

Article

Revealing the Phenomenon of Rank Reversal in Decision-Making by Studying Railway and Road Routes in the Transport Network Using the SIMUS Method

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Abstract

The sustainability of transport planning in intermodal route selection depends on the examination of a set of criteria and alternatives. This paper investigates a dynamic decision-making process by studying the change in ranking when a new alternative is added, or an existing alternative is removed; or when a new criterion is added, or an existing one is removed. The presence of a change in ranking reveals the phenomenon of rank reversal (RR) in multi-criteria analysis. The Sequential Iterative Method for Urban Systems multi-criteria method (SIMUS) is presented as a tool to study the rank reversal problem. The methodology of the research consists of six steps. In the first step, the initial decision-making matrix is compiled. The second step includes the compilation of new decision-making matrices by adding or removing alternatives from the initial decision-making matrix. In the third step, the SIMUS method is applied. In the fourth step, the influence on rank reversal of the addition of alternatives that are very similar or identical to another existing in the system is analyzed. In the fifth step, the influence of a change in the number of criteria on the rank reversal is studied. The sixth step includes determination of the entropy and information of the system. A case for the sustainability of container carriage by railway and road routes between Sofia and Varna in the Bulgarian transport network is demonstrated. Three railway routes and three road routes served by different types of locomotives and heavy trucks have been studied. The routes were evaluated according to the following criteria: CO₂ emissions, operating costs, infrastructure fees, travel time, route length, safety and security factor of transport. The change in the ranking of alternatives was studied by comparison of rankings for cases from 3 to 12 alternatives. A hypothesis based on the assumption that RR in the SIMUS method is a logical consequence of linear transformations along different dimensional spaces, each time an alternative is added or deleted, has been introduced. An invariance coefficient and a rank reversal coefficient have been defined. The values of both coefficients have been classified into groups to explain sustainability in decision-making.



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Keywords: route selection; rank reversal; SIMUS; MCDM; linear programming; railway transport; road transport; intermodal transport; container block train; heavy trucks

1. Introduction

The sustainability of decision-making for intermodal transport route selection between a starting and ending point depends on the analysis of a set of criteria and alternatives, using a scientific-methodological approach.

When analyzing intermodal freight transport along a given corridor with the purpose of selecting the most suitable route, it is common to compare several alternatives, taking into account a set of criteria. The alternatives may include transport with one mode of transport, or the comparison of two or more modes of transport. In the first case, transport with different rolling stock and transport along different routes are compared. In the second case, transport between two points is compared, with alternatives set for each mode of transport. This process is stationary, i.e., with given specific parameters, in which it can be expected that if the operational data remain unchanged, the relative behaviour of the alternatives should also remain unchanged, i.e., the ranking of the transport alternatives is preserved.

The transport process is dynamic. This requires transport operators to observe the changes in the transport infrastructure, transport conditions and transport equipment and to make operational decisions when the transport environment changes. Once a transportation alternative has been selected, a new alternative may appear to be evaluated, or an alternative that has been evaluated may be dropped from consideration. Therefore, the important question is how the choice of the most suitable alternative changes if a new one is added (for example, transport by rolling stock or trucks with different parameters; a new route) or one of the previously studied alternatives is removed from consideration (for example, road repairs, severe weather conditions or other). In these cases, a new ranking must be carried out, in which the new alternative may or may not displace the previous ranking. In decision-making analysis, this behaviour is known as Rank Reversal (RR). It refers to situations in which the relative position of an alternative changes when new alternatives are added to the comparison, even though the original data remain the same.

This study examines the problem of route selection in its dynamics by applying the Sequential Iterative Method for Urban Systems multi-criteria method (SIMUS). The research questions are as follows:

- Is there rank reversal when using the SIMUS method, and what are the reasons for this phenomenon?
- How does adding new alternatives to the initial data or removing existing ones affect the decision-making process?
- How does adding new criteria or removing already set ones affect the ranking of alternatives?

The multi-criteria decision-making methods (MCDM) are widely used in solving problems in various areas, including the selection of an intermodal transport route. In this study, the SIMUS method, which uses linear programming, was chosen to select the most suitable route and assess changes in the ranking of transport alternatives.

The main advantages of the SIMUS method can be summed as follows:

- SIMUS can be classified as a multi-objective optimization method (MOO) rather than MCDM, and solves a variety of problems.
- It works systematically and not algebraically; it acts as a system and is data-driven.
- It does not use weights and personal preferences. The values of the factors are known.
- Expert assessments are not used.
- Both quantitative and qualitative criteria can be explored.
- It is based on the linear programming method (LP). It is an objective method because it applies optimization.

- It works with restrictive conditions, while MCDM assumes that resources have no limits.
- SIMUS is based on LP, but it has a proper algorithm different from LP that allows considering multiple objectives, whereas LP is mono-objective. These methods do not have the same structure. Linear optimization is applied to each criterion separately, with the others being considered as constraints. The results are summarized in a decision matrix to which SIMUS-specific procedures are applied.
- A sensitivity analysis could be performed for input data. As a result of the linear optimization of each criterion, the values of the upper and lower limits of the criteria could be determined.

This paper investigated the case of transportation with block trains vs. heavy trucks on the Sofia–Varna corridor in the Bulgarian transport network. This corridor was chosen because it is of key importance for the economic connection of Western and Eastern Bulgaria with the European transport network. The freight transport can be organized either through block trains carrying containers along the main railway routes, or through heavy trucks operating entirely on the road network. Although both solutions serve the same corridor, they differ significantly in operational behaviour, capacity, gradients, running times and sensitivity to traffic conditions.

The hypothesis of this research is that RR in the SIMUS method is due to linear transformations along different spatial dimensions, which happen every time an alternative is added or deleted. The hypothesis holds that when an alternative is added or deleted, a new and different modelling of the problem takes place, and therefore invariance of results should not be expected when comparing different scenarios.

The purpose of this paper is to investigate the rank reversal problem in the SIMUS method and to show how the analytical behaviour changes when the context changes. For this purpose, the case study for route selection in a transport network for the carriage of containers was examined.

The focus of this study is on the following problems:

- Structural effect: The outcome is influenced not only by the operational values of each alternative, but also by the presence or absence of other alternatives in the comparison. This dependence on the composition of the option set is precisely what gives rise to RR.
- Analytical effect: This is related to the interpretation of results of RR. The purpose is not to recommend a specific route, but to show how the analytical behaviour changes when the context changes.

This paper is structured as follows. Section 2 presents a literature review of RR theory. Section 3 gives the methodology of the research and the hypothesis about RR occurrence. Section 4 presents a detailed analysis based on the methodology of rank reversal for the case of ranking railway and road routes connecting the same starting and ending point in the Bulgarian transport network. The conclusions are given in Section 5.

2. Literature Review

Various methods are used in scientific research to select the most suitable route for intermodal transport between a starting and ending point. Table 1 represents a systematization of the methods applied in the studied scientific research.

Table 1. Approaches for intermodal transport route selection.

Method	Technique	Criteria	Source
MCDM	AHP, SWARA, LOPCOW, MACONT	Infrastructure constraints, road safety, costs and delivery	[1]
	MCDM and SP	Cost, CO ₂ emissions, delivery time, safety, reliability, flexibility, and frequency	[2]
	fuzzy DELPHI, fuzzy DAHP, fuzzy ADAM	Economic, environmental, safety and quality criteria	[3]
	BWM, FHRA, ARAS	Cost, time, freight damage risk, different types of risks	[4]
	AHP, DEA, TOPSIS	Costs, time, operational risks, infrastructure risks, macro risks, and policy and legality risks	[5]
	AHP, DEAMATEL, CP	Environmental protection; economic criteria; additional transportation fare; the duration of transportation; transshipment operations; security; reliability; stability	[6]
GIS, MCDM	fuzzy DEAMATEL, ADAM	Ecological criteria	[7]
MCDM and MOO	DEA, MOO	Cost, time, reliability, and air and noise pollution	[8]
	AHP, ZOGP	Costs, time and freight damages	[9]
	Fuzzy AHP, ANN	Cost, time, risk and unreliability	[10]
	I-NSGA2, TOPSIS	Transportation cost, transshipment cost, tariff cost, time value of cargo, carbon tax cost, in-transit and customs clearance time, and transshipment-related carbon emissions	[11]
	AHP, ZOGP	Costs, time, CO ₂ emissions, integrated quantitative risk	[12]
MOO	MILP	Cost, time	[13]
	(NSGA-II)	Costs, time, carbon dioxide emissions	[14]
SOO	SOOP	Transport cost, transfer cost, and cost of carbon emissions	[15]
	DP	Costs, time, transport quality	[16]

It can be summarized that the following approaches are most often used:

- MCDM methods and Evaluation Tools. The authors used AHP, DEMATEL method, BWM, SWARA, ARAS, ADAM, CP, DEA [1–6];
- Integration between GIS and MCDM methods [7];
- Integration between MCDM and MOO [8–12];
- MOO [13,14];
- SOO Single-objective optimization [15,16].

The MCDM methods are most often applied in scientific research for route selection. In Ref. [1], the authors proposed a complex technique based on the aggregation of Step-wise Weight Assessment Ratio Analysis (SWARA) and Logarithmic Percentage Change-driven Objective Weighting (LOPCOW) for determining the weights of criteria and Mixed

Aggregation by Comprehensive Normalization Technique (MACONT) techniques for route selection in transporting heavy and bulky loads. Infrastructure constraints, road safety, costs and delivery time are taken into account. It was found that fuel consumption is the most important factor in selecting the route of transportation. The problem of determining the most important criteria for intermodal freight transportation has been studied in [2]. The authors analyzed a large database of publications that used MCDM and the Stated Preferences (SP) experiments. The authors found that the criteria cost, CO₂ emissions, delivery time, safety, reliability, flexibility, and frequency were most often used to evaluate intermodal transport.

The choice of the most favourable alternative to the intermodal transport chain was studied in [3]. A multicriteria model was elaborated based on the fuzzy Delphi analytical hierarchy process (fuzzy DAHP) and the fuzzy Axial-Distance-based Aggregated Measurement (fuzzy ADAM) methods. Three criterion groups were defined to assess intermodal routes as follows: economic, environmental and safety and quality criteria. The case of intermodal transportation from China to Bosnia and Herzegovina. It was found that the transport chains that include rail transport are sustainable for planning.

In Ref. [4], the problem of selecting multimodal transport routes was investigated based on a complex approach integrating the best-worst method (BWM), fuzzy hierarchy risk assessment (FHRA), and additive ratio assessment (ARAS). Three hierarchical levels of criteria were defined. Transportation cost, transportation time, freight damage risk, and different types of risks were included in the first level. In the second and third levels, the sub-criteria were defined. A case study of multimodal transport route selection between a factory in Thailand and a distribution centre in Singapore was conducted.

An integrated approach based on the analytic hierarchy process (AHP), data envelopment analysis (DEA), and the technique for order preference by similarity to ideal solution (TOPSIS) for route selection in multimodal transport was elaborated in [5]. The authors proposed the criteria transportation costs, transportation time, operational risks, infrastructure risks, macro risks, and policy and legality risks for assessing the intermodal routes. A case study for transportation of 20-foot containers from the starting point in Thailand to the final point in Vietnam was presented, comparing sixteen routes.

In Ref. [6], the Shannon entropy method, Analytic Hierarchy Process (AHP) method, the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method and Compromise programming method (CP) were integrated into a complex methodology for container route selection. The following criteria have been studied: environmental protection; economic criteria; additional transportation fare; the duration of transportation; transshipment operations; security; reliability; stability. The methodology is tested for routes from the railway and road network of Bulgaria.

A combination of Geographic Information System (GIS) and MCDM is also applied for selecting the most appropriate route. In Ref. [7], GIS was integrated with the fuzzy Delphi and fuzzy Decision-Making Trial and Evaluation Laboratory (fuzzy DEMATEL) methods to determine the weights of the criteria and the Axial Distance-based Aggregated Measurement (ADAM) method was used for ranking the routes. The criteria for evaluating routes were selected with an ecological focus and were determined using GIS data of the European Environment Agency. The methodology was applied to several Trans-European Transport Network (TEN-T) routes.

Some authors used a combination of MCDM and MOO. In Ref. [8], the problem of choosing a route between the initial and final destination when comparing different modes of transport has been studied. The methodological approach consists of the application of Data Envelopment Analysis and a multi-objective model considering cost, time, reliability, and air and noise pollution for optimal route selection. The model was tested for the case

of Thailand and Vietnam. Three origins and three destinations, as well as different vehicles and different routes, were considered. It was found that changing one parameter can lead to changes in the vehicle set.

The AHP and zero-one goal programming (ZOGP) were integrated to determine an optimal multimodal transportation route [9]. The methodology uses the criteria: costs, time and freight damages. A case for route selection between a start point and an end point.

A mathematical model based on artificial neural network (ANN) theory and the Fuzzy Analytic Hierarchy Process (fuzzy AHP) for route selection in multimodal transport was elaborated in [10]. The criteria considered were cost, time, risk and unreliability.

An improved NSGA-II algorithm (I-NSGA2) and entropy weight-TOPSIS method were incorporated to solve the problem of route selection [11]. The model includes transportation cost, transshipment cost, tariff cost, time value of cargo, carbon tax cost, in-transit and customs clearance time, and transshipment-related carbon emissions. A case study on the Nanning–Kuala Lumpur route was presented.

The route selection in multimodal transport based on MCDM methods was investigated in [12]. The criteria costs, time, CO₂ emissions, and integrated quantitative risk were applied for assessment of intermodal routes. A case study for Thailand and Vietnam was presented to demonstrate the methodology. The AHP method was applied to calculate the weights of criteria. The optimal route was determined by using zero-one goal programming. Eleven multimodal transportation routes were compared.

The methods of MOO are also applied to solve the route selection problem. In Ref. [13], a bi-objective mixed integer linear programming model (MILP) based on the normalized normal constraint method that includes transportation cost and transportation time was elaborated for route planning in a multimodal network. The carriage of 20 ft containers by railway, highway and waterway routes was examined. In Ref. [14], the authors used the non-dominated sorting genetic algorithm II (NSGA-II) to solve a problem for route selection with three optimization functions, including transport time, transport costs and carbon dioxide emissions. The case study of alternative routes for container multimodal transport between China and Europe was presented.

In some research, single-objective linear or nonlinear optimization (SOOP) is applied. A mathematical model for route selection based on a single objective function that included transport cost, transfer cost, and the cost of carbon emissions was presented in [15]. Multimodal transportation by rail, road and maritime routes between two major cities in China is examined. The dynamic programming algorithm (DP) is proposed in [16] to solve the problem of route selection of container transportation. The transportation costs, transportation time and transport quality are defined as main factors. The objective function presents the minimization of the generalized transport cost.

All of these studies consider the problem of route selection under specific stationary conditions, i.e., the given criteria and alternatives do not change. The effect of gradually adding alternatives and criteria to the decision-making process, and also the question of how this affects route selection, is not considered. The decision-making process is dynamic, and therefore the task of route selection must be considered in the conditions of a dynamically changing environment. Suitable methods for research are multi-criteria analysis methods, in which the most suitable alternative is selected from a given set of alternatives, taking into account the influence of a given set of criteria.

The RR is a phenomenon that occurs when a new alternative is added, or an existing alternative is deleted, and there is an abundance of theories about it from different researchers. Some authors analyzed the RR occurrence, while others sought to find a way to remove it. Table 2 represents the MCDM methods that have been investigated in the scientific literature on RR.

Table 2. Rank reversal in MCDM.

MCDM Method	Research Method	Cause of RR	Source
AHP	Numerical examples	Modification in normalized values	[17]
BK		Changes in ordinal values	[17]
SAW		Modification in normalized values	[17]
TOPSIS		Modification in normalized values	[17]
DEA		Changes in cross-efficiencies	[17]
TOPSIS	The positive and negative ideal solutions	Modification in normalized values	[18]
EDAS	Indices to analyze RR	Added new information about alternatives	[19]
TOPSIS		Modification in normalized values	[19]
NR—TOPSIS	Normalization of the benefits and costs	Invariant ranking	[20]
PROMETHEE	Pairwise comparison	Net preference flow	[21]
AHP	Effect of aggregation methods	Number of experts	[22]
AHP	Pairwise comparison	Transitivity	[23]
AHP	Gower plots and optimization models	Adding dominating alternatives	[24]
AHP	Pairwise comparison	Adding dominating alternatives	[25]
FAHP	Relative total utility	Change in local priorities	[26]
TOPSIS	Normalization procedures; Shannon entropy	Type of normalization	[27]
AHP	Analysis of publication in the field	Violation of alternatives independence	[28]
TOPSIS			
PROMETHEE			
ELECTRE			
COPRAS			
TOPSIS	Spearman correlation index	Modification in normalized values	[29]
VIKOR			
SAW			
ELEKTRE II	Combination relation digraph; numerator matrix	The change in local priorities of alternatives	[30]
Aggregation methods	Modified score aggregation operators	Inconsistency of information	[31]
RAFSI	Functional mapping approach	Eliminate the rank reversal problem	[32]
SPOTIS	Comparisons with ideal solution	Eliminate the rank reversal problem	[33]

Some multi-criteria methods such as ANP, Borda-Kendall (BK), Simple Additive Weighting method (SAW), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and Data Envelopment Analysis (DEA) were analyzed in [17]. The authors concluded that RR is a normal phenomenon and that the changes are due to modifications in normalized values. In Ref. [18], the TOPSIS method was examined according to RR. The authors proposed an approach to eliminate RR by introducing a modification associated with the positive and negative ideal solutions. They proposed that when new alternatives

are added, the positive and negative ideal solutions should be calculated only for the newly introduced alternatives. In Ref. [19], the authors indicated that the cause of RR is when new information about alternatives or criteria is added to the MCDM problem. They suggest three indices to analyze RR: percentage of the frequency of occurrence, percentage of the frequency of occurrence in first rank and intensity of occurrence. These indices were applied to investigate two multi-criteria methods—the evaluation based on distance from average solution (EDAS) method and TOPSIS method. It was found that RR in both methods is more probable in more than 50 alternatives. The results given by the introduced indices in regard to RR established that EDAS is more efficient than the TOPSIS method. A new improved TOPSIS method, called NR-TOPSIS, which overcomes RR, was introduced in [20]. The extended NR-TOPSIS is based on the normalization of two types of attribute indicators: benefit and cost. Results show that the ranking is invariant.

In Ref. [21], RR in Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), was investigated, and it was established that the cause of RR of two alternatives in the global ranking is when a third alternative is removed. The authors introduce the concept of pairwise rank reversal. They proved that rank reversal depends on the principle of pairwise comparison. Examining the PROMETHEE II method, they show that rank reversal depends on the net preference flow.

In Ref. [22], the authors indicated the distribution of criteria weights and entropy as the probability of RR in the AHP method. The authors concluded that RR depends on the number of experts and that the probability of RR decreases when the number of experts increases. The effect of aggregation methods on RR was investigated. The AHP method and rank reversal were studied in [23]. The authors prove that rank reversal happens when a big decision problem is broken down into smaller sub-problems that take into account the same criteria. It was found that the additive methods are the cause of rank reversal. It is suggested to use the multiplicative version of AHP to avoid this inconvenience. The RR problem in the AHP method was investigated using Gower plots and optimization models [24]. In Ref. [26], the authors introduced the relative total utility in the FAHP method as a criterion for ranking. Thus, it helps to maintain the independence of alternatives and to eliminate the RR problem.

In Ref. [29], it was considered that the addition or elimination of an alternative is a cause of RR; they investigated three MCDM methods, namely Complex Proportional Assessment method (COPRAS), TOPSIS and ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR). The authors proposed a procedure to eliminate RR on the basis of TOPSIS, COPRAS and SAW, based on comparison using the Spearman correlation index. The resemblance between a pair of rankings is analyzed using this index. This procedure was proposed to make decisions against the RR problem. The RR in the ELECTRE II (ELimination and ChoiceExpressingREality) method was analyzed in [30]. The authors investigated the causes of RR in the ELECTRE method. Two concepts have been proposed to analyze the RR in the ELECTRE II method. One is named combination relation digraph, and the other is named numerator matrix.

In Ref. [28], the TOPSIS method was studied, and it was explained that the reasons for RR are the normalization procedures and the choice of a positive ideal solution and an ideal negative solution. The authors analyzed more than 130 papers related to MCDM methods. Based on this analysis, the authors formulated the following cases for RR: Synthesis procedure, normalization, criteria weights, misuse of the method, the uncertainty of the MCDM, and structural dependency between criteria and alternatives. It was found that the main gap regarding RR seems to be the absence of convincing explanations for RR. Inconsistency of information is listed as the main cause for RR in [31]. The authors studied what is possible to happen in different cases, as follows: a new alternative is introduced, an

existing alternative is removed, an alternative is replaced by another one. For each case, the score of each alternative is determined using the sum normalization method proposed by the authors. The rank reversal in the TOPSIS method was studied in [27]. The authors analyzed different normalization techniques and entropy and their influence on ranking. It was found that the choice of normalization can be a cause of differences in preference values and rankings.

In Ref. [32], the authors attempted to remove the inconsistency of information as a cause of RR. The authors introduced a new method, called RAFSI (Ranking of Alternatives through Functional mapping of criterion sub-intervals into a Single Interval), which eliminates the RR. The method proposed a new approach for data normalization based on arithmetic and harmonic means.

In Ref. [33], a new MCDM method called Stable Preference Ordering Towards Ideal Solution (SPOTIS) was proposed, which is rank reversal free. This is due to the fact that the preference ordering established from the decision matrix is performed only by comparisons with respect to the ideal solution.

The AHP rank reversal problem was presented in [25] by Saaty. He defends the thesis that this is a natural phenomenon due to a change in information.

It can be summarized that some authors determine the cause of RR to the type of normalization [27], while others authors assert that the cause is the lack of consistency in the decision-makers [31,32], or to transitivity [23], or to similarity with existing alternatives [17], or to violation of alternatives independence [28,33], or that the performance of a new alternative is unpredictable [19], or by adding a dominating alternative [24,25], or that there is no substantive solution to the problem of rank reversal [20], or change in local priorities [26].

At present, the cause of the RR problem, discovered by Belton and Gear [34], has not been found, since no explanation can elucidate its nature or why sometimes it appears and, in other cases, it is absent.

The SIMUS method has not been investigated regarding the causes of rank reversal.

This paper aims to reveal the phenomenon of rank reversal in decision-making by studying railway and road routes in the transport network using the SIMUS method.

The hypothesis proposed in this paper has no similarity to any of the alleged causes of RR. It claims that rank reversal is a natural consequence of changing the dimensional spaces of a problem when adding or deleting an alternative. This study considers it to be an unavoidable, unpredictable and easily explained phenomenon.

3. Materials and Methods

The methodology for revealing the phenomenon of rank reversal consists of the following steps (Figure 1):

- Step 1. Compiling the initial decision-making matrix $A_{m \times n}$, where $i = 1, \dots, m$ is the number of criteria; $j = 1, \dots, n$ is the number of alternatives. In this step, the decision maker defines the criteria and alternatives for research.
- Step 2. Step-by-step addition of new alternatives or removal of existing alternatives to the initial decision-making matrix. In this step, new matrices are formed: $A_{m \times (n-1)}, A_{m \times (n-2)}, \dots, A_{m \times 2}, A_{m \times (n+1)}, A_{m \times (n+2)}, \dots, A_{m \times (n+N)}$; $n > 2$.
- Step 3. SIMUS (Sequential Iterative Modelling in Urban Systems) method for ranking the alternatives. SIMUS method is applied to the initial decision matrix $A_{m \times n}$ and for the matrices obtained by adding or removing alternatives from the initial matrix. Ranking the alternatives for all matrices separately. Study the main objective of the problem.

- Step 4. Adding alternatives that are very similar or identical to another existing in the system. Study the influence on rank reversal.
- Step 5. Changing the number of criteria without changing the number of alternatives. Study the influence of changing the number of criteria on rank reversal.
- Step 6. Determination of the entropy and information of the system. Study the influence of entropy and information for all matrices formed when adding or removing alternatives on rank reversal.

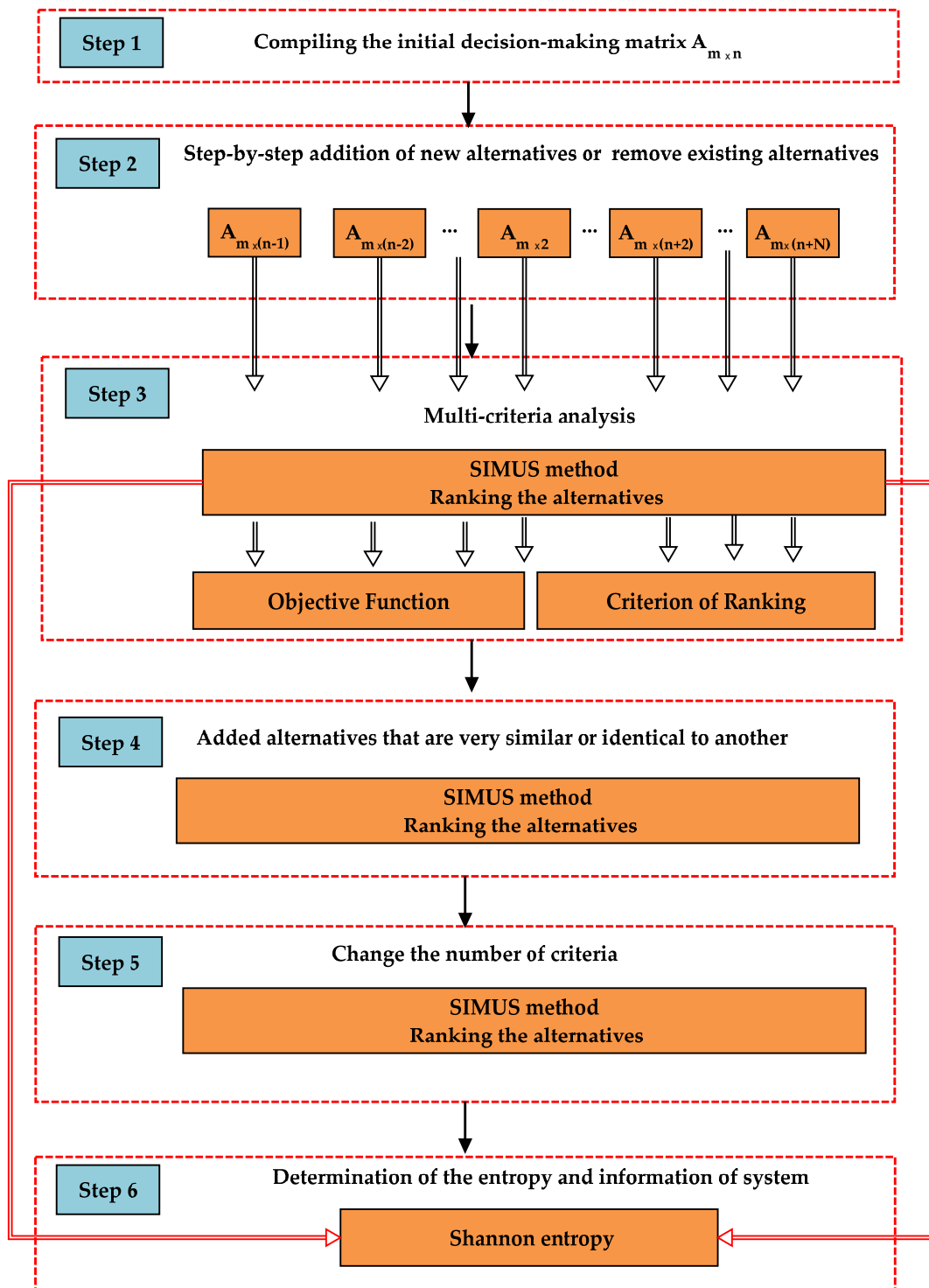


Figure 1. Scheme of methodology.

3.1. Step 1. Compiling the Initial Decision-Making Matrix

The initial decision matrix is formed according to the given alternatives and criteria.

There is a preselected set of n finite alternatives $A_j, j = 1, \dots, n$ in any number and types. A Multi-Criteria Decision-Making (MCDM) problem looks to selecting the most appropriate set of alternatives and ranking them in order of importance. This importance is measured quantitatively by the number of criteria or objectives each alternative result is related to, and the degree of fulfilment of each one. That is, the best solution must be connected to the maximum number of criteria (association) and, for each one of them, as close as possible to their respective goal (satisfaction), either in maximization, minimization or equalization.

The aim of an MCDM method is to find the best alternative that complies with these two requirements and to order them in a ranking.

The objective function, identified as Z_i , shows the variables or alternatives, each one affected by a coefficient that indicates the contribution of each alternative to achieving the general objective of the problem. It may call for maximizing a benefit, minimizing a cost or equalizing a budget, for instance, and it is only an aspiration or a wish, since there is no specific goal to achieve; i.e., it is indefinite. Z_i is a fundamental component of MCDM because it allows finding the best alternative.

Irrelevant to the number of alternatives, but not to their nature, there is a set of criteria $c_{ij}, i = 1, \dots, m$ them. Criteria are also objectives, but they are not indefinite like the objective function, since each criterion needs a numeric goal to achieve; consequently, this is mandatory for each criterion. These goals also involve resources of any kind necessary to execute the project, as well as their limits. This goal may indicate a maximum or minimum.

There could be any number m of criteria, and they must be selected according to the nature and characteristics of each problem, since there are no universal criteria. Criteria may be objective or subjective, and in any mix.

The elements of the decision matrix a_{ij} are the performance values which are objective or subjective, placed in the cells at the intersections of alternatives (in n columns) and criteria (in m rows). They are the contributions from each alternative to each criterion. Each criterion has attributes that refer to its characteristics; for instance, the elements of the decision matrix may be expressed in decimals, integers, or binary. Although each criterion must preserve uniformity in its characteristics, another criterion may have different characteristics, and they can coexist in any mix.

3.2. Step 2. Step-by-Step Addition of New Alternatives or Removal of Existing Alternatives to the Initial Decision-Making Matrix

The alternatives are of two types—initial, those that are included in the initial decision-making matrix, and additional, which are added one by one in the process of researching the rank reversal phenomenon. In this step, new decision matrices are formed $A_{m \times (n+1)}, A_{m \times (n+2)}, \dots, A_{m \times (n+N)}$. N is the total number of alternatives added. In many cases, aggregating or deleting alternatives does not produce RR. For instance, if in a 2D we have a ranking $A_2 > A_1$, aggregating an alternative A_3 does not necessarily mean RR, since A_3 maybe in any position without altering the original precedence. Precedence is a key factor in RR. As an example, if the original ranking is $A_2 > A_1$, and by adding a new alternative A_3 the ranking is $A_2 > A_1 > A_3$. The original precedence of A_2 on A_1 is preserved; therefore, there is no RR. But if the new ranking is $A_1 > A_2 > A_3$, notice that the precedence of A_2 on A_1 is lost, and then there is RR.

Other rankings may appear in this same problem, like:

$$\begin{cases} A_2 > A_3 > A_1 \\ A_2 > A_1 > A_3 \\ A_1 > A_3 > A_2 \\ A_3 > A_1 > A_2 \end{cases} \quad (1)$$

In the first two cases, even when the ranking changed, there is no RR because the original precedence between A_2 and A_1 is preserved. But not in the third and fourth cases, where the precedence indeed changed.

The reason lies in the fact that for a certain problem, the number of alternatives is linked to the amount of information we may have on it; the more alternatives or options in a problem, the greater the information or knowledge we have on it.

With two alternatives, or in the 2D space, we have very scant information, and the two alternatives must satisfy as many criteria as possible. It could also be that there is no solution, i.e., the problem is infeasible.

Adding A_3 , a new scenario opens for us, since now we have more information, and consequently, better results. The number of criteria, their objective and attributes do not vary, but what changes is the number of performance values, due to the A_3 vector.

When adding a new vector, the size of the criteria set changes; for now, there is an additional vector, and with performance values similar to or different from the existing ones, i.e., both, the number of alternatives and the extension of the set of criteria are now different. Therefore, all precedent computation is superseded. That is, increasing alternatives or dimensional spaces will produce more and more information that will modify the precedent rankings and produce RR, since there is no reason for their preservation. It could also be that in some dimensions there is no effect, i.e., the ranking replicates, and this explains why in some opportunities there is RR while in others it is absent.

Adding a third alternative may or may not produce changes. In our case, these special characteristics are the coefficient of the new alternative and the performance value added to each criterion.

Each time an alternative is added, a new scenario is created, and by cracking it, a different problem is solved, depending on the performance values in the added alternatives.

In this step, by adding new alternatives or removing existing alternatives from the initial decision-making matrix, new matrices are formed: $A_{m \times (n-1)}$, $A_{m \times (n-2)}$, $\dots$, $A_{m \times 2}$, $A_{m \times (n+1)}$, $A_{m \times (n+2)}$, $\dots$, $A_{m \times (n+N)}$; $n > 2$. Each addition or removal of an alternative puts the system in a new dimension in multidimensional space that is different from the previous one.

The dimensional space is simply a mathematical space defined by a set of independent axes (dimensions). In LP, it is the Cartesian coordinate system, where axes are the alternatives and criteria that define a figure like a square in 2D and a polygon in 3D.

3.3. Step 3. SIMUS Method for Ranking the Alternatives

In this step, the SIMUS method is applied for the initial decision matrix and for all matrices created by adding alternatives. SIMUS uses linear programming (LP). LP is a mono-objective method and thus considers one objective at a time. For multiple objectives, we use the SIMUS method [35], based on LP, that considers multiple objectives but follows the LP Simplex algorithm; thus, the results can be assumed to represent a balance, a compromise, among all actors of the problem.

The methodology of the SIMUS method consists of the following steps:

- Forming the initial decision matrix $A_{m \times n}$;

- Normalization of the initial decision matrix. For this purpose, different methods of normalization, such as the maximum element, sum of the row, or others, could be used. In this step, a normalized initial decision-making matrix is formed: $LHS_{m \times n}$;
- Determine the type of optimization and the type of constraints for each criterion. The Threshold Values of the Criteria (RHS) are defined according to the type of optimization. RHS presents the Right-Hand Side of the decision matrix, while LHS presents the Left-Hand Side of the decision matrix.

$$RHS_i = \max_j a_{ij}, \text{ when } Z_i \rightarrow \max \quad (2)$$

$$RHS_i = \min_j a_{ij}, \text{ when } Z_i \rightarrow \min \quad (3)$$

- Forming a linear optimization model for each criterion.

For the first criterion, the linear optimization is as follows:

$$Z_1 = \sum_{j=1}^n b_{1j}x_{1j} \rightarrow \max(\min) \quad (4)$$

Constraints:

$$\left| \begin{array}{l} \sum_{j=1}^n b_{2j}x_{2j} \leq (\geq) RHS_2 \\ \dots \\ \sum_{j=1}^n b_{mj}x_{mj} \leq (\geq) RHS_m \\ x_{ij} \geq 0 \end{array} \right. \quad (5)$$

where x_{ij} is the score of alternative j for criterion i ; b_{ij} are the elements of the normalized initial decision-making matrix $LHS_{m \times n}$.

The type of constraint depends on the type of optimization: $\leq$ when $Z_i \rightarrow \max$; $\geq$ when $Z_i \rightarrow \min$.

The linear models for all other criteria are formed in a similar way.

- The results of linear optimization models based on the simplex algorithm are recorded in the efficient results matrix $ERM_{m \times n}$. The number of linear optimizations is equal to the number of criteria. First, the first criterion is used as the objective function, while the constraints are formed by the other criteria. The results are recorded in the first row of ERM. Then the procedure is repeated with the remaining criteria.
- Ranking procedure

In this step, the normalized efficient results matrix is calculated as $NERM_{m \times n}$. The normalization procedure is the same as that for $LHS_{m \times n}$, for example, the sum of the rows.

The score for ranking of alternatives is determined as follows:

$$S_j = NPF_j SC_j \quad (6)$$

$$NPF_j = \frac{PF_j}{n} \quad (7)$$

$$SC_j = \sum_{i=1}^m f_{ij} \quad (8)$$

where NPF_j is the normalized Participation Factor; PF_j is the participation factor of alternative j ; f_{ij} are the elements of the normalized efficient results matrix $NERM_{m \times n}$ in column j ; SC_j are the sum of the column j .

The alternatives are ranked according to the maximal value of the criterion S_j . The best alternative has the maximal values of the criterion S_j .

The SIMUS procedure is applied for all matrices investigated. The alternatives are ranked according to the results of the SIMUS method for each matrix separately. In this step, the change in the objective function in each optimization is examined.

3.4. Step 4. Added Alternatives That Are Very Similar or Identical to Another Existing in the System

In this step, the proposition is investigated that added alternatives that are very similar or identical to another existing in the system (close vectors) may generate RR. The initial decision matrix is examined by adding alternatives with the same values. The matrix $A'_{m \times (n+1)}$ is created from the matrix $A_{m \times n}$ by adding the column $(n+1)$, which is identical to the column n . It is shown in grey in (9). The goal is to examine what the change in ranking is.

$$A_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}; A'_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & a_{1(n+1)} = a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & a_{m(n+1)} = a_{mn} \end{pmatrix} \quad (9)$$

3.5. Step 5. Change the Number of Criteria Without Changing the Number of Alternatives

In this step, new matrices are formed with an identical number of alternatives (columns) but a different number of criteria (rows). The matrix $A'_{(m-1) \times n}$ is created from the matrix $A_{m \times n}$ by removing the criterion row (m) . The goal is to examine what the change in ranking is.

$$A_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix}; A''_{(m-1) \times n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{(m-1)1} & a_{(m-1)2} & a_{(m-1)3} & \cdots & a_{(m-1)n} \end{pmatrix} \quad (10)$$

3.6. Step 6. Determination of the Entropy and Information of the System

This subject aims to confirm, by another means, that entropy, not related to Linear Programming, increases with the number of alternatives. The total normalized entropy of the system is determined by Shannon's entropy (H) formula, that is [36]:

$$H = -k \sum_{i=1}^n p(x)_i \ln p(x)_i \quad (11)$$

$$k = \frac{1}{\ln(n)} \quad (12)$$

$$I = 1 - H \quad (13)$$

where $p(x)_i$ is the probability of the variable x_i ; n is the number of variables (alternatives); k is a factor to compute the average value, I is the information.

4. Results and Discussion

In this paper, the phenomenon of rank reversal in decision-making using the SIMUS method is demonstrated by studying the carriage of containers by railway and road routes between Sofia and Varna for the Bulgarian transport network.

The Sofia—Varna transport route is a main artery between the capital of Bulgaria and the country's largest Black Sea port. The route is part of important regional and trans-European networks connecting the Black Sea with Europe. It allows for the combination of land (train and truck) with sea transport, which is of key importance for supply chains.

The study was conducted under the following conditions:

- The transportation is carried out within the existing capacity of the railway and road routes.
- Containerized cargo is transported.
- The carriers are licenced.
- Real routes from the railway and road networks between Sofia and Varna are compared. The research includes three railway and three road routes.
- The transportation is compared in terms of environmental, financial, and technological aspects.
- The carriage by railway transport is carried out through block freight trains. The train is composed of 20 wagons (container carriages, tare 24 t). The wagons are loaded with 40-foot containers with a gross mass of 20 t. The gross train weight is 1086 t. Gross weight of one wagon—44 t. Two types of locomotives were investigated—electric locomotive series 46 (Electroputere, Craiova, Romania) and electric locomotive Siemens Smartron (Siemens Mobility, Munich, Germany).
- Carriage by road transport is carried out by heavy trucks with a load capacity of 24 tons and a gross weight of 36 tons. Two types of heavy trucks were investigated—diesel and electric.
- Transportation of full containers in the direction from Sofia to Varna is being considered.
- The maximum speed of the road train when driving on a highway is 100 km/h, and when driving on a category II road, it is 70 km/h.
- In order to ensure comparability of the results, the routes and both types of transport are compared for one container.
- The influence of weather conditions is not taken into account.
- The time for loading and unloading containers is not taken into account.
- The transshipment fees are not taken into account.
- The data and fees are from June 2026.

The 46 series locomotive is an electric locomotive that is mainly intended for freight transport. It has been in operation in the Bulgarian Railways since 1986, and in recent years, modernized machines with computer control, new wheel reducers for higher speed and modern electrical and brake systems have been operated. Its maximum power is 5100 kW; the maximum speed is 160 km/h; the weight is 126 tons. The locomotive uses an older generation of power circuits without the possibility of recuperation.

The Siemens Smartron is an electric locomotive, designed primarily for hauling passenger and freight trains. The Siemens Smartron locomotive has been in operation in Bulgaria since 2019. Its power is 5600 kW; the maximum speed is 160 km/h; the weight is 83 tons. The locomotive is equipped with modern transistor converters and a regenerative brake. When stopping, the locomotive returns electrical energy back to the contact network, which reduces electricity consumption compared to series 46 locomotives.

The heavy-duty electric trucks have the following parameters: extended range, capable of travelling up to 700 km on a single charge; Usable battery energy—780 kWh.

The heavy-duty diesel trucks meet the Euro V European environmental standard [37] for harmful emissions.

4.1. Step 1. Initial Decision-Making Matrix

This study begins with six alternatives based on real operational values. These routes represent main options, each with distinct characteristics in terms of distance, gradients, electrification and expected performance. This study does not attempt to reproduce any official ranking of the Sofia–Varna routes. Its purpose is simply to present the operational

characteristics of these alternatives and the initial results obtained from applying the operational conditions considered.

When additional operational scenarios are introduced, the behaviour of the alternatives changes in a way that is not immediately intuitive. The operational data remain the same, yet the relative position of the alternative shifts. This reaction is subtle, but it reveals a deeper structure in the way these comparisons behave. This is a phenomenon that this paper aims to analyze. To illustrate this, additional alternatives are introduced one by one. These new options are constructed in a realistic way. Each one represents a plausible operational scenario. What matters is not their absolute value, but the effect they have on the results when they enter the comparison. As these alternatives are added, the results change. Some routes move up, others move down from their former positions, and in some cases an option that was previously well-positioned may lose its place simply because a new option has been added to the set. The operational data of the original alternatives do not change; only the composition of the set does.

This behaviour is important in transport engineering, because it shows that the relative position of an alternative is not only a function of its own operational attributes, but also of the context created by the set of options considered at the same time, i.e., the structure of the comparison can influence the outcome.

Based on an analysis of the criteria used in the scientific literature for selecting an intermodal transport route, the following criteria have been studied:

- C1—Carbon dioxide emissions during transport, kg/UTI;
- C2—Operational costs for intermodal transportation. They include the cost of power consumption, maintenance and repair of rolling stock, personnel, depreciation allowances and other operational expenses, in euros.
- C3—Infrastructure charges, euros.
- C4—Duration of transportation, h;
- C5—Light on the route, km;
- C6—Security factor, coef.

The initial decision matrix consists of the following alternatives representing railway transportation:

- A1—route Sofia—Gorna Oryahovitsa—Varna, Locomotive Ser. 46.
- A2—route Sofia—Karlovo-Karnobat—Varna, Locomotive Ser. 46v.
- A3—route Sofia—Plovdiv—Karnobat—Varna, Locomotive Ser. 46.
- AS1—route Sofia—Gorna Oryahovitsa—Varna, Locomotive Siemens Smartron.
- AS2—route Sofia—Karlovo-Karnobat—Varna, Locomotive Siemens Smartron.
- AS3—route Sofia—Plovdiv—Karnobat—Varna, Locomotive Siemens Smartron.

The three routes pass through different parts of Bulgaria: A1 and AS1 through the Northern region of Bulgaria, A2 and AS2 through the Central region and A3 and AS3 through the Southern region. The railway route of alternatives A1 and AS1 is fully double-track rail, while for the other rail alternatives it is a one-way-two-way railway line.

The values of criterion C1 (carbon dioxide emissions) were determined based on the amount of CO₂ t/MWh released by the power plants during the production of electricity. A value of 305 g CO₂ eq/kWh was used, according to data from [38]. The type of optimization for this criterion is minimization.

The carbon dioxide emissions for railway transport (for carriage of one container) are determined as follows:

$$C1_{rail,q} = \frac{E_{rail,q} \cdot c_E}{m}, \text{CO}_{2\text{-eq}} \quad (14)$$

where q is the number of railway routes, $q = 1, 2, 3$; $E_{rail,q}$ is the electricity consumed to run the train per route q , kwh; m is the number of wagons in the train, kwh; c_E is the value

of CO₂-eq/kWh, which stands for carbon dioxide equivalent per kilowatt-hour and it is determined according to [38].

The values of electricity consumed to run the train for each of the investigated routes were determined according to data from BDZ “Cargo” Ltd.

The carbon dioxide emissions for diesel heavy trucks are determined as follows:

$$C2_{truck, p}^d = L_{truck, q} \cdot F_{truck, p} \cdot EF, CO_{2-eq} \quad (15)$$

where p is the number of road routes, $p = 1, 2, 3$; $L_{truck, q}$ is the length of the road route p , km; $F_{truck, p}$ is the average fuel consumption per 100 km of the route, l/100 km; EF is the emission factor, which is determined according to [39], CO₂/L.

The carbon dioxide emissions for an electric heavy-duty truck are determined as follows:

$$C2_{truck, p}^e = E_{truck, p} \cdot c_E, CO_{2-eq} \quad (16)$$

where $E_{truck, p}$ is the electricity consumed to run the electric heavy-duty truck per route p , kWh.

The values of the average fuel consumption per 100 km of the investigated road routes and the values of the electricity consumed to run the electric heavy-duty truck per each of the investigated road routes were determined according to data provided by road carriers.

The values of criterion C2 for railway transport present operating costs which include the costs for electricity, depreciation costs, salary and social security costs, costs for maintenance and repair of rolling stock. For road transport, C2 includes fuel consumption, driver wages, taxes and insurance, the costs of tyres, lubricants, maintenance, and other types of costs. The type of optimization for this criterion is minimization. The operating costs were determined according to [40].

The operational costs for intermodal carriage of one container by freight trains on the railway lines are determined as follows:

$$C2_{rail, q} = \frac{L_{rail, q} \cdot n \cdot c_l + L_{rail, q} \cdot c_w + m \cdot c_k}{m}, \text{euro} \quad (17)$$

where n is the number of locomotives in the train; c_l are the cost rate for one locomotive, euro/loc.km; c_w is the cost rate for one wagon, euro/wag.km; c_k are the container handling costs, euro/container.

The cost for carriage of a locomotive and the cost for carriage of a wagon were determined according to data from BDZ “Cargo” Ltd. The container handling costs were determined according to the Unified Tariff for Carriage of Goods by Rail in the Republic of Bulgaria (ETPT) [41].

The operational costs for intermodal carriage of the heavy-duty truck on the routes are determined as follows:

$$C2_{truck, p} = R_{truck, p} + 0.4 \cdot R_{truck, p}, \text{euro} \quad (18)$$

where $R_{truck, p}$ are the fuel cost for a heavy-duty truck, euro; 0.4 is a coefficient for taking into account the other components of the operating costs.

The values of criterion C3 (infrastructure charges) were determined according to the tariff for infrastructure fees, determined by the National Railway Infrastructure Company of Bulgaria [42] and the tariff for using the toll road network in Bulgaria [43]. The type of optimization for this criterion is minimization. The charges for access to and use of the railway infrastructure in Bulgaria include the costs per gross ton kilometre, the costs per train kilometre and also the costs for use of power supply equipment for traction power.

The fee for using the road network in Bulgaria is calculated based on distance travelled (per kilometre) and depends on four main criteria: road category (motorways, first-class roads, or second-class roads), vehicle weight (between 3.5 t and 12 t, or over 12 t); axle count (for heavy trucks); emission class Euro standards (Euro I to Euro VI) [44–49]. All these parameters were taken into account for each section of the railway and road infrastructure when determining the values of the C3 criterion for the studied routes.

The value of criterion C4 (duration of the transportation) was determined according to the timetable of the trains for railway transport and also the permitted speed for trucks along the road infrastructure.

The value of criterion C5 (light on the route) was determined according to data for the section of the transport infrastructure. The type of optimization for this criterion is minimization.

The value of criterion C6 (security factor) was determined by expert assessment of the route in terms of the type of transport infrastructure, road gradients, curves, traffic congestion and accidents. The value of this criterion is between 0 and 1, with the goal being to maximize it. Since, for all the studied criteria, the type of optimization is minimization, the study considers the reciprocal value of this criterion. Thus, the type of optimization is transformed into a minimization problem. To determine the value of C6, an expert assessment was used by 10 experts, who gave ratings from 0 to 1, with the value 1 being the maximum.

The experts are specialists with many years of experience in the field of rail and road freight transport and know the studied transport routes, the condition of the rolling stock and traffic. Five of the experts are specialists from the National Railway Infrastructure Company (3 experts), Executive Agency “Railway Administration” (1 expert), and BDZ Freight Transport (1 expert). They assessed the railway routes. Five experts are specialists in road transport and are from the Road Infrastructure Agency (3 experts), State Agency for Road Safety (1 expert) and the Ministry of Transport and Communications of Bulgaria (1 expert). They assessed the road routes.

A suitable tool for assessing experts’ assessments is the Spearman rank correlation coefficient [50] for cases with no tied values, which is determined as follows:

$$r_s = 1 - \frac{6 - \sum_{i=1}^n d_i^2}{n(n^2 - 1)} \quad (19)$$

where r_s is the Spearman rank correlation coefficient; d_i is the distance between the ranks for each data pair; n is the number of elements in each data series.

The values of the Spearman rank correlation coefficient are between +1 and −1, where +1 indicates that there is perfect correlation in the ranks; the value −1 indicates that the rankings are reversed.

Table 3 represents the values of experts’ assessment for railway and road routes.

To determine the correlation between the experts’ ratings, the table must be arranged so that the alternatives are placed in rows, and the experts are placed in columns.

Since the expert assessments show identical values (equal scores for different criteria by the same expert), the standard Spearman rank correlation coefficient (Equation (19)), based on the squared differences in ranks, is not applicable. For this reason, the Pearson correlation coefficient was used, applied to the previously calculated average ranks [50].

$$r_p = \frac{\sum_{i=1}^n (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \cdot \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (20)$$

where r_p is the Pearson correlation coefficient; x_i, y_i are the variables; $\bar{x}, \bar{y}$ are the average values of the variables; i is the number of observations.

Table 3. Expert assessment for C6 (security factor).

Railway Transport		Alternative				
Experts	A1	A2	A3	AS1	AS2	AS3
1	0.86	0.70	0.80	0.85	0.75	0.85
2	0.82	0.75	0.76	0.95	0.80	0.88
3	0.86	0.75	0.77	0.90	0.77	0.85
4	0.87	0.80	0.83	0.94	0.84	0.86
5	0.84	0.75	0.83	0.88	0.83	0.81
Average (C6)	0.85	0.75	0.80	0.90	0.80	0.85
1/C6	1.18	1.33	1.25	1.11	1.25	1.18
Road Transport		Alternative				
Experts	B1	B2	B3	BE1	BE2	BE3
1	0.70	0.88	0.80	0.70	0.80	0.73
2	0.73	0.85	0.77	0.65	0.76	0.79
3	0.68	0.83	0.81	0.64	0.83	0.73
4	0.71	0.87	0.79	0.65	0.82	0.76
5	0.67	0.82	0.82	0.60	0.79	0.75
Average (C6)	0.70	0.85	0.80	0.65	0.80	0.75
1/C6	1.43	1.18	1.25	1.54	1.25	1.33

Tables 4 and 5 represent the rank of experts' assessment for railway and road routes.

Table 4. Rank of experts' assessment for railway routes.

Alternatives	Experts' Assessment					Rank				
	1	2	3	4	5	1	2	3	4	5
A1	0.86	0.82	0.86	0.87	0.84	1	3	2	2	2
A2	0.70	0.75	0.75	0.80	0.75	6	6	6	6	6
A3	0.80	0.76	0.77	0.83	0.83	4	5	4.5	5	3.5
AS1	0.85	0.95	0.90	0.94	0.88	2.5	1	1	1	1
AS2	0.75	0.80	0.77	0.84	0.83	5	4	4.5	4	3.5
AS3	0.85	0.88	0.85	0.86	0.81	2.5	2	3	3	5

Table 5. Rank of experts' assessment for road routes.

Alternatives	Experts' Assessment					Rank				
	1	2	3	4	5	1	2	3	4	5
B1	0.70	0.73	0.68	0.71	0.67	5.5	5	5	5	5
B2	0.88	0.85	0.83	0.87	0.82	1	1	1.5	1	1.5
B3	0.80	0.77	0.81	0.79	0.82	2.5	3	3	3	1.5
BE1	0.70	0.65	0.64	0.65	0.60	5.5	6	6	6	6
BE2	0.80	0.76	0.83	0.82	0.79	2.5	4	1.5	2	3
BE3	0.73	0.79	0.73	0.76	0.75	4	2	4	4	4

In the event of equal grades, an arithmetic average is calculated from the respective consecutive places.

Table 6 shows the results of the Pearson correlation coefficient for experts' assessments separately for rail and road routes. The average values of the Pearson correlation coefficient are also determined. It can be seen that there is a high correlation between the scores of experts (0.83 for railway routes; 0.86 for road routes). There is homogeneity in the assessments.

Table 6. Pearson correlation coefficient.

Railway Transport Experts	1	2	3	4	5	Road Transport Experts	1	2	3	4	5
1	-	0.75	0.88	0.84	0.65	1	1	0.79	0.94	0.97	0.94
2	0.75	-	0.93	0.94	0.64	2	0.79	1	0.70	0.77	0.78
3	0.88	0.93	-	0.99	0.82	3	0.94	0.70	1	0.99	0.87
4	0.84	0.94	0.99	-	0.81	4	0.97	0.77	0.99	1	0.90
5	0.65	0.64	0.82	0.81	-	5	0.94	0.78	0.87	0.90	1
Average value for experts 1–5: 0.83						Average value for experts 6–10: 0.86					

Table 7 represents the initial decision matrix and the type of optimization. The values in the decision matrix are given for the transport of one container or intermodal transport unit (UTI) for the purpose of comparability.

Table 7. Initial decision matrix.

Criteria		Alternatives						Optimization
		A1	A2	A3	AS1	AS2	AS3	
C1	CO ₂ , kg/UTI	376	356	402	258	245	274	min
C2	EUR/UTI	317	302	365	283	270	327	min
C3	EUR/UTI	53	50	74	47	45	62	min
C4	h	9.7	9	10	9.2	8.5	9.5	min
C5	km	543	514	575	543	514	575	min
C6	coef.	1.18	1.33	1.25	1.11	1.25	1.18	min

4.2. Step 2. Step-by-Step Addition of New Alternatives to the Initial Decision-Making Matrix

To the initial decision matrix, alternatives are successively added that show transportation by heavy trucks on the Bulgarian road network connecting the starting and ending points. The alternatives are as follows:

- B1—Diesel heavy-duty truck on the route Sofia-Veliko Tarnovo-Varna.
- B2—Diesel heavy-duty truck on the route Sofia-Plovdiv-Burgas-Varna.
- B3—Diesel heavy-duty truck on the route Sofia-Plovdiv-Karnobat-Shumen-Varna.
- BE1—Electric heavy-duty truck on the route Sofia-Veliko Tarnovo-Varna.
- BE2—Electric heavy-duty truck on the route Sofia-Plovdiv-Burgas-Varna.
- BE3—Electric heavy-duty truck on the route Sofia-Plovdiv-Karnobat-Shumen-Varna.

The road route of alternatives B1 and BE1 is, for the most part, a secondary road. A lot of the road B2, BE2, B3 and BE3 is a motorway. The routes B1 and BE1 pass through Northern Bulgaria; The routes B2, BE2, B3 and BE3 pass through Southern Bulgaria.

It is assumed that heavy trucks meet European emission standards (Euro V) and have a maximum weight of 36 tons. The electric trucks are Zero-emission vehicles (ZEV) and fall into the highest environmental CO₂.

Table 8 represents the alternatives that are added one by one to the initial decision matrix.

Table 8. Alternatives to add one by one to the initial decision matrix.

Criteria		Alternatives						Optimization
		B1	B2	B3	BE1	BE2	BE3	
C1	CO ₂ , kg/UTI	413	443	497	211	247	260	min
C2	EUR/UTI	411	482	507	335	393	413	min
C3	EUR/UTI	91	112	115	31	37	38	min
C4	h	7.5	8.12	8.62	7.83	8.45	9	min
C5	km	447	524	551	447	524	551	min
C6	coef.	1.43	1.18	1.25	1.54	1.25	1.33	min

4.3. Step 3. Ranking the Alternatives by Applying the SIMUS Method

4.3.1. Rank Reversal in SIMUS Method

In this step, the procedure of the SIMUS method is carried out for all investigated matrices. Table 9 shows the normalized initial decision matrix (LHS), the type of optimization for each criterion, the type of operator for constraints, and the values of the right-hand side of the constraint conditions. The normalization was performed using the sum-of-rows method.

Table 9. Normalized initial decision matrix.

Criteria/ Targets	LHS						Action	Operator	RHS Values
	A1	A2	A3	AS1	AS2	AS3			
	x_1	x_2	x_3	x_4	x_5	x_6			
Z1	0.20	0.19	0.21	0.14	0.13	0.14	min	$\geq$	0.13
Z2	0.17	0.16	0.20	0.15	0.14	0.18	min	$\geq$	0.14
Z3	0.16	0.15	0.22	0.14	0.14	0.19	min	$\geq$	0.14
Z4	0.17	0.16	0.18	0.16	0.15	0.17	min	$\geq$	0.15
Z5	0.17	0.16	0.18	0.17	0.16	0.18	min	$\geq$	0.16
Z6	0.16	0.18	0.17	0.15	0.17	0.16	min	$\geq$	0.15

Table 10 represents the efficient results matrix (ERM), which contains the results of the successive optimizations for each of the criteria. The last column of the table shows the values of the objective function per target.

Table 10. Efficient Results Matrix (ERM).

Objective	Alternative						Objective Function Values per Target
	A1	A2	A3	AS1	AS2	AS3	
Z1	0.00	0.00	0.00	0.61	0.30	0.05	0.13
Z2	0.00	0.00	0.00	0.64	0.30	0.02	0.14
Z3	0.00	0.05	0.00	0.68	0.23	0.00	0.14
Z4	0.00	0.00	0.00	0.00	0.28	0.64	0.15
Z5	0.58	0.32	0.00	0.00	0.00	0.00	0.15
Z6	0.00	0.00	0.01	0.87	0.00	0.06	0.14

The results of the ranking are shown in Table 11. The first part of Table 11 represents the values of the normalized efficient results matrix; the second part of Table 11 includes the parameters of ranking.

Table 11. Normalized Efficient Results Matrix (NERM). Ranking results.

Objective	Normalized Efficient Results Matrix (NERM)					
	A1	A2	A3	AS1	AS2	AS3
Z1	0.000	0.000	0.000	0.634	0.309	0.057
Z2	0.000	0.000	0.004	0.666	0.307	0.022
Z3	0.000	0.050	0.000	0.711	0.240	0.000
Z4	0.000	0.000	0.000	0.000	0.306	0.694
Z5	0.641	0.359	0.000	0.000	0.000	0.000
Z6	0.000	0.000	0.006	0.926	0.000	0.068
SC_j	0.64	0.41	0.01	2.94	1.16	0.84
PF_j	1	2	2	4	4	4
NPF_j	0.17	0.33	0.33	0.67	0.67	0.67
S_j	0.11	0.14	0.00	1.96	0.77	0.56
ERM Ranking: AS1—AS2—AS3—A2—A1—A3						

The ranking results for all studied cases with the addition and subtraction of alternatives are recorded in Table 12. These are cases of different dimensions of the problem studied. The last column of the table shows the change in rank reversal.

Table 12. Rank reversal when adding or removing an alternative.

D	New Alternative Add/Removed	Rank												RR	
		1	2	3	4	5	6	7	8	9	10	11	12	Position	
														First	Other
3D	Remove AS1	A2	A1	A3										yes	no
4D	Remove AS2	AS1	A1	A3	A2									no	yes
5D	Remove AS3	AS1	AS2	A3	A2	A1								no	yes
6D	A1, A2, A3, AS1, AS2, AS3	AS1	AS2	AS3	A2	A1	A3							-	-
7D	Add B1	AS2	A2	AS3	B1	A3	A1	AS1						yes	yes
8D	Add B2	A2	AS2	B2	B1	AS1	A3	AS3	A1					yes	yes
9D	Add B3	A2	AS2	B2	B1	AS1	A1	AS3	A3	B3				no	yes
10D	Add BE1	BE1	B2	A1	A3	AS1	B1	A2	AS2	AS3	B3			yes	yes
11D	Add BE2	BE1	B2	A1	AS2	AS1	BE2	B1	A2	A3	AS3	B3		no	yes
12D	Add BE3	BE1	B2	A1	AS2	AS1	BE3	BE2	B1	A2	A3	AS3	B3	no	no

The initial matrix is in 6D dimension (the alternatives are: A1, A2, A3, AS1, AS2, AS3). This is shown in bold in Table 12. Alternative AS1 is in the first position. This is the carriage with container block trains on the route Sofia—Gorna Oryahovitsa-Varna.

The steps of sequentially adding and removing the alternatives are as follows:

- Remove alternative AS3. This is a 5D dimension. Alternative AS1 is again in the first position. Rank reversal occurs in the other positions of the ranking. For example, alternative A3 from the last position at 6D moves to the 3rd position at 5D.
- Remove alternative AS2. This is the 4D dimension. Alternative AS1 is again in the first position. Rank reversal occurs in the other positions of the ranking. For example, alternative A1 from the last position at 5D moves to the 2nd position at 4D.
- Remove alternative AS1. This is the 3D dimension. There is a rank reversal in the first position. Alternative A2 from the last position at 4D moves to the 1st position at 3D.

- Adding alternative B1 to the initial decision matrix. The dimension is 7D. A rank reversal occurs. The first position is for another alternative—AS2. The rank reversal is observed for the first and other positions in the ranking.
- Adding alternative B2. The dimension is 8D. A rank reversal occurs for all positions.
- Adding alternative B3. The dimension is 9D. There is rank reversal only for alternative A3.
- Adding alternative BE1. The dimension is 10D. A rank reversal occurs for all positions.
- Adding alternative BE2. The dimension is 11D. There is rank reversal for alternatives AS1 and A3.
- Adding alternative BE3. The dimension is 12D. The alternative BE3 takes a position, with no change in the sequence of the remaining alternatives.

It could be summarized that the change in dimension, i.e., the addition or removal of an alternative, causes the rank reversal. Based on the results presented in Table 12, the following changes in rank with respect to the most suitable alternative (the one in first position):

- In the case of ranking only three alternatives (A1, A2 and A3) for carriage by rail transport, locomotive series 46 at the head of the container block train, SIMUS ranked as the most suitable alternative A2. This is the route Sofia-Karlovo-Karnobat-Varna. This route is mountainous, longer than the A1 and has single-track sections. A container block train with a 46 series locomotive is very suitable in this case, as it is suitable for more difficult infrastructure conditions.
- When adding alternatives by railway transport using modern equipment, such as the Siemens Smartron locomotive, new tasks with 4, 5 and 6 alternatives are considered. The results show that the most suitable alternative for all three cases is AS1. This is the route Sofia—Gorna Oryahovitsa—Varna. This route is fully electrified, and most of it is double track. Siemens locomotive is more suitable here because it has less weight and the terrain is not mountainous. Compared to the case with three alternatives using railway transport (A1, A2 and A3), a reversal of rank is observed. A rank reversal is established when transportation with modernized equipment is considered.
- At the next stage, alternatives with road transport are sequentially added. These are the cases of transportation with diesel heavy trucks. The cases with seven, eight and nine alternatives were successively examined. It can be seen in Table 12 that in the first position is again a route with rail transport using locomotive series 46 or Siemens Smartron. This is A2 (AS2)-route Sofia-Karlovo-Karnobat-Varna.
- At the next stage, alternatives with road transport routes are being added sequentially, but with transportation using environmentally friendly electric heavy-duty trucks. These are the options with ten, eleven and twelve alternatives. For all these cases, in the first position is a new alternative—BE1. This is an electric heavy-duty truck on the route Sofia-Veliko Tarnovo-Varna. This route is preferred by logistics companies, as it offers a smaller slope and easier passage through Northern Bulgaria on a category I road and motorway. There is a rank reversal compared to cases where new types of road transport equipment are not considered. The alternatives B2 and A1 remain invariant, i.e., they do not change their position ranking for 10, 11 and 12 alternatives. It can be summarized that the route Sofia-Veliko Tarnovo-Varna is suitable for transporting electric heavy-duty trucks. The route B2 presented the carriage by diesel heavy-duty truck on the route Sofia-Plovdiv-Burgas-Varna as an alternative. In this case, it can be considered a suitable option for transport operators in case they do not have the new type of heavy trucks. The route A1 uses railway transport. It can also be considered suitable for transport operators who do not have new types of equipment for rail and road transport. The results show that transportation through northern

Bulgaria (AS1, BE1) is suitable for the use of new equipment. This is because the terrain conditions allow for greater throughput and higher permitted transport speeds.

- The alternatives A2 and AS2 are invariant in most of the considered cases. This means that transport operators can choose transportation through central Bulgaria as a suitable alternative. The use of a modern Siemens Smartron locomotive will reduce electricity consumption. These routes can also be used as additional ones in case of overloading or repairs along the routes through northern Bulgaria.
- Alternatives A3 and B3 can be considered unsuitable for transport. These are routes through southern Bulgaria. In all cases considered, they are at the bottom of the rankings.

Table 13 represents the change in the rank of the alternatives studied in the different dimensions.

Table 13. Ranking the alternatives in different dimensions.

Alternative	Number	3D	4D	5D	6D	7D	8D	9D	10D	11D	12D
A1	1	2	2	5	5	6	8	6	3	3	3
A2	2	1	4	4	4	2	1	1	7	8	9
A3	3	3	3	3	6	5	6	8	4	9	10
AS1	4	-	1	1	1	7	5	5	5	5	5
AS2	5	-	-	2	2	1	2	2	8	4	4
AS3	6	-	-	-	3	3	7	7	9	10	11
B1	7	-	-	-	-	4	4	4	6	7	8
B2	8	-	-	-	-	-	3	3	2	2	2
B3	9	-	-	-	-	-	-	9	10	11	12
BE1	10	-	-	-	-	-	-	-	1	1	1
BE2	11	-	-	-	-	-	-	-	-	6	7
BE3	12	-	-	-	-	-	-	-	-	-	6

The results of the initial decision matrix give priority to the alternatives using a modern locomotive—Siemens. There is stability in the first position of the ranking for alternative AS1 (route Sofia—Gorna Oryahovitsa—Varna) with the sequential removal of alternatives AS3 and AS2. In the case when the alternatives with a modern locomotive are removed, the ranking changes; there is a reversal of the rank. In this case, the appropriate route is A2—Sofia—Karlovo—Karnobat—Varna. It can be summarized that the change in rail transport technologies leads to a rank reversal.

Adding or removing an alternative changes the dimensionality of the problem. The problem is different; the solution and the ranking are different, and it does not depend on the previous ranking. In the ranking, the new alternative takes a position depending on the ranking criterion. The other alternatives do not remain in their positions or in their order from the previous ranking. They take new positions depending on the value of the ranking criterion. By adding or removing an alternative, the system changes. The occurrence of rank reversal is a natural process related to the mathematical representation of the linear programming problem in multidimensional space. In this process, alternatives are observed that retain their position—such alternatives can be called “invariant”, and even if they are not ranked in first place, they can be considered a priority when making a decision.

The introduction of new technologies in road transport, such as new types of heavy-duty trucks with electric drive (alternatives BE1, BE2, BE3), which reduce harmful emissions, affects the ranking and gives priority to road transport in the considered direction. In these cases, however, alternative A1 and alternative AC1 remain invariant, retaining their position, which is rail transport on the route Sofia—Gorna Oryahovitsa—Varna. The

railway operator should focus its attention on transport on this route in order to respond to competition from new technologies in road transport.

Figure 2 illustrates the rank reversal of alternatives in all studied dimensions.

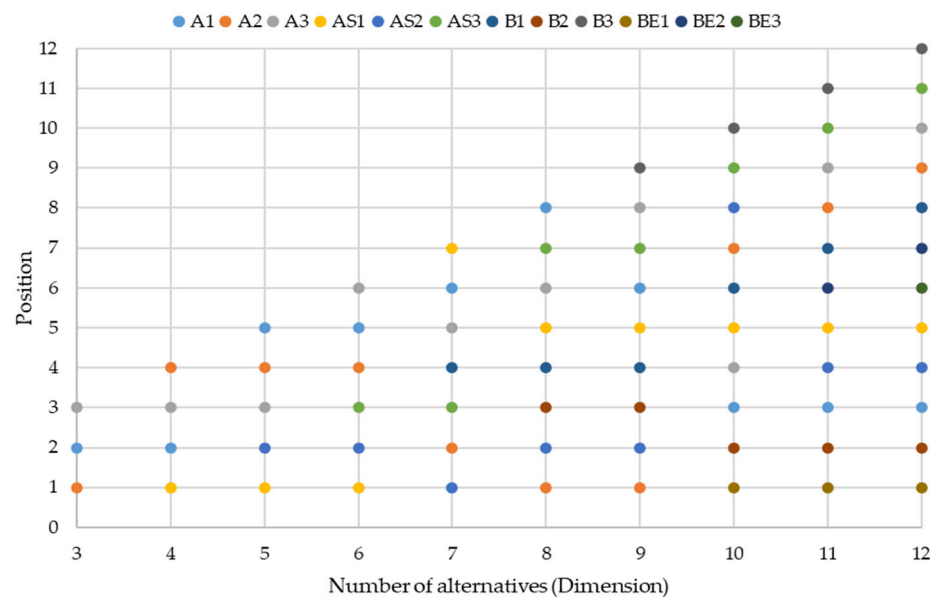


Figure 2. Graphical representation of ranking in rank reversal.

Table 14 represents the values of the ranking criteria for all cases.

Table 14. Values of criterion for ranking S_j .

Alternative	3D	4D	5D	6D	7D	8D	9D	10D	11D	12D
A1	0.96	0.55	0.08	0.11	0.03	0.03	0.10	0.21	0.24	0.24
A2	2.47	0.04	0.37	0.14	0.88	0.88	0.88	0.00	0.00	0.00
A3	0.56	0.21	0.65	0.00	0.21	0.08	0.01	0.17	0.00	0.00
AS1	-	3.52	0.84	1.96	0.00	0.11	0.11	0.14	0.11	0.11
AS2	-	-	0.70	0.77	0.95	0.43	0.43	0.00	0.17	0.17
AS3	-	-	-	0.56	0.56	0.08	0.08	0.00	0.00	0.00
B1	-	-	-	-	0.21	0.25	0.25	0.05	0.05	0.05
B2	-	-	-	-	-	0.30	0.30	0.28	0.30	0.30
B3	-	-	-	-	-	-	0.00	0.00	0.00	0.00
BE1	-	-	-	-	-	-	-	1.20	0.96	0.67
BE2	-	-	-	-	-	-	-	-	0.08	0.08
BE3	-	-	-	-	-	-	-	-	-	0.10
max S_j	2.47	3.52	0.84	1.96	0.95	0.88	0.88	1.20	0.96	0.67
$\sum_{j=1}^n S_j$	3.99	4.33	2.65	3.54	2.84	2.16	2.17	2.05	1.91	1.72

It can be seen that the alternatives AS1 and BE1 remain invariant; they change rank when a new technology is introduced, such as, for rail transport, a new type of modern electric locomotive, and for road transport, a new type of heavy-duty truck drive.

Table 15 shows the relative importance of criteria for ranking. For this purpose, the summary ranking criteria $\sum_{j=1}^n S_j$ is determined. This allows assessing the contribution of

each alternative in the ranking. The change in the relative importance of the criteria causes the rank reversal.

Table 15. Relative importance of the criterion for ranking.

$S_j / \sum_{j=1}^n S_j$	3D	4D	5D	6D	7D	8D	9D	10D	11D	12D
A1	0.24	0.13	0.03	0.03	0.01	0.01	0.04	0.10	0.13	0.14
A2	0.62	0.01	0.14	0.04	0.31	0.41	0.41	0.00	0.00	0.00
A3	0.14	0.05	0.25	0.00	0.07	0.04	0.01	0.08	0.00	0.00
AS1	-	0.81	0.32	0.55	0.00	0.05	0.05	0.07	0.06	0.07
AS2	-	-	0.26	0.22	0.34	0.20	0.20	0.00	0.09	0.10
AS3	-	-	-	0.16	0.20	0.04	0.04	0.00	0.00	0.00
B1	-	-	-	-	0.08	0.12	0.12	0.02	0.02	0.03
B2	-	-	-	-	-	0.14	0.14	0.14	0.16	0.18
B3	-	-	-	-	-	-	0.00	0.00	0.00	0.00
BE1	-	-	-	-	-	-	-	0.59	0.50	0.39
BE2	-	-	-	-	-	-	-	-	0.04	0.05
BE3	-	-	-	-	-	-	-	-	-	0.06
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

As the SIMUS method is based on the linear programming method, for each criterion the Simplex algorithm is solved. The main objective of the problem varies as a function of adding alternatives.

Figure 3 illustrates the change in the summary ranking criteria depending on the dimensionality of the system under study. It can be seen that when the dimensionality increases, the summary ranking criteria decreases.

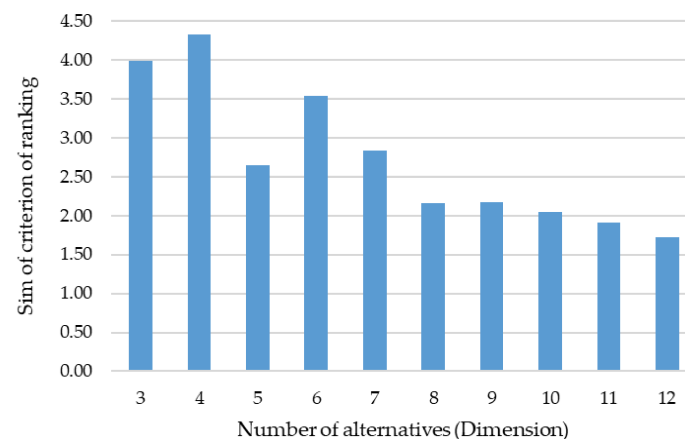


Figure 3. Summary ranking criteria.

Table 16 represents the results of linear optimization models and the values of objective functions when the first criterion C1 is taken as the objective function. Starting with a simple example with two variables, x_1 and x_2 , or in a 2D space, it progressively adds a new alternative. In Table 16, the objective function Z expresses the result of the system as a whole quantitatively. It can be seen that every time a new alternative is added, Z decreases, reaching a value of $Z = 0.05$.

Table 16. Objective function per target for all dimensions for Criterion C1. Rank reversal.

Dimension	Number of Alternatives												Objective Function per Target Z	RR
	-	2D	3D	4D	5D	6D	7D	8D	9D	10D	11D	12D		
	A1	A2	A3	AS1	AS2	AS3	B1	B2	B3	BE1	BE2	BE3		
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}		
2D	0	1	-	-	-	-	-	-	-	-	-	-	0.49	no
3D	0	1	0	-	-	-	-	-	-	-	-	-	0.31	yes
4D	0	0	0	1	-	-	-	-	-	-	-	-	0.19	yes
5D	0	0	0	0	1	-	-	-	-	-	-	-	0.15	yes
6D	0	0	0	0.61	0.30	0.05	-	-	-	-	-	-	0.13	-
7D	0	0	0	0	0.49	0.42	0.00	-	-	-	-	-	0.10	yes
8D	0	0	0	0	0.49	0.42	0	0	-	-	-	-	0.09	no
9D	0	0	0	0	0.49	0.42	0	0	0	-	-	-	0.07	no
10D	0	0	0	0	0	0	0	0	0	1	-	-	0.06	yes
11D	0	0	0	0	0	0	0	0	0	0.47	0.45	-	0.06	no
12D	0	0	0	0	0	0	0	0	0	0.47	0.45	0	0.05	no

Table 17 shows the values of objective functions for individual optimization for each of the criteria. When an alternative is added, a new matrix appears, and since it considers the former one, it incorporates the existing knowledge and adds more knowledge.

Table 17. Objective function per target for all criteria.

Objective Function per Target	Number of Alternatives/Dimensions											
	2D	3D	4D	5D	6D	7D	8D	9D	10D	11D	12D	
C1	0.49	0.31	0.19	0.15	0.13	0.10	0.09	0.07	0.06	0.06	0.05	
C2	0.49	0.31	0.22	0.17	0.14	0.11	0.09	0.08	0.07	0.06	0.05	
C3	0.49	0.28	0.21	0.17	0.14	0.11	0.08	0.07	0.05	0.04	0.04	
C4	0.48	0.31	0.23	0.17	0.15	0.12	0.10	0.09	0.08	0.07	0.07	
C5	0.49	0.31	0.23	0.13	0.15	0.12	0.10	0.09	0.08	0.07	0.07	
C6	0.45	0.30	0.20	0.17	0.14	0.11	0.09	0.08	0.07	0.07	0.06	
Average per criterion	0.48	0.30	0.21	0.16	0.14	0.11	0.09	0.08	0.07	0.06	0.06	

The objective function Z is steadily decreasing, in accordance with the minimization action, but in addition, this decrease is related to a progressively larger amount of information given for the increasing number of alternatives. This example allows us to appreciate the variability of rankings along successive scenarios.

It can be concluded that this is a demonstration of one of the main concepts enunciated in this paper: RR in the SIMUS method is a random process that depends on the values of each new vector corresponding to an added alternative.

The geometric proof is that a system is geometrically represented in a Cartesian coordinate system where alternatives are the axes. Geometrically, if it adds an alternative, it is adding a new dimension. For example, from 2D to 3D, like from a square to a cube. Since the new vector has magnitude, sense and direction, the resulting figure may not be a cube but another shape. In 2D, it is a shape or polygon that contains all feasible solutions of the problem.

Figure 4 illustrates the change in the objective function when the dimension of the system changes.

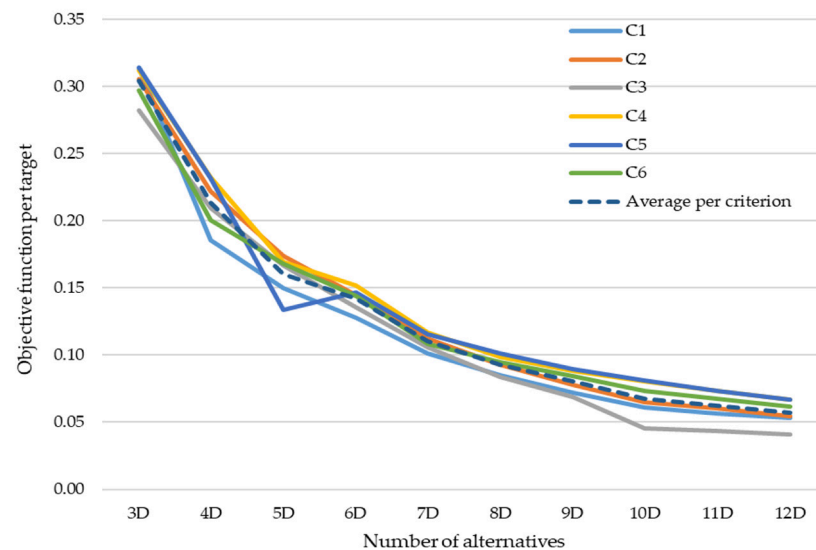


Figure 4. Objective function per target.

The Linear Programming Fundamental Theorem indicates that the vertices of the shape contain the optimal solution, but only one of them. Therefore, when it adds a new alternative, it is adding a vector to the 2D matrix, and only for this, it is a different problem. That is, the original solution space or polytope in 2D is expanded to a three-dimensional body or polyhedron with its corresponding vertices. The optimal solution found in 2D may still be the same in 3D. It depends on the values of the vector added. If these values are similar to the existing values, most probably the selected optimal vertex in 2D is still valid in 3D; that is, invariance, because it slightly modifies the slope of the criteria. Now, the added vector has very different values than in 3D; it involves strong deviations from their original slopes, which conform the polygon, and the resultant polyhedron may have a different geometrical shape compared with 2D. This can cause the original optimal vertex in 2D to be replaced by another vertex whose cardinal coordinates may generate different values in the alternatives and then different rankings, and in so doing destroy any prior invariance. In this circumstance, it can indicate that there is RR.

RR is unavoidable and random, because we do not know the effect that the new vector will have on the matrix transformation. This is our theory, and we have tested it in this paper for the problem of route selection.

The new alternative generates more information and consequently, less uncertainty.

As a summary, RR is a natural phenomenon and not related to methods, normalization, inconsistencies or interpretations. It happens independently of all of them.

It can be concluded as follows:

- A change in space dimensions changes the former problem. This is elemental geometry.
- A new alternative can generate a new polygon with new feasible solutions that may be equal to or different from the former.
- Adding activities or spatial dimensions, we get more information since the old problem is included in the new decision matrix.

4.3.2. Assessment of Invariance and Its Relationship with Sustainability in Decision Making

Based on results represented in Table 13, an invariance condition can be defined by the function of the rank position. An alternative is invariant in its transition from one set of alternatives to another if the following condition is met:

$$r(A_i, V_t) = r(A_i, V_{t+1}) \quad (21)$$

where $r(A_i, V_n)$ is the rank of alternative A_i when considering a set of t alternatives; V_t is the set of t alternatives, $V_t = \{A_1, A_2, \dots, A_t\}$; V_{t+1} is a set with new alternatives added, $V_{t+1} = \{A_1, A_2, \dots, A_t, A_{t+1}\}$. The sets V_t, V_{t+1} correspond also to the dimensional space of the system.

For example, it can be seen in Table 13 for the case with three and four alternatives that: $r(A_1, V_3) = r(A_1, V_4) = 2$; $r(A_3, V_3) = r(A_3, V_4) = 3$; A_1 correspond to A1; A_3 correspond to A3. $V_3 = \{A_1, A_2, A_3\}$; $V_4 = \{A_1, A_2, A_3, AS1\}$. V_3 correspond to the 3D dimensional space, V_4 correspond to the 4D dimensional space.

To analyze the behaviour of all alternatives during all steps of adding and subtracting alternatives with respect to invariance, an invariance matrix is formed.

The elements of the invariance matrix are determined as follows:

$$m_{jq} = \begin{cases} 1, & \text{if } r(A_i, V_t) = r(A_i, V_{t+1}) \\ 0, & \text{if } r(A_i, V_t) \neq r(A_i, V_{t+1}) \end{cases} \quad (22)$$

where j is the number of alternatives in the basic decision matrix, $j = 1, \dots, n$; q is the number of additions of alternatives to the initial matrix, $q = 1, \dots, Q$. In the case under consideration, if it is assumed that the matrix with three alternatives (A1, A2, A3) is the basic one, then when adding alternatives from AS1 to BE3 one by one, 9 additions are made (i.e., from 3D to 12D).

To assess invariance, an invariance coefficient has been introduced as follows:

$$K^{inv}(A_j) = \frac{\sum_{q=1}^Q m_{jq}}{q} \quad (23)$$

$$0 \leq K^{inv}(A_j) \leq 1 \quad (24)$$

where $K^{inv}(A_j)$ is the invariance coefficient of the alternative A_j .

The invariance coefficient depends on the number of steps in which the alternative participated. For example, A1 participates in 9 steps, $q = 9$, but AS1 participates in 8 steps and $q = 8$.

The rank reversal of an alternative A_j throughout the process of adding or removing alternatives can be determined by considering the invariance coefficient as follows:

$$K^{rev}(A_j) = 1 - K^{inv}(A_j) \quad (25)$$

$$0 \leq K^{rev}(A_j) \leq 1 \quad (26)$$

where $K^{rev}(A_j)$ is the rank reversal coefficient of alternative A_j .

Table 18 represents the invariance matrix, invariance coefficient and rank reversal coefficient, taking into account the data in Table 13. The first part of Table 18 shows the invariance matrix; the second part of this table represents the values of the invariance coefficient and rank reversal coefficient for each step of adding a new alternative. The cells with invariance are marked in green. They also have a value of 1. The cells with rank reversal are marked in yellow. They also have a value of 0.

Table 18. Invariance matrix. Invariance coefficient.

Invariance Matrix										Invariance	Rank Reversal		
Step													
1	2	3	4	5	6	7	8	9					
New Alternative Added													
AS1	AS2	AS3	B1	B2	B3	BE1	BE2	BE3					
4D	5D	6D	7D	8D	9D	10D	11D	12D	$\sum_{q=1}^Q m_{jq}$	q	$K^{inv}(A_j)$	$K^{rev}(A_j)$	
A1	1	0	1	0	0	0	0	1	1	4	9	0.44	0.56
A2	0	1	1	0	0	1	0	0	0	3	9	0.33	0.67
A3	1	1	0	0	0	0	0	0	0	2	9	0.22	0.78
AS1	-	1	1	0	0	1	1	1	1	6	8	0.75	0.25
AS2	-	-	1	0	0	1	0	0	1	3	7	0.43	0.57
AS3	-	-	-	1	0	1	0	0	0	2	6	0.33	0.67
B1	-	-	-	-	1	1	0	0	0	2	5	0.40	0.60
B2	-	-	-	-	-	1	0	1	1	3	4	0.75	0.25
B3	-	-	-	-	-	-	0	0	0	0	3	0.00	1.00
BE1	-	-	-	-	-	-	-	1	1	2	2	1.00	0.00
BE2	-	-	-	-	-	-	-	-	0	0	1	0.00	1.00

The position of the invariant alternative is important in decision-making by the transport operator. The value of the invariance coefficient could help the transport operator understand how reliable a route is in the face of dynamic changes in the transport network, and this is also related to the sustainability of transport solutions.

Based on the results given in Table 18, five groups of values of the invariance coefficient can be formed, which can be used to study the sustainability of route selection by the transport operator:

- Absolute sustainability: $K^{inv}(A_j) = 1$. In this case, the alternative is completely sustainable. This alternative is not affected by changes in the model. Such an alternative is BE1. This alternative has been in first position in the ranking since its appearance. Adding new alternatives has no impact on its position. Changing the geometry in the linear programming used in SIMIS cannot affect it. In transport planning, such alternatives provide guaranteed sustainability.
- High sustainability: $0.70 \leq K^{inv}(A_j) < 1$. These are alternatives that manage to maintain their positions in a very large proportion of cases of dynamic change in the number of alternatives. The rank reversal for these alternatives is rare. These are AS1 and B2. This can occur with significant changes in the transport environment, such as the introduction of a competing mode of transport; for example, the rank reversal for AS1 when adding B1 (heavy trucks). These alternatives have a strong presence in the rank matrix and the invariant matrix. The alternative AS1 at the beginning is in the first position, and then retains the fifth position. The alternative B2 retains the third position, then the second position. Both alternatives can guarantee sustainability in transport changes. In transport planning, choosing such alternatives provides reliability and sustainability.
- Medium sustainability: $0.40 \leq K^{inv}(A_j) < 0.70$. These are alternatives that change their sustainability depending on the environment. Under some conditions, they have sustainability, but under others they do not. The probability of the alternative being invariant or receiving a rank reversal is almost equal. These alternatives are A1, AS2 and B1. For example, A1 is in the second position at the beginning, then it

goes through different positions, and finally it is in the third position; i.e., there are periods of sustainability and periods of instability. For such alternatives, the transport operator must monitor whether they are in a period of sustainability or not.

- Low sustainability: $0 < K^{inv}(A_j) < 0.40$. Almost every new alternative changes the rank of these alternatives. This group defines critical dominance of rank reversal. These are alternatives A2, A3 and AS3. These are unsustainable alternatives for the transport operator because they are highly sensitive to changes in the transportation environment.
- Lack of sustainability: $K^{inv}(A_j) = 0$. Each new alternative changes the rank of these alternatives. These are the alternatives BE2 and B3.

The values of the rank reversal coefficient can also be systematized into five groups:

- Absolute rank reversal: $K^{rev}(A_j) = 1$. In this case, the alternative is completely unsustainable. These alternatives are B3 and BE2. These alternatives are highly sensitive to any addition of a new alternative, and this leads to a rank reversal. Each addition of a new alternative leads to a change in the geometry of linear programming, which affects the solution.
- High rank reversal: $0.70 \leq K^{rev}(A_j) < 1$. These are alternatives that change their rank throughout the process of adding new alternatives. The presence of invariance is rare and random. These are alternatives that are particularly sensitive to any change in transportation conditions. The alternative A3 belongs to this group.
- Medium rank reversal: $0.40 \leq K^{rev}(A_j) < 0.70$. In this group are alternatives that remain invariant under some conditions and reverse rank under other conditions. In this group are: A1, A2, AS2, AS3, B1.
- Low rank reversal: $0 < K^{rev}(A_j) < 0.40$. These are alternatives that exhibit invariance, retaining their position in most cases of adding a new alternative. In this group are: AS1, B2.
- Absolute sustainability (Lack of rank reversal): $K^{rev}(A_j) = 0$. These are alternatives where there is no rank reversal. They are invariant. Such is the alternative BE1.

Based on the results given in Tables 13, 14 and 18, it can be concluded that:

- The addition of routes B1 and B2 has caused fluctuations in the transport network, leading to a rearrangement of the ranking of all alternatives. These are cases when routes using a competing mode of transport are introduced for consideration—heavy trucks on road routes versus alternatives with rail transport.
- In the final phase of adding alternatives, four alternatives remain invariant: A1, AS1, B2 and BE1. These are alternatives that stabilize simultaneously. For a transport operator, this is a sign that the system has sustainability; no reversal of rank is observed.
- Alternative AS1 is the longest-lasting invariant alternative from its introduction to the very end of the process of adding new alternatives.
- With the addition of BE1, the system reaches a stage of sustainability. Although new alternatives (BE2, BE3) are introduced, the ranking of the main invariant alternatives BE2, B2, AS1 and A1 remains unchanged.
- Alternative BE1, with its addition, takes the first position and remains absolutely invariant with absolute sustainability. The transport operator can choose the alternative BE1 as the most sustainable. This is the Sofia-Veliko Tarnovo-Varna route of road infrastructure served by electric heavy-duty trucks.
- Alternative A1 is effective in a small network, while in a large network of alternatives it loses its position. In the final phases of the dynamic process, it becomes invariant again. This shows why sequential testing with many alternatives is a necessary condition for sustainable decision-making by the transport operator.

- Since BE1 introduces a new technology with electric heavy-duty trucks, in the absence of such service, the transport operator can choose the alternatives B2, AS1 or A1. B2 is the route Sofia-Plovdiv-Burgas-Varna on the road infrastructure served by diesel heavy-duty trucks. It is in the second position in the ranking. A1 is the route Sofia—Gorna Oryahovitsa—Varna, served by container block-trains with locomotive Ser.46. It is in the third position in the ranking; AS1 is also the route Sofia—Gorna Oryahovitsa—Varna, but served by container block-trains with locomotive Siemens—Smartron. It is an invariant alternative in the fifth position in many cases.
- In SIMUS, when the ranking criterion is equal to 0, this means that the corresponding alternative is not significant for consideration and is rejected as last in the ranking. Such cases occur with alternatives A2, A3, AS3, B3 (Table 14). It can be seen that these are alternatives with a high and medium rank reversal.
- When a new alternative is added, a new variable is actually added to the linear programming model that SIMUS uses. When rank reversal is implemented, the new variable shifts the vertices of the polyhedron in a way that reduces the effectiveness of the previous variable relative to the new one. In the case of invariance, the new variable cannot affect the previous variable and its position in the polyhedron.

When alternatives are added, the ranking depends on the solution of the individual linear optimization models, the formation of the efficient results matrix (ERM) and the ranking criterion (S_j). In order to implement a rank reversal when adding a new alternative, it is necessary that the maximum value of the ranking criterion appears in another alternative, different from the previous ranking.

Lets, B_t and B_{t+1} are the sets of the values of the ranking criterion S_j by the SIMUS method for the following steps: alternatives $(A_1, \dots, A_t)$ and alternatives $(A_1, \dots, A_{t+1})$.

$$B_t = \{S_1^{(t)}(A_1), S_2^{(t)}(A_2), \dots, S_j^{(t)}(A_j), \dots, S_t^{(t)}(A_t)\}$$

$$B_{t+1} = \{S_1^{(t)}(A_1), S_2^{(t)}(A_2), \dots, S_j^{(t)}(A_j), \dots, S_t^{(t)}(A_t), S_{t+1}^{(t)}(A_{t+1})\}$$

The alternatives are ranked according to the maximum value of the ranking criterion. The condition for rank reversal when adding a new alternative is:

$$\operatorname{argmax}(B_t) \neq \operatorname{argmax}(B_{t+1}) \quad (27)$$

This means that the position of the maximum element in the set B_t is different from the position of the maximum element in the set B_{t+1} .

For example, considering the results shown in Table 14, it can be seen that for a set of six alternatives and a set of seven alternatives, Equation (27) is: $\operatorname{argmax}\{S(A1) = 0.11; S(A2) = 0.14; S(A3) = 0.00; S(AS1) = 1.96; S(AS2) = 0.77; S(AS3) = 0.56\} = 4$, and $\operatorname{argmax}\{S(A1) = 0.03; S(A2) = 0.88; S(A3) = 0.21; S(AS1) = 0.00; S(AS2) = 0.95; S(AS3) = 0.56; S(B1) = 0.21\} = 5$. This means that there is rank reversal.

The condition for an invariance at the first position, i.e., one alternative is again in the first position when a new alternative is added, is as follows:

$$\begin{cases} \operatorname{argmax}(B_t) = \operatorname{argmax}(B_{t+1}) = i \\ \max(B_t) = M_1 \\ \max(B_{t+1}) = M_2 \end{cases} \quad (28)$$

where M_1 is the maximum value in the set (B_t) ; M_2 is the maximum value in the set (B_{t+1}) .

For example, considering the results shown in Table 14, it can be seen that for a set of four alternatives and a set of five alternatives, Equation (28) is: $\operatorname{argmax}\{S(A1) = 0.55;$

$S(A2) = 0.04; S(A3) = 0.21; S(AS1) = 3.52\} = 4$ and $\operatorname{argmax}\{S(A1) = 0.08; S(A2) = 0.37; S(A3) = 0.65; S(AS1) = 0.84; S(AS2) = 0.70\} = 4$. This means that the alternative AS1 is invariant in the first position. For the set of four alternatives, the ranking is: $AS1 > A1 > A3 > A2$. For the set of five alternatives, the ranking is: $AS1 > AS2 > A3 > A2 > A1$.

When rank reversal occurs, the new alternative changes the structure of the constraints in the linear programming model in a way that affects the feasible solution set. The new alternative appears as a new column in the effectiveness results matrix. In SIMUS, each criterion is transformed into an objective function, and the remaining criteria form a polyhedron of feasible solutions. When a new alternative is added, this expands the geometric solution space. The alternative that was previously ranked first loses its effectiveness; the new alternative changes the vertices of the solution polyhedron. An example of rank reversal for alternatives AS1 and AS2 presented by the efficient results matrix can be seen for the set of six alternatives in Table 10 and seven alternatives in Table A3 (Appendix A). In Table 10, the alternative AS1 is effective for four criteria (column for AS1—0.61; 0.64; 0.68; 0.0; 0.0; 0.87). When the new alternative B1 is added, it changes the effectiveness of AS1; all elements of the column for AS1 in Table A3 in Appendix A take the value 0, while the alternative AS2 remains much more effective at points where AS1 was effective before the addition of B1. In Table 10, the effectiveness of AS2 can be seen in the column for AS2, it is: (0.30; 0.30; 0.23; 0.28; 0.0; 0.0), while the effectiveness of the same alternative when adding B1 is: (0.49; 0.66; 0.0; 0.53; 0.0; 0.0); it can be seen in Table A3 in Appendix A—the column for alternative AS2. This indicates that the vertices of the solution polyhedron have changed.

4.4. Step 4. Added Alternatives That Are Very Similar or Identical to Another Existing in the System (Close Vectors) May Generate RR

In order to investigate the influence of RR, the addition of similar or identical alternatives in the existing system, the following cases are examined:

- Case 1: Starting from 2D dimension, i.e., alternatives A1 and A2, the SIMUS method was applied.
- Case 2: Adding alternative AA2 similar to alternative A2.
- Case 3: Adding alternative AA1 similar to alternative A1.
- Case 4: Adding a new alternative (A3) to the former matrix with two identical vectors A2 and AA2.
- Case 5: Adding a new alternative (A3) to the former matrix with two identical vectors A1 and AA1.

Table 19 represents the results of the SIMUS procedure for Case 1.

Table 19. Case 1. Initial matrix, SIMUS results for 2 dimensions (A1, A2).

Criteria /Targets	Alternatives		LHS		Action	Operator	RHS	ERM		Z	NERM	
	A1	A2	A1	A2				A1	A2		A1	A2
C1	376.00	356.00	0.51	0.49	min	$\geq$	0.49	0.00	1.00	0.49	0.00	1.00
C2	317.00	302.00	0.51	0.49	min	$\geq$	0.49	0.67	0.29	0.49	0.69	0.31
C3	53.00	50.00	0.51	0.49	min	$\geq$	0.49	0.00	1.00	0.49	0.00	1.00
C4	9.70	9.00	0.52	0.48	min	$\geq$	0.48	0.00	1.00	0.48	0.00	1.00
C5	543.00	514.00	0.51	0.49	min	$\geq$	0.49	0.00	1.00	0.49	0.00	1.00
C6	1.18	1.33	0.47	0.53	min	$\geq$	0.47	0.95	0.00	0.45	1.00	0.00
ERM Ranking: A2—A1							SC_j				1.69	4.31
							PF_j				2.00	5.00
							NPF_j				0.33	0.83
							S_j				0.56	3.59

In case 1, the initial matrix has two alternatives, A1 and A2. The ranking is: $A2 > A1$.

In case 2, when adding a new alternative AA2 identical to the alternative A2, the dimension is changed to 3D. The identical alternatives A2 and AA2 are marked in grey in Table 20. The ranking in this case is: AA2 > A1 > A2, i.e., there is RR.

Table 20. Case 2. Adding alternative AA2 identical to alternative A2. SIMUS results.

Criteria	Initial Matrix			LHS			Type	Operator	RHS	ERM			Z	NERM		
Targets	A1	A2	AA2	A1	A2	AA2				A1	A2	AA2		A1	A2	AA2
C1	376.00	356.00	356.00	0.35	0.33	0.33	min	≥	0.33	0.00	0.00	1.00	0.33	0.00	0.00	1.00
C2	317.00	302.00	302.00	0.34	0.33	0.33	min	≥	0.33	0.67	0.30	0.00	0.33	0.69	0.31	0.00
C3	53.00	50.00	50.00	0.35	0.33	0.33	min	≥	0.33	0.00	0.00	1.00	0.33	0.00	0.00	1.00
C4	9.70	9.00	9.00	0.35	0.32	0.32	min	≥	0.32	0.00	0.00	1.00	0.32	0.00	0.00	1.00
C5	543.00	514.00	514.00	0.35	0.33	0.33	min	≥	0.33	0.00	0.00	1.00	0.33	0.00	0.00	1.00
C6	1.18	1.33	1.33	0.31	0.35	0.35	min	≥	0.31	0.95	0.00	0.00	0.29	1.00	0.00	0.00
ERM Ranking: AA2—A1—A2									SC_j				1.69	0.31	4.00	1.69
									PF_j				2	1	4	2
									NPF_j				0.33	0.17	0.67	0.33
									S_j				0.56	0.05	2.67	0.56
									S_j					0.96	0.44	0.44

The identical alternatives are marked in grey.

Case 3 is also in 3D, but the alternative AA1 is identical to alternative A1. The ranking is: A2 > A1 > AA1. In this case, there is no rank reversal.

In case 2 and case 3, the third alternative is identical to one of the other two, and it should not modify the initial ranking, because the new plane and the existing one are identical, and thus, converting the 3D into a 2D. It could be observed that one of the two identical alternatives disappeared. The reason for this behaviour has already been explained using the dimensional spaces theory, as attributable to missing information from one dimension. Since one hyperplane is identical to another and parallel, they cannot intersect. Table 21 represents the results of ranking for case 3.

Table 21. Case 3. Adding alternative AA1 similar to alternative A1. SIMUS results.

Criteria Targets	Initial Matrix			LHS			Type/ Operator	RHS	ERM			Z	NERM			
	A1	A2	AA1	A1	A2	AA1			A1	A2	AA1		A1	A2	AA1	
C1	376.00	356.00	376.00	0.34	0.32	0.34	min	≥	0.32	0.00	1.00	0.00	0.32	0.00	1.00	0.00
C2	317.00	302.00	317.00	0.34	0.32	0.34	min	≥	0.32	0.67	0.29	0.00	0.32	0.69	0.31	0.00
C3	53.00	50.00	53.00	0.34	0.32	0.34	min	≥	0.32	0.00	1.00	0.00	0.32	0.00	1.00	0.00
C4	9.70	9.00	9.70	0.34	0.32	0.34	min	≥	0.32	0.00	1.00	0.00	0.32	0.00	1.00	0.00
C5	543.00	514.00	543.00	0.34	0.32	0.34	min	≥	0.32	0.00	1.00	0.00	0.32	0.00	1.00	0.00
C6	1.18	1.33	1.18	0.32	0.36	0.32	min	≥	0.32	0.95	0.00	0.00	0.30	1.00	0.00	0.00
ERM Ranking: A2—A1—AA1									SC_j			1.69 4.31 0.00				
									PF_j			2 5 0				
									NPF_j			0.33 0.83 0.00				
									S_i			0.56 3.59 0.00				

The identical alternatives are marked in grey.

Case 4 and Case 5 show what happens when a new one is added to the matrix in which there are two identical alternatives; this is A3. Tables 22 and 23 represent the results of the SIMUS method for both cases. The identical alternatives are marked in grey. There is an RR for both cases.

Table 22. Case 4. Adding a new alternative (A3) to the former matrix A1, A2, AA2. SIMUS results.

Criteria /Targets	Initial Matrix				LHS				NERM			
	A1	A2	A22 = A2	A3	A1	A2	A22 = A2	A3	A1	A2	A22 = A2	A3
C1	376.00	356.00	356.00	402.00	0.25	0.24	0.24	0.27	0.00	0.00	1.00	0.00
C2	317.00	302.00	302.00	365.00	0.25	0.23	0.23	0.28	0.69	0.00	0.31	0.00
C3	53.00	50.00	50.00	74.00	0.23	0.22	0.22	0.33	0.00	1.00	0.00	0.00
C4	9.70	9.00	9.00	10.00	0.26	0.24	0.24	0.27	0.00	0.00	0.32	0.68
C5	543.00	514.00	514.00	575.00	0.25	0.24	0.24	0.27	0.30	0.33	0.00	0.36
C6	1.18	1.33	1.33	1.25	0.23	0.26	0.26	0.25	0.93	0.00	0.00	0.07
ERM Ranking: A1—A22—A3—A2								SC_j	1.93	1.33	1.62	1.11
								PF_j	3	2	3	3
								NPF_j	0.50	0.33	0.50	0.50
								S_j	0.96	0.44	0.81	0.56

The identical alternatives are marked in grey.

Table 23. Case 5. Adding a new alternative (A3) to the former matrix with (A1, A2, AA1). SIMUS results.

Criteria /Targets	Initial Matrix				LHS				NERM			
	A1	A2	AA1 = A1	A3	A1	A2	AA1 = A1	A3	A1	A2	AA1 = A1	A3
C1	376.00	356.00	376.00	402.00	0.25	0.24	0.25	0.27	0.00	1.00	0.00	0.00
C2	317.00	302.00	317.00	365.00	0.24	0.23	0.24	0.28	0.69	0.31	0.00	0.00
C3	53.00	50.00	53.00	74.00	0.23	0.22	0.23	0.32	0.00	1.00	0.00	0.00
C4	9.70	9.00	9.70	10.00	0.25	0.23	0.25	0.26	0.00	0.32	0.00	0.68
C5	543.00	514.00	543.00	575.00	0.25	0.24	0.25	0.26	0.00	0.33	0.30	0.36
C6	1.18	1.33	1.18	1.25	0.24	0.27	0.24	0.25	0.00	0.00	0.93	0.07
ERM Ranking: A2—A3—A11—A1								SC_j	0.69	2.96	1.23	1.11
								PF_j	1	5	2	3
								NPF_j	0.17	0.83	0.33	0.50
								S_j	0.12	2.47	0.41	0.56

The identical alternatives are marked in grey.

Table 24 shows the comparison of RR for all investigated cases. It increases with more additions and decreases with deletions, and this has also been scientifically demonstrated independently by both Linear Programming and Shannon entropy.

Table 24. Comparison of ranking.

D	New Alternative Added	Alternatives	Rank				RR
			1	2	3	4	
2D	-	A1, A2	A2	A1	-	-	-
3D	AA1 = A1	A1, A2, A11 = A1	A2	A1	A11	-	no
4D	A3	A1, A2, A11 = A1, A3	A2	A3	A11	A1	yes
2D	-	A1, A2	A2	A1	-	-	-
3D	AA2 = A2	A1, A2, A22 = A2	A22	A1	A2	-	yes
4D	A3	A1, A2, A11 = A1, A3	A1	A22	A3	A2	yes

Adding more information may provoke that certain criteria values have more effect on certain alternatives than the precedent, and in so doing make possible the occurrence of a new and different ranking.

4.5. Step 5. Study the Influence of Rank Reversal by Changing the Number of Criteria

The rank reversal can appear when changing even one value in a criterion, or changing the number of criteria. The case of five and six criteria and six and seven alternatives is presented with the purpose of studying rank reversal. Table 25 represents the results of the

SIMUS method for five criteria and six alternatives. In this case, the criterion C6 is removed from the decision matrix given in Table 7. The value of the objective function for each linear optimization is shown in the last column of the table. Table 26 represents the Normalized Efficient Results Matrix (NERM) and the results of ranking. The ranking in this case is as follows: AS1 > AS3 > A1 > A2 > A3 > AS2.

Table 25. Efficient Results Matrix (ERM) for the case of five criteria and six alternatives.

Objective	Alternative						Objective Function Values per Target
	A1	A2	A3	AS1	AS2	AS3	
Z1	0.00	0.00	0.00	0.92	0.00	0.08	0.13
Z2	0.00	0.00	0.01	0.96	0.00	0.03	0.14
Z3	0.00	0.07	0.00	0.93	0.00	0.00	0.14
Z4	0.00	0.00	0.00	0.00	0.00	1.00	0.15
Z5	1.00	0.00	0.00	0.00	0.00	0.00	0.15

Table 26. Normalized Efficient Results Matrix (NERM) for the case of five criteria and six alternatives. Ranking results.

Objective	Normalized Efficient Results Matrix (NERM)					
	A1	A2	A3	AS1	AS2	AS3
Z1	0.00	0.00	0.00	0.92	0.00	0.08
Z2	0.00	0.00	0.01	0.96	0.00	0.03
Z3	0.00	0.07	0.00	0.93	0.00	0.00
Z4	0.00	0.00	0.00	0.00	0.00	1.00
Z5	1.00	0.00	0.00	0.00	0.00	0.00
SC_j	1.00	0.07	0.01	2.81	0.00	1.11
PF_j	1	2	1	3	0	3
NPF_j	0.20	0.40	0.20	0.60	0.00	0.60
S_j	0.20	0.03	0.00	1.69	0.00	0.67

ERM Ranking: AS1—AS3—A1—A2—A3—AS2

According to the results represented in Table 11, the ranking for six criteria and six alternatives is as follows: AS1 > AS2 > AS3 > A2 > A1 > A3. It could be concluded that the rank reversal appears for alternatives that are not in the first position. In Appendix A, the ranking results for the case with seven alternatives and five (six) criteria are shown (Tables A1–A6). Table 27 presents a summary of the results.

Table 27. Ranking with different numbers of criteria and alternatives.

Criteria	Alternatives	Rank							RR
		1	2	3	4	5	6	7	
C1–C5	A1, A2, A3, AS1, AS2, AS3	AS1	AS3	A1	A2	A3	AS2	-	yes
C1–C6	A1, A2, A3, AS1, AS2, AS3	AS1	AS2	AS3	A2	A1	A3	-	yes
C1–C5	A1, A2, A3, AS1, AS2, AS3, B1	AS3	A2	A1	B1	AS1	A3	AS2	yes
C1–C6	A1, A2, A3, AS1, AS2, AS3, B1	AS2	A2	AS3	B1	A3	A1	AS1	yes

It can be seen in Table 27 that there is an RR when the number of criteria is changed.

4.6. Step 6. Study the Entropy and Information for All Cases

When using Shannon entropy, the aim is to show how the addition of new alternatives or routes decreases uncertainty. This is another way, not related to Linear Programming, of showing that the information increases with the number of alternatives. There are fluctuations of decrease and increase, but the general trend is that the amount of information in the system increases and improves the main objective. In Appendix A (Tables A7–A9), an example is presented for determining the entropy and the information of the system in the case of 7 alternatives (the initial matrix is shown in Table A1 in Appendix A). Table 28 shows the comparison of entropy and information for all cases from 3 to 12 alternatives and six criteria. Figures 5 and 6 illustrate the change in entropy and information depending on the number of alternatives. A trend line is added, which shows a linear relationship between the number of alternatives and the entropy (information) of the system. The coefficient of determination and the linear relationship equation have been added. The trend is shown with a dotted line.

Table 28. Comparison of entropy and information.

Indicator	Alternatives (Dimension)									
	3	4	5	6	7	8	9	10	11	12
Entropy	5.98	5.96	5.97	5.98	5.96	5.95	5.94	5.93	5.94	5.94
Information	0.02	0.04	0.03	0.02	0.04	0.05	0.06	0.07	0.06	0.06

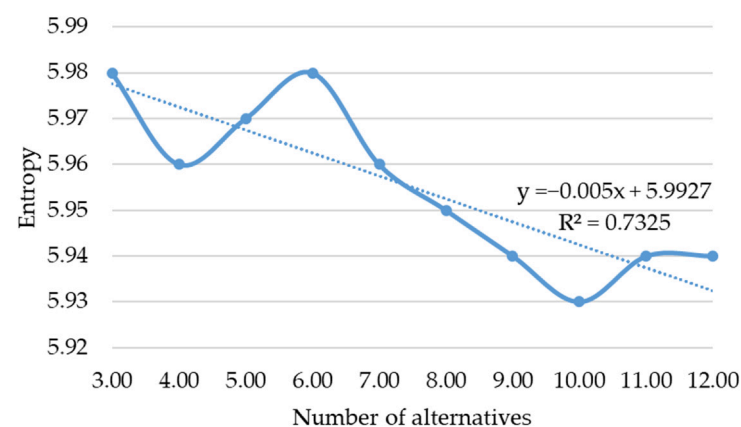


Figure 5. Entropy as a function of dimensions.

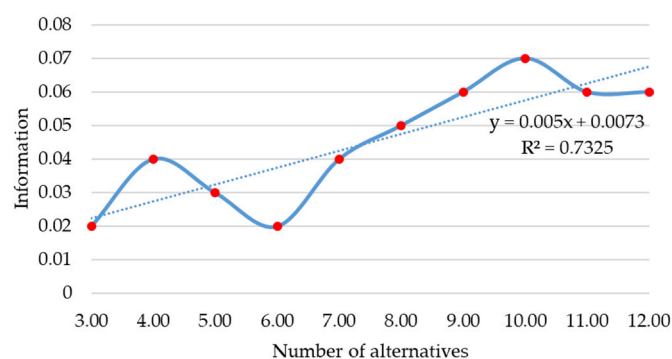


Figure 6. Information as a function of dimensions.

The adjusted coefficient of determination, due to the small number of data points, is 0.70, which shows a strong connection. The standard error of the estimate is 0.0097. The regression analysis performed shows a high degree of statistical significance of the linear model with a number of alternatives ($n = 10$) ($p = 0.002$).

Consequently, it demonstrates that increasing the number of alternatives or spatial dimensions increases information and decreases entropy. Therefore, the more alternatives there are, the less uncertainty there is.

5. Conclusions

The choice of a management strategy for transport companies is of great importance for their sustainable development. Transport is characterized by competition between modes of transport, especially when there is carriage on parallel routes connecting the starting and final points. Conducting an analysis and deciding on the most suitable route is a problem related to sustainability and requires consideration and evaluation of different transport alternatives, taking into account the influence of predefined criteria. The aspects of sustainability when choosing a route for the transport of containers on parallel lines between rail and road transport are related to reducing the environmental footprint, operating costs, infrastructure load and increasing the safety and security of transport.

Sustainability in the transport sector could be considered in two aspects:

- The first aspect is consideration in a stationary environment, when a predefined set of transport alternatives and criteria are evaluated.
- The second aspect is consideration in a dynamic environment. In this case, the impact of introducing new alternatives to a predefined set of alternatives is dynamically and sequentially assessed, and also how this changes the ranking and what the impact is on decision-making.

A stationary environment sustainability study in route selection using the SIMUS method was studied in [51,52]. However, in this case, the impact of the sequential addition of transport alternatives on decision-making cannot be assessed. The results obtained are for the case with a predefined set of alternatives that does not change. In the dynamic environment of studying sustainability, which is the contribution of this research, an assessment is made of the change in ranking when the number of alternatives changes, and how this affects the rank reversal.

The novelty of this study contributes to the sustainability aspects related to environmental friendliness, energy efficiency, safety and security when choosing a transport route in a dynamic environment. This dynamic environment of sustainability assessment is directly related to the rank reversal phenomenon, which is presented in this study, through the SIMUS method.

In this study, an invariance coefficient and a rank reversal coefficient were introduced to assess sustainability. The limits of variation in the introduced coefficients were determined. The values of both defined coefficients have been classified into groups to explain sustainability in decision-making.

This paper examines the phenomenon of rank reversal in decision-making by using the SIMUS method for the studied route selection for intermodal transport. It has been proven, based on the linear programming method and also by Shannon entropy, that adding, or decreasing in the case of deleting, the information content makes the occurrence of a new and different ranking possible. If adding an alternative, more information may provoke that certain criteria values have more effect on certain alternatives than the previous ones and thus makes it possible to implement RR in the SIMUS method.

This research, by studying railway and road routes, explains the reasons for the appearance of rank reversal.

The novelty of this research on rank reversal in the SIMUS method lies in:

- The RR in the SIMUS method is a consequence of dimensional spaces, given by the number of alternatives. When the problem is changed to 3D, it will have three Cartesian coordinates, and there is no guarantee that the original two values will be

preserved. These new coordinates may or may not preserve the two original ones. Therefore, RR in the SIMUS method happens due to the change in dimensional spaces.

- When alternatives are added, the ranking depends on the solution of the individual linear optimization models, the formation of the efficient results matrix (ERM) and the ranking criterion.
- When a new alternative is added, a new variable is actually added to the linear programming model that SIMUS uses. When rank reversal is implemented, the new variable shifts the vertices of the polyhedron in a way that reduces the effectiveness of the previous variable relative to the new one. In the case of invariance, the new variable cannot affect the previous variable and its position in the polyhedron.
- When rank reversal occurs, the new alternative changes the structure of the constraints in the linear programming model in a way that affects the feasible solution set.
- The results are unexpected because the change depends on two unknown factors of the new vector corresponding to the added alternative: the objective coefficient and the performance values of this vector.
- An invariance coefficient and a rank reversal coefficient were defined to assess sustainability in decision-making in a dynamic environment.

This paper presents a hypothesis to explain what RR is in the SIMUS method and why it happens. It is based on a concept; it is grounded in the fact that aggregating or deleting an alternative from an existing scenario implies that the problem ceases to be in its original dimension and transforms into a new one. The paper analyses how this linear transformation may affect an existing ranking by creating a new one that can be equal or different, depending on aspects like modifications in the objective function vector, as well as changes in the objective's vectors or criteria. The paper analyses the consequences of these changes that may or may not change the slopes of the planes and hyperplanes and, in so doing, change the original polytope built, which contains in its space all feasible solutions of the problem.

The proposed hypothesis for rank reversal in the SIMUS method expands the theoretical knowledge about the causes of this phenomenon in MCDM. To the causes for rank reversal in MCDM established in the scientific space, a completely new cause was added, which is valid for the SIMUS method and consists of the logical consequence of linear transformations along different dimensional spaces, each time an alternative is added or deleted. RR in the SIMUS method is a random phenomenon, and as such, it can appear or not, but there is a reason for that randomness, also mathematical, and it is due to the values of the vector that is added to an existing matrix, which depends on the characteristics of the alternative added, like new locomotives, road trains, new roads, etc.

The phenomenon of rank reversal in decision-making by applying the SIMUS method is revealed by studying real railway and road routes in a transport network. It clearly illustrates that RR in the SIMUS method is an unpredictable event. The hypothesis of RR is mathematically justified.

Regarding the investigated real example for container carriage by railway and road transport between Sofia and Varna, the results proposed different transport solutions taking into account the alternatives studied.

When studying rank reversal in different cases, it can be concluded that there are two types of alternatives—some that are ranked first, and others that are not ranked first but retain their positions in the new ranking, i.e., they can be called invariant. These alternatives can also be perceived as the most appropriate when making a decision, since they are not affected by changes occurring. This is important in transport engineering, because it shows that the relative position of an alternative is not only a function of its own

operational attributes, but also of the context created by the set of options considered at the same time, i.e., the structure of the comparison can influence the outcome.

The introduction of new technologies in road transport, such as new types of heavy-duty trucks with electric drive, which reduce harmful emissions, affects the ranking and gives priority to road transport in the considered direction. In these cases, however, alternatives with rail transport on the route Sofia—Gorna Oryahovitsa—Varna remain invariant and retain their position. The railway operator should focus its attention on transport on this route in order to respond to competition from new technologies in road transport.

Several limitations in the conducted research should be acknowledged. First, to reduce problem complexity, some conditions are not considered, such as the influence of weather conditions; the time for loading and unloading containers; the transshipment fees. Second, the influence of the limited types of new rolling stock for rail transport and heavy trucks was investigated.

It could be concluded that in the cases where the rank changes when new alternatives are added, there are alternatives that retain their position, even though they are not in first place in the ranking. These alternatives can be considered invariant. In these cases, the transport operators could select these alternatives as important in the realization of the carriage.

As a consequence of the conducted research and the obtained results, it can be recommended to transport operators, when making a decision for transport planning, to study the impact of the sequential addition of alternatives for evaluation. This would increase the sustainability of the decisions made in the dynamic transport environment.

The connection with sustainability in transport planning, except for the investigated criteria connected to the environment, technology and economics, is precisely in considering the decision-making process in its dynamics, which means that when evaluating a given set of alternatives under given criteria, it is necessary to track the impact of their sequential introduction for consideration in the decision-making matrix, to track whether or not there is a change in the ranking and whether there are invariant alternatives. This would help the decision-maker to analyze the situation in a dynamic environment and to assess how the system changes, depending on the set of alternatives that are considered. As a result, the decision-maker can make an informed choice in a dynamic environment. For comparison, when the process is stationary, i.e., a specific set of alternatives and criteria is examined without tracking the effect of the sequential introduction of the alternatives into the system, it is not possible to track how the change in the set of alternatives affects the ranking. Therefore, the study of the rank reversal phenomenon and the reasons for its occurrence is an important issue related not only to the theory of multi-criteria analysis methods, but also to the issue of sustainability. The SIMUS method, which combines both multi-objective optimization and multi-criteria analysis as a systematic method and does not use expert evaluation, is a suitable means for decision-making in a dynamic environment.

In future studies, the scope of the researched area can be expanded by considering situations in other technological areas.

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Appendix A

Table A1. Initial decision matrix—6 criteria, 7 alternatives.

Criteria	Alternatives							Optimization
	A1	A2	A3	AS1	AS2	AS3	B1	
C1	376	356	402	258	245	274	413	min
C2	317	302	365	283	270	327	411	min
C3	53	50	74	47	45	62	91	min
C4	9.7	9	10	9.2	8.5	9.5	7.5	min
C5	543	514	575	543	514	575	447	min
C6	1.18	1.33	1.25	1.11	1.25	1.18	1.43	min

Table A2. Normalized initial decision matrix—6 criteria, 7 alternatives.

Objective	LHS							Action	Operator	RHS Values
	A1	A2	A3	AS1	AS2	AS3	B1			
	x_1	x_2	x_3	x_4	x_5	x_6	x_7			
Z1	0.16	0.15	0.17	0.11	0.11	0.12	0.18	min	$\geq$	0.11
Z2	0.14	0.13	0.16	0.12	0.12	0.14	0.18	min	$\geq$	0.12
Z3	0.13	0.12	0.18	0.11	0.11	0.15	0.22	min	$\geq$	0.11
Z4	0.15	0.14	0.16	0.15	0.13	0.15	0.12	min	$\geq$	0.12
Z5	0.15	0.14	0.15	0.15	0.14	0.15	0.12	min	$\geq$	0.12
Z6	0.14	0.15	0.14	0.13	0.14	0.14	0.16	min	$\geq$	0.13

Table A3. Efficient Results Matrix (ERM)—6 criteria, 7 alternatives.

Objective	A1	A2	A3	AS1	AS2	AS3	B1	Objective Function Values
Z1	0.00	0.00	0.00	0.00	0.49	0.42	0.00	0.10
Z2	0.00	0.07	0.07	0.00	0.66	0.00	0.07	0.11
Z3	0.00	0.89	0.00	0.00	0.00	0.00	0.00	0.11
Z4	0.00	0.00	0.00	0.00	0.53	0.16	0.18	0.12
Z5	0.14	0.58	0.00	0.00	0.00	0.00	0.12	0.12
Z6	0.00	0.00	0.42	0.00	0.00	0.36	0.00	0.11

Table A4. Normalized Efficient Results Matrix (NERM)—6 criteria, 7 alternatives. Ranking results.

Objective	A1	A2	A3	AS1	AS2	AS3	B1
Z1	0.00	0.00	0.00	0.00	0.54	0.46	0.00
Z2	0.00	0.08	0.08	0.00	0.76	0.00	0.08
Z3	0.00	1.00	0.00	0.00	0.00	0.00	0.00
Z4	0.00	0.00	0.00	0.00	0.61	0.19	0.20
Z5	0.17	0.68	0.00	0.00	0.00	0.00	0.15
Z6	0.00	0.00	0.53	0.00	0.00	0.47	0.00
SC_j	0.17	1.76	0.62	0.00	1.91	1.11	0.43
PF_j	1	3	2	0	3	3	3
NPF_j	0.17	0.50	0.33	0.00	0.50	0.50	0.50
S_j	0.03	0.88	0.21	0.00	0.95	0.56	0.21

ERM Ranking: AS2—A2—AS3—B1—A3—A1—AS1

Table A5. Efficient Results Matrix (ERM)—5 criteria, 7 alternatives.

Objective	Alternative							Objective Function Values per Target
	A1	A2	A3	AS1	AS2	AS3	B1	
Z1	0.00	0.00	0.00	0.00	0.00	0.83	0.00	0.10
Z2	0.00	0.00	0.16	0.44	0.00	0.20	0.00	0.11
Z3	0.00	0.89	0.00	0.00	0.00	0.00	0.00	0.11
Z4	0.00	0.00	0.00	0.00	0.00	0.65	0.16	0.12
Z5	0.66	0.00	0.00	0.00	0.00	0.00	0.15	0.11

Table A6. Normalized Efficient Results Matrix (NERM)—5 criteria, 7 alternatives. Ranking results.

Objective	Normalized Efficient Results Matrix (NERM)							
	A1	A2	A3	AS1	AS2	AS3	B1	
Z1	0.00	0.00	0.00	0.00	0.00	1.00	0.00	
Z2	0.00	0.00	0.20	0.55	0.00	0.25	0.00	
Z3	0.00	1.00	0.00	0.00	0.00	0.00	0.00	
Z4	0.00	0.00	0.00	0.00	0.00	0.00	0.20	
Z5	0.81	0.00	0.00	0.00	0.00	0.00	0.19	
SC_j	0.81	1.00	0.20	0.55	0.00	2.05	0.38	
PF_j	1	1	1	1	0	3	2	
NPF_j	0.20	0.20	0.20	0.20	0.00	0.60	0.40	
S_j	0.16	0.20	0.04	0.11	0.00	1.23	0.15	

ERM Ranking: AS3—A2—A1—B1—AS1—A3—AS2

Table A7. Calculation of Entropy for 7 alternatives. Initial matrix and normalized matrix.

Criteria	$(x)_i$							$p(x)_i$						
	A1	A2	A3	AS1	AS2	AS3	B1	A1	A2	A3	AS1	AS2	AS3	B1
C1	376	356	402	258	245	274	413	0.16	0.15	0.17	0.11	0.11	0.12	0.18
C2	317	302	365	283	270	327	411	0.14	0.13	0.16	0.12	0.12	0.14	0.18
C3	53	50	74	47	45	62	91	0.13	0.12	0.18	0.11	0.11	0.15	0.22
C4	9.7	9	10	9.2	8.5	9.5	7.5	0.15	0.14	0.16	0.15	0.13	0.15	0.12
C5	543	514	575	543	514	575	447	0.15	0.14	0.15	0.15	0.14	0.15	0.12
C6	1.18	1.33	1.25	1.11	1.25	1.18	1.43	0.14	0.15	0.14	0.13	0.14	0.14	0.16

Table A8. Calculation of Entropy for 7 alternatives— $\ln p(x)_i$.

Criteria	$\ln p(x)_i$													
	A1	A2	A3	AS1	AS2	AS3	B1	A1	A2	A3	AS1	AS2	AS3	B1
C1	−1.82	−1.88	−1.75	−2.20	−2.25	−2.14	−1.73							
C2	−1.97	−2.02	−1.83	−2.08	−2.13	−1.94	−1.71							
C3	−2.07	−2.13	−1.74	−2.19	−2.24	−1.92	−1.53							
C4	−1.88	−1.95	−1.85	−1.93	−2.01	−1.90	−2.13							
C5	−1.92	−1.98	−1.86	−1.92	−1.98	−1.86	−2.12							
C6	−2.00	−1.88	−1.94	−2.06	−1.94	−2.00	−1.81							

Table A9. Calculation of Entropy for 7 alternatives—results.

Criteria	$p(x)_i \cdot \ln p(x)_i$							$\sum_{i=1}^n p(x)_i \ln p(x)_i$	$H(x)_i$	$I(x)_i$
	A1	A2	A3	AS1	AS2	AS3	B1			
C1	−0.29	−0.29	−0.30	−0.24	−0.24	−0.25	−0.31	−1.93	0.99	0.01
C2	−0.27	−0.27	−0.29	−0.26	−0.25	−0.28	−0.31	−1.94	1.00	0.00
C3	−0.26	−0.25	−0.31	−0.24	−0.24	−0.28	−0.33	−1.91	0.98	0.02
C4	−0.29	−0.28	−0.29	−0.28	−0.27	−0.28	−0.25	−1.94	1.00	0.00
C5	−0.28	−0.27	−0.29	−0.28	−0.27	−0.29	−0.25	−1.94	1.00	0.00
C6	−0.27	−0.29	−0.28	−0.26	−0.28	−0.27	−0.30	−1.94	1.00	0.00
$k = \frac{1}{\ln(7)} = 0.514$									5.96	0.04

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