrying greater discriminatory information [34]. Following this principle, the normalized Shannon feature entropy h_k is shown as Equation (23). If all prosumers are almost the same on feature k , which indicates $h_k \rightarrow 1$; this feature has almost no clustering information.

$$h_k = -\frac{1}{\ln N} \sum_{i \in \mathcal{N}} \pi_{i,k} \ln \pi_{i,k} \quad (23)$$

The discrimination information carried by feature k is therefore $\delta_k = 1 - h_k$. A larger δ_k indicates greater variation among participants and hence stronger discriminatory information for grouping. Because the five feature blocks have different dimensions, directly normalizing all coordinate-wise entropy weights would allow a high-dimensional block, particularly the demand trajectory, to dominate the distance measure. A hierarchical weighting procedure is therefore adopted. Within each feature block b , the coordinate weights are first normalized as Equation (25), and the block-level discrimination measures determine the relative weight assigned across feature blocks, as shown in Equation (26). The entropy-derived feature weight ω_k is shown as Equation (27). To make sure each feature coordinate belongs to exactly one feature block, the following is defined: $\bigcup_{b \in \mathcal{B}} \mathcal{K}_b = \{1, \dots, K\}$, $\mathcal{K}_b \cap \mathcal{K}_c = \emptyset$ ($b \neq c$).

$$\tilde{\omega}_{b,k} = \begin{cases} \frac{\delta_k}{\sum_{k' \in \mathcal{K}_b} \delta_{k'}}, & \sum_{k' \in \mathcal{K}_b} \delta_{k'} > 0, \\ \frac{1}{K_b}, & \text{otherwise} \end{cases} \quad k \in \mathcal{K}_b \quad (24)$$

$$D_b = \frac{1}{K_b} \sum_{k \in \mathcal{K}_b} \delta_k \quad (25)$$

$$\alpha_b = \begin{cases} \frac{D_b}{\sum_{b' \in \mathcal{B}} D_{b'}}, & \sum_{b' \in \mathcal{B}} D_{b'} > 0, \\ \frac{1}{|\mathcal{B}|}, & \text{otherwise} \end{cases} \quad (26)$$

$$\omega_k = \alpha_b \tilde{\omega}_{b,k}, \quad k \in \mathcal{K}_b \quad (27)$$

where $\delta_k = 1 - h_k$ denotes the feature differentiation information; α_b is the between-block information weight assigned to feature block b ; $\tilde{\omega}_{b,k}$ denotes the entropy-derived weight of coordinate k normalized within feature block b ; and D_b denotes the average discrimination information of feature block b .

$$\mathbf{W} = \text{diag}(\omega_1, \dots, \omega_K), \quad \phi_i = \mathbf{W}^{1/2} \mathbf{z}_i \quad (28)$$

where $\mathbf{W}$ denotes the feature weight matrix; ϕ_i is the weighted participant representation of prosumer i ; and $\mathbf{z}_i$ is the normalization vector of prosumer i .

$$d_{i,j} = \|\phi_i - \phi_j\|_2 = \sqrt{\sum_{k=1}^K \omega_k (z_{i,k} - z_{j,k})^2} \quad (29)$$

where $d_{i,j}$ denotes the physics-economic distance between prosumer i and j .

To prevent high-dimensional feature domains from dominating the clustering distance merely because they contain more coordinates, the entropy weighting is implemented hierarchically. The discrimination information is first normalized within each feature block, while the block-level weight is determined by the average rather than the sum of

its coordinate-wise discrimination information. Consequently, the total contribution of a feature block is governed by its information content rather than its dimensionality.

Clustering-based participant reduction has been shown to provide an effective means of alleviating the computational burden of nucleolus-based prosumer energy management [9]. In this work, DBSCAN is employed to generate candidate partitions from the weighted physics–economic distance because the reduced player dimension does not need to be specified a priori.

Let $\mathcal{K}_{DB}$ denote the set of candidate DBSCAN configurations, where configuration $\kappa \in \mathcal{K}_{DB}$ denotes the candidate-configuration index. $\theta^{(\kappa)} = (\varepsilon^{(\kappa)}, N_{min}^{(\kappa)})$ denotes the DBSCAN parameter pair associated with configuration κ . The DBSCAN neighborhood $\mathcal{N}_{\varepsilon^{(\kappa)}}^{(\kappa)}(i)$ can be expressed as Equation (30):

$$\mathcal{N}_{\varepsilon^{(\kappa)}}^{(\kappa)}(i) = \left\{ j \in \mathcal{N} : d_{ij} \leq \varepsilon^{(\kappa)} \right\}, \left| \mathcal{N}_{\varepsilon^{(\kappa)}}^{(\kappa)}(i) \right| \geq \mathcal{N}_{min}^{(\kappa)} \quad (30)$$

where $\varepsilon^{(\kappa)}$ is the DBSCAN neighborhood radius, and $\mathcal{N}_{min}^{(\kappa)}$ denotes the minimum neighborhood size.

DBSCAN may classify isolated participants as noise rather than assigning them to a density-based cluster. Since every prosumer must remain represented in the cooperative game, each noise participant is retained as a singleton cluster. The resulting candidate partition is therefore shown in Equation (31):

$$\mathcal{C}^{(\kappa)} = \mathcal{C}_{dens}^{(\kappa)} \cup \left\{ \{i\} : i \in \mathcal{O}^{(\kappa)} \right\} = \left\{ C_1^{(\kappa)}, C_2^{(\kappa)}, \dots, C_{G^{(\kappa)}}^{(\kappa)} \right\} \quad (31)$$

$$G^{(\kappa)} = \left| \mathcal{C}^{(\kappa)} \right|, \kappa \in \mathcal{K}_{DB}$$

$$\bigcup_{g=1}^{G^{(\kappa)}} C_g^{(\kappa)} = \mathcal{N}, C_g^{(\kappa)} \cap C_h^{(\kappa)} = \emptyset, g \neq h \quad (32)$$

where $\mathcal{C}_{dens}^{(\kappa)}$ denotes the density-generated clusters; $\mathcal{O}^{(\kappa)}$ is the DBSCAN noise/outlier set under candidate configuration κ ; $C_g^{(\kappa)}$ is the g -th participant cluster; $\mathcal{C}^{(\kappa)}$ is the κ -th candidate partition; and $G^{(\kappa)}$ is the number of final clusters under configuration κ .

The physical, economic, and network heterogeneity are quantified among participants within a cluster:

$$\bar{\phi}_g^{(\kappa)} = \frac{1}{\left| C_g^{(\kappa)} \right|} \sum_{i \in C_g^{(\kappa)}} \phi_i \quad (33)$$

$$r_g^{(\kappa)} = \max_{i \in C_g^{(\kappa)}} \left\| \phi_i - \bar{\phi}_g^{(\kappa)} \right\|_2 \quad (34)$$

where $\bar{\phi}_g^{(\kappa)}$ is the cluster feature centroid of cluster g ; $\left| C_g^{(\kappa)} \right|$ denotes the cardinality of cluster g ; and $r_g^{(\kappa)}$ is the physical–economic radius of cluster g .

4.4. Game-Theoretic Cluster Consistency

Feature space compactness ensures that participants within a candidate cluster have similar observable physical and economic characteristics. It does not, however, guarantee that they contribute similarly when embedded in different cooperative coalitions. A second screening layer is therefore introduced to evaluate each candidate partition directly from cooperative game behavior.

Define $\mathcal{G}^{(\kappa)} = \{1, \dots, G^{(\kappa)}\}$ as the cluster-level player set.

$$\mathcal{R}^{(\kappa)}(A) = \bigcup_{g \in A} C_g^{(\kappa)}, A \subseteq \mathcal{G}^{(\kappa)} \quad (35)$$

where A is the cluster-level coalition, and $\mathcal{R}^{(\kappa)}(A)$ is the original participant primitive-restoration operator.

For each candidate partition $C^{(\kappa)}$, the contextual contribution of a participant is evaluated over a calibration set of cluster-level coalition contexts. Specifically, let $\mathcal{A}_g^{cal,(\kappa)} \subseteq 2^{\mathcal{G}^{(\kappa)} \setminus \{g\}}$ denote the calibration-context set associated with cluster g . To avoid concentrating the calibration on coalition sizes that are combinatorially dominant, the candidate contexts are stratified according to coalition cardinality. For a given size s , the corresponding context stratum is $\mathcal{A}_{g,s}^{(\kappa)} = \{A \subseteq \mathcal{G}^{(\kappa)} \setminus \{g\} : |A| = s\}$. When the total number of admissible contexts is sufficiently small, all contexts are included in $\mathcal{A}_g^{cal,(\kappa)}$. Otherwise, a prespecified number of contexts is sampled approximately evenly across the nonempty cardinality strata, with uniform sampling within each stratum. The calibration set is fixed before evaluating the game-consistency index and is not adjusted according to the subsequently obtained nucleolus or allocation accuracy.

Originally, μ_i only considered the grand coalition, but $m_i(A)$, which indicates the contextual marginal contribution of prosumer i , can examine whether participants have similar contributions across multiple cooperative contexts, as shown in Equation (36). Equation (38) addresses the question of whether similarity in load/network characteristics implies similarity in cooperative-game behavior.

$$m_i^{(\kappa)}(A) = v(\mathcal{R}^{(\kappa)}(A) \cup \{i\}) - v(\mathcal{R}^{(\kappa)}(A)), i \in C_g^{(\kappa)}, g \notin A \quad (36)$$

$$\bar{m}_g^{(\kappa)}(A) = \frac{1}{|C_g^{(\kappa)}|} \sum_{i \in C_g^{(\kappa)}} m_i^{(\kappa)}(A) \quad (37)$$

$$\delta_g^{game,(\kappa)} = \max_{A \in \mathcal{A}_g^{cal,(\kappa)}} \frac{\sqrt{\frac{1}{|C_g^{(\kappa)}|} \sum_{i \in C_g^{(\kappa)}} [m_i^{(\kappa)}(A) - \bar{m}_g^{(\kappa)}(A)]^2}}{|\bar{m}_g^{(\kappa)}(A)| + \varepsilon_m} \quad (38)$$

where $\bar{m}_g^{(\kappa)}(A)$ denotes the mean contextual marginal contribution of cluster g ; $\mathcal{A}_g^{cal,(\kappa)}$ is the game-consistency calibration coalition-context set; $\delta_g^{game,(\kappa)}$ represents the game-theoretic heterogeneity of cluster g ; and $\varepsilon_m > 0$ is a small regularizer expressed on the same payoff scale as $m_i^{(\kappa)}(A)$, preventing numerical instability when $|\bar{m}_g^{(\kappa)}(A)|$ is close to zero.

The measure in Equation (38) tests whether participants assigned to the same cluster exhibit similar marginal contribution patterns across the calibration contexts. This member-wise consistency is necessary but not sufficient for aggregation. Even if individual marginal contributions are similar, the joint contribution of all members may differ from their additive counterpart because of network coupling, shared operating constraints, or complementary flexibility. To capture this additional source of reduction error, the aggregation interaction index is defined as

$$\delta_g^{agg,(\kappa)} = \max_{A \in \mathcal{A}_g^{cal,(\kappa)}} \frac{|v(\mathcal{R}^{(\kappa)}(A) \cup C_g^{(\kappa)}) - v(\mathcal{R}^{(\kappa)}(A)) - \sum_{i \in C_g^{(\kappa)}} m_i^{(\kappa)}(A)|}{|v(\mathcal{N})| + \varepsilon_v} \quad (39)$$

where $\varepsilon_v > 0$ is a small numerical constant. The first two terms in the numerator give the contextual marginal value of the restored cluster $C_g^{(\kappa)}$ as a whole, whereas the summation gives the additive counterpart obtained from the member-wise contextual marginal contributions defined in Equation (36) under the same restored context. Their difference therefore measures the non-additive interaction hidden by representing the cluster as a

single reduced player. A smaller $\delta_g^{agg,(\kappa)}$ indicates greater aggregation consistency. For a singleton cluster, $\delta_g^{agg,(\kappa)} = 0$.

A cluster must satisfy both physical-economic/network homogeneity and cooperative game homogeneity simultaneously, as shown in Equation (40). Accordingly, a candidate partition is considered admissible only when every cluster simultaneously satisfies the feature-space compactness, member-wise game-consistency, and aggregation-interaction requirements in Equation (40). The tolerances τ_ϕ , τ_m , and τ_a control the allowable feature-space dispersion, contextual marginal-contribution heterogeneity, and aggregation-induced interaction distortion, respectively.

$$r_g^{(\kappa)} \leq \tau_\phi, \delta_g^{game,(\kappa)} \leq \tau_m, \delta_g^{agg,(\kappa)} \leq \tau_a, \forall g \in \mathcal{G}^{(\kappa)} \quad (40)$$

where τ_ϕ , τ_m and τ_a denote the prescribed feature-space, member-wise game-consistency, and aggregation-interaction tolerances, respectively. These thresholds are fixed before reduced-game construction and are not optimized according to the resulting nucleolus or allocation accuracy. Their effects are examined separately through sensitivity analysis in Section 6.6 rather than used to tune the evaluation results.

Among the admissible candidates, the partition with the smallest number of clusters is selected according to Equation (41). Hence, the reduction stage seeks the greatest compression permitted by the prescribed fidelity tolerances rather than minimizing a weighted combination of compression and approximation metrics. The full-space nucleolus is not used in either candidate screening or partition selection.

$$\begin{aligned} \kappa^* = & \underset{\kappa \in \mathcal{K}_{DB}}{\operatorname{argmin}} G^{(\kappa)} \\ \text{s.t.} \quad & \max_{g=1, \dots, G^{(\kappa)}} r_g^{(\kappa)} \leq \tau_\phi, \\ & \max_{g=1, \dots, G^{(\kappa)}} \delta_g^{game,(\kappa)} \leq \tau_m \\ & \max_{g=1, \dots, G^{(\kappa)}} \delta_g^{agg,(\kappa)} \leq \tau_a \end{aligned} \quad (41)$$

The two game-related criteria in Equation (40) serve different purposes. The member-wise criterion $\delta_g^{game,(\kappa)}$ evaluates whether participants grouped into the same cluster exhibit consistent marginal-contribution behavior across restored contexts. In contrast, $\delta_g^{agg,(\kappa)}$ evaluates whether representing these participants collectively as a single reduced player introduces excessive non-additive interaction. The latter is required because similarity among individual participants does not by itself ensure that their aggregate can be represented consistently in the reduced cooperative game.

$$\mathcal{C}^* = \mathcal{C}^{(\kappa^*)} = \{C_1^*, \dots, C_G^*\}, G = G^{(\kappa^*)} \quad (42)$$

For notational simplicity, the superscript $(\cdot)^*$ is omitted hereafter. Accordingly, C_g , G , $\mathcal{G}$ and $\mathcal{R}(A) = \bigcup_{g \in A} C_g$ refer to the selected admissible partition, its reduced player set, and the corresponding restoration mapping.

4.5. Whole-Reduction Mapping and Primitive Restoration

The participant contribution mass is defined as $a_i = \max\{\mu_i, 0\} + \varepsilon_\mu$, where $\varepsilon_\mu > 0$ is a small positive regularizer ensuring $a_i > 0$. The truncation is used only to construct a nonnegative contribution mass for inverse payoff mapping; the signed marginal contribution μ_i itself is retained in the participant signature and game-consistency analysis. The normalized participant mass is then defined as $p_i = \frac{a_i}{\sum_{j \in N} a_j}$ and cluster-level contribution

mass $P_g = \sum_{i \in C_g} p_i$, and the conditional intra-cluster contribution share $q_{i|g}$ is shown as Equation (43).

$$q_{i|g} = \frac{p_i}{P_g}, i \in C_g \quad (43)$$

The retained pair $(P_g, q_{i|g})$ reconstructs the original participant contribution mass through $p_i = P_g q_{i|g}$. Its associated Shannon information decomposition is provided in Appendix B.

The contribution shares above retain the information required for the later inverse payoff mapping. They do not determine the characteristic value of a reduced coalition. Coalition valuation instead follows a restore-before-evaluation procedure so that participant aggregation does not alter the underlying physical optimization. For any $A \subseteq \mathcal{G}$, the reduced coalition is restored to the original participant set $\mathcal{R}(A)$:

$$\bar{v}(A) = v(\mathcal{R}(A)) \quad (44)$$

where $\bar{v}(A)$ denotes the reduced game characteristic value.

5. Proposed Stability-Certified Reduced-Game Nucleolus and Payoff Recovery

Section 4 determines which participants can be represented jointly and constructs the corresponding reduced characteristic function. This section addresses the remaining allocation problem. The nucleolus is first obtained in the reduced player space and mapped back to individual participants. Because the reduced game explicitly represents only coalitions formed by complete participant clusters, a subsequent correction and independent validation procedure are introduced to address coalitions that split one or more clusters.

5.1. Reduced Cooperative Game and Nucleolus

Reduction does not change the total distributable benefit of the entire cooperative system. By construction, the grand-coalition value is preserved because $\mathcal{R}(\mathcal{G}) = \mathcal{N}$ and the proof is in Appendix C.

$$\sum_{g \in \mathcal{G}} \bar{x}_g = \bar{v}(\mathcal{G}), \bar{x}_g \geq \bar{v}(\{g\}), \forall g \quad (45)$$

where $\bar{x}_g$ denotes the payoff of cluster g .

$$\bar{e}(A, \bar{x}) = \bar{v}(A) - \sum_{g \in A} \bar{x}_g \quad (46)$$

where $\bar{x}$ denotes the cluster payoff vector; $\bar{e}(A, \bar{x})$ is the reduced coalition excess.

The reduced-game nucleolus $\bar{x}^{nuc}$ is shown as Equation (47), and the actual sequential LP is in Appendix D.

$$\bar{x}^{nuc} = \arg \min_{\bar{x} \in \bar{\mathcal{I}}(\bar{v})} \text{lex sort}_{\downarrow} \{ \bar{e}(A, \bar{x}) : \emptyset \neq A \subsetneq \mathcal{G} \} \quad (47)$$

The set of cluster-level payoff vectors satisfying Equation (45) is denoted by $\bar{\mathcal{I}}(\bar{v})$, i.e., the reduced-game imputation set.

5.2. Inverse Mapping for Participant-Level Payoff Recovery

Before inverse mapping, the reduced nucleolus satisfies $\bar{x}_g^{nuc} \geq 0$ for every cluster under the reduced imputation constraints. Hence, each cluster payoff can be decomposed into individually rational, nonnegative participant-level payoffs.

The initial inverse-mapped payoff $x_i^{(0)}$ is shown as Equation (48). This is the first inverse mapping from reduced space to original participant space, and it is not the final allocation.

$$x_i^{(0)} = q_{i|g} \bar{x}_g^{nuc}, i \in C_g \quad (48)$$

Since $\sum_{i \in C_g} q_{i|g} = 1$, the mapping in (48) automatically preserves the cluster-level payoff, i.e., $\sum_{i \in C_g} x_i^{(0)} = \bar{x}_g^{nuc}$.

5.3. Stability-Corrected De-Clustering

The inverse mapping in Equation (48) preserves the payoff assigned to each reduced cluster. It therefore preserves the total payoff of every coalition that can be expressed as a union of complete clusters. The reduced game, however, does not explicitly contain coalitions that include only part of one or more clusters. These omitted coalitions are referred to as split coalitions and are collected in Equation (49).

$$\mathfrak{S}_{split} = \{S \subseteq \mathcal{N} : \exists g, 0 < |S \cap C_g| < |C_g|\} \quad (49)$$

where $\mathfrak{S}_{split}$ is the split-coalition family.

Minimize the most severe dissatisfaction among sampled split coalitions, under the condition that the cluster-level nucleolus remains completely unchanged. Let $\mathfrak{S}_{tr} \subset \mathfrak{S}_{split}$ denote the split coalition training set generated from the prespecified stratified sampling distribution described in Section 5.4.

$$\begin{aligned} \eta^* &= \min_{x, \eta} \eta \\ \text{s.t.} \quad & v(S) - \sum_{i \in S} x_i \leq \eta, \forall S \in \mathfrak{S}_{tr}, \\ & \sum_{i \in C_g} x_i = \bar{x}_g^{nuc}, \forall g \in \mathcal{G}, \\ & x_i \geq 0, \forall i \in \mathcal{N}, \\ & \eta \geq 0 \end{aligned} \quad (50)$$

where η is a nonnegative upper bound on training split-coalition positive excess, and $\eta^* = 0$ indicates that no training split coalition has a positive excess.

$$\begin{aligned} (x^{SC}, s^*) &= \underset{\tilde{x}, s}{\operatorname{argmin}} \sum_{i \in \mathcal{N}} s_i \\ \text{s.t.} \quad & -s_i \leq \tilde{x}_i - x_i^{(0)} \leq s_i, \forall i, \\ & v(S) - \sum_{i \in S} \tilde{x}_i \leq \eta^*, \forall S \in \mathfrak{S}_{tr}, \\ & \sum_{i \in C_g} \tilde{x}_i = \bar{x}_g^{nuc}, \forall g \in \mathcal{G}, \\ & \tilde{x}_i \geq 0, s_i \geq 0 \end{aligned} \quad (51)$$

where x^{SC} is the stability-corrected final payoff, and the optimal value of the decision variable $\tilde{x}$ is the Stage-2 participant-payoff decision vector; s_i is the absolute-deviation auxiliary variable, $s = [s_i]_{i \in \mathcal{N}}$.

Consequently, the final allocation satisfies $e(S, x^{SC}) \leq \eta^*$ for every training split coalition.

The two objectives are treated lexicographically rather than through a weighted-sum scalarization. Equation (50) first determines the minimum achievable positive-excess violation level η^* . With this stability level fixed, Equation (51) minimizes the L_1 -deviation from the contribution-guided initial allocation. This sequential construction avoids introducing an additional subjective trade-off coefficient between stability and allocation fidelity.

Theorem 1. *Exact preservation of representable-coalition excess.*

For all, $A \subseteq \mathcal{G}$, the stability-corrected allocation exactly preserves the excess of the corresponding reduced coalition, as shown in Equation (52). This property establishes exact excess equivalence for representable coalitions; it does not by itself imply that their excess is nonpositive. The complete proof is in Appendix C.

$$e(\mathcal{R}(A), x^{SC}) = \bar{e}(A, \bar{x}^{mc}), \forall A \subseteq \mathcal{G} \quad (52)$$

The final allocation maintains both budget balance and individual rationality:

$$\begin{aligned} \sum_{i \in \mathcal{N}} x_i^{SC} &= v(\mathcal{N}) \\ x_i^{SC} &\geq 0 = v(\{i\}) \end{aligned} \quad (53)$$

5.4. Structural and Statistical Stability Guarantees

The stability-correction model in Section 5.3 explicitly controls the split coalitions included in the training set $\mathfrak{S}_{tr}$. Since exhaustive enumeration of all split coalitions may again become computationally prohibitive, the stability of the final allocation is further assessed using an independent out-of-sample validation procedure. To avoid concentrating the assessment on coalition structures that are combinatorially dominant, the split-coalition space is stratified according to coalition cardinality, and the number of partially intersected participant clusters. For a split coalition S , let $c(S) = |\{g \in \mathcal{G} : 0 < |S \cap C_g| < |C_g|\}|$ denote the number of clusters partially intersected by S . The corresponding stratum is defined as $\mathfrak{S}_{s,c} = \{S \in \mathfrak{S}_{split} : |S| = s, c(S) = c\}$, and $\mathcal{H} = \{(s,c) : \mathfrak{S}_{s,c} \neq \emptyset\}$ denotes the set of nonempty strata. A fixed split-coalition sampling distribution is then defined as Equation (54), where $\omega_{s,c} > 0$ and $\sum_{(s,c) \in \mathcal{H}} \omega_{s,c} = 1$.

$$\mathbb{P}_{split}(S) = \sum_{(s,c) \in \mathcal{H}} \omega_{s,c} \frac{1\{S \in \mathfrak{S}_{s,c}\}}{|\mathfrak{S}_{s,c}|} \quad (54)$$

where $\omega_{s,c}$ denotes the probability mass assigned to stratum $\mathfrak{S}_{s,c}$; $|\mathfrak{S}_{s,c}|$ is the number of coalitions in that stratum; and $1\{\cdot\}$ denotes the indicator function.

Equal stratum weights, $\omega_{s,c} = 1/|\mathcal{H}|$, are adopted in this study so that coalition structures with fewer combinatorial realizations are not overwhelmed by the more numerous medium-sized coalitions. Conditional on a selected stratum, coalitions are sampled uniformly.

The equal stratum distribution is used as a structure balanced assessment rather than as an empirical model of coalition formation behavior. In the present application, no exogenous behavioral data are available to assign different formation probabilities to coalition sizes or split patterns. Uniform sampling over all individual split coalitions would place substantially more probability mass on combinatorially abundant structures, which may underrepresent less frequent but structurally distinct deviations. Equal stratum weighting therefore gives each combination of coalition size and number of partially intersected clusters comparable representation in the stability assessment. Accordingly, $p_{viol}(\tau)$ should be interpreted as the violation probability conditional on the prescribed distribution $\mathbb{P}_{split}$, rather than as an unconditional real-world probability that a coalition forms and deviates.

To examine the dependence of the reported stability results on this weighting choice, Section 6.4 additionally considers a uniform over split coalitions weighting, for which $\omega_{s,c}^{uni} = \frac{|\mathfrak{S}_{s,c}|}{|\mathfrak{S}_{split}|}$. Under this weighting, every individual split coalition has the same probability $1/|\mathfrak{S}_{split}|$, irrespective of its stratum. This alternative weighting is used only for robustness evaluation and does not alter the participant partition, reduced nucleolus, stability correction, or final allocation.

The training and validation coalition sets are generated independently from the pre-specified distribution $\mathbb{P}_{split}$. The training set is generated before solving the stability-correction models in Equation (50) and Equation (51), whereas validation coalitions are drawn only after the final allocation $\mathbf{x}^{SC}$ has been obtained. The validation samples are not used in participant grouping, candidate-partition selection, stability correction, or parameter selection.

For M_{val} independent validation coalitions $S_m^{val} \stackrel{i.i.d}{\sim} \mathbb{P}_{split}$, the out-of-sample violation probability at excess tolerance τ is defined as Equation (55). Validation coalitions are sampled independently with replacement from $\mathbb{P}_{split}$; repeated coalition evaluations are retrieved from the cache.

$$p_{viol}(\tau) = \Pr_{S \sim \mathbb{P}_{split}} [e(S, \mathbf{x}^{SC}) > \tau] \quad (55)$$

$$\hat{p}_M(\tau) = \frac{1}{M_{val}} \sum_{m=1}^{M_{val}} 1\{e(S_m^{val}, \mathbf{x}^{SC}) > \tau\} \quad (56)$$

where $p_{viol}(\tau)$ denotes the out-of-sample split-coalition violation probability and $\tau \geq 0$ is the prescribed excess tolerance; $\hat{p}_M(\tau)$ denotes the empirical violation frequency based on M_{val} independent validation coalitions.

Accordingly, Hoeffding's inequality gives the following one-sided out-of-sample certificate, as shown in Equation (57). A short derivation of the probabilistic certificate in Equation (57) is provided in Appendix E.

$$p_{viol}(\tau) \leq \min \left\{ 1, \hat{p}_{M_{val}}(\tau) + \sqrt{\frac{\ln(1/\beta)}{2M_{val}}} \right\} \quad (57)$$

where τ is the prescribed excess tolerance, with $\tau = 0$ corresponding to a strict positive excess violation; M_{val} denotes the number of independent validation coalitions, and $\beta \in (0,1)$ denotes the confidence tail probability. The bound in Equation (57) holds with probability at least $1 - \beta$. This certificate is probabilistic with respect to the prespecified distribution $\mathbb{P}_{split}$ and does not provide a deterministic guarantee for all split coalitions.

5.5. Computational Reduction

$$\rho_{coal} = \frac{N_{coal}^{red}}{N_{coal}^{full}} = \frac{2^G - 2}{2^N - 2}, \quad (58)$$

where ρ_{coal} denotes the coalition-space retention ratio, and N_{coal}^{red} is the number of reduced-game coalitions.

Coalition space reduction alone does not account for all physical optimization calls required by the proposed method. Before reduced game construction, the participant signatures require the grand coalition value, the N stand-alone coalition evaluations, and up to N leave-one-out evaluations used to obtain μ_i . After cache reuse, the reduced characteristic function contributes at most $2^G - 2$ additional proper coalition evaluations. Further evaluations may be required by the game consistency calibration and split coalition correction. Accordingly, the number of unique physical solves required before validation is bounded by (59).

$$\begin{aligned} N_{eval}^{solve} &\leq 2N + 1 + (2^G - 2) + N_{cal}^{add} + M_{tr}^{add} \\ N_{eval}^{total} &\leq N_{eval}^{solve} + M_{val}^{add} \end{aligned} \quad (59)$$

where N_{eval}^{solve} denotes the number of unique physical coalition evaluations required to obtain the proposed allocation before out-of-sample validation. N_{eval}^{total} additionally includes

the evaluations required for independent validation. $N_{\text{cal}}^{\text{add}}$ denotes the additional unique coalition evaluations required for the member-wise game consistency and aggregation interaction assessments after cache reuse; $M_{\text{tr}}^{\text{add}}, M_{\text{val}}^{\text{add}}$ denote the additional unique coalition evaluations required for training and validation, respectively, that are not already available in the cache.

6. Case Study

6.1. System Description and Overall Solution Procedure

The case studies are conducted on the IEEE 33-bus radial distribution system. Real residential demand and rooftop-PV profiles are obtained from an Australian distribution-network dataset reported in [35], which contains separately measured residential load and rooftop-PV generation profiles at 30 min intervals. The selected profiles are aggregated and scaled to construct participant-level operating profiles while preserving their original temporal characteristics. The resulting cooperative energy-sharing system contains heterogeneous participants connected at different buses of the distribution network. The locations of participants are determined using a deterministic feeder spanning procedure rather than being manually selected. The procedure sequentially selects load buses according to a greedy farthest point rule based on the resistance-weighted radial path distance, thereby promoting electrical dispersion across the feeder. For the primary $N = 10$ benchmark, the resulting participant buses are 22, 18, 33, 9, 25, 14, 28, 2, 7, and 12. This deterministic configuration is used as the primary benchmark for the detailed fidelity comparison in Section 6.2. To examine whether the observed performance depends on a particular participant realization, an additional multi-instance assessment with independently generated participant profiles and locations is presented in Section 6.3.

The distribution network configuration and participant locations are shown in Figure 2a. Red circles indicate buses at which participating prosumers are connected, whereas blue circles indicate the remaining buses without participating prosumers. The aggregate load, PV generation, and resulting net demand profile are presented in Figure 2b. For the $N = 10$ benchmark, the peak aggregate load is 2.093 MW, while the peak aggregate PV generation reaches 2.368 MW. The resulting net demand ranges from -1.357 MW during periods of high PV production to 2.093 MW during periods of net consumption. Before storage dispatch and cooperative optimization are applied, the corresponding network operating conditions are evaluated in Figure 2c,d. The minimum voltage magnitude is approximately 0.947 p.u. The maximum absolute branch power flow is 2.093 MW, corresponding to 83.3% of the calibrated branch limit of 2.512 MW. These results characterize the electrical operating conditions underlying the cooperative game and confirm that coalition values are evaluated under nontrivial network constraints rather than through an unconstrained economic allocation model.

To distinguish the effects of feature-based aggregation, game-consistency screening, and the subsequent stability correction, four comparison cases are considered, as summarized in Table 1.

Case 1: Full-space nucleolus. All participants are retained individually, and the nucleolus is evaluated in the original participant space without any WRT reduction. This case provides the exact allocation benchmark for problem sizes where exhaustive coalition evaluation remains computationally tractable.

Case 2: Feature-based WRT reduced nucleolus. Participants are grouped using the entropy-weighted physical-economic signatures and the feature-radius criterion. The original participants and network model are restored when evaluating the reduced game, after which the reduced-space nucleolus is mapped back to individual participants. No game-consistency screening or split-coalition correction is applied.

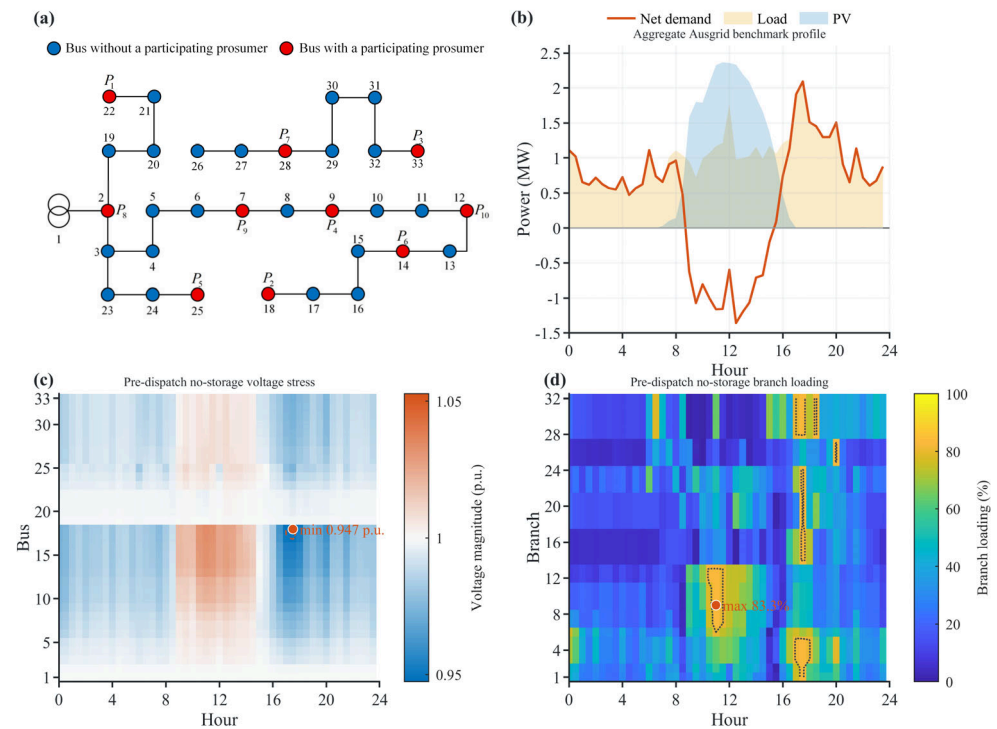


Figure 2. Physical benchmark configuration and pre-dispatch operating conditions: (a) IEEE 33-bus distribution network and participant locations; (b) aggregate load, PV generation, and net-demand profiles; (c) pre-dispatch voltage magnitudes across buses; (d) pre-dispatch branch loading levels.

Case 3: Game-consistent WRT reduced nucleolus. In addition to the feature-radius requirement of Case 2, candidate clusters are required to satisfy both the member-wise game-consistency and aggregation-interaction criteria. The admissible partition with the smallest reduced-player dimension is then used to construct the restored reduced game and determine the reduced-space nucleolus. No post-allocation correction is performed.

Case 4: Stability-corrected WRT nucleolus. Case 4 uses the same admissible partition and reduced-space nucleolus construction as Case 3. The mapped individual allocation is subsequently refined by the split-coalition stability correction while preserving the payoff assigned to each reduced cluster. This case represents the complete proposed framework.

Table 1. Comparison cases and ablation settings.

Setting	Case 1	Case 2	Case 3	Case 4
Full participant space	✓	-	-	-
Feature-based WRT clustering	-	✓	✓	✓
Feature-radius screen $r_g \leq \tau_\phi$	-	✓	✓	✓
Member-wise game-consistency screen $\delta_g^{game} \leq \tau_m$	-	-	✓	✓
Aggregation-interaction screen $\delta_g^{agg} \leq \tau_a$	-	-	✓	✓
Restore-before-evaluate reduced game	-	✓	✓	✓
Reduced-space nucleolus	-	✓	✓	✓
Inverse allocation mapping	-	✓	✓	✓
Split-coalition stability correction	-	-	-	✓
Cluster-total preservation in correction	-	-	-	✓

The four cases therefore form a progressive ablation structure rather than four independent solution schemes. Case 1 establishes the full-space reference, whereas the comparison between Cases 2 and 3 isolates the contribution of game-aware cluster screening. Cases 3

and 4 use the same reduced game and cluster-level allocation, so their difference specifically identifies the effect of the post-allocation stability correction. For larger participant populations, where full coalition enumeration is no longer practical, Cases 2–4 are evaluated directly to examine the scalability of the reduced formulation.

For each problem size, the procedure begins by constructing the participant signatures and candidate WRT partitions. Physical and game-consistency conditions are then applied according to the selected case. Once the partition is fixed, reduced-coalition values are obtained by restoring the original participants associated with each reduced coalition and solving the corresponding physical scheduling problem. The reduced-space nucleolus is subsequently calculated and mapped back to the original participants. Only Case 4 performs the additional split-coalition correction. Independent out-of-sample coalition tests are carried out after the final allocation has been obtained and are not used in clustering, partition selection, or correction parameter determination.

Unless otherwise stated, all numerical experiments use the same prespecified computational settings. The game consistency calibration budget is $M_{\text{cal}} = 32$ contexts per candidate cluster, the split coalition correction uses $M_{\text{tr}} = 1000$ training samples, and the independent validation uses $M_{\text{val}} = 3000$ samples with $\beta = 0.05$. The random seeds for participant profile selection, game consistency calibration, split coalition training, and independent validation are 20260825, 20260826, 20260827, and 20260828, respectively. The detailed DBSCAN grid, numerical regularizers, solver tolerances, cache rules, and computing environment are summarized in Appendix F, with the main numerical and reproducibility settings listed in Table A1.

To separate parameter specification from performance evaluation, the case study follows a fixed calibration and evaluation protocol. The nominal fidelity tolerances and the set of DBSCAN candidate configuration are specified before the independent evaluation stage. The primary $N = 10$ configuration is used to illustrate the behavior of the reduction criteria and to evaluate sensitivity around the prescribed nominal settings. The five independently generated $N = 10$ instances introduced in Section 6.3 are reserved for evaluation and do not feed back into threshold selection, DBSCAN configuration, or any other model setting. The same nominal settings are subsequently retained in the larger participant studies.

6.2. Reduction Fidelity and Nucleolus Recovery

The $N = 10$ system is used for the fidelity assessment because the full space nucleolus can be obtained by evaluating all proper coalitions, providing an exact reference for the reduced formulations. Table 2 gives the partition characteristics and game consistency diagnostics. Case 2 compresses the original system into four reduced players and retains only 1.37% of the proper coalition space. Its maximum feature radius is 0.2133 and satisfies $\tau_\phi = 0.25$. However, the corresponding δ_g^{game} and δ_g^{agg} are 1.2365 and 0.5716, both above their prescribed limits. Participants that are sufficiently close in the feature space can therefore still exhibit different marginal contributions or aggregation effects when they participate in different coalitions.

Table 2. Partition and game-consistency diagnostics for the $N = 10$ benchmark.

Metric	Case 2	Case 3/4	Threshold
Reduced players, G	4	6	-
Coalition-space retention, $\rho_{\text{coal}}(\%)$	1.37	6.07	-
Maximum feature radius	0.2133	0.1839	$\tau_\phi = 0.25$
Maximum δ_g^{game}	1.2365	0.8694	$\tau_m = 1.00$
Maximum δ_g^{agg}	0.5716	0.1685	$\tau_a = 0.20$

Introducing the member-wise game consistency and aggregation interaction conditions changes the selected partition from four to six reduced players. For Cases 3 and 4, the maximum feature radius becomes 0.1839, while δ^{game} and δ^{agg} decrease to 0.8694 and 0.1685, respectively. These values satisfy $\tau_\phi = 0.25$, $\tau_m = 1.00$, and $\tau_a = 0.20$. The retained coalition space increases from 1.37% to 6.07%, which reflects the additional representation required to preserve coalition-related behavior rather than feature proximity alone.

The resulting difference in allocation fidelity is evident in Figure 3a,b. The normalized L_1 allocation error decreases from 14.31% in Case 2 to 3.67% in Case 3, while the maximum individual error decreases from 4.61% to 1.05%. These reductions correspond to approximately 74.4% and 77.2%, respectively. The improvement is obtained without using the full space nucleolus in the partition selection, since the admissibility tests are constructed from feature radius, contextual marginal consistency, and aggregation interaction. Differences between the two reduced formulations also appear in coalition stability. The maximum normalized coalition excess in Figure 3c decreases from 3.43% for Case 2 to 1.17% for Case 3. At the tolerance $\tau = 1\%v(N)$, the out-of-sample violation probability shown in Figure 3d decreases from 6.83% to 0.900%. The reduced allocation obtained after game consistency screening therefore reproduces the stability characteristics of the original cooperative game more closely, in addition to its lower participant-level allocation error.

Case 4 uses the same six-player reduced game as Case 3 and applies the split coalition correction after the reduced nucleolus has been mapped back to the original participants. Since cluster totals remain fixed during this correction, its effect is mainly observed in the coalition stability measures. The maximum coalition excess decreases from 1.17% to 1.03%, and the out-of-sample violation probability decreases from 0.900% to 0.433%. The latter represents a reduction of approximately 51.9%. The normalized L_1 error changes from 3.67% to 3.96%, whereas the maximum individual error remains 1.05%. This behavior is consistent with the correction formulation, which adjusts the distribution within each preserved cluster total to reduce unfavorable coalition excess rather than minimizing the distance from the full space allocation.

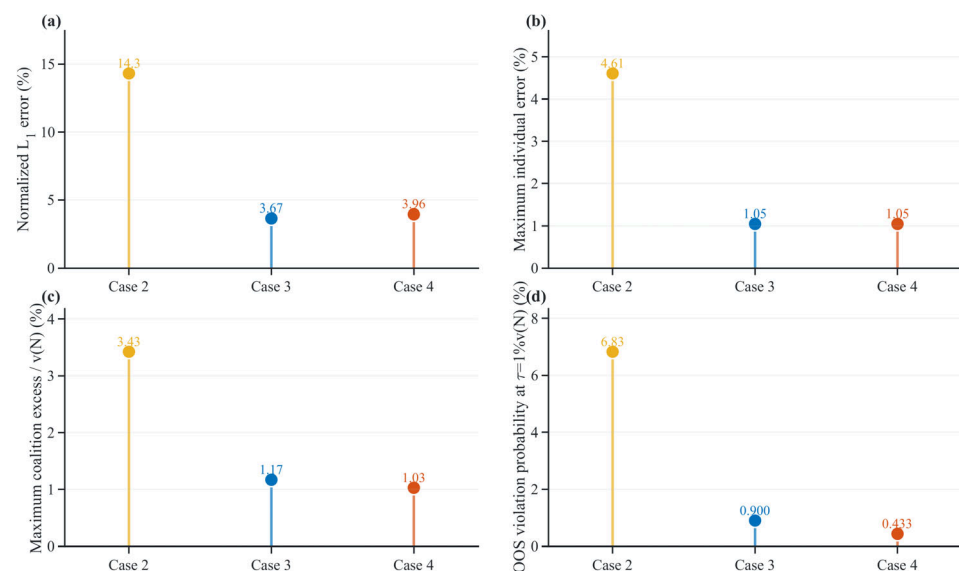


Figure 3. Exact-benchmark comparison of allocation fidelity and coalition stability: (a) normalized L_1 allocation error; (b) maximum individual allocation error; (c) maximum normalized coalition excess; (d) out-of-sample violation probability at $\tau = 1\%v(N)$.

Table 3 provides the computational results together with the fidelity and stability measures. Full enumeration in Case 1 requires 1023 physical evaluations and 618.75 s.

Case 2 reduces these values to 29 evaluations and 19.18 s, but the stronger compression is accompanied by considerably larger allocation and coalition errors. Case 3 requires 228 physical evaluations and 114.56 s, corresponding to a $5.40\times$ speedup relative to Case 1, while its normalized L_1 error is limited to 3.67%. The stability correction in Case 4 increases the computational requirement to 595 evaluations and 356.68 s because additional split coalitions are evaluated. Its runtime remains below that of the full space solution, with lower maximum coalition excess and out-of-sample violation probability than the other reduced cases.

Table 3. Performance of the four cases under the $N = 10$ benchmark.

Metric	Case 1	Case 2	Case 3	Case 4
Reduced players, G	10	4	6	6
Coalition retention (%)	100	1.37	6.07	6.07
Normalized L_1 allocation error (%)	0	14.31	3.67	3.96
Maximum individual error (%)	0	4.61	1.05	1.05
Maximum coalition excess/ $v(\mathcal{N})$ (%)	≈ 0	3.43	1.17	1.03
OOS violation probability at $\tau = 1\%v(\mathcal{N})$ (%)	0	6.83	0.900	0.433
95% Hoeffding upper bound (%)	-	9.068	3.134	2.668
Physical evaluations	1023	29	228	595
Runtime (s)	618.75	19.18	114.56	356.68
Speed-up vs. Case 1	1.00 $\times$	32.25 $\times$	5.40 $\times$	1.73 $\times$

From a sustainability perspective, the numerical results should be interpreted as evidence of enabling conditions for sustained cooperative participation. The reduction in coalition evaluation burden improves the practical scalability of benefit allocation as the number of prosumers increases, while the lower coalition excess and out-of-sample violation frequency reduce the economic incentive for participants to leave the cooperative arrangement. Combined with the network-constrained coalition valuation, these properties support the continued operation of larger renewable-rich prosumer communities.

6.3. Robustness Across Independent Exact Benchmark Instances

The primary $N = 10$ benchmark in Section 6.2 provides a detailed comparison against the exact full space nucleolus. To assess whether the observed performance depends on a particular participant configuration, the fidelity assessment is further repeated over five independently generated $N = 10$ instances. Different participant profiles and electrical locations are used across these instances, while the network model and all methodological settings are kept unchanged. The prescribed fidelity tolerances, clustering settings, and stability correction parameters are not retuned for individual instances.

Figure 4 presents the instance-wise results for allocation fidelity, coalition stability, and computational cost, while Table 4 summarizes their distributions using the median, interquartile range, and range where appropriate. Across the five instances, the median normalized L_1 allocation error is 9.79% for Case 3 and 8.74% for Case 4. The variation among instances indicates that the fidelity of the reduced allocation depends on the particular participant composition, but the stability correction does not introduce a systematic deterioration in allocation fidelity.

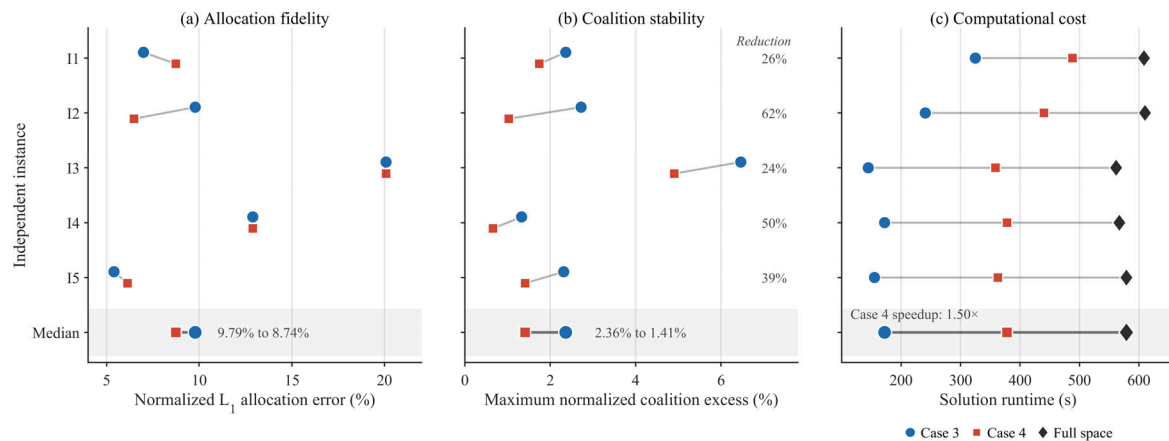


Figure 4. Robustness assessment across five independent $N = 10$ benchmark instances.

Table 4. Performance distribution across five independent $N = 10$ benchmark instances.

Metric	Case 1	Case 3	Case 4
Reduced players G , median	10	7 [7,8]	7 [7,8]
Normalized L_1 allocation error, median (%)	0	9.79 [6.99–12.90]	8.74 [6.48–12.90]
Maximum normalized excess, median (%)	≈ 0	2.364 [2.316–2.723]	1.412 [1.028–1.745]
Runtime, median (s)	579.10 [567.03–608.58]	172.31 [155.32–240.79]	378.08 [362.89–440.24]
Speedup vs. full space, median	1.00×	3.29× [1.87–3.88]	1.50× [1.25–1.60]

The effect of the stability correction is more consistent in terms of coalition excess. The median maximum normalized excess decreases from 2.364% in Case 3 to 1.412% in Case 4, and Case 4 achieves a lower maximum excess in all five independent instances. This improvement is obtained with additional computation relative to Case 3, but the complete Case 4 framework remains faster than the exact full space solution in every instance. The median runtime is 579.10 s for Case 1, 172.31 s for Case 3, and 378.08 s for Case 4. Accordingly, Case 4 provides a median speedup of $1.50\times$ over the full space calculation, with a range of $1.25\times - 1.60\times$. Accordingly, Case 4 provides a median speedup of $1.50\times$ over the full space calculation, with a range of $1.25\times - 1.60\times$. The median normalized L_1 allocation error of Case 4 is 8.74% across the five instances, compared with 3.96% in the primary benchmark, indicating noticeable variation in allocation fidelity across participant realizations. In terms of coalition stability, Case 4 achieves a lower maximum normalized excess than Case 3 in all five instances. These results show that the complete framework consistently improves coalition stability while maintaining a computational advantage over the exact full space solution across the examined $N = 10$ instances.

6.4. Split-Coalition Stability and Out-of-Sample Validation

The effect of the split-coalition correction is further examined as the number of participants increases. Case 3 is taken as the allocation before correction, while Case 4 preserves the same partition and reduced nucleolus and only redistributes the payoff within the corresponding cluster totals. As reported in Figure 5a, the maximum excess over the training split coalitions decreases from 5.82 to 1.47 for $N = 15$ and from 4.77 to 1.71 for $N = 20$. A stronger effect is observed for the larger systems. The value of 17.42 at $N = 25$ is reduced to nearly zero, and a similar result is obtained at $N = 30$, where the initial maxi-

excess of 5.64 is almost completely removed. These results are consistent with the objective of the correction model, which directly limits the most unfavorable excess among the sampled split coalitions without altering the reduced game solution. The improvement is also retained for coalitions that are not used in the correction procedure. At a tolerance of $0.5\%v(N)$, the out-of-sample violation probability decreases from 0.47% to 0.10% for $N = 15$ and from 5.73% to 1.77% for $N = 20$, as shown in Figure 5b. At the tolerance of $0.5\%v(N)$, no empirical violation is observed among the 3000 validation coalitions for Case 4 at $N = 25$ and $N = 30$, whereas the corresponding probabilities before correction are 9.53% and 4.20%. Since these coalitions are generated independently after the correction has been completed, the reduction cannot be attributed to direct fitting of the validation set.

To assess whether the stability comparison depends on the equal stratum weighting, the same independent validation samples are additionally evaluated using uniform weighting over individual split coalitions.

As shown in Table 5, the absolute violation probabilities change under the alternative weighting, confirming that the numerical probability is distribution dependent. However, the qualitative comparison between Cases 3 and 4 is preserved across all examined participant scales. Under uniform weighting over individual split coalitions, the violation probability decreases from 0.11% to 0.04% for $N = 15$ and from 7.26% to 1.47% for $N = 20$. For $N = 25$ and $N = 30$, the corresponding Case 3 probabilities are 12.89% and 6.80%, whereas no empirical Case 4 violation is observed in the validation sample. These results indicate that the absolute probability values remain conditional on the selected weighting scheme and should therefore be interpreted as complementary stability assessments.

The amount of participant payoff redistribution required to obtain this stability improvement is given in Figure 5c. The normalized adjustment D_x remains between 2.3% and 3.2% for $N = 15, 20$, and 30. A larger adjustment of 15.0% occurs at $N = 25$, which coincides with the substantially larger pre-correction excess observed in Figure 5a. This quantity measures the aggregate participant level redistribution relative to $v(N)$; it does not represent a loss of cooperative surplus because the correction preserves the total allocation of each reduced cluster. The computational cost of the correction varies considerably with system size. The relative runtime overhead in Figure 5d is 208.4% and 217.6% for $N = 15$ and $N = 20$, respectively. It decreases to 58.5% at $N = 25$ and 6.9% at $N = 30$. Thus, although the correction can dominate the reduced solution time for smaller instances, its relative contribution becomes much less pronounced once the computational burden of constructing and solving the reduced game increases.

A broader view of the validation results is provided in Figure 6, where the out-of-sample violation probability is evaluated jointly over participant scale N and the normalized excess tolerance $\tau/v(N)$. The surfaces for Cases 2–4 in Figure 6a–c all decay as the admissible tolerance increases, since fewer independently sampled coalitions exceed a less restrictive stability threshold. Case 3 is not uniformly below Case 2 over every (N, τ) combination. This is expected because the game consistency screens used in Case 3 are designed to improve representation of coalition interactions and recovery of the nucleolus rather than to minimize the violation probability at every individual tolerance. The effect of the stability correction is more directly observed by comparing Figure 6b,c. The Case 4 surface is substantially suppressed over several regions in which Case 3 retains noticeable violation probabilities, particularly at larger participant scales and relatively small excess tolerances. Figure 6d reports this difference directly as the reduction in out-of-sample violation probability, $P_{vio}^{C3} - P_{vio}^{C4}$, in percentage points. Positive regions therefore indicate improved stability after correction. The largest gains occur in the more demanding low-to-moderate tolerance range, while the difference approaches zero once both cases have very small violation probabilities.

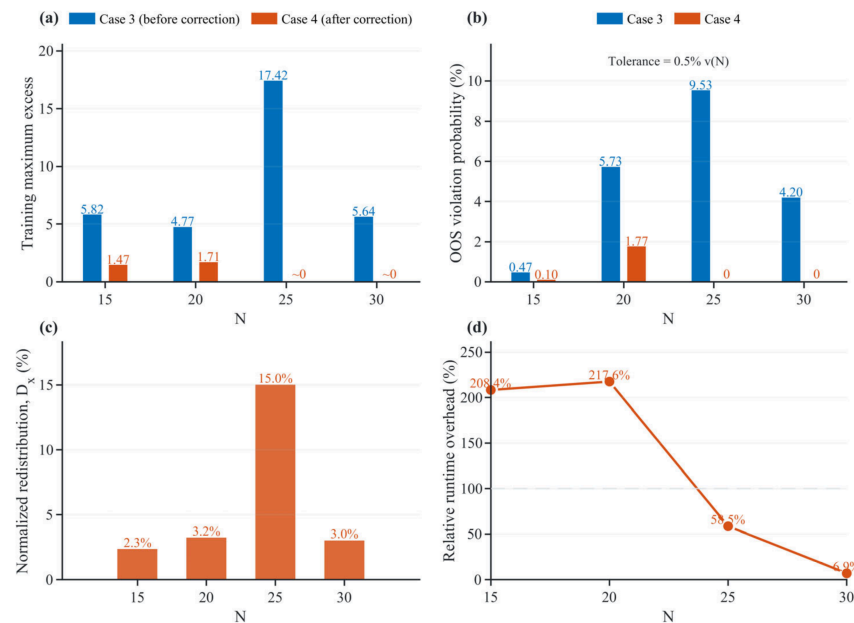


Figure 5. Effect of split-coalition stability correction across participant scales: (a) maximum excess over the training split coalitions before and after correction; (b) out-of-sample violation probability at $\tau = 0.5\%v(N)$; (c) normalized participant-payoff adjustment D_x ; (d) relative runtime overhead of the stability correction.

Table 5. Out-of-sample violation probability under alternative split coalition weighting at $\tau = 0.5\%v(N)$.

N	Equal Stratum Case 3 (%)	Equal Stratum Case 4 (%)	Uniform Split Coalition Case 3 (%)	Uniform Split Coalition Case 4 (%)
15	0.47	0.10	0.11	0.04
20	5.73	1.77	7.26	1.47
25	9.53	0.00	12.89	0.00
30	4.20	0.00	6.80	0.00

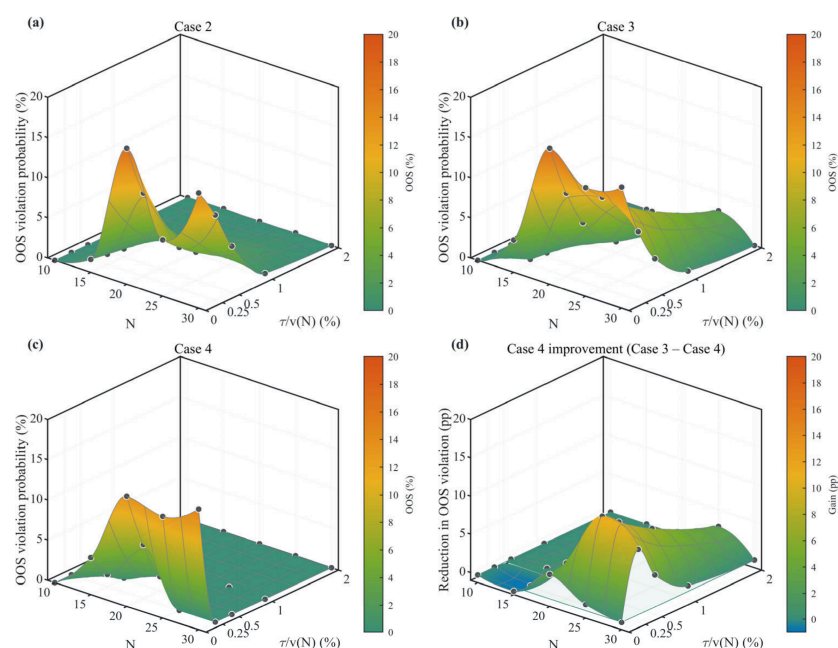


Figure 6. Out-of-sample coalition-stability landscape across participant scale and excess tolerance.

6.5. Nonlinear AC Power Flow Validation

To directly assess the physical approximation used in the coalition model, selected optimized coalition schedules from the $N = 10$ benchmark are evaluated ex post using nonlinear AC power flow on the same IEEE 33 bus network. The optimized storage schedules are retained unchanged, and the nonlinear AC calculation is performed over all 48 operating intervals. The validation includes the grand coalition together with four coalitions associated with the most critical voltage and active branch-loading conditions identified by the linear model.

As shown in Table 6, the nonlinear AC power flow results remain close to those obtained from the linearized network model. Across the selected coalition schedules, the maximum voltage magnitude mismatch is 0.00091 p.u., while the maximum active branch loading mismatch is 2.14 percentage points. All nonlinear AC power flow calculations converge over the examined operating intervals. These results provide a direct physical check of the linearized voltage and branch flow representations used in the coalition model.

Table 6. Nonlinear AC power flow validation of selected coalition schedules.

Selected Coalition	Maximum Voltage Mismatch (p.u.)	Maximum Active Branch Loading Mismatch (p.p.)
Grand coalition	0.00091	2.14
Voltage critical coalition 1	0.00088	1.99
Voltage critical coalition 2	0.00091	2.01
Loading critical coalition 1	0.00089	2.11
Loading critical coalition 2	0.00091	2.13

6.6. Sensitivity to Fidelity Tolerances

The sensitivity of the WRT reduction to the three fidelity tolerances is examined using the primary $N = 10$ configuration. Each tolerance is varied individually while the remaining fidelity thresholds and methodological settings are retained at their nominal values. Table 7 reports the sensitivity to the feature space tolerance τ_ϕ and member wise game consistency tolerance τ_m in terms of the selected reduced dimension, retained coalition space, and normalized L_1 allocation error. Figure 7 reports the sensitivity to the aggregation interaction tolerance τ_a . For Table 7, partition selection follows the structural fidelity criteria in (40) and (41), and the L_1 error is evaluated after the selected partition is obtained.

Table 7. Sensitivity to the feature space and member-wise game consistency tolerances.

Parameter	Value	Selected Reduction G/Retention (%)	Normalized L_1 Error (%)
τ_m	0.60	8/24.85	≈ 0
	0.80	7/12.33	4.42
	1.00	6/6.07	3.67
	1.20	6/6.07	3.67
	1.40	6/6.07	3.67
τ_ϕ	0.15	8/24.85	≈ 0
	0.20	6/6.07	3.67
	0.25	6/6.07	3.67
	0.30	6/6.07	3.67
	0.35	6/6.07	3.67

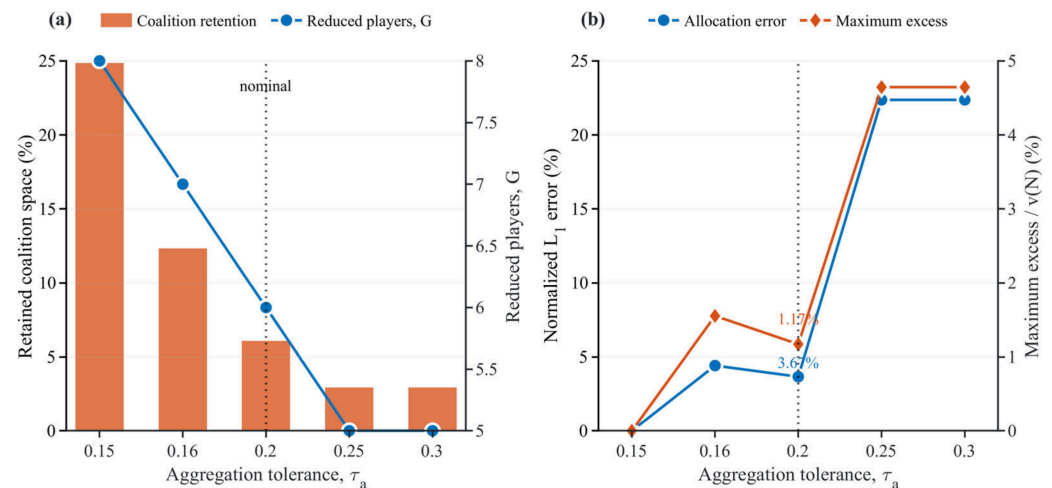


Figure 7. Sensitivity of WRT reduction to the aggregation-interaction tolerance τ_a : (a) retained coalition space and reduced-player dimension; (b) normalized L_1 allocation error and maximum normalized coalition excess.

Table 7 shows a stepwise response of the selected partition to the two tolerances. Tightening τ_m from 1.00 to 0.80 and 0.60 increases the reduced dimension from $G = 6$ to $G = 7$ and $G = 8$, with coalition retention increasing from 6.07% to 12.33% and 24.85%, respectively. Relaxing τ_m above 1.00 leaves the selected $G = 6$ partition unchanged. For τ_ϕ , the stricter value 0.15 retains $G = 8$ players, whereas values from 0.20 to 0.35 select the same $G = 6$ partition. The corresponding L_1 error varies with the selected partition and is not monotonic across all tolerance values. The nominal settings $\tau_m = 1.00$ and $\tau_\phi = 0.25$ both lie in regions where moderate relaxation does not further change the selected partition.

The sensitivity to the aggregation interaction tolerance τ_a is shown in Figure 7. As τ_a is relaxed from 0.15 to 0.30, fewer reduced players are retained and the corresponding coalition space decreases. At the nominal value $\tau_a = 0.20$, the $N = 10$ game is represented by $G = 6$ reduced players with 6.07% of the original coalition space retained.

The resulting change in allocation fidelity is shown in Figure 7b. The normalized L_1 allocation error is 3.67% at $\tau_a = 0.20$, while the maximum coalition excess is 1.17% of $v(N)$. Both measures increase sharply when τ_a is relaxed to 0.25, indicating that the additional reduction begins to merge participants whose aggregation interactions are no longer sufficiently consistent. By contrast, reducing τ_a below 0.20 increases the reduced-game dimension without producing a corresponding improvement in both accuracy measures. These observations illustrate the expected trade-off between compression and fidelity around the specified nominal setting $\tau_a = 0.20$; they are not used in selecting or readjusting this value in subsequent evaluation cases.

6.7. Computational Scalability and Comparison with Existing Approaches

Table 8 provides a structured comparison with representative scalable cooperative energy allocation approaches in terms of participant scale, allocation objective, network treatment, stability treatment, and reported computational performance.

Table 8. Comparison with representative scalable cooperative energy allocation approaches.

Approach/Scale	Allocation Objective/Network Treatment	Stability Treatment	Reported Computational Performance
Han et al. [9], >100 prosumers	Nucleolus estimation through prosumer clustering; feeder-level network constraints not explicitly reported in the scalable comparison	Nucleolus-based allocation	Scale extended from <15 to >100 players
Raja and Grammatico [25], 8 prosumers	Bilateral assignment and energy trading; no feeder-level power-flow model	Stable bilateral contracts	Approx. 40× faster negotiation step than benchmark
Cremers et al. [27], up to 200 prosumers	Shapley value approximation through sampling and clustering; feeder constraints not included	Shapley-based fair allocation	Relative Shapley error below 1% at larger scales; lower computational cost than adaptive sampling
Bossu et al. [28], 10 households	Shapley-based allocation with cooperative-game subdivision; feeder-level voltage/branch constraints not explicitly reported	Stability treatment not specifically targeted	Approx. 100-fold reduction in reported computation time
Proposed framework, $N = 10\text{--}30$	Reduced-space nucleolus with participant-level recovery; IEEE 33-bus voltage and branch-flow constraints	Split-coalition correction with independent OOS validation	Case 4: 1.73× speedup at primary $N = 10$; $N = 30$: $G = 14, 16,382$ reduced coalitions

The reported computational results are obtained under different cooperative solution concepts, physical models, datasets, hardware platforms, and stopping criteria, so their absolute runtimes are not directly comparable. References [9,27] primarily address approximation of the nucleolus and Shapley value, respectively, while [25] solves a bilateral assignment game, and [28] applies game subdivision for Shapley-based allocation. The present framework additionally evaluates coalition values through a network-constrained operating model and applies participant-level split-coalition stability treatment after payoff recovery. The comparison therefore emphasizes the reported computational scale and methodological scope rather than raw execution time across studies.

The computational scalability of the proposed framework is evaluated by increasing the number of participants from $N = 10$ to $N = 30$. The corresponding growth of the original and reduced coalition spaces is reported in Figure 8a. While the number of proper coalitions in the full game increases exponentially with N , the reduced games remain several orders of magnitude smaller. At $N = 30$, Case 3/4 retains $G = 14$ reduced players, corresponding to 16,382 proper reduced coalitions, compared with more than 10^9 coalitions in the original game. This represents a reduction of approximately 6.55×10^4 in the coalition space. The reduced-player dimensions in Figure 8b increase much more slowly than the original participant population, although the game-consistency conditions used in Cases 3 and 4 generally require a moderately larger G than the feature-based reduction in Case 2.

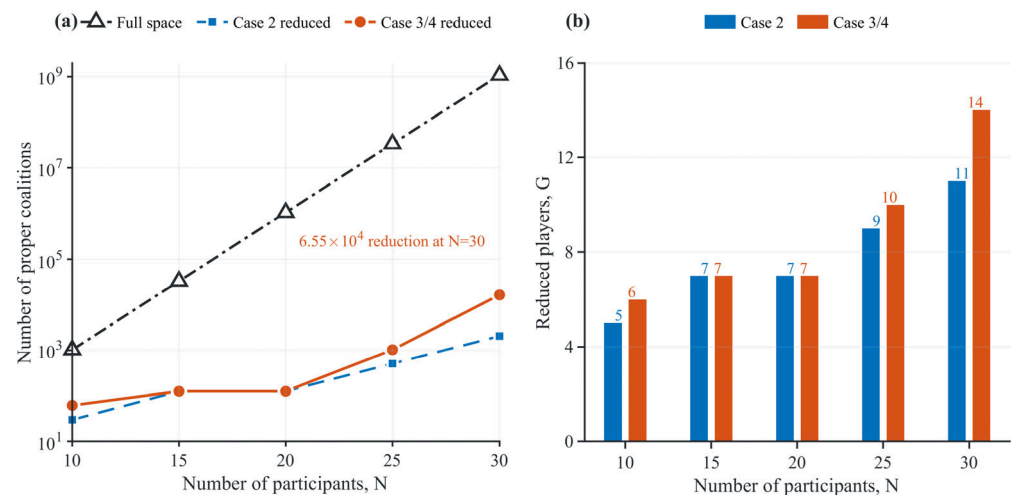


Figure 8. Scalability of coalition-space reduction and reduced-player dimension: (a) numbers of proper coalitions in the full and reduced games; (b) reduced-player dimension G for Cases 2 and 3/4.

The computational effect of this reduction can be observed in Figure 9a. Runtime increases with participant scale for all reduced formulations. At $N = 30$, Case 3 requires 11.54 h, while the additional stability correction increases the total runtime to 12.34 h. As shown in Figure 9b, the relative correction overhead decreases to 6.9% at $N = 30$, indicating that the main computational burden at this scale arises from constructing and evaluating the reduced cooperative game. The 12.34 h runtime also indicates that further computational acceleration would be beneficial for applications requiring frequent allocation updates. Parallel coalition evaluation, more extensive cache reuse, and faster coalition valuation strategies provide potential directions for reducing this computational burden.

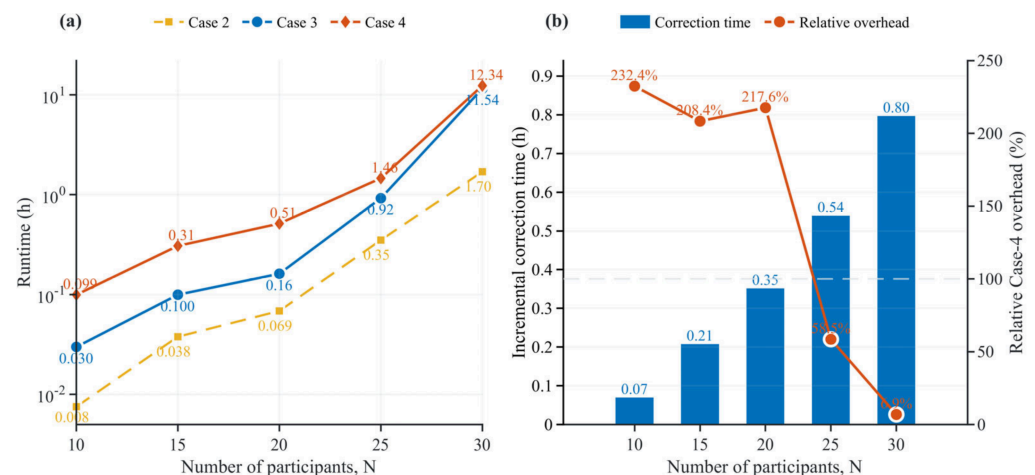


Figure 9. Computational runtime and correction overhead with increasing participants: (a) runtimes of Cases 2–4; (b) incremental stability-correction time and relative runtime overhead.

7. Conclusions

This paper develops a fidelity-controlled WRT-inspired reduction framework for scalable nucleolus allocation in network-constrained cooperative energy sharing. Rather than reducing the cooperative game according to participant similarity alone, the proposed method evaluates candidate partitions using physical–economic compactness, contextual marginal-contribution consistency, and aggregation-interaction consistency. The selected reduced game is constructed through a restore-before-evaluate procedure, so that coalition values continue to be obtained from the original participants and network-constrained

operating model. The reduced-space nucleolus is subsequently recovered at the participant level, followed by a split-coalition correction that addresses stability conditions not explicitly represented in the reduced coalition space.

The numerical results demonstrate the tradeoff between dimensional reduction, allocation fidelity, and coalition stability. In the primary $N = 10$ benchmark, the game-consistent reduced formulation (Case 3) retains 6.07% of the original proper coalition space, achieves a normalized L_1 allocation error of 3.67%, and provides a $5.40\times$ speedup over the full-space nucleolus. After the split coalition correction, the complete Case 4 framework reduces the out-of-sample violation probability from 0.900% to 0.433% while retaining a $1.73\times$ speedup. Across five independently generated $N = 10$ instances, Case 4 achieves a median normalized L_1 allocation error of 8.74% and a median speedup of $1.50\times$. Its maximum normalized excess is lower than that of Case 3 in all five instances, showing consistent improvement in coalition stability across different participant realizations. At $N = 30$, the selected 14-player representation contains 16,382 proper coalitions, corresponding to an approximately 6.55×10^4 -fold reduction in coalition space. The complete Case 4 computation requires 12.34 h, with the stability correction accounting for a 6.9% overhead relative to the preceding reduced game computation. Further computational acceleration and evaluation on larger and more diverse distribution networks are therefore important directions for extending the framework to broader applications.

The proposed framework provides an explicit mechanism for controlling the information lost during participant reduction and for assessing stability after participant-level payoff recovery. The present formulation relies on prescribed fidelity tolerances, for which an explicit analytical relation to the resulting full-space nucleolus error remains to be established. Moreover, stability for non-representable split coalitions is certified probabilistically rather than deterministically over the complete coalition space. Future work will investigate error-bounded fidelity criteria, adaptive critical coalition identification, and computational acceleration of repeated coalition valuation. The framework will also be evaluated on larger and structurally different distribution networks to examine its computational and allocation performance under a broader range of network conditions.

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Appendix A. Derivation of the Network-Sensitivity Matrices

This appendix derives the network-sensitivity matrices used in Equation (8), Equation (9), and Equation (18). For the radial distribution network considered in this study, the active- and reactive-power flow on each branch can be obtained by accumulating the downstream

nodal withdrawals along the corresponding slack-to-bus paths. Using the bus-to-branch path matrix M defined in Equation (17), the branch-flow vectors can be expressed as

$$f_t^P = Mp_t, f_t^Q = Mq_t \quad (A1)$$

where f_t^P and f_t^Q denote the active- and reactive-power branch-flow vectors at time t , respectively. The active branch-flow vector f_t^P corresponds to f_t used in the main text.

Under the linearized radial power-flow approximation, the squared voltage drop from the slack bus to all non-slack buses is obtained by accumulating the voltage drops across the branches on their corresponding radial paths. Neglecting the second-order loss terms gives

$$u_t = u_0 \mathbf{1} - 2M^T \text{diag}(r) f_t^P - 2M^T \text{diag}(x^{line}) f_t^Q \quad (A2)$$

Substituting Equation (A1) into Equation (A2):

$$u_t = u_0 \mathbf{1} - 2M^T \text{diag}(r) Mp_t - 2M^T \text{diag}(x^{line}) Mq_t \quad (A3)$$

Therefore, the topology- and impedance-dependent sensitivity matrices are

$$\begin{aligned} R &= 2M^T \text{diag}(r) M, \\ X &= 2M^T \text{diag}(x^{line}) M, \\ H &= M \end{aligned} \quad (A4)$$

Equations (A3) and (A4) directly recover the voltage and active branch flow relations in Equation (8) and Equation (9). Hence, R , X and H are determined by the feeder topology and line parameters rather than by exogenously selected sensitivity coefficients.

Appendix B. Hierarchical Information Reconstruction

The cluster-level contribution mass P_g and the conditional intra-cluster contribution share $q_{i|g}$ retain the information required to reconstruct the original participant-level contribution distribution. Define the Shannon entropy of the normalized participant contribution masses as

$$\mathcal{H}(p) = -\sum_{i \in \mathcal{N}} p_i \ln p_i \quad (A5)$$

where $\mathcal{H}(p)$ denotes the Shannon entropy of the participant contribution-mass distribution $p = [p_i]_{i \in \mathcal{N}}$.

From the hierarchical mapping introduced in Section 4.5,

$$p_i = P_g q_{i|g}, \quad i \in C_g \quad (A6)$$

with

$$\sum_{g \in \mathcal{G}} P_g = 1, \sum_{g \in \mathcal{G}} q_{i|g} = 1, \forall g \in \mathcal{G} \sum_{g \in \mathcal{G}} P_g = 1, \sum_{g \in \mathcal{G}} q_{i|g} = 1, \forall g \in \mathcal{G} \quad (A7)$$

Substituting Equation (A6) into Equation (A5) and using Equation (A7) gives Equation (A8):

$$\mathcal{H}(p) = -\sum_{g \in \mathcal{G}} \sum_{i \in C_g} P_g q_{i|g} \ln(P_g q_{i|g}) \quad (A8)$$

$$\mathcal{H}(p) = -\sum_{g \in \mathcal{G}} P_g \ln P_g - \sum_{g \in \mathcal{G}} P_g \sum_{i \in C_g} q_{i|g} \ln q_{i|g} \quad (A9)$$

Define $\mathcal{H}(P) = -\sum_{g \in \mathcal{G}} P_g \ln P_g$ and $\mathcal{H}(q_g) = -\sum_{i \in C_g} q_{i|g} \ln q_{i|g}$, where $\mathcal{H}(P)$ denotes the entropy of the cluster-level contribution-mass distribution and $\mathcal{H}(q_g)$ denotes the conditional contribution entropy within cluster g .

Accordingly,

$$\mathcal{H}(p) = \mathcal{H}(\mathbf{P}) + \sum_{g \in \mathcal{G}} P_g \mathcal{H}(q_g) \quad (\text{A10})$$

Equation (A10) is an exact hierarchical decomposition of the participant-level contribution information. It does not imply that the cluster-level entropy $\mathcal{H}(\mathbf{P})$ alone is equal to the original participant-level entropy. Rather, the original information is represented jointly by the inter-cluster component $\mathbf{P}$ and the intra-cluster conditional components $\{q_g\}_{g \in \mathcal{G}}$. This hierarchical representation provides the information required for the inverse payoff mapping in Equation (48).

Appendix C. Proof of Exact Excess Preservation

Since the selected clusters form a disjoint and complete partition of the original participant set, $\mathcal{R}(\mathcal{G}) = \mathcal{N}$. Combined with Equation (44), this immediately gives $\bar{v}(\mathcal{G}) = v(\mathcal{N})$, i.e., the grand-coalition value is preserved by the reduced-game construction.

For any reduced coalition $A \subseteq \mathcal{G}$, the cluster payoff conservation constraint in Equation (51) gives

$$\sum_{i \in \mathcal{R}(A)} x_i^{\text{SC}} = \sum_{g \in A} \sum_{i \in C_g} x_i^{\text{SC}} = \sum_{g \in A} \bar{x}_g^{\text{muc}} \quad (\text{A11})$$

Therefore, using the reduced characteristic function in Equation (44) and the excess definitions in Equations (13) and (46):

$$\begin{aligned} e(\mathcal{R}(A), \mathbf{x}^{\text{SC}}) &= v(\mathcal{R}(A)) - \sum_{i \in \mathcal{R}(A)} x_i^{\text{SC}} \\ &= v(A) - \sum_{g \in A} \bar{x}_g^{\text{muc}} \\ &= e(A, \bar{\mathbf{x}}^{\text{muc}}), \quad \forall A \subseteq \mathcal{G} \end{aligned} \quad (\text{A12})$$

Equation (A12) proves Theorem 1 in Equation (52), showing that the stability corrected participant-level allocation exactly preserves the excess of every coalition representable as a union of complete clusters. Taking $A = \mathcal{G}$ further gives $\sum_{i \in \mathcal{N}} x_i^{\text{SC}} = v(\mathcal{N})$, while the nonnegativity constraint in Equation (51), together with $v(\{i\}) = 0$, establishes individual rationality. Hence, the final allocation also satisfies the budget balance and individual rationality properties reported in Equation (53).

Appendix D. Sequential LP Computation of the Reduced Nucleolus

The lexicographic minimization in Equation (47) is implemented through a finite sequence of linear programs. All reduced-coalition characteristic values $\bar{v}(A)$ are evaluated and cached before this procedure and therefore enter the following LPs as fixed coefficients. Let $\mathfrak{A} = \{A: \emptyset \neq A \subsetneq \mathcal{G}\}$ denote the set of nonempty proper reduced coalitions. The first lexicographic stage minimizes the maximum reduced-coalition excess:

$$\begin{aligned} \epsilon_1^* &= \min_{\mathbf{x} \preceq \epsilon_1} \epsilon_1 \\ \text{s.t.} \quad &\bar{v}(A) - \sum_{g \in A} x_g \leq \epsilon_1, \quad \forall A \in \mathfrak{A}, \\ &\mathbf{x} \in \mathcal{I}(\bar{\mathbf{v}}) \end{aligned} \quad (\text{A13})$$

where ϵ_1 denotes the first-stage excess bound.

After each lexicographic stage k , let $\mathcal{T}_k$ denote the set of coalitions whose excess is fixed at the optimal level ϵ_k^* on the resulting optimal face. A maximal linearly independent subset $\mathcal{F}_k \subseteq \mathcal{T}_k$ is retained as equality constraints in subsequent stages using a standard rank test. For $k \geq 2$, the subsequent LP is Equation (A14). Thus, the excess levels of

the linearly independent coalitions fixed at earlier stages are retained exactly, while the maximum excess of the remaining coalitions is minimized at the current stage.

$$\begin{aligned} \epsilon_k^* &= \min_{\bar{x}, \epsilon_k} \epsilon_k \\ \text{s.t.} \quad & \bar{v}(A) - \sum_{g \in A} \bar{x}_g = \epsilon_r^*, \quad \forall A \in \mathcal{F}_r, \quad r = 1, \dots, k-1, \\ & \bar{v}(A) - \sum_{g \in A} \bar{x}_g \leq \epsilon_k, \quad \forall A \in \mathfrak{A} \setminus \bigcup_{r=1}^{k-1} \mathcal{T}_r, \\ & \bar{x} \in \bar{\mathcal{I}}(\bar{v}) \end{aligned} \quad (\text{A14})$$

where ϵ_k is the excess bound optimized at stage k .

Let $A_{fix}^{(k)}$ denote the coefficient matrix formed by the reduced-game efficiency equality in (45) and all linearly independent coalition equalities retained through stage k . The procedure terminates when

$$\text{rank}(A_{fix}^{(k)}) = G \quad (\text{A15})$$

Once Equation (A15) is satisfied, the corresponding payoff vector is unique and gives the reduced-game nucleolus $\bar{x}^{nuc}$ in (47). Since all $\bar{v}(A)$ are fixed before this procedure, Equation (A13)–Equation (A15) constitute a sequence of ordinary LPs and introduce no feedback optimization with the preceding clustering or coalition-operation stages.

Appendix E. Derivation of the Out-of-Sample Stability Certificate

For each independent validation coalition $S_m^{val} \sim \mathbb{P}_{split}$, define the Bernoulli violation indicator

$$Y_m(\tau) = \mathbf{1}\{e(S_m^{val}, \mathbf{x}^{SC}) > \tau\}, \quad m = 1, \dots, M_{val} \quad (\text{A16})$$

where $Y_m(\tau)$ denotes whether the m -th validation coalition violates the prescribed excess tolerance τ .

Since the validation coalitions are sampled independently and identically from the prespecified distribution $\mathbb{P}_{split}$, $\mathbb{E}[Y_m(\tau)] = p_{viol}(\tau)$, and their empirical mean is

$$\frac{1}{M_{val}} \sum_{m=1}^{M_{val}} Y_m(\tau) = \hat{p}_{M_{val}}(\tau) \quad (\text{A17})$$

Hoeffding's inequality for independent bounded random variables gives

$$\Pr[p_{viol}(\tau) - \hat{p}_{M_{val}}(\tau) \geq \epsilon_H] \leq \exp(-2M_{val}\epsilon_H^2) \quad (\text{A18})$$

where $\epsilon_H > 0$ denotes a one-sided Hoeffding deviation margin.

Setting the right-hand side of Equation (E3) equal to the prescribed confidence-tail probability β gives

$$\epsilon_H = \sqrt{\frac{\ln(1/\beta)}{2M_{val}}} \quad (\text{A19})$$

Consequently, with probability at least $1 - \beta$,

$$p_{viol}(\tau) \leq \hat{p}_{M_{val}}(\tau) + \sqrt{\frac{\ln(1/\beta)}{2M_{val}}} \quad (\text{A20})$$

Since a probability cannot exceed one, the bound can be written as Equation (57). The guarantee in Equation (57) is distribution-dependent: it bounds the violation probability with respect to the prespecified split-coalition sampling distribution $\mathbb{P}_{split}$. It therefore does not constitute a deterministic worst-case guarantee over every coalition in $\mathfrak{S}_{split}$.

Appendix F. Numerical and Reproducibility Settings

This appendix summarizes the numerical settings used in the reported experiments. Random sampling uses the MATLAB 2024b Mersenne Twister generator with the seeds reported below. Unless otherwise noted, the same settings are retained across participant scales. All optimization problems are solved using Gurobi through MATLAB. All computations were performed on a laptop equipped with a 13th Gen Intel(R) Core(TM) i7-13650HX processor at 2.60 GHz and 16 GB of RAM.

Table A1. Numerical and reproducibility settings used in the case studies.

Item	Setting
Calibration contexts	$M_{\text{cal}} = 32$ per candidate cluster
Split coalition training	$M_{\text{tr}} = 1000$
Independent validation	$M_{\text{val}} = 3000$
Confidence tail	$\beta = 0.05$, corresponding to 95% confidence
Profile selection seed	20260825
Calibration seed	20260826
Training seed	20260827
Validation seed	20260828
DBSCAN N_{min}	{2,3,4}
Neighborhood radius candidates	k -distance quantiles {0.20,0.30,0.40,0.55,0.70,0.85}
Numerical regularization	$\epsilon_{\pi} = \epsilon_m = \epsilon_v = 10^{-8}$, $\epsilon_{\mu} = 10^{-9}$
Feasibility tolerance	10^{-8}

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