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Adaptive Design Reasoning of Pre-Service Mathematics Teachers in Instructional Materials Design: A Conjecture Mapping Study

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Abstract

The sustainability of education systems depends on teachers' capacity to adapt to changing curriculum expectations and diverse student needs. However, there is limited information on how adaptive design reasoning develops in the iterative process of designing and adapting instructional materials by pre-service mathematics teachers. This study reinterprets the Dynamic Capabilities Theory in the context of individual teacher learning, examining how the processes of sensing, seizing, and transformation emerge as teacher candidates develop materials suitable for the new national mathematics curriculum. The research was conducted using a design-based research approach. Conjecture mapping was used to articulate relationships among design features, mediating processes, and observable outcomes. The study group consisted of 23 pre-service mathematics teachers, and in-depth analyses were conducted with two purposefully selected focus participants who exhibited different developmental patterns. The data were obtained from teaching materials, design plans, interviews, and feedback processes; they were analyzed using an inductive-deductive analysis approach. The findings indicate that iterative material design cycles made the adaptive design reasoning of pre-service mathematics teachers visible. Two focus cases, although following different developmental paths in terms of contextual awareness, representational clarity, digital tool usage, feedback-based reflection, and pedagogical justification, have both been observed to have design decisions becoming more conscious and curriculum-sensitive over time. The findings may provide a course-based context for making curriculum-responsive design reasoning visible and open to reflection. Findings should be interpreted within the study context.

Keywords: adaptive design reasoning; dynamic capabilities; curriculum reform; design-based research; mathematics teacher education; instructional material design

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1. Introduction

Sustainable curriculum implementation requires teachers who can respond to changing curricular, technological, and learner-related demands by reconsidering their instructional decisions and resources. Such adaptation involves more than the technical adoption of new materials; it constitutes a professional learning process in which teachers interpret emerging demands, mobilize pedagogical resources, and revise their practices [1,2]. Initial teacher education, therefore, needs to provide pre-service teachers with structured opportunities to practice and articulate this form of adaptation before entering the profession.

Research on teachers' curriculum use and instructional-material design has provided important accounts of how teachers interpret, select, adapt, and mobilize pedagogical resources [3,4]. Related perspectives, including pedagogical design capacity, professional noticing, adaptive expertise, and professional agency, foreground different but complementary dimensions of teachers' work, such as resource use, attention to pedagogically significant situations, flexible application of knowledge, and intentional action within contextual and institutional conditions [5,6]. However, these perspectives do not always make explicit the temporal sequence through which pre-service teachers move from identifying a curricular or learner-related demand to mobilizing resources in response and subsequently reorganizing their design routines. In this study, adaptive design reasoning refers to the reasoning through which pre-service teachers notice relevant curricular, learner-related, technological, or contextual demands; select and coordinate pedagogical resources in response to those demands; and revise their design decisions and underlying assumptions through feedback and reflection.

Instructional-material design provides a particularly relevant context for examining adaptive reasoning in initial teacher education. When pre-service teachers select, modify, and justify materials, they are required to coordinate curricular goals, mathematical content, anticipated student thinking, available technologies, and contextual constraints. These activities position them as active designers rather than passive implementers of curriculum materials [3,4,7]. Design tasks can also create opportunities for pre-service teachers to develop pedagogical knowledge, professional agency, and readiness for teaching by making their instructional assumptions available for examination and revision [5,8]. Open-ended and iterative design experiences may therefore provide an appropriate setting for investigating how prospective teachers respond to curricular change and reconsider their pedagogical decisions [9,10]. Dynamic Capabilities Theory was originally developed to explain how organizations identify changes in their environments, mobilize resources, and reconfigure established routines in response to those changes [11,12]. Its use at the level of individual teacher learning therefore requires theoretical caution because organizations and individuals differ in their units of action, resources, decision-making structures, and temporal scales. In the present study, the framework is not transferred as a complete theory of individual cognition, nor are sensing, seizing, and transforming treated as stable personal traits. Instead, these constructs are used as analytically distinguishable categories for tracing observable forms of adaptive design reasoning across participants' design plans, revised artifacts, feedback discussions, and reflective explanations. In this way, Dynamic Capabilities Theory complements existing perspectives by foregrounding the sequence connecting the noticing of a demand, the mobilization of pedagogical resources, and the reconfiguration of a design decision.

Although Education for Sustainable Development commonly emphasizes the integration of sustainability-related knowledge, values, and practices into curricula and teacher preparation [13], sustainability is used more narrowly in the present study. It refers to the professional capacity required to support the continued implementation of curriculum reform through the ongoing reconsideration of instructional resources and pedagogical decisions. Adaptive design reasoning is therefore examined as a potential professional resource for sustainable curriculum implementation, rather than as direct evidence of long-term professional development, durable classroom change, or student learning outcomes.

Previous research has established that pre-service teachers draw on multiple forms of knowledge when interpreting, selecting, and adapting instructional materials [3,4]. Research has also examined professional vision, pedagogical knowledge, technological competence, and agency in teacher education [5,6,14]. Nevertheless, less is known about how pre-service mathematics teachers' adaptive reasoning can be traced across successive

versions of instructional materials and how feedback, reflection, and design-rationale articulation mediate movement from noticing a curricular or learner-related demand to revising an instructional design. Moreover, few studies have used Dynamic Capabilities Theory as a complementary analytical lens at the individual level or combined this lens with conjecture mapping to connect course design features, mediating processes, and observable changes in design reasoning. Addressing this gap can contribute theoretically by refining the individual-level interpretation of sensing, seizing, and transforming and practically by identifying course features through which teacher educators can make adaptive design reasoning visible and discussable.

Accordingly, this study examines how sensing, seizing, and transforming become visible as forms of adaptive design reasoning while pre-service mathematics teachers design and revise instructional materials aligned with a new national curriculum. It also examines how specific course design features (curriculum-aligned tasks, explicit prompts, guided feedback, and written reflection) mediate observable changes in participants' design reasoning. The study traces evidence across initial plans, feedback discussions, revised artifacts, interviews, and reflective explanations.

The study addresses the following research questions:

1. How do sensing, seizing, and transforming become visible as forms of adaptive design reasoning while pre-service mathematics teachers design and revise instructional materials aligned with a new national curriculum?
2. How do curriculum-aligned design tasks, explicit prompts, guided feedback, and reflective activities mediate observable changes in pre-service mathematics teachers' design reasoning across iterative material-design cycles?
3. What forms of professional learning, curricular noticing, and changing understandings of instructional materials are evident in participants' design artifacts and reflective explanations during the course?

The remainder of this paper first develops the individual-level interpretation of Dynamic Capabilities Theory and its relationship to adaptive design reasoning. It then describes the Design-Based Research context, the use of conjecture mapping, the design intervention, and the inductive–deductive analysis. The findings are presented through two focal cases and a cross-case comparison, followed by a discussion of the study's theoretical, methodological, practical, and sustainability-related implications.

2. Conceptualization of the Study

Instructional-material design provides a practice-based context in which pre-service mathematics teachers make explicit decisions about content representation, task sequencing, pedagogical possibilities, and anticipated student thinking. Structured instructional-design tasks can create opportunities for pre-service mathematics teachers to make their pedagogical content knowledge visible and to reconsider how mathematical content, instructional strategies, and anticipated student thinking are coordinated within a material or task [15,16]. Previous research indicates that pre-service and novice teachers draw on multiple domains of mathematical and pedagogical knowledge when planning instruction, interpreting student thinking, and making design decisions [17,18]. Taken together, these studies suggest that instructional material design can function as a context in which pre-service mathematics teachers articulate, coordinate, and reconsider their mathematical and pedagogical reasoning.

Dynamic Capabilities Theory: From the Organizational to the Individual Level

Dynamic Capabilities Theory originated in strategic management to explain how organizations sense environmental changes, mobilize resources, and reconfigure routines

in response to changing conditions [1,12]. The framework comprises three interrelated dimensions: sensing changes, opportunities, or constraints; seizing them through the selection and coordination of resources; and transforming established routines and structures to support adaptation [12].

In the present study, these dimensions are cautiously reinterpreted at the individual level as analytical categories for examining pre-service mathematics teachers' observable design reasoning. Sensing refers to identifying curricular, learner-related, technological, or contextual demands; seizing refers to selecting and coordinating pedagogical resources in response to those demands; and transforming refers to revising design routines, decisions, or underlying pedagogical assumptions across successive stages of the design process. Sensing was operationalized as evidence that a participant identified a pedagogically relevant curricular, learner-related, technological, or contextual demand that could affect the design or use of an instructional material. This operationalization is related to professional vision, defined as the capacity to notice and interpret pedagogically significant aspects of instructional situations [6]. Instructional-material design offers a context in which such noticing can become observable because participants are required to articulate the curricular, learner-related, and contextual considerations underlying their design decisions.

Building on this framework, the three dimensions were reinterpreted as analytical categories for tracing changes in pre-service mathematics teachers' instructional-material design reasoning. This adaptation does not assume that organizational capabilities and individual cognition are equivalent; rather, it uses the sensing–seizing–transforming sequence as a complementary analytical structure for connecting noticed demands, resource-based responses, and subsequent design revisions. Table 1 presents the individual-level operationalization of the three dimensions, the indicators used in the analysis, the relevant data sources, and the forms of change examined across time.

Table 1. Operationalization of dynamic capabilities at the individual level in this study.

Dimension	Individual-Level Operationalization	Analytical Indicators	Primary Data Sources	Indicators of Change Over Time
Sensing	Curricular, learner-related, and contextual demands are noticed	Curriculum goals, student difficulties, constraints, and tool affordances	Initial plans, interview excerpts, and discussion notes	From general noticing to specific anticipation of student thinking
Seizing	Resources and strategies are selected and coordinated	Task modification, digital tools, historical resources, and design justifications	Revised materials, design rationales, and peer-review notes	From isolated resource use to pedagogically justified resource orchestration
Transforming	Design routines and pedagogical assumptions are reorganized	Revision after feedback, rethinking teacher/student roles, future design principles	Successive versions, reflective interviews, and final design plan	From implicit practice to explicit design principle

To distinguish theoretically meaningful evidence from general commentary, specific inclusion and exclusion criteria were applied. A statement or design action was coded as sensing only when it identified a curricular, learner-related, technological, or contextual demand and connected that demand to a possible consequence for instructional design or implementation. General statements such as "students may find this difficult" were not coded as sensing unless the participant specified what the difficulty involved and how it affected the material or task. Seizing was coded when a participant selected, adapted, or coordinated a resource and articulated, or demonstrated through the artifact, a pedagogical

rationale for that choice. Merely mentioning a tool or resource was not considered sufficient. Transforming was coded when evidence showed a revision in a design routine, decision, or pedagogical assumption across at least two timepoints or data sources. Participation in a revision activity alone was not treated as evidence of transformation.

The enactment of sensing, seizing, and transforming may vary across participants and design tasks because adaptive reasoning is shaped by task demands, prior experience, available resources, and the instructional context [14]. Guided practice, collaborative interaction, and scaffolded opportunities for experimentation may create conditions in which pre-service teachers can articulate and reconsider their professional reasoning [19]. Within this study, transforming does not refer to the permanent consolidation of a new professional practice. It refers more narrowly to observable revisions in participants' design routines, pedagogical decisions, or underlying assumptions across successive design artifacts and reflective explanations.

In this study, sensing, seizing, and transforming are treated as analytically distinguishable forms of adaptive design reasoning rather than as learning mechanisms in themselves. Feedback-based reflection, articulation of design rationales, comparison between initial and revised artifacts, collaborative discussion, and engagement with curricular criteria are conceptualized as mediating processes through which these forms of reasoning became observable. A single statement or isolated design action was not treated as sufficient evidence of change; interpretations were based on evidence chains connecting initial decisions, feedback interactions, revised artifacts, and later explanations.

When pre-service mathematics teachers engage in instructional design, they make explicit decisions about mathematical content, pedagogy, and anticipated student learning, thereby making aspects of their existing knowledge and beliefs available for reflection and revision [20]. This interpretation is also compatible with practice-based perspectives on teacher learning, which examine how professional reasoning becomes visible through engagement with concrete instructional tasks, student work, and reflective activity [21].

Thus, the study uses a pedagogical reinterpretation of Dynamic Capabilities Theory at the individual level as a complementary analytical lens rather than as a comprehensive theory of teacher cognition or professional development [12]. The framework was used to trace how pre-service mathematics teachers identified relevant demands, mobilized available resources, and revised design decisions while working with instructional materials aligned with the new curriculum.

Participation in a design cycle was not treated as evidence of capability development in itself. Evidence of change required a traceable difference in reasoning, resource use, design routines, or pedagogical assumptions across multiple timepoints or data sources. Participants' pedagogical statements and design decisions were coded as indicators of sensing, seizing, or transforming only when they met the operational criteria presented above and were supported by related evidence from another data source or timepoint. This operationalization made it possible to examine how participants interpreted the new curriculum's emphases, selected design strategies and resources, and revised their instructional-material decisions through cycles of feedback and reflection.

Applied to the present study, this perspective provides a structured way to examine how pre-service mathematics teachers notice demands in the new curriculum, experiment with design strategies, and revise their pedagogical reasoning while generating and adapting instructional materials. The analysis therefore focuses on observable enactments and changes within the course rather than claiming the stable development of dynamic capabilities beyond the study context.

This study contributes to teacher education research by using Dynamic Capabilities Theory as a complementary analytical lens for examining individual-level adaptive design

reasoning among pre-service mathematics teachers [11,12]. Applications of Dynamic Capabilities Theory in education remain limited and have largely focused on institutional or higher-education contexts rather than on the adaptive reasoning of individual pre-service teachers [22]. The study offers an individual-level operationalization for examining how pre-service mathematics teachers notice curricular and learner-related demands, mobilize pedagogical resources, and revise teaching material decisions in response to curricular change.

In this study, adaptation to the new curriculum is interpreted through observable forms of sensing, seizing, and transforming, while feedback, reflection, design-rationale articulation, and artifact revision are treated as mediating processes. The findings are used to examine how iterative cycles of design, feedback, revision, and reflection made participants' adaptive design reasoning observable within the course. This perspective may inform the design of mathematics teacher-education courses by emphasizing the integration of instructional design, guided feedback, reflection, and opportunities to justify design decisions.

Previous research has examined pre-service teachers' pedagogical content knowledge, curriculum literacy, and instructional material use [3,4,8]. However, less is known about how different forms of knowledge and reasoning are reorganized across successive stages of material design, feedback, and revision. Therefore, there is a need to understand how pre-service mathematics teachers notice curricular and learner-related demands, make resource-based design decisions, and revise their pedagogical assumptions through feedback and reflection. Addressing this gap is important because material-design courses can function not only as settings for producing instructional artifacts but also as structured contexts for making adaptive professional reasoning visible, discussable, and open to revision. The broader sustainability relevance of this perspective lies in its potential to support curriculum-responsive professional reasoning, although the present study does not examine long-term classroom enactment or student outcomes.

3. Methodology

3.1. Research Design

This study was conducted within a broader Design-Based Research (DBR) project examining how pre-service mathematics teachers engaged in the design and adaptation of instructional materials aligned with a new national curriculum in Türkiye. The broader project comprised two iterative DBR cycles implemented across consecutive course iterations. Each cycle involved the design, implementation, analysis, and refinement of the instructional materials design course offered within the mathematics teacher-education program. In the first cycle of the research, the initial design was implemented, and the operation of the application, participants' experiences during the process, and areas for improvement were examined. Based on the findings from the first cycle, the improvements include the reorganization of instructional activities, making design tasks clearer, planning to support participants' roles in the design process, and enhancing the data collection process. The second cycle represents a more developed and refined application design, developed based on the feedback obtained from the first cycle. For example, in the first cycle, participants were expected to perform an adaptation task directly, but it was observed that analyzing an existing instructional material before this task could help participants notice design features and make subsequent design decisions more consciously. Therefore, in the second cycle, an activity focused on material analysis was moved before the adaptation task, and the sequence of activities was rearranged. Thus, the current study focuses on the second cycle, which is more directly related to the research questions and provides richer analytical evidence regarding the design process. Although the first cycle is not directly

analyzed in this study, it has formed a fundamental component of the research's iterative nature as a formative stage shaping the design of the second cycle. This emphasizes the importance of articulating clear learning goals, understanding the mechanisms of learning, and evaluating instructional effectiveness, thereby fostering a coherent program [23].

Within the Design-Based Research approach, the Conjecture Mapping methodology [24] is used, which allows for the explicit representation of hypothetical relationships between design elements, mediating processes, and observable outcomes of PMTs' learning. In this study, DBR was used to create the methodological framework in which the design process is structured, while Conjecture Mapping was employed as an analytical tool to examine the underlying assumptions of the design decisions that emerged during this process. For example, having participants explain why they changed or retained a specific feature during the redesign process of an instructional material helped uncover the assumptions behind the design decisions. In this context, while DBR reveals how the design process progresses, Conjecture Mapping has allowed for a detailed examination of the design thinking and decision-making that emerge during this process.

The application of conjecture mapping is used to enhance visualizing and navigating the complexities of the design process [25]. The Conjecture Map provides a systematic way to link the design of an educational intervention with the theoretical assumptions that underpin it. In this study, the analysis was structured using a Conjecture Map [24], which enabled linking design decisions to learning mechanisms and observable outcomes in cases of Alya and Yaman. Within this framework, Dynamic Capabilities Theory [12] provided a lens for identifying the mediating processes that explain how Alya and Yaman transformed their pedagogical knowledge and practice throughout the design cycles. Figure 1 shows the initial conjecture map for the alignment of design and theoretical conjectures in this study.

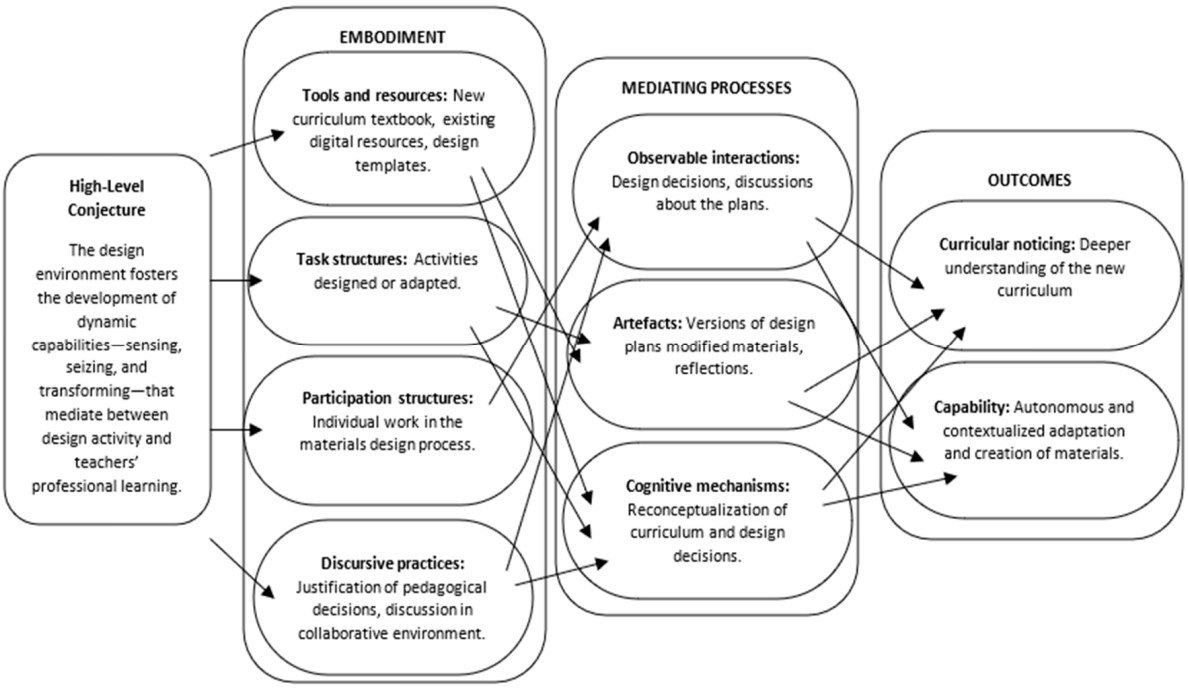


Figure 1. Initial conjecture map for the alignment of design and theoretical conjectures for this study.

As shown in Figure 1, the conjecture map developed for this study enabled alignment between theoretical conjectures and design decisions.

3.2. Design Intervention

The design under investigation was the structured material design environment embedded in the mathematics teacher education course. This environment consisted of four interrelated design features: (a) curriculum-aligned material design tasks, (b) explicit prompts for identifying learner needs and contextual constraints, (c) guided peer and instructor feedback sessions, and (d) written reflection and revision activities requiring PMTs to justify changes in their materials. Therefore, the focus of the design-based research was the learning environment that was designed to support adaptive design reasoning through cycles of planning, feedback, revision, and reflection. The four design features have not been applied independently or linearly. Design features have been used at different intensities in various stages of the material design process and have been integrated in a way that supports each other throughout the process. Table 2 shows the timing of the design features within the course flow and the activities in which they are implemented.

Table 2. Timeline of learning activities, design features, and assessments.

Week	Topic	Design Feature(s)
1	Types of materials	Explicit prompts
2	Principles of material design	Curriculum-aligned design tasks + Explicit prompts
3	Material selection	Explicit prompts + Written reflection
4–5	Analysis of instructional materials	Explicit prompts + Written reflection
6–7	Adapting ready-made materials	Curriculum-aligned task + Peer/Instructor feedback + Reflection
8–11	Designing teacher-created materials	All four design features
12	Collaboration with peers	Guided peer/instructor feedback
13	Implementation	Revision + Reflection
14	Presentation	Peer feedback + Written reflection

3.3. Participants and Context

The study was carried out in a mathematics teacher education program at a university in Türkiye. The focal context was a semester-long compulsory course which aimed to develop PMTs' instructional materials design for mathematics. The course lasted for 14 weeks, with two 2 h sessions per week, combining theoretical discussions, hands-on design activities, and peer review sessions. In total, 23 PMTs (18 women and 5 men) studying in the second year of the Mathematics Education undergraduate program at a university. The participants were between 20 and 21 years of age. By the time the research was conducted, they had completed the courses on the History of Mathematics, Middle School Mathematics, Principles and Methods of Teaching, and Approaches to Learning and Teaching Mathematics. In the semester during which the research was conducted, they were taking the Instructional Technologies course concurrently with this course. However, the participants do not have previous systematic experiences in instructional material design and instructional design, nor do they have school-based teaching practice (internship) experiences.

From this group, two PMTs—Alya and Yaman—were selected as focal participants for in-depth analysis [26]. Data from all participants were subjected to a preliminary review, and as a result of this review, two participants who exhibited distinctly different development patterns in their design processes were selected for in-depth case analysis. Other participants were also evaluated within the scope of this preliminary review. However, they

were not included in the in-depth analysis. The focal cases were selected retrospectively following a preliminary review of the complete dataset. The selection criteria were defined before the focal case analysis but after data collection. The primary criterion predetermined in the selection process was that participants exhibit distinctly different patterns in their design processes. Their design trajectories represented contrasting forms of adaptive design reasoning within the same course environment. Yaman's work foregrounded historical narratives, low-cost physical resources, and task structures for conceptual exploration, whereas Alya's work foregrounded visual clarity, digital tools, accessibility, and differentiation. This contrast allowed us to examine how sensing, seizing, and transforming processes were enacted through different resource orientations and design priorities, rather than treating PMTs' material design as a uniform process. This approach has allowed for a detailed and contextual examination of how dynamic capabilities develop during the design process.

3.4. Data Collection

Although the course lasted 14 weeks, the data analyzed in the present study were collected during the eight-week period in which participants engaged directly in the focal material-analysis, adaptation, feedback, revision, and reflection activities. The remaining course weeks involved introductory instruction and activities outside the analytic scope of the present study. Data were collected over a period of eight weeks using three primary sources: (a) design plans and related documents, including successive versions of the materials created by the participants; (b) semi-structured interviews conducted after each design cycle; and (c) discussion and guided review sessions. To make changes in participants' reasoning traceable, the data sources were organized chronologically for each focal participant. This organization enabled us to compare initial design decisions, feedback-related reflections, revised materials, and later explanations, rather than relying on isolated interview statements as evidence of capability development. Table 3 shows the chronological organization of the data sources and their analytic functions.

Table 3. Chronological organization of data sources and their analytic functions.

Data Source	Timing	Purpose	Role in Analysis
Initial design plan	Before feedback	Initial reasoning	Baseline for sensing/seizing
Revised material	After feedback	Design change	Evidence of seizing/transforming
Interview	After the first design	Design rationale	Participant reasoning
Guided review notes	During feedback	Interaction and mediation	Feedback-based reflection
Final reflection	After revision	Professional learning	Transforming evidence

Semi-structured interviews were conducted after the teacher candidates completed the design process. The interviews were conducted in English, consistent with the language of instruction, and lasted 43 and 48 min, respectively. Interviews were audio-recorded and transcribed, and these transcripts were directly used in the analysis process. The feedback process includes structured feedback from different sources. In this context, participants received verbal feedback from instructors and peers during in-class presentations; they were supported with written feedback on online tasks conducted via Zoom; and they revised their materials by evaluating them according to predetermined assessment rubrics. One of the researchers implemented the teaching process as the course instructor. The same researcher was responsible for the coordination of the data collection process.

To make changes in participants' reasoning traceable, data were organized as temporal sequences for each participant. For each case, we compared initial design plans, feedback-

session records, revised materials, and post-cycle interview reflections. This chronological organization allowed us to distinguish between isolated demonstrations of capability and evidence of change across the design process. As part of the study, participants developed three types of design plans focused on the use, adaptation, and creation of instructional materials for mathematics. In this study, only the existing material adaptation plan was used as a reference, since it allows for clearer observation of the analysis processes, pedagogical decision-making, and application of design principles. Below is the short description of the aims of each task.

Task 1: To choose a specific mathematics instructional material and analyze its appropriateness for various instructional purposes. To design a visually engaging and informative design plan on a selected learning outcome from the new mathematics curriculum that outlines how the material aligns with diverse learning needs.

Task 2: To create a design plan for adapting an existing mathematics instructional material appropriate for a specific classroom environment. The plan will consider the unique characteristics of the class and individual student needs.

Task 3: To develop a lesson plan for teaching a particular math concept, integrating various instructional materials, and aligning with differentiation principles.

To guide the development of these design plans, participants were provided with a document containing detailed instructions, a rubric, and supplementary explanations on the ethical use of digital resources and artificial intelligence. The document is presented in Appendix A.

Participants worked individually in iterative cycles of design, feedback, and redesign. Each cycle included three main phases. First, the initial planning and materials design phase was conducted. In this phase, PMTs selected a curricular topic, identified learning objectives, and proposed a first version of their instructional material. In the second phase, during discussions and guided review, they analyzed their decisions, received feedback, and reformulated their designs based on new interpretations of the curriculum. In the last phase, they documented the changes they made and justified their design reasoning.

As another data collection tool, the semi-structured interviews were conducted, audio-recorded, and transcribed verbatim to ensure the accuracy of the content and the fidelity to the participants' original discourse. The semi-structured interview guide was based on five main ideas about how to make and design instructional materials. The interview protocol included probing questions designed to elicit participants' pedagogical rationales behind their design decisions. For example, participants were asked not only what they changed in their materials, but also why these changes were pedagogically necessary, how they expected the changes to influence students' mathematical understanding, and how feedback shaped their subsequent design decisions. The questions were about (1) where the ideas for the material came from and how the first decisions were made about its development; (2) the design criteria, creativity, and use of digital tools; (3) the problems that came up and how they were solved; (4) how the needs of the students were taken into account and how they were involved in the design; and (5) personal reflection, professional learning, and future plans for using the materials. In addition to these guiding questions, follow-up probes were used to elicit participants' pedagogical rationales more deeply. Participants were asked what they changed in their materials and why these changes were pedagogically necessary. Also, they were asked how they expected the changes to influence students' mathematical understanding, and how feedback shaped their subsequent design decisions. For example, when participants referred to exploration, enjoyment, accessibility, or differentiation, they were prompted to explain how these considerations were related to mathematical learning, student engagement, conceptual understanding, or the demands of the new curriculum.

These follow-up questions were important for interpreting participants' statements not as isolated opinions, but as part of broader design reasoning processes.

Data sources were organized chronologically to trace how sensing, seizing, and transforming processes became visible across participants' design decisions, feedback responses, revised artifacts, and reflective explanations. Taken together, these sources provided a basis for examining the mediating role of feedback-based reflection, design rationale articulation, and artifact revision in participants' adaptive design reasoning. The use of multiple data sources also enabled methodological triangulation.

3.5. Data Analysis

Data were analyzed iteratively using an inductive–deductive approach that was closely aligned with the conjecture mapping framework. This analytic combination was necessary because the study aimed to remain open to patterns emerging from participants' actual talk and artifacts while also examining whether and how these patterns corresponded to the theoretically defined processes of sensing, seizing, and transforming. This analysis process was conducted in accordance with the principles of iterative coding in qualitative content analysis. The unit of analysis was defined as chains of evidence that include participants' initial design decisions related to a specific design focus, feedback on these decisions, revised design products, and subsequent explanations. Thus, the comments were based on the development of a design decision throughout the process rather than on individual statements.

The inductive phase helped identify situated concerns, such as clarity, accessibility, exploration, and contextual constraints, whereas the deductive phase enabled us to interpret these concerns as part of broader adaptive design processes. First, all interviews, discussion sessions, and design documents were organized chronologically. Transcripts were produced for the interviews and discussion sessions, and all written plans and adapted materials were compiled in a single corpus. Open coding was conducted by the second researcher on the entire dataset. To support the reliability of the coding, the first researcher independently coded 25% of the dataset. The coding results were compared, and codes that were interpreted differently were discussed among the researchers to reach consensus. At the end of this process, the code definitions were reviewed, and the codebook was iteratively developed.

In the first analytic cycle, we conducted open coding to identify recurring patterns in the participants' talk and design actions. Codes captured, for example, references to curriculum objectives, recognition of student difficulties, references to contextual constraints, and justifications for design choices. In the second cycle, these preliminary codes were grouped into broader categories such as "attention to curriculum," "attention to learners," "task structure," and "use of tools and resources." In the third cycle, we re-examined these categories through the lens of dynamic capabilities theory and re-coded the data with a focus on sensing, seizing, and transforming. The created categories were reinterpreted in terms of the dimensions of sensing, seizing, and transforming in accordance with the theoretical framework. This approach aimed to reduce the risk of theory guiding data interpretations at an early stage.

For each participant, we then constructed a case-specific conjecture map that linked (a) the designed tasks and learning environment, (b) observed mediating processes (e.g., noticing, reasoning, reflecting), and (c) emerging learning outcomes in terms of individual-level dynamic capabilities. Finally, we conducted a cross-case analysis to identify convergences and divergences between the two cases regarding how sensing, seizing, and transforming capabilities emerged and developed over time. Throughout the analysis process, coding decisions and emerging comments were discussed in regular researcher

meetings; based on these discussions, code definitions were clarified, similar codes were merged, and the codebook was developed iteratively. Although some data points appear to be associated with multiple dynamic capability dimensions, the interpretations are based on a chain of evidence formed by relevant design documents, feedback records, and reflective explanations rather than a single statement. Each design focus has been interpreted within the dynamic capability dimension, representing the dominant analytical function. Figure 2 shows two samples from the open coding process.

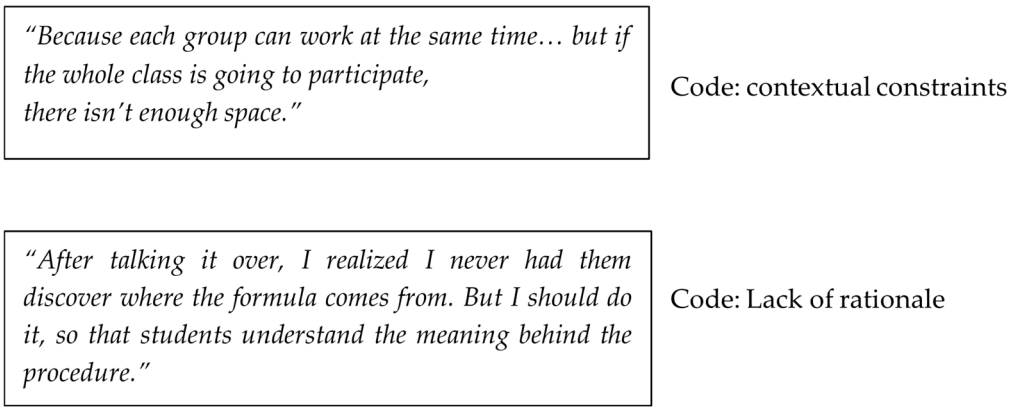


Figure 2. Sample open coding.

In the first example, Yaman identified contextual constraints that influence the implementation of the design. In the second example, Alya identified a limitation in her past practice and reconfigured her pedagogical understanding. These cases exemplify how, through interaction with the context and reflection on practice, the participants mobilize different dimensions of dynamic capabilities. The first excerpt is categorized as *sensing*, whereas the second example is categorized as *transforming*. Table 4 shows examples of code-capability matching.

Table 4. Code-capability matching examples.

Code	Capability Dimension	Example Interpretation
Anticipating student difficulty	Sensing	Participant identifies a possible conceptual or procedural difficulty before implementation
Using visual/digital tools for clarity	Seizing	Participant selects resources and justifies their pedagogical role
Changing instructions after peer discussion	Transforming	Participant reorganizes design routine based on critique

For the evidence of adaptive design reasoning, a statement was interpreted as evidence of sensing, seizing, or transforming only when it could be connected to at least one additional data source, such as an initial design plan, a revised artifact, a feedback-session record, or a later reflective explanation. The open codes created in the first analysis cycle were compared based on semantic similarities and grouped under higher-level categories. As the analysis progressed, the code definitions were reviewed, similar codes were merged, and new codes were added as necessary, resulting in an iterative development of the codebook. The coding process initially produced 28 open codes. The final analytic framework consisted of nine categories that represented the participants' adaptable instructional design reasoning. In the analysis process, comments were not based on singular quotes; each comment was constructed through chains of evidence established between participants'

design plans, revised materials, feedback sessions, and statements. In this way, the analysis focused on evidence chains rather than single excerpts. For example, a participant's statement about avoiding rote memorization was examined together with changes in the task structure and the participant's subsequent justification for those changes. Similarly, a statement about differentiation was interpreted in relation to how differentiation was or was not made visible in the initial and revised versions of the material.

3.6. Trustworthiness

To enhance the trustworthiness of the findings, we triangulated multiple data sources (design plans, adapted materials, interviews, and discussion sessions), maintained an explicit audit trail of coding and mapping decisions, and engaged in peer debriefing throughout the analytic process. Thick description was used in presenting the cases to enable readers to judge the transferability of the findings. Since the study involved interpretive qualitative analysis, researchers acknowledge that their theoretical commitments may have shaped how participants' statements and artifacts were interpreted. To mitigate this risk, coding decisions were discussed among the researchers, discrepant interpretations were revisited, and claims about capability development were checked against multiple data sources, including design plans, revised artifacts, interviews, and discussion records. The researchers also avoided treating single quotations as sufficient evidence of development unless they were supported by changes in artifacts or later reflections.

The research team has academic studies in the fields of mathematics teacher education, curriculum development, and teacher professional learning. In the design of the study and the interpretation of the findings, the researchers, aware that this theoretical orientation could influence data interpretation, used both inductive and deductive coding during the analysis process, ensuring that the codes were directly supported by the data. Additionally, throughout the analysis process, regular reflective analytic memos were maintained, and discussions were held to allow researchers to question their potential biases.

One of the researchers has served as the course instructor in the implementation of the learning environment. To mitigate the potential impact of this situation on data interpretation, coding decisions were regularly reviewed with the other researcher, differing interpretations were discussed, and all analytical inferences were supported by evidence obtained from multiple data sources. Throughout the analysis process, the researcher who analyzed the entire data kept reflective analytical notes to document emerging interpretations, analytical decisions, and possible alternative explanations. These notes were used to support the traceability of the coding process. Informed consent was obtained from all participants in the study, and they were informed about the purpose of the research, the data collection process, and the principles of confidentiality.

4. Findings

Findings are presented through two focal cases, Yaman and Alya, followed by a cross-case analysis. In each case, evidence is organized chronologically around participants' initial design decisions, feedback-related reflections, revised artifacts, and later explanations. Therefore, interview quotations are not treated as sufficient evidence of capability development by themselves. Rather, they are interpreted in relation to changes in participants' design products and the pedagogical rationales they articulated across the design cycle. This structure allows us to trace how sensing, seizing, and transforming processes became visible through participants' adaptive design reasoning. Therefore, in each case analysis, evidence of the sensing seizing and transformation processes exhibited by the participants at specific design moments was first presented, and then the development of

these processes throughout the design cycle was interpreted using chains of evidence from different data sources.

4.1. Yaman's Case

From contextual noticing to feasible task design. Yaman's design process initially foregrounded contextual and logistical features of classroom implementation. Rather than beginning only with the mathematical content of the activity, he attended to the physical conditions under which the material would be used. His statement, *"The classroom might be too crowded; I think it might be difficult for the students to move around during the activity,"* was interpreted as evidence of sensing because it identified a possible constraint before implementation and connected this constraint to students' participation in the activity. This contextual noticing also shaped how Yaman considered the feasibility of the material. When he explained, *"We wanted them to use sticks and rods as radii… things they could easily find at school,"* he was connecting limited classroom resources with a possible design solution. In this sense, the evidence for sensing was in the relationship between his anticipation of classroom constraints and his attempt to make the task workable with accessible materials.

Yaman also anticipated possible student difficulties during the activity. For example, his comment that *"Since they might not draw the circles correctly, the teacher needs to remind them how to do so"* indicated that he was beginning to consider how students might engage with the mathematical and practical demands of the task. When these pieces of evidence are considered together, it is observed that Yaman's initial design decisions were shaped around the physical conditions of the classroom, available materials, and the practical difficulties students might encounter. At this stage, sensing emerges not only as the recognition of potential problems but also as the means by which design decisions are related to contextual requirements.

From history to pedagogical resource organization. After identifying contextual constraints, Yaman began to mobilize resources in relation to a more explicit pedagogical purpose. His decision to foreground the historical emergence of π was interpreted as part of a seizing process because he connected a mathematical resource to a specific instructional concern. He stated, *"The origin of the number π is something that is rarely explored in class; we wanted to break away from that traditional way of teaching it."* This statement suggests that Yaman did not want students to encounter π only as a ready-made numerical value; instead, he aimed to create conditions for students to examine how the idea could be explored. This explanation shows that Yaman positions the historical context as a pedagogical resource that supports students in exploring the concept of π. Therefore, the process of seizing becomes visible through the holistic organization of historical content, physical materials, and task structure in line with a specific educational purpose.

Yaman also considered additional tools such as a worksheet, a video, and artificial intelligence. However, these were not interpreted as evidence of strategic resource use. They were treated as supporting evidence only insofar as they were connected to his stated intention to enrich students' exploration and clarify the task structure. Therefore, digital tools have been interpreted as part of the seizing process not only because they have been used but also because they have been integrated into the design to support a specific teaching purpose. Figure 3 presents the instructional material designed to support students' exploration of the relationship between circumference, diameter, and π.

The historical perspective served as the pedagogical rationale behind this design decision and was reflected primarily in Yaman's explanations rather than in the visual artifact itself. Additionally, he also considered the desire to enrich the use and design of the tools: *"A worksheet can be added; a video can be added." Additionally, we can also use artificial intelligence."* These statements indicate that Yaman considers different tools as

pedagogical resources that support the learning experience. His justification for selecting and integrating technology in a way that supports specific teaching objectives has been interpreted as an indicator of the seizing process. Thus, Yaman strategically defines, selects, and integrates resources in line with pedagogical objectives.

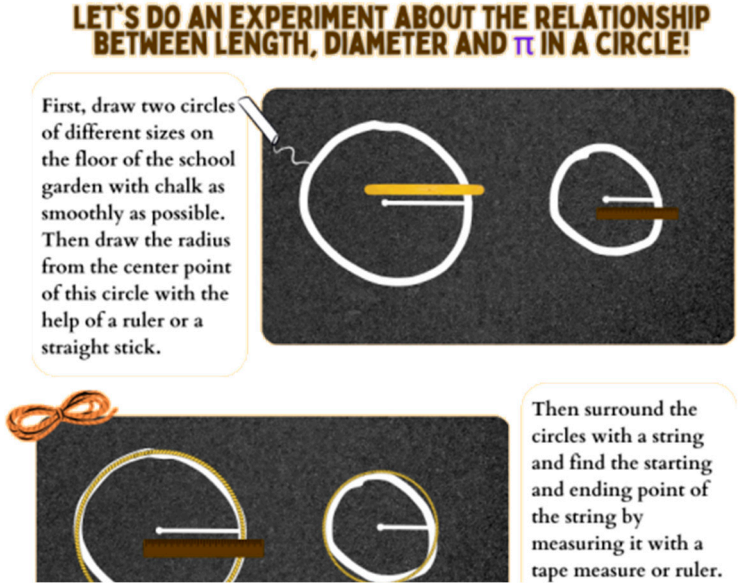


Figure 3. Yaman's instructional material for exploring the relationship between circumference, diameter, and π.

After designing, justifying, and testing his ideas, Yaman began to see his work from a new perspective, recognized the limitations, and reformulated his decisions. This stage shows that he reevaluated his previous decisions based on the feedback received and design experience. Therefore, this process reflects the transition from the conceptualization phase to the transformation phase. *"The activity I prepared was already based on a previous lesson... but I realized I could make improvements,"* he said. Yaman is reconstructing his previous experience by reinterpreting it: *"I used to have just an image; now I've turned it into a table full of instructions." "The change Yaman made is more about restructuring the cognitive organization of the material than altering the mathematical content."* This situation shows that Yaman transformed his design decisions by re-evaluating the relationship between instructional clarity and student autonomy at the end of the feedback and reflection process.

From procedural support to student reasoning. The transformation process in Yaman's case has become apparent through the relationship between the revised material and the subsequent explanations regarding the rationale for this revision. In his initial design, the task was structured in a way that was closer to the existing course structure and provided more direct guidance to students. After the feedback and reflection process, Yaman began to question again whether this structure sufficiently supported students in producing mathematical explanations. He expressed this process with the words, "The activity I prepared was already based on a previous lesson... but I realized that I could make improvements". This reassessment has also been reflected in the revised material. Indeed, his explanation, *"I used to have just an image; now I've turned it into a worksheet with instructions"*, shows that he restructures design decisions. Thus, the transformation has become evident in the structural differences between the initial and revised materials. The revised structure has created a clearer learning process that supports students' mathematical reasoning by reorganizing the cognitive organization of the task. Therefore, the transformation has been interpreted not as minor adjustments to the existing material, but as a restructuring of design principles that support the student's learning process.

A second indicator of the transformation process has emerged in Yaman's reassessment of the role of studied examples. His statements, *"I wanted the students to create their own expressions... They can say X, Y, or Z"* and *"I didn't want to give a solved example... it would lead to rote memorization,"* indicate that he aimed to provide students with more space for mathematical thinking. However, these statements have not been considered as evidence of transformation on their own. The commentary on the transformation process is based on the evaluation of these statements along with the revised task structure and the participant's subsequent reflections. When these pieces of evidence are considered together, it is observed that Yaman's design approach has evolved from adapting an existing lesson structure to designing a learning environment that encourages students to generate mathematical explanations. Figure 4 shows Yaman's adaptation of the existing material.

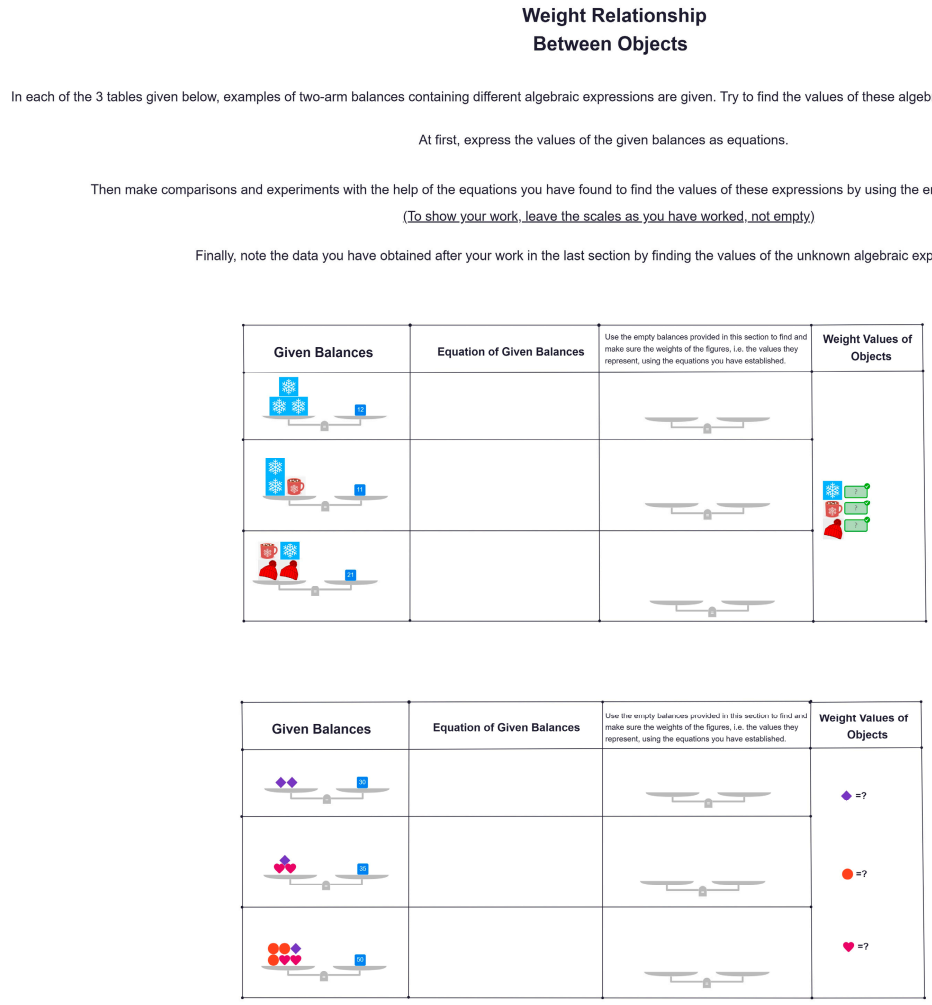


Figure 4. Yaman's adaptation of the existing material. Available at: https://polypad.amplify.com/p/oGZBc38SePYgvA (accessed 10 August 2026).

When evaluated in general, Yaman's transformation process is not limited to mere formal changes made to the material. When the initial design decisions, the revised material, feedback, and subsequent reflections are considered together, it is observed that he developed a new design approach that re-evaluates design decisions and centers on students' mathematical reasoning. In this respect, Yaman's transformation process provides strong evidence that he developed adaptive design reasoning by restructuring design routines. Table 5 summarizes Yaman's findings on adaptive design reasoning.

Table 5. Yaman's adaptive design reasoning.

Design Focus	Initial Design/Reasoning	Feedback-Related Reflection	Revised Artifact/Later Explanation	Capability Interpretation
Contextual feasibility	He anticipated that classroom crowding could make movement difficult.	He considered how task organization would work in real classroom conditions.	He proposed using accessible objects such as sticks and rods.	Sensing became visible through attention to contextual constraints and available resources.
Historical exploration of π	π was initially connected to a conventional activity structure.	He questioned whether students usually encounter π only as a given value.	He integrated a historical explanation to foreground how π could be explored.	Seizing became visible through the pedagogical use of historical resources.
Task organization	The material was initially based on a previous lesson/image.	He recognized that the task could be improved for clarity.	He transformed the image into a table with instructions.	Transforming became visible through the reorganization of the artifact.
Student reasoning	The earlier structure risked procedural imitation.	He reconsidered the use of solved examples.	He avoided giving a solved example and asked students to produce their own statements.	Transforming became visible through a shift toward student explanation.

A case-specific conjecture map was constructed for Yaman. The map was used to represent how Yaman's design reasoning moved across design features, mediating learning processes, and outcomes. In Yaman's case, the main mediating processes were contextual noticing, pedagogical use of historical resources, articulation of design rationale, and feedback-based reorganization of the artifact. Figure 5 shows the final conjecture map for Yaman's case.

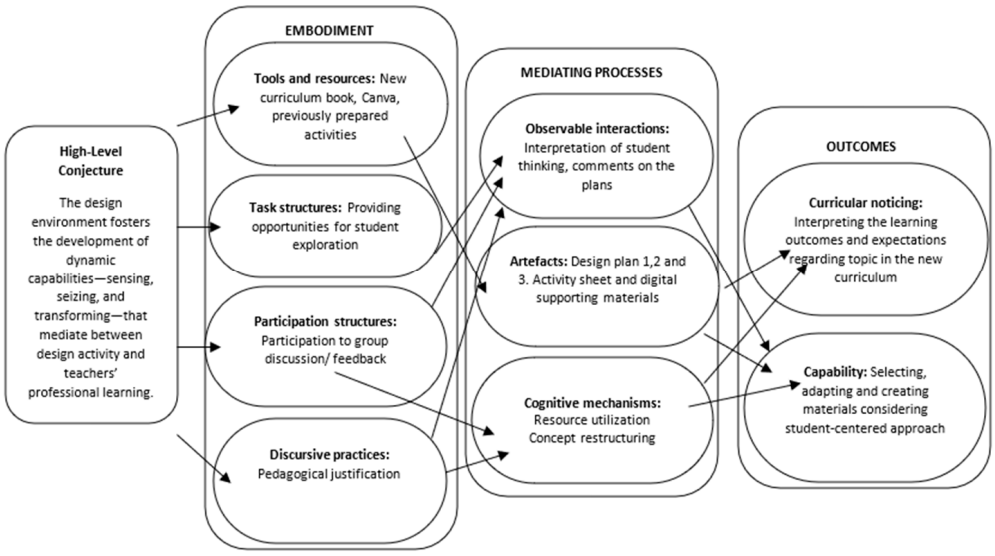


Figure 5. Final conjecture map for Yaman's case.

As shown in Figure 5, Yaman's adaptive design reasoning has been shaped around the class's applicability, available resources, and task features that support students' conceptual exploration. These design features have been supported by recognizing contextual constraints, using historical context to give pedagogical meaning to the concept of π, and reorganizing the task structure in line with the feedback and reflection process. In this

process, cognitive processes such as evaluating the classroom context, purposefully selecting pedagogical resources, and restructuring the task to support students' mathematical reasoning have been decisive. As a result, Yaman has demonstrated adaptive design reasoning by consciously selecting, adapting, and developing materials that are sensitive to the classroom context and support students' conceptual exploration.

4.2. Alya's Case

From representational confusion to accessibility-oriented noticing. Alya's design process began with attention to representational clarity and accessibility. Her initial noticing emerged from her experience with a previous activity on π, where she observed that confusion was experienced. She stated, *"In the material my colleague prepared, even we got confused, so I decided to change my material."* This statement was interpreted as evidence of sensing because it pointed to a problem in how the mathematical idea was represented and communicated through the material. Here, sensing manifests as the anticipation that this problem could affect students' processes of constructing mathematical meaning.

Alya further explained, *"The students don't understand where the value of π came from; the activity doesn't give them an opportunity to explore it."* This quotation was interpreted as an initial indication that she was beginning to connect students' conceptual difficulty with the design of the instructional sequence. Therefore, the sensing has become apparent by recognizing the inconsistency between the intended mathematical idea and the instructional design that enables students to access this idea.

Alya has also associated accessibility with students' cognitive and affective engagement. Her expression, *"If they have fun, they learn better; this leads me to think about making activities more accessible"*, indicates that she considers accessibility as a pedagogical design principle that supports students' engagement in the learning process. When these pieces of evidence are considered together, it is seen that Alya's initial sensing process is shaped around clarity and accessibility. The realization that students' mathematical understanding can be supported by visual organization, language use, and opportunities for exploration has formed the fundamental starting point for subsequent design decisions.

From digital tool use to pedagogically justified redesign. After identifying the issues of clarity and accessibility, Alya began utilizing visual and digital resources to redesign her material. The statement *"The concept of ten is generally not well understood; I added lines to make it clearer"* indicates that she transformed an observed learning difficulty into a concrete design decision. This change has been interpreted as a pedagogical decision aimed at making the mathematical structure more visible and understandable for students, beyond being merely an aesthetic choice in visual arrangement.

Evidence of Alya's use of Canva, Mathigon, and Polypad has been considered an indicator of the seizing process in cases where these tools were associated with specific pedagogical purposes. She used these tools to enhance visual organization, reduce unnecessary text density, and represent mathematical relationships more clearly. Her explanation, *"I made a clean design, without excessive text or bold colors"*, shows that she considers how the visual features of the material can affect students' processes of constructing mathematical meaning.

She also experimented with generative AI to generate alternative question ideas and rephrase instructions, explaining, *"I used AI to think of other ways to ask the same thing."* The statement shows that she used artificial intelligence not only to generate content but also to present the same mathematical idea through different teaching methods. Therefore, the use of artificial intelligence has been interpreted as part of a pedagogical reasoning aimed at enhancing openness and accessibility. Figure 6 shows Alya's use of technology and the accompanying visual design decisions.

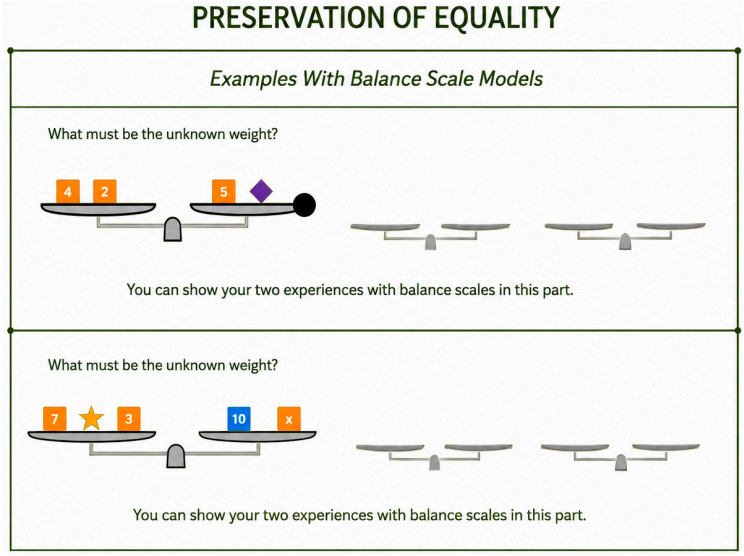


Figure 6. Alya's use of technology and visual decisions. Available at: https://polypad.amplify.com/p/oGZBc38SePYgvA (accessed 10 August 2026).

Alya facilitated the exploration of the principle of preserving equality using balance-scale models. Visual representations show how to identify the unknown weight by maintaining balance on both sides, offering students a concrete way to experiment with equality relationships. Additionally, she experimented with generative AI to generate question ideas and rephrase instructions: *"I used AI to think of other ways to ask the same thing."* When these pieces of evidence are evaluated together, it can be said that Alya's seizing process becomes more visible through her ability to justify and integrate these tools for specific pedagogical purposes rather than her ability to select digital tools.

From implicit differentiation to explicit design principle. Alya's transformation process became visible over time as her initially implicit concerns about accessibility and representation evolved into an understanding of differentiation and curriculum-based planning. In the initial design decisions, she preferred a more guided structure and justified it by saying, *"I used a more guiding format because they did not yet have sufficient experience."* This statement shows that the level of students' readiness is included in design decisions. At the same time, her statement *"If the group is more autonomous, I like them to discover the ideas themselves"* indicates that she has developed an awareness of the ability to reorganize the design structure according to the characteristics of the students. This situation indicates that Alya has started to develop design decisions that can be adapted according to student characteristics rather than a one-size-fits-all teaching approach.

In this transformation, peer and instructor feedback have played an important role as a tool. Alya said, *"I hadn't realized that some instructions could be confusing until we discussed them."* and stated that the feedback prompted a reevaluation of design decisions. This statement has not been considered as evidence of transformation on its own. It has been addressed in conjunction with the changes made in the revised material and their subsequent implications. Especially the simplification of the instructions and the addition of explanatory questions not only indicate a change in the visual layout of the material but also suggest a restructuring of the pedagogical assumptions regarding how students will interpret the material.

The transformation process has also become evident in Alya's interpretation of the curriculum. The statement *"I had never seen the elements of the new curriculum so clearly; this made me think more deeply about what planning means"* indicates that the curriculum has transformed from being a referenced official document into a pedagogical framework that guides design decisions. This change is further highlighted by her statement, *"I didn't*

mention differentiation in my plans, but I realized I was implementing it". This statement, when evaluated in conjunction with the initial accessibility-focused intuitions, revisions made based on feedback, and reflections developed on the curriculum, demonstrates a transition from an implicit design understanding to an explicit and conscious pedagogical design understanding. Indeed, her expression *"Not everything needs to be new anymore; what matters is how we adapt what we have for our students"* reveals that she reconceptualizes material design as a process of consciously adapting to students' needs rather than producing innovation. Therefore, Alya's transformation has become evident in the redefinition of the pedagogical principles that guide design decisions. Table 6 summarizes Alya's findings on adaptive design reasoning.

Table 6. Alya's adaptive design reasoning.

Design Focus	Initial Design/Reasoning	Feedback-Related Reflection	Revised Artifact/Later Explanation	Capability Interpretation
Representational clarity	She noticed that the previous material caused confusion.	She reconsidered whether the material gave students enough opportunity to understand the idea.	She redesigned visual elements and clarified representations.	Sensing became visible through attention to representational access.
Accessibility	She associated student engagement with making activities more accessible.	She reflected on how students might interpret instructions.	She simplified visual structure and reduced excessive text/color.	Seizing became visible through pedagogically justified visual redesign.
Instructional guidance	She initially preferred a guided format.	Feedback highlighted possible confusion in instructions.	She added or reorganized questions and instructions.	Transforming became visible through feedback-based revision.
Differentiation	Differentiation was initially implicit in her concern for clarity.	She realized that some adaptations addressed different learner needs.	She began to name differentiation explicitly in relation to her design.	Transforming became visible through explicit articulation of an implicit design principle.
Curricular planning	Curriculum was initially treated as a design requirement.	Reflection made curriculum elements more visible.	She connected planning decisions to the new curriculum.	Transforming became visible through deeper curricular noticing.

A case-specific conjecture map was constructed to represent the trajectory of her adaptive design reasoning. Alya's map foregrounds representational clarity, accessibility, digital tool use, and differentiation. The mediating processes in her case were representational noticing, visual redesign, feedback-based clarification, and explicit articulation of differentiation. Figure 7 presents the conjecture map for Alya's case.

As shown in Figure 7, Alya's adaptive design reasoning has been shaped around design features that center on visual clarity, student accessibility, and curriculum expectations. These design features have been supported by her recognizing representation issues, restructuring digital tools for pedagogical purposes, and reevaluating design decisions based on peer and instructor feedback. In this process, cognitive processes such as identifying conceptual difficulties, reorganizing resources in accordance with pedagogical purposes, and justifying design decisions by centering on students' learning experiences have been decisive. As a result, Alya has deepened her interpretation of the curriculum

and demonstrated a stronger adaptive design reasoning by consciously selecting, adapting, and developing materials that support student-centered learning.

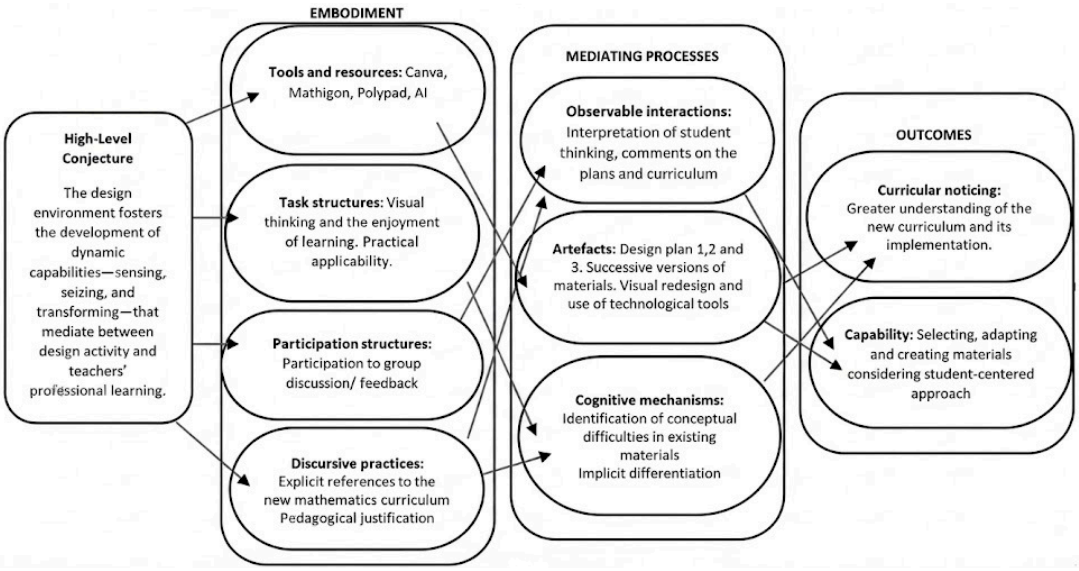


Figure 7. Final conjecture map for Alya's case.

4.3. Cross-Case Analysis

Different starting points of adaptive design reasoning. The cross-case analysis shows that Yaman and Alya developed adaptive design reasoning by starting from different pedagogical priorities. In both cases, although sensing, seizing, and transformation processes were observed, the design focuses that initiated these processes differed. Yaman's design process began with his noticing of contextual elements such as the physical conditions of the classroom, the available materials, and the feasibility of implementation. In contrast, Alya's initial focus was on how students construct mathematical meaning through representation, the visual arrangement of the material, and its accessibility. This difference shows that the sensing can develop through different starting points that preservice teachers find pedagogically problematic.

Mobilizing resources in different forms. The second fundamental difference between the two cases emerged in how the resources were mobilized for pedagogical purposes. Yaman reorganized the historical context, low-cost materials, and task structure as pedagogical resources to support students in exploring the concept of π. Alya, on the other hand, used digital tools, visual design principles, and generative artificial intelligence in a way that allows students to see mathematical relationships more clearly. Despite these differences, in both cases, the resources gained significance not only because they were used but also to the extent that they were associated with specific learning needs and pedagogical purposes. This situation indicates that the process of understanding is more related to the pedagogical justification of the tools used than to the tools themselves.

Different patterns of transformation. The transformation process was observed in different ways in the two cases. Yaman's transformation progressed from adapting an existing material for application to creating task structures that support students' mathematical reasoning. In contrast, Alya's transformation became visible by explicitly expressing the pedagogical design principles of accessibility and differentiation, which were initially implicit. Therefore, while Yaman has been reorganizing the task structure, Alya has been restructuring the pedagogical principles that guide design decisions. This finding suggests that the transformation process can be understood through the development of the pedagogical reasoning behind the design decisions.

Common mechanisms. Despite different development paths, some common mediation processes have been identified in both cases. Especially feedback-based reflection, justifying design decisions, comparing initial and revised materials, and the explanations made throughout the design process have emerged as common mechanisms that connect the processes of sensing, seizing, and transformation.

The findings also indicate that three dimensions of competence do not follow a linear sequence but rather feed into each other iteratively. However, it is observed that the pedagogical issues participants initially focused on significantly guided their subsequent resource selections, feedback interpretations, and material revisions. Therefore, the study reveals that for adaptive design reasoning, different pedagogical starting points can make the same dynamic capability dimensions visible in different ways. Table 7 shows the case of Yaman and Alya's adaptive design reasoning.

Table 7. Cross-case comparison.

Dimension	Yaman	Alya	Cross-Case Interpretation
Sensing	Classroom feasibility and contextual constraints.	Representational clarity and accessibility.	Adaptive design reasoning emerged from different initial pedagogical concerns.
Seizing	Historical resources, physical materials, and task structures.	Digital tools, visual representations, and AI-supported redesign.	Resources became meaningful when linked to explicit pedagogical purposes rather than merely being used.
Transforming	Reorganized tasks to promote student-generated reasoning.	Made implicit design principles explicit through redesign and curriculum alignment.	Transformation involved changes in pedagogical reasoning, not only modifications of artifacts.
Mediating processes	Feedback, reflection, and task revision.	Feedback, reflection, and artifact revision.	Iterative reflection and justification connected sensing, seizing, and transforming across both cases.

5. Discussion

The findings show how sensing, seizing, and transforming became visible as analytically distinguishable forms of adaptive design reasoning across two contrasting cases. Yaman's reasoning initially centered on contextual feasibility, accessible physical resources, and opportunities for student-generated mathematical explanation, whereas Alya's reasoning centered on representational clarity, digital tools, accessibility, and the explicit articulation of differentiation. These findings do not demonstrate the complete or stable development of individual dynamic capabilities. Rather, they show how participants' design reasoning changed within the boundaries of a semester-long instructional-materials course as they compared initial ideas, received feedback, revised artifacts, and articulated the rationales underlying those revisions.

Interpreting these patterns through Dynamic Capabilities Theory offers a complementary perspective on teacher learning. The framework draws attention to the sequence through which participants noticed a pedagogically relevant demand, mobilized resources in response, and subsequently revised a design decision or pedagogical assumption [11,12]. In this respect, the findings extend existing accounts of teacher knowledge and curriculum use by emphasizing adaptive movement across stages of design reasoning rather than treating knowledge, noticing, agency, or resource use as isolated constructs. The findings are also consistent with research showing that pre-service and novice teachers draw on multiple domains of mathematical and pedagogical knowledge when interpreting student thinking and making instructional decisions [17,18].

The sustainability relevance of the findings should be interpreted cautiously. The present study did not investigate Education for Sustainable Development learning out-

comes, classroom enactment, student achievement, or the long-term durability of participants' professional practices. Its contribution to sustainability therefore lies at the level of professional preparedness for curriculum-responsive adaptation. Structured opportunities to notice emerging demands, justify resource choices, and revise instructional materials may help teacher-education programs prepare prospective teachers to engage with curriculum reform as an ongoing process rather than as the one-time adoption of a fixed program. In this limited sense, adaptive design reasoning may be considered a potential professional resource for the continued implementation of curriculum innovation [13].

The first research question concerned how sensing, seizing, and transforming became visible during instructional material design. Sensing was evident when participants identified specific curricular, learner-related, representational, technological, or contextual demands that could affect the design. Seizing was evident when they selected or coordinated resources in relation to an articulated pedagogical purpose. Transforming was evident when changes across artifacts and reflective explanations showed a revision in a design routine, instructional decision, or pedagogical assumption. This operational distinction is theoretically useful because it connects noticing, resource mobilization, and revision without treating them as interchangeable forms of teacher learning.

At the same time, the transfer of Dynamic Capabilities Theory to the individual level has clear limitations. Organizational capabilities involve collective routines, institutional resources, and strategic adaptation over extended periods, whereas the present study examined two individuals' reasoning within a bounded course context. Consequently, sensing, seizing, and transforming should be understood here as analytical categories describing observable design activity, not as stable personal attributes or evidence of enduring professional capability. The contribution of the framework is therefore complementary: it provides a temporal and relational structure for interpreting how noticing became connected to resource use and subsequent revision.

Methodologically, conjecture mapping provided a structure for connecting the design features of the course with mediating processes and observable outcomes. The case-specific maps made it possible to compare how the same design environment operated differently for Yaman and Alya. For Yaman, contextual noticing and the pedagogical use of historical and physical resources were prominent mediating processes; for Alya, representational noticing, digital redesign, feedback-based clarification, and the articulation of differentiation were more prominent. The value of conjecture mapping, therefore, lies not only in summarizing the intervention but also in making visible the different pathways through which course features were related to participants' design reasoning. Future design-based studies may use this approach to test, revise, or reject design conjectures across broader participant groups and different teacher-education contexts.

The second research question concerned how design features, feedback opportunities, and reflective tasks mediated observable changes in participants' reasoning. Across both cases, the instructional artifact functioned as a shared object around which initial assumptions, resource choices, and later revisions could be examined. The requirement to justify design decisions made participants' reasoning available for discussion, while guided feedback and comparison between initial and revised artifacts created opportunities to identify inconsistencies between intended pedagogical purposes and the actual structure of the materials.

These findings suggest that iterative instructional material design may be most productive when it includes four connected elements: an initial design requiring explicit rationale, structured feedback focused on curricular and learner-related considerations, an opportunity to revise the artifact, and a final reflection explaining why particular changes were made. The importance of this sequence lies not in assuming that each cycle automatically develops a capability, but in making changes in noticing, resource coordination, and

design routines empirically traceable. This interpretation is compatible with the view that teacher learning is context-sensitive, task-specific, and shaped by opportunities for guided experimentation and reflection [14,20].

The third research question concerned what forms of professional learning, curricular noticing, and changing understandings of instructional materials became evident during the course. Across the two cases, professional learning was visible not as a stable or long-term outcome, but as a change in how participants articulated the purposes of instructional materials and justified their design decisions. Yaman moved from treating the material primarily as an adapted classroom activity toward considering how task structure could support students' own mathematical explanations. Alya moved from an implicit concern with clarity and accessibility toward a more explicit articulation of differentiation and curriculum-responsive planning. These changes suggest that instructional material design can make pedagogical assumptions visible and open to reconsideration when participants are required to explain, revise, and compare their decisions across time.

The findings have several implications for mathematics teacher education. First, instructional material design tasks should be organized as iterative sequences rather than as one-time assignments. Such sequences should require participants to produce an initial design, explain its pedagogical rationale, receive structured feedback, revise the artifact, and document the reasoning underlying the revision. This structure enables educators to examine how participants notice demands, mobilize resources, and reconsider design decisions. Also, feedback should direct attention to specific dimensions of instructional design, including curricular alignment, anticipated student thinking, accessibility, contextual feasibility, representational clarity, and differentiation. Additionally, comparing initial and revised artifacts can help make implicit pedagogical assumptions visible and provide a basis for discussing why particular resources, representations, instructions, or task structures were retained, modified, or removed.

For curriculum designers and teacher-education program developers, the findings indicate that curriculum reform should be accompanied by structured opportunities for prospective teachers to interpret, adapt, and justify instructional resources. Merely introducing new curricular expectations may not be sufficient for developing curriculum-responsive reasoning. Program designs should include tasks that require pre-service teachers to identify learner-related and contextual demands, compare alternative resources, and explain how revisions are connected to curricular intentions. At the policy level, this suggests that implementation support should address teachers' capacity to interpret and adapt curriculum materials, rather than focusing exclusively on procedural compliance with new curriculum documents.

Limitations and Future Research

This study has several limitations. First, the analysis focuses on two purposively selected cases from a single mathematics teacher-education course at one university in Türkiye. The cases were selected for their contrasting design trajectories and were intended to support theoretical comparison rather than statistical generalization. Consequently, the findings are context-specific and have limited transferability to other institutions, disciplines, national curricula, or teacher-education systems. Also, the study examined participants' design artifacts, interviews, guided review discussions, and reflective explanations during a relatively short course-based period. The analysis therefore cannot establish whether the observed revisions became stable professional routines or whether they were sustained beyond the course. Furthermore, the study did not examine classroom enactments or student outcomes. Further research should examine whether adaptive design reasoning is reflected in classroom implementation, student participation, mathematical understanding, or differentiated learning opportunities. In addition, the interpretive appli-

cation of Dynamic Capabilities Theory may have introduced confirmation bias by directing attention toward sensing, seizing, and transforming. The use of inductive coding, multiple data sources, evidence chains, peer debriefing, and attention to discrepant interpretations reduced this possibility. For future research, longitudinal studies following pre-service teachers into classroom practice are needed to determine whether the forms of adaptive design reasoning observed during coursework become sustained professional practices. Comparative studies could also investigate whether different course designs, feedback structures, or types of instructional material tasks produce different pathways of sensing, seizing, and transforming. Finally, future work should examine negative or non-confirming cases in which feedback and revision do not lead to observable changes in design reasoning.

6. Conclusions

This study examined how sensing, seizing, and transforming became visible as forms of adaptive design reasoning in two pre-service mathematics teachers' instructional material design processes. The findings showed contrasting but complementary trajectories. Yaman's reasoning emphasized contextual feasibility, accessible physical and historical resources, and task structures intended to support student-generated mathematical explanations. Alya's reasoning emphasized representational clarity, digital tools, accessibility, feedback-based revision, and the explicit articulation of differentiation.

The study's theoretical contribution lies in using Dynamic Capabilities Theory as a complementary individual-level analytical lens for connecting the noticing of curricular or learner-related demands, the mobilization of pedagogical resources, and the revision of design decisions. Its methodological contribution lies in combining this framework with conjecture mapping and chronologically organized evidence chains to relate course design features, mediating processes, and observable changes in reasoning. The findings do not establish stable capability development, long-term professional learning, or sustainable classroom change. Rather, they show that iterative design, guided feedback, artifact revision, and explicit reflection can make pre-service teachers' adaptive design reasoning visible within a teacher-education course. In this bounded sense, adaptive design reasoning may serve as a professional resource for curriculum-responsive teacher preparation and for the continued implementation of curriculum reform.

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Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

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Appendix A. Sample Task

Instructional Material Adaptation Task

Aim: To create a brief plan for adapting existing mathematics instructional material appropriate for a specific classroom environment. The plan will consider the unique characteristics of the class and individual student needs.

Steps:

1. **Select a Mathematics Instructional Material** (Choose a specific mathematics instructional material-activity, etc.)
2. **Analyze Material Appropriateness** (Consider factors such as content alignment, level of difficulty, engagement level, accessibility for diverse learners, and suitability for various teaching methods, etc., discussed in the class)
3. **Adapt the material** (Propose effective and innovative strategies for adapting the instructional material to meet the needs of the class and prepare the adapted version of the original material)
4. **Provide information about considerations regarding adaptation** (outline how the adapted material will cater to diverse learning needs through differentiation with your reasons behind these considerations)
5. **Reflect on potential challenges** (anticipate potential challenges regarding the use of material in the classroom and propose thoughtful solutions)
6. **Provide a short note about your adaptation process regarding the important issues in our course (sustainability, using generative AI, design thinking, service learning, etc.)**

Rubric:

Criteria	Excellent (4 Points)	Good (3 Points)	Fair (2 Points)	Needs Improvement (1 Point)
Selection of Mathematics Instructional Material (1)	The existing instructional material chosen is highly relevant.	The existing instructional material chosen is relevant.	The existing instructional material chosen is somewhat relevant.	The existing instructional material chosen is not suitable for adaptation.
Adaptation Strategies (4)	Proposes highly effective and innovative strategies for adapting the instructional material to meet the needs of the class.	Proposes effective strategies for adapting the instructional material to meet the needs of the class.	Proposes some strategies, but there is room for improvement in effectiveness.	Fails to propose effective strategies for adaptation.
Considerations for Adaptation (4)	Clearly outlines how the adapted material will cater to diverse learning needs through differentiation.	Outlines how the adapted material will cater to diverse learning needs through differentiation.	Somewhat outlines differentiation, with room for improvement in clarity.	Fails to outline how the adapted material will cater to diverse learning needs effectively.
Reflection on Potential Challenges (4)	Clearly anticipates potential challenges and proposes thoughtful solutions.	Anticipates potential challenges and proposes solutions.	Identifies some potential challenges, but there is room for improvement in solutions.	Fails to anticipate potential challenges or propose effective solutions.
Grammar and Mechanics (1)	No grammatical or mechanical errors; the plan is polished and professional.	Few grammatical or mechanical errors; the plan is mostly polished and professional.	Some grammatical or mechanical errors that slightly impact professionalism.	Numerous grammatical or mechanical errors that significantly impact professionalism.
Time management (1)	Student effectively managed time and submits the assignment on time and completed.	Student submitted their poster with a one-day delay.	Student submitted their poster with a two-day delay.	Student submitted their poster with a three-day delay.

Some Insights:

1. Competencies supported in this task:

Integrated problem-solving competency	This task encourages problem-solving skills and the ability to customize materials for diverse learning settings.

2. SDGs supported in this task:



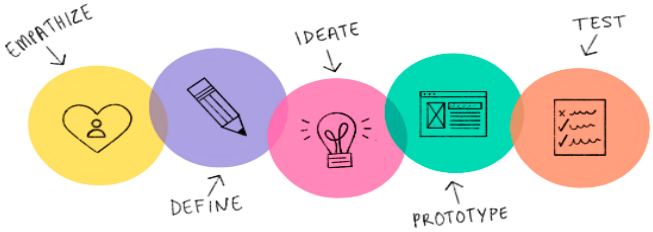
This task involves creating a plan to adapt an existing mathematics instructional material to provide students with a more suitable and effective learning experience. This is directly linked to the goal of Quality Education, as it aims to improve the quality of education by focusing on the specific needs of students.



Taking into account the unique characteristics of the classroom and individual student needs can help reduce inequality in education by promoting equal opportunities. This step is aligned with the Reduced Inequalities goal.

3. Design thinking in this task:

Design thinking is an innovative problem-solving approach that involves understanding the needs of users, brainstorming creative ideas, prototyping solutions, and testing them to improve and refine the final outcome.



In the context of this task, the design thinking process can be experienced as follows:

1. **Empathy with Learners:** You will empathize with diverse learners by understanding their unique needs, challenges, and preferences. This will help you design instructional material that is more personalized and effective for the specific classroom environment.
2. **Define the Problem:** You will define the problem statement clearly, identifying specific challenges or gaps in the current instructional material that need to be addressed for better adaptation to the classroom environment.
3. **Ideation and Brainstorming:** You will brainstorm and generate multiple creative ideas for adapting the instructional material, exploring various possibilities, and thinking outside the box.
4. **Prototyping and Testing:** You will create prototypes or drafts of your adapted instructional material to visualize and test your ideas, identify strengths and areas for improvement, and then iterate on your designs to refine them further.

4. Guideline for using resources, including generative AI, ethically and effectively in this task:

As you work on this task, I would like to remind you to use resources including generative AI correctly, beneficially, and ethically. By using resources including generative AI in a responsible and ethical manner, you can enhance your learning experience and create high-quality posters for your assignment. Here are some tips for using these resources responsibly:

1. Use resources including generative AI only when necessary.

2. Verify the information you get from resources including generative AI and always cite your sources properly. Use it ethically by avoiding practices like plagiarism or copyright infringement.
3. Evaluate the suggestions and tips from resources including generative AI critically and add your own thoughts to enhance the information.

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