

Article

Heuristic Optimization Rules Applied for the Sustainable Design of Lightweight Engineering Structures Under Loads Subject to Random Changes

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Abstract

In engineering design, optimization is crucial for achieving sustainable goals. This involves creating environmentally responsible structures. Optimizing the design is the first step in reducing the environmental impact of construction. Topology optimization (TO) is one way to do this. TO is the process of designing the material layout in the design domain according to selected criteria. The criteria can be explicitly defined to promote sustainability. As a result, a new structure topology is proposed to make the structure both lightweight and durable, with the aim of improving its functionality and reducing its environmental impact. In optimal engineering design, it is particularly important to take into account the structure's special operating conditions, e.g., loads subject to random changes. Predicting topologies under such conditions is important since random load changes can significantly affect the resulting topologies. In this paper, an easy to implement numerical method for this kind of problem is proposed. The basic idea is to transform a random loads case into the deterministic problem of multiple loads. A heuristic method of Cellular Automata is proposed as a numerical optimization tool. The examples of topology optimization have been performed to illustrate the concept, confirming the efficiency, versatility, and ease of its implementation.

Keywords: sustainable design; optimal design; lightweight design; sustainability; topology optimization; compliance minimization; Cellular Automata; heuristic methods; random loads



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1. Introduction

The practice of optimization in engineering design is actively involved in the construction of structures that prioritize environmental responsibility and are crucial to achieving sustainable goals. The first step in reducing the environmental impact of construction is optimization at the design stage. Conventional production of building materials contributes to CO₂ emissions and energy demand. The construction sector, which uses around 40% of all energy to produce materials such as steel and cement, is the largest consumer of energy [1]. For example, the cement manufacturing industry is responsible for ~7% of the world's total CO₂ emissions [2]. For comprehensive studies on the carbon footprint of the construction industry, i.e., the total amount of greenhouse gas emissions arising directly or indirectly from the construction industry, readers are referred to [1]. Reducing the demand for building materials (modern and traditional) will contribute to environmental protection

by reducing extraction and quarrying, transportation, material production, processing, construction operations, and waste storage. The problem concerns not only civil construction but the entire industry. The reduction in impact on ecology can be achieved, e.g., through the responsible use of materials by optimizing the design of structures to create stronger, more reliable but lighter constructions at the product development process stage. As an example of reducing the impact on the environment through weight reduction in various industries, reducing the weight of a car by 10% reduces fuel consumption by 6% [3]. The importance of reducing the volume of used materials and thus the extraction of materials is shown in studies [4,5], which indicate key issues to support low-carbon design. Of course, the footprint is not just about the amount of material that is used. To reduce the weight of the structure, lighter materials can be used for the same project, but this may not be enough—for example, a part made of an aluminum alloy is much more difficult to process than a part made of steel [6], which may lead to higher energy consumption at the manufacturing stage. It is worth underlining that, in general, the lightweight design (i.e., minimization of the mass of the product) is not equivalent to the minimization of the carbon footprint [7]. In the best scenario, the combination of material selection and minimization of the weight needs to be processed simultaneously. Another paper showing how complex it is to calculate the actual CO₂ footprint from the production and use of products is [6], but it also draws attention to the importance of optimizing for the volume of the structure. For example, in [8], reducing the mass of the structure by only about 1% resulted in a significant reduction in the carbon footprint. More about the attempts to measure the product carbon footprint calculation model for the product design stage and the entire product life cycle can be found, e.g., in [4,8].

The innovative optimization techniques, such as topology optimization (TO), are very promising for supporting a problem's solution. A detailed investigation on achieving sustainability via TO can be found in [9,10]. The process of optimal design via topology optimization involves designing the material layout in the design domain under consideration according to selected criteria, for example, minimizing the amount of material used. It means that topology optimization is a tool that allows for the modification of material arrangement within a structure, thereby enabling the creation of a new optimized shape. During the iterative process, material from the parts of the design domain that are not required is redistributed to the parts that are essential for the carriage of the specified loads. The result of the process is a structure that can exhibit the desired properties. For instance, it can be as light as possible, but its stiffness is still sufficiently large. The lightweight structures can be an attractive alternative to traditional solutions, while the load-bearing capacity of both can be comparable.

In recent years, a significant number of papers addressing the application of topology optimization in a broad context have been published. For example, publications on important and emerging topics in a broadly understood adaptive topology optimization [11–17] may serve as an illustration. This represents only a portion of the research conducted worldwide on this topic, which suggests that it is both current and important.

Although topological optimization is a well-established tool, it continues to be actively developed, proposing new applications and methods for solving problems of sustainable design. Nowadays, topology optimization is more often considered as an eco-friendly design tool, while considering the reduction in the carbon footprint in the optimization formulation process, e.g., [18]. It means that during the process of optimization, the weight of the structure (i.e., the material consumption) is not the only factor taken into consideration. Indeed, the energy consumption and greenhouse gas emissions resulting from the structure during its service life are also treated as the objective function to be minimized.

The concept covers investigations from the design at the micro level (material designing) to the macro level of designing large civil engineering structures. The environmental benefits of using topology optimization are highlighted in many papers (e.g., industrial machinery construction [6], in the material design process [19], in the automotive industry [20], aerospace industry [21], repair and restoration [22,23]). The innovative trend of using topology optimization in civil engineering allows for the design of the layout of large civil structures (e.g., [24–26]), where structural and functional goals can be included in optimized concepts. The outer structure's features, such as the structure's topology to reduce the mass of the structure, have been identified as one of the main goals of sustainable designs in civil engineering [27]. Special attention is paid to the concrete constructions. The material quantity and reinforcement efficiency are under consideration in [27–29], including those studies, where embodied carbon emission and cost optimization are considered, complementing studies under sustainable design.

There is no doubt that optimization is needed at every stage of a product's life; however, this paper focuses on the design stage due to the volume of material used according to the assumption that the minimization of the compliance of the structure allows for maximum use of material to maximize the efficiency of the structure. Either exact mathematical programming, heuristics or hybrid approaches are used to achieve building sustainability through the use of optimization tools. This article follows this trend by utilizing a versatile but simple heuristic method of structure topology optimization, applied to the design of lightweight and durable structures. A key benefit of the described method, known as Cellular Automata, is its capacity to generate optimized topologies in a manner that is transparent to the designer. The multiple applications of the method have demonstrated its versatility. It has great potential because only a small number of iterations are required to achieve manufacturable topologies (i.e., without gray-scale and checkerboard effects). The realistic, complicated geometries of engineering structures under computationally demanding, design-dependent or random loads can be optimized by applying a synergy between a simple decision-making (optimization) process and a powerful analysis tool, e.g., Ansys—a commercial analysis package based on the Finite Element Method [30,31].

This paper aligns with contemporary trends in engineering, particularly in the field of engineering optimization, by employing heuristic and metaheuristic methodologies. Heuristic and metaheuristic methods are global optimization techniques inspired by the natural world. They use biological evolution, physical phenomena and swarm intelligence to explore possible solutions efficiently. These algorithms are particularly useful for tasks that are difficult to solve using traditional methods. The utilization of these powerful tools in engineering applications dates back to the mid-twentieth century. As the examples of current research on the application of this class of algorithms in engineering tasks may serve [32,33] or the review publication [34].

The paper makes an additional contribution to the development of engineering topology optimization by proposing a simple scheme for predicting structural topologies in the presence of random loads, where a random position of load application is considered. In the field of civil engineering, random loads can be observed as unpredictable loading, where the magnitude, direction, or point of application cannot be directly defined. As an example of random loads in civil engineering may serve wind gusts, seismic ground movements, ocean waves, loads from human and vehicle traffic, operating loads (e.g., those found in production halls). In the industrial context, random loads may be observed as impacts, collisions, variable cutting or machining forces, mechanical vibrations. It has been demonstrated in the literature, e.g., [35,36], that the application of random loads results in topology optimization of different final layouts from those obtained for deterministic loads.

The objective of the present paper is to propose a straightforward and readily implementable numerical approach that facilitates the prediction of such topologies for randomly defined point of load application. The proposed novelty is a simple idea that complex calculations can be replaced by an understandable and clear approach. The proposed concept can contribute to the simple, rapid design of lightweight, robust and therefore sustainable structures in civil engineering and industry in general, without sophisticated software. This idea can be combined with any optimization algorithm capable of dealing with multiple load case problems in topology optimization. In addition, this paper contributes to the development of the CA method by showing its new application to a problem rarely encountered in the literature, namely the modeling of the topology of structures subjected to loads with random points of application. It thus demonstrates and reaffirms its versatility. The CA method is reformulated into an adaptive method with limited need for parameter tuning. The technique under discussion is tested using multiple simple examples, thereby expanding the base of examples illustrating the method barely mentioned in the existing literature.

In order to provide a clear presentation of the idea, the structure of the paper is as follows: In Section 2, the topology optimization problem of minimization of compliance under total volume constraint has been formulated, and the algorithm of Cellular Automata used to solve it has been described. The numerical technique for implementing random loading in the topology generation process as a set of multiple load cases is presented. Section 3 provides a series of illustrative examples that serve to demonstrate the implementation of the approach that has been outlined in the preceding sections. In light of the findings from the numerical experiments conducted, Section 4 of the paper offers concluding remarks and provides a comprehensive overview of the research results and their implications.

2. Methods

2.1. Lightweight Design Through the Utilization of Topology Optimization

Topology optimization is a powerful design tool for sustainability and ecological purposes, as evidenced by numerous publications cited in Section 1 and others, e.g., [37–45]. The idea of topology optimization is to minimize the defined objective function by designing the considered structure's layout. The material is redistributed inside the design domain, complying with the restrictions of the constraints. A broad discussion of the subject can be found in the groundbreaking book, [10], as well as in many review articles, e.g., [46,47] or one of the latest [48].

The process of optimal design in the basis of the TO is a discretization of the considered domain into the lattice of finite elements, which allows us to execute the analysis using finite element method (different approaches not utilized in this paper can be found in, e.g., [40–50] where the discrete element method, finite volume, and boundary element method are utilized, respectively). Each finite element is represented by the design variable d_n referred to as the relative density of the material. The presence of real material, i.e., a solid element, is represented by the design variable equal to 1, when d_n is 0 or very near to 0, that means that the element can be treated as the void (in practice, the lower bound for the design variable is defined to avoid the instabilities in the finite element method). The intermediate values of d_n have no physical interpretation, so the reduction in those is necessary and can be reached by penalization. In this paper, the intermediate values of d_n are penalized based on the SIMP approach (Solid Isotropic Material with Penalization [10,51,52]), where the elastic modulus E_n of each finite element is modeled as a function of design variable using power law $E_n = d_n^p E_0$, where the power p (usually $p = 3$) penalizes intermediate densities to drive a design to a material/void structure. E_0 represents the elastic modulus of a solid material.

The formulation of the optimization problem is defined, for this paper, as the minimization of the compliance subject to a total volume constraint, where κ is the available material volume fraction (see Equations (1)–(4)):

$$\text{minimize } c(\mathbf{d}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{i=1}^N d_n^p \mathbf{u}_n^T \mathbf{k}_n \mathbf{u}_n, \quad (1)$$

$$\text{subject to } V = \kappa V_0, \quad (2)$$

$$\mathbf{K} \mathbf{u} = \mathbf{F}, \quad (3)$$

$$0 < d_{\min} \leq d_n \leq 1. \quad (4)$$

In Equations (1)–(4), $\mathbf{u}$ and $\mathbf{F}$ stand for the global displacement and force vectors, $\mathbf{K}$ is the global stiffness matrix. The element displacement vector and element stiffness matrix are defined as $\mathbf{u}_n$ and $\mathbf{k}_n$, respectively.

2.2. Heuristic Optimization Tool for Use in Sustainable Design

The process of generating the topology rearranges the distribution of material within the design domain, leading to the elimination of elements that are not relevant to the design objectives. This process, for the purpose of this paper, is governed by the Cellular Automaton optimization tool. Cellular Automaton (CA) is defined as the mathematical idealization of a physical system in which a domain of design is divided into a grid of cells. All cells are described by the states updated in a synchronous manner in discrete timesteps according to local rules. The principle of CA states that the global behavior of the system is governed by cells that only interact with their neighbors. Inou et al. proposed the application of Cellular Automata to structural design in [53,54].

In the most elementary implementation of Cellular Automata in optimal design, the grid of cells of the Automaton is equivalent to the lattice of finite elements used in the finite element method (FEM) of analysis. The states of cells described by a design variable d_n are updated according to the defined local rule $d_n^{\text{new}} = d_n + \Delta d_n$. The choice of Δd_n is crucial to the effectiveness of the rule. The present article opts for the effective method of adaptive rule proposed in [55]. The method was tested in [55] for elastic two-dimensional and three-dimensional structures, in [56] for graded multi-material structures and in [30] for periodic structures subject to self-weight loading. The details and validation of the method can be found in the literature mentioned above, and this paper focuses on the implementation of the method in the original idea of designing of topology of sustainable design of lightweight engineering structures subject to random loading conditions.

The chosen optimization procedure is a straightforward process that can be completed in a few simple steps. The simplicity and clarity of the method are its great advantages. The process begins with a FEM structural analysis applied to determine the local compliances for all cells. Following this, the compliance values of the N cells are sorted in ascending order, enabling the identification of two key subsets: the N_1 cells with the lowest compliance and the N_2 cells with the highest. Fixed values are then assigned to these subsets: $F(n) = -C$ for $n < N_1$ and $F(n) = C$ for $n > N_2$, where n denotes the cell index and C is a constant set to 1. It can thus be concluded that in regions where the algorithm has high confidence in the design decision, either to remove material (low compliance) or to add material (high compliance)—the choice can be directly set to its limiting value of C (see Equation (5)). For the remaining cells within the intermediate interval $N_1 \leq n \leq N_2$, a monotonically

increasing function $f(n)$ can be defined to smoothly represent the varying compliance levels of the elements as given by Equation (6):

$$F_n = \begin{cases} -C & n < N_1 \\ f_n & N_1 \leq n \leq N_2, \\ C & n > N_2 \end{cases} \quad (5)$$

$$f(n) = C \frac{\tanh\left[\beta\left(\frac{n-N_1}{N_2-N_1} - \frac{1}{2}\right)\right]}{\tanh\left(\frac{1}{2}\beta\right)}. \quad (6)$$

The CA method assumes the local interaction between cells; consequently, the update is based on the states of the adjacent cells. The rule is thus formulated to include the influence of the M neighboring cells, see Equation (7):

$$d_n^{new} = d_n + [F(n) + \sum_{k=1}^M F(k)] \frac{m}{M+1}, \quad (7)$$

where m is a move limit set to 0.2.

In Figure 1, the influence of parameter β on the shape of the function $f(n)$ is presented. For small β values the function is a linear one, whereas for its increasing values the function tends to a step one. The selection of β affects the above update rule and plays an important role in the design process allowing adjustment of a type of $f(n)$ to the considered design problems.

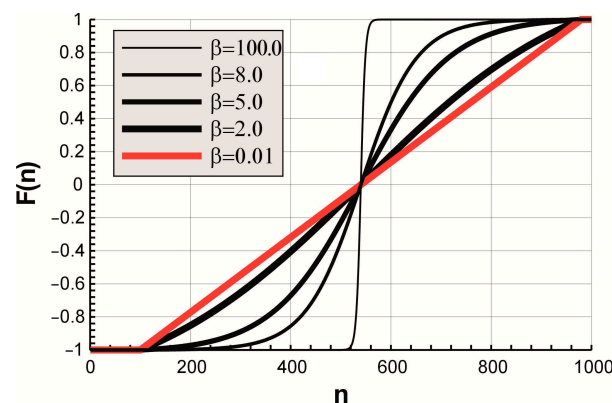


Figure 1. Function $F(n)$ for selected values of β .

The detailed studies on neighborhood types in Cellular Automata theory can be found in [57]. In [58], a discourse has been presented on the neighborhood types in the context of topology optimization. In this paper, the Moore neighborhood is implemented, so, for two-dimensional examples, the neighborhood is composed of nine cells: a central cell and the eight cells that surround it. As previously stated, the cells of Automata are equivalent to finite elements implemented in structural analysis.

The limited number of parameters in optimization algorithms has many practical advantages, especially in the context of engineering optimization. A reduction in the number of parameters can simplify the implementation and configuration of the algorithm, resulting in a decreased necessity for calibration. The simple modification allows for the introduction of an adaptive process that will help to improve the convergence and reduce the number of parameters to adjust. In its current form, the algorithm has 3 parameters, namely N_1 , N_2 , and β .

The threshold values N_1 and N_2 can be adjusted during the iteration process. The algorithm starts with a relatively wide initial interval $[N_1, N_2]$, which allows for a preliminary outline of the structure layout by eliminating void cells. The interval is then successively

reduced which leads to the elimination of so-called gray cells, i.e., cells of intermediate densities. Finally, a distinct solid/void structure can be obtained. The discussion regarding strategy for adjusting the N_1 and N_2 has been included in the introductory paper [55]. It is worth pointing out that the proposed strategy to some extent resembles the well-known simulated annealing method.

The self-adapting technique for choosing β is presented here. The parameter β is a key factor in determining the shape of the function $f(n)$ as it is presented in Figure 1. The necessity of choosing the value in the original idea [55] is now eliminated by a self-adjusting adaptation scheme: at each iteration, the design variables are updated using two heuristically selected values of β ($\beta = 0.01$ and $\beta = 8.0$). Subsequently, the algorithm identifies the lowest value of the objective function and progresses to the next iteration with the solution that exhibits the smallest compliance. The development of self-adapting techniques has rendered the discussed algorithm easier to adapt, eliminating the necessity of adjusting the parameters.

In order to facilitate comprehension of the calculation process, a flowchart is provided in Figure 2.

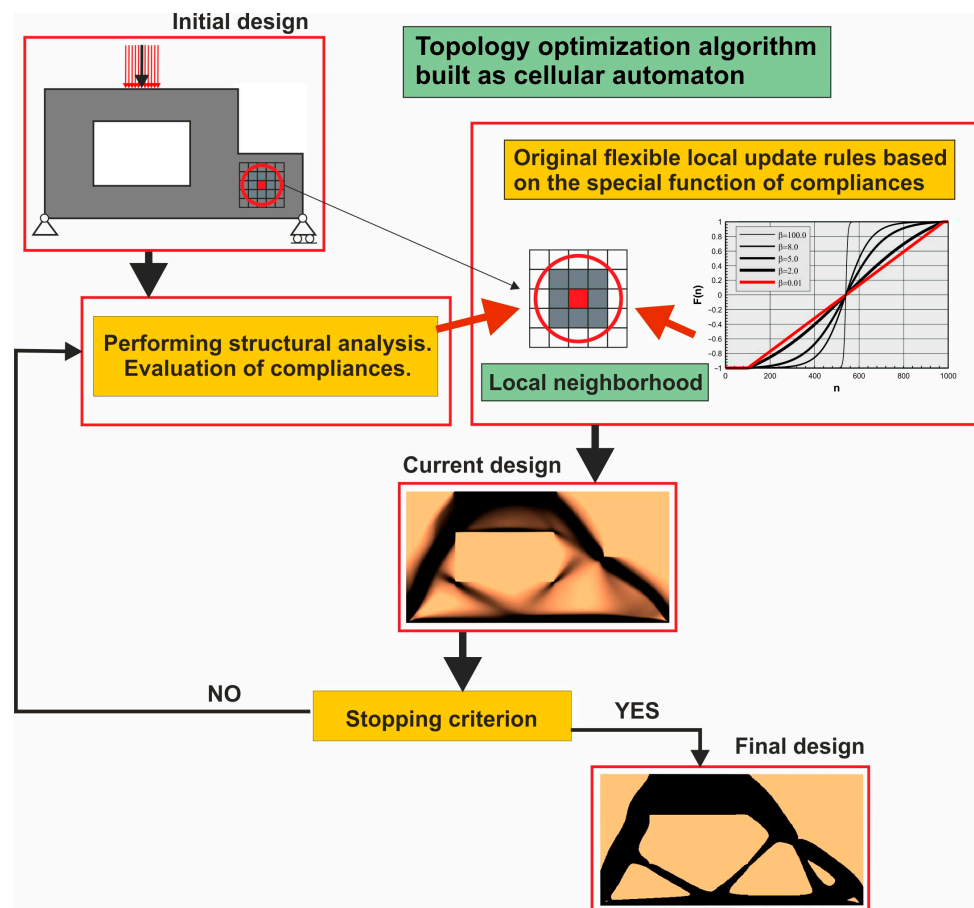


Figure 2. The flowchart of the topology generation process.

2.3. Simulation of Loads Applied at Randomly Selected Positions

The objective of the present paper is to propose a straightforward and readily implementable numerical approach that facilitates the prediction of topologies of structures under loads, for which the random positions of application are under consideration. Random loads are inherent to all realistic structural systems and must be considered during the design optimization process. Both this paper and the existing literature demonstrate that the structural topologies resulting from random loading conditions can differ significantly

from those derived from deterministic loading scenarios. Therefore, it is crucial to consider randomness when designing durable and sustainable constructions. The idea presented in this paper may significantly simplify this process.

The preliminary concept involves adapting a technique for generating topologies for multiple load cases. In the process of minimizing the compliance of a structure in the case of multiple load cases, the total number of compliances calculated for all load cases is taken into account. The first utilization of the concept is presented in [31] for randomly generated angles of load application. It has been demonstrated that if the number of load cases considered is sufficiently large, the final topology, which aligns with the solutions of such problems documented in the literature (i.e., [59,60]), can be obtained. Numerical research has confirmed that the application of loads at randomly selected angles, when treated as a multiple load case, facilitates the generation of minimal compliance topologies for non-deterministic scenarios.

The present concept considers the randomization of application load position (see Figure 3).

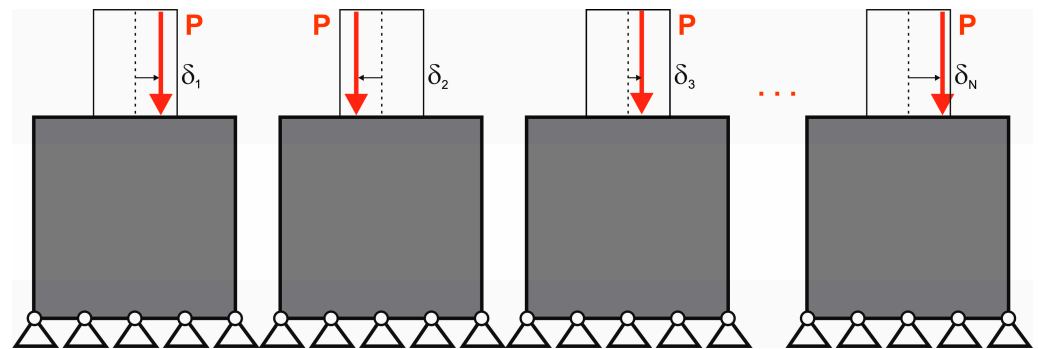


Figure 3. Multiple load case for the introductory example mimicking the load applied at randomly selected positions, conceptual diagram.

The position of load application is randomly selected, and the load applied at this position is treated as a particular load case. The set of such load cases forms the basis for the topology optimization performed as multiple load case.

The introductory example (see Figure 4) is discussed first. It is a square structure (200×200 elements/cells) loaded with deterministic force and supported as it is presented in Figure 4 (right). The topology optimization was performed to minimize the compliance subject to a total volume constraint, where the volume fraction was defined as $\kappa = 0.2$ (see Figure 4 (left)). The material data, i.e., the Young modulus $E = 1$ and Poisson ratio $\nu = 0.3$ have been used.

In the next step, the same structure is considered, but in this case, the load position is not fixed, but it is randomly selected from the range of $[-\Delta, \Delta]$. It is assumed that the load position varies only in the horizontal direction. The algorithm randomly selects 100 positions of force application and performs the structural analysis for each one. Like for the multiple load cases in topology optimization, the resulting compliance at each iteration step is calculated as the average of the compliance values for each of the load cases. This process simulates the action of the load with a random point of application. In such a case, the optimization process leads to the creation of a topology different from that in the deterministic case (Figure 5(right)).

The uncomplicated integration of multiple load cases into the optimization process has encouraged the adaptation of this technique to generate a topology for loads applied in a random manner. With this technique, one can perform the topology optimization of structures where some kind of uncertainty in load position needs to be analyzed. The

presented idea can be considered as a simple and straightforward technique to predict optimized structural topologies under loads subject to random changes.

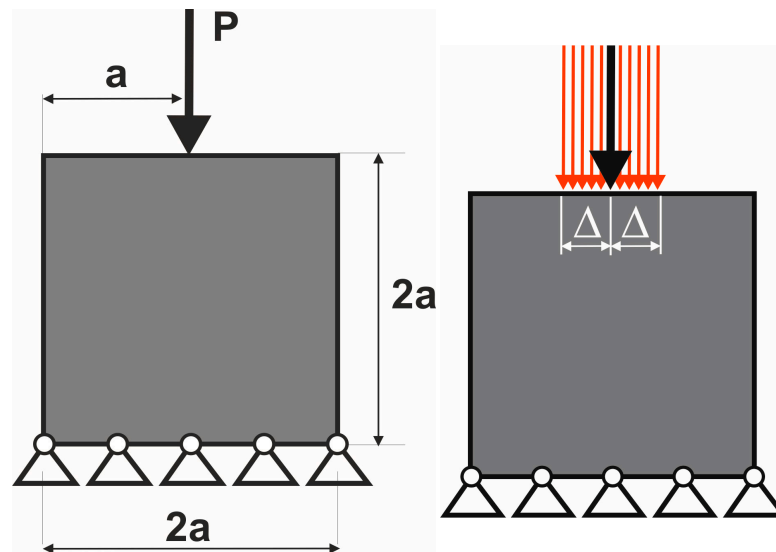


Figure 4. The square structure under single deterministic load (**left**) and under load applied at randomly selected positions (**right**), conceptual diagram.



Figure 5. The square structure under a single deterministic load (**left**) and under a load applied at randomly selected positions (**right**).

3. Results

To clarify the concept presented in this work, a number of test problems have been addressed. The algorithm, developed in MATLAB R2020b, was used to perform the computations. The self-adapting technique for choosing β has been implemented. The strategy for selecting values of parameters N_1 and N_2 is proposed as follows: the computations start with $N_1 = N \cdot 0.02$, and then from iteration 25 $N_1 = N \cdot 0.5$, from iteration 50 $N_1 = N \cdot 0.7$, and finally from iteration 75 $N_1 = N \cdot 0.9$, while $N_2 = N \cdot 0.98$ is kept for all iterations.

3.1. The Test Structure 1

In this example, a simply supported plane structure, as shown in Figures 6 and 7, is considered to demonstrate the effectiveness of the proposed approach. The structure under a single deterministic load is loaded with a force $P = 1 \text{ N}$ applied at the center of the upper edge. For the structure under load, subject to random changes, the 100 load cases are considered, and the randomly selected positions of load application vary within the range $[-\Delta, \Delta]$.

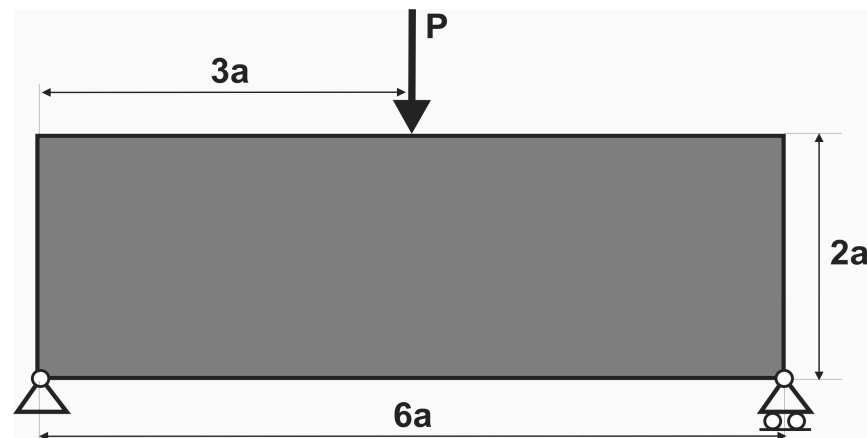


Figure 6. The test structure 1 under a single deterministic load. The regular mesh of 120,000 (600×200) elements/cells ($1 \text{ mm} \times 1 \text{ mm}$) has been applied. The parameter $a = 100 \text{ mm}$.

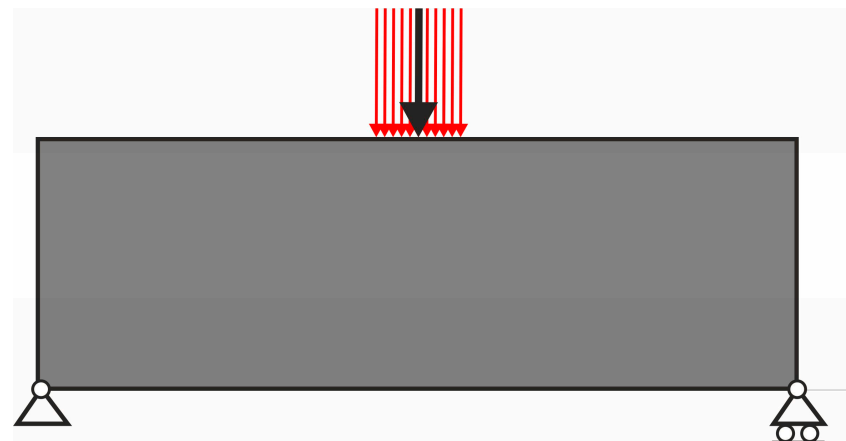


Figure 7. The test structure 1 under load applied at randomly selected positions, conceptual diagram.

The regular mesh of 120,000 (600×200) elements/cells of $1 \text{ mm} \times 1 \text{ mm}$ has been applied. The parameter $a = 100 \text{ mm}$. The structural analysis and topology generation for $E = 1 \text{ N/mm}^2$, $\nu = 0.3$ and the volume fraction $\kappa = 0.5$ have been performed. Figure 8 presents the final topology for the structure under a single deterministic load whereas Figures 9 and 10 show the results of topology generation under randomly selected positions of load application. Hence, the selected intermediate topologies obtained during the optimization process along with the illustration of the compliance convergence history are presented in Figure 9 while the final topology is given in Figure 10.



Figure 8. The final topology for the structure under a single deterministic load.

One can observe that the topologies generated for the deterministic case and for the load applied in a random manner significantly differ from each other.

The value of compliance for the deterministic case is equal to 25.85 Nmm whereas for the load applied at randomly selected positions $\Delta = 100$ mm the average compliance is found to be 26.96 Nmm.

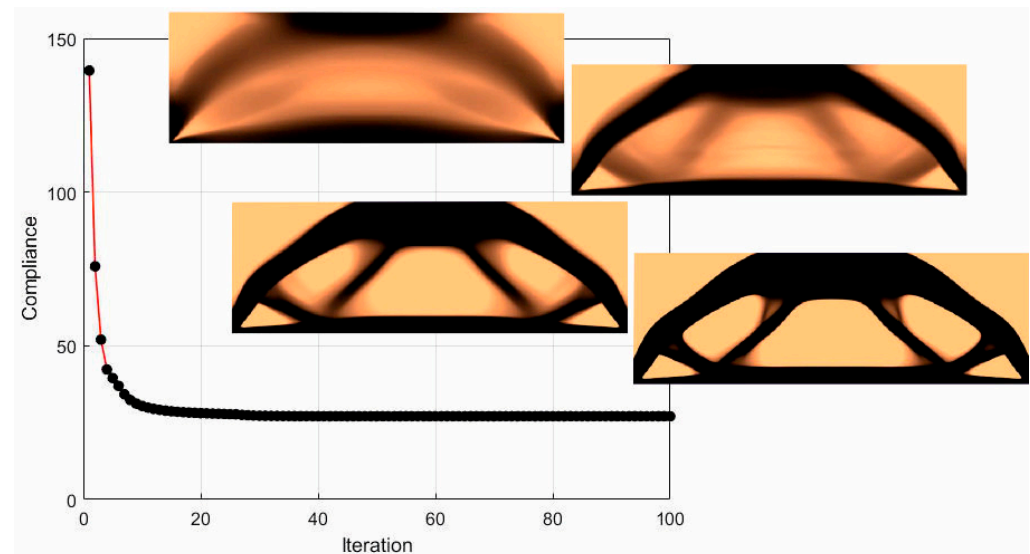


Figure 9. The selected intermediate topologies obtained during the optimization process for the structure under randomly selected positions of load application together with the compliance convergence history.



Figure 10. The final topology for the structure under a load applied at randomly selected positions.

3.2. The Test Structure 2

The second test plane structure is presented in Figure 11 for deterministic load $P = 1$ N and for the load applied at randomly selected positions in Figure 12. The regular mesh of 240,000 (1200×200) elements/cells of $1 \text{ mm} \times 1 \text{ mm}$ have been applied. The parameter $a = 200$ mm. The calculations for the volume fraction $\kappa = 0.5$ and $E = 1 \text{ N/mm}^2$, $\nu = 0.3$ have been performed.

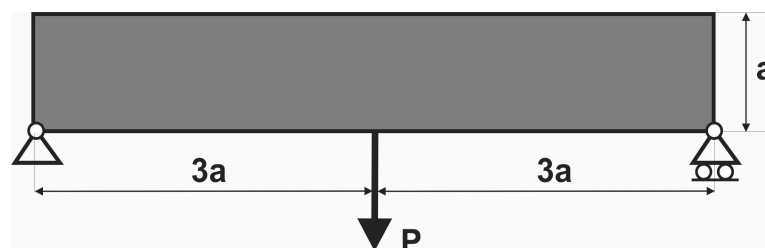


Figure 11. The test structure 2 under a single deterministic load. The regular mesh of 240,000 (1200×200) elements/cells ($1 \text{ mm} \times 1 \text{ mm}$) has been applied. The parameter $a = 200$ mm.

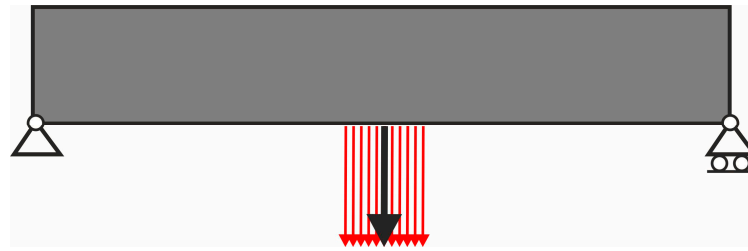


Figure 12. The test structure 2 under load applied at randomly selected positions, conceptual diagram.

The admissible change in load position is defined as the range $[-\Delta, \Delta]$. As with all examples, the load position varies only in the horizontal direction.

As was the case with the previous example, calculations were performed for test structure 2 for those two load conditions. The final topology for the structure subjected to a single deterministic load is shown in Figure 13. Figure 14 illustrates selected intermediate designs from the optimization process and the corresponding compliance convergence under randomly selected positions of load application.



Figure 13. The final topology for the structure under a single deterministic load.

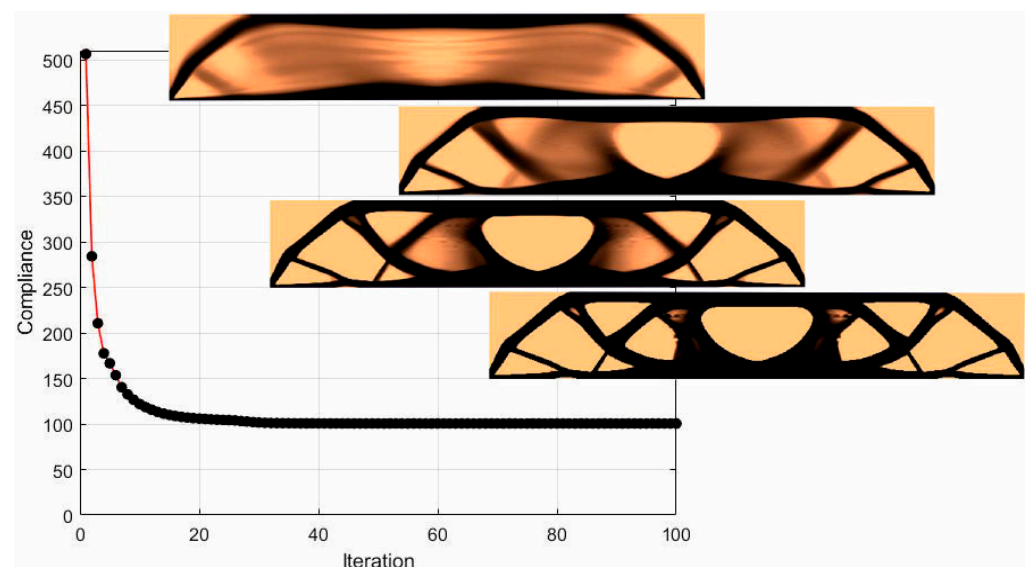


Figure 14. The selected intermediate topologies obtained during the optimization process for the structure under randomly selected positions of load application together with the compliance convergence history.

The resulting topology of the structure under randomly positioned loads is illustrated in Figure 15.



Figure 15. The final topology for the structure under a load applied at randomly selected positions.

The value of compliance for the deterministic case is equal to 99.70 Nmm whereas for the load applied at randomly selected positions $\Delta = 200$ mm the average compliance is found to be 100.28 Nmm.

3.3. The Test Structure 3

The structure shown in Figures 16 and 17 has been selected as the test example 3. The regular mesh of 204,800 (640×320) elements/cells of $1 \text{ mm} \times 1 \text{ mm}$ has been applied. The parameter $a = 40$ mm. The structural analysis and topology generation for $P = 1 \text{ N}$, $E = 1 \text{ N/mm}^2$, $\nu = 0.3$ and the volume fraction $\kappa = 0.4$ have been performed.

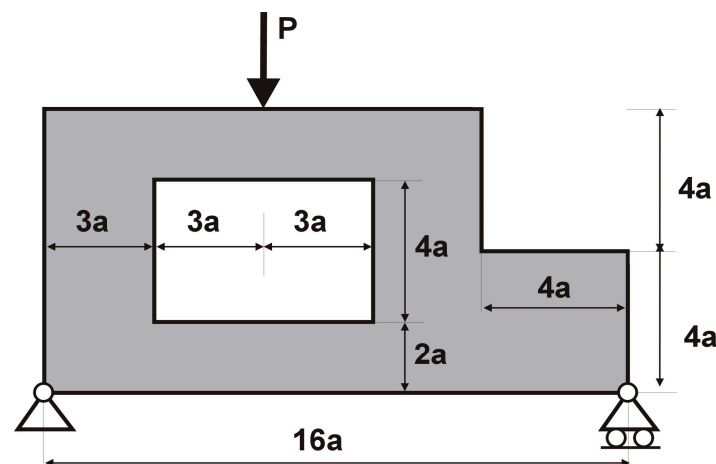


Figure 16. The test structure 3 under a single deterministic load. The regular mesh of 204,800 (640×320) elements/cells ($1 \text{ mm} \times 1 \text{ mm}$) has been applied. The parameter $a = 40$ mm.

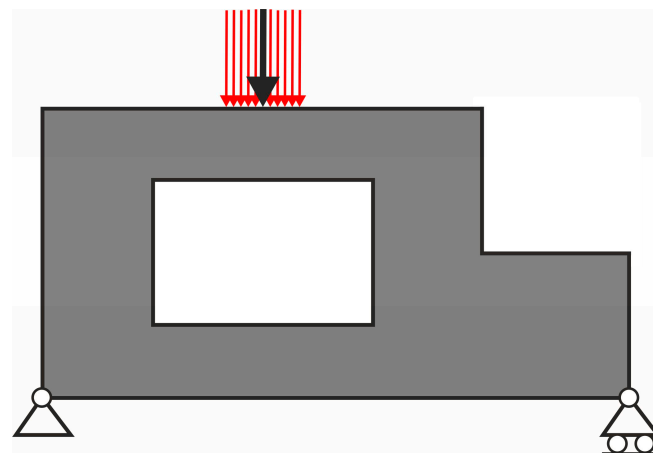


Figure 17. The test structure 3 under load applied at randomly selected positions, conceptual diagram.

The topology generation has been performed for a single deterministic load, see Figure 18, as well as for the 100 load cases, where the randomly selected positions of load application vary within the range $[-\Delta, \Delta]$, see Figures 19 and 20.

The value of compliance for the deterministic case is equal to 20.26 Nmm, whereas for the load applied at randomly selected positions $\Delta = 120$ mm, the average compliance is found to be 21.38 Nmm.

As illustrated by the test examples in this section, the implemented strategy, which mimics random loads, facilitates the generation of topologies by making it very easy to implement. The introduction of loads with randomly selected application positions, modeled using the newly proposed scheme, has an impact on the final topology. As demonstrated by the above test examples, it is vital to predict any potential uncertainties

that may arise from the load position application in the exploitation of constructions during the design process.

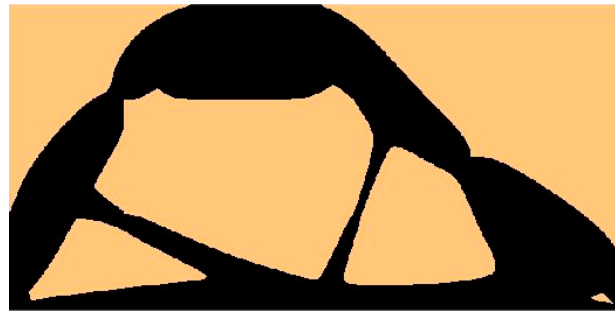


Figure 18. The final topology for the structure under a single deterministic load.

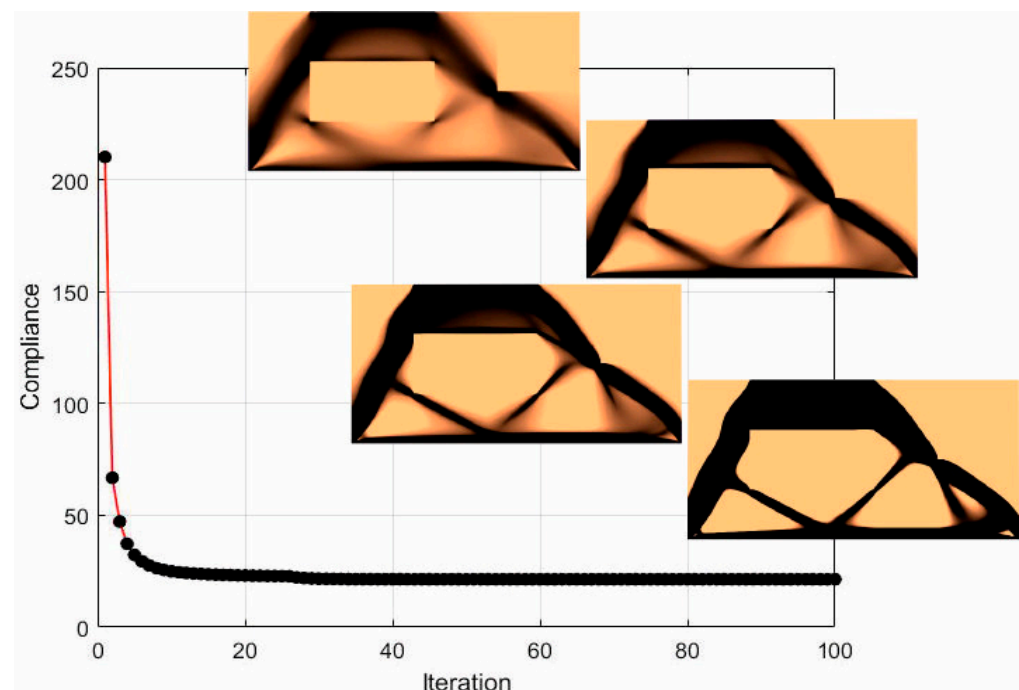


Figure 19. The selected intermediate topologies obtained during the optimization process for the structure under randomly selected positions of load application together with the compliance convergence history.



Figure 20. The final topology for the structure under a load applied at randomly selected positions.

4. Discussion and Conclusions

This paper constitutes a contribution to the contemporary research trend, engaging a heuristic method for optimizing structures that is both versatile and simple. The method has been designed for the purpose of creating lightweight and robust structures. By developing

the optimization of structural designs that can take into account resource constraints, this paper contributes to a more sustainable development strategy in engineering. A particular strength of the described method, known as Cellular Automata, is its ability to generate optimized topologies in a transparent manner for the designers. The technique has been demonstrated to have numerous applications in both present and former papers, thereby confirming its versatility. The method demonstrates considerable potential, as it requires only a limited number of iterations to produce designs of structures that are easy to interpret for manufacture. In other words, it produces designs that are free from ‘gray-scale’ and chequerboard effects. The simple decision mechanism of the optimization process can be easily combined with commercial software based on the finite element method, allowing for the optimization of realistic and complex geometries of engineering structures, even in complicated cases of non-deterministic loads. Although Cellular Automata algorithms have many advantages, it is important to underline that they also have some limitations. Their performance is based on local information exchange, which is well suited to locally formulated problems such as those dealing with local compliance or stresses, but limits their direct application to problems with global objectives. On the other hand, some global constraints can be imposed on the local solution. A typical example is the constraint of a fixed volume of material. This can be easily satisfied by a simple scaling of the local values of the design variables.

Loads subject to random changes are present in every realistic type of structure to some extent, and they must be considered in the process of optimal design. The present paper and the literature on the subject have proven that the resulting topologies in the presence of random load changes are significantly different from those for deterministic load cases. This paper demonstrates an easy and straightforward technique to implement randomization of the loading conditions in engineering daily practice, while the multiple load schemes are very easy to implement without the necessity of engaging additional specialized software.

Further development within this area may cover various issues. The combination of random positions with random angles of load application can broaden area of application of the proposed approach. Another important issue regards the simplification of computations. In this respect the idea of ELS—equivalent loading scheme, recently introduced in [31] for random angles of load application, may be extended to the case of randomized load positions. The expected benefit of that is the reduction in a numerical effort of computations since the selection of a few representative load cases may replace the necessity of generation of hundreds of them, simultaneously preserving nearly the same final topologies. There is no doubt that the next research step should also involve the design of real-world engineering structures. It is worth mentioning that the proposed topology optimization approach is a versatile one and can easily be combined with a professional structural analysis tool like, for example, ANSYS.

It is essential to enable optimal design for every type of product, even by small and medium-sized design companies. From the perspective of sustainable development, developing simple, fast methods for optimal design contributes to reducing the impact of human activity on the environment. The present paper makes a contribution to the field of sustainable design by proposing the concept of optimal design of lightweight and durable structures that are subject to random changes in load, as is often the case in practical applications. A key strategy for reducing the carbon footprint of engineering projects is to restrict material usage without compromising the strength properties of constructions in realistic working conditions. The design of more durable structures has the effect of reducing the necessity for frequent repairs, reinforcements, or early replacement. This can result in lower energy consumption during production, storage, and transportation.

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