

Optimal Sustainable Manufacturing for Product Family Architecture in Intelligent Manufacturing: A Hierarchical Joint Optimization Approach

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Abstract: As consumers and governments prioritize cost-effectiveness and ecological sustainability, the limitations of traditional manufacturing paradigms become apparent in the context of constrained resources. The adverse effects of these paradigms on the environment and society hinder the achievement of a sustainable product life cycle. Intelligent manufacturing processes offer a solution by efficiently gathering meaningful data, such as usage and product recycling information, from previous product generations to enhance product design and subsequent sustainable manufacturing processes (SMPs). Modular product family architecture (PFA) design holds promise in promoting product sustainability and addressing diverse consumer needs. PFA design and SMPs are inherently interconnected within intelligent manufacturing frameworks. This paper aims to integrate the decision-making processes underlying PFA with SMPs. We model integrated PFA and SMP decisions as a Stackelberg game, which involves hierarchical joint optimization (HJO) for assessing product modularity and sustainable manufacturing fulfillment. We develop a bilevel 0–1 integer nonlinear programming model to represent the HJO decision-making process and propose a nested genetic algorithm (NGA) to solve the HJO problem. A case study with a laptop is conducted to validate the feasibility and potential of the proposed HJO model for joint optimization problems in PFA design and SMPs.

Keywords: bilevel programming; hierarchical joint optimization; product family architecture; sustainable

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1. Introduction

The government is increasingly prioritizing environmental concerns and has enacted a series of pertinent laws aimed at holding original equipment manufacturers (OEMs) accountable for the sustainability of products throughout their lifecycle. This requires enterprises to continually adapt for their own development and to uphold a positive societal image. They must not only produce products that meet consumer demands but also consider the sustainability of these products [1]. For designers and manufacturers, it is essential to consider the sustainability of the product throughout its lifecycle [2]. Product lifecycles involve several stages, from pre-manufacturing (material selection and processing) through manufacturing (component manufacture and assembly) to use and post-use (recycling and reuse of products) [3]. In the context of the current availability of limited resources, the negative impact of traditional manufacturing paradigms on the environment and society is not conducive to achieving a sustainable product lifecycle [4].

Intelligent manufacturing has the potential to significantly enhance sustainable manufacturing competitiveness. By efficiently capturing relevant data, such as usage and recycling information from previous product generations, it can improve product design

and subsequent sustainable manufacturing processes (SMPs) [5]. Intelligent manufacturing is often categorized into two main types: base and front-end technologies [6]. The base technologies, including cloud computing, internet of things (IoT), and big data analytics, provide connectivity and intelligence for the front-end technologies. The front-end technologies involve the restructuring of manufacturing activities using the intelligent technologies derived from the base technologies. The reconfiguration of manufacturing activities is largely influenced by product design. Therefore, it is essential to consider product design in conjunction with sustainable manufacturing activities.

Modular product family architecture (PFA) offers several advantages in achieving product sustainability and meeting the diverse needs of consumers [7]. PFA relies on product platforms organized in family formats, where a variety of modules are selected to create product variations tailored to different market segments [8]. This approach not only reduces development time and costs but also facilitates product dismantling and module reuse [9].

In the realm of intelligent manufacturing, this paper explores the interrelation and optimization of modular PFA with SMPs in response to customer needs and product usage data. The solutions for each stage of PFA and SMPs are inherently interconnected. The inclusion of various module types within the product family—such as common, differentiated, and unique modules—significantly influences the selection of sustainable solutions [10]. When designing PFAs, it is essential to consider the rationality of product architecture and the selection of module types based on functional, technical, and structural coupling, while also meeting the sustainability requirements of consumers and government regulators. For SMP activities, product sustainability should align with the company's overall development objectives. Therefore, sustainable solutions must strike a balance between production costs, enterprise profitability, and environmental sustainability, rather than solely prioritizing the latter.

Limited articles currently address the design of joint optimization for PFA and SMPs [11]. These articles often employ multi-objective programming methods [12–14]. However, adopting this method faces limitations. Firstly, it often overlooks the coupling relationship between PFA and SMP scheme selection. Secondly, it fails to address conflicting objectives among different decision-makers [15]. Specifically, technical challenges encompass the following facets.

Hierarchical joint optimization. The complexity lies in establishing the hierarchical joint optimization (HJO) decision-making framework. The PFA design and SMP selection involve distinct decision-makers: the PFA designers and the sustainable manufacturers of the product family, respectively. Enterprises may have different decision variables and goals, which can sometimes conflict [15]. On one hand, PFA designers aim to maximize both customer-perceived utility and sustainable utility per unit cost. They can influence the choice of SMP scheme through modular architecture and configuration scheme design. On the other hand, sustainable manufacturers strive to make sustainable manufacturing solutions based on comprehensive evaluation indicators, which in turn feed back to the PFA designers and impact PFA decisions. The choice of PFA design and SMP selection constitutes an HJO problem, with the designer acting as the leader and the sustainable manufacturer as the follower. This necessitates an HJO decision-making framework that seeks a balanced solution through noncooperative games. Despite numerous studies on bilevel optimization, explicitly formulating an HJO decision-making framework to describe the PFA and SMP problems in engineering remains challenging [16].

Heterogeneous decision criteria. Another challenge addressed in this paper is the construction of a heterogeneous decision criterion. PFA decisions primarily involve multiple levels of modules, whereas SMP decisions focus more on sustainability considerations. For instance, PFA design entails combining compound modules and configuring basic modules [17]. Sustainable solutions encompass the selection of raw materials, processing methods, recycling methods, and end-of-life processing methods. Furthermore, these two types of decision problems entail diverse decision criteria, such as customer

utilities, purchase preferences, modularization, and comprehensive sustainable certification standards, which encompass cost, quality, emissions, and energy consumption, among others [18]. Dealing with multi-dimensional and heterogeneous decision criteria in the optimization problem presents another challenge in this paper.

The objective of this study is to establish an HJO model by which to address the hierarchical optimization problem of PFA and SMPs. Firstly, we must delve into the decision mechanism and achieve a hierarchical joint optimization process for PFA and SMP decisions based on a Stackelberg game. Secondly, a bilevel programming model implementing the joint optimization of PFA and SMPs must be designed, establishing heterogeneous decision criteria for different decision-makers. Thirdly, a nested genetic algorithm (NGA) that aligns with the HJO model solving mechanism must be developed, and its effectiveness and reliability must be validated through comparative experiments.

In light of these considerations, this paper employs the HJO method to analyze the PFA and SMP problems. Section 2 provides a summary of previous research on PFA, SMPs, and bilevel programming. Section 3 establishes a conceptual model for PFA and SMP solutions. Section 4 formulates the mathematical model of bilevel 0–1 integer nonlinear programming. Section 5 develops a nested genetic algorithm (NGA) based on the established HJO model. Section 6 presents a laptop case study, demonstrating the applicability of the HJO model and NGA algorithm proposed in this paper. Finally, Section 7 summarizes the research findings and outlines future research prospects.

2. Literature Review

2.1. Product Family Architecture

A product family shares a common platform with specific features and functionalities to cater to different sets of customers [19]. Product Architecture refers to how the functional elements of a product are organized into physical units that interact accordingly [8]. PFA design enables the creation of a diverse array of products tailored to individualized needs within a unified framework. This approach facilitates economies of scale, allowing the fulfillment of customer demands across various market segments [19].

In articles regarding PFA, different evaluation metrics are applied. Tyagi et al. [20] research product family design and multiple platform architecture by fuzzy goal programming, with the objectives of maximizing the overall utility and minimizing the total production cost. Trentin and Alessio [21] aim to improve configuration capabilities to increase consumers' perceived benefits of the mass customization experiences. Yoo et al. [22] have considered increasing consumer-perceived benefits and consumer-perceived values. Ma et al. [23] have studied products' family-driven module design, the objective function of which is to maximize the sum of modularity satisfaction. In recent years, scholars have gradually begun to investigate the integration of product family design within the sustainable implications in Industry 4.0. Ceschin and Idil [24] have proposed an evolutionary framework of design for sustainability and mapped the reviewed design for sustainability approaches onto this framework. Xiao et al. [25] have increased carbon emissions to evaluate a low-carbon product family architecture. Tao et al. [26] have presented a digital twin-driven product design framework. Lim et al. [27] have reviewed engineering product lifecycle management from the perspective of a digital twin.

At present, research that considers PFA design and SMP solutions in the context of intelligent manufacturing has focused mainly on the theoretical level. Few scholars have considered establishing a joint decision-making framework that combines PFA design and SMPs.

2.2. Sustainable Manufacturing Processes

Sustainability encompasses economic, environmental, and social dimensions [28]. Many scholars have attempted to quantify sustainability criteria from these perspectives [29]. Jayal et al. [2] focused on societal aspects, such as worker health, safety, and

ergonomics. Kremer et al. [30] advocate for a consideration of sustainability throughout the entire lifecycle activity, aiming to minimize total costs and carbon footprint. Madani and Rasti-Barzoki [31] argue that sustainable management should take into account pricing, greening efforts, and governmental roles, with governments playing a leadership role in these areas. Mangla et al. [32] have evaluated sustainability based on cost, energy efficiency, and material utilization in production techniques.

Some scholars assess sustainability across various stages of product manufacturing. However, the majority of analytical sustainable product modeling has, for many decades, relied on qualitative approaches [33]. Garbie [34] proposes to cut down the material, the energy consumption and the emission throughout the product lifecycle. Yan and Feng [35] have established a structure matrix considering materials, manufacturability, end-of-life stages and so on. Kim and Moon [36] have developed a modular product architecture while considering manufacturing and recovering processes at the product design stage. Kim and Moon [10] have developed a model involving materials production, transportation, use, and end-of-life stages. Guo et al. [37] believe that the green performance of a fashion product relates to the material(s) used in the manufacturing process.

Currently, few researchers have developed a quantitative optimization framework for SMPs and integrated it with PFA design. This paper proposes a joint optimal decision-making framework for PFA and SMPs.

2.3. Bilevel Programming

Bilevel programming involves two decision-makers and is well-suited for addressing leader–follower optimization decision-making problems [38]. It has garnered significant attention in academia and has been applied across various fields, including reconfigurable process planning [39], crowdsourcing service operations, product family design of personalized services [40], and the data-driven newsvendor problem [41]. Unlike ordinary mathematical programming, bilevel programming presents unique challenges. Because the upper-level model incorporates the optimal solution or optimal value function of the lower-level model, the problem becomes non-smooth [42]. Even linear bilevel programming is NP-hard, and when the upper-level constraint includes decision variables from the lower-level model, the feasible region may become disconnected [43].

Many scholars are devoted to studying the solution method of bilevel programming. When the lower-level model encompasses convex programming with a continuous variable, the KKT condition can be used to replace the lower-level model, and the solution method of single-level mathematical programming can then be used to solve this model [44]. The K-degree best method involves obtaining the optimal solution of a problem by implicitly enumerating all poles of the constrained domain of a linear bilevel programming problem [45]. In addition, models that do not meet the above types adopt approximation algorithms to solve bilevel programming models [38]. Condition types that are often used include the following: Firstly, the expression of $y(x)$ is estimated and then substituted into the upper level to transform the model into common single-level mathematical programming, which can then be solved by the solution algorithms of single-level mathematical programming [46]. Secondly, one must develop a heuristic algorithm that conforms to the solution framework of the bilevel programming in order to perform iterative calculations to obtain approximate solutions [43]. Currently, there is a lack of a solution algorithm for solving bilevel 0–1 integer nonlinear programming established in this paper.

3. Consideration Sustainable Product Family Design

3.1. Research Methodology

Based on the methodology of management science, a research framework is presented to better explain the development steps of our study.

First and foremost, the identification of the research problem is crucial. Through empirical research on manufacturing enterprises, it has been discovered that there is a mutual influence between PFA design and SMPs, and that a hierarchical relationship exists between product designers and sustainable manufacturers. Literature review reveals that the hierarchical relationship between the two agents has not been studied yet. Therefore, the problem is refined and formulated into key optimization decision issues, accompanied by an analysis and description of the problem.

Subsequently, the establishment of the research model is essential. A bilevel optimization model based on Stackelberg games is developed to characterize the relationship between PFA design and SMPs. The upper-level PFA design solutions are modeled based on product design theory and methods. The MNL model is used to estimate the probability of the product selection. SMPs are modeled based on the theory of the lifecycle manufacturing processes and the definition of sustainable development.

Furthermore, in line with the established 0-1 nonlinear bi-level optimization model, we develop an NGA tailored to meet the requirements of the bilevel optimization solving mechanism. Through case studies that focus on laptop design and sustainable manufacturing processes, we validate the efficacy of our proposed model and algorithm. Employing a conjoint analysis method in the case study has enabled us to quantitatively ascertain consumer preferences. Comparative experiments with integrated optimization methods and two-stage optimization methods corroborated the superiority of our proposed HJO model. The reliability of our proposed algorithm is emphasized through experiments that assess optimality and stability.

3.2. Problem Description

When an enterprise possesses a significant volume of product usage data, maintenance data, and customer demand information, it can intelligently optimize PFA and SMPs based on these data. This enables the development of the most optimal product family design and sustainable manufacturing plan in alignment with the current data landscape. In different markets, consumers exhibit distinct preferences for various product attributes. Accordingly, the size of each market segment, denoted as Q_i , represents the number of consumers in the i -th segment. There are J product variants in the product families of each market segment, offering the possibility to meet consumers' individualized product needs. At the same time, there are N_c competitive products in the market. As shown in Figure 1, each product variant is composed of compound modules $CM_r (r = 1, \dots, R)$, each compound module is composed of several basic modules $SM_k (k = 1, \dots, K)$. Some basic modules have some optional module instances $p_{kl}^* (l = 1, \dots, L_k)$, which represent different product attributes. Some compound modules (M_1) and basic modules (M_2 and M_N) are standard parts that cannot be selected, and which are often the core part of the product.

We analyze sustainability across the product lifecycle through three key aspects: cost, energy consumption, and emissions. This is divided into four stages. The first stage involves selecting raw materials for the module instance, followed by the second stage, which encompasses the production methods of the compound modules. The third stage focuses on the selection of recycling methods for the product variant, while the final stage involves selecting end-of-life processing methods of the product family.

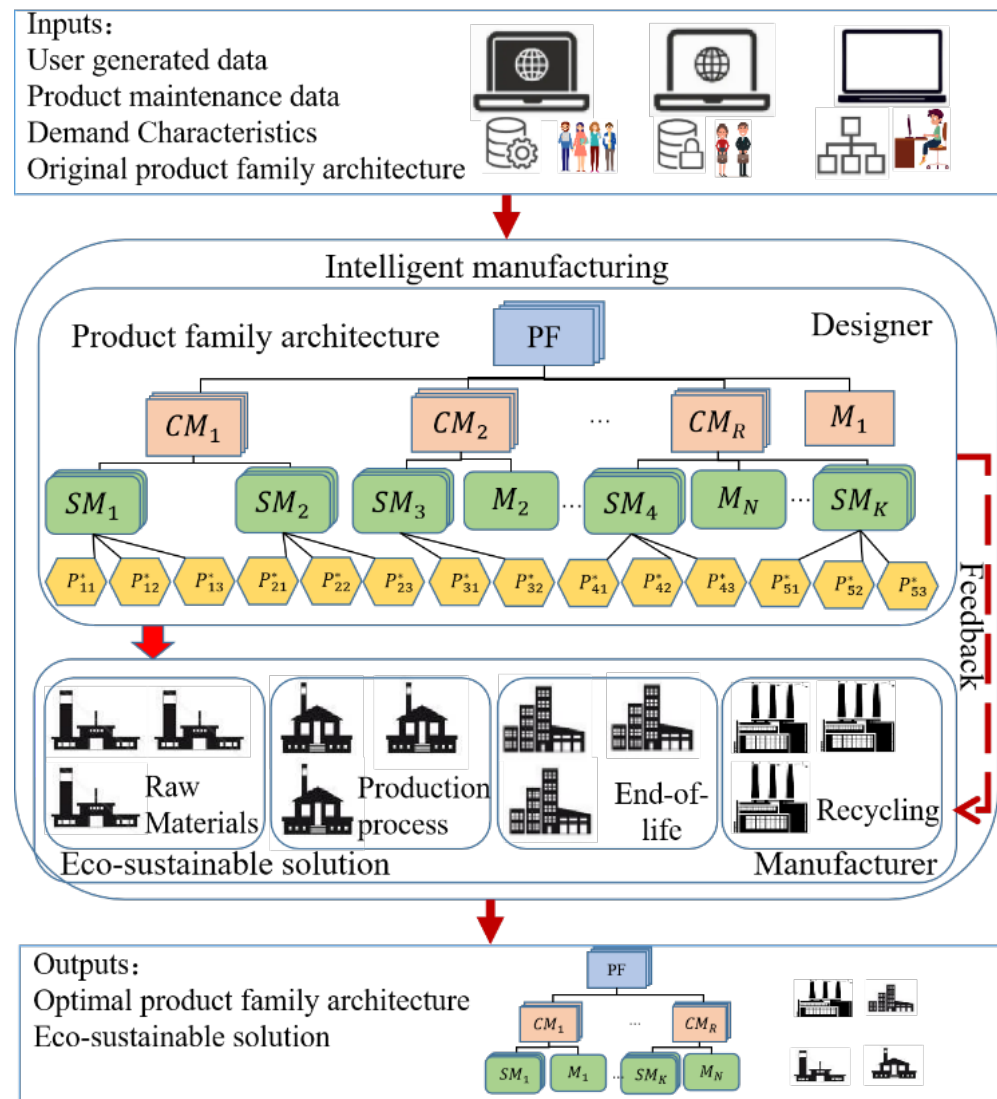


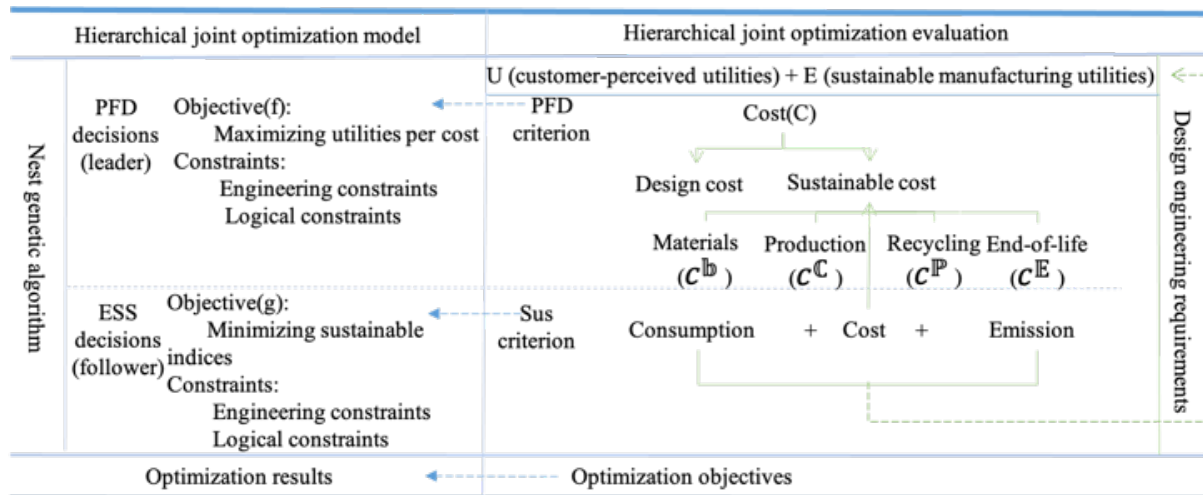
Figure 1. HJO decision making of PFA and SMPs.

3.3. Hierarchical Interactive Decisions for Dynamic Evaluation

Assuming both designers and manufacturers are rational decision-makers. Designers hold a dominant position as they investigate market demands and production costs, they formulate product variant combinations for segmented markets. Manufacturers are subordinate because they need to implement SMPs at different stages based on PFA designs. The decisions made by designers can influence the decision-making of manufacturers regarding SMPs, thereby altering the costs, energy consumption, and emissions associated with SMPs. Simultaneously, sustainable decision-making can impact the utility and costs of a product family. Thus, a Stackelberg game emerges between the designer and manufacturer. An HJO problem arises between PFA and SMP schemes.

As shown in Figure 2, an HJO decision mechanism is presented. The left is the HJO model, and the right is the HJO evaluation mechanism. The leader of the model is the design of the PFA and configuration in each market segment (i). Its objective is to maximize unit cost (C) utilities ($U + V$). Costs include product design cost (c^{DC}) and sustainable costs (c^S), which include raw material costs (c^b), manufacturing costs (c^c), recycling costs (c^p), and processing costs (c^E). Utilities cover customer-perceived utilities and sustainable manufacturing utilities which are determined by the combination of energy consumption (S) and emissions (D). The sustainable decision-making of the follower model has three aspects that need to be considered: costs, energy consumption and carbon emissions. The

lower objective function is to minimize the comprehensive evaluation indices of the SMPs. Both the upper and lower constraints of the model include two parts: the logical constraints and the functional constraints. The right-to-left arrows indicate that our model is based on a Stackelberg game.



Product family design: PFD

Sustainable manufacture processes: SMP

Figure 2. Hierarchical joint optimization decision mechanism.

3.4. Illustrative Example

Taking a brand laptop as a case for discussion and analysis. According to the research of the market segment and the functional constraints of the computer architecture, the product type and the number of compound modules for a certain market segment are pseudo-located as follows: $J = \{2, 3\}$; $R = \{2, 3, 4\}$. After the arrangement and combination, there are six cases, which are $J = 2, R = 2$; $J = 2, R = 3$; $J = 2, R = 4$; $J = 3, R = 2$; $J = 3, R = 3$; $J = 3, R = 4$. Based on the calculations of the final model, the optimal combination of PFA is obtained. For the sake of simplicity and without loss of generality, this paper properly handles the structure and data of the laptop. The PFA is shown in Figure 3. Assume a laptop is divided into 12 basic modules: shell, display, speaker, motherboard, graphics card, CPU, RAM, keyboard, hard disk, battery, fan, and optical drive. Each basic module corresponds to several module instances, with each module instance representing an attribute feature. For example, the shell's three module instances are black, red and white. In addition to the optimal PFA solution, the upper-level model will calculate the optimal combination of product configurations.

We provide an example to illustrate the lower-level model. If a market segment corresponds to a product family PF, there are three types of products: PV_1, PV_2, PV_3 . There are two compound modules, CM_1^2 and CM_2^2 , in some product variants, three basic modules, M_4, M_8 , and M_{10} , are in CM_1^2 , while M_{10} has three attributes, m_{101}, m_{102} , and m_{103} . As each module instance is composed of different materials, decisions need to be made regarding the types of raw materials at the module instance level. Assume that there are two materials to choose from for m_{101} , each corresponding to different fixed costs, variable costs, energy consumption, and carbon emissions. A judgement must be made based on these data in order to provide a satisfactory materials selection for the module instance m_{101} . Assume that for compound module CM_1^2 , there are two production methods to choose, each of which corresponds to different production design costs, manufacturing costs, energy consumption and carbon emissions. In the same way, one must choose the most suitable production method. After the product variant PV is used from the consumer group, the corresponding recycling mechanism is formulated. At this stage,

each PV has several recycling methods. It is assumed that PV_1 has two recycling methods. A similar method is used to find the most reasonable recycling way. According to the characteristics of a product family, we give several treatments at a product's end-of-life stage, such as incineration, reuse, landfill, etc. At this level, one or more processing methods are reasonably determined according to the characteristics of the product family. In conjunction with the other three levels of SMP choices, one must select the optimal approach for addressing sustainability.

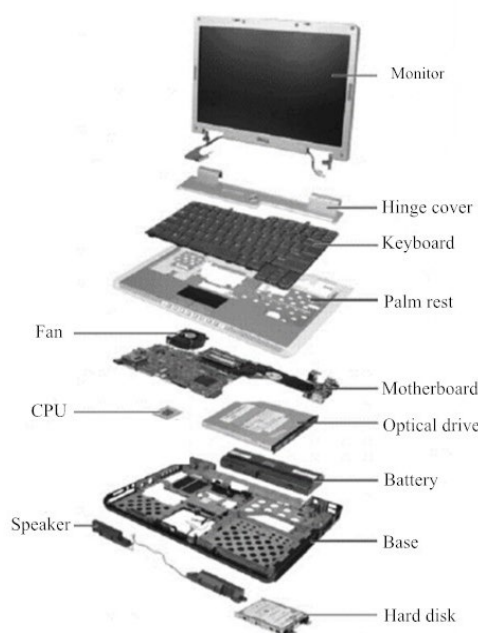


Figure 3. The laptop architecture.

4. Hierarchical Joint Optimization

We propose an HJO mechanism for PFA and SMPs based on Stackelberg game. The upper-level model decides the PFA scheme, and the lower-level model decides the SMPs of the product family.

4.1. Parameters

The parameters used in the HJO model are shown in Table 1.

Table 1. Notations for model parameters.

Parameters	Parameter Description
U_{ij}	Utility for the j -th product variant in the i -th market segment.
w_{jk}	Weight for the k -th basic module of the j -th product variant.
u_{ikl}	Utility for the l -th module instance of the k -th basic module in the i -th market.
π_{ij}	Constant related to the comprehensive utility for the j -th product variant in i -th market segments.
ε_{ij}	Error term for the j -th product variant in the i -th market segment.
E_{ij}	Sustainable utility for the j -th product variant in the i -th market segment.
e_{ijk}	Sustainable utility for the l -th module instance of the k -th basic module in i -th market segments.
P_{ij}	Probability for the j -th product variant in the i -th market segment.
C	Enterprise total cost.
C^D	Enterprise design cost.
C^S	Enterprise sustainable cost.
Q_i	Number of customers in the i -th market segment.
c^{DF}	Fixed design cost.

c_{jrklm}^{DB}	Sustainable design cost for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant.
c_{jrp}^{DC}	Sustainable design cost for selecting the p -th production method of the r -th compound module of the j -th product variant.
c_{jc}^{DP}	Sustainable design cost for selecting the c -th recycling method of the j -th product variant.
c_e^{DE}	Sustainable design cost for selecting the e -th end-of-life processing method of the product family.
c_{klm}^{MF}	Fixed cost for selecting the m -th raw material of the l -th module instance of the k -th basic module.
c_{klm}^{MV}	Unit variable cost for selecting the m -th raw material of the l -th module instance of the k -th basic module.
c_{rp}^{CF}	Fixed manufacturing cost for selecting the p -th production method of the r -th compound module.
c_{rp}^{CV}	Unit variable manufacturing cost for selecting the p -th production method of the r -th compound module.
c_{jc}^{PF}	Fixed recycling cost for selecting the c -th recycling method of the j -th product variant.
c_{jc}^{PV}	Unit variable recycling cost for selecting the c -th recycling method of the j -th product variant.
c_e^{EF}	Fixed process cost for selecting the e -th end-of-life processing method.
c_e^{EV}	Unit variable process cost for selecting the e -th end-of-life processing method.
d_{jrklm}	Emission for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant.
d_{jrp}	Emission for selecting the p -th production method of the r -th compound module of the j -th product variant.
d_{jc}	Emission for selecting the c -th recycling method of the j -th product variant.
d_e	Emission for selecting the e -th end-of-life processing method of the product family.
s_{jrklm}	Consumption for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant.
s_{jrp}	Consumption for selecting the p -th production method of the r -th compound module of the j -th product variant.
s_{jc}	Consumption for selecting the c -th recycling method of the j -th product variant.
s_e	Consumption for selecting the e -th end-of-life processing method of the product family.
τ_j	Recovery probability of the j -th product variant.
β_e	Proportion of e -th end-of-life processing method of the product family.
J^+	Maximal number of product variants.

4.2. Decision Variables

The decision variables used in the model are as follows:

x_{jrkl} : Binary integer = 1 or 0, depending on whether the k -th basic module of the r -th compound module of the j -th product selects the l -th module instance.

y_{jrklm} : Binary integer = 1 or 0, depending on whether the l -th module instance of the k -th basic module of the r -th compound module of the j -th product selects the m -th raw material.

y_{jrp} : Binary integer = 1 or 0, depending on whether the r -th compound module of the j -th product selects the p -th production method.

y_{jc} : Binary integer = 1 or 0, depending on whether the j -th product selects the c -th recycling method.

y_e : Binary integer = 1 or 0, depending on whether the product family selects the e -th end-of-life processing method of the product family.

x_{jrkl} is the upper-level decision variable and y_{jrklm} , y_{jrp} , y_{jc} , and y_e are the lower-level decision variables. Figure 4 is a structural of the decision variables. The upper part describes the choice of PFA (x), and the lower part describes the choice of SMPs. The upper-level decision variable x is passed to the lower-level model by calculating the upper-level objective function, and the lower-level decision variable y feeds back to the upper-level model through the result of the lower-level objective function value, thus looping until the optimal solution satisfying the upper and lower-level constraints is obtained.

$$\alpha_{1jklm} = \sum_{n=1}^N \vartheta_{njklm} (b_{njklm}^e m_{jklm}^e + b_{njklm}^g m_{jklm}^g), \quad (4)$$

where n represents the type of consumption, for example lighting and heating, ventilation, or air conditioning; ϑ_{njklm} describes consumption performance of the n -th type for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant; b_{njklm}^e indicates the power consumption of group n for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant; and b_{njklm}^g is fuel (natural gas/coals) consumption for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant. m_{jklm}^e is total annual power consumption for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant, m_{jklm}^g is total annual fuel consumption for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant.

Equation (5) represents the emission index.

$$\alpha_{2jklm} = \sum_{n=1}^N \xi_{njklm} R_{njklm} t_{njklm}, \quad (5)$$

where n represents the type of emission. ξ_{njklm} describes emission performance of the n -th type for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant. R_{njklm} is reuse percentage of the n -th type for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant. t_{njklm} is total disposal amount of the n -th type for selecting the m -th raw material of the l -th module instance of the k -th basic module of the r -th compound module of the j -th product variant, tons/yr.

The multinomial logit (MNL) selection rules are known for providing a more realistic representation of consumer decision-making processes and are used to simulate customer selection probabilities for products in product design [50]. Specifically, they are expressed as in Equation (6).

$$P_{ij} = \frac{\exp [\mu(U_{ij} + E_{ij})]}{\sum_j^J \exp [\mu(U_{ij} + E_{ij})] + \sum_j^{N_c} \exp [\mu(U_{ij} + E_{ij})]}, \quad (6)$$

where μ represents a positive scaling parameter of the MNL model. As μ tends to infinity, the model approaches a deterministic selection rule. Conversely, as μ tends to 0, the model approximates a uniformly distributed selection rule.

The total cost is composed of design cost and sustainable cost, which is as follows:

$$C = c^D + c^S. \quad (7)$$

The design cost includes the following components: fixed design cost, raw material design cost, production method design cost, recycling method design cost, and end-of-life processing method design cost. The design cost is specifically expressed as follows:

$$c^D = c^{DF} + \sum_{i=1}^I Q_i \left\{ \sum_{j=1}^J P_{ij} \left[\sum_r^R \left(\sum_{k=1}^K \sum_{l=1}^{L_k} x_{jrkl} \sum_{m=1}^{M_{lk}} y_{jrklm} c_{jrklm}^{DB} + \sum_{p=1}^{P_r} y_{jrpp} c_{jrpp}^{DC} \right) + \sum_{c=1}^C y_{jc} c_{jc}^{DP} \right] + \sum_{e=1}^E y_e c_e^{DE} \right\}. \quad (8)$$

4.4. Lower-Level Optimization

The lower-level model selects the sustainable solution for the product. In accordance with the upper decision-making scheme, the lower-level decision schemes are optimized to minimize the sustainable comprehensive evaluation indices.

For the lower level of the model, the following assumptions are made.

- (1) Product family cost is divided by direct and indirect cost;
- (2) The SMPs of the product family only consider the economic, environmental and consumer demands.

Equation (9) is the SMP cost for a product family. The sustainable cost of the product primarily encompasses the expenses associated with raw materials, production methods, recovery methods, and end-of-life processing methods.

$$c^S = c^b + c^c + c^p + c^e, \quad (9)$$

Equations (10)–(13) represent the engineering cost of the four stages of the product family.

$$c^b = \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R \sum_{k=1}^K \sum_{l=1}^L x_{jrkl} \sum_{m=1}^{M_{lk}} y_{jrklm} (c_{klm}^{MF} + Q_i P_{ij} c_{klm}^{MV}), \quad (10)$$

$$c^c = \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R \sum_{p=1}^{P_r} y_{jr p} (c_{rp}^{CF} + Q_i P_{ij} c_{rp}^{MV}), \quad (11)$$

$$c^p = \sum_{i=1}^I \sum_{j=1}^J \sum_{c=1}^C y_{jc} (c_{jc}^{PF} + Q_i P_{ij} c_{jc}^{PV}) \tau_j, \quad (12)$$

$$c^e = \sum_{i=1}^I \sum_{j=1}^J \sum_{e=1}^E y_e (c_e^{EF} + Q_i P_{ij} c_e^{EV}) \beta_e. \quad (13)$$

Equation (10) indicates that the module instances take both fixed and variable costs into account when selecting raw materials. Equation (11) represents the manufacturing cost of the compound modules, which is composed of fixed and variable costs. Equation (12) represents the recycling cost of product variants, including fixed and variable costs. Equation (13) represents the cost of different processing methods at the end-of-life stage of the product family, including fixed and variable costs.

Equation (14) is emissions.

$$D = \sum_{j=1}^J \left\{ \sum_{r=1}^R \left(\sum_{k=1}^K \sum_{l=1}^L \sum_{m=1}^{M_{lk}} y_{jrklm} d_{jrklm} + \sum_{p=1}^{P_r} y_{jr p} d_{jr p} \right) + \sum_{c=1}^C y_{jc} d_{jc} \tau_j \right\} + \sum_{e=1}^E y_e d_e \beta_e. \quad (14)$$

Equation (15) represents the energy consumption.

$$S = \sum_{j=1}^J \left\{ \sum_{r=1}^R \left(\sum_{k=1}^K \sum_{l=1}^L \sum_{m=1}^{M_{lk}} y_{jrklm} s_{jrklm} + \sum_{p=1}^{P_r} y_{jr p} s_{jr p} \right) + \sum_{c=1}^C y_{jc} s_{jc} \tau_j \right\} + \sum_{e=1}^E y_e s_e \beta_e. \quad (15)$$

4.5. HJO Decision Making of PFA and SMPs

The HJO model can be obtained as follows:

$$\text{Max } F = \frac{U_{ij} + E_{ij}}{C} P_{ij} Q_i \quad (16)$$

$$s. t. \sum_{k=1}^K \sum_{l=1}^{L_k} (x_{jrkl} - x_{j'_{rkl}}) \geq 0 \quad j \neq j' \quad (17)$$

$$\sum_{l=1}^{L_k} x_{jrkl} = 1 \quad (18)$$

$$\sum_j x_j \leq J^+ \quad (19)$$

$$x_{jrkl} \in \{0,1\} \quad (20)$$

$$\text{Min } f = \frac{1}{\hat{K}} [(\hat{K}k_1C + 1)(\hat{K}k_2D + 1)(\hat{K}k_3S + 1) - 1] \quad (21)$$

$$s. t. 1 + \hat{K} = \prod_{i=1}^3 (1 + \hat{K}k_i) \quad (22)$$

$$x_{jrkl} = \sum_{m=1}^M y_{jrklm} \quad (23)$$

$$\sum_{p=1}^P y_{jrp} = 1 \quad (24)$$

$$\sum_{c=1}^C y_{jc} \geq 1 \quad (25)$$

$$\sum_{e=1}^E y_e \geq 1 \quad (26)$$

$$y_{jrklm}, y_{jrp}, y_{jc}, y_e \in \{0,1\} \quad (27)$$

Equation (16) is the objective function of the PFA design problem. Equation (17) represents the unique constraint of the product variant. Equation (18) indicates that every basic module only selects one module instance. Equation (19) represents the product quantity constraint. Equation (20) represents the range constraint of the upper-level decision variable.

Equation (21) is the objective function of the SMP problem. $k_i (i = 1,2,3)$ indicates a single attribute scaling constant and \hat{K} is a normalising constant range from 0 to 1. Equation (22) represents the relationship between \hat{K} and k_i . Equation (23) represents the relationship constraint for the instance selection variable and the raw material selection variable. Equation (24) indicates that only one production method can be selected for each compound module. Equation (25) indicates at least one or more recycling methods are available for each product variant. Equation (26) indicates at least one or more end-of-life process methods are available for the product family. Equation (27) is the range constraints of the lower-level decision variables.

In the HJO model, the upper-level PFA design determines the PFA and configuration scheme for a certain market segment. According to the decision results of the upper level, the lower level decides on the choice of raw materials y_{jrklm} , the production method y_{jrp} , the recycling method y_{jc} , and the end-of-life processing method y_e . The lower-level results determined according to the sustainable comprehensive evaluation indices are

passed to the upper-level model. According to the results of the lower-level feedback, the upper level will adjust the product architecture, and then reevaluate the decision variable x_{jrkl} to maximize the upper-level objective function. The loop continues until it reaches the Stackelberg equilibrium. The optimal solution of the model is reached when neither decision-maker is willing to change their decisions further, and the upper- and lower-level objective function values are calculated based on this optimal solution.

5. Solution of the Model

5.1. Algorithm Construction and Evaluation

The above bilevel 0–1 integer nonlinear programming model established, based on the actual engineering characteristics, is an NP-hard problem [43], which makes the model solution very difficult. The genetic algorithm is highly effective when efficiently discovering a global near-optimal solution and has the capability to evade local optima while overcoming the multimodality of the objective function [51]. Given the complexity of our model, we have developed a nested framework aligned with the bilevel programming solution mechanism and applying genetic algorithms to this framework, forming an NGA. Figure 5 illustrates the specific process of the NGA, with the detailed steps outlined as follows:

Step 1: Set the parameters: Set the parameters of PFA optimization, contain the upper-level and lower-level genetic algorithm population size N and M and the maximum number of iterations in upper and lower model GN and GM .

Step 2: Upper-level population initialization: According to the different PFA design and the upper-level bounds of the product variant configuration, the coding strategy of the variables are determined and the upper-level product variant is encoded.

Step 3: Determine that the upper-level constraints are satisfied: First, one must judge whether the individual population of the upper-level PFA design satisfies the upper-level constraint conditions. If it is satisfied, the individuals are passed to the lower-level and proceed to the next step. If not, set the upper-level fitness function to zero and jump to step 7. Secondly, the utility of unit cost of the whole product family is taken as the fitness function of the upper-level model.

Step 4: Lower-level population initialization: According to the different SMP schemes and the bounds of the lower-level decision variants, the coding strategy of the variables are determined and the lower-level SMP variants are encoded.

Step 5: Judgement of the lower-level constraints: Evaluate the parent populations of the lower-level model, setting their fitness values to zero if the populations do not satisfy the constraints. For populations that meet the constraints, utilize the lower-level sustainable comprehensive evaluation indices value as the fitness function values.

Step 6: Lower-level termination checking: Check whether the current number of iterations in the lower-level has reached the maximum limit set by GM . If the limit has been reached, record the optimal solutions along with their corresponding values. Then, feed back the decision variables for SMPs to the upper-level model. If the maximum iteration limit has not been reached, proceed with the selection, crossover, and mutation of the lower-level population individuals and move to step 5.

Step 7: Termination checking: Check whether the upper-level genetic algorithm has reached the maximum number of iterations, denoted as GN . If the maximum number of iterations has been reached, record the upper-level optimal solutions and their corresponding optimal value. If the maximum iteration limit has not been reached, continue with the selection, crossover, and mutation of individuals in the upper-level population and proceed to step 3.

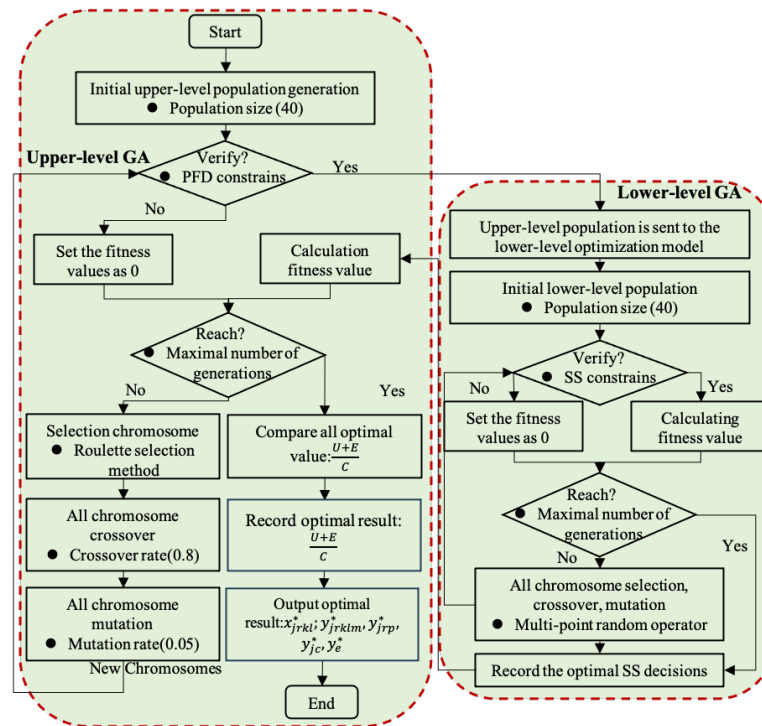


Figure 5. NGA process of HJO.

5.2. Encoding

To facilitate the calculation, we encode the upper- and lower-level decision variables into two chromosomes of finite length. The upper chromosomes represent the PFA design, and its length is the sum of all modules and corresponding module instances in the product variants. As shown in Figure 6a, the first layer represents the product family, and the different colors correspond with the products in the product family. Different PFAs result in different lengths of this layer. The second layer represents the product variants, and the product variant consists of several compound modules. As shown, the product variant $j \in J$ consists of $r \in R$ compound modules. The third layer represents the compound modules, which consists of several basic modules. The fourth layer represents module instances, with each corresponding number representing the selection of a specific module instance by a basic module. The upper-level code length is $J * R * L$. The lower-level chromosomes represent the choice of product family, product variants, compound modules and basic modules for the corresponding SMP solutions. The coding of the lower-level decision variables selects the 0–1 coding mode, 0 when a certain method is not selected and 1 when a certain method is selected. The specific coding method is shown in Figure 6b.

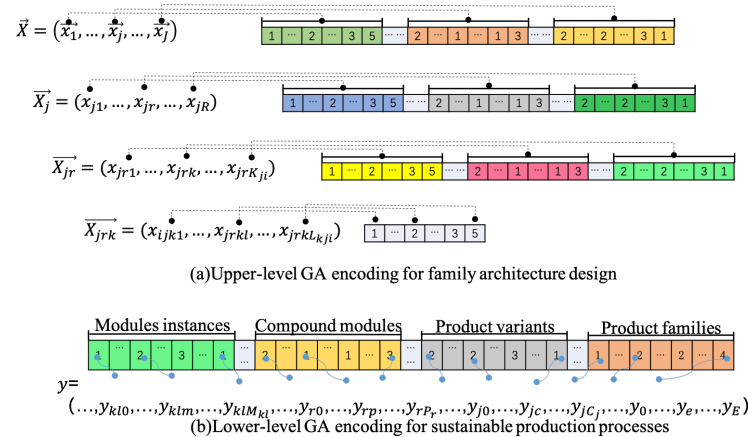


Figure 6. NGA encoding. (a) Upper-level GA encoding for PFA. (b) Lower-level GA encoding for SMPs.

5.3. Crossover and Mutation

The selection process involves the use of an operator to choose individuals with robust vitality from the population, thus generating new populations. This can be undertaken in order to obtain the best child chromosomes for survival in the evolution [52]. In this study, the operator is generated by roulette. This means that the probability of each chromosome being selected for the next generation is determined by the ratio of its fitness value to the total fitness value of all individuals in the population. Consequently, individuals with higher fitness values are more likely to be selected for the next generation.

Crossover is the primary method for generating new chromosomes. Illustrated in Figure 7, crossover involves two parent chromosomes exchanging part of their genes at a certain position to produce two new offspring chromosomes. Mutation, on the other hand, involves altering a portion of the chromosome's gene with a small probability after the crossover operation. Through the mutation operation, the corresponding module selections, recycling methods, etc., can be randomly altered.

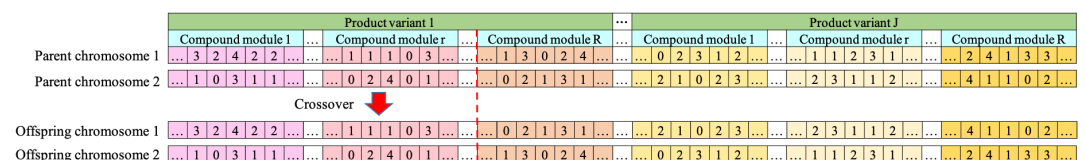


Figure 7. Crossover operations.

6. Case Study

6.1. Background Description

To verify the effectiveness of the proposed HJO model and NGA, it was applied to the design of a brand of a laptop computer product family. The laptop is a typical modular product. Its PFA is shown in Figure 8. To ensure the company's benign development while meeting the requirements of consumers, the company must determine the optimal PFA and corresponding configuration.

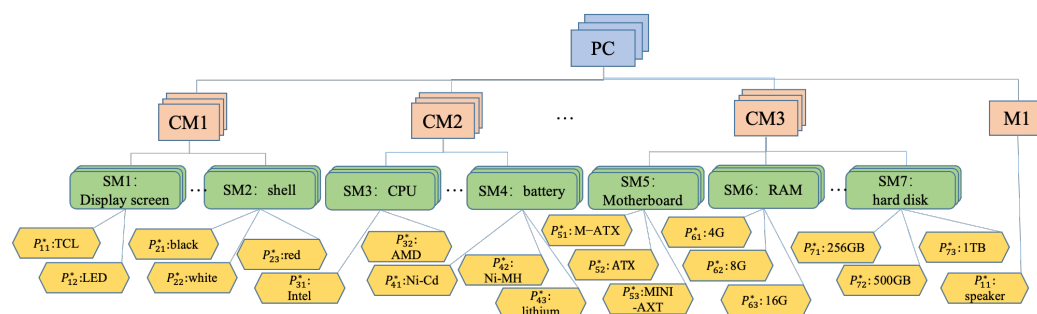


Figure 8. Structure of laptop.

The modules information for the laptop family is presented in Table 2. With a total of 12 modules, the number of potential options amounts to 46,656 ($4 \times 3^6 \times 2^4 \times 1$). Given the considerable number of selected product variants, conducting individual market research for each one is impractical. Consequently, to gauge consumer preferences across product segments, orthogonal analysis is employed on the alternatives, yielding 32 orthogonal product profiles (detailed in Table 3). Subsequently, a group of 50 consumers was selected to assess and rank the 32 product variants. Through conjoint analysis, the utility values of different modules within the product variants were determined for the market segment, as illustrated in Table 4. The estimated sizes of these market segments were 5000, 6000 and 8000. The population size for both upper and lower levels was fixed at 40 generations, with crossover and mutation probabilities set to 0.8 and 0.05, respectively. Parameters k_1 , k_2 and k_3 in the multi-attribute utility function are assigned values of 0.3, 0.35 and 0.35, respectively. The parameter μ can be set to 0.6.

Table 2. Modules in the laptop family.

ID	Name	m_{kl}	Attribute
M_1	Shell	m_{11}	Black
		m_{12}	White
		m_{13}	Red
M_2	Display	m_{21}	LCD
		m_{22}	LED
M_3	Speaker	m_{31}	Mono
		m_{32}	Stereo
M_4	Motherboard	m_{41}	ATX
		m_{42}	M-ATX
M_5	Graphics card	m_{51}	Standalone graphics
		m_{52}	Integrated graphics
M_6	CPU	m_{61}	Intel
M_7	RAM	m_{71}	4G
		m_{72}	8G
		m_{73}	16G
M_8	Keyboard	m_{81}	Mechanical
		m_{82}	Plastic film
		m_{83}	Conductive rubber
		m_{84}	Capacitive
M_9	Hard disk	m_{91}	1TB
		m_{92}	500GB
		m_{93}	256GB
M_{10}	Battery	m_{101}	Nickel-cadmium
		m_{102}	NiMH
		m_{103}	Lithium

M_{11}	Fan	m_{111}	60
		m_{112}	80
		m_{113}	120
M_{12}	Optical drive	m_{121}	CD
		m_{122}	CD/DVD

Table 3. Orthogonal product profiles with conjoint analysis.

ID	M_1	M_2	M_3	M_4	...	M_{12}
1	Red	LCD	Mono	ATX	...	DVD
2	White	LED	Mono	ATX	...	CD
3	Black	LCD	Stereo	ATX	...	DVD
4	Red	LCD	Mono	ATX	...	CD
...
31	Black	LCD	Stereo	M-ATX	...	DVD
32	Red	LCD	Mono	ATX	...	CD

Table 4. Part-worth utilities.

m_{kl}	u_{ikl}	m_{kl}	u_{ikl}
m_{11}	1.41	m_{71}	5.11
m_{12}	2.53	m_{72}	−1.35
m_{13}	6.42	m_{73}	2.46
m_{21}	6.47	m_{81}	−3.23
m_{22}	7.45	m_{82}	1.46
...
m_{61}	3.58	m_{122}	3.16

The compound module configuration classification is shown in Table 5. For the SMP problem information for the module instances, the compound modules, the product variants and the product family can be introduced in Tables 6–10. We assume that the information regarding the module instances of different products in each product family are the same.

Table 5. Compound modules configuration classification.

R	CM_r	m_{kl}
R = 2	CM_1^2	Motherboard, graphics card, CPU and hard disk are mandatory; the others are to be optimized
	CM_2^2	Shell, RAM, keyboard and battery are mandatory; the others are to be optimized
R = 3	CM_1^3	Motherboard, graphics card and RAM are mandatory; the others are to be optimized
	CM_2^3	Shell, CPU and hard disk are mandatory; the others are to be optimized
	CM_3^3	Display, speaker and optical drive are mandatory; the others are to be optimized
R = 4	CM_1^4	Display, speaker and RAM are mandatory; the others are to be optimized
	CM_2^4	Motherboard, graphics card and CPU are mandatory; the others are to be optimized
	CM_3^4	RAM and hard disk are mandatory; the others are to be optimized
	CM_4^4	Keyboard is mandatory; the others are to be optimized

Table 6. Raw material information for module instances.

m_{kl}	m	c_{jrklm}^{DB}	c_{klm}^{MF}	c_{klm}^{MV}	s_{jrklm}	d_{jrklm}
m_{11}	1	20	80	9	3	2
	2	22	70	8	6	4
m_{12}	1	25	80	8	8	3
	2	23	60	9	1	10

m_{13}	1	20	70	6	9	8
	2	21	90	9	10	2
m_{21}	1	260	300	20	50	32
	2	260	310	21	20	23
m_{22}	1	100	210	23	20	21
	2	120	290	25	19	40
...
m_{121}	1	200	190	10	10	17
m_{122}	2	180	200	12	16	6

Table 7. Sustainable utilities information for module instances.

m_{kl}	m	m_{jklm}^e	m_{jklm}^g	n	ϑ_{njklm}	b_{njklm}^e	b_{njklm}^g	ξ_{njklm}	R_{njklm}	t_{njklm}
m_{11}	1	0.9	0.5	1	0.5	2	1	1	23	5
			
				4	0.2	7	0	0.9	25	6
	2	0.7	0.9	1	0.4	6	4	0.8	20	4
			
				4	0.7	4	1	1.1	23	5
m_{12}	1	0.9	0.2	1	0.6	8	3	1.2	22	4
			
				4	0.9	5	0	0.6	25	6
	2	0.6	0.4	1	0.9	1	1	0.7	10	20
			
				4	0.8	7	5	0.9	22	4
m_{13}	1	0.7	0.6	1	0.7	9	8	1.1	10	3
			
				4	0.6	10	4	1.2	11	1
	2	0.9	0.9	1	0.7	4	2	1.2	10	30
			
				4	0.5	3	6	1.0	12	27
m_{21}	1	0.3	0.9	1	0.4	5	2	0.6	21	22
			
				4	0.9	2	2	0.7	19	8
	2	0.3	0.8	1	0.5	2	2	0.8	10	10
			
				4	0.2	3	0	0.9	12	6
m_{22}	1	0.7	0.2	1	0.4	2	2	1.0	22	9
			
				4	0.2	2	3	0.9	18	7
	2	0.8	0.5	1	0.2	1	4	1.2	10	2
			
				4	0.3	2	3	0.9	8	1
...
m_{121}	1	0.8	0.8	1	0.5	1	7	0.8	25	9
			
	2	0.7	0.9	1	0.4	1	6	1.0	24	3
			
				4	0.6	8	2	0.9	29	7
m_{122}	2	0.7	0.9	1	0.4	1	6	1.0	24	3
			
				4	0.6	5	7	1.1	28	8

Table 8. Production information for compound modules.

CM_r	P	c_{jrp}^{DC}	c_r^{CF}	c_r^{CV}	s_{jrp}	d_{jrp}
CM_{11}^2	1	900	500	25	90	50
	2	910	510	22	120	40
	3	800	450	23	80	30
CM_{21}^3	1	700	520	24	40	80
...
CM_{41}^4	1	500	300	11	30	25
	2	550	310	9	50	20
	3	400	350	12	10	17
	4	390	270	10	90	32

Table 9. Products recycling information.

Stage	PV	c	c_{jc}^{DP}	c_j^{PF}	c_j^{PV}	s_{jc}	d_{jc}
Recycling	1	1	27	300	10	9	3
		2	15	390	19	10	1
		3	83	330	14	15	2
	2	1	32	320	17	15	3
		2	37	290	17	22	4
		3	83	330	14	15	2
	3	1	26	290	13	13	3
		2	56	280	12	11	1
		3	37	470	12	8	1
	4	1	12	160	12	12	3
		2	57	590	12	17	4
		3	37	470	12	8	1

Table 10. End-of-life processing information for product family.

Stage	e	c_e^{DE}	c_j^{EF}	c_j^{EV}	s_e	d_e
End of life processing	1	23	300	73	130	34
	2	24	500	70	150	32
	3	25	100	68	200	19
	4	22	300	77	150	22
	5	20	200	79	180	47

6.2. Results of HJO Model

The developed solution method is adopted to solve the HJO model. The NGA is realized by MATLAB 2023b on an Intel(R) Core (TM)i5 and 16 GB RAM 3733 MHz with the following parameters: initial population scale is capped at 40; the crossover probability is set at 0.80; the mutation probability is 0.05; the maximal number of iterations is 150; and the precision of the binary code is set to 0.01, these settings are derived from experience in the domain of computational experiments. Figure 9 displays the optimal results for PFA design in SMPs. Various colors represent different PFA combinations across distinct product variants. Figure 9 comprises an x-axis representing the number of iterations and a y-axis representing the optimal value of the upper-level objective function for each scenario. The iteration count is set to 150 generations with a computational time of 2593 s. Figure 10 presents the results of the optimal SMPs for PFA scheme ($J = 3$, $R = 3$). The x-axis represents the number of iterations. The left y-axis illustrates the change in optimization results at the upper-level model, while the right y-axis depicts the change in optimization results at the lower-level model. Throughout the entire optimization process, there is a mutual influence between the upper and lower levels. The optimization outcomes of both

the upper-level and lower-level models fluctuate with each generation and begin to converge around the 95th generation, continuing until the end of the iterations. Both the leader and follower are reluctant to further alter their decisions, which suggests that the two decision-makers have reached a state of equilibrium.

Table 11 shows the optimal SMPs for the laptop PFA design. For example, the first product variant consists of three compound modules, while CM_{11}^3 consists of four basic modules. Each basic module selects the corresponding product configuration m_{42} , m_{51} , m_{73} and m_{84} , and each module instance selects the corresponding raw material in the fourth column. m_{42} selects the corresponding second raw material. CM_{11}^3 selects the third production method, product variant 1 selects the second recycling method, and the product family selects the third end-of-life processing method.

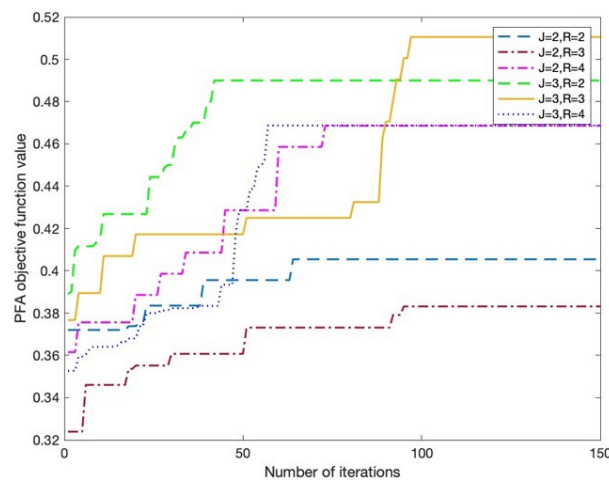


Figure 9. Evolution processes with respect to different settings of (J, R).

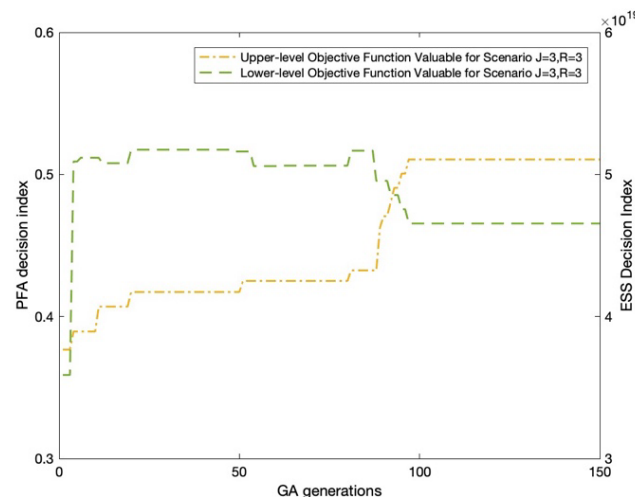


Figure 10. Evolution processes for both upper-level and lower-level NGA in scenarios $J = 3$ and $R = 3$.

Table 11. Optimal laptop PFA considering SMPs.

PV	CM_r	m_{kl}	y_{jrklm}	y_{jrp}	y_{jc}	y_e
1	CM_{11}^3	m_{42}	2	3	2	
		m_{51}	1			
		m_{73}	3			
		m_{84}	1			
	CM_{12}^3	m_{13}	1	2		
		m_{61}	2			
		m_{93}	1			
		m_{102}	2			
	CM_{13}^3	m_{111}	1	2		
		m_{21}	1			
		m_{31}	1			
		m_{83}	1			
		m_{103}	2			
		m_{122}	1			
2	CM_{21}^3	m_{41}	1	3	3	3
		m_{52}	2			
		m_{71}	2			
		m_{102}	2			
		m_{113}	1			
	CM_{22}^3	m_{12}	2	2		
		m_{62}	2			
		m_{82}	2			
		m_{93}	1			
		m_{103}	2			
	CM_{23}^3	m_{113}	1	2		
		m_{22}	2			
		m_{32}	3			
		m_{84}	1			
		m_{101}	2			
	m_{112}	1				
	m_{121}	2				
3	CM_{31}^3	m_{41}	1	3	2	
		m_{51}	1			
		m_{73}	3			
		m_{81}	1			
		m_{101}	2			
	CM_{32}^3	m_{111}	1	2		
		m_{13}	1			
		m_{62}	2			
		m_{83}	1			
		m_{92}	2			
	CM_{33}^3	m_{101}	2	2		
		m_{113}	1			
		m_{22}	2			
		m_{32}	3			
		m_{102}	2			
	m_{112}	1				
	m_{121}	2				

6.3. Sustainability Analysis for Problem

To validate the advantages of our proposed sustainable product design and manufacturing process, we conducted comparative experiments between our proposed optimization design problem and a non-sustainable optimization design problem. In the non-sustainable optimization design model (NS-HJO), our upper-level optimization objective is to maximize the utility of unit cost. The lower-level optimization objective is to minimize the lifecycle cost. This means that the upper-level no longer considers sustainable

manufacturing utilities, while the lower-level no longer considers consumption and emissions.

The experimental results are shown in Table 12. One can observe that the optimal solutions for sustainable PFA schemes are different from those without sustainability considerations. For the former, the optimal solution involves three product variants in a product family, each with three combinations of compound modules. For the latter, the optimal PFA design involves three product variants in a product family, each with two combinations of compound modules. Additionally, compared with the non-sustainability optimization problem, the utility of unit cost obtained by considering the sustainability problem is increased by 155% from 2.0×10^{-1} to 5.1×10^{-1} . This suggests that considering sustainable product development activities not only meets consumer demands for the product family but also fulfills the environmental requirements of society, government, and customers.

Table 12. Result comparison of different methods for different settings of (J; R).

Approach	(J, R)	J = 2; R = 2	J = 2; R = 3	J = 2; R = 4	J = 3; R = 2	J = 3; R = 3	J = 3; R = 4
HJO	U/C ($\times 10^{-1}$)	4.1	3.8	4.6	4.9	5.1	4.6
	Index ($\times 10^{19}$)	4.9	4.8	4.8	4.9	4.7	4.7
NS-HJO	U/C ($\times 10^1$)	1.8	1.6	1.5	2.0	1.9	1.8
	Index ($\times 10^6$)	1.5	1.4	1.4	1.5	1.5	1.4
IOM	U/C ($\times 10^{-1}$)	4.0	3.7	4.2	4.7	4.9	4.4
	Index ($\times 10^{19}$)	4.8	4.7	4.5	4.6	4.8	4.6
TSM	U/C ($\times 10^{-1}$)	3.6	3.5	4.1	4.6	4.7	4.4
	Index ($\times 10^{19}$)	4.9	4.8	4.5	4.5	4.8	4.9

6.4. Performance Analysis for Model

To demonstrate the superiority of the HJO model when addressing the optimal SMPs for PFA design, we compare the results obtained from this approach with those from two commonly used methods: the integrated optimization method (IOM) [52] and the two-stage method (TSM) [17]. The comparison results are presented in Table 12.

The IOM method is not based on the HJO mechanism of PFA and SMPs. The two problems are combined into one problem and the PFA and SMP solutions are decided at the same time. The leader and follower objective functions of the HJO act as the objective functions in IOM method, and the decision variables and constraints of the leader and follower models are used in the calculation of the IOM method. After obtaining the value calculated by the IOM method, it is taken into the lower-level objective function in order to calculate the lower-level model result. Compared with the integrated optimization method in $J = 3, R = 3$, the utility of unit cost obtained by the HJO method is increased by 4.1% from 4.9×10^{-1} to 5.1×10^{-1} , while the comprehensive evaluation index is decreased by 2.13% from 4.8×10^{19} to 4.7×10^{19} . This is mainly because the HJO design method prioritizes the cost-to-utility ratio of the leader, placing the comprehensive evaluation indices of the follower after the PFA design. The first-mover advantage of PFA design in bilevel decision-making promotes excellent quality and a favorable cost-to-utility ratio. Conversely, in the IOM method, PFA design and SMP solutions are equally important. Consequently, the product design department loses the first-mover advantage, resulting in a lower utility of unit cost.

The TSM divides PFA design and SMP solutions into two stages. Firstly, the PFA is optimized, and then SMP solutions are determined according to the PFA and configuration results. In the first stage, based on historical data and existing data of channels such as second-hand market, the product family manufacturing cost and utility are estimated and optimized. In the second stage, according to the obtained PFA and configuration

results, the corresponding raw materials, production methods, recovery methods and end-of-life process methods are selected, and sustainable comprehensive evaluation indices are calculated. In the first stage, the objective function is still the upper-level objective function, and the lower-level constraints are incorporated into the upper-level constraints. Compared with the two-stage method in $J = 3$; $R = 3$, the unit cost-utility ratio obtained by the HJO method increases by 8.5%, from 4.7×10^{-1} to 5.1×10^{-1} . Additionally, the sustainable comprehensive evaluation indices increase by 2.13%, from 4.8×10^{19} to 4.7×10^{19} . This is primarily attributed to the fact that the PFA cost and utility values are estimated solely through historical and secondary market data analysis, without considering the impact of SMPs on cost and utility. As a result, the PFA solution cannot be adjusted promptly when the SMP plan changes.

6.5. Reliability Analysis for Algorithm

We validate the reliability for the NGA algorithm through two sets of experiments.

- **Optimality Analysis:** Particle swarm optimization (PSO) is a relatively recent heuristic algorithm inspired by the social behavior of crowded species like bird flocking and fish schooling, which has demonstrated success across a wide range of optimization tasks [53]. Consequently, we undertake sensitivity analysis of the parameter μ in the MNL choice rule on the PFA objective function value using both our proposed NGA algorithm and the PSO algorithm within a bilevel solving framework in order to verify the reliability of our proposed algorithm in terms of optimality. An experiment is conducted by fixing μ as a series of constants ranging from 0 to 9 in increments of 1. The parameter settings of PSO are as follows: The number of swarms is 1, the number of particles is set to 20, and the number of generations is set to 100. The inertia weight (w) is set to 0.4 and 0.6. The acceleration coefficients ($c1$ and $c2$) are set to 2, and $rand1$ and $rand2$ follow a standard normal distribution with a mean of 0 and a variance of 1. As illustrated in Figure 11, it is apparent that the PFA objective function value calculated by the nested PSO algorithm for 10 points ranging from 0 to 9 is inferior to those obtained by the NGA. Consequently, we can infer that the reliability of our proposed NGA in terms of optimality surpasses that of the PSO algorithm.
- **Stability Analysis:** The second set of experiments involves conducting multiple trials for each value of the parameter μ in the MNL choice rule to determine deviation values. Subsequently, we assess the model's stability by comparing the range of deviation intervals for each value. The shaded area depicts the error band from multiple trials. As illustrated in Figure 11, the NGA displays varying levels of fluctuation across different parameter values. There is a smaller range of fluctuation compared with the PSO algorithm. Hence, we can infer that the reliability of our proposed NGA in terms of stability outperforms that of the PSO algorithm.

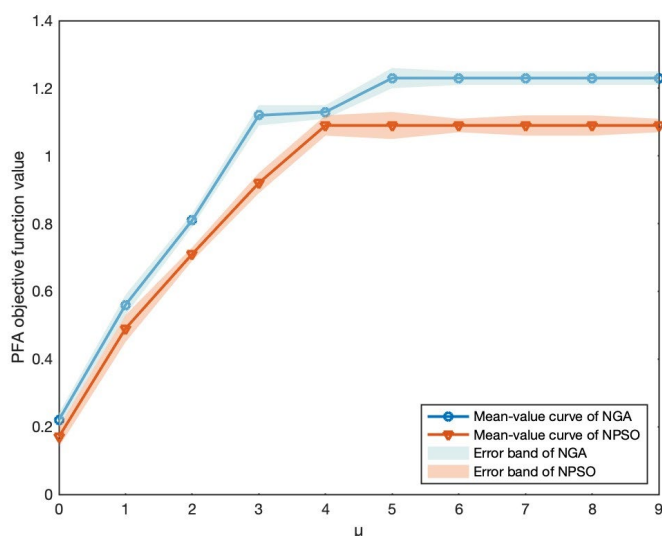


Figure 11. The reliability of μ on the MNL choice rule.

7. Conclusions, Limitations and Future Research Directions

7.1. Conclusions

This study focuses on the HJO of PFA and SMP solutions. We delve into the decision mechanism and achieve an HJO process for PFA and SMP decisions. An HJO model implementing the joint optimization of PFA and SMPs is designed, establishing heterogeneous decision criteria for different decision-makers. The PFA plays a leader's role, and it is intended to select the optimal PFA that meets the needs of a specific market niche. The choices of SMP solution are based on the design of the PFA and play a follower's role. It is intended to arrive at an optimal sustainable solution to meet government, consumer and business requirements for sustainable development. An NGA that aligns with the HJO model solving mechanism is developed. A laptop case is designed for the application of our proposed model and algorithm. The effectiveness of the model and the reliability of the algorithm are validated through comparative experiments. Based on the study results, some valuable management insights can be concluded, as follows:

- (1) Considering sustainable product family design and manufacturing processes is valuable. Drawing from the experimental outcomes in Section 6.3, it becomes evident that, while integrating sustainability into the product family could potentially lead to additional manufacturing and sustainable costs, these expenses pale in comparison with the considerable boost in market share and competitive advantage. This result is consistent with the prevailing research findings on green product innovation, further validating the correctness and necessity of our consideration of sustainable PFA design and manufacturing processes [54,55].
- (2) Sustainable PFA design can be achieved by introducing sustainable utility functions. The incorporation of sustainability can alter the module configuration choices in PFA design schemes [25]. Because the objective of the PFA design is often to maximize utility per unit cost, the impact of sustainability on PFA can be described by constructing sustainable utility models. The form of sustainable utility models (such as linear or nonlinear) typically depends on the actual problem and decision preferences.
- (3) The HJO mechanism is advantageous. The proposed HJO model is robust and excels when dealing with the complex tradeoffs between the optimal PFA design decision and SMP decision. Compared with the IOM and TSM, the HJO approach tends to obtain better PFA solutions leveraging with the SMP decision. The study findings

provide an approach for the industry to address the joint optimization problem concerning PFA design and SMPs.

- (4) NGA is reliable. Through the comparative experiments in Section 6.5, we observe that our algorithm exhibits favorable performance in both optimality and stability. The nested algorithm framework we designed aligns with the approach for bi-level optimization and is easily scalable. These research findings can serve as a basis for subsequent studies on nested algorithms.

The proposed model and algorithm in this paper are especially suitable for the mechanical or electronic modular PFA design. For manufacturers that outsource SMPs to outsourcers, our HJO model and NGA provides a new effective approach to handle the joint optimization of PFA and SMPs.

7.2. Limitations and Future Research Directions

Several avenues for future research emerge from the limitations of the current study.

Initially, the focus of this study was the joint optimization of PFA design and of SMPs, which constitutes a deterministic optimization problem. However, practical scenarios often involve randomness, such as the case of the stochastic nature of product demands [19]. Therefore, further research could incorporate stochastic considerations into the HJO model.

Furthermore, our lower-level decisions focus on the selection of various schemes, leading to the utilization of discrete decision variables. However, there are also important sustainability-related decisions concerning continuous production, especially in processing raw materials, such as metals, oil, and ore [56]. Ignoring the sustainability implications of these factors is a limitation of this study. Therefore, we can further enhance the generalizability of the problem by incorporating decisions related to continuous production into the model.

Finally, genetic algorithms are sensitive to parameter settings, and optimizing these parameters often requires multiple experiments, which is a limitation of this study. Currently, artificial intelligence algorithms are gaining popularity. Some researchers have combined artificial intelligence algorithms with heuristics to improve their performance [57]. We can further consider integrating artificial intelligence algorithms, such as reinforcement learning algorithms, into the NGA we designed in order to automatically optimize parameter selections.

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