

Article

Sectoral and Regional Allocation of Initial Water Rights of Reservoirs: A Two-Dimensional Method Based on Matter-Element Extension Theory

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Abstract

As an important surface water source, the rational water rights allocation of reservoirs plays a crucial role in alleviating the contradiction between water supply and demand in surrounding areas. However, the theoretical framework for water rights allocation mostly focuses on the watershed scale, which is different from water rights allocation of reservoirs. Research on the water rights allocation of reservoirs is relatively scarce and still at a preliminary stage. Therefore, this study developed a two-dimensional method for sectoral and regional allocation of initial water rights of a reservoir, which was applied to Nishan Reservoir in northern China. The reservoir's utilizable storage capacity, total initial water rights, and water rights of various sectors were determined using the water balance principle and chronological method. Subsequently, based on the constructed system of indices and assessment criteria for the regional allocation of initial water rights, an allocation model for the reservoirs' initial water rights was established using the matter-element extension theory to subdivide the sectoral water rights allocations into regional initial water rights allocations. The results show that the total initial water rights of Nishan Reservoir are 20.64 million m³, with the water rights allocations for agricultural irrigation, industry/domestic use, and ecological needs at 4.24, 10.4, and 6.00 million m³, respectively. The 6.00 million m³ allocation for ecological water use is solely managed by the prefecture-level administration of Jining City. The remaining allocations of the reservoir's initial water rights are 9.67 million m³ for Qufu City, 1.63 million m³ for Sishui County, and 3.34 million m³ for Zoucheng City. This allocation scheme has been accepted by all stakeholders of the Nishan Reservoir's water rights. The method proposed in this study can provide support for the construction of a reservoir water rights allocation system. However, it has a limitation in that it fails to consider the robustness of the allocation scheme and the dynamic adjustment mechanism under future variable conditions, such as extreme hydrological scenarios and changes in water demand. This serves as a direction for future research.

Keywords: China; initial water rights; matter-element extension theory; continuous regulation computations; water rights allocation indices; Nishan Reservoir



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1. Introduction

Water is the source of life and the foundation of vital ecosystems such as wetlands, rivers, and lakes, making it indispensable for socioeconomic development [1–3]. The allocation and management of water resources are pivotal to sustainable water resources utilization [4,5]. The allocation of water rights is the institutional foundation for water resources allocation and also an effective and powerful tool for water resources management [6–8]. Research on water rights allocation in different dimensions (e.g., watershed, regional, and sectoral) is important for sustainable development and conservation of water resources [9,10]. Reservoirs are vital surface water sources, supporting domestic, industrial, agricultural, and ecological water needs, especially in arid and semi-arid regions of northern China. Due to the scarcity of water resources, the imbalance between water supply and demand in the areas surrounding reservoirs has become increasingly prominent. Disputes and scrambles for water often occur between the upper and lower reaches of reservoirs, the left and right banks, different administrative regions, and different sectors, which have seriously hindered the healthy and harmonious socioeconomic development as well as the protection of ecological environments [4,11]. Superficially, these issues seem to stem from a mismatch between the available water supply from reservoirs and the growing water demands of the population. Upon further inspection, however, it becomes clear that these problems are actually caused by inadequacies in the allocation of reservoir water rights. With the continuous socioeconomic development of arid and semi-arid regions in northern China, the water supply targets of reservoirs have shifted from the original single agricultural water supply to a diversified pattern covering industrial, ecological, and agricultural water supply. In addition, the area being served by each reservoir has expanded from the direct jurisdiction of the dam to the downstream and upstream areas of the reservoir. Consequently, the allocation of reservoir water is now an issue that involves a multitude of stakeholders. To promote the fair and efficient utilization of regional water resources, it is necessary to research and develop methods for the initial water rights allocation of reservoirs in multi-stakeholder scenarios [12].

The allocation of initial water rights is an extremely complex decision-making process involving multiple stakeholders (e.g., governments, water users, and ecosystems) [13] and requires consideration of multiple factors (e.g., economic benefits, market dynamics, societal needs, management requirements, and decision-making factors) [14,15]. Many studies have been conducted on the theoretical and practical allocation of water rights in river basins. To summarize, water rights allocation methodologies may generally be categorized as marginal cost-based methods [14,16], public administration methods [8,17–19], market-based methods [7,20,21], and water-demand-based methods [22–24]. Each method has the following strengths and limitations: (1) Marginal cost-based methods use the marginal cost of water resources as a basis for the reallocation of water rights. Although these methods effectively prevent the overuse of water resources, they face difficulties in determining the pricing of marginal costs in practical applications. An example of this approach is the method developed by Kelman et al. [16], who proposed an allocation model based on the opportunity cost of water for different users and analyzed the linear and economic benefit-based allocation schemes. (2) Public administration methods pertain to the use of governmental means to implement water rights allocation systems, including the riparian rights and prior appropriation systems [25,26]. Their key advantage lies in ensuring fairness: they prioritize water supply to water-scarce areas while meeting environmental needs, without relying on pricing mechanisms. However, these methods neglect resource scarcity and require substantial public investment, which may result in water resource wastage. A representative example is Gopalakrishnan's method [25], which developed a water rights allocation framework based on the prior appropriation doctrine.

(3) Market-based methods facilitate the transfer of water resources from low-efficiency to high-efficiency users through water trading in water markets, thereby enhancing water use efficiency. The allocation of resources through water markets is efficient and effective, enabling water rights to be transferred from one stakeholder to another. However, this type of method tends to prioritize economic benefits over the needs of marginalized groups and the general public. For instance, Howe [27] constructed an allocation method for water rights based on water markets. In another study, Zhao et al. [20] constructed an agent-based modeling framework incorporating variables specific to administrative and market-based water allocation systems, including water trade, trading prices, water-use violations, and penalties/subsidies, to compare water-user behaviors under these two systems. (4) The allocation of water rights based on water demand involves constructing a water allocation index system through comprehensive analysis of stakeholder relationships [28–30], followed by deriving precise water rights allocation solutions via mathematical modeling [24,31–34]. This type of method can be flexibly applied to meet the water demands of different regions and sectors, but it requires a transparent negotiation-based allocation mechanism, which is challenging to establish, and the coordination of multiple interests. Examples include the hierarchical or graded water rights allocation models developed by Wang et al. [31] and Wu et al. [33]. In summary, existing studies on water rights allocation have generally focused on the watershed scale [9,29,33,35–37]. Typically, the water rights of river basins are first allocated across regions and then subdivided among sectors within each region. The methods for initial water rights allocation of reservoirs differ from those for river basins. Moreover, research on the water rights allocation of reservoirs is relatively scarce and still at a preliminary stage [38]. Rational water rights allocation of reservoirs can effectively alleviate the contradiction between water supply and demand in surrounding areas; therefore, the rationality of water rights allocation schemes of reservoirs is of crucial importance. Each of the four aforementioned water rights allocation methodologies has its own strengths and limitations. Among them, the water-demand-based methods are the most widely adopted, owing to their practicality and user-friendliness. The core of these methods lies in the selection of a decision-making model that can derive a water rights allocation scheme.

To address the shortcomings of existing research on water rights allocation of reservoirs, this study developed a two-dimensional method for sectoral and regional allocation of initial water rights of a reservoir, which was applied to Nishan Reservoir in northern China. Firstly, using the long-series chronological method, the initial allocatable capacity of the reservoir and the sectoral initial water rights corresponding to the required water supply reliability are determined by continuous regulation computations [39,40]. Then, a system of indices and assessment criteria for the regional initial water rights allocation is established. This is followed by applying matter-element extension theory to construct a two-dimensional allocation model for the sectoral and regional initial water rights. Finally, on the basis of the sectoral water rights allocation, the matter-element extension model is used to allocate the regional initial water rights. Matter-element extension theory, proposed by Cai [41] in 1983, provides a framework for solving complex multi-factor decision problems, with applications in control systems, information technology, and artificial intelligence. However, its use for the initial water rights allocation of reservoirs remains unreported, and a method validated through practical application would be particularly noteworthy.

The remainder of this paper is organized as follows: Section 2.1 introduces the overview of the study area, Nishan Reservoir, as well as the stakeholders involved in water rights allocation and their water demand; Section 2.2 outlines the steps of the two-dimensional allocation of initial water rights of reservoirs; Section 3 presents the re-

sults of water rights allocation for Nishan Reservoir; Section 4 discusses the rationality of the water rights allocation results for Nishan Reservoir; and Section 5 provides a summary of the conclusions.

2. Materials and Methods

2.1. Study Area

2.1.1. Overview of Nishan Reservoir

Nishan Reservoir, a large reservoir in northern China ($117^{\circ}11'11.98''$ E, $35^{\circ}28'39.76''$ N), is located in Jining City, Shandong Province. Situated in the upper-middle reaches of the Yi River basin, it was primarily built for flood control, while also serving functions such as agricultural irrigation, industrial and domestic water supply, and maintaining downstream river ecosystems. The dam controls a watershed area of 687.14 km^2 , with the dead water level of 116.19 m (corresponding to the dead storage capacity of 8.31 million m^3), the normal storage level of 124.59 m (corresponding to the utilizable storage capacity of 61.25 million m^3), and the check flood level of 127.91 m (corresponding to the total storage capacity of 112.8 million m^3). The Yi River basin flows through Qufu City, Sishui County, and Zoucheng City, accounting for 343.96 km^2 (50.1%), 82.04 km^2 (11.9%), and 261.14 km^2 (38.0%) of Nishan Reservoir's controlled watershed area, respectively. Based on the boundaries of the reservoir-controlled area, the segments of the Yi River basin within Qufu City and Zoucheng City can be divided into upstream and downstream regions, whereas the entire segment within Sishui County lies upstream of the reservoir. The downstream region of Qufu City encompasses 1444 km^2 of farmland, which relies on surface water from Nishan Reservoir for irrigation. The location of Nishan Reservoir is shown in Figure 1. Figure 1 is created by GIS based on the Digital Elevation Model, which is obtained free of charge from the website <https://www.gscloud.cn/> (22 September 2025).

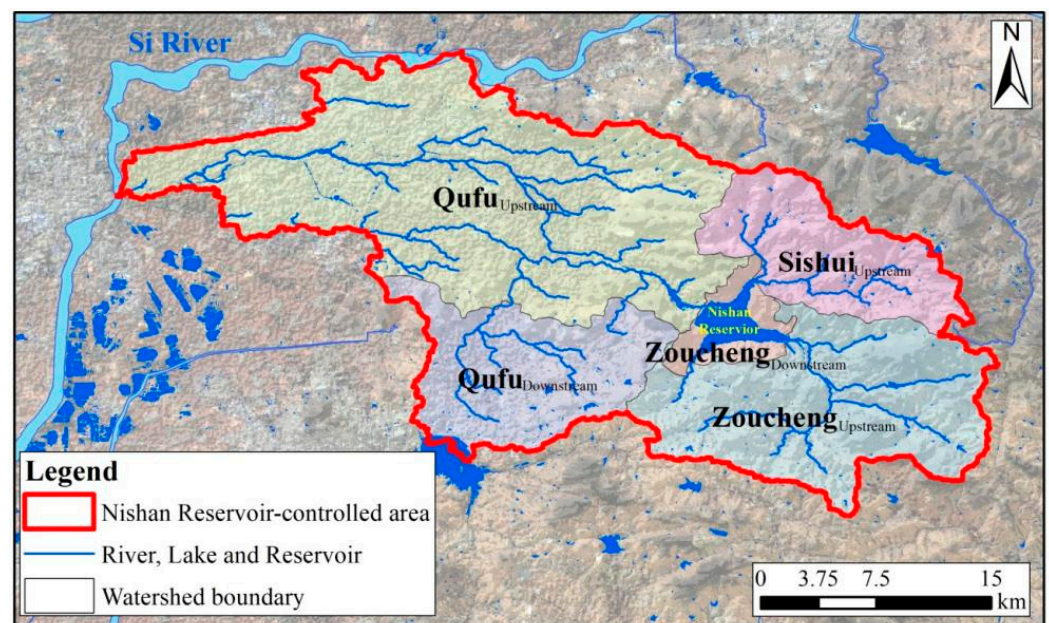


Figure 1. Location of Nishan Reservoir.

2.1.2. Stakeholders and Water-Demand Analysis

The current water users of Nishan Reservoir include the reservoir irrigation district (1444 km^2 of farmland) and a power plant in Qufu City, with an annual water consumption of 3.7 million m^3 . In recent years, the water demand in the area surrounding the reservoir has increased due to socioeconomic development. Qufu City has proposed utilizing the reservoir's water to partially meet its domestic water demand and increase the water

supply to its power stations and industries. Sishui County and Zoucheng City have also filed a claim for a portion of Nishan Reservoir’s surface water to meet their domestic and industrial water demands. In response, the administration of the reservoir irrigation district and farmers’ association have emphasized that while water supply for domestic and industrial use can be increased, this must not compromise farmers’ irrigation rights. Furthermore, since the downstream reaches of the reservoir are a scenic area, the downstream management authority has also stated that sufficient instream flow must be maintained to preserve ecological functions and scenic values. Therefore, the allocation of Nishan Reservoir’s water rights involves multiple stakeholders, including industrial users, urban and rural residential users, the water user association of the agricultural irrigation district, the reservoir management authority, and river ecological conservation representatives (encompassing public spokespersons and government agencies). Each stakeholder represents distinct water user groups with varying interests. The relationships among the stakeholders and their water demands are detailed in Table 1.

Table 1. An overview of Nishan Reservoir’s stakeholders and their water demands.

Region		Stakeholders	Water Demands
Qufu City	Upstream	Industrial users, urban and rural residential users, reservoir management authorities, and government	Industrial and domestic
	Downstream	Industrial users, urban and rural residential users, water user associations of irrigation districts, reservoir management authorities, ecosystem conservation spokespersons, and government	Industrial, domestic, agricultural, and ecological
Sishui County	Upstream	Industrial users, urban and rural residential users, reservoir management authorities, and government	Industrial and domestic
Zoucheng City	Upstream	Industrial users, urban and rural residential users, reservoir management authorities, and government	Industrial and domestic
	Downstream	Industrial users, urban and rural residential users, reservoir management authorities, ecosystem conservation spokespersons, and government	Industrial, domestic, and ecological
Jining City	Downstream reaches	Ecosystem conservation spokespersons and government	Ecological

Considering the stakeholders and water demands of each region, the sectoral and regional water rights of Nishan Reservoir should be allocated to the administrative districts of Qufu City, Sishui County, and Zoucheng City. Meanwhile, the ecological water rights pertaining to the downstream reaches of Nishan Reservoir shall be managed by the prefectural government of Jining City.

2.2. Methods

The two-dimensional allocation of initial water rights of reservoirs comprises two components: the allocation of sectoral water rights and that of regional initial water rights. For the allocation of sectoral water rights, the available water supply for each

sector is determined to meet its designed water supply reliability, based on the multi-year continuous regulation of Nishan Reservoir. The available water supply then constitutes the sector's allocated water rights. As for the allocation of regional initial water rights, it involves subdividing the sectoral water rights allocations into different regions.

2.2.1. Total Initial Water Rights of a Reservoir

The initial water rights of a reservoir refer to the volume of available water supply to users following reservoir regulation. The initial water rights capacity of a reservoir is typically taken from its utilizable storage capacity. In other words, the utilizable storage capacity of a reservoir is equivalent to its allocatable initial water rights. The main characteristic water levels and characteristic storage capacities of the reservoir are illustrated in Figure 2.

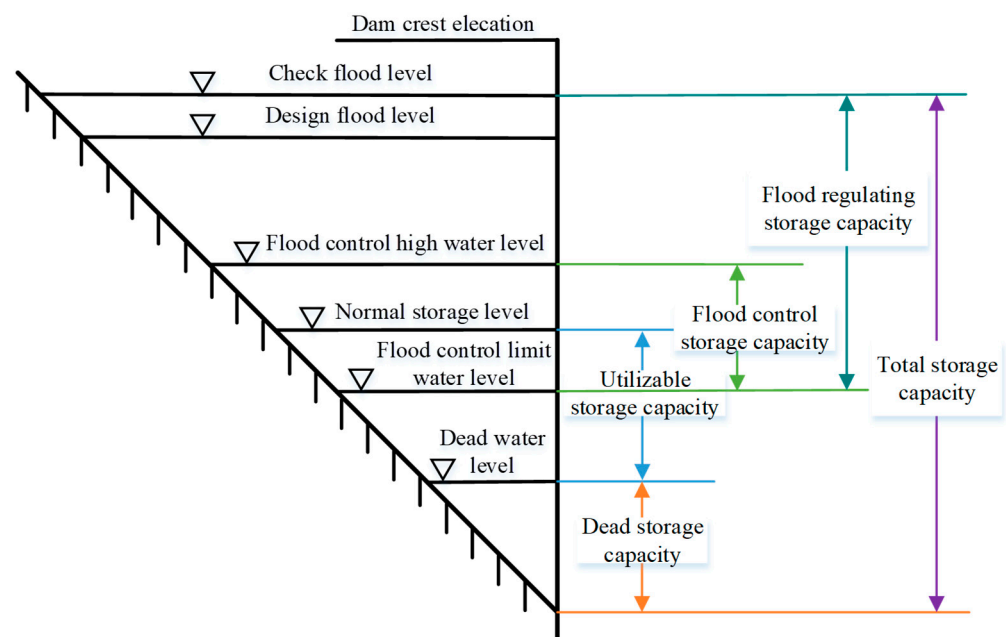


Figure 2. Schematic diagram of the storage spaces of a reservoir's initial water rights.

2.2.2. Method for Sectoral Allocation of Initial Water Rights

Sectoral initial water rights are determined by water supply tasks of the reservoir, and the designed water supply reliability of each sector. Since reservoirs with only one water supply task cannot allocate their water rights to multiple sectors, only those with multiple water supply tasks are considered. Therefore, the sectoral initial water rights of a reservoir can be expressed as follows:

$$WR = f(ID, P) = \sum_{i=1}^N WR_i \quad (1)$$

where WR is the total initial water rights of a reservoir (in 10^4 m^3); ID is the type of water supply task; $P = 1 - m/(n + 1)$ is the water supply reliability of each sector; i is the i -th water supply task; N is the number of water supply tasks; WR_i is the initial water right for the i -th water supply task; n is the number of hydrological years; and m is the number of years with water supply shortfall.

For large multi-year regulated reservoirs such as Nishan Reservoir, their initial water rights should be calculated through long-series continuous regulation computations. These calculation methods are diverse and can be categorized into chronological methods and statistical methods. The former includes trial-and-error, tabular, and graphical methods. Given that Nishan Reservoir has a long series of measured data from 1956 to 2014, including

monthly inflow to the reservoir, monthly storage capacity, monthly evaporation, and monthly water consumption of various water-use sectors, the trial-and-error method was adopted to iteratively calculate initial water rights of Nishan Reservoir.

The procedures of the trial-and-error method are as follows:

(1) Collect the reservoir's design parameters (including characteristic storage capacity, water level-storage capacity relationship, etc.), long-series measured hydrological data (including monthly inflow to the reservoir, monthly storage capacity, and monthly evaporation), monthly water consumption of various water-use sectors, and water supply reliability of each water-use sector.

(2) Based on the water balance principle, the monthly water balance iterative calculation is carried out starting from the time when the reservoir is full or empty. The water balance equation for the continuous regulation computations of a reservoir is expressed as follows:

$$\Delta W = V_{j+1} - V_j = W_{i,j} - W_{s,j} - W_{e,j} - W_{l,j} \quad (2)$$

where ΔW is the change in water storage (in 10^4 m^3); V_j is the storage volume at time j (in 10^4 m^3); $W_{i,j}$ is the inflow of the reservoir (in 10^4 m^3); $W_{s,j}$ is the water supply volume from the reservoir (in 10^4 m^3); $W_{e,j}$ is the evaporation loss of the reservoir (in 10^4 m^3); and $W_{l,j}$ is the leakage loss of the reservoir (in 10^4 m^3).

(3) After completing the iterative calculation for the entire hydrological year series, the number of years with water supply shortfall for each sector is counted to determine the water supply reliability (P) of each sector. Then, the following two critical conditions are checked:

① Whether the initial storage capacity of the first year was equal to the end-of-year storage capacity of the last year.

② Whether the water supply reliability of each sector met its design requirements.

If any one of the conditions is not met, adjust the water consumption of each water-use sector and the water release from the reservoir, and repeat the calculation process of the previous step until the above two conditions are satisfied.

For Nishan Reservoir, where long-term high-quality measured data are available, the trial-and-error method's ability to reflect real hydrological dynamics ensures that the water supply processes and storage changes calculated in the regulation computations are consistent with historical operational realities, thereby enhancing the reliability of the subsequent water rights allocation results. The trial-and-error process follows a standardized cycle (initialization → regulation computation → validation → adjustment) with quantifiable convergence criteria, thus featuring reproducibility.

2.2.3. Allocation Method of Regional Initial Water Rights

The matter-element extension model [41] applied to allocate the regional initial water rights of a reservoir can be summarized as follows: (1) The regions in need of initial water rights allocation are identified, and then a system of indices and criteria for allocating initial water rights at the region level is established. (2) The classic domains and matter-elements to be assessed of the water rights allocation matter-elements are determined, their closeness degrees are calculated, and thereby the initial water rights allocation weights of regions are determined. (3) Finally, the initial water rights allocation weights of regions are substituted into the equation to allocate the sectoral water rights, thus ensuring that the sectoral water rights are reasonably allocated to each region.

A System of Indices and Assessment Criteria for the Regional Allocation of Initial Water Rights

Reservoirs are water conservancy structures functioning to control floods and regulate flows. The allocation of their initial water rights, in essence, involves allocating their available water supply among different regions, sectors, and users. Therefore, such allocations must depend on the reservoir's water inflow, storage, conveyance, and allocation processes. By considering both engineering factors of the reservoir and water usage status and demands of the relevant region, there are four factors determining the allocation of water rights:

- (1) Hydrological factors. These include precipitation, catchment area, underlying surfaces, and water quality of the reservoir, all of which directly influence the volume of inflows and the water quality of the reservoir.
- (2) Construction and management factors. These include utilizable storage capacity, normal storage level, submerged area, construction costs, factors related to the construction of the reservoir's water storage and conveyance systems as well as water supply pipelines, and the management of reservoir releases under both normal operating conditions and flood control scenarios. These factors affect the available water supply of the reservoir, as well as its water supply costs, operational costs, and management costs.
- (3) Supply and demand factors. These include the current water demands of the reservoir's existing water users (for agricultural irrigation, industrial use, domestic consumption, and ecological purposes) and the water demand in the planned water supply regions of the reservoir. These factors determine the proportion of the available water supply allocated to each user.
- (4) Socioeconomic factors. These include the level of socioeconomic development, population size, water price, and water use efficiency in the upstream and downstream regions of the reservoir. Such factors dictate the socioeconomic benefits derived from the water supply by the reservoir.

An overly simplistic approach to water rights allocation can lead to imbalances. For example, if allocations are based solely on catchment area, upstream regions would receive a large allocation of water, while downstream regions would receive very little, despite most traditional water users of the reservoir being located downstream. This is highly inequitable for downstream regions, which are often tasked with both water supply tasks and flood control responsibilities. Similarly, allocating water purely based on socioeconomic benefit would likely prioritize industrial use at the expense of agricultural and ecological water demands. Therefore, the allocation of water rights must balance the needs of multiple stakeholders and take all relevant factors into comprehensive consideration.

An index system for water rights allocation plays a crucial role in allocating initial water rights of a reservoir, and it should be representative, systematic, comparable, universal, and concise. Consequently, the selected indices ought to be widely applicable, objective, comprehensive, and recognizable. However, there are no existing literature findings that can be directly adopted for this purpose. After comprehensively analyzing the aforementioned factors influencing the allocation of initial water rights, the specific conditions of the study area, and currently available indices for the allocation of watershed or regional water rights [32,34], 10 sensitive indices were selected from the four factors determining the allocation of water rights discussed earlier to construct an index system for allocating initial water rights (Table 2). Through the establishment of this index system, the available water supply of the reservoir can be allocated according to the weight assigned to each index.

Table 2. Index system for the allocation of a reservoir’s initial water rights.

Factors	Indices	
	Names	Numbers
Hydrological	Catchment area	C ₁
	Annual runoff depth	C ₂
Supply and demand	Degree of water shortage with 95% water supply reliability	C ₃
	Water consumption for each 10,000 yuan in industrial added value	C ₄
	Water consumption for domestic water use	C ₅
Construction and management	Construction and management costs of the reservoir	C ₆
	Matching rate of the water supply pipelines	C ₇
Socioeconomic	Productivity per cubic meter of water	C ₈
	Cost of domestic water supply	C ₉
	Cost of industrial water supply	C ₁₀

It should be noted that the index system can be optimized in accordance with the specific requirements of the water rights allocation process. For example, if there are no objections to the water rights of the agricultural sector, the corresponding indices may be excluded from the analysis of sectoral water rights allocations—given that such allocations would then only apply to other sectors.

Given the absence of a unified standard for classifying these indices, three priority levels—Grades I, II, and III, in descending order of priority—have been established to differentiate regions. Based on the Jining Statistical Yearbook and the Jining Water Resources Bulletin, the values of 10 indices for five regions—Qufu_{Upstream}, Qufu_{Downstream}, Sishui_{Upstream}, Zoucheng_{Upstream}, and Zoucheng_{Downstream}—for the period 2010–2014 were calculated. Using an empirical cumulative frequency analysis of each index, the 25th and 75th percentiles were set as the classification thresholds (Table 3).

Table 3. Grading criteria for the indices.

Indices	Units	Priority Levels		
		I	II	III
C ₁	km ²	>250	250–60	<60
C ₂	mm	>200	200–100	<100
C ₃	%	>20	20–9	<9
C ₄	m ³ /10 ⁴ CNY	<8.5	8.5–15	>15
C ₅	10 ⁴ m ³	>500	500–80	<80
C ₆	10 million CNY	>1.5	1.5–0.1	<0.1
C ₇	%	>40	40–2.5	<2.5
C ₈	10 ⁴ CNY/m ³	>300	300–200	<200
C ₉	CNY/m ³	<2.0	2.0–3.0	>3.0
C ₁₀	CNY/m ³	<0.4	0.4–0.8	>0.8

Matter-Element Extension Theory

Proposed by Chinese scholar Cai Wen [41], the matter-element extension theory centers on “matter-element” (a basic unit describing things, including object, characteristic, and value). It resolves contradictory problems via extension transformation, converting abstract contradictions into quantifiable matter-element issues. Based on the matter-element extension theory, the matter-elements of water rights allocation, classic domain, matter-elements to be assessed, closeness, and weights for the allocation of regional initial water rights are established as follows [42]:

- (1) Matter-elements of water rights allocation

In matter-element theory, all matter-elements are described by a (N, C, V) triad, where N is the matter, C is the characteristic, and V is the measure of N with respect to C . This can be expressed as follows:

$$R = (N, C, V) \quad (3)$$

where R is the matter-element of water rights allocation; N is the water right to be allocated or is allocatable; C is the index influencing the allocation of water rights; and V is the specified range of index values for N .

(2) Classic domain

In extension theory, the classic domain is a core concept used to describe the range of values that a characteristic of a matter-element can take within a specific, well-defined category of objects. Formally, it is typically expressed as a matter-element structure:

$$R_0 = \begin{bmatrix} N & N_1 & N_2 & \dots & N_m \\ C & V_1 & V_2 & \dots & V_m \end{bmatrix} = \begin{bmatrix} N & N_1 & N_2 & \dots & N_m \\ C_1 & \langle a_{11}, b_{11} \rangle & \langle a_{12}, b_{12} \rangle & \dots & \langle a_{1m}, b_{1m} \rangle \\ C_2 & \langle a_{21}, b_{21} \rangle & \langle a_{22}, b_{22} \rangle & \dots & \langle a_{2m}, b_{2m} \rangle \\ \dots & \dots & \dots & \dots & \dots \\ C_n & \langle a_{n1}, b_{n1} \rangle & \langle a_{n2}, b_{n2} \rangle & \dots & \langle a_{nm}, b_{nm} \rangle \end{bmatrix} \quad (4)$$

where R_0 represents the m matter-elements with identical characteristics; $N_j (j = 1, 2, \dots, m)$ is the j -th priority level for water rights allocation; and $V_{ij} = (a_{ij}, b_{ij})$ is the value range of the i -th index within the j -th grade, i.e., the classic domain.

(3) Matter-elements to be assessed

In the extension theory, the matter-element to be assessed refers to a specific matter-element that needs to be assessed or analyzed within a given problem context. It is a fundamental concept used to characterize the object of evaluation. Specifically, the matter-element is compared or matched against reference matter-elements (such as classic domains) to determine its suitability, classification, or performance in relation to predefined criteria. For example, in the allocation of water rights, the matter-element to be assessed could represent a particular region, along with its key indices and their values, which are then analyzed to determine its priority. The matter-element to be assessed is expressed as follows:

$$R_k = (N_k, C, V_k) = \begin{bmatrix} N & C_1 & V_{k1} \\ & C_2 & V_{k2} \\ & \dots & \dots \\ & C_n & V_{kn} \end{bmatrix} \quad (5)$$

where N_k is the k -th region to be assessed; $C_i (i = 1, 2, \dots, n)$ is the i -th index affecting the allocation of water rights; and V_{ki} is the value of the i -th index corresponding to the k -th region to be assessed.

(4) Calculation of closeness

It has been proposed that asymmetric closeness could replace the maximum subordination principle [43]. More specifically, it was proposed that the concept of distance in real analysis can be generally applied to describe the distance between points and regions, as formulated by the following equation:

$$\rho(V_{ki}, V_{ij}) = |V_{ki} - (a_{ij} + b_{ij})/2| - (b_{ij} - a_{ij})/2 \quad (6)$$

Suppose the weight of the index C_i is β_i , and let $K_{kj}(V_{ki}) = \rho(V_{ki}, V_{ij})$. Closeness can then be expressed as follows:

$$K_{kj}(N_k) = 1 - \frac{1}{n(n+1)} \sum_{i=1}^n \beta_i K_{kj}(V_{ki}), (j = 1, 2, \dots, m) \quad (7)$$

where $K_{kj}(N_k)$ is the closeness of the k -th region to be assessed to grade j . The weight of the index, β_i , is calculated via the Analytic Hierarchy Process (AHP).

In the course of these calculations, it is possible to obtain closeness values less than 0. In such cases, the closeness value must be corrected as follows: if $K_{kj}(N_k) < 0$, set $K_{kj}(N_k) = 0$ and increase the closeness of the k -th region to be assessed to all other grades by $|K_{kj}(N_k)|$. This ensures that the corrected closeness values do not affect the grade-based differentiation or assessment of the regions. The corrected equation can be expressed as follows:

$$\begin{cases} K_{kj'}(N_k) = 0, \text{ if } K_{kj}(N_k) < 0, j' = j \\ K_{kj'}(N_k) = K_{kj'}(N_k) + |K_{kj}(N_k)|, j' \neq j \end{cases} \quad (8)$$

(5) Weights for the allocation of regional initial water rights

According to the principle of asymmetric closeness, the greater the overall closeness of a region to be assessed (N_k), the higher its water demand and the greater its weight for water rights allocation. To ensure that the closeness of each region to be assessed relative to each grade is given adequate consideration, a contribution coefficient (α_j) is assigned to each grade. The overall closeness of N_k is then given by:

$$K_k^T = \sum_j^m \alpha_j K_{kj}(N_k) \quad (9)$$

The overall closeness of each N_k may then be normalized to derive its weight for water rights allocation, denoted as ω_k :

$$\omega_k = K_k^T / \sum K_k^T \quad (10)$$

Multiplying these weights by the allocatable water capacity of the reservoir for each sector, WR_i , then yields the initial water rights allocation for each region, denoted as WR_i^k :

$$WR_i^k = WR_i \times \omega_k \quad (11)$$

In summary, a flowchart illustrating the two-dimensional allocation of initial water rights of reservoirs is shown in Figure 3. In Figure 3, V_0 and V_n are the storage capacity at the initial moment and the last moment (in 10^4 m^3), respectively; W_i , W_s , W_e , and W_l are the initial inflow of the reservoir (in 10^4 m^3), the water supply volume from the reservoir (in 10^4 m^3), the evaporation loss of the reservoir (in 10^4 m^3), and the leakage loss of the reservoir (in 10^4 m^3), respectively; R_a , $R_{i,d}$, and R_e are the water supply reliability for agricultural irrigation (in %), industrial/domestic water use (in %), and ecological water use (in %), respectively; WR_a , $WR_{i,d}$, and WR_e are the initial water rights for agricultural irrigation (in 10^4 m^3), industrial/domestic water use (in 10^4 m^3), and ecological water use (in 10^4 m^3), respectively; C_i is the i -th evaluation index; the meanings of other symbols are the same as above. The two-dimensional allocation of initial water rights of reservoirs develops a water rights allocation index system that comprehensively measures the interests of all stakeholders by incorporating hydrological, construction and management, supply and demand, and socioeconomic factors. Compared with marginal cost-based and market-based methods, which prioritize economic efficiency while neglecting equity, and public administration methods, which focus solely on water scarcity while ignoring economic

considerations, the method proposed in this paper achieves a more equitable water rights allocation that balances the interests of all stakeholders.

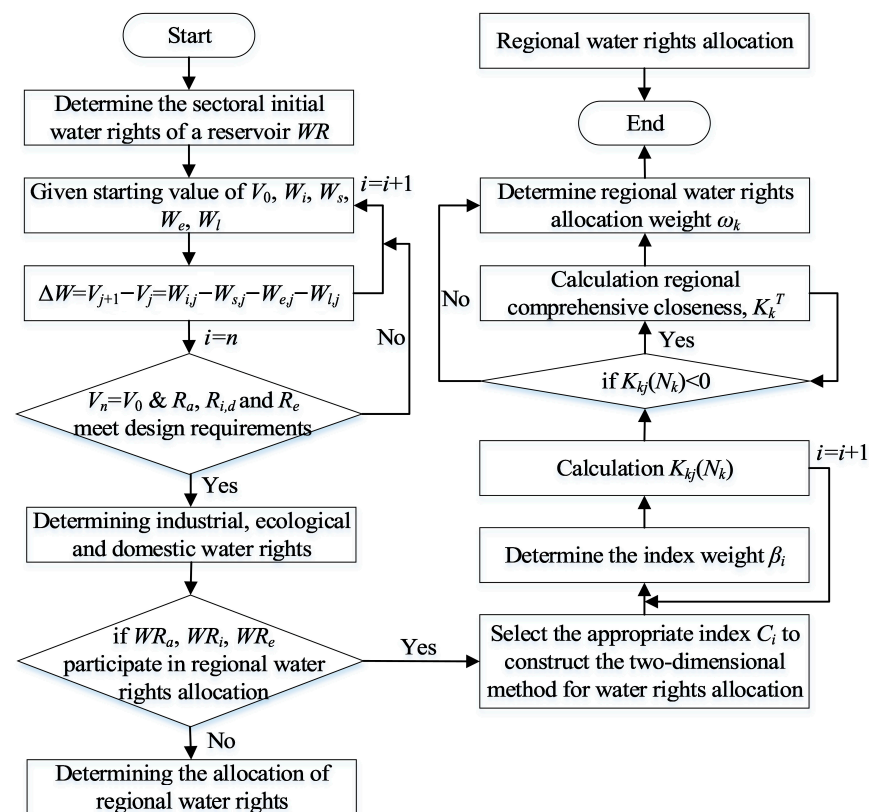


Figure 3. Flowchart of the two-dimensional allocation of initial water rights of reservoirs.

3. Results

3.1. Sectoral Allocation of the Initial Water Rights of Nishan Reservoir

The allocation of reservoir water rights is inherently based on the reservoir's inflows and available water supply. Hydrological data for Nishan Reservoir were reconstructed using measured runoff of the reservoir from 1956 to 2014, while the reservoir's available water supply was calculated using multi-year regulation computations based on the long-series chronological method. The reservoir's utilizable storage capacity was determined from the hydrological year time series. For each hydrological year, the water level and storage capacity in June were defined as the reservoir's dead water level and dead storage capacity, respectively. Using the water balance equation (Equation (2)), the utilizable storage capacity was iteratively calculated for each year until two conditions were satisfied: the initial storage capacity in the first year equaled the end-of-year storage capacity of the last year, and the water supply reliability of each sector met its design requirements. Additionally, the upper limit of the regulation range was treated as the reservoir's normal storage level (it is assumed that all water above this level is released). The water released from the reservoir was also maintained at a level sufficient to meet the ecological water demands of the downstream reaches. The designed minimum water supply reliability values for agricultural, ecological, and industrial/domestic water use were 50%, 70%, and 90%, respectively. The results of the multi-year regulation of Nishan Reservoir are shown in Table 4.

Table 4 illustrates that the annual average water use in the downstream reaches of Nishan Reservoir for ecological demands and agricultural irrigation is 6 million m^3 and 4.24 million m^3 , respectively. The annual average water supply from the Yi River basin for domestic and industrial use is 10.4 million m^3 . The total of these water usages

(20.64 million m³) may be treated as the available water supply of Nishan Reservoir for the allocation of sectoral initial water rights. It should be noted that the sectoral water allocations calculated based on the reservoir's inflows and regulation performance are contingent upon reliability. For instance, in drought years, industrial and domestic water demands will take priority over ecological and agricultural ones, given that the latter have lower reliability levels.

Table 4. Results of continuous regulation computations for Nishan Reservoir (in 10⁴ m³).

Hydrological Year	Inflow	Evaporation and Leakage Losses	Agricultural Irrigation		Industrial and Domestic Water Use		Ecological Water Use		Surplus Water Volume	End-of-Year Storage Capacity
			Water Supply Volume	Water Shortage Volume	Water Supply Volume	Water Shortage Volume	Water Supply Volume	Water Shortage Volume		
1956–1957	3065	1388	603	−61	1064	0	712	0	0	3612
1957–1958	16,891	1928	687	0	1064	0	712	0	11,654	4458
1958–1959	4090	1544	301	0	1064	0	712	0	70	4857
1959–1960	1694	1343	488	−43	1064	0	712	0	0	2944
1960–1961	3069	1122	443	−140	1064	0	652	−60	0	2732
1961–1962	7423	1893	581	0	1064	0	712	0	561	5344
1962–1963	9058	1690	360	0	1064	0	712	0	3951	6625
1963–1964	13,116	1822	445	0	1064	0	712	0	8967	6732
1964–1965	14,324	1825	505	0	1064	0	712	0	11,203	5747
1965–1966	9717	1870	545	0	1064	0	712	0	6080	5192
1966–1967	2058	1507	722	−48	1064	0	712	0	0	3245
1967–1968	7445	1786	504	0	1064	0	712	0	1472	5152
1968–1969	704	1056	172	−253	1064	0	712	0	0	2852
1969–1970	4299	1254	465	−197	1064	0	712	0	0	3655
1970–1971	15,633	1561	576	0	1064	0	712	0	8361	7014
1971–1972	12,932	1888	609	0	1064	0	712	0	9911	5762
1972–1973	4051	1423	407	0	1064	0	712	0	0	6207
1973–1974	8595	2043	657	0	1064	0	712	0	4088	6238
1974–1975	14,003	1689	411	0	1064	0	712	0	10,072	6294
1975–1976	10,009	1765	638	0	1064	0	712	0	6515	5610
1976–1977	1092	1299	238	−317	1064	0	712	0	0	3390
1977–1978	7676	1713	633	0	1064	0	712	0	684	6260
1978–1979	6591	1348	419	0	1064	0	712	0	2383	6925
1979–1980	11,338	1367	478	0	1064	0	712	0	7628	7014
1980–1981	6267	1641	652	0	1064	0	712	0	3955	5257
1981–1982	401	1246	0	−674	1064	0	0	−712	0	3347
1982–1983	393	674	14	−466	1064	0	0	−712	0	1988
1983–1984	1904	399	9	−387	805	−259	178	−534	0	2502
1984–1985	4412	1063	212	−165	1064	0	712	0	0	3863
1985–1986	3714	1294	0	−516	1064	0	712	0	0	4507
1986–1987	414	1081	0	−632	1064	0	240	−472	0	3537
1987–1988	330	795	0	−624	1064	0	0	−712	0	2007
1988–1989	232	441	0	−778	799	−265	0	−712	0	1000
1989–1990	1306	270	0	−595	178	−886	0	−712	0	1858
1990–1991	2315	593	311	−214	1064	0	298	−414	0	1906
1991–1992	15,614	1526	643	0	1064	0	712	0	9069	4505
1992–1993	824	822	340	−132	1064	0	538	−174	0	2565
1993–1994	12,944	1294	485	0	1064	0	712	0	6477	5478
1994–1995	3822	1377	484	0	1064	0	712	0	674	4988
1995–1996	8928	1452	588	0	1064	0	712	0	5590	4510
1996–1997	5004	1313	433	0	1064	0	712	0	815	5177
1997–1998	7374	1267	341	0	1064	0	712	0	2153	7014
1998–1999	6690	1569	639	0	1064	0	712	0	4932	4788
1999–2000	1605	1029	526	−152	1064	0	298	−414	0	3475
2000–2001	2218	868	446	−190	1064	0	653	−59	0	2661
2001–2002	5696	1540	551	−63	1064	0	712	0	0	4491
2002–2003	734	988	565	−80	1064	0	238	−474	0	2369
2003–2004	10,240	1284	453	0	1064	0	712	0	4129	4967
2004–2005	8669	1291	458	0	1064	0	712	0	4048	6063
2005–2006	9661	1447	490	0	1064	0	712	0	6236	5775
2006–2007	4197	1341	513	0	1064	0	712	0	1592	4750
2007–2008	11,096	1310	472	0	1064	0	712	0	7621	4667
2008–2009	1278	1130	545	−29	1064	0	359	−353	0	2847
2009–2010	5818	1240	534	0	1064	0	712	0	593	4522
2010–2011	9455	1499	569	0	1064	0	712	0	5748	4384
2011–2012	4572	1485	574	0	1064	0	712	0	104	5018
2012–2013	4719	1546	432	0	1064	0	712	0	926	5058
2013–2014	2866	1403	430	−122	1064	0	712	0	0	4314
Average values	6131	1339	424	−119	1040	−24	600	−112	2729	−

Based on the number of water shortfall years for each sector obtained through chronological regulation computations, the water supply reliability values for agricultural irrigation, industrial/domestic water use, and ecological water use are calculated to be 59.3%, 94.9%, and 74.6%, respectively, which are consistent with their designed reliability values.

3.2. Regional Allocation of the Initial Water Rights of Nishan Reservoir

The sectoral water rights of Nishan Reservoir also need to be allocated to counties and cities. It should be noted that the agricultural irrigation district supplied by Nishan Reservoir is entirely within Qufu City, and no objections to the water rights allocation for this district have been raised by other surrounding counties and cities. Therefore, the water rights for agricultural irrigation may first be allocated from the reservoir's calculated utilizable storage capacity, ensuring that Qufu City receives 4.24 million m³ of agricultural irrigation water with a 50% reliability level. This district will be excluded from subsequent water rights allocations and the established index system. In addition, since the management of downstream ecological water use falls under the jurisdiction of the Jining Prefectural Government, the 6 million m³ allocated for ecological water use will also be excluded from the allocation of regional water rights (all ecological water use will be allocated to Jining City). Consequently, the regional allocations will only involve 10.4 million m³ of available water supply allocated for domestic and industrial use.

To this end, the Yi River basin is divided into five upstream and downstream regions, demarcated by the dam of Nishan Reservoir and the administrative regions within the basin. These regions are Qufu_{Upstream}, Qufu_{Downstream}, Sishui_{Upstream}, Zoucheng_{Upstream}, and Zoucheng_{Downstream}. The water rights allocation indices for each region are then calculated, as shown in Table 5, while the spatial distribution of index values in the five regions is shown in Figure 4. In Figure 4, UPS stands for upstream, DWS stands for downstream, and C1 to C10 represent the indices listed in Table 2.

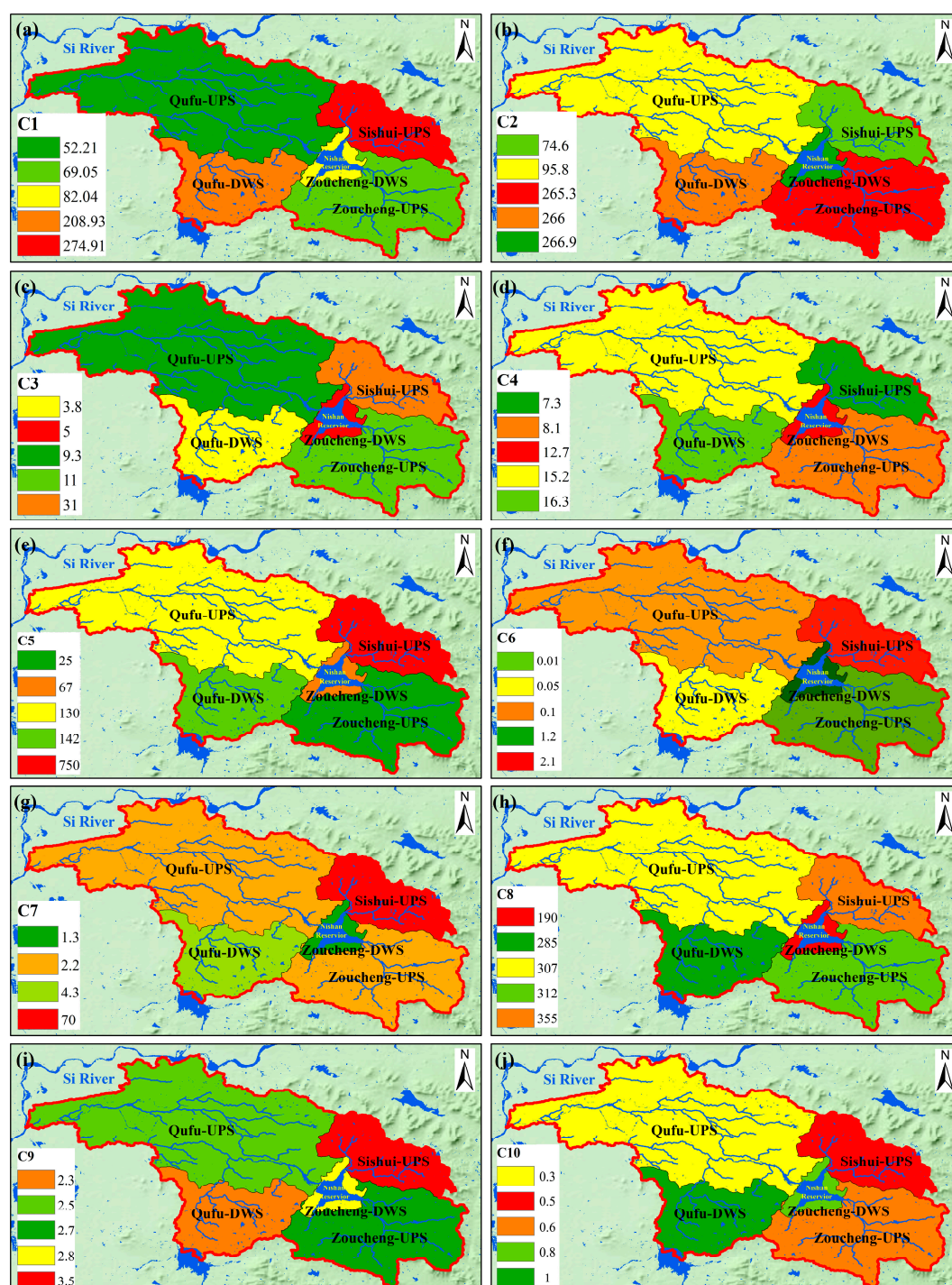
The classic domain (R_0) and the matter-elements to be assessed (R_k) for the allocation of water rights from Nishan Reservoir were constructed using Equations (4) and (5).

$$R_0 = \begin{bmatrix} & N_1 & N_2 & N_3 \\ C_1 & \langle 250, 300 \rangle & \langle 60, 250 \rangle & \langle 0, 60 \rangle \\ C_2 & \langle 200, 300 \rangle & \langle 100, 200 \rangle & \langle 0, 100 \rangle \\ C_3 & \langle 20, 35 \rangle & \langle 9, 20 \rangle & \langle 0, 9 \rangle \\ C_4 & \langle 0, 8.5 \rangle & \langle 8.5, 15 \rangle & \langle 15, 20 \rangle \\ C_5 & \langle 500, 800 \rangle & \langle 80, 500 \rangle & \langle 0, 80 \rangle \\ C_6 & \langle 1.5, 2.5 \rangle & \langle 0.1, 1.5 \rangle & \langle 0, 0.1 \rangle \\ C_7 & \langle 40, 75 \rangle & \langle 2.5, 40 \rangle & \langle 0, 2.5 \rangle \\ C_8 & \langle 300, 400 \rangle & \langle 200, 300 \rangle & \langle 0, 200 \rangle \\ C_9 & \langle 0, 2 \rangle & \langle 2, 3 \rangle & \langle 3, 4 \rangle \\ C_{10} & \langle 0, 0.4 \rangle & \langle 0.4, 0.8 \rangle & \langle 0.8, 1.2 \rangle \end{bmatrix}$$

$$R_k = \begin{bmatrix} & V_1 & V_2 & V_3 & V_4 & V_5 \\ C_1 & 69.05274.9182.04208.9352.21 \\ C_2 & 265.3 & 74.6 & 266.9 & 266 & 95.8 \\ C_3 & 11 & 31 & 5 & 3.8 & 9.3 \\ C_4 & 8.1 & 7.3 & 12.7 & 16.3 & 15.2 \\ C_5 & 25 & 750 & 67 & 142 & 130 \\ C_6 & 0.05 & 2.1 & 0.01 & 0.01 & 1.2 \\ C_7 & 2.2 & 70 & 1.3 & 4.3 & 2.2 \\ C_8 & 312 & 355 & 190 & 285 & 307 \\ C_9 & 2.7 & 3.5 & 2.8 & 2.3 & 2.5 \\ C_{10} & 0.6 & 0.8 & 1.0 & 0.3 & 0.5 \end{bmatrix}$$

Table 5. Values of water rights allocation indices of each region.

Index	QufuUpstream	QufuDownstream	SishuiUpstream	ZouchengUpstream	ZouchengDownstream
C ₁	69.05	274.91	82.04	208.93	52.21
C ₂	265.30	74.6	266.9	266.0	95.8
C ₃	11.0	31.0	5.0	3.8	9.3
C ₄	8.10	7.3	12.7	16.3	15.2
C ₅	25.0	750.0	67.0	142.0	130.0
C ₆	0.05	2.1	0.01	0.01	1.2
C ₇	2.20	70	1.3	4.3	2.2
C ₈	312.0	355	190	285	307
C ₉	2.70	3.5	2.8	2.3	2.5
C ₁₀	0.60	0.8	1.0	0.3	0.5

**Figure 4.** Spatial distribution of index values in the five regions.

To address the implementation details of the AHP for calculating index weights (β_1), a total of 50 questionnaires were distributed. The questionnaires were designed based on the 9-point pairwise comparison of the importance of the 10 indices, as required by the AHP algorithm. The respondents included experts from universities, local technical institutions, and research academies, who mainly specialized in fields such as water conservancy, natural sciences, engineering, and finance. Using the survey data, pairwise comparison matrices for the average values of each index in Nishan Reservoir were constructed. Subsequently, statistical analysis and consistency checks were conducted to ensure the reliability of the matrices. Finally, the calculated index weights were obtained as $\beta_1 = [0.118, 0.021, 0.079, 0.027, 0.198, 0.238, 0.218, 0.087, 0.004, 0.010]$. These weights were substituted into Equation (7) to calculate the closeness values, and those approaching 0 were corrected using Equation (8). The corrected closeness values are shown in Table 6.

Table 6. Closeness values for regions to be assessed.

Region	$K_I(N_k)$	$K_{II}(N_k)$	$K_{III}(N_k)$	K_k^T
Q _{Upstream}	0	1.008 6	1.031 8	0.811 5
Q _{Downstream}	1.854 8	1.117 8	0	2.154 5
S _{Upstream}	0	1.108 6	1.114 3	0.888 0
Z _{Upstream}	0.218 1	1.154 7	0.628 1	0.992 9
Z _{Downstream}	0.021 1	1.075 5	0.832 4	0.828 6

Note: In the table, Q, S, and Z represent Qufu City, Shishui County, and Zoucheng City, respectively.

The contribution coefficient of each grade is defined within the range [0,1] and follows the “closeness priority principle” of extension theory—grades with a stronger association with the index system are assigned higher coefficients. In the context of the grading system adopted above (Grade I, Grade II, and Grade III), the contribution coefficients should decrease in sequence from Grade I to Grade III. Based on these principles, the contribution coefficients were set as $\alpha_i = [0.8, 0.6, 0.2]$. The overall closeness of the regions to be assessed, K_k^T , was calculated using Equation (9), with the results presented in Table 6. As shown in this table, Qufu_{Downstream} exhibits the highest closeness, while Qufu_{Upstream} has the lowest closeness, thus indicating that these regions should receive the largest and smallest water rights allocations, respectively.

The closeness values of the regions were then normalized using Equation (10) to derive the initial water rights allocation weights of Qufu_{Upstream}, Qufu_{Downstream}, Shishui_{Upstream}, Zoucheng_{Upstream}, and Zoucheng_{Downstream}, which are 14.3%, 38.0%, 15.6%, 17.5%, and 14.6%, respectively. The 10.4 million m³ available water supply of the reservoir was accordingly allocated to these five regions for industrial and domestic use, with the allocations for each region shown in Table 7.

Table 7. Regional allocation of Nishan Reservoir’s water rights for industrial and domestic use.

Region		Percentage of Allocation	Water Rights/10 ⁴ m ³	
Qufu	Qufu _{Upstream}	14.3	148	543
	Qufu _{Downstream}	38.0	395	
Sishui	Shishui _{Upstream}	15.6	163	163
Zoucheng	Zoucheng _{Upstream}	17.5	182	334
	Zoucheng _{Downstream}	14.6	152	
Total		100	1040	1040

4. Discussion

4.1. Analysis of Initial Water Rights Allocations

The initial water rights allocations of the Nishan Reservoir were derived from the aforementioned sectoral and regional initial water rights allocations (Table 8). The total allocatable water rights of Nishan Reservoir, including downstream ecological water use, amount to 20.64 million m³. Specifically, the water rights allocations for Qufu City, Sishui County, Zoucheng City, and Jining City are 9.67 million m³, 1.63 million m³, 3.34 million m³, and 6 million m³, respectively.

Table 8. Initial water rights allocations of Nishan Reservoir.

Sector/Region	Qufu City			Sishui County	Zoucheng City	Jining City	Total
	Agricultural Irrigation	Industrial and Domestic Water Use	Subtotal	Industrial and Domestic Water Use	Industrial and Domestic Water Use	Ecological Water Use	
Water rights/10 ⁴ m ³	424	543	967	163	334	600	2064
Water supply reliability	≥50	≥90	—	≥90	≥90	≥70	—

Analysis of sectoral water rights allocations (Tables 6–8) shows that domestic and industrial water use received the largest allocation of water rights (10.4 million m³), followed by ecological water use for the downstream reaches (6 million m³). Agricultural irrigation received the smallest allocation of water rights, at 4.24 million m³. In recent years, urban expansion has led to increases in urban water use and urban scenic water use. Conversely, the irrigation district has shrunk, reducing the water demand for agricultural irrigation. Thus, the water rights allocations align with the realities of the study area, reflecting the application of game theory to balance urban, rural, industrial, and agricultural water demands. In terms of regional water rights allocations, Qufu City received the largest allocation of water rights (encompassing agricultural irrigation and domestic/industrial water use), totaling 9.67 million m³. This is consistent with Qufu City's responsibilities and contributions: as a traditional hub for irrigation and urban water use, it has long overseen the construction, management, and operation of Nishan Reservoir. Sishui County, located upstream of the reservoir, currently uses no water from it. Nonetheless, this region is a catchment area that contributes toward the reservoir's inflows, and it plays an important role in protecting the water quality of the reservoir's water sources. For these reasons, Sishui County was allocated 1.63 million m³ of the reservoir's initial water rights. This allocation will enable Sishui County to construct water supply pipelines to nearby towns in the future. Zoucheng City, whose administrative jurisdiction includes both upstream and downstream reaches of the reservoir, was allocated 3.34 million m³ for similar reasons. These water rights can be used for the upstream regions of the reservoir and also to meet industrial and domestic water demands in the downstream regions. In summary, our water rights allocation method has fully considered the water demands of both upstream and downstream regions, as well as their current and future needs, resulting in a fair and objective allocation scheme. This scheme has been approved by the relevant administrative authorities and implemented in 2019.

Before the implementation of this scheme, the Water Affairs Bureau of Jining City organized a special consultation meeting to brief all stakeholders—including the Water Affairs Bureau of Jining City, Qufu City, Sishui County, and Zoucheng City, as well as representatives from agricultural irrigation districts, industrial enterprises, and urban water supply companies—on the allocation principles, sectoral/regional allocation quotas, and implementation safeguards of the water rights allocation scheme. After the scheme was formally implemented in 2019, the local Water Affairs Bureaus of Qufu City, Sishui County,

and Zoucheng City have conducted annual routine consultations with stakeholders to track water use satisfaction, identify potential conflicts, and collect optimization suggestions. During the 6-year implementation period (2019–2024), the water rights allocation scheme has achieved stable operation, with no water use disputes reported among sectors (agriculture, ecology, domestic, and industry) and regions (Qufu City, Sishui County, and Zoucheng City). This outcome is clearly reflected in the quantitative indicators of water supply guarantee:

- (1) Agricultural irrigation water supply: Despite the relatively small water rights allocation (4.24 million m³), the water supply reliability for irrigation has reached 82% (higher than the designed target of 50%), effectively supporting the production of key crops (wheat and corn) in the irrigation districts. During the critical irrigation periods (spring sowing and summer drought), the actual irrigation water volume per hectare of farmland has remained at 3000–3300 m³, ensuring a stable grain yield (average annual yield of 7.5–9 t/ha).
- (2) Ecological water supply: The water supply reliability of ecological water use has reached 90%, which is also closely related to the above-average precipitation since 2019. The water quality at monitoring points in the downstream river section has remained consistently at Grade III or above in accordance with the surface water quality standard of China.
- (3) Domestic water supply: The water supply reliability of domestic water use in Qufu City and Zoucheng City has remained 100% (consistent with the designed target of 90%), with no water supply interruptions. The per capita daily domestic water consumption has stabilized at 115–130 L, meeting the national standard for medium-sized cities.
- (4) Industrial water supply: The water supply reliability of industrial water use for key industrial enterprises in the study area reaches 98.5% (exceeding the designed target of 90%), and water shortage has not affected normal production and operation.

4.2. Comparison of Matter-Element Extension Theory with Other Multi-Criteria Decision-Making Tools

Compared with other multi-criteria decision-making tools commonly used in water resource management, such as fuzzy logic, the entropy method, and AHP-TOPSIS hybrids, the matter-element extension theory exhibits distinct advantages in addressing complex evaluation problems, especially those involving uncertainty, contradiction, and multi-dimensional indices.

Firstly, a prominent limitation of fuzzy logic, the entropy method, and AHP-TOPSIS hybrids lies in their focus on “consistent” evaluation scenarios—they assume that the indices, standards, and evaluation objects are logically compatible and struggle to resolve contradictory relationships (e.g., an object meeting some excellent standards but failing to meet basic requirements for other key indices). In contrast, the matter-element extension theory is built on the concept of “extension” and “matter-element”. It explicitly models contradictory problems through tools like the extension set (distinguishing the “positive domain,” “negative domain,” and “extension domain” of evaluation standards) and correlation function (quantifying the degree to which an object belongs to both acceptable and unacceptable domains). This enables it to not only identify contradictory evaluation scenarios but also provide a quantitative basis for resolving such contradictions (e.g., adjusting index weights or revising standards), which is unavailable in the other three methods.

Secondly, unlike fuzzy logic, which primarily relies on membership functions to handle uncertainty and often struggles with quantifying ambiguous qualitative indicators, extension theory constructs matter-element models to systematically integrate both quanti-

tative data and qualitative criteria, enabling a more comprehensive representation of the complex water right allocation system.

Thirdly, the entropy method, while effective in objectively determining index weights, lacks the ability to flexibly adjust to dynamic changes in allocation scenarios (e.g., sudden droughts or policy shifts); in contrast, extension theory's extension transformation mechanism allows for real-time modification of evaluation parameters, enhancing the adaptability of the decision-making process.

Fourthly, the AHP-TOPSIS hybrid model, though combining subjective weight determination (AHP) and objective alternative ranking (TOPSIS), tends to simplify conflicting criteria into a single optimal solution and fails to fully capture the trade-offs between multiple objectives. Extension theory, however, excels in addressing such conflicts—especially regarding sectoral and regional priority conflicts—through its unique correlation function and extension set theory.

In conclusion, the application of extension theory to sectoral and regional water right allocation not only quantifies the degree of compatibility between different stakeholders' demands (e.g., agricultural water use vs. domestic water use; upstream regions vs. downstream regions) but also identifies feasible "extension paths" to reconcile conflicts and further proposes balanced solutions that minimize trade-off losses. By breaking through the limitations of traditional multi-criteria decision-making tools in handling complexity, dynamics, and conflicts, extension theory provides a more scientific and practical decision-making support system for reservoir water right allocation.

4.3. Sensitivity Analysis of Indices for the Allocation of a Reservoir's Initial Water Rights

To analyze the sensitivity and robustness of the water rights allocation index system, one index at a time is repeatedly removed, and the remaining nine indices are used to calculate their weights using AHP. With these nine new indices and their new weights, the water rights allocation results are computed following the procedure outlined in Section 2.2.3. These results are then compared with those obtained using all ten indices. The differences between the two sets of results and the mean absolute deviation (MAD) are presented in Table 9. The MAD value measures the average magnitude of change in the water rights allocation results across regions after removing a specific index. A lower MAD value indicates that removing the index results in minimal changes to the water rights allocation across regions, suggesting the index has a relatively small impact on the outcome—meaning the result is insensitive to changes in that index. Conversely, a higher MAD value means removing the index causes significant changes in water rights allocation for certain regions, indicating the index has a substantial influence on the result and is a key driving factor—meaning the result is highly sensitive to changes in that index.

Table 9. Differences in water rights allocation results calculated using different indices (in 10^4 m^3).

Regions	Remove C ₁	Remove C ₂	Remove C ₃	Remove C ₄	Remove C ₅	Remove C ₆	Remove C ₇	Remove C ₈	Remove C ₉	Remove C ₁₀
Q _{Upstream}	−7.2186	1.6078	0.2817	0.2743	−1.6134	−0.0692	0.2584	12.2022	−0.0048	−0.0039
Q _{Downstream}	2.9457	0.0274	−0.1296	−0.0534	−52.8767	−0.0696	0.4697	−19.8936	0.0160	0.0069
S _{Upstream}	−3.0866	1.4098	0.0736	−0.0094	−21.7889	−0.0298	1.3288	−7.1235	−0.0051	−0.0067
Z _{Upstream}	16.8729	2.0397	−0.3817	−0.1332	91.4572	−0.1551	−2.0537	4.0222	−0.0019	0.0046
Z _{Downstream}	−9.5134	−5.0847	0.1560	−0.0784	−15.1782	0.3236	−0.0032	10.7927	−0.0042	−0.0010
MAD	7.9274	2.0339	0.2045	0.1097	36.5829	0.1295	0.8228	10.8068	0.0064	0.0046

Note: In the table, Q, S, and Z represent Qufu City, Sishui County, and Zoucheng City, respectively.

As shown in Table 9, the three most sensitive indices are C₅ ($\text{MAD} = 36.5829 \times 10^4 \text{ m}^3$), C₈ ($\text{MAD} = 10.8068 \times 10^4 \text{ m}^3$), and C₁ ($\text{MAD} = 7.9274 \times 10^4 \text{ m}^3$). This indicates that the indices of water consumption for domestic water use, productivity per cubic meter of water, and catchment area are the key drivers of the results and should be the

focus of attention. The three least sensitive indicators are C_{10} ($MAD = 0.0046 \times 10^4 \text{ m}^3$), C_9 ($MAD = 0.0064 \times 10^4 \text{ m}^3$), and C_4 ($MAD = 0.1097 \times 10^4 \text{ m}^3$), suggesting that the cost of industrial water supply, the cost of domestic water supply, and water consumption for each 10,000 yuan in industrial added value have minimal impact on the results. From a regional perspective, $Qufu_{Upstream}$ is primarily positively influenced by C_8 ($+12.20 \times 10^4 \text{ m}^3$) and negatively influenced by C_1 ($-7.22 \times 10^4 \text{ m}^3$); $Qufu_{Downstream}$ is extremely sensitive to changes in C_5 ($-52.88 \times 10^4 \text{ m}^3$) and C_8 ($-19.89 \times 10^4 \text{ m}^3$), both acting as negative drivers; $Shishui_{Upstream}$ is mainly negatively affected by C_5 ($-21.79 \times 10^4 \text{ m}^3$) and C_8 ($-7.12 \times 10^4 \text{ m}^3$); $Zoucheng_{Upstream}$ is highly sensitive to changes in C_5 ($+91.46 \times 10^4 \text{ m}^3$) and C_1 ($+16.87 \times 10^4 \text{ m}^3$), both acting as positive drivers; $Zoucheng_{Downstream}$ is influenced by multiple indices, with the most significant negative impacts from C_5 ($-15.18 \times 10^4 \text{ m}^3$) and C_2 ($-5.08 \times 10^4 \text{ m}^3$), while C_8 ($+10.79 \times 10^4 \text{ m}^3$) has a substantial positive influence.

5. Conclusions

Methods for allocating a reservoir's initial water rights are crucial to resolving imbalances between water supply and demand and form an important component of water rights management systems. However, the allocation of a reservoir's water rights is a complex undertaking. It requires balancing regional and sectoral water allocations while ensuring fairness among stakeholders, making it imperative that such allocations are based on scientific and rational principles. The conclusions of this study are as follows:

A framework was developed for the two-dimensional allocation of initial water rights from a reservoir, encompassing both sectoral and regional dimensions. Firstly, long-series chronological continuous regulation calculation computations were employed to iteratively determine the reservoir's available water supply, under the condition that the calculated water supply reliability for each sector matched its designed reliability. The converged results yielded the reservoir's sectoral initial water rights. Subsequently, based on an analysis of the factors influencing the allocation of initial water rights from Nishan Reservoir, a system of indices and assessment criteria for initial water rights allocation from the reservoir was established. Finally, a matter-element extension model was constructed for allocating the initial water rights of Nishan Reservoir. The water rights allocation weights of each region were calculated using corrected closeness values, thereby forming a new model-based method for the initial water rights allocation of reservoirs.

This method can be deeply integrated with China's existing water governance system. In terms of legal frameworks, this method strictly abides by the Water Law of the People's Republic of China and aligns with the requirements of "total water resources control" and "differentiated water use management", thus preventing legal conflicts. In terms of law enforcement mechanisms, its index system matches the supervision indices of water conservancy departments, which helps reduce the cost of verification and inspection. In terms of compatibility with market-based water trading schemes, this method clarifies the clear and measurable ownership of water rights for different entities (e.g., agricultural, industrial, and domestic water users) at both regional and sectoral levels, thereby addressing the core issue of "unclear property rights" that restricts the development of water trading in some regions. Therefore, the water rights allocation method can be used as a reference by water resources management authorities such as water conservancy bureaus, river basin management agencies, and provincial water resources departments.

Given that the research and application of initial water rights allocation methods are still in the exploratory stage, the two-dimensional initial water rights allocation method for reservoirs proposed in this paper—covering both sectoral and regional aspects—requires expanding the scope of case studies, such as to transboundary basins, to verify the method's

applicability. Additionally, it has a limitation in that it fails to consider the applicability and robustness of the allocation scheme and the dynamic adjustment mechanism under future variable conditions, such as extreme hydrological scenarios and changes in water demand. These serve as directions for future research.

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