

Article

Research on Dynamic Optimization and Coordination Strategy of Value Co-Creation in Digital Innovation Ecosystems

Huimin Ji, Hua Zou * and Baotong Liu

School of Management, Shenyang University of Technology, Shenyang 110870, China; jihm272@smail.sut.edu.cn (H.J.); liubaotong6106@smail.sut.edu.cn (B.L.)

* Correspondence: suo-2001@163.com

Abstract: Based on the theory of differential games and guided by the realization of value co-creation, this paper discusses the value co-creation of a technology innovation platform, scientific innovation layer, and support layer in the digital innovation ecosystem. Given the dynamic change characteristics of digital technology innovation and resource integration, this paper constructs a differential game decision model. The conclusions are as follows: (1) The Stackelberg master–slave game and collaborative model have incentive effects. The returns in both methods increase over time and finally reach a stable value. In the collaborative game model, the effort level of participants is the highest and realizes the Pareto optimality. (2) The digital technology innovation capacity coefficient, digital technology assimilation capacity and absorption capacity coefficient, and resource integration cost coefficient are the key factors affecting the optimal return of the innovation ecosystem. (3) The two-way cost-sharing path can balance the innovation ecosystem, in which the technology innovation platform shares the cost and provides incentives to the scientific innovation layer and the support layer. The sharing ratio and incentive degree are positively correlated with the benefit of value co-creation. However, if the income distribution coefficient is not appropriately set, the participants' income will decrease.

Keywords: digital innovation ecosystem; value co-creation; differential game; sustainable innovation; resource integration



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1. Introduction

With the incremental innovation and breakthrough of the new generation of information technology, intelligence, networking, and digitalization are developing deeply. The digital economy has become a new driving force for economic development, disrupting how enterprises obtain sustainable competitive advantage [1]. The innovation ecosystem guides the transition from individual value creation to multi-actor value co-creation by coordinating the interests of multiple parties for cross-border cooperation and enhancing the efficiency of technological innovation and resource integration in the system [2]. The innovation ecosystem value co-creation is based on the value galaxy and value network theory and emphasizes the discontinuous, interactive, and non-linear characteristics of the value co-creation process. Focusing on inter-actor collaboration, resource sharing, and symbiotic evolution, it has become a new paradigm for studying complex innovation phenomena [3]. With the rapid development of digital technology, the operation and development of innovation ecosystems have also incorporated digital elements, not only emphasizing the use of digital resources, digital technologies, and digital technology facilities as critical elements of innovation but also reshaping the value co-creation logic of innovation subjects [4]. Building a digital innovation ecosystem is a crucial channel to breaking industry, regional, and enterprise boundaries and realizing digital value co-creation. Nowadays, more and more enterprises make digital innovation ecosystems to achieve interactive delivery of value [5], further promoting the breakthrough of digital technology innovation barriers and

boundary extension. Haier, Facebook, Google, Amazon, and many other digital natives have formed open digital innovation ecosystems to gain competitive advantages continuously. Digital innovation ecosystems are loosely interconnected networks formed by firms and other innovation agents. All participating agents collaborate to participate in value co-creation through shared technologies, knowledge, and skills. Still, significant differences exist in the roles these innovation agents assume and play [6,7].

Although the business and academic circles have reached a consensus on the digital innovation ecosystem's core role and development prospects [8], the research is still in development. The current research results must provide more mature guidance for enterprises' digital innovation practice. The study results on the dynamic optimization and coordination strategy of value co-creation of the digital innovation ecosystem are rare. Existing research mainly focuses on the value co-creation mechanism of the digital innovation ecosystem. Still, to a certain extent, there is path dependence, which makes it challenging to cross disciplinary barriers in this field and needs to be more conducive to the comprehensive deepening of the problem and theoretical formation of practice. There is an urgent need to adopt a decision-making approach to comprehensively and systematically grasp the nature of value co-creation decisions in the digital innovation ecosystem. Accordingly, this paper raises the following questions: In the digital innovation ecosystem, when the collaborative value co-creation of the scientific innovation layer, the support layer, and the technology innovation platform reach the steady-state equilibrium process, how do the three innovation subjects decide to achieve the optimal individual income and the optimal system overall income? How can resource allocation be adjusted and the process of value co-creation be promoted under different cooperation modes? How does digital technology's absorption and assimilation ability affect the value co-creation equilibrium of the three innovation subjects? Existing literature has yet to explore these issues thoroughly.

In view of this, this paper takes the enterprise digital innovation ecosystem as the research object and divides its value co-creation core innovation subjects into technology innovation platform, science and innovation layer, and support layer. We construct a differential game model for the interaction of the three innovation subjects, introduce the core variables of innovation subsidy, digital technology assimilation, and digital technology absorption, and explore the optimal benefits under different modes of cooperation. First, the optimal strategies, returns, and overall returns of the digital innovation ecosystem are calculated under the Nash non-cooperative game, Stackelberg master-slave game, and collaborative game model. Second, the optimal strategies of the technology innovation platform, scientific innovation layer, and support layer in the value co-creation process are obtained, and the income distribution mechanism of value co-creation is discussed. The effects of innovation subsidy, digital technology assimilation, and digital technology absorption on ecosystem benefits are further analyzed. Third, the optimal returns of the three cooperation modes are compared, and the critical parameters are simulated by numerical simulation to verify the validity of the differential game model.

The research is an essential extension of the digital innovation ecosystem and value co-creation theory. In addition, it enriches the differential game theory of value co-creation, which helps clarify the operation and change laws of the digital innovation ecosystem. It guides innovation agents in the system to better enhance digital technology innovation capability and access to innovation resources. It is essential guidance for enterprise transformation and value co-creation in the new era. Moreover, this paper divides the core innovation subjects of the digital innovation ecosystem into three levels. It uses the differential game model to make decisions on their value co-creation, which provides a reference for subsequent scholars to conduct related research on the perspective of how to divide innovation subjects and decision-making methods.

2. Theoretical Basis and Literature Review

2.1. Enterprise Digital Innovation

2.1.1. Evolution Analysis of Enterprise Digitalization Research

The keyword time zone view reveals the evolutionary trends and development trends in the field [9], and the time zone view of digital innovation in enterprises is shown in Figure 1. Combined with the keyword evolution, the research hot spots in the field of enterprise digital innovation are divided into three stages. Phase I (2003–2010): theory construction and development period. Scholars have conducted in-depth research on digital technology and thus laid the foundation for developing enterprise digital technology. At this stage, research hot spots have been reflected in information systems, data diffusion, and collaborative innovation. Phase II (2010–2016): digital technology application period. Technology development and data technology advancement have driven the R&D capability of enterprise digital technology. The hot spots of research in the period are reflected in digital technology, business models, digital products, and technology governance. Phase III (2017–2022): digital technology transformation period. Superimposed on the Industry 4.0 context, enterprises face urgent digital technology transformation challenges. The hot spots of research in this period are reflected in digital platforms, innovation ecosystems, value co-creation of innovation subjects, and digital services.



Figure 1. Cont.

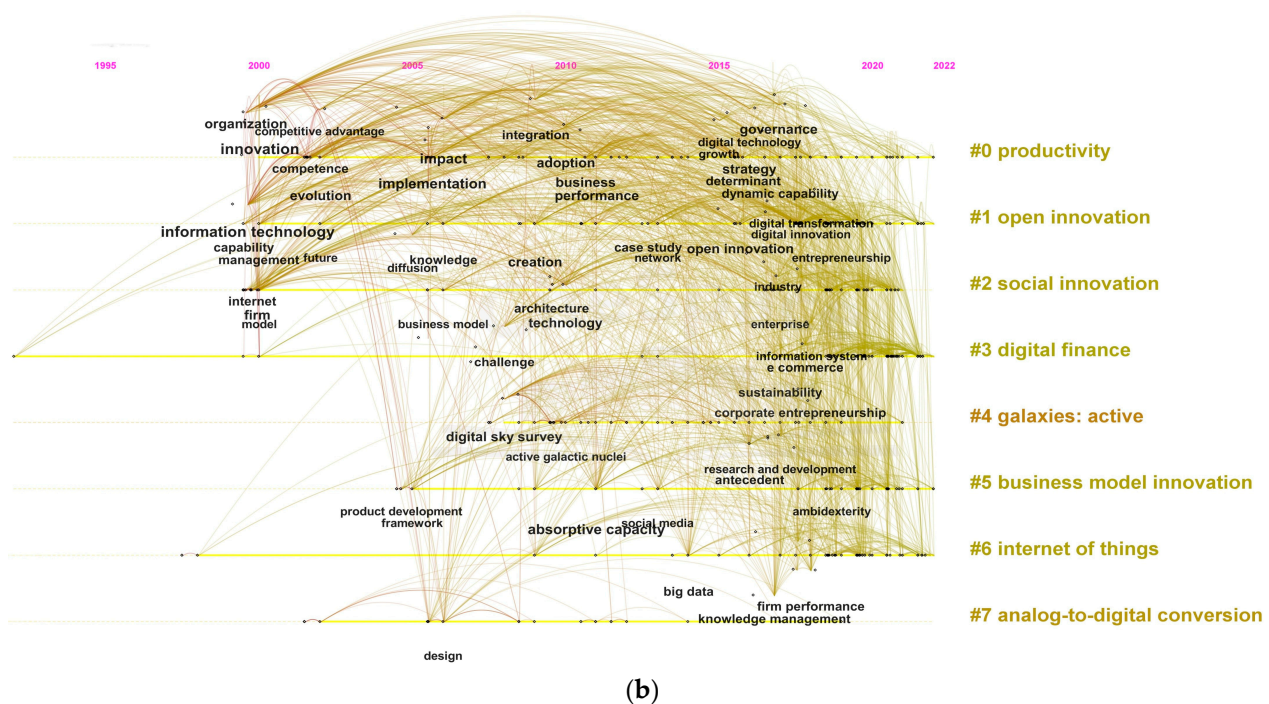


Figure 1. Keyword Time Zone View for Enterprise Digital Innovation. (a) Keyword co-occurrence network. (b) Evolution path knowledge map.

2.1.2. Current Status of Research on Enterprise Digitalization

In the era of the digital economy, digital innovation has become an essential engine for enterprises to cultivate new dynamic energy. Facebook, Google, Amazon, etc., have been making breakthroughs in product, process, and business model innovation under digital technology, facilitating the acquisition and continuation of competitive advantages. Yoo and Henfridsson et al. first defined the concept of digital innovation [10]. They proposed that digital innovation is implementing new combinations of digital and non-digital resources to produce new products. Fichman and Santos et al. further suggested that the digital innovation process is divided into four stages: discovery, development, diffusion, and impact [11]. Porter and Heppelmann integrated digital technologies and traditional physical products and proposed that digital innovations have convergent properties [12]. Compared with the conventional innovation management theory logic, the digital innovation process is characterized by a looser innovation process, difficulty in determining the innovation subject in advance, and blurred boundaries between the innovation process and the innovation result. Digital innovation can face more severe challenges as digital technologies are embedded in traditional products and services, capable of generating new products and services that fundamentally change the nature of products and services [13,14]. To access and integrate different resources, enterprises rely on the unique environment of innovation incentives provided by digital technology platforms, which may harm competition and innovation [15,16]. Technologies such as intelligent Internet, cloud computing, and big data in the digital context are changing the industry's structure and competitive nature while impacting organizational change, performance, and digital entrepreneurship. Digital platforms and ecosystems are places where enterprises create and capture value, facilitating new ways for participating actors to build knowledge and relationships and providing new ways for global customers to develop and deliver value [17].

2.2. Digital Innovation Ecosystem

The innovation ecosystem concept originates from the ecology field, an organic combination of ecology, system science, and innovation theory. After Tansley introduced the concept of "ecosystem" in 1935 [18], scholars have enriched and extended the idea of an

innovation ecosystem. Moore first introduced the concept of corporate ecosystems in 1993 by the analogy of biological systems to corporate competition [19]. Adner subsequently pointed out that innovation ecosystems are open systems that link the individual and the whole [20]. Since then, the innovation ecosystem has undergone an iterative spiral from “sustainable development” to “open innovation” and from “value creation” to “value co-creation”.

The digital innovation ecosystem is a complex network system of several subjects and organizations relying on digital technology for products and services, featuring digitalized innovation elements, virtualized participating issues, and ecological inter-subject relationships [21]. The circular flow of data elements promotes the heterogeneity of innovation subjects in the digital innovation ecosystem, the complexity of resource integration and sharing process, and the challenge of technological innovation research and development [22]. Because of these significant differences, the cross-domain flow of innovation resources and value distribution becomes possible, so the value co-creation of the digital innovation ecosystem has a more significant trend of iteration and upgrading [23]. Different innovation subjects and environments in the ecosystem rely on digital technology to realize the combination of digital and non-digital resources, stimulate continuous endogenous innovation within the system, and make digital innovation more stable and sustainable [24,25]. On the one hand, it reflects the inherent characteristics of the innovation ecosystem; on the other hand, it highlights the emerging technologies in the digital innovation environment [26]. Scholars have revealed the internal logic and importance of multiple innovation subjects participating in value co-creation in the digital innovation ecosystem from different perspectives. Suseno and Laurell et al. proposed that a digital innovation ecosystem is an organization that uses digital technology to create new products and values and has a strong interactive relationship with stakeholders [27]. Senyo and Liu et al. believed that the digital innovation ecosystem reflects the impact of digital transformation on the innovation ecosystem. Digital technology is not only the object of innovation but also the promoter of innovation activities [28]. Beliaeva and Ferasso et al. conducted an exploratory case study of an IT company in Brazil [29]. They concluded that there are significant differences in the participants and relationships supporting the innovation ecosystem during the evolution of enterprise digital capability from low to vigorous. Based on the background of Industry 4.0, Benitez, Ayala, and Frank concluded that the innovation ecosystem promotes the resource integration of SMEs. Moreover, with the support of the Internet of Things digital technology, innovation ecosystem members collaborate to realize value creation [30].

2.3. Multi-Subject Value Co-Creation

2.3.1. The Evolution of Multi-agent Value Co-Creation Research

The keyword time zone view of value co-creation in industry is shown in Figure 2. Combined with the evolution of keywords, the research on value co-creation shows a trend of “binary relationship–simple system–complex system network relationship”. The research results of value co-creation of the innovation ecosystem are increasingly abundant. Meanwhile, the current research results can be divided into stages: the subject and initiation method of value co-creation, co-creation channels, and co-creation forms and processes.

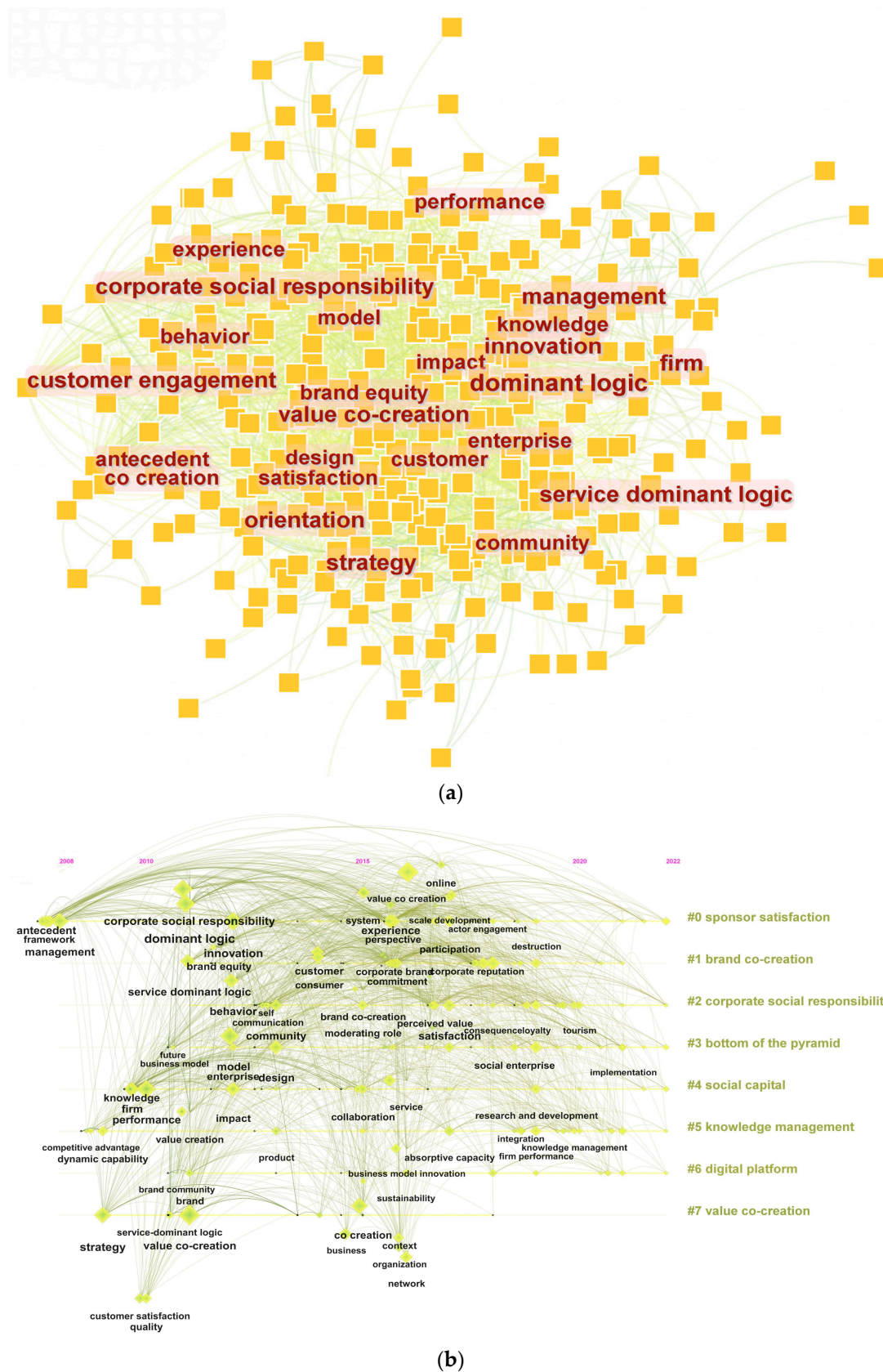


Figure 2. Keyword time zone view for enterprise value co-creation. (a) Keyword co-occurrence network. (b) Evolution path knowledge map.

2.3.2. Current Status of Existing Research on Value Co-Creation

In the 19th century, the germ of the idea of value co-creation arose in the field of marketing management [3]. In the 1960s, scholars gradually described value co-creation behavior based on the customer production theory. Customers are believed to participate in product production or service use based on their own needs, which promotes the enterprise's technology research and development innovation and rational allocation of resources to realize sustainable value creation. Ramirez first introduced the concept of value co-creation [31], which has been widely discussed in academia. The current concept of value co-creation can be divided into three levels: One is the relational interaction level. Tóth and Peters et al. argued that value co-creation is a process in which multiple actors participate and create value in an interactive approach and suggested that strengthening the relational governance of innovation agents is conducive to achieving value co-creation [32]. The second is the environmental support level. Payne and Dahl et al. believed that the value creation of a service ecosystem in the digital environment is based on the interaction between people [33]. There are differences in the mechanisms of value co-creation behaviors among different types of participants, so the innovation ecosystem leaders need to adopt different governance approaches. The third is the value guidance level. Holbrook earlier defined and delineated customer value in value co-creation, arguing that value co-creation is mainly divided into economic, hedonic, and social values, including intrinsic and extrinsic values and self-directed and other-directed values [34]. Ye and Kankanhalli found that enterprises increased value co-creation opportunities and improved digital technology service performance using service-led logic for system design and IT services [35].

With the evolution of the research on the co-creation of enterprise value, the duality relationship between enterprise and customer develops into a multi-stakeholder relationship. It extends to the collaborative participation of multiple subjects in the innovation ecosystem. The behavioral ecology of value co-creation has evolved from the traditional offline interaction to the online dynamics of the online environment and then to a digital innovation ecosystem that combines online and offline. As new resources for value creation, the latest digital technologies have created a strong link between the interaction methods and interaction environment of value co-creation and technological development by overturning the traditional rules of linking resources in industrial organizations [36]. Aluri and McIntyre et al. found that effective and purposeful value co-creation between customers and firms stimulates new knowledge integration and positively influences future innovation outcomes [37]. Chi and Bi et al. argued that the more frequent and closer the customer–firm interaction is, the more it helps to improve the efficiency of new product development [38] for the application of the research model of enterprise value co-creation. Demirezen and Shetty developed a differential game model to study value co-creation between customers and suppliers, arguing that the level of effort of both parties is dynamic in supply chain and information technology environments [39]. Broeke and Paparoidamis used mathematical methods to conclude that enterprises would derive more excellent value from collaborating with customers than from independent innovation [40]. Ketonen-Oksi and Valkokari constructed a participant structure model to study the factors influencing value co-creation in innovation ecosystems. Still, they ignored the dynamic nature of value co-creation and the lagging nature of outcomes [41].

2.4. Research Review and Innovation Points

In summary, existing research on value co-creation has mainly focused on theoretical mechanisms, primarily exploring the modes and processes of value co-creation, with only a small amount of literature considering the enabling utility of digital technologies, and likewise not thoroughly investigating the laws of value co-creation in the context of digital innovation ecosystems. Moreover, the existing literature focuses on the factors influencing customer participation in value co-creation from a single customer perspective. It needs more exploration at the firm level and the level of the service ecosystem as a whole. In addition, the existing literature needs to be narrower in perspective. It lacks rigorous

derivative verification of inter-subjective decision logic, which does not have a certain degree of universality and has limited the development of empirical research on value co-creation. Regarding value co-creation research methods, more scholars use case studies for qualitative research and rarely use mathematical methods for quantitative analysis. Some scholars use mathematical models to study the game relationships of the subjects involved in value co-creation. Still, only the binary relationship between enterprises and customers is considered, ignoring the multiple cooperative relationships between enterprises and other innovation subjects. Most studies on enterprise value co-creation are based on specific subject areas, and there are fewer studies on enterprise digital innovation ecosystem contexts. Given the significant differences in scholars' research perspectives and the inconsistency in model construction, this paper selected some representative pieces of literature to summarize and put forward solutions to the research purpose. The specific research results are summarized in Table 1.

Table 1. A summary of some representative studies.

Author	Year	Research Perspective	Research Limitations	Proposed Solution
Endres and Huesig et al. [16]	2022	Digital innovation management for entrepreneurial ecosystems.	a. Lack of in-depth research on the enabling effect of digital technology. b. The law of value co-creation in the context of digital innovation ecosystem is not fully considered.	Construct general enterprise digital innovation ecosystem, and explain the logical system of value co-creation by digital technology.
Beliaeva and Ferasso et al. [29]	2019	The dynamic process of digital innovation ecosystem.		
Tóth and Peters et al. [32]	2018	Network case study of value co-creation.		
Holbrook [34]	2006	Customer experience and value in the process of value co-creation.	Considering the influence factors of value co-creation only from the perspective of a single customer, there is a lack of discussion on the whole level of innovation ecosystem.	The core members of the enterprise digital innovation ecosystem are divided into a technology innovation platform, scientific innovation layer, and support layer. Then the three are incorporated into the value co-creation system, and the income analysis mechanism is established to analyze factors such as time, subsidies, digital assimilation, and absorption.
Ye and Kankanhalli [35]	2020	The impact of customer engagement and mobile application relationships on service innovation value co-creation.		
Fu and Zhang et al. [42]	2022	The influence of customer fit on innovation performance of value co-creation of small and micro technology enterprises.		
Aluri and Price et al. [37]	2019	Value co-creation between customers and enterprises promotes knowledge integration.		
Demirezen and Shetty [39]	2020	A differential game model is constructed to analyze the value co-creation decision behavior of customers and suppliers.	The research perspective is narrow, the decision logic between ecosystem subjects lacks rigorous derivation and verification, and the universality of model simulation results needs to be improved.	The three-party differential game model of the technology innovation platform, scientific innovation layer, and support layer is constructed to analyze the interrelation of the decision-making process among members effectively, and the simulation analysis is carried out based on the realistic situation.
Broeke and Paparoidamis [40]	2021	Mathematical models are used to analyze the impact of joint value co-creation between enterprises and customers.		
Ketonen-Oksi and Valkokari [41]	2019	Construct the participant structure model and explore the influencing factors of value co-creation.		

Unlike the existing research literature, the innovations of this paper mainly include the following: (1) A differential game model of independent decision-making and collaborative decision-making among innovation entities of a technology innovation platform, scientific innovation layer, and support layer is constructed. The dynamic digital technology innovation strategy and value co-creation strategy under different conditions are calculated using two income modes: technology R&D cost and resource integration cost. (2) Considering

the influence of time factors on value co-creation, the differential game method is employed to analyze collaborative technology innovation in the digital innovation ecosystem. (3) Digital technology is deconstructed into assimilation and absorption, and the random evolution law of digital technology absorption and assimilation ability on value co-creation is considered.

3. Problem Description and Model Assumption

3.1. Problem Description

In the digital innovation ecosystem, the technology innovation platform, scientific innovation layer, and support layer are interdependent in sharing and exchanging resources and collaborating in digital technology innovation and research and development, thus achieving long-term balanced and mutually beneficial value co-creation. Among them, the technology innovation platform, as the core of the innovation ecosystem, dominates and leads the overall operation of the system while playing the role of risk sharing and accelerating the innovation process [43]. The scientific innovation layer is the critical R&D subject in the innovation ecosystem with sufficient innovation momentum to accelerate product, service, and business model innovation through science and technology innovation and technology development, enhancing the system's digital technology innovation capability [44]. The support layer connects the internal and external environment of the innovation ecosystem, provides services, resources, and information for other innovation subjects, and then solves the problems of the innovation ecosystem subject-to-resource integration, technology absorption, and transformation.

The three types of actors in the digital innovation ecosystem have a high degree of autonomy to choose value co-creation strategies from a dynamic perspective. The degree of technological innovation and resource sharing of actors also changes continuously and dynamically over time. The continuous time change is an essential factor affecting the decision making of actors in the ecosystem [45]. The differential game is an effective tool for analyzing the dynamic decision problem of the game parties' behavior in continuous time, considering the influence of time variables on the actors' decisions [46]. Figure 3 shows the structure of value co-creation in the digital innovation ecosystem. We divide the core innovation subjects in the system into the technology innovation platform, scientific innovation layer, and support layer. Driven by digital technology elements such as technology, information, and capital, innovation subjects achieve the optimal equilibrium state of value co-creation after making static and dynamic innovation decisions through technology assimilation, technology absorption, resource integration, and other technology innovation means, which eventually improves the participation of innovation subjects, overall system revenue, technology innovation level, etc., and promotes the sustainable development of the digital innovation ecosystem.

Considering the dynamic equilibrium relationship among the technology innovation platform, scientific innovation layer, and support layer, the value co-creation coordination strategy can be divided into three models: (1) The technology innovation platform does not take any incentive measures to the scientific innovation layer, and the support layer and the three innovation bodies make independent decisions, which is represented by the Nash non-cooperative game model. (2) The technology innovation platform shares a certain proportion of costs with the scientific innovation and support layers and gives incentive measures. In this case, the technology innovation platform is the leader, while the scientific innovation layer and the support layer are the followers, which is represented by the master-slave game model. (3) The technology innovation platform, the scientific innovation layer, and the support layer cooperate in research and development and jointly determine the optimal effort level and the optimal income function to maximize the overall income of the digital innovation ecosystem. This situation is represented by the collaborative game model.

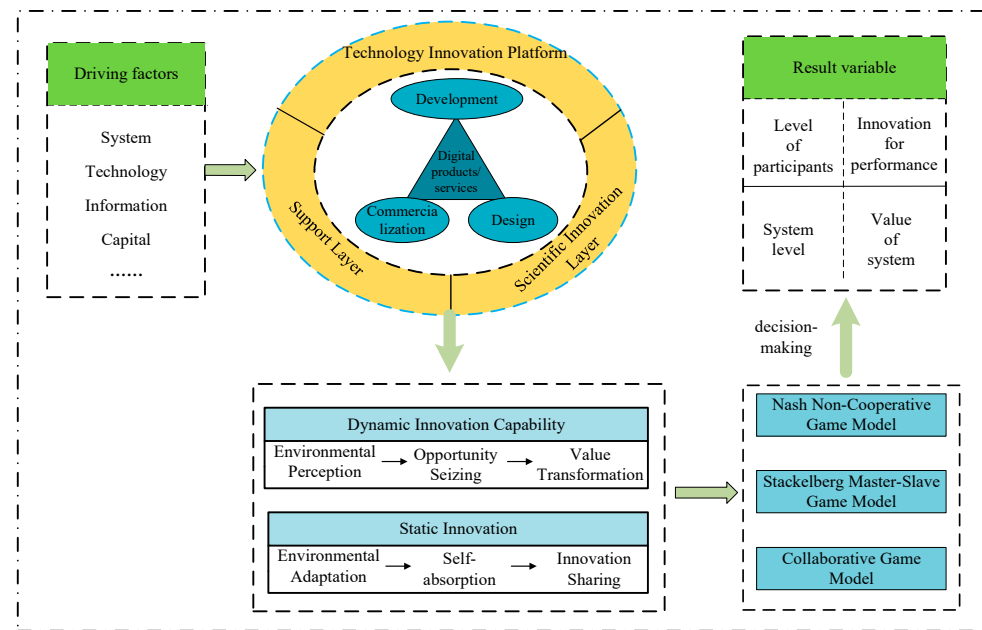


Figure 3. Digital innovation ecosystem value co-creation structure.

3.2. Model Assumptions

This paper assumes that the cost of digital technology innovation in the scientific innovation layer, the support layer, and the technology innovation platform is related to the level of effort involved in value co-creation. Effort refers to innovation resource inputs and specific actions of participating agents. The cost of innovation is considered in two main aspects: digital technology development and resource integration. The expression of the tripartite cost function is shown in (1).

$$\begin{aligned}
 C_M(t) &= \frac{1}{f_i + g_i} S_M^2(t); \\
 C_N(t) &= \frac{\varepsilon_i}{2} S_N^2(t); \\
 C_P(t) &= \left(\frac{\varepsilon_i}{2} + \frac{1}{f_i + g_i} \right) S_P^2(t)
 \end{aligned} \tag{1}$$

where f_i and g_i denote digital technology assimilation capacity and digital technology absorption capacity, respectively. ε_i denotes the resource integration factor. S_M , S_N , and S_P denote the level of effort put into participating in value co-creation by the scientific innovation layer, the support layer, and the technology innovation platform, respectively. $C_M(t)$, $C_N(t)$, and $C_P(t)$ respectively represent the value co-creation of R&D cost of the scientific innovation layer, support layer, and technology innovation platform at time t , which is an increasing function of effort degree; that is, the effort paid is proportional to the total cost.

Hypothesis $I'(t)$ represents the innovation level of the digital innovation system at moment t , which is a dynamic changing process and is determined by the efforts of the scientific innovation layer, the support layer, and the technology innovation platform in value co-creation. The scientific innovation layer conducts digital technology research and development according to the core element resources provided by the support layer and optimizes the resource allocation of value co-creation. Furthermore, it is assumed that the development process of digital technology satisfies (2).

$$I'(t) = \eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t) \tag{2}$$

In Equation (2), η_i is the effort coefficient of participation in value co-creation and represents the influence degree of effort level on digital technology R&D. σ represents the

natural decay rate of digital technology in the process of value co-creation. $I(0)$ represents the initial innovation level of digital technology.

The efforts of the scientific innovation layer, support layer, and technology innovation platform in value co-creation directly affect the overall innovation income of the system. Moreover, the level of digital technology research and development and the transformation of research and development results will coordinate the participation of innovation subjects in the value co-creation strategy. Therefore, considering the joint influence of effort level, R&D level, and achievement transformation level, the total revenue $\pi(t)$ of the digital innovation ecosystem at the moment t is assumed to be as shown in (3).

$$\pi(t) = \delta_M S_M(t) + \delta_N S_N(t) + \delta_P S_P(t) + (\mu + \varphi)I(t) \quad (3)$$

Here, δ represents the degree of influence of effort level on value co-creation, and μ and φ represent the degree of influence of digital technology R&D level and achievement transformation level on value co-creation, respectively.

It is assumed that in the value co-creation of the digital innovation ecosystem, the innovation subject distributes the overall income in a certain proportion, and the distribution coefficients are λ , γ , and $1 - \lambda - \gamma$, which are determined by the efforts and contributions of the innovation subject. The technology innovation platform shares R&D costs for the scientific innovation layer and the support layer, thus generating an incentive effect, with the ratio of α and β . It is assumed that all innovation subjects are rational decision-makers with complete information, the discount rate ρ is the same at any time, and all can obtain the maximum profit within an infinite time interval.

The objective function of the scientific innovation layer is shown in (4).

$$\max_{S_M(t)} J_M = \int_0^\infty e^{-\rho t} \left[\lambda \pi(t) - (1 - \alpha(t)) \frac{1}{f_M + g_M} S_M^2(t) \right] dt \quad (4)$$

The objective function of the support layer is shown in (5).

$$\max_{S_N(t)} J_N = \int_0^\infty e^{-\rho t} \left[\gamma \pi(t) - (1 - \beta(t)) \frac{\varepsilon_N}{2} S_N^2(t) \right] dt \quad (5)$$

The objective function of the technology innovation platform is shown in (6).

$$\max_{S_P(t)} J_P = \int_0^\infty e^{-\rho t} \left\{ [(1 - \gamma - \lambda) \pi(t) - (\frac{\varepsilon_P}{2} + \frac{1}{f_P + g_P}) S_P^2(t) - \alpha(t) \frac{1}{f_M + g_M} S_M^2(t) - \beta(t) \frac{\varepsilon_N}{2} S_N^2(t)] \right\} dt e^{-\rho t} \quad (6)$$

The coordination strategy game model involves state variables $I(t)$ and control variables $S_M(t)$, $S_N(t)$, $S_P(t)$, α , and β , and other parameters are constants independent of time. In the process of value co-creation, the optimal behavior of the innovation subject is determined by the coordination strategy. For the convenience of writing, t is omitted below.

4. Model Construction

4.1. Nash Non-Cooperative Game Model

Under this coordination strategy, the technology innovation platform does not share any cost for the scientific innovation layer and the support layer. That is, it does not provide any incentive measures. In this case, $\alpha = 0$ and $\beta = 0$; innovative agents independently make the optimal decision simultaneously and coordinate the combination of strategies to form a Nash equilibrium solution. The objective functions of the support layer, scientific innovation layer, and technology innovation platform are shown in (7)–(9).

$$\max_{S_M(t)} J_M = \int_0^\infty e^{-\rho t} \left[\lambda \pi(t) - \frac{1}{f_M + g_M} S_M^2(t) \right] dt \quad (7)$$

$$\max_{S_N(t)} J_N = \int_0^\infty e^{-\rho t} \left[\gamma \pi(t) - \frac{\varepsilon_N}{2} S_N^2(t) \right] dt \quad (8)$$

$$\max_{S_P(t)} J_P = \int_0^\infty e^{-\rho t} \left\{ e^{-\rho t} \left[(1 - \gamma - \lambda) \pi(t) - \left(\frac{\varepsilon_P}{2} + \frac{1}{f_P + g_P} \right) S_P^2(t) \right] \right\} dt \quad (9)$$

There exist continuously differentiable revenue functions $V_M(I)$, $V_N(I)$, and $V_P(I)$ for the support layer, the scientific innovation layer, and the technology innovation platform, all of which satisfy the following Hamilton–Jacobi–Bellman (HJB) equations when $I \geq 0$:

$$\rho V_M(I) = \max_{S_M \geq 0} \left\{ \lambda \pi(t) - \frac{1}{f_M + g_M} S_M^2(t) + V'_M(I) [\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \right\} \quad (10)$$

$$\rho V_N(I) = \max_{S_N \geq 0} \left\{ \gamma \pi(t) - \frac{\varepsilon_N}{2} S_N^2(t) + V'_N(I) [\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \right\} \quad (11)$$

$$\rho V_P(I) = \max_{S_P \geq 0} \left\{ (1 - \lambda - \gamma) \pi(t) - \left(\frac{1}{f_P + g_P} + \frac{\varepsilon_P}{2} \right) S_P^2(t) + V'_P(I) [\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \right\} \quad (12)$$

Calculate the first partial derivatives of S_M , S_N , and S_P on the right side of (10)–(12) and set them equal to zero. The solutions are as follows:

$$\begin{aligned} S_M &= \frac{(\lambda \delta_M + V'_M(I) \eta_M)(f_M + g_M)}{2} \\ S_N &= \frac{\gamma \delta_N + V'_N(I) \eta_N}{\varepsilon_N} \\ S_P &= \frac{(f_P + g_P)[(1 - \lambda - \gamma) \delta_P + V'_P(I) \eta_P]}{\varepsilon_P(f_P + g_P) + 2} \end{aligned} \quad (13)$$

Substitute (13) into (10)–(12), and simplify:

$$\begin{aligned} \rho V_M(I) &= I[(\mu + \varphi) \lambda - \sigma V'_M(I)] + \frac{(\gamma \delta_N + \eta_N V'_N(I))(\lambda \delta_N + \eta_N V'_M(I))}{\varepsilon_N} \\ &\quad + \frac{(\lambda \delta_P + \eta_P V'_M(I))(f_P + g_P)[(1 - \lambda - \gamma) \delta_P + V'_P(I) \eta_P]}{\varepsilon_P(f_P + g_P) + 2} + \frac{(f_M + g_M)[\lambda \delta_M + V'_M(I) \eta_M]^2}{4} \end{aligned} \quad (14)$$

$$\begin{aligned} \rho V_N(I) &= I[(\mu + \varphi) \gamma - \sigma V'_N(I)] + \frac{(\gamma \delta_N + \eta_N V'_N(I))^2}{2 \varepsilon_N} + \frac{(\gamma \delta_P + \eta_P V'_N(I))(f_P + g_P)[(1 - \lambda - \gamma) \delta_P + V'_P(I) \eta_P]}{\varepsilon_P(f_P + g_P) + 2} \\ &\quad + \frac{(\lambda \delta_M + \eta_M V'_M(I))(f_M + g_M)[\gamma \delta_M + V'_N(I) \eta_M]}{2} \end{aligned} \quad (15)$$

$$\begin{aligned} \rho V_P(I) &= I[(\mu + \varphi)(1 - \lambda - \gamma) - \sigma V'_P(I)] + \frac{(\gamma \delta_N + \eta_N V'_N(I))[(1 - \lambda - \gamma) \delta_N + \eta_N V'_P(I)]}{\varepsilon_N} \\ &\quad + \frac{(f_P + g_P)[(1 - \lambda - \gamma) \delta_P + V'_P(I) \eta_P]^2}{2[\varepsilon_P(f_P + g_P) + 2]} + \frac{(\lambda \delta_M + \eta_M V'_M(I))(f_M + g_M)[(1 - \lambda - \gamma) \delta_M + V'_P(I) \eta_M]}{2} \end{aligned} \quad (16)$$

As can be seen from (14)–(16), the unary function with I as the independent variable is the solution of the HJB equation; let

$$V_M(I) = a_1 I + b_1, \quad V_N(I) = a_2 I + b_2, \quad V_P(I) = a_3 I + b_3 \quad (17)$$

where a_1 , a_2 , a_3 , b_1 , b_2 , and b_3 are constants, and substituting $V_M(I)$, $V_N(I)$, $V_P(I)$, and their first-order partial derivatives into (14)–(16), we can obtain the following:

$$\begin{aligned} \rho(a_1 I + b_1) &= I[(\mu + \varphi) \lambda - \sigma a_1] + \frac{(\gamma \delta_N + \eta_N a_2)(\lambda \delta_N + \eta_N a_1)}{\varepsilon_N} \\ &\quad + \frac{(\lambda \delta_P + \eta_P a_1)(f_P + g_P)[(1 - \lambda - \gamma) \delta_P + a_3 \eta_P]}{\varepsilon_P(f_P + g_P) + 2} + \frac{(f_M + g_M)[\lambda \delta_M + \eta_M a_1]^2}{4} \end{aligned} \quad (18)$$

$$\begin{aligned} \rho V_N(a_2 I + b_2) &= I[(\mu + \varphi)\gamma - \sigma a_1] + \frac{(\gamma \delta_N + \eta_N a_2)^2}{2\varepsilon_N} \\ &+ \frac{(\gamma \delta_P + \eta_P a_2)(f_P + g_P)[(1 - \lambda - \gamma)\delta_P + \eta_P a_3]}{\varepsilon_P(f_P + g_P) + 2} + \frac{(\lambda \delta_M + \eta_M a_1)(f_M + g_M)[\gamma \delta_M + \eta_M a_2]}{2} \end{aligned} \quad (19)$$

$$\begin{aligned} \rho V_P(a_3 I + b_3) &= I[(\mu + \varphi)(1 - \lambda - \gamma) - \sigma a_3] + \frac{(\gamma \delta_N + \eta_N a_2)[(1 - \lambda - \gamma)\delta_N + \eta_N a_3]}{\varepsilon_N} \\ &+ \frac{(f_P + g_P)[(1 - \lambda - \gamma)\delta_P + \eta_P a_3]^2}{2[\varepsilon_P(f_P + g_P) + 2]} + \frac{(\lambda \delta_M + \eta_M a_1)(f_M + g_M)[(1 - \lambda - \gamma)\delta_M + \eta_M a_3]}{2} \end{aligned} \quad (20)$$

Since (18)–(20) should be satisfied for all $V_M(I) \geq 0$, $V_N(I) \geq 0$, and $V_P(I) \geq 0$, the parameter values of a_1 , a_2 , a_3 , b_1 , b_2 , and b_3 can be calculated:

$$a_1 = \frac{\lambda(\mu + \varphi)}{\rho + \sigma} \quad (21)$$

$$a_2 = \frac{\gamma(\mu + \varphi)}{\rho + \sigma} \quad (22)$$

$$a_3 = \frac{(1 - \lambda - \gamma)(\mu + \varphi)}{\rho + \sigma} \quad (23)$$

$$\begin{aligned} b_1 &= \frac{\lambda\gamma[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{\varepsilon_N\rho(\rho + \sigma)^2} + \frac{\lambda(1 - \gamma - \lambda)(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{[\varepsilon_P(f_P + g_P) + 2]\rho(\rho + \sigma)^2} \\ &+ \frac{\lambda^2(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} \end{aligned} \quad (24)$$

$$\begin{aligned} b_2 &= \frac{\gamma^2[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho + \sigma)^2} + \frac{\gamma(1 - \gamma - \lambda)(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{[\varepsilon_P(f_P + g_P) + 2]\rho(\rho + \sigma)^2} \\ &+ \frac{\lambda\gamma(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{2\rho(\rho + \sigma)^2} \end{aligned} \quad (25)$$

$$\begin{aligned} b_3 &= \frac{\gamma(1 - \gamma - \lambda)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{\varepsilon_N\rho(\rho + \sigma)^2} + \frac{(1 - \gamma - \lambda)^2(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{2\rho[\varepsilon_P(f_P + g_P) + 2](\rho + \sigma)^2} \\ &+ \frac{\lambda(1 - \lambda - \gamma)(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{2\rho(\rho + \sigma)^2} \end{aligned} \quad (26)$$

By substituting a_1 , a_2 , and a_3 into (13), the optimal effort levels of the scientific innovation layer, support layer, and technology innovation platform can be obtained:

$$S_{M_1}^* = \frac{\lambda(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]}{2(\rho + \sigma)} \quad (27)$$

$$S_{N_1}^* = \frac{\gamma[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]}{\varepsilon_N(\rho + \sigma)} \quad (28)$$

$$S_{P_1}^* = \frac{(1 - \lambda - \gamma)(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]}{[\varepsilon_P(f_P + g_P) + 2](\rho + \sigma)} \quad (29)$$

By substituting a_1 , a_2 , a_3 , b_1 , b_2 , and b_3 into (17), the optimal income functions of the scientific innovation layer, support layer, and technology innovation platform are respectively obtained as follows:

$$\begin{aligned} V_{M_1}(I)^* &= \frac{\lambda(\mu + \varphi)}{\rho + \sigma} I + \frac{\lambda\gamma[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{\varepsilon_N\rho(\rho + \sigma)^2} + \frac{\lambda(1 - \gamma - \lambda)(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{[\varepsilon_P(f_P + g_P) + 2]\rho(\rho + \sigma)^2} \\ &+ \frac{\lambda^2(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} \end{aligned} \quad (30)$$

$$V_{N_1}(I)^* = \frac{\gamma(\mu+\varphi)}{\rho+\sigma}I + \frac{\gamma^2[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho+\sigma)^2} + \frac{\gamma(1-\gamma-\lambda)(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2}{[\varepsilon_P(f_P+g_P)+2]\rho(\rho+\sigma)^2} \\ + \frac{\lambda\gamma(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2}{2\rho(\rho+\sigma)^2} \quad (31)$$

$$V_{P_1}(I)^* = \frac{(1-\lambda-\gamma)(\mu+\varphi)}{\rho+\sigma}I + \frac{\gamma(1-\gamma-\lambda)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2}{\varepsilon_N\rho(\rho+\sigma)^2} + \frac{(1-\gamma-\lambda)^2(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2}{2[\varepsilon_P(f_P+g_P)+2]\rho(\rho+\sigma)^2} \\ + \frac{\lambda(1-\lambda-\gamma)(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2}{2\rho(\rho+\sigma)^2} \quad (32)$$

Thus, the optimal income of value co-creation of the digital innovation ecosystem under this model can be obtained as follows:

$$V_1(I)^* = \frac{(\mu+\varphi)}{\rho+\sigma}I + \frac{\gamma(2-\gamma)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho+\sigma)^2} + \frac{[1-(\gamma+\lambda)^2](f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2}{2\rho[\varepsilon_P(f_P+g_P)+2](\rho+\sigma)^2} \\ + \frac{\lambda(2-\lambda)(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2}{4\rho(\rho+\sigma)^2} \quad (33)$$

4.2. Stackelberg Master–Slave Game Model

Under this coordinated strategy mode, the technology innovation platform leads digital technology research and development, while the scientific innovation layer and support layer are followers. The technology innovation platform bears a certain proportion of the cost for the scientific innovation layer and the support layer. It can effectively motivate them to carry out collaborative research and development. The incentive factors are assumed to be α and β . After observing the decision of the technology innovation platform, the scientific innovation layer and the support layer will make the following corresponding strategies to maximize the benefits. The technology innovation platform should be able to predict the coping strategy of the scientific innovation layer and the support layer before making the final decision. At this time, the objective functions of the scientific innovation layer, the support layer, and the technology innovation platform are as follows:

$$\max_{S_M(t)} J_M = \int_0^\infty e^{-\rho t} \left[\lambda \pi(t) - (1-\alpha(t)) \frac{1}{f_M+g_M} S_M^2(t) \right] dt \quad (34)$$

$$\max_{S_N(t)} J_N = \int_0^\infty e^{-\rho t} \left[\gamma \pi(t) - (1-\beta(t)) \frac{\varepsilon_N}{2} S_N^2(t) \right] dt \quad (35)$$

$$\max_{S_P(t)} J_P = \int_0^\infty e^{-\rho t} \left\{ e^{-\rho t} [(1-\gamma-\lambda)\pi(t) - (\frac{\varepsilon_P}{2} + \frac{1}{f_P+g_P}) S_P^2(t) - \alpha(t) \frac{1}{f_M+g_M} S_M^2(t) - \beta(t) \frac{\varepsilon_N}{2} S_N^2(t)] \right\} dt \quad (36)$$

It is assumed that there are continuously differentiable and bounded revenue functions $V_M(I)$, $V_N(I)$, and $V_P(I)$ of the scientific innovation layer, support layer, and technology innovation platform, which satisfy the HJB equation for all $I \geq 0$. By backward induction, the optimal decision problem of the scientific innovation layer and the support layer is solved, and we can obtain the following:

$$\rho V_M(I) = \max_{S_M \geq 0} \left\{ \lambda \pi(t) - (1-\alpha) \frac{1}{f_M+g_M} S_M^2(t) \right. \\ \left. + V'_M(I) [\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \right\} \quad (37)$$

$$\rho V_N(I) = \max_{S_N \geq 0} \left\{ \gamma \pi(t) - (1-\beta) \frac{\varepsilon_N}{2} S_N^2(t) + V'_N(I) [\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \right\} \quad (38)$$

Calculating the first partial derivatives on the right side of (37) and (38) and setting them equal to zero, we can obtain the following:

$$S_M = \frac{(\lambda\delta_M + V'_M(I)\eta_M)(f_m + g_m)}{2(1 - \alpha)} \quad (39)$$

$$S_N = \frac{\gamma\delta_N + V'_N(I)\eta_N}{(1 - \beta)\varepsilon_N} \quad (40)$$

As a rational decision-maker, a technology innovation platform can predict the optimal strategy selection of the scientific innovation layer and support layer in advance. The optimal strategy and subsidy ratio are determined according to the reaction function formula (37) and (38) of the scientific innovation and support layers. The HJB equation can be expressed as follows:

$$\begin{aligned} \rho V_P(I) = & \max_{S_P \geq 0} \{ (1 - \lambda - \gamma)\pi(t) - (\frac{\varepsilon_P}{2} + \frac{1}{f_P + g_P})S_P^2(t) - \alpha \frac{1}{f_M + g_M} S_M^2(t) - \beta \frac{\varepsilon_N}{2} S_N^2(t) \\ & + V'_P(I)[\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \} \end{aligned} \quad (41)$$

By substituting (39) and (40) into (41) and calculating the first-order partial derivative of the right side of (41) based on first-order conditions, and making it equal to zero, the optimal strategy of technology innovation platform can be obtained:

$$S_P = \frac{[(1 - \lambda - \gamma)\delta_P + V'_P(I)\eta_P](f_P + g_P)}{\varepsilon_P(f_P + g_P) + 2} \quad (42)$$

$$\alpha = \frac{(2 - 2\gamma - 3\lambda)\delta_M + \eta_M(2V'_P(I) - V'_M(I))}{(2 - 2\gamma - \lambda)\delta_M + \eta_M(2V'_P(I) + V'_M(I))} \quad (43)$$

$$\beta = \frac{(2 - 3\gamma - 2\lambda)\delta_N + \eta_N(2V'_P(I) - V'_N(I))}{(2 - \gamma - 2\lambda)\delta_N + \eta_N(2V'_P(I) + V'_N(I))} \quad (44)$$

By substituting (39), (40) and (42)–(44) into the HJB equation, we can obtain the following:

$$\begin{aligned} \rho V_M(I) = & I[\lambda(\mu + \varphi) - \sigma V'_M(I)] + \frac{(f_M + g_M)(\lambda\delta_M + \eta_M V'_M(I))[(2 - 2\gamma - \lambda)\delta_M + \eta_M(V'_M(I) + 2V'_P(I))]}{8} \\ & + \frac{(f_P + g_P)(\lambda\delta_P + \eta_P V'_M(I))[(1 - \gamma - \lambda)\delta_P + \eta_P V'_P(I)]}{\varepsilon_P(f_P + g_P) + 2} + \frac{(\lambda\delta_N + \eta_N V'_M(I))[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2V'_P(I) + V'_N(I))]}{2\varepsilon_N} \end{aligned} \quad (45)$$

$$\begin{aligned} \rho V_N(I) = & I[\lambda(\mu + \varphi) - \sigma V'_N(I)] + \frac{(f_M + g_M)(\gamma\delta_M + \eta_M V'_N(I))[(2 - 2\gamma - \lambda)\delta_M + \eta_M(V'_M(I) + 2V'_P(I))]}{4} \\ & + \frac{(f_P + g_P)(\lambda\delta_P + \eta_P V'_M(I))[(1 - \gamma - \lambda)\delta_P + \eta_P V'_P(I)]}{\varepsilon_P(f_P + g_P) + 2} + \frac{(\gamma\delta_N + \eta_N V'_M(I))[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2V'_P(I) + V'_N(I))]}{4\varepsilon_N} \end{aligned} \quad (46)$$

$$\begin{aligned} \rho V_P(I) = & I[\lambda(\mu + \varphi) - \sigma V'_P(I)] + \frac{(f_M + g_M)[(2 - 2\gamma - \lambda)\delta_M + \eta_M(V'_M(I) + 2V'_P(I))]^2}{16} \\ & + \frac{(f_P + g_P)[(1 - \gamma - \lambda)\delta_P + \eta_P V'_P(I)]^2}{2\varepsilon_P(f_P + g_P) + 2} + \frac{[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2V'_P(I) + V'_N(I))]^2}{8\varepsilon_N} \end{aligned} \quad (47)$$

It can be obtained from (45)–(47) that the unary one-time function with the independent variable is the solution of the HJB equation; let

$$V_M(I) = a_1 I + b_1, \quad V_N(I) = a_2 I + b_2, \quad V_P(I) = a_3 I + b_3 \quad (48)$$

where a_1, a_2, a_3, b_1, b_2 , and b_3 are constants. By substituting $V_M(I)$, $V_N(I)$, and $V_P(I)$ and their first-order partial derivatives into (45)–(47), we can obtain the following:

$$\begin{aligned} \rho(a_1 I + b_1) &= [\lambda(\mu + \varphi) - \sigma a_1]I + \frac{(f_M + g_M)(\lambda\delta_M + \eta_M a_1)[(2 - 2\gamma - \lambda)\delta_M + \eta_M(a_1 + 2a_3)]}{8} \\ &\quad + \frac{(f_P + g_P)(\lambda\delta_P + \eta_P a_1)[(1 - \gamma - \lambda)\delta_P + \eta_P a_3]}{\varepsilon_P(f_P + g_P) + 2} \\ &\quad + \frac{(\lambda\delta_N + \eta_N a_1)[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2a_3 + a_2)]}{2\varepsilon_N} \end{aligned} \quad (49)$$

$$\begin{aligned} \rho(a_2 I + b_2) &= [\gamma(\mu + \varphi) - \sigma a_2]I + \frac{(f_M + g_M)(\gamma\delta_M + \eta_M a_1)[(2 - 2\gamma - \lambda)\delta_M + \eta_M(a_1 + 2a_3)]}{4} \\ &\quad + \frac{(f_P + g_P)(\gamma\delta_P + \eta_P a_1)[(1 - \gamma - \lambda)\delta_P + \eta_P a_3]}{\varepsilon_P(f_P + g_P) + 2} \\ &\quad + \frac{(\lambda\delta_N + \eta_N a_1)[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2a_3 + a_2)]}{4\varepsilon_N} \end{aligned} \quad (50)$$

$$\begin{aligned} \rho(a_3 I + b_3) &= [(1 - \lambda - \gamma)(\mu + \varphi) - \sigma a_3]I + \frac{(f_M + g_M)[(2 - \gamma - 2\lambda)\delta_M + \eta_M(a_1 + 2a_3)]^2}{16} \\ &\quad + \frac{(f_P + g_P)[(1 - \gamma - \lambda)\delta_P + \eta_P a_3]^2}{2[\varepsilon_P(f_P + g_P) + 2]} + \frac{[(2 - \gamma - 2\lambda)\delta_N + \eta_N(2a_3 + a_2)]^2}{8\varepsilon_N} \end{aligned} \quad (51)$$

Equations (49)–(51) satisfy all $V_M(I) \geq 0$, $V_N(I) \geq 0$, and $V_P(I) \geq 0$, so the parameter values of a_1, a_2, a_3, b_1, b_2 , and b_3 can be calculated:

$$a_1 = \frac{\lambda(\mu + \varphi)}{\rho + \sigma} \quad (52)$$

$$a_2 = \frac{\gamma(\mu + \varphi)}{\rho + \sigma} \quad (53)$$

$$a_3 = \frac{(1 - \lambda - \gamma)(\mu + \varphi)}{\rho + \sigma} \quad (54)$$

$$\begin{aligned} b_1 &= \frac{(f_M + g_M)\lambda(2 - 2\gamma - \lambda)[(\rho + \sigma)\delta_M + \eta_M(\mu + \varphi)]^2}{8\rho(\rho + \sigma)^2} + \frac{(f_P + g_P)\lambda(1 - \lambda - \gamma)[\delta_P(\rho + \sigma) + \eta_P(\mu + \varphi)]^2}{\rho[\varepsilon_P(f_P + g_P) + 2](\rho + \sigma)^2} \\ &\quad + \frac{\lambda(2 - 2\lambda - \gamma)[(\rho + \sigma)\delta_N + \eta_N(\mu + \varphi)]^2}{2\varepsilon_N\rho(\rho + \sigma)^2} \end{aligned} \quad (55)$$

$$\begin{aligned} b_2 &= \frac{(f_M + g_M)\gamma(2 - 2\gamma - \lambda)[(\rho + \sigma)\delta_M + \eta_M(\mu + \varphi)]^2}{4\rho(\rho + \sigma)^2} + \frac{(f_P + g_P)(1 - \gamma - \lambda)^2[\delta_P(\rho + \sigma) + \eta_P(\mu + \varphi)]^2}{\rho(\rho + \sigma)^2[\varepsilon_P(f_P + g_P) + 2]} \\ &\quad + \frac{(2 - \gamma - 2\lambda)^2[(\rho + \sigma)\delta_N + \eta_N(\mu + \varphi)]^2}{4\varepsilon_N\rho(\rho + \sigma)^2} \end{aligned} \quad (56)$$

$$\begin{aligned} b_3 &= \frac{(f_M + g_M)(2 - 2\gamma - \lambda)^2[\delta_M(\rho + \sigma) + \eta_M(\mu + \varphi)]^2}{16\rho(\rho + \sigma)^2} + \frac{(f_P + g_P)(1 - \gamma - \lambda)^2[\delta_P(\rho + \sigma) + \eta_P(\mu + \varphi)]^2}{2[\varepsilon_P(f_P + g_P) + 2]\rho(\rho + \sigma)^2} \\ &\quad + \frac{(2 - \gamma - 2\lambda)^2[(\rho + \sigma)\delta_N + \eta_N(\mu + \varphi)]^2}{8\varepsilon_N\rho(\rho + \sigma)^2} \end{aligned} \quad (57)$$

By substituting a_1, a_2 , and a_3 into (39) and (40)–(42), the optimal effort level of the scientific innovation layer, support layer, and technology innovation platform, as well as the optimal incentive coefficient of the technology innovation platform on the scientific innovation layer and support layer, can be obtained as follows:

$$S_{M_2}^* = \frac{(f_M + g_M)(2 - 2\gamma - \lambda)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]}{4(\rho + \sigma)} \quad (58)$$

$$S_{N_2}^* = \frac{(2 - 2\lambda - \gamma)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]}{2\varepsilon_N(\rho + \sigma)} \quad (59)$$

$$S_{P_2}^* = \frac{(1-\lambda-\gamma)(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]}{[\varepsilon_P(f_P+g_P)+2](\rho+\sigma)} \quad (60)$$

$$\alpha = \frac{2-2\gamma-3\lambda}{2-2\gamma-\lambda} \quad (61)$$

$$\beta = \frac{2-3\gamma-2\lambda}{2-\gamma-2\lambda} \quad (62)$$

By substituting a_1, a_2, a_3, b_1, b_2 , and b_3 into (48), the optimal revenue function of the scientific innovation layer, support layer, and digital technology innovation platform can be obtained:

$$V_{M_2}(I)^* = \frac{\lambda(\mu+\varphi)}{\rho+\sigma} I + \frac{\lambda(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2(2-2\gamma-\lambda)}{8\rho(\rho+\sigma)^2} \\ + \frac{\lambda(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2(1-\gamma-\lambda)}{\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} + \frac{\lambda[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2(2-\gamma-2\lambda)}{2\varepsilon_N\rho(\rho+\sigma)^2} \quad (63)$$

$$V_{N_2}(I)^* = \frac{\gamma(\mu+\varphi)}{\rho+\sigma} I + \frac{\gamma(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2(2-2\gamma-\lambda)}{4\rho(\rho+\sigma)^2} \\ + \frac{\gamma(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2(1-\gamma-\lambda)}{\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} + \frac{\gamma[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2(2-\gamma-2\lambda)}{4\varepsilon_N\rho(\rho+\sigma)^2} \quad (64)$$

$$V_{P_2}(I)^* = \frac{(1-\lambda-\gamma)(\mu+\varphi)}{\rho+\sigma} I + \frac{(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2(2-2\gamma-\lambda)^2}{16\rho(\rho+\sigma)^2} \\ + \frac{(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2(1-\gamma-\lambda)^2}{2\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} + \frac{[(\rho+\sigma)\delta_N+(\mu+\varphi)\eta_N]^2(2-\gamma-2\lambda)^2}{8\varepsilon_N\rho(\rho+\sigma)^2} \quad (65)$$

So, the optimal return of the innovation system in this model can be obtained:

$$V_2(I)^* = \frac{\mu+\varphi}{\rho+\sigma} I + \frac{(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2[4-(\lambda+2\gamma)^2]}{16\rho(\rho+\sigma)^2} \\ + \frac{(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2[1-(\lambda+\gamma)^2]}{2\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} + \frac{[(\rho+\sigma)\delta_N+(\mu+\varphi)\eta_N]^2[4-(2\lambda+\gamma)^2]}{8\varepsilon_N\rho(\rho+\sigma)^2} \quad (66)$$

4.3. Collaborative Game Model

In this case, the scientific innovation layer, the support layer, and the technology innovation platform cooperate to coordinate and dynamically optimize the digital innovation ecosystem's coordinated strategy of value co-creation. As an organic whole, the innovation subject will take the overall revenue maximization as the goal and jointly determine the optimal decision of the three parties. At this point, the objective function of value co-creation of the digital innovation ecosystem is as follows:

$$\max_{S_{M(t)}S_{N(t)}S_{P(t)}} J = \int_0^\infty e^{-\rho t} [\pi(t) - (\frac{\varepsilon_T}{2} + \frac{1}{f_P+g_P})S_P^2(t) - (\frac{1}{f_M+g_M})S_M^2(t) - \frac{\varepsilon_T}{2}S_N^2(t)] dt \quad (67)$$

Assume that there exists a continuous arguably bounded revenue function $V(I)$ for the digital innovation ecosystem, which satisfies the following HJB equation for all $I \geq 0$:

$$\rho V(I) = \max_{S_{M(t)}S_{N(t)}S_{P(t)} \geq 0} \{ \pi(t) - (\frac{\varepsilon_T}{2} + \frac{1}{f_P+g_P})S_P^2(t) - (\frac{1}{f_M+g_M})S_M^2(t) - \frac{\varepsilon_T}{2}S_N^2(t) \\ + V'(I)[\eta_M S_M(t) + \eta_N S_N(t) + \eta_P S_P(t) - \sigma I(t)] \} \quad (68)$$

After calculating the first partial derivatives of S_M, S_N , and S_P in (68) and setting them equal to zero, we can obtain the following:

$$S_M = \frac{(f_M + g_M)(\delta_M + \eta_M V'(I))}{2}, S_N = \frac{\delta_N + \eta_N V'(I)}{\varepsilon_N}, S_P = \frac{(f_P + g_P)(\delta_P + \eta_P V'(I))}{\varepsilon_N(f_P + g_P) + 2} \quad (69)$$

Substitute (69) into (68), and simplify as follows:

$$\rho V(I) = I[(\mu + \varphi) - \sigma V'] + \frac{(f_P + g_P)(\delta_P + \eta_P V'(I))^2}{2[\varepsilon_N(f_P + g_P) + 2]} + \frac{(f_P + g_P)(\delta_M + \eta_M V'(I))}{4} + \frac{(\delta_N + \eta_N V'(I))^2}{2\varepsilon_N} \quad (70)$$

Equation (70) is a function of one unary function as an independent variable; let

$$V(I) = a_1 I + b_1 \quad (71)$$

where a_1 and b_1 are constants to be solved. By substituting (71) into (70), we can obtain the following:

$$\rho(a_1 I + b_1) = I[(\mu + \varphi) - \sigma a_1] + \frac{(f_P + g_P)(\delta_P + \eta_P a_1)^2}{2[\varepsilon_N(f_P + g_P) + 2]} + \frac{(f_P + g_P)(\delta_M + \eta_M a_1)}{4} + \frac{(\delta_N + \eta_N a_1)^2}{2\varepsilon_N} \quad (72)$$

According to the above assumptions, $V_M(I) \geq 0$, $V_N(I) \geq 0$, and $V_P(I) \geq 0$ are all satisfied, so a_1 and b_1 can be calculated:

$$a_1 = \frac{\mu + \varphi}{\rho + \sigma} \quad (73)$$

$$b_1 = \frac{(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{2\rho[\varepsilon_N(f_P + g_P) + 2](\rho + \sigma)^2} + \frac{(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} + \frac{[\delta_N(\rho + \sigma) + \eta_N(\mu + \varphi)]^2}{2\varepsilon_N\rho(\rho + \sigma)^2} \quad (74)$$

By substituting a_1 into (69), the optimal effort levels of the scientific innovation layer, support layer, and digital technology innovation platform can be obtained:

$$S_{M_3}^* = \frac{(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]}{2(\rho + \sigma)} \quad (75)$$

$$S_{N_3}^* = \frac{\delta_N(\rho + \sigma) + \eta_N(\mu + \varphi)}{\varepsilon_N(\rho + \sigma)} \quad (76)$$

$$S_{P_3}^* = \frac{(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]}{(\rho + \sigma)[\varepsilon_P(f_P + g_P) + 2]} \quad (77)$$

By substituting a_1 and b_1 into (71), the optimal income function of the scientific innovation layer, the supporting layer, the technology innovation platform, and the digital innovation ecosystem under this coordination strategy mode can be obtained:

$$V_{M_3}(I)^* = \frac{\lambda(\mu + \varphi)}{\rho + \sigma} I + \frac{\lambda(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} + \frac{\lambda(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{2\rho(\rho + \sigma)^2[\varepsilon_P(f_P + g_P) + 2]} + \frac{\lambda[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho + \sigma)^2} \quad (78)$$

$$V_{N_3}(I)^* = \frac{\gamma(\mu + \varphi)}{\rho + \sigma} I + \frac{\gamma(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} + \frac{\gamma(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{2\rho(\rho + \sigma)^2[\varepsilon_P(f_P + g_P) + 2]} + \frac{\gamma[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho + \sigma)^2} \quad (79)$$

$$V_{P_3}(I)^* = \frac{(1-\lambda-\gamma)(\mu+\varphi)}{\rho+\sigma} I + \frac{(1-\lambda-\gamma)(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2}{4\rho(\rho+\sigma)^2} \\ + \frac{(1-\lambda-\gamma)(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2}{2\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} + \frac{(1-\lambda-\gamma)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho+\sigma)^2} \quad (80)$$

So, the optimal return of the innovation system in this model can be obtained:

$$V_3(I)^* = \frac{(\mu+\varphi)}{\rho+\sigma} I + \frac{(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]^2}{4\rho(\rho+\sigma)^2} + \frac{(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]^2}{2\rho(\rho+\sigma)^2[\varepsilon_P(f_P+g_P)+2]} \\ + \frac{[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]^2}{2\varepsilon_N\rho(\rho+\sigma)^2} \quad (81)$$

4.4. Comparative Analysis of Model Results

By comparing the optimal strategies and benefits of the scientific innovation layer, the support layer, and the technology innovation platform, as well as the innovation level and the overall benefits of the digital innovation ecosystem system under the three coordination strategy games, this paper draws the following propositions.

Proposition 1.

$$S_{M_1}^* < S_{M_2}^* < S_{M_3}^*, S_{N_1}^* < S_{N_2}^* < S_{N_3}^*, S_{P_1}^* < S_{P_2}^* < S_{P_3}^*, \alpha^* = \frac{S_{M_2}^* - S_{M_1}^*}{S_{M_2}^*} (0 < \alpha < \frac{2}{3}), \\ \beta^* = \frac{S_{N_2}^* - S_{N_1}^*}{S_{N_2}^*} (0 < \beta < \frac{2}{3}).$$

Proof.

$$S_{M_2}^* - S_{M_1}^* = \frac{(f_M+g_M)(2-2\gamma-3\lambda)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]}{4(\rho+\sigma)} \\ = \frac{2-2\gamma-3\lambda}{2-2\gamma-\lambda} \frac{(f_M+g_M)(2-2\gamma-\lambda)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]}{4(\rho+\sigma)} \\ = \alpha^* S_{M_2}^* > 0 \quad (82)$$

$$S_{N_2}^* - S_{N_1}^* = \frac{(2-2\lambda-3\gamma)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]}{2\varepsilon_N(\rho+\sigma)} \\ = \frac{2-3\gamma-2\lambda}{2-\gamma-2\lambda} \frac{(2-2\lambda-\gamma)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]}{2\varepsilon_N(\rho+\sigma)} \\ = S_{N_2}^* \beta^* > 0 \quad (83)$$

$$S_{M_3}^* - S_{M_2}^* = \frac{(2\gamma+\lambda)(f_M+g_M)[\delta_M(\rho+\sigma)+(\mu+\varphi)\eta_M]}{4(\rho+\sigma)} > 0 \quad (84)$$

$$S_{N_3}^* - S_{N_2}^* = \frac{(\lambda+2\gamma)[\delta_N(\rho+\sigma)+(\mu+\varphi)\eta_N]}{2\varepsilon_N(\rho+\sigma)} > 0 \quad (85)$$

$$S_{P_3}^* - S_{P_2}^* = \frac{(\lambda+\gamma)(f_P+g_P)[\delta_P(\rho+\sigma)+(\mu+\varphi)\eta_P]}{2\varepsilon_N(\rho+\sigma)} > 0 \quad (86)$$

□

Corollary 1. In both the Nash non-cooperative game and the Stackelberg master–slave game, the effort level of the digital technology innovation platform remains unchanged, but the cost sharing of the digital technology innovation platform can significantly improve the effort level of the scientific innovation layer and the support layer. The increased range equals the optimal cost-sharing ratio of the digital technology innovation platform to the scientific innovation layer and the support layer, namely the optimal incentive coefficient.

Proposition 2. The optimal benefit of the digital innovation ecosystem is most prominent in the cooperative game model, next in the Stackelberg master–slave game model, and most minor in the Nash non-cooperative game model. That is, $V_3(I)^* > V_2(I)^* > V_1(I)^*$.

Proof.

$$V_2(I)^* - V_2(I) = \frac{(2\gamma + 3\lambda - 2)(\lambda - 2\gamma - 2)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{16\rho(\rho + \sigma)^2} + \frac{(3\gamma + 2\lambda - 2)(-2\lambda + \gamma - 2)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{4\epsilon_N\rho(\rho + \sigma)^2} > 0 \quad (87)$$

$$\begin{aligned} V_3(I)^* - V_2(I)^* &= \frac{(2\gamma + \lambda)^2(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{16\rho(\rho + \sigma)^2} + \frac{(\gamma + 2\lambda)^2[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{8\epsilon_N\rho(\rho + \sigma)^2} \\ &\quad + \frac{(\lambda + \gamma)^2(f_P + g_P)[\delta_P(\rho + \sigma) + (\mu + \varphi)\eta_P]^2}{2\rho(\rho + \sigma)^2[\epsilon_P(f_P + g_P) + 2]} > 0 \end{aligned} \quad (88)$$

□

Corollary 2. *The higher the revenue ratio of the scientific innovation layer and the support layer, the smaller the optimal incentive coefficient of the digital technology innovation platform for the two types of subjects. At this time, the platform can flexibly adjust the subsidy coefficient to maximize the benefits of the three parties.*

Proposition 3. *The optimal payoffs for all three types of subjects in the Stackelberg master–slave game model are more significant than those in the Nash non-cooperative model. That is $V_{M_2}(I) > V_{M_1}(I)$, $V_{N_2}(I) > V_{N_1}(I)$, $V_{P_2}(I) > V_{P_1}(I)$.*

Proof.

$$V_{M_2}(I) - V_{M_1}(I) = \frac{\lambda(2 - 2\gamma - 3\lambda)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{8\rho(\rho + \sigma)^2} + \frac{\lambda(2 - 3\gamma - 2\lambda)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{2\epsilon_N\rho(\rho + \sigma)^2} > 0 \quad (89)$$

$$V_{N_2}(I) - V_{N_1}(I) = \frac{\lambda(2 - 2\gamma - 3\lambda)(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{4\rho(\rho + \sigma)^2} + \frac{\lambda(2 - 3\gamma - 2\lambda)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{4\epsilon_N\rho(\rho + \sigma)^2} > 0 \quad (90)$$

$$V_{P_2}(I) - V_{P_1}(I) = \frac{(2\gamma + 3\lambda - 2)(f_M + g_M)[\delta_M(\rho + \sigma) + (\mu + \varphi)\eta_M]^2}{16\rho(\rho + \sigma)^2} + \frac{(3\gamma + 2\lambda - 2)[\delta_N(\rho + \sigma) + (\mu + \varphi)\eta_N]^2}{8\epsilon_N\rho(\rho + \sigma)^2} > 0 \quad (91)$$

□

Corollary 3. *In the Stackelberg master–slave game, the optimal returns of value co-creation by innovation subjects are greater than those of the Nash non-cooperation game.*

5. Simulation Analysis of Algorithms

According to the above analysis, under the three cooperative strategy game modes, the optimal strategy and income of the scientific innovation layer, the support layer, and the technology innovation platform, as well as the technology innovation ability, the resource integration ability, and the overall income of the digital innovation ecosystem, all depend on the parameter setting in the model. Literature research and data analysis were conducted on digital innovation ecosystems such as Haier and Facebook to fit the realistic situation. The parameter location logic in numerical simulation refers to the parameter setting in [42,47].

$I(0) = 0$, $\epsilon_M = 0.7$, $\epsilon_N = 0.5$, $\epsilon_P = 0.3$, $\eta_M = 0.4$, $\eta_N = 0.3$, $\eta_P = 0.2$, $\delta_M = 0.7$, $\delta_N = 0.6$, $\delta_P = 0.5$, $f_M = 0.15$, $f_P = 0.1$, $g_M = 0.15$, $g_P = 0.1$, $\mu = 0.2$, $\varphi = 0.2$, $\rho = 0.1$, $\lambda = 0.4$, $\gamma = 0.3$, and $\sigma = 0.1$.

The following is obtained:

$$\begin{aligned} S_M^N &= 0.09, S_N^N = 0.72, S_P^N = 0.0262, V_M^N = 11.2444, V_N^N = 7.7718, V_P^N = 9.0324, \\ V^N &= 23.7246, S_M^S = 0.1125, S_N^S = 1.08, S_P^S = 0.0262, V_M^S = 10.4651, V_N^S = 8.6859, \end{aligned}$$

$$V_P^S = 9.3733, V^S = 23.9269, S_M^C = 0.225, S_N^C = 2.4, S_P^C = 0.1748, V_M^C = 14.5923, V_N^C = 10.9443, V_P^C = 10.9443, \text{ and } V^C = 36.4807.$$

Therefore, Propositions 1–3 are proven.

To further verify the conclusion, according to the expression of the particular solution function of the first-order differential equation, we can obtain the following:

$$V_M^N = 3.5324 - 0.288e^{-0.1t}, V_N^N = 1.8032 - 0.0314e^{-0.1t}, V_P^N = 3.0638 - 0.0314e^{-0.1t}, \text{ and the overall income is } V^N = 8.8694 - 5.14486e^{-0.1t}.$$

$$V_M^S = 4.7351 - 0.27e^{-0.1t}, V_N^S = 4.6299 - 1.944e^{-0.1t}, V_P^S = 3.4047 - 0.0314e^{-0.1t}, \text{ and the overall income is } V^S = 11.4117 - 7.4848e^{-0.1t}.$$

$$V_M^C = 7.3123 - 0.72e^{-0.1t}, V_N^C = 9.2643 - 4.32e^{-0.1t}, V_P^C = 5.1541 - 0.2098e^{-0.1t}, \text{ and the overall income is: } V^C = 33.3797 - 16.8992e^{-0.1t}.$$

The simulation diagrams of the above model results are shown in Figures 4–7.

Figures 4–7 show the trend of optimal returns over time, with the horizontal axis being time and the vertical axis being optimal returns. The optimal returns of the scientific innovation layer, the support layer, and the digital technology innovation platform increase with time and stabilize after reaching equilibrium. The cooperative game model is better for the individual participants and the digital innovation ecosystem as a whole than the Stackelberg master–slave game model. Both are better than the non-cooperative model, and the platform can contribute to the increase in the overall returns of the system by sharing the costs for the scientific innovation layer and the support layer. The comprehensive benefits of the system increase, verifying the conclusions drawn from Propositions 2–3.

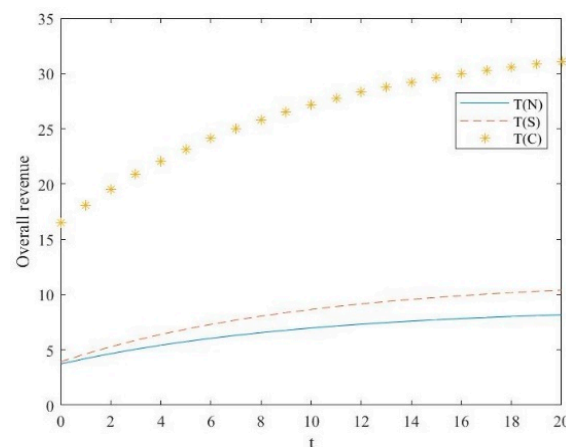


Figure 4. Optimal returns for the digital innovation ecosystem.

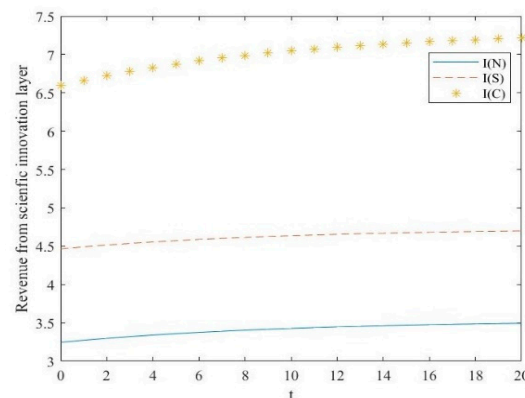


Figure 5. Optimal returns of scientific innovation layer.

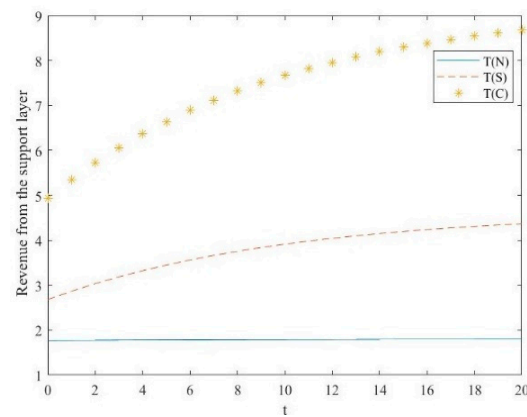


Figure 6. Optimal returns of support layer.

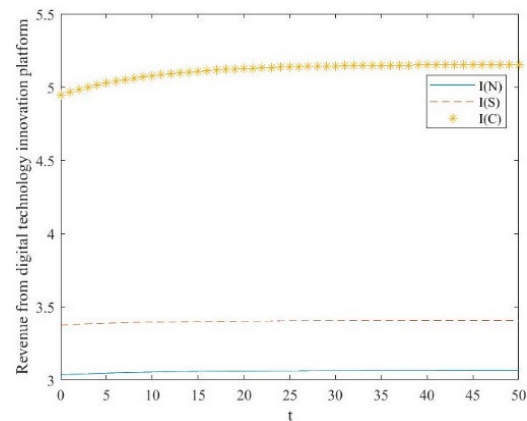


Figure 7. Optimal returns of digital technology innovation platform.

Figures 8–12 are three-dimensional diagrams of the effects of the digital technology innovation capability coefficient, digital technology assimilation and absorption capability coefficient, and resource integration cost coefficient on the optimal returns of the innovation R&D system under the synergistic symbiotic cooperation model, with the horizontal axis showing the time and influence variables and the vertical axis showing the optimal returns.

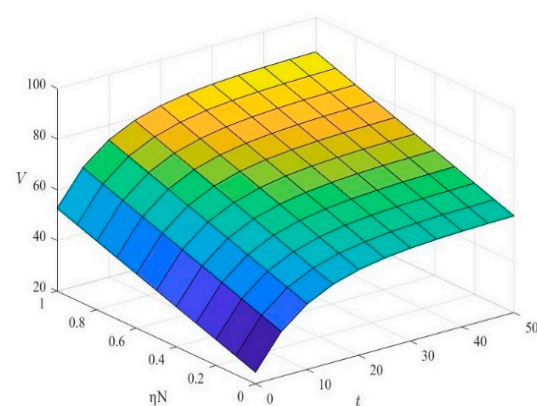


Figure 8. The effect of η_N and t on $V_3(I)^*$.

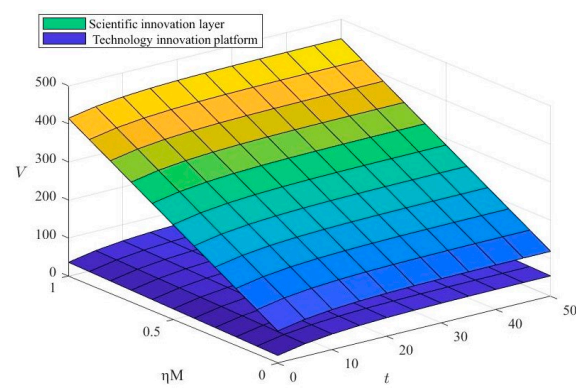


Figure 9. The effect of η_M and t on $V_3(I)^*$.

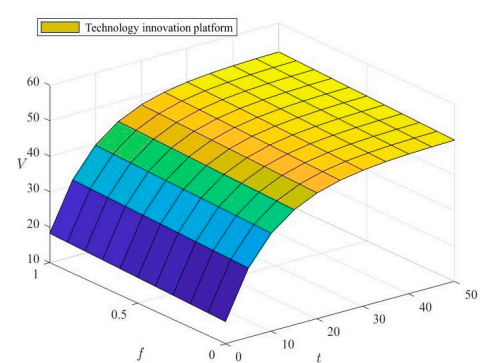
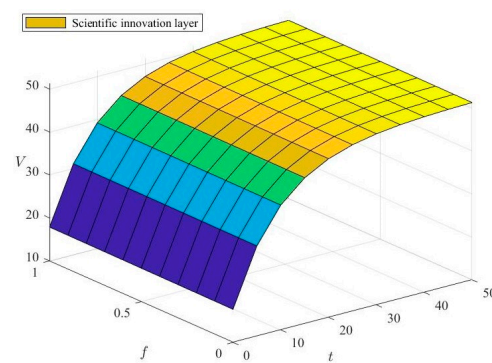


Figure 10. The effect of f_M , f_P , and t on $V_3(I)^*$.

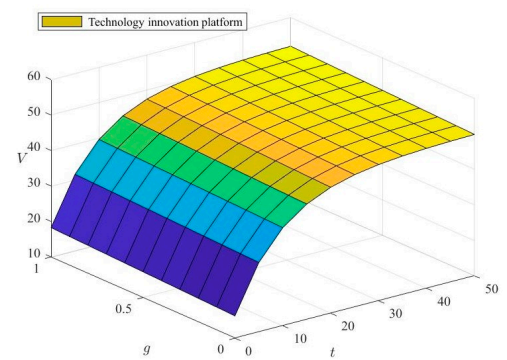
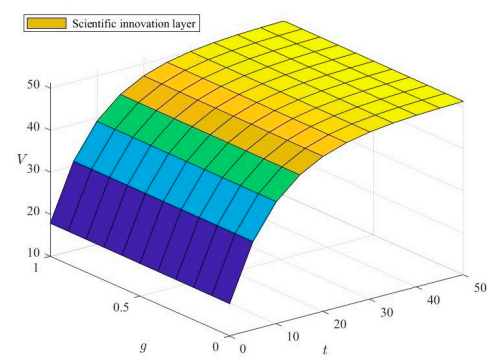


Figure 11. The effect of g_M , g_P , and t on $V_3(I)^*$.

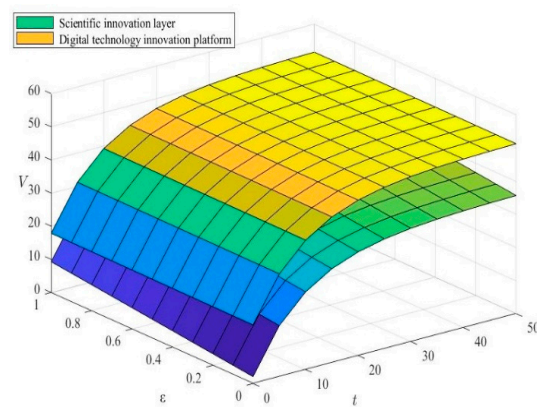


Figure 12. The effect of ε_i and t on $V_3(I)^*$.

As shown in Figure 8, the influence of the digital technology innovation R&D capability coefficient on the returns of the digital innovation ecosystem is positive, and the amplitude increases with time. As seen in Figure 9, the innovation R&D capability of the scientific innovation layer has the most significant impact on earnings. In contrast, the innovation R&D capability of the technology innovation platform has the most negligible effect on the payments. For the digital innovation ecosystem, the scientific innovation layer needs to use digital technology to break the boundaries and eliminate the barriers of technology and experience. By relying on advanced digital technology and utilizing data sharing, multiple factors such as product, service, technology, management, and market are fully interconnected to optimize innovation, research, development, and resource integration capabilities and realize sustainable value co-creation.

As seen in Figures 10 and 11, digital technology's assimilation and absorption capacity coefficients are consistent, but the impact on income needs to be made apparent. Over time, there is a certain degree of decline in returns. The enterprise-centered technology innovation platform should pay attention to the circuitous changes in the structure of digital products and services and take measures to guide and cope with the indirect changes actively. In addition, the iterative upgrading of digital technology should be considered in digital technology absorption and assimilation in the digital innovation ecosystem. The value co-creation strategy of digital products and services should be coordinated to optimize technology absorption and assimilation channels.

As seen in Figure 12, the impact of the resource integration cost of the support layer on the income is more significant than that of the technology innovation platform. Still, the impact degree gradually decreases with the evolution of time. Under the digital innovation ecosystem paradigm, integrating digital resources considers resources and pays attention to the construction of digital resources and their sharing platform. The digital innovation ecosystem should explore how to reorganize and integrate resource-related applications and services, expand their availability, improve utilization efficiency, and inject momentum into the value co-creation of innovation subjects.

6. Discussions and Implications

6.1. Theoretical Implications

Our study contributes to digital innovation research in the following ways: First, the factors involved in multi-subject value co-creation in digital innovation ecosystems were identified based on literature visualization. Although most studies have mentioned the complexity and synergy of digital innovation ecosystems [29], there needs to be a more systematic exploration of value co-creation participation in decision making. Recent scholars have focused on the involvement of innovation subjects in co-creation decision making, but mainly on the dynamic decision-making process of both issues [48]. In contrast, research on subject participation is rare and involves fewer factors. The factors related to technology assimilation, absorption, and resource integration are systematically identified through literature visualization and the construction of a general digital innovation ecosystem structure. These factors can support future research in value co-creation in digital innovation ecosystems.

This paper's second contribution is assessing the critical role of innovation subjects in decision making under different modes of collaboration. In particular, the category of innovation subjects is proposed based on a hierarchical perspective, and the structure outperforms the collaboration effect between single issues. Thus, the hierarchical structure is more applicable [42]. The digital innovation ecosystem is divided into multiple evolutionary stages [41]. Collaboration among innovation agents is divided into different modes, in which innovation agents are subject to significantly different effects of influencing factors and their corresponding equilibrium benefits. The additional decision models imply that the digital innovation ecosystem will continue to evolve. Therefore, this decision measure provides insights for analyzing and evaluating the whole ecosystem from a continuity perspective.

6.2. Managerial Implications

These findings have several managerial implications for enterprises participating in the digital innovation ecosystem. First, the cooperative game mode is optimal for realizing sustainable value co-creation in the digital innovation ecosystem. While the scientific and innovation layers and the support layers dynamically optimize technological innovation and resource supply, they should also actively conduct digital core technology research and development activities to enhance the vitality of value-co-creation innovation and development. In addition to the research and development of digital technology innovation, technology innovation platforms should also reduce the risk of the scientific innovation layer and the support layer participating in value co-creation by sharing the technology cost. At the same time, the technology innovation platform needs to flexibly adjust the intensity of subsidies based on the actual needs of customers and changes in the market environment to ensure that the costs borne by the scientific innovation layer and the support layer are in a positive proportional relationship with the excess returns.

In particular, for the technology innovation platform, the government should strengthen the regulation and guide the technology innovation platform with enterprise as the core to incentivize other participants to make the procedure and proportion of intellectual property ownership and profit distribution of result transformation more apparent. For scientific and technological innovation, we should optimize the scientific and technical environment, regulate the innovation subjects, detail the rights and responsibilities of the issues, and stimulate the market vitality of the value co-creation of all participants. We should improve the resource integration capability and value-added digital technology innovation for the support layer and promote the collaborative drive of value co-creation in the digital environment. Furthermore, we should build an excellent macro support environment for digital technology, coordinate and promote the construction of new infrastructure such as the Internet of Things, cloud computing, and artificial intelligence, and give full play to information technology-enabled value co-creation through new infrastructure. We should break down the barriers of digital technology expertise, provide an entire space for the cluster effect and technology accumulation advantages, share the cost of digital technology innovation, and inject vitality into the value co-creation of the digital innovation ecosystem.

7. Conclusions and Future Research

This paper constructs a differential game model of value co-creation coordination strategy by considering the strategic behaviors among the three innovation entities: the scientific and innovation layer, the support layer, and the technology innovation platform. The optimal effort level, the optimal income, and the optimal income of the digital innovation ecosystem are obtained under the Nash non-cooperative game mode, the Stackelberg master–slave game mode, and the collaborative game model. The following conclusions are obtained by comparing the equilibrium solution and simulation results under the three coordination strategy modes.

- (1) The Stackelberg master–slave and cooperative game modes have pronounced incentive effects, improving innovation subjects' optimal effort level to varying degrees. In the collaborative game model, the effort level, optimal return, and overall return of value co-creation of the digital innovation ecosystem are the highest. In Nash's non-cooperative mode, they are the lowest.
- (2) In Nash's non-cooperative game and Stackelberg's master–slave game, the willingness to participate in value co-creation of the technology innovation platform does not change. In the Stackelberg master–slave game mode, the willingness to participate in value co-creation between the scientific innovation layer and the support layer was higher than that in the Nash non-cooperative game model, and the difference between the two was equal to the level of technological incentive subsidy. This indicates that cost sharing and resource allocation as a regulatory mechanism can promote the value co-creation willingness of the scientific innovation and support layers. In collaborative game mode, the willingness of the technology innovation platform,

scientific innovation layer, and support layer to participate in value co-creation is higher than that in the Nash non-cooperative game model.

- (3) When the initial innovation level of digital technology research and development is high, but the innovation subject's effort level and participation willingness are low, the digital technology research and development innovation will not improve the overall income level of the digital innovation ecosystem. In addition, with the evolution of time, the optimal effort level and comprehensive income of innovation subjects will show a downward trend.

The limitations of this paper are as follows: The value co-creation process of the digital innovation ecosystem is affected by exogenous variables such as government regulation and subsidies, and relevant variables are not included in the model construction of this paper. The realization process of value co-creation involves many vital links. This paper only focuses on the strategy choice behavior of the innovation subject. The readiness of differential game simulation assignments directly affects the rationality of the conclusions. For example, the reward coefficient of the technological innovation center to the scientific innovation layer and the support layer under the Stackelberg master–slave game is constant. Still, the coefficient is dynamic and changing in the digital innovation ecosystem value co-creation collaboration process. The other parameters in the algorithm analysis are assigned to real situations, not actual data, and only indirectly reflect the actors' effort level and optimal benefits. Therefore, the setting of variables in the model and numerical simulation may generate errors.

This paper can be expanded in the following aspects in follow-up research: Combined with regional policy guidance, based on this paper, the influence of local government regulation and subsidies, as well as social welfare, on the digital innovation ecosystem, can be considered. Because the cooperative game can increase individual and system benefits, future research may consider using utility theory to deconstruct the cooperative game in the full link of value co-creation. Considering the marginal diminishing effect of resource input, future research can solve the changes in resource input in the process of value co-creation of innovative subjects under different game modes and discuss the differences in income when the value approaches marginal conditions in combination with the development status of the digital innovation ecosystem of specific enterprises.

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