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A New Resource Allocation Multiple Criteria Decision-Making Method in a Two-Stage Inverse Data Envelopment Analysis Framework for the Sustainable Development of Chinese Commercial Banks

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Abstract: The resource allocation of commercial banks is a multiple-criteria decision-making issue with complex internal structure, and traditional inverse data envelopment analysis cannot meet its decision-making needs. A two-stage structure with undesirable outputs is constructed to describe the operations of a Chinese commercial bank, and then a new two-stage inverse data envelopment analysis with undesirable outputs is proposed to address its resource allocation multiple criteria decision-making issue. The new method can be used to calculate the minimum input increment required to achieve the goals of desirable and undesirable output under a certain efficiency, and then a specific resource allocation plan can be obtained to promote the sustainable development of commercial banks. Finally, the new method is applied to the resource allocation of 16 Chinese listed commercial banks in 2013, and the application results fully demonstrate the effectiveness of the new method.

Keywords: inverse data envelopment analysis; resource allocation; two-stage; undesirable outputs; commercial bank

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1. Introduction

Commercial banking is an important component of China's economic system, and it can provide strong power for the growth of China's economy through financial intermediary services. According to Banker magazine's Global Banking 1000 2021 list, Chinese commercial banks have developed rapidly and are in a leading position among the world banks [1], and more and more resources are transferred to Chinese commercial banks. Although Chinese commercial banks have performed outstandingly in recent years, they still face a series of risks and challenges. Among them, the most critical risk is the continuous increase in non-performing loans. Data from the World Bank show that the non-performing loan ratio of Chinese commercial banks has increased from 0.954 in 2012 to 1.833 in 2018 [2].

Sustainable development is defined as meeting the needs of the present without compromising the ability of the future to meet its own needs. According to this concept, the sustainable development of commercial banks can be defined as meeting the current needs of profit growth without compromising the ability to obtain future profits. However, non-performing loans are undesirable outputs generated by the pursuit of current profit growth in the operation process of commercial banks, which directly undermines the bank's ability to obtain profits in the future. Therefore, the non-performing loan is not only a key risk in the operation of commercial banks but also directly affects their sustainable development.

Scientific efficiency evaluation is an effective means for achieving sustainable development in commercial banks, which has received widespread attention. Evaluation is often a multiple-criteria decision-making problem. For example, Zhang and Li (2023) proposed a consensus-based multiple-criteria decision-making method to evaluate green buildings [3]; Kundu et al. (2023) constructed an integrated fuzzy multiple-criteria group decision-making model to evaluate the public transportation systems for sustainable cities [4]; and Khazaei et al. (2023) employed a multiple-criteria decision-making method to evaluate suppliers [5]. Data envelopment analysis (DEA) is one of the most popular multiple-criteria decision-making methods to evaluate the efficiency of commercial banks. Puri and Yadav (2014) proposed a fuzzy DEA model with undesirable outputs to evaluate the efficiency of the banking sector in India [6], while Wu et al. (2023) utilized a two-stage network DEA to assess the overall efficiency, fund-raising efficiency, and fund-using efficiency of Chinese commercial banks [7]. Li et al. (2022) claimed that the internal structure of the bank should be considered in the process of evaluating bank efficiency [8]. A large number of scholars support this viewpoint and have evaluated the efficiency of commercial banks under the network DEA framework. Omrani et al. (2023) proposed a mixed-integer network DEA with shared inputs and undesirable outputs for evaluating the efficiency of internet banking [9]. Tong et al. (2023) used network DEA to evaluate the relative performance of 19 Taiwanese banks from 2018 to 2021 [10]. Xie et al. (2022) constructed a network DEA model with a multi-period leader-follower model for evaluating the efficiency of 16 representative Chinese commercial banks from 2009 to 2018 [11]. In general, considering the internal structure of DMU has become a consensus in evaluating the efficiency of commercial banks.

Efficiency evaluation is an effective guide for resource allocation. Soltanifar et al. (2022) proposed a DEA model with common set weights to create a new procedure for resource allocation [12]. Chu et al. (2022) used a DEA-based approach with non-regressive production technology to promote the resource allocation of emergency medical care among hospitals [13]. Zhu et al. (2023) developed a cross-two-stage data envelopment analysis model with a nested parallel structure to optimize innovation resource allocation in industrial enterprises [14]. However, decision-makers are unable to obtain specific resource allocation plans through pure efficiency evaluation. Essentially, the resource allocation of commercial banks is a multiple-criteria decision-making issue that not only needs to consider the allocation relationships between multiple different input resources but also needs to consider different output goals. Inverse DEA, as proposed by Wei et al. (2000) [15], is an effective tool to solve this issue and can be used to develop specific resource allocation plans to achieve specific output goals with a certain efficiency. Due to its objectivity and effectiveness, inverse DEA is widely used in the decision-making of resource allocation in different fields. Amin et al. (2019) proposed a combined goal programming and inverse DEA method for the target setting of 42 banks in the Gulf Cooperation Council [16]. Chen et al. (2021) proposed a new inverse DEA to determine the path for the safety objective of China's road transportation [17]. Ghiyasi et al. (2022) proposed a novel inverse DEA-R model for the resource allocation of 130 public hospitals in Iran [18].

As a resource allocation multiple criteria decision-making method, the research on inverse DEA theory has been increasing in recent years. Lu and Li (2022) proposed an extended inverse DEA model with frontier changes for the resource allocation of China's high-tech industry [19]. Emrouznejad et al. (2023) claimed that inverse DEA is a post-DEA sensitivity analysis approach developed initially for solving resource allocation [20]. However, most of the research has the following limitations: First, the internal structure of decision-making units (DMUs) is often overlooked; second, undesirable outputs often cannot be considered. Wang et al. (2014) claimed that the Chinese commercial banking system has a two-stage internal structure [21]; Azad et al. (2021) [22], Tan et al. (2021) [23], and Yang et al. (2023) [24] all emphasized that it is necessary to consider the internal system structure of banks when evaluating their efficiency; otherwise, the accuracy of commercial bank efficiency evaluation would be affected. Safiullah and Shamsuddin (2022) [25], Shah et al. (2022) [26], and Wanke et al. (2023) [27] all claimed that non-performing loans should be considered undesirable outputs in the efficiency evaluation of commercial banking because they represent the risks of commercial banking and hinder its sustainable development. Therefore, considering the internal structure and undesirable outputs are of great significance in the resource allocation of commercial banks based on efficiency evaluation, but they are often overlooked in existing research.

In recent years, some scholars have also paid attention to this issue. An et al. (2019) proposed a two-stage inverse DEA with undesirable outputs, but their method may have infeasible solutions [28]. Kazemi and Galagedera (2023) attempted to introduce network structure into the inverse DEA framework, but they did not consider the role of undesirable outputs in the production process [29]. Actually, there is currently no convincing inverse DEA method with both internal structure and undesirable outputs to allocate resources for promoting the sustainable development of commercial banks.

In response to the shortcomings of existing research, this paper analyzes the operational characteristics of commercial banks and introduces their internal structure with undesirable outputs to an inverse DEA framework to develop a new resource allocation multiple-criteria decision-making method for promoting the sustainable development of commercial banks. Finally, 16 Chinese commercial banks are selected as an example to demonstrate the effectiveness of the new method.

The rest of this paper is organized as follows: Section 2 reviews the traditional two-stage DEA model and the inverse DEA model. Section 3 proposes a new two-stage inverse DEA resource allocation method, while Section 4 discusses the effectiveness of the new method. Section 5 applies new methods to the resource allocation of Chinese commercial banks, and some conclusions are provided in Section 6.

2. Preliminaries

2.1. Two-Stage DEA Model

Assume that there are n-evaluated DMUs with the two-stage structure shown in Figure 1. In this structure, each DMU has m inputs denoted by x_i ($i = 1, \ldots, m$), h intermediate elements denoted by z_f ($f = 1, \ldots, h$), and s desirable outputs denoted by y_r ($r = 1, \ldots, s$). According to Chen et al. (2009) [30], the additive two-stage DEA model can be constructed as follows:

$$
\begin{aligned}
& \text{Max} \;\; \theta_d = \alpha_1 * \frac{\sum\limits_{f=1}^{h} w_d z_{fd}}{\sum\limits_{i=1}^{m} v_d x_{id}} + \alpha_2 * \frac{\sum\limits_{r=1}^{s} u_d y_{rd}}{\sum\limits_{f=1}^{h} w_d z_{fd}} \\
& \text{s.t.} \;\; \frac{\sum\limits_{f=1}^{h} w_d z_{fj}}{\sum\limits_{i=1}^{m} v_d x_{ij}} \le 1; \quad j = 1, \ldots, n; \\
& \qquad \frac{\sum\limits_{r=1}^{s} u_d y_{rj}}{\sum\limits_{f=1}^{h} w_d z_{fj}} \le 1; \quad j = 1, \ldots, n; \\
& \qquad v_d,\; u_d,\; w_d \ge 0.
\end{aligned} \tag{1}
$$

where θ_d represents the efficiency of the evaluated DMU$_d$, and v_d, u_d, and w_d are decision variables, which represent the weights of inputs, intermediate elements, and desirable outputs, respectively.

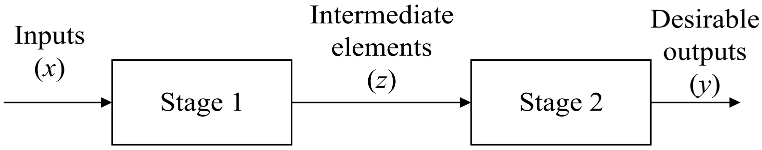

Figure 1. Traditional two-stage structure.

Chen et al. (2009) [30] assumed that the proportion of investment in each stage to the total investment can be used as its weight, and then the values of α_1 and α_2 can be obtained as follows:

$$\alpha_1 = \frac{\sum_{i=1}^{m} v_d x_{id}}{\sum_{i=1}^{m} v_d x_{id} + \sum_{f=1}^{h} w_d z_{fd}},\ \alpha_2 = \frac{\sum_{f=1}^{h} w_d z_{fd}}{\sum_{i=1}^{m} v_d x_{id} + \sum_{f=1}^{h} w_d z_{fd}} \tag{2}$$

Based on model (2), model (1) can be transformed into the following linear programming solution:

$$\begin{aligned} &\text{Max } \ \theta_d = \sum_{r=1}^{s} u_d y_{rd} + \sum_{f=1}^{h} w_d z_{fd} \\ &\text{s.t. } \ \sum_{f=1}^{h} w_d z_{fj} - \sum_{i=1}^{m} v_d x_{ij} \le 0; \quad j = 1, \ldots, n; \\ &\qquad \sum_{r=1}^{s} u_d y_{rj} - \sum_{f=1}^{h} w_d z_{fj} \le 0; \quad j = 1, \ldots, n; \\ &\qquad \sum_{f=1}^{h} w_d z_{fj} + \sum_{i=1}^{m} v_d x_{ij} = 1; \\ &\qquad v_d,\ u_d,\ w_d \ge 0. \end{aligned} \tag{3}$$

2.2. Traditional Inverse DEA Model

Traditional inverse DEA only considers inputs and desirable outputs, and its basic model proposed by Wei et al. (2000) [15] is as follows:

$$\begin{aligned} &\text{Min } (\Delta x_{1d}, \ldots, \Delta x_{md}) \\ &\text{s.t. } \ \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_d^* (x_{id} + \Delta x_{id}); \quad i = 1, \ldots, m; \\ &\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rd} + \Delta y_{rd}^*; \quad r = 1, \ldots, s; \\ &\qquad \lambda_j,\ \Delta x_{id} \ge 0. \end{aligned} \tag{4}$$

where θ_d^* represents the optimal efficiency of DMU$_d$, λ_j and Δx_{id} are decision variables, and Δx_{id} and Δy_{rd}^* represent the input increment and output increment of DMU$_d$. Among them, θ_d^* and Δy_{rd}^* are the known constants. The former can be obtained by the DEA model, and the latter can be determined by the decision-maker's goals.

Model (4) is a multiple-criteria decision-making method that originates from the dual model of DEA. It is used to determine the minimum input increment Δx_{id} required to complete the output increment Δy_{rd}^* when the efficiency of DMU$_d$ is equal to θ_d^*. Model (4) can be solved by assigning weights to the objective function, and its specific model is as follows:

$$\begin{aligned} &\text{Min } W^T (\Delta x_{1d}, \ldots, \Delta x_{md}) \\ &\text{s.t. } \ \text{The same constraints as model } (4)\,. \end{aligned} \tag{5}$$

3. New Two-Stage Inverse DEA Resource Allocation Method

3.1. Two-Stage DEA Model with Undesirable Outputs

The biggest difference between sustainable development and traditional development is the existence of undesirable outputs. Therefore, the two-stage structure in Figure 1 should be expanded to the following structure shown in Figure 2:

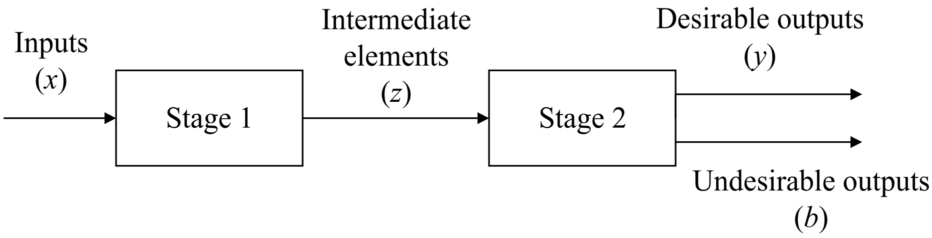

Figure 2. Two-stage structure with undesirable outputs.

DMUs with this structure not only have x_i, z_f, and y_r but also have q undesirable outputs denoted by b_k (k = 1, . . ., q). Therefore, model (3) can be extended to the following model for evaluating the efficiency of DMUs with the new structure:

$$\begin{aligned} \text{Max } \theta_d &= \sum_{r=1}^{s} u_d y_{rd} - \sum_{k=1}^{q} g_d b_{kd} + \sum_{f=1}^{h} w_d z_{fd} \\ \text{s.t. } & \sum_{f=1}^{h} w_d z_{fj} - \sum_{i=1}^{m} v_d x_{ij} \leq 0; \quad j = 1, \ldots, n; \\ & \sum_{r=1}^{s} u_d y_{rj} - \sum_{k=1}^{q} g_d b_{kd} - \sum_{f=1}^{h} w_d z_{fj} \leq 0; \quad j = 1, \ldots, n; \\ & \sum_{f=1}^{h} w_d z_{fj} + \sum_{i=1}^{m} v_d x_{ij} = 1; \\ & v_d, u_d, w_d, g_d \geq 0. \end{aligned} \tag{6}$$

where g_d is also the decision variable, which represents the weight of undesirable outputs. The first and second constraints are used to ensure that the two stages' efficiencies of all DMUs are not greater than 1, while the objective function is used to maximize the overall system efficiency of the evaluated DMU$_d$. However, undesirable outputs b_k violate the fundamental principle of maximizing outputs in DEA theory, and the convexity of the model has changed because of undesirable outputs, so that the reliability of the optimal solution has been affected.

The most direct way to overcome this problem is to address undesirable outputs appropriately. Seiford and Zhu (2002) claimed that data transformation functions can be used to translate undesirable outputs into desirable outputs [31], and their method has been widely adopted because it is not only simple and easy to implement but also can effectively reflect the production relationship among inputs, desirable outputs, and undesirable outputs [32,33]. Therefore, the data transformation function proposed by Seiford and Zhu (2002) [31] will also be adopted in this section to address undesirable outputs, which is as follows:

$$\overline{b}_{rd} = M - b_{rd} \tag{7}$$

where M is a positive number that can make all b_k positive. Model (7) can convert b_{rd} to $\overline{b}_{rd}$ value, which is a value that is as great as possible. Based on model (7), model (6) can be transformed into the following form:

$$\begin{aligned} \text{Max } \theta_d &= \sum_{r=1}^{s} u_d y_{rd} + \sum_{k=1}^{q} \overline{g}_d \overline{b}_{kd} + \sum_{f=1}^{h} w_d z_{fd} \\ \text{s.t. } & \sum_{f=1}^{h} w_d z_{fj} - \sum_{i=1}^{m} v_d x_{ij} \leq 0; \quad j = 1, \ldots, n; \\ & \sum_{r=1}^{s} u_d y_{rj} + \sum_{k=1}^{q} \overline{g}_d \overline{b}_{kd} - \sum_{f=1}^{h} w_d z_{fj} \leq 0; \quad j = 1, \ldots, n; \\ & \sum_{f=1}^{h} w_d z_{fj} + \sum_{i=1}^{m} v_d x_{ij} = 1; \\ & v_d, u_d, w_d, g_d \geq 0. \end{aligned} \tag{8}$$

According to model (4), inverse DEA originates from the dual model of DEA. In order to construct the two-stage inverse DEA model more conveniently, model (8) is thus transformed into the following dual form:

$$
\begin{aligned}
&\text{Min } \theta_d \\
&\text{s.t.} \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_d x_{id}; \quad i = 1, \ldots, m; \\
&\quad \sum_{j=1}^{n} \mu_j y_{rj} \ge y_{rd}; \quad r = 1, \ldots, s; \\
&\quad \sum_{j=1}^{n} \mu_j \bar{b}_{kj} \ge \bar{b}_{kd}; \quad r = 1, \ldots, s; \\
&\quad \sum_{j=1}^{n} \lambda_j z_{fj} - \sum_{j=1}^{n} \mu_j z_{fj} \ge z_{fd} - \theta_d z_{fd}; \\
&\quad \lambda_j,\ \mu_j \ge 0.
\end{aligned} \tag{9}
$$

where λ_j and μ_j are decision variables.

3.2. Two-Stage Inverse DEA Model

Assume that n DMU has the two-stage structure shown in Figure 2. If decision-makers want to achieve the output increment Δy^*_{rd}, how much minimum input increment does DMU require under a certain efficiency? This is a resource allocation multiple criteria decision-making issue, but the traditional inverse DEA method is not suitable for this decision-making situation because a two-stage structure and undesirable outputs need to be considered.

Therefore, based on traditional inverse DEA and the dual model of two-stage DEA, a new inverse DEA model with a two-stage structure and undesirable outputs is constructed as follows:

$$
\begin{aligned}
&\text{Min } (\Delta x_{1d}, \ldots, \Delta x_{md}) \\
&\text{s.t.} \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta^*_d (x_{id} + \Delta x_{id}); \quad i = 1, \ldots, m; \\
&\quad \sum_{j=1}^{n} \mu_j y_{rj} \ge (y_{rd} + \Delta y^*_{rd}); \quad r = 1, \ldots, s; \\
&\quad \sum_{j=1}^{n} \mu_j \bar{b}_{kj} \ge (\bar{b}_{kd} + \Delta \bar{b}^*_{kd}); \quad r = 1, \ldots, s; \\
&\quad \sum_{j=1}^{n} \lambda_j z_{fj} - \sum_{j=1}^{n} \mu_j z_{fj} \ge (z_{fd} + \Delta z_{fd}) - \theta^*_d (z_{fd} + \Delta z_{fd}); \\
&\quad \lambda_j,\ \mu_j,\ \Delta x_{dj},\ \Delta z_{fd}\ \ge 0.
\end{aligned} \tag{10}
$$

where Δx_{id} and Δz_{fd} are decision variables, which represent the input increment and intermediate element increment of the evaluated DMU$_d$, while θ^*_d, Δy^*_{rd}, and $\Delta \bar{b}^*_{kd}$ are known constants, which represent the optimal efficiency, desirable output increment, and undesirable output decrement of the evaluated DMU$_d$. θ^*_d can be obtained by model (9), and Δy^*_{rd} and $\Delta \bar{b}^*_{kd}$ can be determined based on the decision-maker's established goals or their subjective preferences. The constraints of model (10) are used to ensure the new DMU with $\left(x_{id} + \Delta x^*_{id}, z_{fd} + \Delta z^*_{fd},\ y_{rd} + \Delta y^*_{rd}\ ,\ \bar{b}_{kd} + \Delta \bar{b}^*_{kd} \right)$ is still in the production possibility set composed of all DMUs, while its objective function is to minimize the input increments as much as possible.

Unlike traditional inverse DEA models, the new two-stage anti DEA model differs in that it adds the third and fourth constraints of model (10). According to model (7), the decrement of b_{kj} is also the increment of $\bar{b}_{kd}$; the third constraint is thus used to ensure that the decrement of undesirable outputs is not less than $\Delta \bar{b}^*_{kd}$. The fourth constraint is used to limit the variation range of the intermediate element, and it also reflects the efficiency relationship between the two stages. Note that Δz_{fd} is the variable that is greater than 0.

Model (10) is a multiple-criteria decision-making method because different input increments represent different criteria. According to the common practices of inverse DEA [34], $(\Delta x_{1d}, \ldots, \Delta x_{md})$ can be regarded as $W^T(\Delta x_{1d}, \ldots, \Delta x_{md})$. Let $W^T = [w_1, \ldots, w_m]^T$, then model (10) can be transformed into the following model:

$$\begin{aligned}
&\text{Min } w_1 \Delta x_{1d} + \ldots + w_m \Delta x_{md} \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} - \theta_d^* \Delta x_{id} \le \theta_d^* x_{id}; \quad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} \mu_j y_{rj} \ge y_{rd} + \Delta y_{rd}^*; \quad r = 1, \ldots, s; \\
&\sum_{j=1}^{n} \mu_j \overline{b}_{kj} \ge \overline{b}_{kd} + \Delta \overline{b}_{kd}^*; \quad r = 1, \ldots, s; \\
&\sum_{j=1}^{n} \lambda_j z_{fj} - \sum_{j=1}^{n} \mu_j z_{fj} - (1 - \theta_d^*) \Delta z_{fd} \ge (1 - \theta_d^*) z_{fd}; \\
&\lambda_j,\ \mu_j,\ \Delta x_{dj},\ \Delta z_{fd} \ \ge 0.
\end{aligned} \tag{11}$$

where the value of $W^T = [w_1, \ldots, w_m]^T$ can be obtained by the subjective preferences of decision-makers.

Model (11) is a linear programming problem that can be directly solved, and the optimal solution $(\Delta x_{1d}, \ldots, \Delta x_{md})$ can be obtained. If decision-makers want to achieve sustainable development on the premise that optimal efficiency remains unchanged, they need to both increase Δy_{rd}^* and decrease $\Delta \overline{b}_{kd}^*$ at the same time. Moreover, the minimum input x_{ij}^* of resources can be obtained as follows:

$$x_{ij}^* = x_{ij} + \Delta x_{ij} \tag{12}$$

4. Discussion on the Effectiveness of the New Method

According to models (11) and (12), the new $\text{DMU}_{\hat{d}}$ can be obtained, and its input-output situation is $\left(x_{id} + \Delta x_{id}^*, z_{fd} + \Delta z_{fd}^*, \quad y_{rd} + \Delta y_{rd}^*, \quad \overline{b}_{kd} + \Delta \overline{b}_{kd}^*\right)$. Among them, Δx_{id}^* and Δz_{fd}^* are the optimal values of model (10), while Δy_{rd}^* and $\Delta \overline{b}_{kd}^*$ are the known values determined by sustainable development's goal for decision-makers. Based on model (9), the optimal efficiency $\theta_{\hat{d}}^*$ of $\text{DMU}_{\hat{d}}$ relative to the original production frontier can be obtained by the following model:

$$\begin{aligned}
&\text{Min } \theta_{\hat{d}} \\
&\text{s.t. } \sum_{j=1}^{n} \hat{\lambda}_j x_{ij} \le \theta_{\hat{d}} (x_{id} + \Delta x_{id}^*); \qquad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} \hat{\mu}_j y_{rj} \ge y_{rd} + \Delta y_{rd}^*; \qquad r = 1, \ldots, s; \\
&\sum_{j=1}^{n} \hat{\mu}_j \overline{b}_{kj} \ge \overline{b}_{kd} + \Delta \overline{b}_{kd}^*; \qquad r = 1, \ldots, s; \\
&\sum_{j=1}^{n} \hat{\lambda}_j z_{fj} - \sum_{j=1}^{n} \hat{\mu}_j z_{fj} \ge \left(z_{fd} + \Delta z_{fd}^*\right) - \theta_{\hat{d}} \left(z_{fd} + \Delta z_{fd}^*\right); \\
&\hat{\lambda}_j, \quad \hat{\mu}_j \ge 0.
\end{aligned} \tag{13}$$

Because model (11) is used to obtain the minimum input increment $x_{id} + \Delta x_{id}^*$ for producing $y_{rd} + \Delta y_{rd}^*$ and $\overline{b}_{kd} + \Delta \overline{b}_{kd}^*$ on the premise that the optimal efficiency is θ_d^*. Whether the optimal efficiency $\theta_{\hat{d}}^*$ of $\text{DMU}_{\hat{d}}$ obtained by model (13) is equal to θ_d^* is thus an important indicator for testing the effectiveness of the new method.

Theorem 1. *There must be* $\theta_d^* = \theta_{\hat{d}}^*$.

Proof. Because Δx_{id}^* and Δz_{fd}^* are the optimal solution of model (10), $(\theta_{\hat{d}}, \lambda_{\hat{j}}, \mu_{\hat{j}}) = (\theta_d^*, \lambda_j^*, \mu_j^*)$ must be a feasible solution of model (13); then there must be $\theta_{\hat{d}}^* \leq \theta_d^*$. Assume that there is $\theta_{\hat{d}}^* < \theta_d^*$ and let $\theta_{\hat{d}}^* = k\theta_d^*$, then $k < 1$ and $(k\theta_d^*, \lambda_j^*, \mu_j^*)$ is a feasible solution of model (13). Therefore, based on the first constraint of model (13), there is $\sum_{j=1}^{n} \lambda_j x_{ij} \leq k\theta_d^*(x_{id} + \Delta x_{id}^*)$. However, Δx_{id}^* is obtained by model (10), and $\sum_{j=1}^{n} \lambda_j x_{ij} \leq k\theta_d^*(x_{id} + \Delta x_{id}^*)$ cannot be established when $k < 1$. Therefore, it must have $k = 1$, and then there must be $\theta_d^* = \theta_{\hat{d}}^*$. □

According to Theorem 1, the optimal efficiency $\theta_{\hat{d}}^*$ of DMU$_{\hat{d}}$ obtained by model (13) is equal to the optimal efficiency θ_d^* of DMU$_d$, and the new method is thus effective.

5. Application in the Resource Allocation of Chinese Commercial Banks

To better illustrate the effectiveness of the new method, 16 Chinese listed commercial banks in 2013 were selected as examples to show the resource allocation and multiple criteria decision-making for their sustainable development.

5.1. Case Description

According to the study of An et al. (2019) [28], the two-stage structure of Chinese listed commercial banks is shown in Figure 3.

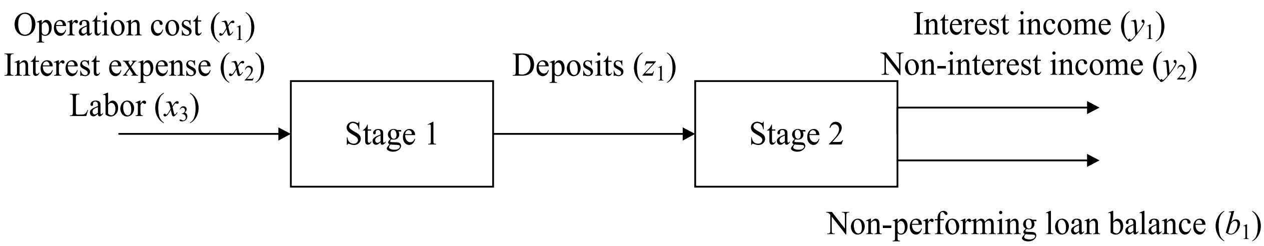

Figure 3. Two-stage structure of a commercial bank.

In Figure 3, the operation of commercial banks is divided into Stages 1 and 2, which represent deposits and loans, respectively. In Stage 1, the operation cost (x_1), interest expense (x_2), and labor (x_3) are selected as the inputs of Chinese listed commercial banks, which represent the inputs of operations, capital, and labor, respectively. These are the basics of commercial bank operations [35,36]. Deposits (z_1) is selected as the intermediate element, and it is both the input of Stage 1 and the output of Stage 2, which is the most important intermediate variable within the bank [37,38]. Interest income (y_1) and non-interest income (y_2) are selected as desirable outputs, which represent the returns of commercial banks [1], while non-performing loan balance (b_1) is selected as undesirable outputs, which is the key indicator to reflect the risks of commercial banks [25–27]. Specifically, x_1, x_2, and x_3 are inputs in Stage 1 of the commercial bank to produce z_1, and then z_1 is regarded as the inputs in Stage 2 of the commercial bank to produce y_1, y_2, and b_1.

The annual data of 16 Chinese listed commercial banks in 2013 are from the study of An et al. (2019) [28], which was obtained by "the Listed Commercial Bank of China Financial Reporting Database of the China Merchants Bank". The specific data are shown in Table 1.

Table 1. Data of Chinese-listed commercial banks.

DMU	x_1	x_2	x_3	z_1	y_1	y_2	b_1
Industrial and Commercial Bank of China	1768.29	3237.76	441,902	146,208.3	7671.11	1477.93	191.14
Agricultural Bank of China	1693.97	2371.82	473,766	118,114.1	6133.84	875.85	19.33
Bank of China	1478.42	2354.1	251,617	100,977.9	5189.95	1245.09	78.23
China Construction Bank	1557.79	2567.09	368,410	122,230.4	6462.53	1208.98	106.46
Bank of Communications	538.12	1286.34	99,919	41,578.33	2592.92	341.7	73.15
China Merchants Bank	458.96	745.82	51,667	27,752.76	1734.95	342.05	66.38
China CITIC Bank	328.45	776.47	38,803	26,516.78	1633.35	191.34	77.11
Shanghai Pudong Development Bank	266.05	926.27	38,976	24,196.96	1778.04	151.64	41.39
Industrial Bank Co., Ltd.	291.9	1037.57	33,134	21,703.45	1896.02	236.25	50.45
China Minsheng Banking	380.9	991.21	53,064	21,466.89	1821.54	332.01	28.81
Ping An Bank	212.79	524.14	28,369	12,170.02	931.02	115.86	6.75
Huaxia Bank	176.23	373.51	25,043	11,775.92	762.53	63.62	11.04
China Everbright Bank	207.81	692.2	31,464	16,052.78	1200.82	145.8	24.16
Bank of Beijing	78.41	315.96	9193	8344.8	578.81	44.32	8.44
Bank of Nanjing	32.55	116.72	4357	2601.49	207.68	14.29	2.64
Bank of Ningbo	44.5	122.36	6310	2339.38	234.95	14.16	4.17

Source: These data are from the study of An et al. (2019) [28].

5.2. Result Analysis

Let M = 300, then the efficiencies of 16 Chinese listed commercial banks in 2013 can be obtained based on model (3), which is shown in Table 2.

Table 2. Efficiency evaluation result of 16 Chinese listed commercial banks in 2013.

DMU	θ_d^*	DMU	θ_d^*
Industrial and Commercial Bank of China	0.827	Industrial Bank Co., Ltd.	0.845
Agricultural Bank of China	0.791	China Minsheng Banking	0.791
Bank of China	0.899	Ping An Bank	0.746
China Construction Bank	0.820	Huaxia Bank	0.770
Bank of Communications	0.797	China Everbright Bank	0.798
China Merchants Bank	0.898	Bank of Beijing	0.854
China CITIC Bank	0.833	Bank of Nanjing	0.842
Shanghai Pudong Development Bank	0.837	Bank of Ningbo	0.745

As shown in Table 2, all DMUs are inefficient because there is no DMU that is efficient in two stages simultaneously. The most efficient DMU is Bank of China, which has θ_d^* = 0.899; and the most inefficient DMU is Bank of Ningbo, which has θ_d^* = 0.745. It means that the overall operational performance of these banks does not differ significantly.

To achieve sustainable development, commercial banks should increase their desirable outputs and decrease their undesirable outputs. Assume that 15% desirable output increment and 10% undesirable output decrement are the goals of sustainable development, i.e., Δy_{rd}^* = $0.15 y_{rd}^*$ and Δz_{rd}^* = $0.10 z_{fd}^*$. Meanwhile, assume that all inputs have the same importance, and then there is $w_1 =, \ldots, = w_m = \frac{1}{m}$. Based on model (11), the results of resource allocation by 16 Chinese commercial banks in 2013 are shown in Table 3.

Note that P1, P2, and P3 represent the proportion of incremental inputs to total inputs, respectively.

As shown in Table 3, if 16 commercial banks want to achieve their goals of sustainable development, they need to make some increases in different inputs. For example, the three input increments of Bank of Beijing are 9.75, 39.28, and 1142.8, and the proportion of these incremental inputs to total inputs is 12.43%. These input increments are the minimum amount of resource increments required for these banks to achieve their goals of sustainable development. In addition, x_1 and x_2 are resources that generally need to be increased for most Chinese commercial banks, while only two banks need to increase x_3.

Table 3. Resource allocation results of 16 Chinese-listed commercial banks in 2013.

DMU	Δx_1	Δx_2	Δx_3	P1	P2	P3
Industrial and Commercial Bank of China	304.25	469.51	0	17%	15%	0%
Agricultural Bank of China	0	369.33	0	0%	16%	0%
Bank of China	0	536.83	0	0%	23%	0%
China Construction Bank	172.33	530.60	0	11%	21%	0%
Bank of Communications	109.77	103.86	0	20%	8%	0%
China Merchants Bank	0	140.79	0	0%	19%	0%
China CITIC Bank	0.92	203.49	0	0%	26%	0%
Shanghai Pudong Development Bank	64.82	66.06	0	24%	7%	0%
Industrial Bank Co., Ltd.	10.39	180.54	2307.47	4%	17%	7%
China Minsheng Banking	68.43	108.06	0	18%	11%	0%
Ping An Bank	27.46	69.90	0	13%	13%	0%
Huaxia Bank	13.14	58.00	0	7%	16%	0%
China Everbright Bank	58.91	25.23	0	28%	4%	0%
Bank of Beijing	9.75	39.28	1142.80	12%	12%	12%
Bank of Nanjing	4.40	0	0	14%	0%	0%
Bank of Ningbo	8.94	10.20	0	20%	8%	0%

5.3. Methods Comparison

To further illustrate the effectiveness of the new method, its results are compared with the results obtained by two different methods. One is the traditional inverse DEA method, and the other is the traditional inverse DEA method with undesirable outputs. Before that, traditional DEA, traditional DEA with undesirable outputs, and model (3) were used to evaluate the efficiency of 16 Chinese listed commercial banks in 2013, and their results are shown in Figure 4.

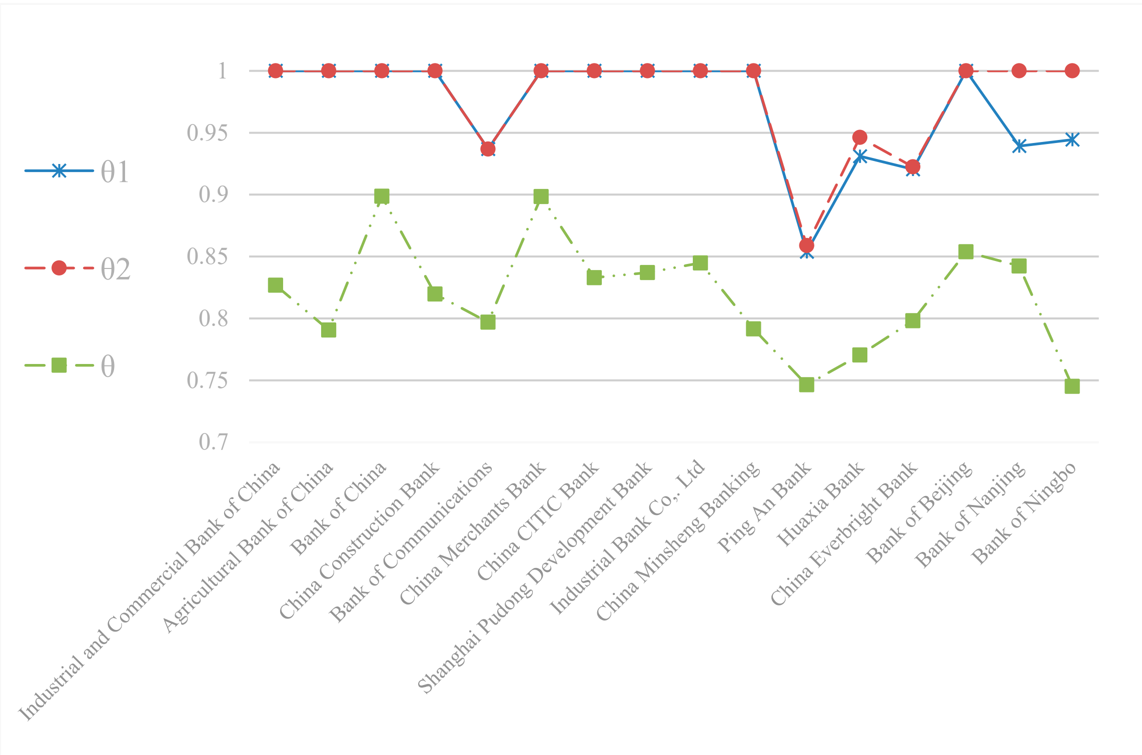

Figure 4. Comparison of efficiency evaluation results. Note that θ1, θ2, and θ represent the optimal efficiencies obtained by traditional DEA, traditional DEA with undesirable outputs, and model (3), respectively.

As shown in Figure 4, if the internal structure of two-stage is not considered, most commercial banks would be efficient DMUs. This means that traditional DEA models have weak recognition abilities for inefficiency. For example, the efficiencies of the Bank of China obtained by traditional DEA, traditional DEA with undesirable outputs, and model (3) are 1, 1, and 0.899. It means that the Bank of China is the efficient DMU in the first two methods, but it is an efficient DMU in the new method because its efficiency is influenced by intermediate elements.

According to the efficiency evaluation results, traditional inverse DEA, traditional inverse DEA with undesirable outputs, and model (11) are used to make resource allocation plans for commercial banks, and the results are shown in Table 4.

Table 4. Comparison of resource allocation results.

DMU	Δx_1			Δx_2			Δx_3		
	TP1	**UP1**	**P1**	**TP2**	**UP2**	**P2**	**TP3**	**UP3**	**P3**
Industrial and Commercial Bank of China	24%	24%	17%	11%	11%	15%	0%	0%	0%
Agricultural Bank of China	0%	0%	0%	18%	18%	16%	0%	0%	0%
Bank of China	23%	23%	0%	24%	24%	23%	0%	0%	0%
China Construction Bank	17%	17%	11%	18%	18%	21%	0%	0%	0%
Bank of Communications	23%	23%	20%	14%	14%	8%	0%	0%	0%
China Merchants Bank	6%	6%	0%	132%	132%	19%	7%	7%	0%
China CITIC Bank	3%	3%	0%	22%	22%	26%	0%	0%	0%
Shanghai Pudong Development Bank	15%	15%	24%	16%	16%	7%	0%	0%	0%
Industrial Bank Co., Ltd.	15%	15%	4%	15%	15%	17%	15%	15%	7%
China Minsheng Banking	24%	24%	18%	69%	69%	11%	1%	1%	0%
Ping An Bank	15%	14%	13%	17%	16%	13%	0%	0%	0%
Huaxia Bank	0%	0%	7%	22%	20%	16%	0%	0%	0%
China Everbright Bank	32%	32%	28%	9%	9%	4%	0%	0%	0%
Bank of Beijing	15%	15%	12%	15%	15%	12%	15%	15%	12%
Bank of Nanjing	8%	12%	14%	17%	14%	0%	0%	9%	0%
Bank of Ningbo	0%	0%	20%	19%	19%	8%	0%	0%	0%

Note that TP, UP, and P represent the results obtained by traditional DEA, traditional DEA with undesirable outputs, and model (11), respectively.

As shown in Table 4, the input increments of the three methods are relatively consistent with the overall trend, which reflects the reliability of the new method proposed in this manuscript. From a local trend perspective, the new method is more focused on increasing investment in x_1, while two traditional methods emphasize more on the increase of x_2 and x_3. For example, Shanghai Pudong Development Bank needs to increase x_1 and x_2 by 15% and 16% in both traditional DEA and traditional DEA with undesirable outputs, respectively, while these inputs are required to increase by 15% and 4% in model (11), respectively.

Overall, the new method requires fewer input increments because it has a stronger recognition ability for inefficiencies, and it also considers the internal structure of DMUs and their sustainable development. Therefore, the results of the new method are more reasonable.

6. Conclusions

Scientific resource allocation by commercial banks is an important way to achieve sustainable development. Due to commercial banks having multiple inputs and outputs, their resource allocation is thus a multiple-criteria decision-making issue, and the inverse DEA method is an effective theoretical tool to address this issue. Traditional inverse DEA regards the production activities of DMUs as a black box, and it makes resource allocation decisions only for the purpose of increasing desirable outputs. However, the operation of commercial banks not only involves complex internal structures but also has to face serious risks. Therefore, traditional inverse DEA cannot meet the resource allocation needs of commercial banks for achieving sustainable development.

In order to better promote the sustainable development of commercial banks, two problems need to be addressed during their resource allocation process. One is how to balance development and risk in resource allocation, and the other is how to reflect the role of internal structure in the resource allocation process. This paper introduces two-stage structure and undesirable outputs into the inverse DEA framework and innovatively constructs a new resource allocation multiple criteria decision-making method, i.e., the two-stage inverse DEA method. The new method can be used to calculate the minimum input increment required to achieve the goals of desirable and undesirable output under a certain efficiency, and then a specific multiple-criteria resource allocation scheme can be obtained for the sustainable development of commercial banks. Finally, the new method was applied to the resource allocation of 16 Chinese-listed commercial banks in 2013, and the application results fully demonstrate the effectiveness of the new method.

Although the new method can develop specific resource allocation plans with unchanged efficiency to achieve the sustainable development of commercial banks, there are still some limitations: first, the new method is constructed on the basis of constant returns to scale without considering the possibility of variable returns to scale; second, there is a lack of reliable methods to test the validity of the results. Therefore, to construct the two-stage inverse DEA with undesirable outputs under variable returns to scale and to explore the testing methods for guaranteeing the reliability of its results are the future research directions.

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