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The Dual-Channel Low-Carbon Supply Chain Network Equilibrium with Retailers' Risk Aversion Under Carbon Trading

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Abstract: Carbon emissions from human activities such as production and consumption have exacerbated climate deterioration. A common worldwide objective is to create a low-carbon economy by implementing carbon reduction measures in production, consumption, and other processes. To this end, this paper explores the production, price, carbon reduction rate, and profit or utility for a dual-channel low-carbon supply chain network (DLSCN) that includes numerous competing suppliers, manufacturers, risk-averse retailers, and demand markets under carbon trading. In order to create an equilibrium model for the DLSCN, risk-averse retailers are characterized using the mean-CVaR method, and each member's optimal decision-making behavior is described using variational inequalities. A projection contraction algorithm is used to solve the model, and numerical analysis is presented to investigate how risk aversion, carbon abatement investment cost coefficients, and carbon trading prices affect network equilibrium. The results indicate that increasing retailers' risk aversion can enhance supply chain members' profits and carbon reduction rates. Retailers prioritize expected profits, while other members prefer them to focus more on CVaR profits. When retailers are more risk-averse and value CVaR, traditional retail channels become more popular. Increasing the carbon reduction investment cost coefficients for suppliers and manufacturers can boost their profits, and retailers also support this move to charge more for low-carbon products and enhance utility. When carbon trading prices rise, suppliers and manufacturers opt to increase carbon reduction rates to generate more profits from selling carbon allowances. This study provides decision-making references for achieving both economic and environmental benefits for members of DLSCN.

Keywords: dual-channel low-carbon supply chain network; carbon trading; risk aversion; mean-CVaR; variational inequality

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1. Introduction

Emissions of greenhouse gases worldwide have been increasing at an accelerated rate in the past few decades, and the state of the climate is getting worse. Global carbon dioxide emissions rose by 1.1 percent in 2023 and hit a record-breaking high of 37.4 billion tons, according to the International Energy Agency's "CO2 Emissions in 2023" report. The "Emissions Gap Report 2024: Stop the Rhetoric", released by the United Nations Environment Programme, clearly states that the 1.5 °C target of the Paris Agreement will be difficult to achieve if countries do not step up their actions to reduce emissions. Climate deterioration poses a serious threat to social stability and economic growth, forcing countries to actively pursue emission reduction options [1,2]. China presently emits more carbon than any other country in the world [3,4], and its emissions are increasing annually [2], increasing by

over 565 million tons in 2023—by far the biggest rise globally. In 2020, China announced a solemn commitment to the world community to "peak carbon emissions by 2030 and achieve carbon neutrality by 2060", so as to rapidly cut carbon emissions while taking economic and social growth into consideration. In 2021, China established a nationwide carbon trading market [4,5]. The carbon trading market has produced impressive outcomes after four years of development, offering a useful incentive mechanism to encourage the growth of a low-carbon economy [6]. In order to enable more enterprises to engage in low-carbon building, the "2024–2025 Energy Conservation and Carbon Reduction Action Plan" reiterates the necessity of gradually broadening the scope of the national carbon trading market. How enterprises decide output and carbon reduction under the carbon trading system has grown in importance in this setting.

Since the 21st century, competition among enterprises has gradually shifted to competition among supply chains. With changes in economic development and market structures, supply chains have increasingly exhibited complex network characteristics, giving rise to supply chain networks (SCN). A carbon trading mechanism will drive suppliers and manufacturers to explore investing in carbon reduction technologies [7], manufacture low-carbon raw materials and finished products, and sell low-carbon products throughout the value chain. It has been observed in various studies that, compared to other products, low-carbon products are more appealing to consumers [8,9], leading to a greater demand for them, and, consequently, the number of products circulating in the SCN increases. As a result, the more suppliers and manufacturers invest in carbon reduction, the higher their returns may be, including extra revenue from raw material or product sales, as well as income from selling extra carbon allowances through the carbon trading market [10]. Both suppliers and manufacturers seek to maximize their profits; therefore, a key concern is whether the increased returns can offset the cost of carbon reduction investments, which ultimately determines their involvement in such investments.

The global e-commerce industry has entered a period of tremendous growth due to the internet and information technology's rapid growth. E-commerce's explosive growth has significantly changed traditional market structures and customer behavior, particularly in China [11]. It has brought much convenience to consumers in acquiring product information and transaction activities online. E-commerce sites such as Taobao, JD.com, and Pinduoduo have become the main channels for daily shopping. Traditional enterprises have progressively moved from single offline channels to a dual-channel sales strategy that combines online and offline techniques to attain wider market reach in response to changing market demands [12]. For instance, Apple's dual-channel model in China relies not only on the offline Apple Store, but also on e-commerce platforms such as Apple's official website and JD.com for online sales. In order to accomplish coordination and optimization across both channels, enterprises must reevaluate how they distribute resources and advantages between online and physical channels while using this dual-channel approach [13,14]. Studying how to coordinate the interests of online and offline channels, optimize decision-making at all levels of dual-channel SCN, and achieve the synergistic growth of SCN is therefore extremely important from both an academic and practical application standpoint.

In an increasingly complex market environment, the uncertainty of market demand has intensified the volatility and uncontrollability of profit expectations among supply chain members. As a result, enterprises tend to adopt risk-averse strategies. Prospect theory suggests that when facing choices involving risk and returns, firms often exhibit "asymmetric risk preferences" [15]. In a market experiencing frequent variations in demand, the risk-averse behavior of firms affects not only overall SCN performance but may also

alter its structure and dynamic evolution process [16]. Therefore, it is both theoretically and practically crucial to incorporate risk-averse behavior into SCN' study.

Considering the preceding context, this paper intends to investigate the equilibrium issues of a dual-channel low-carbon supply chain network (DLSCN) in the setting of carbon trading, with consideration given to the retailer's risk aversion. Examining how risk aversion, carbon reduction investment cost coefficients, and carbon trading prices affect DLSCN members' transaction volumes, prices, carbon reduction rates, and profits or utilities is the main goal of this study. The findings aim to provide relevant management insights for decision-makers in addressing low-carbon production, managing risks, and adapting to carbon trading mechanisms. The following are this paper's key contributions:

- This paper integrates carbon trading mechanisms, risk aversion of retailers, low-carbon production, dual-channel supply chains, and SCN, thereby broadening the research boundaries of related fields.
- This study employs the mean-CVaR to describe the risk-averse retailers within the supply chain network. While existing research predominantly uses the mean-variance and CVaR methods to describe risk-averse members, the mean-CVaR approach has only been applied to individual supply chains and not extended to SCN.
- We find that raising the carbon abatement investment cost coefficient can, counterintuitively, improve supply chain participants' utilities or profits. This outcome reflects the demand market's favorable inclination toward low-carbon products and its relatively low sensitivity to price changes.
- We also find that, in a four-tier DLSCN, all supply chain members can achieve both environmental and economic benefits. This conclusion differs from that of Zou et al., whose study suggests that retail pricing cannot compensate for the reduction in demand, and does not examine the effect of carbon trading prices on the equilibria of supply chain participants [17].

The remainder of the paper is organized in the following manner. We examine the literature in Section 2. Based on the scientific issues, model assumptions, and notations discussed in Section 3, we have constructed a four-tier DLSCN. Section 4 is dedicated to the model formulation. In Section 5, we solve the model using a projection contraction algorithm and conduct a numerical analysis to explore the impact of retailers' trade-off weight and risk aversion degree, suppliers' and manufacturers' carbon emission reduction investment cost coefficients, and carbon trading prices on the equilibrium decisions within the DLSCN. Section 6 distills managerial insights based on the conclusions drawn from Section 5. Finally, Section 7 summarizes the entire paper and points out potential directions for future research expansion.

2. Literature Review

Three areas of the literature are directly connected to this study: the equilibrium models of SCN under carbon trading, equilibrium decisions in dual-channel supply chains, and the application of risk aversion in supply chains. We will review these areas and highlight the existing research gaps.

2.1. The Equilibrium Model of Supply Chain Network Under Carbon Trading

Nagurney et al. pioneered the use of variational inequalities and equilibrium theory to establish a SCN equilibrium model containing numerous manufacturers, retailers, and demand markets, which provided an effective method for solving the equilibrium decision-making issue in the competitive game of supply chain members [18]. Subsequently, Prof. Nagurney's team has carried out corresponding extension studies in the fields of food, finance, and electricity [19–21]. Inspired by the above studies and the development of

carbon emission reduction policies, several academics have included carbon trading and investments in low-carbon technologies in the research of SCN equilibrium. For example, when comparing SCN equilibrium decisions under various carbon policies, Mohammed et al. and Li et al. discovered that the carbon trading policy is the most flexible and effective one [22,23]. Rezaee et al. constructed a green SCN in a carbon trading environment and discovered that supply chain greening and carbon trading prices were positively correlated [24]. Pathak et al. put up a dual SCN model under the carbon trading system, advancing the idea that the carbon quota and trading policy can help achieve both environmental and economic goals [25]. Ma et al. constructed a low-carbon tourism SCN model and investigated how member cooperation and competition affected equilibrium decision-making [26]. Zou et al., in a low-carbon SCN, discovered that the carbon trading system can encourage enterprise to make investments in reducing carbon emissions and enhance the low-carbon quality of products [17]. Manufacturers can also utilize the carbon trading price to its fullest potential in order to optimize profits. Although the aforementioned research analyzes the SCN equilibrium model with low-carbon technology investment and carbon trading, it did not take into account the cost coefficient of investment in carbon emission reduction as a choice variable to examine its impact on SCN equilibrium.

2.2. Equilibrium Decisions in Dual-Channel Supply Chains

Most of the existing literature on dual-channel supply chains is limited to for supply chains. For example, Yao and Liu reveal that competitive-pricing behavior exists between the online direct sales channel and the traditional retail channel, and that manufacturer's online direct sales channel affects the traditional retail channel [27]. Zhang et al. study how a dual-channel supply chain's price decisions are influenced by service level and return rate [28]. A higher carbon trading price reduces carbon emissions and increases profits by selling more carbon allowances, according to Qi et al., who analyzed the dual-channel supply chain under the carbon quota trading coordination problem [29]. Many academics have examined the equilibrium decision-making in dual-channel SCN, taking into account the environment's complexity and variability. For example, Askarian-Amiri et al. considered the effects of demand dependence and discounts on dual-channel SCN [30]. Zhang et al. evaluated production price and carbon reduction in dual-channel SCN under a progressive carbon tax mechanism [4]. Kazancoglu et al. constructed a green dual-channel closed-loop SCN model to optimize economic and environmental objectives [31]. Although a few works in the literature have considered carbon trading in the research of dual-channel SCN, no literature work has yet included risk aversion behavior in the research of dual-channel SCN equilibrium.

2.3. Risk Aversion in the Supply Chain

Most of the literature assumes that supply chain participants are risk-neutral. However, the uncertainty of expected returns for supply chain participants also rises with market demand uncertainty, which frequently causes businesses to display risk-averse traits [15]. Numerous researchers have incorporated risk aversion into the analysis of equilibrium decisions in individual supply chains and found that the level of risk aversion or the conduct of decision-makers can have a substantial impact on supply chain equilibrium [32–35]. Building on this foundation, Rahimi et al. incorporated risk aversion behavior into sustainable SCN design, utilizing the Conditional Value at Risk (CVaR) method to evaluate risk aversion, and discovered that greater risk aversion in the model leads to lower target profits [36]. Tarei et al. analyzed design and uncertainty issues in oil SCN, maximizing the minimization of supply chain costs and risks, and characterized members' risk aversion using the mean-variance method, which indicated a conflict between total supply chain

costs and risk aversion [37]. However, traditional methods of measuring risk aversion do not effectively capture decision-makers' risk-averse characteristics. The mean-variance method treats gains and losses symmetrically; hence, it is not suitable for low-probability events. In addition, the CVaR method can make the decision-making target too low and reduce the expected profit [38]. However, the mean-CVaR technique is a convex function of expected profits and CVaR, and it can maximize the expected profits of risk-averse decision-makers while limiting downside profit risks; therefore, we use it to measure the risk aversion of retailers [39]. Xie et al. analyzed the impact of risk aversion on supply chain coordination using the mean-CVaR technique [39]. Liu et al. characterized the risk aversion behavior of retailers using the mean-CVaR method in information sharing based on blockchain technology and studied its effect on supply chain coordination and decision-making [40]. Jammernegg et al. investigated the role of risk-averse newsvendors, modeled using mean-CVaR, in supply chain coordination [41]. Al of the literature described above evaluates the impact of risk aversion decision-makers on supply chain equilibrium decision making using the mean-CVaR method, but there is a lack of research on supply chain network equilibrium decision-making using this approach.

2.4. Research Gaps

Table 1 compares our study with the existing literature from the four dimensions of carbon trading, supply chain structure, risk aversion, and low-carbon technology investment, and finds that the existing literature has the following four limitations:

- Although most of the studies on SCN equilibrium under carbon trading consider low-carbon technology investment, none of them takes the carbon abatement investment cost coefficient as a decision-making variable, they lack the study of its impact on SCN equilibrium, and they neglect the members' risk aversion behavior.
- Although some related dual-channel supply chain studies have addressed the network level, none of them have considered the carbon trading mechanism or the risk-averse behavior of members.
- The existing literature on risk aversion mainly focuses on a single supply chain, with only one article studying how risk aversion affects supply chains under carbon trading, and only two articles include it in the research on SCN equilibrium.
- Although there are various methods for portraying risk aversion of existing supply chain members, only the mean-variance and CVaR methods are used in SCN, and the mean-CVaR method has not yet been applied to the study of risk aversion in SCN.

Table 1. Comparison of this study with previous research.

Author	Carbon Trading	Supply Chain Structure		Risk Aversion	Low-Carbon Technology Investment
		Network	Dual-Channel	Measurement Method	
Nagurney et al. (2002) [18]		√			
Mohammed et al. (2018) [22]	√	√			√
Li et al. (2020) [23]	√	√			√
Rezaee et al. (2017) [24]	√	√			√
Pathak et al. (2020) [25]	√	√			√
Ma et al. (2023) [26]	√	√			√
Zou et al. (2023) [17]	√	√			√
Zhang et al. (2022) [28]			√		
Qi et al. (2018) [29]	√		√		

Table 1. *Cont.*

Author	Carbon Trading	Supply Chain Structure		Risk Aversion	Low-Carbon Technology Investment
		Network	Dual-Channel	Measurement Method	
Askarian-Amiri et al. (2021) [30]		√	√		
Zhang et al. (2021) [4]		√	√		
Kazancoglu et al. (2022) [31]		√	√		
Tao et al. (2020) [32]				CVaR	
Zhang et al. (2022) [28]	√			Mean-VaR	
Chen et al. (2023) [34]				Mean-VaR	
Xu et al. (2023) [35]				CVaR	
Rahimi et al. (2019) [36]		√		CvaR	
Tarei et al. (2022) [37]		√		Mean-VaR	
Xie et al. (2018) [39]				Mean-CVaR	
Liu et al. (2019) [40]				Mean-CVaR	
Jammernegg et al. (2024) [41]				Mean-CVaR	
This paper	√	√	√	Mean-CVaR	√

3. Problem Statement

3.1. Problem Description

The DLSCN considered in the paper consists of $n(n = 1, 2, \cdots, N)$ suppliers who supply homogeneous raw materials and compete with each other, $i(i = 1, 2, \cdots, I)$ manufacturers who produce homogeneous products and compete with each other, and $j(j = 1, 2, \cdots, J)$ risk-averse retailers who also compete with each other, catering to $d(d = 1, 2, \cdots, D)$ demand markets. Suppliers are responsible for providing raw materials to manufacturers; manufacturers are in charge of producing products and selling low-carbon items directly online; and risk-averse retailers are in charge of selling low-carbon products through traditional sales channels. Manufacturers and suppliers are investing in carbon-reducing technologies for low-carbon production. Suppliers and manufacturers make optimal decisions in conjunction with government-allocated carbon allowances, and retailers make optimal decisions considering risk-averse behavior. Suppliers, manufacturers, and retailers in the same tier follow a non-cooperative competitive game. The specific DLSCN is displayed in Figure 1.

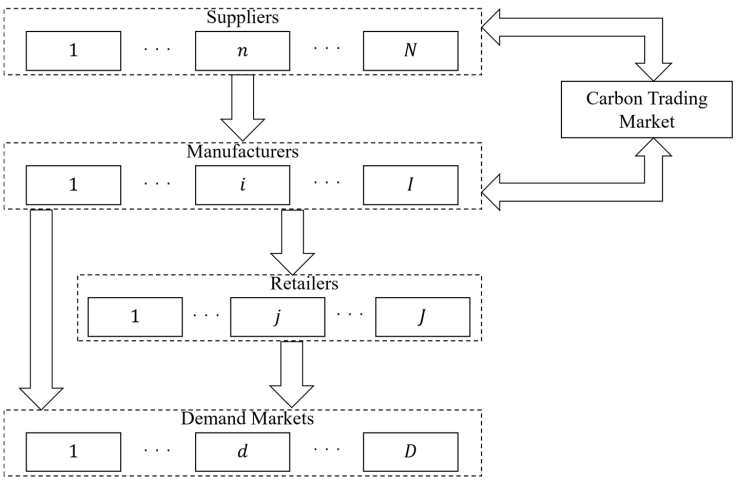


Figure 1. The dual-channel low-carbon supply chain network model.

3.2. Conditional Assumptions

To facilitate the investigation, we have made the following assumptions:

- There is complete information symmetry among the different levels of the DLSCN, and the decision-makers at each level are in a non-cooperative competitive relationship [31].
- Manufacturers and suppliers make investments in carbon reduction to lower the carbon emissions generated during the manufacturing of raw materials and completed items. According to research by Kang et al. and Zou et al., there is a concave function in the link between company earnings and investments in carbon reduction [17,42]. The carbon abatement cost coefficient and the carbon reduction rate per unit of raw material (or product) can be used to calculate the suppliers' (manufacturers') carbon reduction investment costs, represented as $\frac{1}{2}T_n{r_n}^2\left(\frac{1}{2}T_i{r_i}^2\right)$. This indicates that a higher carbon reduction rate for raw materials or products results in greater investment costs.
- All DLSCN members make risk-neutral decisions, with the exception of retailers, who make risk-averse decisions.
- Assume that all cost functions are continuously differentiable convex functions.

3.3. Notations

Table 2 lists the notations used in this paper, along with their descriptions.

Table 2. Notations and descriptions.

Notations	Descriptions
q_{ni}	Quantity of raw materials provided by supplier n to manufacturer i; the entire set of quantities forms a column vector, $Q_1 \in R_+^{NI}$.
q_i	Production quantity of manufacturer i; the production quantities of all manufacturers are represented as a column vector, $q \in R_+^{I}$.
q_{ij}	Quantity of products provided by manufacturer i to retailer j; the entire set of quantities forms a column vector, $Q_2 \in R_+^{IJ}$.
q_{id}^e	Quantity of products sold by manufacturer i to demand market d; the entire set of quantities forms a column vector, $Q_3 \in R_+^{ID}$.
q_j	Total order quantity of retailer j from all manufacturers; $q_j = \sum_{i=1}^{I} q_{ij}$.
w_{ni}	Unit wholesale price charged by supplier n to manufacturer i.
w_{ij}	Unit wholesale price charged by manufacturer i to retailer j.
p_i^e	Selling price set by manufacturer i in the online direct sales channel.
p_j	Selling price set by retailer j in the traditional retail channel.
$f_n(Q_1)$	Cost function for producing raw materials by supplier n, which depends on the quantity of raw materials provided at the supplier level.
$f_i(q)$	Cost function for producing products by manufacturer i, which depends on transaction quantities at the manufacturer and supplier levels.
$f_i(Q_3)$	Exhibition cost function for selling products by manufacturer i, which depends on product transaction quantities between the manufacturer and the demand market.
$f_j(Q_2)$	Exhibition cost function for selling products by retailer j, which depends on product transaction quantities between the retailer and manufacturers.

Table 2. *Cont.*

Notations	Descriptions
d_j	Market demand of retailer j, which is stochastic: $d_j \in \left[0, \frac{b_j}{p_j}\right]$, where $b_j > 0$. The probability density function is $f_j(x)$, and the cumulative distribution function is $F_j(x)$, both of which are continuous, differentiable, and monotonically increasing functions.
d_i^e	Market demand of manufacturer i, which is stochastic: $d_i^e \in \left[0, \frac{c_i}{p_j}\right]$, where $c_i > 0$. The probability density function is $f_i(x)$, and the cumulative distribution function is $F_i(x)$, both of which are continuous, differentiable, and monotonically increasing functions.
c_{ni}	Unit raw material transaction cost between supplier n and manufacturer i.
c_{ij}	Unit product transaction cost between manufacturer i and retailer j.
r_n	Carbon reduction rate of supplier n.
r_i	Carbon reduction rate of manufacturer i.
T_n	Carbon reduction investment cost coefficient for supplier n.
T_i	Carbon reduction investment cost coefficient for manufacturer i.
A_n	Carbon allowance freely allocated by the government to supplier n.
A_i	Carbon allowance freely allocated by the government to manufacturer i.
e_n	Initial unit carbon emissions by supplier n.
e_i	Initial unit carbon emissions by manufacturer i.
P_n	Unit carbon trading price for supplier n.
P_i	Unit carbon trading price for manufacturer i.
v	Quantile η of a random variable.
η	Risk aversion degree of the decision-maker, $\eta \in (0, 1]$.
λ	Trade-off weight, $\lambda \in [0, 1]$.
$(\cdot)^*$	the equilibrium value of each variable

4. Model Formulation

4.1. Competitive Decision-Making Behavior and Equilibrium Conditions at the Supplier Level

Suppliers must decide on the wholesale price for each manufacturer for the raw materials, what level of carbon reduction investment to make based on the carbon abatement investment cost coefficient, and at what carbon trading price to engage in transactions in the carbon trading market.

The profit maximization model for supplier n is as follows:

$$max\pi_n = \sum_{i=1}^{I} w_{ni} q_{ni} + P_n \left[A_n - e_n(1 - r_n) \sum_{i=1}^{I} q_{ni} \right] - \sum_{i=1}^{I} c_{ni}(q_{ni}) - f_n(Q_1) - \frac{1}{2} T_n {r_n}^2 \quad (1)$$

$$\text{s. t. } q_{ni} \geq 0, \forall i \quad (2)$$

The term $\sum_{i=1}^{I} w_{ni} q_{ni}$ represents the revenue from raw material sales by supplier n, while $f_n(Q_1)$ and $\sum_{i=1}^{I} c_{ni}(q_{ni})$ denote the production cost and transaction cost with all manufacturers for supplier n, respectively. Focusing on the government's carbon quota policy, suppliers can sell the leftover carbon quota on the carbon trading market if their carbon emissions fall below the allotted amount. On the other hand, the supplier will have

to buy the necessary carbon quotas in the carbon trading market if its carbon emissions are above the government-specified quota. Thus, $P_n\left[A_n - e_n(1-r_n)\sum_{i=1}^{I} q_{ni}\right]$ gives the revenue (or expense) from supplier n's carbon quota trading. The carbon abatement investment cost for supplier n is $\frac{1}{2}T_n{r_n}^2$ to achieve the low-carbon supply chain goal.

Proposition 1. *The profit function of the original supplier n is a jointly concave function of the decision variables, q_{ni} and r_n . In other words, the equilibrium solution that maximizes the profit of supplier n exists and is unique.*

Proof. All proofs of the conclusions in this paper are provided in the Appendix A. □

Theorem 1. *The competition between suppliers constitutes a non-cooperative Nash equilibrium game. The conditions under which all suppliers achieve optimality are equivalent to the following variational inequality, such that $\left(Q_1^*, r_n^*\right) \in R_+^{NI+N}$ satisfies*

$$\sum_{n=1}^{N}\left\{P_n e_n(1-r_n) + \sum_{i=1}^{I}\left[-w_{ni}^* + \frac{\partial c_{ni}\left(q_{ni}^*\right)}{\partial q_{ni}} + \frac{\partial f_n\left(Q_1^*\right)}{\partial q_{ni}}\right]\right\} \times \left(q_{ni} - q_{ni}^*\right) + \sum_{n=1}^{N}\left[-P_n e_n \sum_{i=1}^{I} q_{ni} + T_n r_n\right] \times \left(r_n - r_n^*\right) \geq 0, \quad \forall\left(Q_1^*, r_n^*\right) \in R_+^{NI+N} \tag{3}$$

The economic interpretation of variational inequality (3) is as follows: When suppliers in the DLSCN reach equilibrium through mutual competition, if there is a raw material transaction between supplier n and manufacturer i, then the sum of supplier n's marginal transaction cost, marginal production cost, and the loss in revenue due to incomplete carbon reduction precisely equals the maximum unit price that manufacturer i is ready to pay for the raw materials. If supplier n engages in carbon reduction, then the marginal cost of carbon reduction for supplier n exactly equals the carbon trading revenue lost due to not implementing full carbon reduction.

4.2. Competitive Decision-Making Behavior and Equilibrium Conditions at the Manufacturer Level

Manufacturers must decide how much raw material to buy from each supplier and at what price; how much to sell wholesale to each retailer and at what price; how much to sell to the demand market and at what price; how much to invest in carbon abatement and at what price; and how much to trade on the carbon market and at what price.

The profit maximization model for manufacturer i is as follows:

$$max\pi_i = \sum_{j=1}^{J} w_{ij}q_{ij} - \sum_{n=1}^{N} w_{ni}q_{ni} + P_i\left[A_i - e_i(1-r_i)\sum_{j=1}^{J} q_{ij}\right] - \sum_{j=1}^{J} c_{ij}\left(q_{ij}\right) - f_i(q) + p_i^e\sum_{d=1}^{D} q_{id}^e - f_i(Q_3) - \frac{1}{2}T_i{r_i}^2 \tag{4}$$

$$\begin{gathered} s.t\ \sum_{j=1}^{J} q_{ij} + \sum_{d=1}^{D} q_{id} \leq q_i; \\ q_i \leq \sum_{n=1}^{N} q_{ni}; \\ q_i \geq 0, q_{ni} \geq 0, q_{ij} \geq 0, q_{id} \geq 0, \forall i, j, n, d \end{gathered} \tag{5}$$

The term $\sum_{j=1}^{J} w_{ij}q_{ij}$ represents the revenue for manufacturer i from wholesaling products to retailers. $\sum_{n=1}^{N} w_{ni}q_{ni}$ represents the cost for manufacturer i to purchase raw materials from suppliers. $f_i(q)$ and $\sum_{j=1}^{J} c_{ij}\left(q_{ij}\right)$ denote the production cost and transaction costs with all retailers for manufacturer i, respectively. The profit from selling products through the online direct sales channel for manufacturer i is given by $p_i^e\sum_{d=1}^{D} q_{id}^e - f_i(Q_3)$. The income (or cost) from trading carbon allowances for manufacturer i is given by $P_n\left[A_n - e_n(1-r_n)\sum_{i=1}^{I} q_{ni}\right]$. The carbon reduction investment cost for manufacturer i is $\frac{1}{2}T_i{r_i}^2$.

Proposition 2. *The profit function of the original manufacturer i is a jointly concave function of the decision variables $q_i, q_{ni}, q_{ij}, q_{id}$, and r_i. In other words, the equilibrium solution that maximizes the profit of manufacturer i exists and is unique.*

Theorem 2. *The manufacturers are engaged in a non-cooperative Nash equilibrium game. The conditions under which all manufacturers achieve optimality are equivalent to the following variational inequality, such that $\left(q^*, Q_1^*, Q_2^*, Q_3^*, r_i^*, \varepsilon_i^*, \alpha_i^*\right) \in R_+^{NI+IJ+ID+4I}$ satisfies*

$$\begin{aligned}
&\sum_{i=1}^{I}\left(\frac{\partial f_i(q^*)}{\partial q_i} - \varepsilon_i^* + \alpha_i^*\right) \times \left(q_i - q_i^*\right) + \sum_{n=1}^{N}\sum_{i=1}^{I}\left(w_{ni}^* - \alpha_i^*\right) \times \left(q_{ni} - q_{ni}^*\right) \\
&+ \sum_{i=1}^{I}\sum_{j=1}^{J}\left[-w_{ij}^* + P_i e_i(1 - r_i) + \frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} + \varepsilon_i^*\right] \times \left(q_{ij} - q_{ij}^*\right) \\
&+ \sum_{i=1}^{I}\sum_{d=1}^{D}\left[-p_i^{e*} + \frac{\partial f_i(Q_3^*)}{\partial q_{id}} + \varepsilon_i^*\right] \times \left(q_{id} - q_{id}^*\right) + \sum_{i=1}^{I}\left[-P_i e_i \sum_{j=1}^{J} q_{ij} + T_i r_i\right] \times \left(r_i - r_i^*\right) \\
&+ \sum_{i=1}^{I}\left(q_i{}^* - \sum_{j=1}^{J} q_{ij}^* - \sum_{d=1}^{D} q_{id}^*\right) \times \left(\varepsilon_i - \varepsilon_i^*\right) + \sum_{i=1}^{I}\left(\sum_{n=1}^{N} q_{ni}^* - q_i{}^*\right) \times \left(\alpha_i - \alpha_i^*\right) \\
&\geq 0, \forall\left(Q_1^*, Q_2^*, Q_3^*, r_i^*, \varepsilon_i^*\right) \in R_+^{NI+IJ+ID+I+I} \forall\left(Q_1^*, r_n^*\right) \in R_+^{NI+N}
\end{aligned} \tag{6}$$

where ε_i and α_i are Lagrange multipliers for manufacturer i related to the constraint conditions in Equation (6).

The economic interpretation of variational inequality (6) is as follows: When the manufacturers within the DLSCN reach equilibrium through mutual competition, if manufacturer i produces products, then ε_i^* is exactly equivalent to the total of the marginal production cost and α_i^*. If manufacturer i and supplier n engage in raw material transactions, then α_i^* is exactly equal to the maximum raw material wholesale price that manufacturer i is ready to pay. If there are product transactions between manufacturer i and retailer j, then the maximum unit product wholesale price that retailer j is ready to pay is precisely equal to ε_i^*, manufacturer i's marginal transaction cost, plus the benefits lost as a result of manufacturer i's incomplete carbon abatement. The marginal sales costs of manufacturer i and ε_i^* are exactly equivalent to the maximum unit product sales price that demand market d is ready to pay via the online direct sales channel if manufacturer i has product interactions with demand market d through this channel. If manufacturer i engages in carbon abatement, then the marginal cost of carbon abatement for manufacturer i is exactly equal to the loss of carbon trading benefits due to not engaging in carbon abatement.

4.3. Competitive Decision-Making Behavior and Equilibrium Conditions at the Retailer Level

Retailers must choose the price and amount of goods to purchase from manufacturers, and the price and quantity of products to offer through traditional retail channel to each demand market.

In the case where the market demand for retailer j is stochastic, the profit function for retailer j is as follows:

$$\pi_j = p_j q_j - \sum_{i=1}^{I} w_{ij} q_{ij} - f_j(Q_2) - p_j\left(q_j - d_j\right)^+ \tag{7}$$

where $\left(q_j - d_j\right)^+ = \max\left[\left(q_j - d_j\right), 0\right]$. $p_j q_j - p_j\left(q_j - d_j\right)^+$ represents the revenue from retailer j selling products to the demand market, $\sum_{i=1}^{I} w_{ij} q_{ij}$ represents the cost for retailer j to purchase products from manufacturers, and $f_j(Q_2)$ represents the exhibition and promotion cost for retailer j.

Conditional Value at Risk (CvaR) calculates the average profit below the quantile η, addressing shortcomings of Variance (VaR) in failing to reflect tail-end losses, while also exhibiting subadditivity. Equation (8) is a more generalized definition of CvaR put forth and persuasively defended by Rockafellar and Uryasev [43]. The CvaR of retailer j is represented computationally simply in this paper using Equation (8), where E is the decision variable's expected value, v is the random variable's η-quantile, and η is the decision-maker's risk aversion, $\eta \in (0,1]$. The smaller η is, the more risk-averse the decision-maker is.

$$CVaR_{\eta}(\pi_j) = \max_{v \epsilon R}\left\{ v + \frac{1}{\eta} E\left[min(\pi_j) - v, 0\right] \right\} \tag{8}$$

CvaR ignores excess and only calculates the mean of returns below the quantile η. However, the mixed conditional value-at-risk (mean-CvaR) overcomes the drawbacks of CvaR by including the mean of returns below and above the quantile η, which is a mixed weighted average of expected profit and CvaR that maximizes the expected profit of the risk-averse decision-maker and minimizes the profit of the downside risk, allowing for a more holistic and realistic decision making [39]. Thus, this study employs the mean-CvaR metric to assess the utility function of risk-averse retailers, as expressed by the following formula:

$$G_n(\pi_j) = \lambda E(\pi_j) + (1-\lambda) CVaR_{\eta}(\pi_j) \tag{9}$$

where λ is the weight of the trade-off between the two, $\lambda \in [0,1]$. If $\lambda = 0$, the decision-maker is extremely risk-averse and bases their choice only on the CvaR measure's downside profit, ignoring the expected profit; if $\lambda = 1$, they are risk-neutral and only take the expected profit into account; and if $0 < \lambda < 1$, they consider both the expected profit and the CvaR.

Substituting Equation (7) into Equation (8), we obtain

$$\begin{aligned} CVaR_{\eta}(\pi_j) = v \quad & -\frac{1}{\eta}\int_0^{q_j}\left[v - \left(p_j x - \sum_{i=1}^{I} w_{ij} q_{ij}\right) + f_j(Q_2)\right]^{+} f_j(x)dx \\ & -\frac{1}{\eta}\int_{q_j}^{\frac{b_j}{p_j}}\left[v - \left(p_j q_j - \sum_{i=1}^{I} w_{ij} q_{ij}\right) + f_j(Q_2)\right]^{+} f_j(x)dx \end{aligned} \tag{10}$$

$$E(\pi_j) = p_j q_j - \sum_{i=1}^{I} w_{ij} q_{ij} - f_j(Q_2) - p_j \int_0^{q_j} (q_j - x) f_j(x)dx \tag{11}$$

Substituting Equations (10) and (11) into Equation (9), the utility maximization model for the retailer is obtained as follows:

$$\begin{aligned} & max G\left(\pi_j\right) \\ & = \lambda\left[p_j q_j - \sum_{i=1}^{I} w_{ij} q_{ij} - f_j(Q_2) - p_j \int_0^{q_j}\left(q_j - x\right) f_j(x)dx\right] \\ & +(1-\lambda)\left\{ v - \frac{1}{\eta}\int_0^{q_j}\left[v - \left(p_j x - \sum_{i=1}^{I} w_{ij} q_{ij}\right) + f_j(Q_2)\right]^{+} f_j(x)dx - \frac{1}{\eta}\int_{q_j}^{\frac{b_j}{p_j}}\left[v - \left(p_j q_j - \sum_{i=1}^{I} w_{ij} q_{ij}\right) + f_j(Q_2)\right]^{+} f_j(x)dx \right\} \end{aligned} \tag{12}$$

$$s.t. q_j \geq 0, q_{ij} \geq 0, \forall i, j \tag{13}$$

Proposition 3. *The utility function of the original retailer j is a jointly concave function of the decision variable, q_{ij}. In other words, the equilibrium solution that maximizes the utility of retailer j exists and is unique.*

Theorem 3. *The retailers are engaged in a non-cooperative Nash equilibrium game. The condition under which all retailers achieve optimality is equivalent to the following variational inequality, such that* $(Q_2^*) \in R_+^{IJ}$ *satisfies*

$$\sum_{j=1}^{J}\left\{\left(-\lambda-(1-\lambda)\frac{1}{\eta}\right)\left[p_j^*\left(1-F_j\left(q_j,p_j^*\right)\right)\right]+\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda\left(w_{ij}^*+\frac{\partial f_j(Q_2^*)}{\partial q_{ij}}\right)+(1-\lambda)\frac{1}{\eta}\left[\left(w_{ij}^*+\frac{\partial f_j(Q_2^*)}{\partial q_{ij}}\right)^+\right]\right\} \times\left(q_{ij}-q_{ij}{}^*\right),\forall(Q_2^*)\in R_+^{IJ} \tag{14}$$

The economic interpretation of the variational inequality (14) is as follows: When the retailors in the DLSCN reach an equilibrium through mutual game-play, and retailer j and manufacturer i have product transactions: (1) When $w_{ij}^* + \frac{\partial f_j(Q_2^*)}{\partial q_{ij}} > 0$, the maximum amount that retailer j is ready to pay to manufacturer i for each unit of raw material wholesale price, plus retailer j's marginal sales promotion cost, is precisely equivalent to the product of the maximum unit price the demand market d is ready to pay in the traditional retail channel and the complement probability of the demand market's willingness to buy the product at the market demand quantity q_j and sales price p_j^*, denoted as $\left[1-F_j\left(q_j,p_j^*\right)\right]$. (2) When $w_{ij}^* + \frac{\partial f_j(Q_2^*)}{\partial q_{ij}} \leq 0$, the maximum amount that retailer j is ready to pay to manufacturer i for each unit of raw material wholesale price, plus retailer j's marginal sales promotion cost, multiplied by λ, is precisely equivalent to the product of the maximum unit price the demand market d is ready to pay in the traditional retail channel and $\lambda+(1-\lambda)\frac{1}{\eta}\left(1-F_j\left(q_j,p_j^*\right)\right)$.

4.4. Competitive Decision-Making Behavior and Equilibrium Conditions at the Demand Market Level

Consumers in the demand market must decide whether to purchase goods through the online direct sales channel or the traditional retail channel, at what price and in what quantity to purchase products from manufacturers or retailers.

The demand market's equilibrium condition can be stated as follows, using Nagurney et al.'s spatial price equilibrium condition as a guide [18]:

$$d_j\begin{cases}=\sum_{i=1}^{I} q_{ij}{}^*, p_j{}^* > 0\\ <\sum_{i=1}^{I} q_{ij}{}^*, p_j{}^* = 0\end{cases} \tag{15}$$

$$d_i^e\begin{cases}=q_i{}^*-\sum_{j=1}^{J} q_{ij}{}^*, p_i^{e*} > 0\\ <q_i{}^*-\sum_{j=1}^{J} q_{ij}{}^*, p_i^{e*} = 0\end{cases} \tag{16}$$

The equilibrium between the quantity of goods ordered by retailer j and its market demand, d_j, determines the equilibrium of the demand market in the traditional sales channel, as demonstrated by Equation (15). When the price that customers are ready to pay in the conventional retail channel is more than zero, the quantity of products that retailer j orders and its demand are precisely identical. Equation (16) shows that the equilibrium in the online direct sales channel demand market depends on whether the difference between the product production quantity of manufacturer i and the quantity wholesaled to retailers is balanced with its market demand quantity, d_i^e. When consumers are ready to pay a price greater than zero in the online direct sales channel, the difference between the quantity of

the product produced by manufacturer i and the quantity wholesaled to retailers is exactly equal to its demand quantity.

Theorem 4. *The demand markets are involved in a non-cooperative Nash equilibrium game. The condition for all demand markets to reach optimality, based on the equilibrium conditions for stochastic demand, is equivalent to the following variational inequality, such that* $\left(p_j{}^*, p_i^{e*}\right) \in R_+^{J+I}$ *satisfies*

$$\sum_{j=1}^{J}\left(\sum_{i=1}^{I} q_{ij}{}^* - d_j\right)\times\left(p_j - p_j{}^*\right) + \sum_{i=1}^{I}\left(q_i{}^* - \sum_{j=1}^{J} q_{ij}{}^* - d_i^e\right)\times\left(p_i^e - p_i^{e*}\right) \geq 0, \forall\left(p_j, p_i^{e*}\right) \in R_+^{I+I} \tag{17}$$

4.5. The Dual-Channel Low-Carbon Supply Chain Network Equilibrium Model

When the DLSCN reaches an equilibrium state, the decision-making behaviors of all decision-makers in the network need to jointly reach an equilibrium state, that is, the equilibrium trading volumes, carbon emission reduction rates, and prices must simultaneously satisfy the variational inequalities (3), (6), (14), and (17).

Theorem 5. *The equilibrium solution* $\left(q^*, Q_1^*, Q_2^*, Q_3^*, r_n^*, r_i^*, p_j{}^*, p_i^{e*}, \varepsilon_i^*, \alpha_i^*\right) \in R_+^{NI+IJ+ID+N+5I+J}$ *of the dual-channel low-carbon supply chain network considering retailer risk aversion under carbon quotas satisfies the variational inequality as follows:*

$$\begin{aligned}
&\sum_{n=1}^{N}\left\{P_n e_n(1-r_n) + \sum_{i=1}^{I}\frac{\partial c_{ni}\left(q_{ni}^*\right)}{\partial q_{ni}} + \frac{\partial f_n\left(Q_1^*\right)}{\partial q_{ni}} - \alpha_i^*\right\}\times\left(q_{ni} - q_{ni}^*\right) + \sum_{i=1}^{I}\left(\frac{\partial f_i\left(q^*\right)}{\partial q_i} - \varepsilon_i^* + \alpha_i^*\right)\times\left(q_i - q_i^*\right)\\
&+\sum_{j=1}^{J}\sum_{i=1}^{I}\left\{\left[-w_{ij}^* + P_i e_i(1-r_i) + \frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} + \varepsilon_i^*\right] + \lambda\left(w_{ij}^* + \frac{\partial f_j\left(Q_2^*\right)}{\partial q_{ij}}\right) + (1-\lambda)\frac{1}{\eta}\left[\left(w_{ij}^* + \frac{\partial f_j\left(Q_2^*\right)}{\partial q_{ij}}\right)^+ F_j\left(\frac{b_j}{p_j}, p_j^*\right)\right] - \sigma_j^*\right\}\\
&\times\left(q_{ij} - q_{ij}{}^*\right) + \sum_{i=1}^{I}\sum_{d=1}^{D}\left[-p_i^{e*} + \frac{\partial f_i\left(Q_3^*\right)}{\partial q_{id}} + \varepsilon_i^*\right]\times\left(q_{id} - q_{id}^*\right) + \sum_{j=1}^{J}\left\{\left(-\lambda - (1-\lambda)\frac{1}{\eta}\right)\left[p_j^*\left(1 - F_j\left(q_j, p_j^*\right)\right)\right] + \sigma_j^* + \sigma_j^*\right\}\\
&\times\left(q_j - q_j{}^*\right) + \sum_{n=1}^{N}\left[-P_n e_n\sum_{i=1}^{I} q_{ni} + T_n r_n\right]\times\left(r_n - r_n^*\right) + \sum_{i=1}^{I}\left[-P_i e_i\sum_{j=1}^{J} q_{ij} + T_i r_i\right]\times\left(r_i - r_i^*\right) + \sum_{j=1}^{J}\left(\sum_{i=1}^{I} q_{ij}{}^* - d_j\right)\\
&\times\left(p_j - p_j{}^*\right) + \sum_{i=1}^{I}\left(q_i{}^* - \sum_{j=1}^{J} q_{ij}{}^* - d_i^e\right)\times\left(p_i^e - p_i^{e*}\right) + \sum_{i=1}^{I}\left(q_i{}^* - \sum_{j=1}^{J} q_{ij}^* - \sum_{d=1}^{D} q_{id}^*\right)\times\left(\varepsilon_i - \varepsilon_i^*\right) + \sum_{i=1}^{I}\left(\sum_{n=1}^{N} q_{ni}^* - q_i{}^*\right)\\
&\times\left(\alpha_i - \alpha_i^*\right) \geq 0, \forall\left(q^*, Q_1^*, Q_2^*, Q_3^*, r_n^*, r_i^*, p_j{}^*, p_i^{e*}, \varepsilon_i^*, \alpha_i^*\right) \in R_+^{NI+IJ+ID+N+5I+J}
\end{aligned} \tag{18}$$

In the model, the supplier's wholesale price to the manufacturer, w_{ni}, and the manufacturer's wholesale price to the retailer, w_{ij}, are endogenous variables. According to variational inequality (3), if $q_{ni}^* > 0$, the equilibrium value of w_{ni} can be obtained as w_{ni}^*=$P_n e_n(1-r_n) + \frac{\partial c_{ni}\left(q_{ni}^*\right)}{\partial q_{ni}} + \frac{\partial f_n\left(Q_1^*\right)}{\partial q_{ni}}$. According to variational inequality (6), if $q_{ij}^* > 0$, the equilibrium value of w_{ij} can be obtained as $w_{ij}^* = P_i e_i(1-r_i) + \frac{\partial c_{ij}\left(q_{ij}^*\right)}{\partial q_{ij}} + \varepsilon_i^*$.

5. Numerical Analysis

All feasible regions for the decision variables in this paper are non-negative, enabling the use of He's projective shrinkage approach to solve for the equilibrium solution of the DLSCN, i.e., to solve Equation (18) [44]. Consider a numerical example analysis based on a DLSCN consisting of two suppliers, two manufacturers, two retailers, and two demand markets. The relevant cost functions and parameter values are referenced from the studies by Nagurney et al., Xu and Jianbin, and Zou et al., while also being set in accordance with real conditions [17,18,45]. The specific forms are as follows:

The cost functions for the suppliers are

$$f_1(Q_1) = 0.5\left(\sum_{i=1}^{2} q_{1i}\right)^2 + 0.2\sum_{i=1}^{2} q_{1i}\sum_{i=1}^{2} q_{2i} + \sum_{i=1}^{2} q_{1i}, f_2(Q_1) = 0.5\left(\sum_{i=1}^{2} q_{2i}\right)^2 + 0.2\sum_{i=1}^{2} q_{1i}\sum_{i=1}^{2} q_{2i} + \sum_{i=1}^{2} q_{2i}, i = 1,2 \quad (19)$$

The transaction cost functions between suppliers and manufacturers are

$$c_{ni}(q_{ni}) = 0.1(q_{ni})^2, n = 1,2; i = 1,2 \quad (20)$$

The cost functions for manufacturers are

$$f_1(q) = 0.5(q_1)^2 + 0.2q_1q_2 + q_1, f_2(q) = 0.5(q_2)^2 + 0.2q_1q_2 + q_2,\ n = 1,2 \quad (21)$$

The transaction cost functions between manufacturers and retailers are

$$c_{ij}(q_{ij}) = 0.1(q_{ij})^2, i = 1,2; j = 1,2 \quad (22)$$

The exhibition cost functions for manufacturers are

$$f_1(Q_3) = 0.2\left(\sum_{d=1}^{2} q_{1d}^e\right)^2, f_2(Q_3) = 0.2\left(\sum_{d=1}^{2} q_{2d}^e\right)^2, d = 1,2 \quad (23)$$

The exhibition cost functions for retailers are

$$f_1(Q_2) = 0.2\left(\sum_{i=1}^{2} q_{i1}\right)^2, f_2(Q_3) = 0.2\left(\sum_{i=1}^{2} q_{i2}\right)^2, i = 1,2 \quad (24)$$

The demand for products from customers who favor the online direct sales channel is also assumed to be separate from the demand in the conventional market. The demand function d_j for retailer j has $b_j = 100$, which follows a uniform distribution of $\left[\frac{0,100}{p_j}\right]$, while the demand function i for manufacturer i has $c_i = 40$, which follows a uniform distribution of $\left[\frac{0,40}{p_j}\right]$. The initial carbon emissions for supplier n and manufacturer i are set at $e_n = e_i = 0.2$. The carbon quotas allocated by the government to supplier n and manufacturer i are $A_n = A_i = 1$.

5.1. Impact of Changes in Retailer Trade-Off Weight and Risk Aversion Degree

To find out how changes in the retailer's risk aversion degree, η, and trade-off weight, λ, affect the DLSCN's equilibrium, we set the carbon trading prices for supplier n and manufacturer i as $P_n = P_i = 1$, and the carbon abatement investment cost coefficients for supplier n and manufacturer i as $T_n = T_i = 2$. When the trade-off weight, λ, is set to (0.5,0.6,0.7), respectively, the impact of varying the risk aversion degree, η, on the equilibrium solution of the DLSCN game is examined. Simultaneously, when η is fixed at (0.5,0.6,0.7), respectively, the influence of changing λ on the equilibrium solution of the DLSCN game is also investigated.

As shown in Figure 2, the order quantity, q_{ij}, from retailers to manufacturers is inversely proportional to both the trade-off weight, λ, and the risk aversion degree, η. That implies that if the more risk-averse retailer places more emphasis on CVaR profits, a retailer orders more from the manufacturer. In contrast, it means that if the retailer is more risk-neutral, setting more emphasis on expected profit, the quantity ordered from the manufacturer will be low. This conclusion differs from the research findings of Liu et al. on risk aversion and coordination among supply chain entities. Liu et al. discovered that retailers' order quantities are directly proportional to the trade-off weight, and that order

quantities are only positively correlated with the degree of risk aversion at lower levels, while at higher levels, they are not directly related [40]. This discrepancy may stem from differences in research focus, supply chain structure, and the diversity of factors considered. Liu et al.'s research placed greater emphasis on supply chain coordination, whereas we prioritize sensitivity analysis. In terms of supply chain structure, they examined a simple two-tier supply chain structure, whereas we delved into a more complex four-tier supply chain network. Additionally, beyond risk factors, they explored the impact of blockchain technology, while we incorporated carbon trading mechanisms into our considerations.

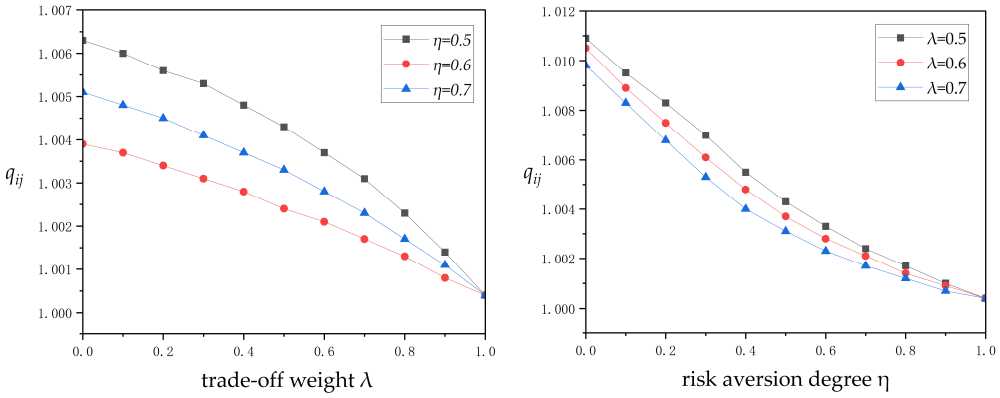


Figure 2. The impact of retailers' trade-off weight and risk aversion degree on the order quantity of retailers. (**a**) The impact of λ on q_{ij}; (**b**) The impact of η on q_{ij}.

From Figure 3, it can be seen that when the trade-off weight, λ, is fixed, the expected utility, $G(\pi_j)$, of the retailer is inversely proportional to the risk aversion degree, η. But, given the risk aversion degree, η, $G(\pi_j)$ is directly proportional to λ. That is, the expected utility will be greater when the retailer is more risk-averse and places greater emphasis on expected profit. However, if the retailer is more risk-neutral and places a greater emphasis on CVaR profits, the expected utility will decrease, potentially even becoming negative, which would be highly detrimental to the retailer. This finding supports the results obtained by Xie et al. with respect to the coordination between supply chains and retailers under the mean-CVaR criterion [39].

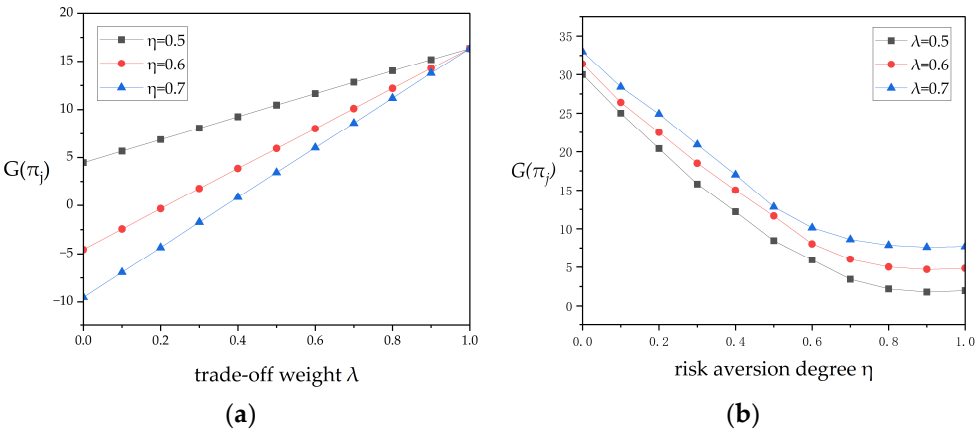


Figure 3. The impact of retailers' trade-off weight and risk aversion degree on the expected utility of the retailers. (**a**) The impact of λ on $G(\pi_j)$. (**b**) The impact of η on $G(\pi_j)$.

From Figure 4a,b, it can be seen that the market demand for traditional retail channels is inversely proportional to both the trade-off weight, λ, and the risk aversion degree, η. From Figure 4c,d, it can be observed that the demand for online direct-sales channels

increases proportionally with both λ and η. The reason may be that as the retailer's trade-off weight and risk aversion degree increase, the retailer's ordering amount from the manufacturer decreases. In the demand market, it means that a drop in supply will drive up the sales price of traditional channels (as shown in Figure 5a,b), thereby lowering demand for traditional retail channels. However, the demand market's requirement of online direct-sales channels goes up. This means that for the manufacturer, because of the retailer's decreased demand, the wholesale price is lowered, which in turn decreases the price of online direct-sales channels (as shown in Figure 5c,d) and thus raises demand for them.

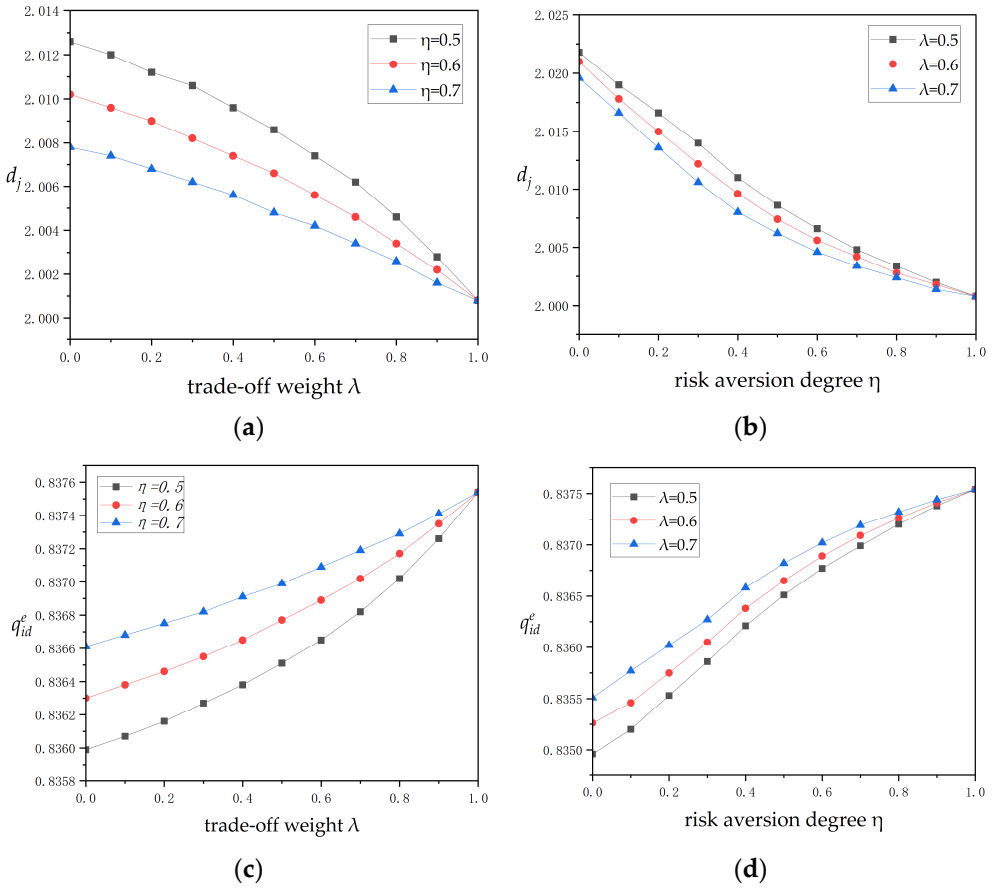


Figure 4. The impact of retailers' trade-off weight and risk aversion degree on the traditional channel and online retail market demands. (**a**) The impact of λ on d_j. (**b**) The impact of η on d_j. (**c**) The impact of λ on q_{id}^e. (**d**) The impact of η on q_{id}^e.

From Figure 6a–h, we can observe that both the profits and carbon reduction rates of manufacturers and suppliers are inversely proportional to the trade-off weight, λ, and risk aversion, η. This suggests that retailers' attitudes toward risk and their trade-offs between the two types of profits have corresponding impacts on the profits and carbon emission reduction policies of suppliers and retailers. For suppliers and manufacturers, when retailers are more risk-averse and place greater emphasis on CVaR profit, their profits and carbon reduction rates both increase, achieving both economic and environmental benefits. Therefore, it is necessary for suppliers and manufacturers to design mechanisms to incentivize risk-averse retailers and increase investment in carbon reduction, thereby enhancing the circulation of low-carbon products within the entire DLSCN and boosting their own profits and carbon reduction rates. This finding differs from the conclusions drawn by Xie et al. [39]. Their research indicates that suppliers' profits are directly proportional to their risk aversion degree, η, but inversely proportional to their trade-off weight, λ. Their study

focuses on a single two-tier supply chain comprising suppliers and retailers, and it does not consider network conditions. The sales channels examined are limited to traditional retail channels, excluding the dual-channel sales model. Therefore, differences in supply chain structure and sales channels may collectively contribute to the differing trends in the impact of retailers' trade-off weights on the profits of suppliers and manufacturers.

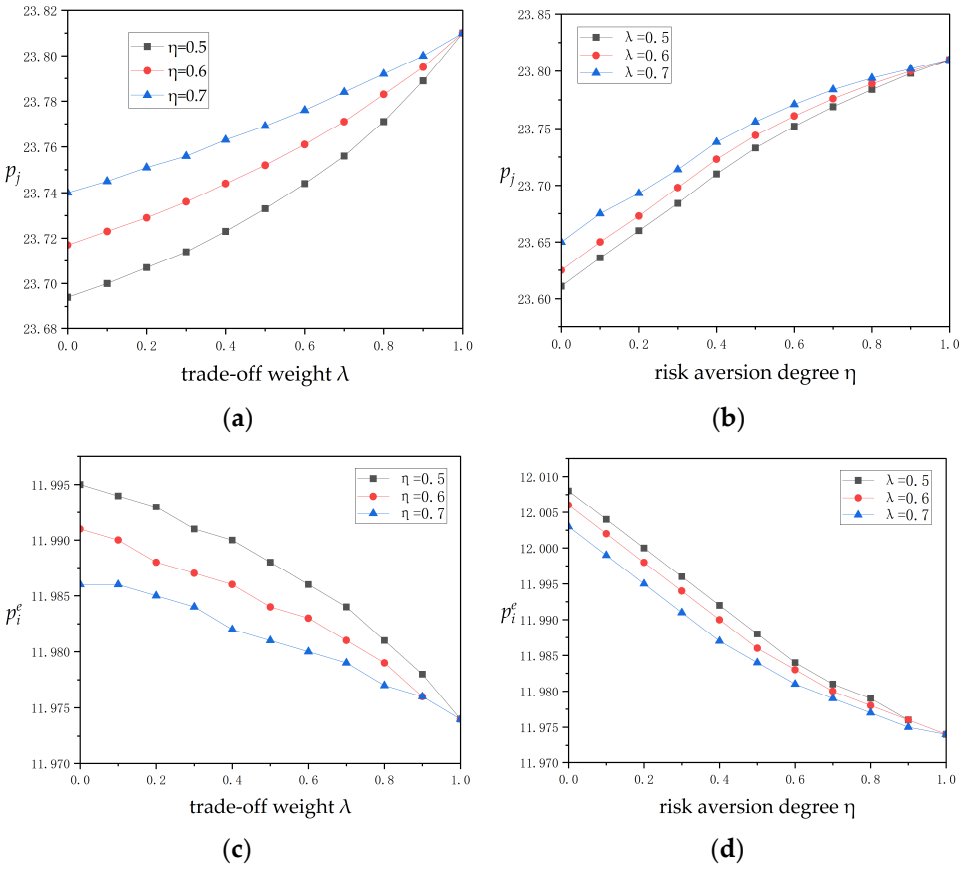


Figure 5. The impact of retailers' trade-off weight and risk aversion degree on the sales prices of traditional and online retail channels. (**a**) The impact of λ on p_j. (**b**) The impact of η on p_j. (**c**) The impact of λ on p_i^e. (**d**) The impact of η on p_i^e.

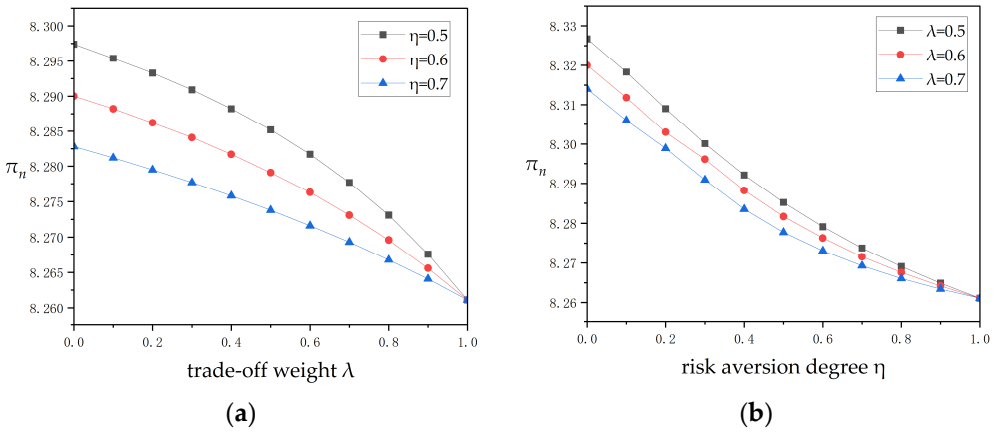


Figure 6. *Cont.*

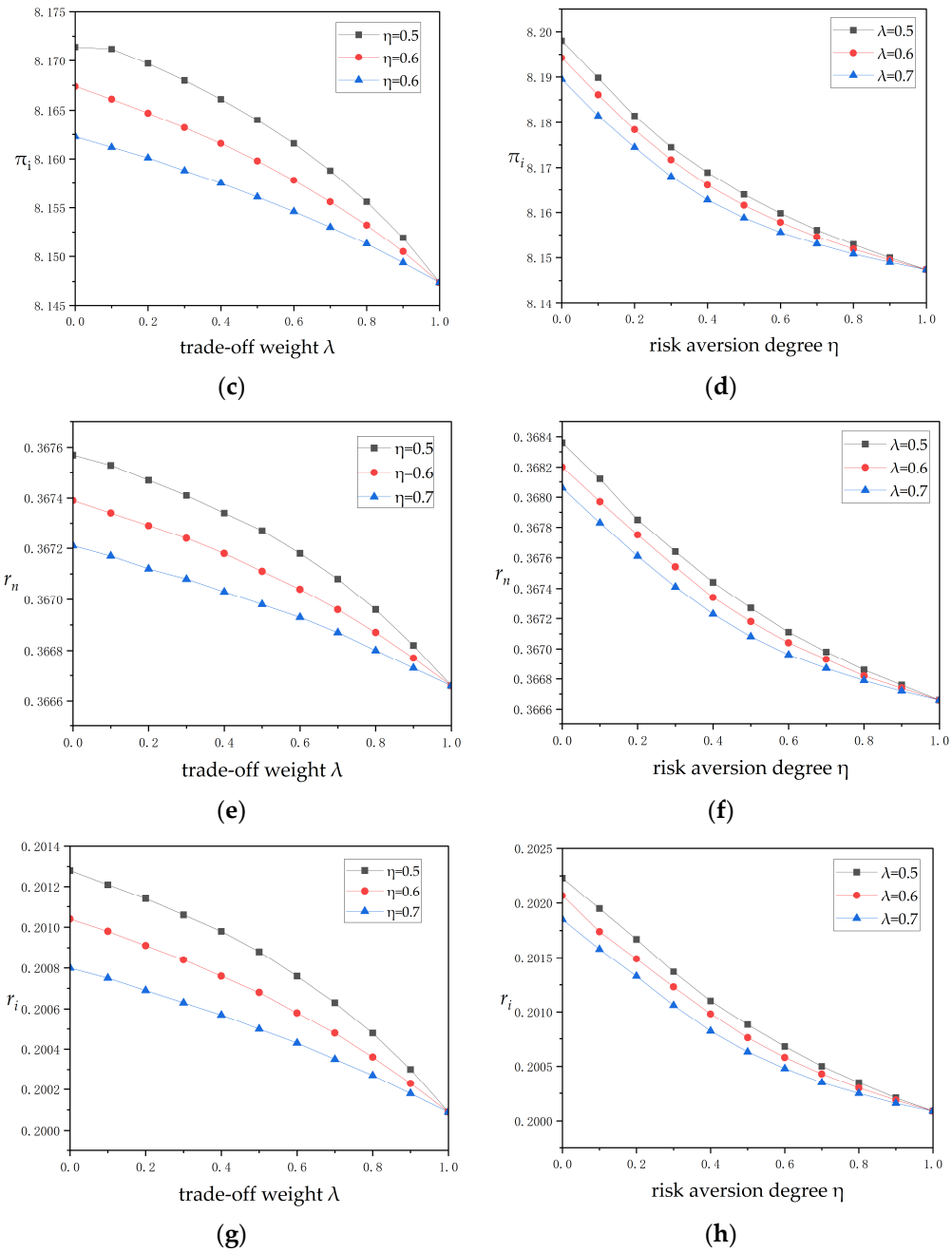


Figure 6. The impact of retailers' trade-off weight and risk aversion degree on the profits and carbon reduction rates of suppliers and manufacturers. (**a**) The impact of λ on π_n. (**b**) The impact of η on π_n. (**c**) The impact of λ on π_i. (**d**) The impact of η on π_i. (**e**) The impact of λ on r_n. (**f**) The impact of η on r_n. (**g**) The impact of λ on r_i. (**h**) The impact of η on r_i.

5.2. Impact of Changes in Carbon Abatement Investment Cost Coefficient

To investigate the impact of varying carbon abatement investment cost coefficients on the equilibrium state of the DLSCN, we set the carbon trading prices for supplier n and manufacturer i as $P_n = P_i = 1$. The retailer j's risk aversion degree and trade-off weight are $\eta = 0.5$ and $\lambda = 0.5$, respectively. The carbon abatement investment cost coefficients of supplier n and manufacturer i are assumed to be $T_n = T_i = (1, 2, 3, 4, 5)$.

From Figure 7, it can be seen that the carbon reduction rate and the amount of products provided by suppliers and manufacturers are inversely proportional to the carbon abatement investment cost coefficients, T_n and T_i, of suppliers and manufacturers, and their own carbon abatement investment cost coefficients have a greater impact on their own carbon

reduction rate and the amount of products provided. That is, when the cost coefficient of carbon abatement investment rises, manufacturers and suppliers will control their own carbon reduction rate to keep the overall cost of carbon reduction from surpassing a certain threshold. At the same time, they will decrease the production of high-cost products, reduce the quantity of products they produce, and have more carbon allowances left over for carbon trading to maximize profits.

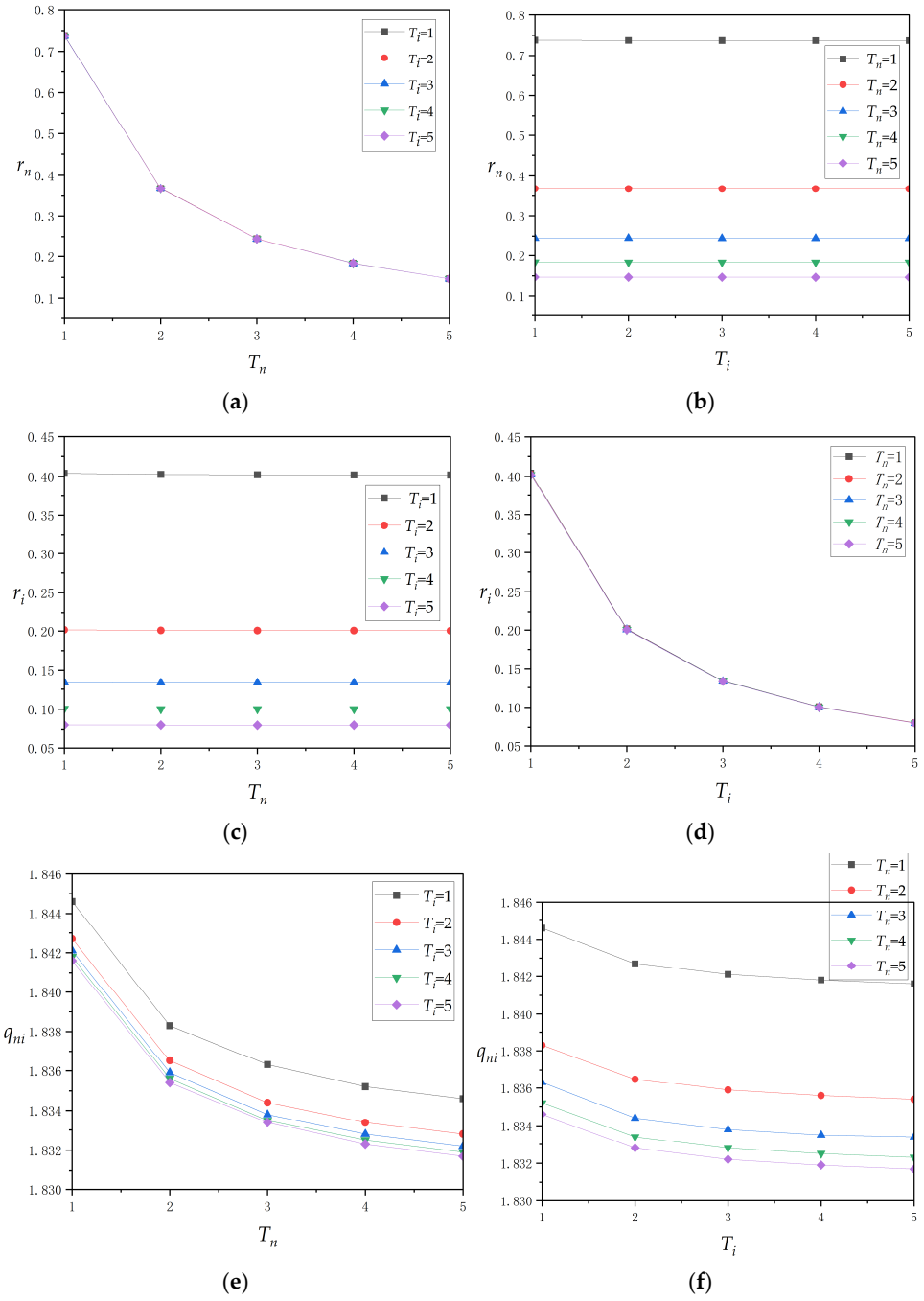


Figure 7. The impact of the carbon abatement investment cost coefficients of suppliers and manufacturers on their carbon reduction rates and the amount of products. (**a**) The impact of T_n on r_n. (**b**) The impact of T_i on r_n. (**c**) The impact of T_n on r_i. (**d**) The impact of T_i on r_i. (**e**) The impact of T_n on q_{ni}. (**f**) The impact of T_i on q_{ni}.

As shown in Figure 8, when the supplier's carbon abatement investment cost coefficient, T_n, is constant, the supplier's wholesale price is inversely proportional to the

manufacturer's carbon abatement investment cost coefficient, T_i, and the manufacturer's wholesale price is proportional to T_i. When T_i is constant, both suppliers' and manufacturers' wholesale prices are proportional to T_n. It is evident that the supplier and manufacturer will raise their wholesale prices to offset the increased costs stemming from the higher cost factor associated with their own carbon reduction efforts. The rise in wholesale prices of the supplier and the raise in raw material purchasing costs for the manufacturer will be balanced by increasing the wholesale prices of their products and the prices of their sales in the network channel to make a profit; in the meantime, the manufacturer hopes that in the case that their own carbon abatement investment cost coefficient increases, the supplier can reduce the wholesale price, and then for the manufacturer, the wholesale price will be increased. Meanwhile, the manufacturer hopes that the supplier can lower the wholesale price in the event that their own carbon abatement investment cost coefficient rises, after which the manufacturer will raise the wholesale price. At this moment, suppliers will decide to lower the wholesale price to maintain the number of transactions and amount of profits because, if the wholesale price is not lowered, the demand will decline.

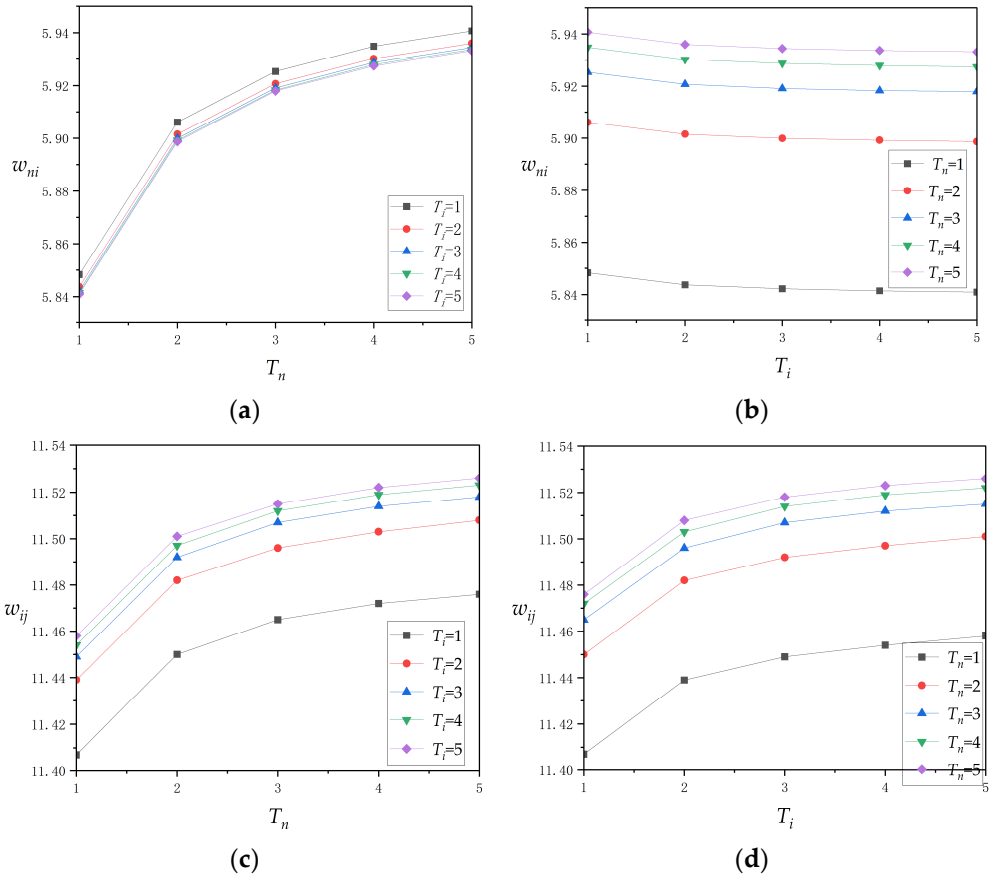


Figure 8. The impact of the carbon abatement investment cost coefficients of suppliers and manufacturers on their wholesale prices. (**a**) The impact of T_n on w_{ni}. (**b**) The impact of T_i on w_{ni}. (**c**) The impact of T_n on w_{ij}. (**d**) The impact of T_i on w_{ij}.

From Figure 9, we can observe that suppliers and manufacturers will increase their own profits by increasing their own carbon reduction investment cost coefficients, while upstream or downstream carbon reduction investment cost coefficients will decrease their profits. Therefore, manufacturers and suppliers do not need to worry too much about the rise in carbon abatement investment cost coefficient, and they can control the cost of carbon reduction by controlling carbon reduction rate, but they also need to be mindful of the

upstream or downstream carbon reduction when they reduce carbon reduction, which will affect their own profits.

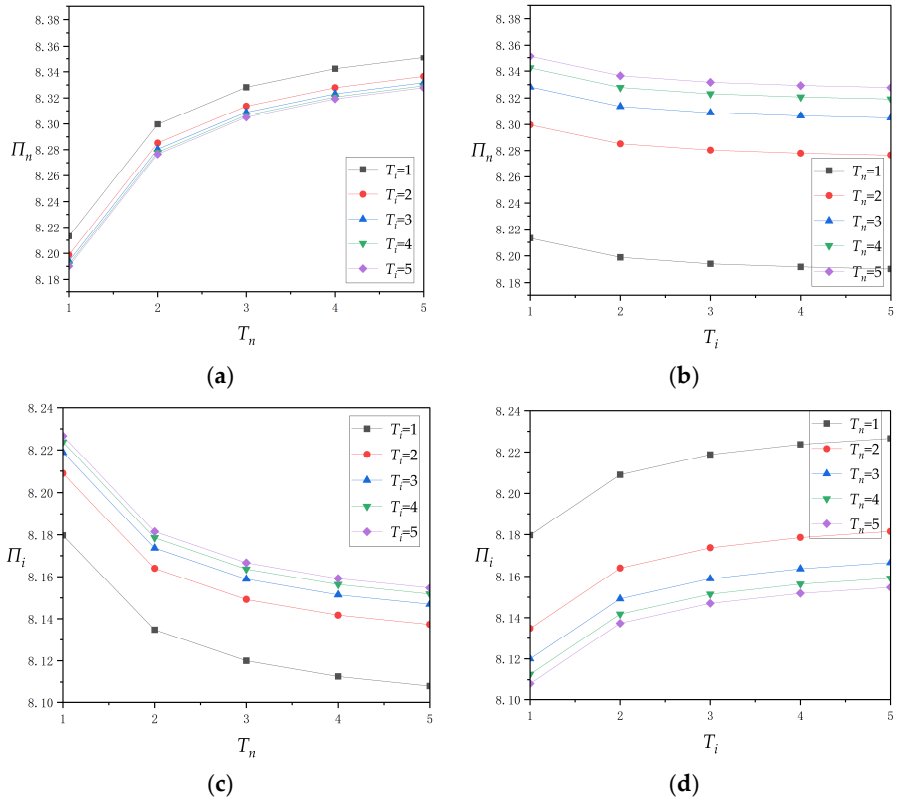


Figure 9. The impact of the carbon abatement investment cost coefficients of suppliers and manufacturers on their profits. (**a**) The impact of T_n on π_n. (**b**) The impact of T_i on π_n. (**c**) The impact of T_n on π_i. (**d**) The impact of T_i on π_i.

As shown in Figure 10, the expected utility of retailers, $G(\pi_j)$, is proportional to both suppliers and manufacturers carbon abatement investment cost coefficients, T_n and T_i. This suggests that retailers are more inclined to sell low-carbon products, and for them, larger carbon abatement investment cost coefficients of suppliers and manufacturers will result in larger purchasing costs, and retailers can increase utility by selling low-carbon products at higher prices to make up for the loss of increased costs.

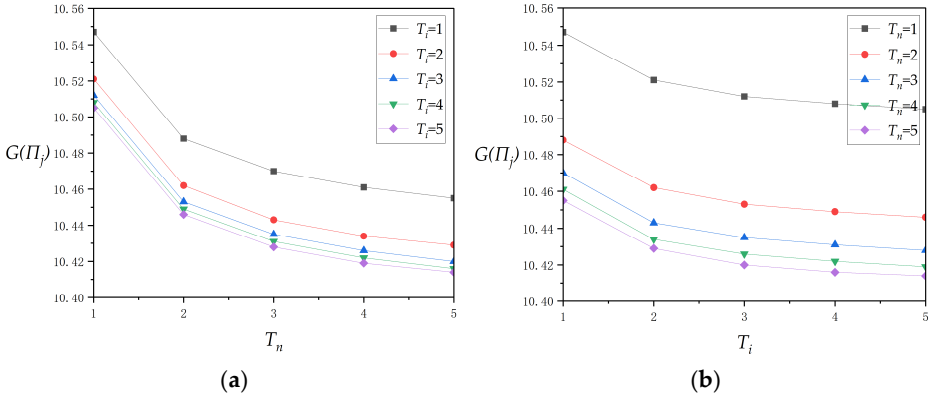


Figure 10. The impact of the carbon abatement investment cost coefficients of suppliers and manufacturers on the expected utility of retailers. (**a**) The impact of T_n on $G(\pi_j)$. (**b**) The impact of T_i on $G(\pi_j)$.

5.3. Impact of Carbon Trading Price Changes

To investigate the impact of changes in the government-allocated carbon quota, A_n for suppliers n and A_i for manufacturers i, on the equilibrium of the DLSCN, we set the risk aversion degree and the balance weight for retailer j to $\eta = 0.5$ and $\lambda = 0.5$, respectively. The carbon abatement investment cost coefficients for supplier n and manufacturer i are $T_n = T_i = 2$. Furthermore, based on the carbon trading price data since the establishment of the national carbon trading market in China in 2021, it has been found that the carbon trading price range across various regions is 0.0385–0.106 RMB/kg. For the convenience of statistics and analysis, the unit carbon trading price is set as $P_n = P_i = (0.5, 1, 1.5)$. The equilibrium solutions for the DLSCN are presented in Table 3.

Table 3. Impact of carbon trading price change.

Variables	P_n = 0.5			P_n = 1			P_n = 1.5		
	P_i = 0.5	P_i = 1	P_i = 1.5	P_i = 0.5	P_i = 1	P_i = 1.5	P_i = 0.5	P_i = 1	P_i = 1.5
$(q_{ni})_{2\times2}$	1.8434	1.8403	1.8380	1.8397	1.8365	1.8343	1.8390	1.8358	1.8336
$(q_i)_{1\times2}$	3.6868	3.6806	3.6760	3.6794	3.6730	3.6686	3.6780	3.6716	3.6672
$(w_{ni})_{2\times2}$	5.8744	5.8663	5.8604	5.9096	5.9015	5.8957	5.9160	5.908	5.9023
$(w_{ij})_{2\times2}$	11.4010	11.4560	11.4940	11.4280	11.4820	11.5210	11.4320	11.4870	11.5260
$(p_j)_{1\times2}$	23.6130	23.6940	23.7510	23.6520	23.7330	23.790	23.6620	23.7430	23.8000
$(p_i^e)_{1\times2}$	11.9780	11.9630	11.9530	12.0030	11.9880	11.9770	12.0070	11.9920	11.9820
$(r_n)_{1\times2}$	0.1843	0.1840	0.1838	0.3679	0.3673	0.3668	0.5517	0.5507	0.5500
$(r_i)_{1\times2}$	0.1011	0.2019	0.30103	0.1009	0.2009	0.3004	0.1008	0.2008	0.3003
$(\pi_n)_{1\times2}$	7.9420	7.9165	7.8984	8.3103	8.2852	8.2673	8.6361	8.6115	8.5940
$(\pi_i)_{1\times2}$	7.7426	8.1926	8.6284	7.7139	8.1640	8.6000	7.7078	8.1579	8.5938
$(G(\pi_j))_{1\times2}$	10.5180	10.4790	10.4520	10.5010	10.4620	10.4350	10.5100	10.4710	10.4440

From Table 3, the following observations can be made:

Both suppliers' and manufacturers' carbon reduction rates and profits are positively proportional to their own carbon trading prices and inversely proportional to upstream or downstream carbon trading prices, and their own carbon reduction rates and profits are more influenced by their own carbon trading prices. This suggests that when the suppliers' or manufacturers' carbon trading price rises, suppliers or manufacturers will choose to obtain more surplus carbon allowances by increasing their own carbon reduction rate, hoping to make more profits by selling carbon allowances at a high price, rather than incurring a large amount of carbon costs by overproducing and buying carbon allowances at a high price, which will reduce their profits.

The number of raw materials supplied by suppliers and the number of products produced by manufacturers are inversely proportional to the carbon trading prices, P_n and P_i, for both suppliers and manufacturers. This suggests that when the carbon trading prices of suppliers and manufacturers increase, both suppliers and manufacturers will choose to reduce their production to prevent high carbon expenses as a result of surpassing their carbon allowances, and they can also sell their remaining carbon allowances at a profit.

When the supplier carbon trading price, P_n, is constant, the supplier's wholesale price is inversely proportional to the manufacturer's carbon trading price, P_i, while the manufacturer's wholesale price is proportional to P_i; when P_i is constant, the supplier's and the manufacturer's wholesale price are both proportional to P_n. This suggests that both supplier and manufacturer will raise their wholesale prices because of the increase in their own carbon trading prices: both supplier and manufacturer will seek to make a profit by increasing their wholesale prices when lowering their output; when the manufacturer's

wholesale price increases, for the retailer, the volume of products ordered by retailers from the manufacturer will decrease, and the retailer will seek to make more profits by increasing the selling price in the traditional retail channel, and the manufacturer will choose a price-reduction strategy to maximize profits from the increase in surplus products. At this point, the manufacturer's remaining products are sold through the online direct sales channel, and as the number of remaining products rises, the manufacturer will choose a price-reduction strategy to make more profit.

When the supplier's carbon trading price, P_n, remains constant, the retailer's expected utility is inversely proportional to the manufacturer's carbon trading price, P_i. When the manufacturer's carbon trading price, P_i, is constant, the retailer's expected utility initially increases and then decreases as the supplier's carbon trading price, P_n, rises. This indicates that retailers are also affected by carbon trading prices, with suppliers and manufacturers exerting distinct influences. Overall, however, higher carbon trading prices from suppliers and manufacturers lead to increased wholesale procurement costs and reduced product supply for retailers, lowering market demand and thereby diminishing the retailer's expected utility. These findings align with the conclusions of Zou et al. in their study on low-carbon supply chain equilibrium pricing mechanisms under carbon cap-and-trade policies, where they similarly observed that retailer profits are inversely proportional to the manufacturer's carbon trading price, P_i [17]. However, while their supply chain model did not incorporate suppliers, this paper extends their framework by constructing a four-tier SCN that includes suppliers and further investigates the impact of supplier carbon trading-price fluctuations on network equilibrium.

6. Managerial Insights

Based on the above numerical analysis from three different perspectives, we obtain management insights as follows:

First, retailers are more risk-averse, which favors DLSCN members' profit and CVaR. However, it seems that DLSCN members cannot reach a consensus on whether retailers pay more attention to expected profit or CVaR downside profit. Suppliers and manufacturers prefer their downstream retailers to pay more attention to CVaR downside profit, while retailers prefer expected profit, and at this time, members of the supply chain network need to negotiate to maximize the win-win situation of economic and environmental benefits of the DLSCN.

Second, as retailers become more risk-neutral and prioritize expected profits, consumers increasingly opt for the Internet direct sales channel for shopping, resulting in a decreased demand for traditional retail channels. Conversely, the change in sales price exhibits an opposite trend, yet during this period, the retailer's utility remains relatively low, indicating that the current sales price is inadequate to compensate for the loss due to the decline in demand. In other words, the dual-channel sales model exerts a significant impact on retailers, necessitating that they carefully consider whether the associated losses fall within their affordable range and select a manufacturer that better aligns with their needs.

Third, for suppliers or manufacturers who face greater difficulty in carbon emission reduction, they do not have to worry too much about carbon emission reduction due to the difficulty of carbon emission reduction, technical difficulties, and other issues resulting from the high cost of carbon emission reduction investment coefficient; however, in their own carbon emission reduction investment, they should always pay attention to the upstream or downstream carbon emission reduction investment, which will have an impact on their own profits. Because the suppliers and manufacturers of the carbon emission reduction rate are always negatively correlated with the suppliers and manufacturers of the carbon emission reduction investment cost coefficient, their own carbon emission reduction investment cost

coefficient has a greater impact on their own carbon emission reduction rate, which can effectively balance their overall carbon emission reduction costs; and their own carbon emission reduction investment cost coefficient increases will increase their own profits and retailer's expectations of the utility of the upstream or downstream carbon emission reduction investment situation on their own profits and the impact on their own profits and the retailer's expectations of the utility of the upstream or downstream carbon emission reduction investment situation. Their abatement investment situation has a different but smaller impact on their own profits. Therefore, even if suppliers or manufacturers think that it is difficult for enterprises to carry out carbon emission reduction, they should still make reasonable investment, and the investment will be more rewarding.

Fourth, to a certain extent, the government's carbon trading mechanism contributes to achieving both economic and environmental benefits. When the carbon trading prices of suppliers or manufacturers rise, they should simultaneously increase their own emission reduction rates and obtain more surplus carbon allowances, thereby earning more profits from the sale of these allowances rather than incurring high carbon costs due to overpurchasing. However, they also need to observe changes in upstream or downstream carbon trading prices to make rational choices. Nevertheless, when both carbon trading prices reach their peaks, the retailer's utility is relatively low, which is detrimental to the retailer. In other words, while the mechanism does help achieve dual economic and environmental benefits to a certain extent, further exploration is needed to better safeguard the interests of retailers.

7. Conclusions

This study establishes a four-layer DLSCN model composed of suppliers, manufacturers, retailers, and demand markets, based on a carbon trading mechanism. The utility function for risk-averse retailers was constructed using the mean-CVaR method, and a DLSCN equilibrium model was developed using variational inequalities. This model explores optimal pricing, optimal quantity, carbon trading decisions, and optimal profit or utility of a product in the DLSCN, and further identifies the conditions under which equilibrium is achieved for decision-makers at all levels and for the DLSCN as a whole. The projective contraction algorithm is used to solve it, and numerical analysis is conducted to assess the effects of changes in key parameters on the network equilibrium, thus providing insights for management. The following are the primary conclusions.

Under certain conditions, the DLSCN can enhance both economic and environmental benefits. Firstly, retailers should be more risk-averse and make rational trade-offs with all members in terms of weights. Secondly, suppliers and manufacturers should increase the cost coefficients of their own carbon emission reduction investments. Lastly, under the government's carbon trading mechanism, higher carbon trading prices for suppliers and manufacturers can increase their profits and carbon reduction rates, but considering the retailers' utility, the carbon trading prices should not be excessively high.

Future research could be expanded in the following three areas: First, the DLSCN will be enlarged to include various carbon policies, including progressive carbon taxation, carbon offsets, and carbon taxes, and various effects of these policies on DLSCN equilibrium will be investigated. Second, other behaviors, such as fairness concerns, should be incorporated into the model, and the research should talk about how risk avoidance and fairness concerns both affect the DLSCN equilibrium. Third, an effective contractual model for the DLSCN should be designed to raise the overall target profit of DLSCN and to achieve Pareto optimization for each member.

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curation, C.L.; writing—original draft preparation, H.W. and C.L.; writing—review and editing, H.W. and C.L.; visualization, C.L.; supervision, H.W.; project administration, H.W.; funding acquisition, H.W. and C.L. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Proof of Proposition 1. Based on Equation (1), the Hessian matrix concerning q_{ni} and r_n is derived as follows:

$$\begin{bmatrix} -\frac{\partial^2 c_{ni}(q_{ni})}{\partial^2 q_{ni}} - \frac{\partial^2 f_n(Q_1)}{\partial^2 q_{ni}} & 0 \\ 0 & -T_n \end{bmatrix}.$$

The cost functions are assumed to be convex, and T_n represents the carbon emission reduction investment cost coefficient for supplier n, with $T_n > 0$. Consequently, the Hessian matrix is negative definite. Therefore, the profit function of supplier n is jointly concave with respect to the decision variables, q_{ni} and r_n. □

Proof of Proposition 2. According to Equation (4), the Hessian matrix with respect to q_i, q_{ni}, q_{ij}, q_{id}, and r_i is given by

$$\begin{bmatrix} -\frac{\partial^2 f_i(q)}{\partial^2 q_i} & & & & \\ & 0 & & & \\ & & -\frac{\partial^2 c_{ij}(q_{ij})}{\partial^2 q_{ij}} & & \\ & & & -\frac{\partial^2 f_i(Q_3)}{\partial^2 q_{id}} & \\ & & & & -T_i \end{bmatrix}.$$

Since it is assumed that the cost functions are convex and T_i is the carbon abatement investment cost coefficient for supplier i with $T_i > 0$, the Hessian matrix is negative semi-definite. Therefore, the profit function of manufacturer i with respect to the decision variables q_i, q_{ni}, q_{ij}, q_{id}, and r_i is a jointly concave function. □

Proof of Theorem 2. By taking the negative of the profit maximization problem (4) and combining it with constraints (5) and their Lagrange multipliers, ε_i and α_i, the Lagrangian function for each entity is formed as follows:

$$\begin{aligned} L_i = &-\sum_{j=1}^{J} w_{ij}q_{ij} + \sum_{n=1}^{N} w_{ni}q_{ni} - P_i\left[A_i - e_i(1-r_i)\sum_{j=1}^{J} q_{ij}\right] + \sum_{j=1}^{J} c_{ij}(q_{ij}) + f_i(q) - p_i^e\sum_{d=1}^{D} q_{id}^e + f_i(Q_3) + \frac{1}{2}T_i{r_i}^2 \\ &+\varepsilon_i\left(\sum_{j=1}^{J} q_{ij} + \sum_{d=1}^{D} q_{id} - q_i\right) + \alpha_i\left(q_i - \sum_{n=1}^{N} q_{ni}\right) \end{aligned} \tag{A1}$$

From Equation (A1), the first-order partial derivatives of the Lagrangian function, L_i, with respect to $q_i, q_{ni}, q_{ij}, q_{id}, r_i, \varepsilon_i$, and α_i, are respectively, $\frac{\partial L_i}{\partial q_i} = \frac{\partial f_i(q)}{\partial q_i} - \varepsilon_i + \alpha_i$, $\frac{\partial L_i}{\partial q_{ni}} = w_{ni} - \alpha_i$, $\frac{\partial L_i}{\partial q_{ij}} = -w_{ij} + P_i e_i(1 - r_i) + \frac{\partial c_{ij}(q_{ij})}{\partial q_{ij}} + \varepsilon_i$, $\frac{\partial L_i}{\partial q_{id}} = -p_i^e + \frac{\partial f_i(Q_3)}{\partial q_{id}} + \varepsilon_i$, $\frac{\partial L_i}{\partial r_i} = -P_i e_i \sum_{j=1}^{J} q_{ij} + T_i r_i$, $\frac{\partial L_i}{\partial \varepsilon_i} = \sum_{j=1}^{J} q_{ij} + \sum_{d=1}^{D} q_{id} - q_i$, $\frac{\partial L_i}{\partial \alpha_i} = q_i - \sum_{n=1}^{N} q_{ni}$. Based on the first-order partial derivatives, the complementary slackness conditions are derived as shown in Equation (6). □

Proof of Proposition 3. According to Equation (12), the second-order derivative of $G(\pi_j)$ with respect to q_{ij} is given by $-\lambda \frac{\partial^2 f_j(Q_2)}{\partial^2 q_{ij}} - (1 - \lambda)\frac{1}{\eta}\frac{\partial^2 f_j(Q_2)}{\partial^2 q_{ij}}$. Since it is assumed that the cost function is convex, and $\lambda \in [0, 1], \eta \in (0, 1)$, the Hessian matrix is negative definite, which implies that the profit function of retailer j with respect to the decision variable q_{ij} is a concave function. □

Proof of Theorem 5. By summing Equations (3), (6), (14), and (17), Theorem 5 can be obtained. □

Proof of Theorem 1, Theorem 3, and Theorem 4. The proofs of these three theorems can refer to the proof of Theorem 2. □

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