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The Optimal Remanufacturing Strategy of the Closed-Loop Supply Chain Network under Government Regulation and the Manufacturer's Design for the Environment

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Abstract: To solve the problem of global warming and resources crisis, we adopt two remanufacturing strategies, denoted 'In-House Remanufacturing Strategy' and 'Outsourcing Remanufacturing Strategy,' respectively, for recycling and reusing waste products. However, to study the optimal remanufacturing strategy of a closed-loop supply chain (CLSC) network under government regulations and the manufacturer's design for the environment, we use variational inequality to construct a CLSC network equilibrium model based on these two strategies. By using a comparative analysis of the decision-makers' profits, carbon emissions, and carbon taxes, we show how the decision-makers should choose the optimal remanufacturing strategies under different government regulations and the manufacturer's levels of design for the environment. The findings of the study show that the manufacturer's design for the environment is conducive to resource recovery and promotes the development of remanufacturing activities. When manufacturers' levels of design for the environment are high, although manufacturers will adopt the outsourcing remanufacturing strategy to obtain high profits, they will lose environmental benefits. The findings also show that the new product handling fee policy in government regulations can promote energy conservation and emission reduction, and the reproduction subsidy policy can encourage product remanufacturing. Moreover, when the government's subsidy for remanufactured products increases to a threshold, it will prompt manufacturers to adopt the outsourcing remanufacturing strategy; and the remanufacturing subsidy threshold is negatively correlated with the manufacturer's levels of design for the environment.

Keywords: government regulation; design for the environment; remanufacturing strategy; closed-loop supply chain

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Citation: Zhou, Y.; Hou, M.; Wong, K.-H. The Optimal Remanufacturing Strategy of the Closed-Loop Supply Chain Network under Government Regulation and the Manufacturer's Design for the Environment. *Sustainability* **2023**, *15*, 7342. https://doi.org/10.3390/su15097342

Academic Editor: Claudia Colicchia

Received: 4 April 2023
Revised: 16 April 2023
Accepted: 26 April 2023
Published: 28 April 2023

1. Introduction

With the development of world industry, resource shortages and environmental pollution problems are becoming increasingly prominent. So product remanufacturing activities are attracting more attention as governments and enterprises are seeking new methods to solve resource utilization and environmental pollution problems. Product remanufacturing activities refer to the dismantling and repairing of useful parts of a recycled second-hand product and producing a new product from repaired products, which can obtain more economic benefits than ordinary recycling [1]. Compared to the production of new products, remanufacturing reduces air pollution from waste disposal by saving resources and extending the life cycle of the product [2]. Thus, enterprises have also invested in remanufacturing interaction, such as Kodak, BMW, Xerox, HP, Bosch, etc., all of which have obtained economic and environmental benefits through remanufacturing [3]. For example, the remanufacturing project implemented by Bosch since 1980 has saved 40% of the cost and has reduced carbon emissions by 23,000 tons [4].

Product remanufacturing strategies can be classified into 'in-house remanufacturing strategy (IHRS)' and 'outsourcing remanufacturing strategy (ORS),' respectively, which have different product recycling and remanufacturing methods. ORS refers to the strategy that the third-party remanufacturer (3PR) recycles and remanufactures waste products, but the original equipment manufacturer (OEM) has the right to sell the product and is responsible for the payment of the outsourcing fees [5]. ORS is a popular strategy among remanufacturers in Europe and other countries. For example, the manufacturers of Land Rover outsource their product remanufacturing operations to Caterpillar, and the manufacturers of BMW also outsource their remanufacturing operations to many enterprises [5]. However, under different conditions, manufacturers will choose different remanufacturing strategies according to the profitability of remanufactured products and government policy [6,7]. Thus, the study of the remanufacturing strategies of OEM is an important issue.

On the other hand, to support the national sustainable development strategy and alleviate the pressure on resources and the environment, the government promotes the development of the remanufacturing industry by imposing government regulations for manufacturers. For electronic products, WEEE requires companies to start collecting 65% of end-of-life electrical and electronic equipment in 2019 [8]. Governments have adopted policies to charge environmental treatment fees and subsidies for new products; for example, the WEEE Treatment Fund Collection and Subsidy Management Measures were issued by the Chinese government in 2012 [9]. Subsidies are provided to recycling units through a fund levied on manufacturers. The government promotes the implementation of the Extended Producer Responsibility (EPR) principle through regulatory policies to encourage manufacturers to take responsibility for product recycling and remanufacturing [8]. At the same time, the manufacturer's design for the environment (DFE) means that manufacturers consider the environmental impact of the product throughout its life cycle during the design phase of the product. By using the most efficient method for resource utilization and polluting emissions reduction in the production and recycling of products, we can improve the economic and environmental benefits [10]. DFE is very popular among manufacturers in many companies. For example, Canon and HP consider the convenience provided by product recycling and use recyclable materials for production [11] in such a way that the DFE of various products brings benefits to both OEMs and 3PR.

Since the closed-loop supply chain (CLSC) network equilibrium model can depict the competition between multiple manufacturers (multiple retailers), it has a lot of real-life applications [12–16]. Thus, this paper aims to analyze the impact of government regulations and OEM's DFE on the CLSC network remanufacturing strategy. Under the IHRS and ORS, a CLSC network equilibrium model is constructed considering the government regulations and OEMs' DFE, respectively. By using the comparative analysis of the decision-makers' profits, carbon emissions, and carbon taxes based on these two remanufacturing strategies, we show how the decision-makers should choose the optimal remanufacturing strategies under different government regulations and the manufacturer's levels of DFE. The contributions of this paper are as follows:

(1) Considering the impact of government regulations and OEM's DFE, the CLSC network equilibrium model under the government regulations and OEM's DFE is constructed based on IHRS and ORS, respectively.
(2) The effect of government regulations and OEM's DFE on the equilibrium decisions, profits, carbon emissions, and carbon taxes are analyzed qualitatively and quantitatively, which provides a scientific basis for OEMs to choose their optimal remanufacturing strategy.
(3) By using the comparison analysis, we show how the decision-makers should choose the optimal remanufacturing strategy under different government regulations and the OEM's levels of DFE.

The remainder of this paper is structured as follows: The next section summarizes the research results related to this paper. Section 3 describes the research problem and provides relevant assumptions; the two remanufacturing models are established in Section 4; Section 5 provides numerical analysis; Section 6 provides the conclusion of this paper.

2. Literature Review

2.1. Government Regulations

At the end of product life, resource recovery and environmental protection in the process of end-of-life product treatment have become more and more important issues in the world. Thus, a series of regulatory policies on remanufacturing has been formulated to regulate supply chain recycling and remanufacturing to promote the development of the remanufacturing industry [17–25]. Research on government policies on manufacturers' production emission reduction is as follows: Cao et al. [17] found that government regulations can stimulate the enthusiasm of the reverse supply chain members to recycle waste products and implement environmental protection measures. Ding et al. [18] found that the government's high remanufacturing target harmed the interests of consumers and was not necessarily conducive to manufacturers' emission reduction. The analysis results of Zhang et al. [19] showed that the tax subsidy policy did not always promote remanufacturing. Liu et al. [20] introduced a dual regulatory system with a minimum product recovery rate to deal with the deficit of government regulations in the traditional reward and punishment system for new (remanufactured) products and analyzed the optimal recycling and remanufacturing activities. Wei and Wang [21] studied the impact of enterprises' production emissions on government regulatory measures and concluded that government regulation could form a good feedback relationship. Wang et al. [22] studied the influence of the assimilation effect on OEMs' remanufacturing strategies under the government's cap-and-trade regulation. Liu et al. [23] found that the government's strict control measures against excessive emitting enterprises can affect the "free rider" behavior of enterprises. Kushwaha et al. [24] studied the government's recycling regulations on manufacturers' remanufacturing activities; they found that the government can assist companies in recycling more remanufactured products by imposing its recycling regulations. Zhao et al. [25] considered the factors of government subsidies and customers' green preferences in the car sales platform and analyzed the effects of these two factors under the background of carbon reduction. All the above research on government regulations mainly focuses on their impact on environmental efficiency and enterprise production; the results show that the implementation of government regulations can play a good regulatory effect on product recycling and remanufacturing in the reverse supply chain, as well as carbon emission reduction. In view of the above discussion, we concentrate on the study of the impact of government regulations on OEMs' remanufacturing strategy in this paper.

2.2. Manufacturers' Design for the Environment

In response to governments' regulatory policies, manufacturers found that the implementation of DFE can effectively reduce products' costs and pollution emissions in the supply chain. The research concerning OEMs' DFE from the perspective of the product life cycle can be described as follows: Chen [26] proposed a concept of DFE in which manufacturers need to consider environmental factors in the design stage only. Raz et al. [27] then extended this concept in such a way that manufacturers should analyze the impact of DFE on the company's revenue at different stages of the product life cycle. Chen et al. [28] analyzed the impact of remanufacturing design on remanufacturing activities under the retailers' remanufacturing model. Wang et al. [29] studied how OEMs can actively participate in product design to improve product recyclability and reduce carbon emissions. However, it was found that a higher level of remanufacturing is not necessarily conducive to remanufacturing; Hu et al. [30] analyzed the impact of two different remanufacturing design methods on the forward and reverse flow of a CLSC. In terms of the relationship between DFE and government regulations, Pazoki and Samarghandi [31] examined whether the government's recycling regulation is beneficial to remanufacturing or eco-design. They found that if eco-design can sufficiently reduce production costs, then more polluting products do not need recycling regulation; however, environmentally friendly products still need recycling regulation. Wang et al. [32] studied the impact of take-back legislation and product eco-design on OEMs' remanufacturing strategies. The above studies show that DFE, as a key part of the product life cycle, has also attracted more and more attention to the recycling and remanufacturing activities of enterprises. However, there is no literature to analyze whether OEMs should choose to share technology or adopt ORS. Therefore, this paper introduces DFE into the CLSC network equilibrium for the first time to explore its impact on OEMs' remanufacturing strategy choices.

2.3. Remanufacturing Strategy

The OEM's remanufacturing strategy is affected by a variety of factors [6,7]. Recently, research on the impact of government policies on OEMs' remanufacturing strategies has been studied in [33–38]. Zhang et al. [33] discussed the impact of government funding on OEM's IHRS and retailers' remanufacturing strategy. They found that in the absence of government funding policies, manufacturers preferred to use IHRS. Qiao and Su [34] discussed the impacts of government subsidies on the choice of OEM's IHRS and ORS. Feng et al. [35] studied the impact of government subsidies on the choice of OEM's IHRS and authorized remanufacturing strategy. They found that government subsidies were ineffective for authorized remanufacturing when the cost of remanufacturing was low. Zhou et al. [36] studied the impact of the production cost of new products on OEM remanufacturing strategies. Chen et al. [37] studied the influence of the remanufacturing threshold on the choice of OEM's IHRS and ORS. They found that when the entry threshold of the remanufacturing industry was low, the manufacturers could achieve more profits by using the IHRS. Liu et al. [38] considered the conditions under which OEMs should choose the ORS when recycled products can be sold, rented, and remanufactured. Fang et al. [39] compared and analyzed the two strategies (i.e., the IHRS and the ORS) and found that OEMs' remanufacturing strategy was related to the remanufacturing cost and the product quality. Li et al. [40] analyzed the impact of the tax and tariff regulations on OEMs' choice between the IHRS and the ORS. However, there is no research in the literature that studies the effect of government regulation and DFE on the OEMs' remanufacturing strategy choices; thus, this paper focuses on analyzing OEMs' remanufacturing strategies in the situation that OEMs can choose either the IHRS or the ORS according to different government regulations and OEM's levels of DFE.

2.4. Closed-Loop Supply Chain Network Equilibrium

Since the research work on the CLSC network equilibrium problem can depict the competitive relationship between multiple manufacturers or multiple retailers very well, it has attracted interest among a lot of researchers. Hammond and Beullens [12] were the first to study a CLSC network equilibrium problem. Yang et al. [13] further introduced features such as recycling rates to characterize remanufacturing activities. Chan et al. [14] established a dynamic equilibrium model considering a temporal demand. Duan et al. [15] studied retail marketing and manufacturers' social responsibility in establishing multi-period CLSC equilibrium. Fu et al. [16] constructed a CLSC network model which is not the same as those consisting of ordinary recycling and remanufacturing only; more precisely, they studied how the waste products from the forward supply chain can be recycled to produce other types of products. Regarding the impact of governments' policies and manufacturers' carbon emission reduction measures on network equilibrium, Yang et al. [41] studied the equilibrium decisions of a CLSC network under the carbon cap trading scheme. Based on the theory of vibrational inequality, Cheng et al. [42] considered carbon quotas, consumers' low-carbon preferences, and recycling rates for both low-carbon-emission and high-carbon-emission remanufacturers. Regarding the analysis of remanufacturing decisions, Zhou et al. [43] constructed an ORS model, considering recovery rates, remanufacturing costs, and environmental impact.

Although the above research works have investigated the impact of various carbon emission reduction measures implemented by the government and enterprises on the equilibrium decision of CLSC networks, they do not analyze the impact of the government regulations and the OEM's DFE on the remanufacturing strategy. Thus, we study a CLSC network equilibrium problem that considers both the government regulations and the OEM's DFE under the IHHS and ORS, respectively; the optimal remanufacturing strategy for OEMs is also analyzed.

2.5. Research Methods

From the above research, government regulations have obvious regulatory effects on the production and carbon-emission reduction of remanufactured products; thus, they have become a key external factor in controlling the recycling and remanufacturing activities of decision-makers in the CLSC. Secondly, OEMs' DFE is also a key factor in controlling the remanufacturing activities of the decision-makers in the CLSC very much. Therefore, we introduce the above two features into the CLSC network to construct equilibrium models under both the IHRS and the ORS, which are studied in Sections 4.1 and 4.2, respectively. For each model of the CLSC network, we obtain the equilibrium decisions of all the decision-makers by using variational inequality (VI) [44]. Finally, in Section 5, the optimal government regulations, the optimal DFE input of the OEM, and the marginal conditions for the choice of OEMs' remanufacturing strategies are analyzed based on their profits and carbon emissions via solving numerical examples. By using the comparison analysis, we show how the OEMs should choose the optimal remanufacturing strategies under different government regulations and DFE levels. Table 1 shows the differences in character between this article and the existing literature.

Table 1. The character of this paper.

Reference	IHRS		ORS		Government Regulations		DFE		CLSC Network	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Zheng et al. (2019) [11]	√			√		√	√			√
Pozaki and Samarghandi.(2020) [31]	√			√	√		√			√
Zhang et al. (2020) [19]	√			√	√			√		√
Fu et al. (2021) [16]	√			√		√		√	√	
Wang et al. (2021) [29]	√			√		√	√			√
Feng et al. (2021) [35]	√			√	√			√		√
Zhou et al. (2021) [43]	√			√		√		√	√	
Wang et al. (2022) [32]	√			√	√		√			√
Cheng et al. (2022) [42]		√		√	√			√	√	
Li et al. (2023) [40]	√		√		√			√		√
This study	√		√		√		√		√	

3. Model and Assumption

This paper investigates a competitive CLSC network that involves two remanufacturing strategies: IHRS (Figure 1) and ORS (Figure 2).

As shown in Figure 1, CLSC under the IHRS consists of I OEMs, $(i = 1, 2, \ldots, I) J$ retailers, $(j = 1, 2, \ldots, J)$ and K demand markets $(k = 1, 2 \ldots, K)$ in which the decision-makers at the same level are competing non-cooperatively [45]. Under IHRS, OEMs are responsible for producing new products and selling them to demand markets through retailers. They are also responsible for collecting waste products and producing remanufactured products from usable materials, taking environmental factors into account in the design of remanufactured products, which are also sold to the demand markets. The government coordinates the production and sales of the products by implementing regulatory measures for OEMs (new products' handling fees, recycling products' subsidies) to achieve the best environmental benefits.

As shown in Figure 2, CLSC under ORS consists of I OEMs, $(i = 1, 2, \ldots, I)$ J retailers, $(j = 1, 2, \ldots, J)$ and K demand markets $(k = 1, 2 \ldots, K)$ and O 3PRs $(o = 1, 2 \ldots, O)$, in which the decision-makers at the same level are competing non-cooperatively for the sale of two products [45]. Under ORS, OEMs invest in the manufacturing and DFE of new products and share DFE technology with 3PRs, while 3PRs recycle and produce the remanufactured products. Like IHRS, OEMs are still responsible for the sales of the two products [5], and the production and sales of these products are still subject to government regulations. The parameters are shown in Table 2.

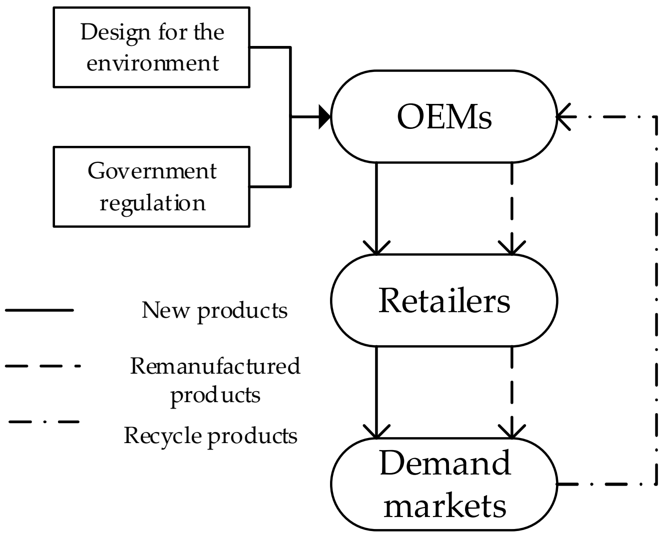

Figure 1. Schematic diagram of IHRS transactions.

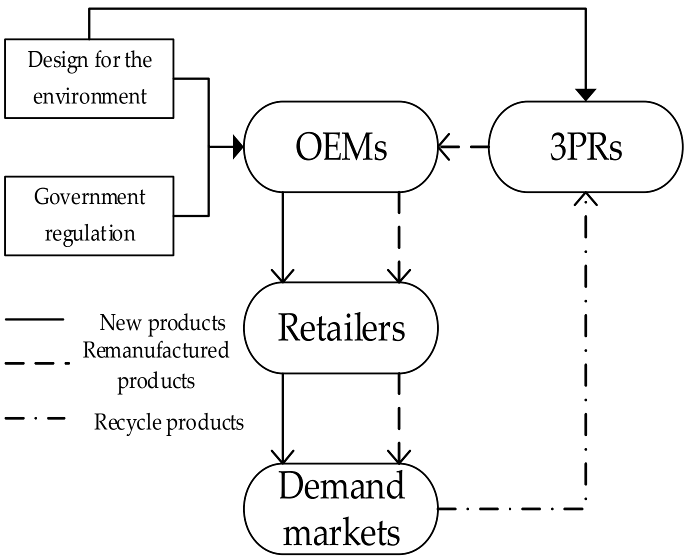

Figure 2. Schematic diagram of ORS transactions.

Table 2. Parameters.

Parameter	Definition
α	Scrap product availability ratio $\alpha \in R_+$
θ	OEMs′ levels of DFE $\theta \in R_+$
t_1	Carbon emissions tax per unit of new product production $t_1 \in R_+$
t_2	Carbon emissions tax per unit of remanufactured product production $t_2 \in R_+$
η	OEMs' unit waste landfill cos t $\eta \in R_+$
v	The reduction of carbon emissions of new or remanufactured products per unit OEMs′ DFE level $\nu \in R_+$
k_n	Retailers' new product promotion effort coefficient $k_n \in R_+$
k_r	Retailers' remanufacturer product promotion effort coefficient $k_r \in R_+$
s_n	Governments' unit new product handling fee (unit: yuan) $s_n \in R_+$
s_r	Governments' unit remanufacturing subsidies (unit: yuan) $s_r \in R_+$
e_{new}	Carbon emissions per unit of new products $e_{new} \in R_+$
e_r	Carbon emissions per unit of remanufactured products $e_r \in R_+$
e_{nn}	Carbon emissions per unit of new products under DFE $e_{nn} \in R_+$
e_{rr}	Carbon emissions per unit of remanufacturing products under DFE $e_{rr} \in R_+$

For the convenience of research, the following hypothesis is proposed:

Hypothesis 1. *The two products, i.e., new products and remanufactured products, can be used and recycled by the demand market, but due to the skepticism of remanufactured products in the demand market, the sales price of the remanufactured products should be reduced, and the environmental performance should be improved to attract customers in the demand markets to purchase these products [46].*

Hypothesis 2. *Due to the two products will compete in the demand market [2], we assume that the demand for new products (remanufactured products) is a decreasing function of the new products (remanufactured products) but an increasing function of the remanufactured products (new products).*

Hypothesis 3. *Due to the fact that OEM can use DFE to reduce the production cost and carbon emissions units and hence enhance its own competitiveness [11], we assume that the production cost function of the new products (remanufactured products) is a decreasing function of θ and the DFE input cost function is an increasing function of θ, carbon emissions per unit of the new product $e_{nn} = e_{new} - v * \theta$, and carbon emission coefficient per unit of remanufactured products $e_{rr} = e_r - v * \theta$ [11]. Since the carbon emissions per unit of remanufactured products are less than those of new products, we assume that $t_1 > t_2$ [47].*

4. Model Formulation

4.1. IHRS Equilibrium Model

The non-negative decision variables required in this section are shown in Table 3.

Table 3. Decision variables.

Decision Variable	Definition
q_i^n	The new products produced by OEM i. All these products form a vector $Q^1 \in R_+^I$
q_{ij}^n	The sale of new products from OEM i to retailer j. All these sales form a matrix $Q^2 \in R_+^{IJ}$
q_{ij}^r	The sale of remanufactured products from OEM i to retailer j. All these sales form a matrix $Q^3 \in R_+^{IJ}$
q_{ki}	The waste products collected by OEM i from the demand market k All these products form a matrix $Q^4 \in R_+^{KI}$.
q_{jk}^n	The transactions of new products from retailer j to demand market k. All these transactions form a matrix $Q^5 \in R_+^{JK}$
q_{jk}^r	The transactions of remanufactured products from retailer j to demand market k. All these transactions form a matrix $Q^6 \in R_+^{JK}$
ρ_k^n	The price for purchasing 1 item of new products at demand market k. All these prices form a vector $\rho^n \in R_+^K$
ρ_k^r	The price for purchasing 1 item of the remanufactured products at demand market k. All these prices form a vector $\rho^n \in R_+^K$

4.1.1. OEMs' Equilibrium Decisions

I competing OEMs use the IHRS to produce and sell new products to J retailers and are responsible for collecting waste products for remanufacturing; the remanufactured products are resold to demand markets. By Hypothesis 3, OEMs use DFE to reduce production costs and carbon emissions. At the same time, the governments regulate the production of the two products of OEMs through regulatory policies, promote resource reuse, and reduce polluting emissions. Suppose that the transaction revenue between OEMs i and J retailers on two products is $\sum_{j=1}^{J}(\rho_{ij}^{n}q_{ij}^{n}+\rho_{ij}^{r}q_{ij}^{r})$ and the waste recycling cost between OEMs i and K demand markets is $\sum_{k=1}^{K}\rho_{ki}q_{ki}$ ($\rho_{ij}^{n}\rho_{ij}^{r}$, and ρ_{ki} are the endogenous price of new products, remanufactured products, and waste products, respectively.). Suppose that only $\alpha\sum_{k=1}^{K}q_{ki}$ of the recycled products can be remanufactured. Moreover, OEMs i also have to pay a variety of costs, including production costs of the two products $f_i^n(q_i^n,\theta)$ and $f_i^r(q_{ki},\theta)$, new product DFE input costs $c_i(q_i^n,\theta)$, waste product collection transaction costs $c_{ij}^n(q_{ij}^n)$, $c_{ij}^r(q_{ij}^r)$, and $c_{ki}(q_{ki})$, and waste disposal costs $\eta(1-\alpha)\sum_{k=1}^{K}q_{ki}$ (All these cost functions are differentiable and convex [48]). On the other hand, when the government levies a carbon emission tax on remanufactured product subsidy fees $s_r\alpha\sum_{k=1}^{K}q_{ki}$, a new product processing fee $s_nq_i^n$ and a carbon emission tax on the two products $t_1e_{nn}q_i^n+t_2e_{rr}\alpha\sum_{k=1}^{K}q_{ki}$ from life cycle assessment (LCA) emerge [49]. Then, the maximum profit of OEM i (denoted by π_i) is:

$$\max\pi_i=\sum_{j=1}^{J}(\rho_{ij}^{n}q_{ij}^{n}+\rho_{ij}^{r}q_{ij}^{r})-\sum_{k=1}^{K}\rho_{ki}q_{ki}-f_i^n(q_i^n,\theta)-f_i^r(q_{ki},\theta)-c_i(q_i^n,\theta)-\sum_{j=1}^{J}c_{ij}^n(q_{ij}^n)-\sum_{j=1}^{J}c_{ij}^r(q_{ij}^r)-\sum_{k=1}^{K}c_{ki}(q_{ki}) \\ -\eta(1-\alpha)\sum_{k=1}^{K}q_{ki}+s_r\alpha\sum_{k=1}^{K}q_{ki}-s_nq_i^n-t_1e_{nn}q_i^n-t_2e_{rr}\alpha\sum_{k=1}^{K}q_{ki} \tag{1}$$

$$s.t\sum_{j=1}^{J}q_{ij}^{n}\le q_i^n \tag{2}$$

$$\sum_{j=1}^{J}q_{ij}^{r}\le\alpha\sum_{k=1}^{K}q_{ki} \tag{3}$$

$$q_i^n,\ q_{ij}^n,\ q_{ij}^r,\ q_{ki}\ge 0\ \forall j,k \tag{4}$$

where inequalities (2) and (3) represent the sales volume constraints of the two products of OEM i, respectively.

Proposition 1. *The profit of OEM i, π_i is a concave function of q_i^n q_{ij}^n q_{ij}^r, and q_{ki}.*

Proof. See Appendix A. □

Theorem 1. *Under the IHRS, all the OEMs at the same level are competing non-cooperatively in the Nash manner [45]. From Proposition 1, the equilibrium of the OEMs can be determined* *by solving the VI (5) as follows [12]: Determine* $(Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, \lambda_1^*, \lambda_2^* \in R_+^{I+2IJ+KI+2I}$ *which satisfies:*

$$\begin{aligned}
&\sum_{i=1}^{I}[\frac{\partial f_i^n(q_i^{n*},\theta)}{\partial q_i^n}+\frac{\partial c_i(q_i^{n*},\theta)}{\partial q_i^{n*}}+s_n+t_1^* e_{nn}-\lambda_{1i}^*]\times[q_i^n-q_i^{n*}]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n}-\rho_{ij}^{n*}+\lambda_{1i}^*]\times[q_{ij}^n-q_{ij}^{n*}]+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r}-\rho_{ij}^{r*}+\lambda_{2i}^*]\times[q_{ij}^r-q_{ij}^{r*}]\\
&+\sum_{k=1}^{K}\sum_{i=1}^{I}[\frac{\partial f_i^r(q_{ki}^*,\theta)}{\partial q_{ki}}+\frac{\partial c_{ki}(q_{ki}^*)}{\partial q_{ki}}+\eta(1-\alpha)+t_2^*\alpha e_{rr}-s_r\alpha-\alpha\lambda_{2i}^*+\rho_{ki}^*]\times[q_{ki}-q_{ki}^*]\\
&+\sum_{i=1}^{I}[q_i^{n*}-\sum_{j=1}^{J}q_{ij}^{n*}]\times[\lambda_{1i}-\lambda_{1i}^*]+\sum_{i=1}^{I}[\alpha\sum_{k=1}^{K}q_{ki}^*-\sum_{j=1}^{J}q_{ij}^{r*}]\times[\lambda_{2i}-\lambda_{2i}^*]\geq 0,\\
&\forall(Q^1,Q^2,Q^3,Q^4,\lambda_1,\lambda_2)\in R_+^{I+2IJ+KI+2I}.
\end{aligned} \tag{5}$$

where λ_{1i} *and* λ_{2i} *are the Lagrange multipliers of constraints (2) and (3), respectively.*

Proof. See Appendix B. □

Property 1. *When* $t_1\nu - \frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^n \partial\theta} - \frac{\partial^2 c_i(q_i^n,\theta)}{\partial q_i^n \partial\theta} > 0$, q_i^n *and* q_{ij}^n *are increasing functions of* θ*, where* θ *is the DFE level of OEM i; when* $t_2\alpha v - \frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki}\partial\theta} > 0$, q_{ki} *and* q_{ij}^r *are increasing functions of* θ*.*

Proof. See Appendix C. □

The economic interpretation of Property 1 is: When the OEMs' carbon tax is large enough, i.e., $t_1\nu \geq \frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^n\partial\theta} + \frac{\partial^2 c_i(q_i^n,\theta)}{\partial q_i^n\partial\theta}$ $(t_2\alpha v - \frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki}\partial\theta} > 0)$, OEMs will increase the level of the DFE to reduce the cost of the carbon emissions tax and increase the production and trading volumes of the new products; hence, the trading volume of the remanufactured products also increases.

Property 2. q_i^n *and* q_{ij}^n *are decreasing functions of* s_n*.* q_{ki} *and* q_{ij}^r *are increasing functions of* s_r*.*

Proof. The proof is like that given for Property 1. □

The economic explanation of Property 2 follows: When governments increase the processing cost of new products, it will prompt OEMs to reduce the production and sales of new products. When governments increase the reproduction subsidies, OEMs are encouraged to recycle more waste products, which in turn increases the sales of remanufactured products.

4.1.2. Retailers' Equilibrium Decisions

J competing retailers purchase two products from I OEMs and advertise and sell both products to K demand markets. Retailer j obtains the revenue $\sum_{k=1}^{K}(\rho_{jk}^n q_{jk}^n + \rho_{jk}^r q_{jk}^r)$ from the transaction of the two products with K demand markets (ρ_{jk}^n and ρ_{jk}^r are the endogenous sales prices) but needs to pay the purchase cost $\sum_{i=1}^{I}(\rho_{ij}^n q_{ij}^n + \rho_{ij}^r q_{ij}^r)$, the exhibition cost $c_j^n(q_{ij}^n)$ + $c_j^r(q_{ij}^r)$, and the advertising cost $c_j^n(k_n) + \bar{c}_j^r(k_r)$ (All these cost functions are differentiable and convex [48]). Then, the maximum profit of retailer j (denoted by π_j) is:

$$\max \pi_j = \sum_{k=1}^{K}(\rho_{jk}^n q_{jk}^n + \rho_{jk}^r q_{jk}^r) - \sum_{i=1}^{I}(\rho_{ij}^n q_{ij}^n + \rho_{ij}^r q_{ij}^r) - c_j^n\left(q_{ij}^n\right) - c_j^r(q_{ij}^r) - \bar{c}_j^n(k_n) - \bar{c}_j^r(k_r) \tag{6}$$

$$s.t\sum_{k=1}^{K} q_{jk}^{n} \le \sum_{i=1}^{I} q_{ij}^{n} \tag{7}$$

$$\sum_{k=1}^{K} q_{jk}^{r} \le \sum_{i=1}^{I} q_{ij}^{r} \tag{8}$$

$$q_{ij}^{n},\ q_{ij}^{r}, q_{jk}^{n},\ q_{jk}^{r} \ge 0\ \forall i,\ k \tag{9}$$

where inequalities (7) and (8) represent the sales volume constraints of the two products of retailer j, respectively.

Proposition 2. *The profit of retailer* j, π_j, *is a concave function of* q_{ij}^{n} q_{ij}^{r}, q_{jk}^{n}, *and* q_{jk}^{r}.

Proof. The proof is similar to that given for Proposition 1. □

Theorem 2. *Under the IHRS, all the retailers at the same level are competing non-cooperatively in the Nash manner [45]. From Proposition 2, the equilibrium of all retailers can be determined by solving the following VI (10) [12]: Determine* $(Q^{2*}, Q^{3*}, Q^{5*}, Q^{6*}, \mu_1^*, \mu_2^*) \in R_+^{2IJ+2JK+2J}$ *which satisfies*:

$$\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{J}[\rho_{ij}^{n*} + \frac{\partial c_j^n(q_{ij}^{n*})}{\partial q_{ij}^{n}} - \mu_{1j}^*] \times [q_{ij}^{n} - q_{ij}^{n*}] + \sum_{i=1}^{I}\sum_{j=1}^{J}[\rho_{ij}^{r*} + \frac{\partial c_j^r(q_{ij}^{r*})}{\partial q_{ij}^{r}} - \mu_{2j}^*] \times [q_{ij}^{r} - q_{ij}^{r*}] \\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}[-\rho_{jk}^{n*}+\mu_{1j}^*] \times [q_{jk}^{n} - q_{jk}^{n*}] + \sum_{j=1}^{J}\sum_{k=1}^{K}[-\rho_{jk}^{r*}+\mu_{2j}^*] \times [q_{jk}^{r} - q_{jk}^{r*}] + \sum_{j=1}^{J}[\sum_{i=1}^{I} q_{ij}^{n*} - \sum_{k=1}^{K} q_{jk}^{n*}] \times [\mu_{1j} - \mu_{1j}^*] \\
&+\sum_{j=1}^{J}[\sum_{i=1}^{I} q_{ij}^{r*} - \sum_{k=1}^{K} q_{jk}^{r*}] \times [\mu_{2j} - \mu_{2j}^*] \ge 0, \forall (Q^2, Q^3, Q^5, Q^6, \mu_1, \mu_2) \in R_+^{2IJ+2JK+2J},
\end{aligned} \tag{10}$$

where μ_{1j} *and* μ_{2j} *are the Lagrange multipliers of the constraints (7) and (8), respectively.*

Proof. The proof is similar to that given for Theorem 1. □

4.1.3. Demand-Markets' Equilibrium Decisions

K competing demand markets purchase the two products from J retailers. The demand functions for the two products in demand market k are $d_k^n(\rho_k^n, \rho_k^r, k_n)$ and $d_k^r(\rho_k^n, \rho_k^r, k_r)$, respectively, and the transaction costs are c_{jk}^n and c_{jk}^r, respectively. Then the transaction volume and the price of the demand market k should satisfy the following complementary conditions:

New products:

$$\rho_{jk}^{n*} + c_{jk}^{n*}(q_{jk}^{n*}) \begin{cases} = \rho_k^{n*},\ if\ q_{jk}^{n*} > 0 \\ \ge \rho_k^{n*},\ if\ q_{jk}^{n*} = 0 \end{cases} \tag{11}$$

$$d_k^n(\rho_k^{n*}, \rho_k^{r*}, k_n) \begin{cases} = \sum_{j=1}^{J} q_{jk}^{n*}, if\ \rho_{jk}^{n*} > 0 \\ \le \sum_{j=1}^{J} q_{jk}^{n*}, if\ \rho_{jk}^{n*} = 0 \end{cases} \tag{12}$$

Remanufactured products:

$$\rho_{jk}^{r*} + c_{jk}^{r*}(q_{jk}^{r*}) \begin{cases} = \rho_k^{r*},\ if\ q_{jk}^{r*} > 0 \\ \ge \rho_k^{r*},\ if\ q_{jk}^{r*} = 0 \end{cases} \tag{13}$$

$$d_k^r(\rho_k^{n*},\rho_k^{r*},k_r)\begin{cases} = \sum_{j=1}^{J} q_{jk}^{r*},\ if\ \rho_{jk}^{r*} > 0 \\ \leq \sum_{j=1}^{J} q_{jk}^{r*},\ if\ \rho_{jk}^{r*} = 0 \end{cases} \tag{14}$$

Complementary condition (11) indicates that when the demand market k purchases a new product from retailer j, the product price that it is willing to pay for the new product from the retailer j is equal to ρ_{jk}^n plus the transaction cost c_{jk}^n. The interpretation of complementary condition (13) for remanufactured products is the same as that of complementary condition (11). Complementary condition (12) indicates that when ρ_k^n is greater than 0, the demand for the new products is equal to $\sum_{j=1}^{J} q_{jk}^n$. The interpretation of complementary condition (14) for remanufactured products is the same as that of complementary condition (12).

Under the IHRS, demand market k in reverse logistics sells the waste product to I OEMs. The aversion function of the lost waste product is $\alpha_k(q_{ki})$. Then, the equilibrium decision in the reverse logistics needs to satisfy the constraints:

$$\alpha_k(q_{ki}^*)\begin{cases} = \rho_{ki}^*,\ if\ q_{ki}^* > 0 \\ \geq \rho_{ki}^*,\ if\ q_{ki}^* = 0 \end{cases} \tag{15}$$

$$\sum_{i=1}^{I} q_{ki} \leq \sum_{j=1}^{J} (q_{jk}^n + q_{jk}^r) \tag{16}$$

$$q_{jk}^n, q_{jk}^r, q_{ki} \geq 0\ \forall i, j \tag{17}$$

Inequality (16) represents the scrap products constraint of demand market k.

Theorem 3. *The complementarity condition (11–16) is equivalent to the following VI (18) [50], i.e., the equilibrium of all the demand markets can be determined from VI (18): Determine* $\left(Q^{4*}, Q^{5*}, Q^{6*}, \rho^{n*}, \rho^{r*}, v^*\right) \in R^{KI+2JK+3K}$ *which satisfy:*

$$\begin{aligned}
&\sum_{k=1}^{K}\sum_{i=1}^{I}[\alpha_k(q_{ki}^*) - \rho_{ki}^* + v_k^*] \times [q_{ki} - q_{ki}^*] + \sum_{j=1}^{J}\sum_{k=1}^{K}[\rho_{jk}^{n*} + c_{jk}^n(q_{jk}^{n*}) - \rho_k^{n*} - v_k^*) \times [q_{jk}^n - q_{jk}^{n*}] \\
&+ \sum_{j=1}^{J}\sum_{k=1}^{K}[\rho_{jk}^{r*} + c_{jk}^r(q_{jk}^{r*}) - \rho_k^{r*} - v_k^*) \times [q_{jk}^r - q_{jk}^{r*}] + \sum_{k=1}^{K}[\sum_{j=1}^{J} q_{jk}^{n*} - d_k^{n*}(\rho_k^{n*}, \rho_k^{r*}, k_n)] \times [\rho_k^n - \rho_k^{n*}] \\
&+ \sum_{k=1}^{K}[\sum_{j=1}^{J} q_{jk}^{r*} - d_k^{r*}(\rho_k^{n*}, \rho_k^{r*}, k_r)] \times [\rho_k^r - \rho_k^{r*}] + \sum_{k=1}^{K}[\sum_{j=1}^{J}(q_{jk}^{n*} + q_{jk}^{r*}) - \sum_{i=1}^{I} q_{ki}^*] \times [v_k - v_k^*] \geq 0, \\
&\forall (Q^4, Q^5, Q^6, \rho^n, \rho^r, v) \in R^{KI+2JK+3K}.
\end{aligned} \tag{18}$$

where v_k is the Lagrange multiplier of constraint (17).

Proof. Equations ((11)–(16)) represent the complementary conditions of the decisions of the demand markets. From the equivalence of complementary conditions and VI [50], we obtain VI (18). □

4.1.4. CLSC Network Equilibrium in IHRS

Suppose that all the CLSC decision-makers reach equilibrium in IHRS, i.e., the equilibrium of all the OEMs, retailers, and demand markets satisfy VI (5), VI (10), and VI (18) simultaneously. Then, by adding all the VIs and subtracting all the endogenous prices ρ_{ij}^{n*},ρ_{ij}^{r*},ρ_{jk}^{n*}, and ρ_{ki}^*, we obtain the network equilibrium, that is, determine $(Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, Q^{5*}, Q^{6*}, \rho^{n*}, \rho^{r*}, \lambda_1^*, \lambda_2^*, \mu_1^*, \mu_2^*, v^*) \in \mathrm{R}^{I+2IJ+KI+2JK+2K+2I+2J+K}$, which satisfies the following VI:

$$
\begin{aligned}
&\sum_{i=1}^{I}[\frac{\partial f_i^n(q_i^{n*},\theta)}{\partial q_i^n}+\frac{\partial c_i(q_i^{n*},\theta)}{\partial q_i^n}+s_n+t_1^*e_{nn}-\lambda_{1i}^*]\times[q_i^n-q_i^{n*}]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n}+\frac{\partial c_j^n(q_{ij}^{n*})}{\partial q_{ij}^n}-\mu_{1j}^*+\lambda_{1i}^*]\times[q_{ij}^n-q_{ij}^{n*}]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r}+\frac{\partial c_j^r(q_{ij}^{r*})}{\partial q_{ij}^r}-\mu_{2j}^*+\lambda_{2i}^*]\times[q_{ij}^r-q_{ij}^{r*}]\\
&+\sum_{k=1}^{K}\sum_{i=1}^{I}[\frac{\partial f_i^r(q_{ki}^*,\theta)}{\partial q_{ki}}+\frac{\partial c_{ki}(q_{ki}^*)}{\partial q_{ki}}+\eta(1-\alpha)+t_2^*\alpha e_{rr}-s_r\alpha-\alpha\lambda_{2i}^*+\alpha_k(q_{ki}^*)+v_k^*]\times[q_{ki}-q_{ki}^*]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}[c_{jk}^n(q_{jk}^{n*})-\rho_k^{n*}-v_k^*+\mu_{1j}^*]\times[q_{jk}^n-q_{jk}^{n*}]+\sum_{j=1}^{J}\sum_{k=1}^{K}[c_{jk}^r(q_{jk}^{r*})-\rho_k^{r*}-v_k^*+\mu_{2j}^*]\times[q_{jk}^r-q_{jk}^{r*}]\\
&+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{n*}-d_k^{n*}(\rho_k^{n*},\rho_k^{r*},k_n)]\times[\rho_k^n-\rho_k^{n*}]+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{r*}-d_k^{r*}(\rho_k^{n*},\rho_k^{r*},k_r)]\times[\rho_k^r-\rho_k^{r*}]\\
&+\sum_{i=1}^{I}[q_i^{n*}-\sum_{j=1}^{J}q_{ij}^{n*}]\times[\lambda_{1i}-\lambda_{1i}^*]+\sum_{i=1}^{I}[\alpha\sum_{k=1}^{K}q_{ki}^*-\sum_{j=1}^{J}q_{ij}^{r*}]\times[\lambda_{2i}-\lambda_{2i}^*]+\sum_{j=1}^{J}[\sum_{i=1}^{I}q_{ij}^{n*}-\sum_{k=1}^{K}q_{jk}^{n*}]\times[\mu_{1j}-\mu_{1j}^*]\\
&+\sum_{j=1}^{J}[\sum_{i=1}^{I}q_{ij}^{r*}-\sum_{k=1}^{K}q_{jk}^{r*}]\times[\mu_{2j}-\mu_{2j}^*]+\sum_{k=1}^{K}[\sum_{j=1}^{J}(q_{jk}^{n*}+q_{jk}^{r*})-\sum_{i=1}^{I}q_{ki}^*]\times[v_k-v_k^*]\geq 0\,,\\
&\forall(Q^v,Q^1,Q^2,Q^3,Q^4,Q^5,\rho^n,\rho^r,\lambda_1,\lambda_2,\mu_1,\mu_2,v)\in \mathrm{R}^{I+2IJ+KI+2JK+2K+2I+2J+K}.
\end{aligned}
\tag{19}
$$

We get the endogenous prices $\rho_{ij}^{n*}=\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n}+\lambda_{1i}^*$ and $\rho_{ij}^{r*}=\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r}+\lambda_{2i}^*$ from VI (5) [50]. We also get $\rho_{jk}^{n*}=\mu_{1j}^*, \rho_{jk}^{r*}=\mu_{2j}^*$ from VI (10) and $\rho_{ki}^*=\alpha_k(q_{ki}^*)+v_k^*$ from VI (18).

4.2. ORS Equilibrium Model

The non-negative decision variables that need to be added under the ORS are shown in Table 4.

Table 4. Decision variables.

Decision Variable	Definition
q_{oi}	The sale of new products from 3PR o to OEM i. All these products form a matrix $Q^7\in R_+^{OI}$
q_{ko}	The waste products collected by 3PR o from the demand market k. All these products recovered from the demandsform a matrix $Q^8\in R_+^{KO}$.

4.2.1. OEMs' Equilibrium Decisions

Comparing Figure 1 with Figure 2, I competing OEMs using the ORS no longer need to recycle and remanufacture scrap products. That is, they do need to pay the recycling and remanufacturing costs, but they still need to pay the carbon emission tax and obey all the government regulations. More precisely, the OEM i pays the cost ρ_{oi} via outsourcing their waste products for remanufacturing and the transaction cost $\sum_{o=1}^{O}\rho_{oi}q_{oi}$ to the O 3PRs for the remanufacturing of waste products. The remaining decision variables, parameters, and cost functions are the same as those described in Section 4.1.1. Thus, the profit of OEM i (denoted by π_i) is:

$$
\begin{aligned}
\max\,\pi_i=&\sum_{j=1}^{J}(\rho_{ij}^nq_{ij}^n+\rho_{ij}^rq_{ij}^r)-f_i^n(q_i^n,\theta)-\sum_{o=1}^{O}\rho_{oi}q_{oi}-c_i(q_i^n,\theta)-\sum_{j=1}^{J}c_{ij}^n(q_{ij}^n)-\sum_{j=1}^{J}c_{ij}^r(q_{ij}^r)\\
&+s_r\sum_{o=1}^{O}q_{oi}-s_nq_i^n-t_1e_{nn}q_i^n-t_2e_{rr}\sum_{o=1}^{O}q_{oi}
\end{aligned}
\tag{20}
$$

$$
s.t\sum_{j=1}^{J}q_{ij}^n\leq q_i^n \tag{21}
$$

$$\sum_{j=1}^{J} q_{ij}^{r} \leq \sum_{o=1}^{O} q_{oi} \tag{22}$$

$$q_i^n, q_{ij}^n, q_{ij}^r, q_{oi} \geq 0 \; \forall j, o \tag{23}$$

Inequalities (21) and (22) represent the constraints on the sales of the two products, respectively.

Proposition 3. *The profit of OEM I, π_i, is a concave function of $q_i^n q_{ij}^n$ q_{ij}^r, and q_{oi}.*

Proof. The proof is like that of Proposition 1. □

Theorem 4. *Under the ORS, all the OEMs at the same level are competing non-cooperatively in the Nash manner [45]. From Proposition 3, the equilibrium of the OEMs can be determined by solving the VI (24) as follows [12]: Determine $(Q^{1*}, Q^{2*}, Q^{3*}, Q^{7*}, \lambda_1^{o*}, \lambda_2^{o*}) \in R_+^{I+2IJ+OI+2I}$ which satisfies:*

$$\begin{aligned}
&\sum_{i=1}^{I} [\frac{\partial f_i^n(q_i^{n*},\theta)}{\partial q_i^n} + \frac{\partial c_i(q_i^{n*},\theta)}{\partial q_i^{n*}} + s_n + t_1^* e_{nn} - \lambda_{1i}^{o*}] \times [q_i^n - q_i^{n*}] \\
&+ \sum_{i=1}^{I} \sum_{j=1}^{J} [\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n} - \rho_{ij}^{n*} + \lambda_{1i}^{o*}] \times [q_{ij}^n - q_{ij}^{n*}] + \sum_{i=1}^{I} \sum_{j=1}^{J} [\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r} - \rho_{ij}^{r*} + \lambda_{2i}^{o*}] \times [q_{ij}^r - q_{ij}^{r*}] \\
&+ \sum_{o=1}^{O} \sum_{i=1}^{I} [t_2^* e_{rr} - s_r - \lambda_{2i}^{o*} + \rho_{oi}^*] \times [q_{oi} - q_{oi}^*] + \sum_{i=1}^{I} [q_i^{n*} - \sum_{j=1}^{J} q_{ij}^{n*}] \times [\lambda_{1i}^o - \lambda_{1i}^{o*}] \\
&+ \sum_{i=1}^{I} [\sum_{o=1}^{O} q_{oi}^* - \sum_{j=1}^{J} q_{ij}^{r*}] \times [\lambda_{2i}^o - \lambda_{2i}^{o*}] \geq 0, \forall (Q^1, Q^2, Q^3, Q^7, \lambda_1^o, \lambda_2^o) \in R_+^{I+2IJ+OI+2I}.
\end{aligned} \tag{24}$$

where λ_{1i}^o and λ_{2i}^o are the Lagrange multiplier of constraints (21) and (22), respectively.

Proof. The proof is similar to that given for Theorem 1. □

From Figures 1 and 2, the retailers' equilibrium conditions under the ORS are the same as those under the IHRS. Therefore, from Theorem 2, VI (5) in this section is still satisfied.

4.2.2. Demand-Markets' Equilibrium Decisions

From Figure 2, trading in the demand markets in the forward supply chain under the ORS is similar to that under the IHRS; thus, the equilibrium decisions (11)–(14) are satisfied. In the reverse logistics, K demand markets choose to sell waste products to O 3PR for remanufacturing, and the aversion function for losing scrap products is $\alpha_k(q_{ko})$. Thus, the equilibrium of demand market k should satisfy the following constraints:

$$\alpha_k(q_{ko}^*) \begin{cases} = \rho_{ko}^*, \; if \; q_{ko}^* > 0 \\ \geq \rho_{ko}^*, \; if \; q_{ko}^* = 0 \end{cases} \tag{25}$$

$$\sum_{k=1}^{K} q_{ko} \leq \sum_{j=1}^{J} (q_{jk}^n + q_{jk}^r) \tag{26}$$

$$q_{jk}^n, q_{jk}^r, q_{ko} \geq 0 \; \forall j, o \tag{27}$$

Inequality (26) represents the scrap products constraint of demand market k in the demand market.

Theorem 5. *From the equivalence of complementary conditions (11)–(14), (25)–(26) and VI [50], the equilibrium of all demand markets should satisfy the following VI, i.e., determine $(Q^{4*}, Q^{5*}, Q^{8*}, \rho^{n*}, \rho^{r*}, v^{o*}) \in R^{2JK+KO+3K}$ which satisfies:*

$$\begin{aligned}&\sum_{j=1}^{J}\sum_{k=1}^{K}[\rho_{jk}^{n*}+c_{jk}^{n}(q_{jk}^{n*})-\rho_{k}^{n*}-v_{k}^{o*})\times[q_{jk}^{n}-q_{jk}^{n*}]+\sum_{j=1}^{J}\sum_{k=1}^{K}[\rho_{jk}^{r*}+c_{jk}^{r}(q_{jk}^{n*})-\rho_{k}^{r*}-v_{k}^{o*})\times[q_{jk}^{r}-q_{jk}^{r*}]\\&+\sum_{k=1}^{K}\sum_{o=1}^{O}[\alpha_k(q_{ko}^{*})-\rho_{ko}^{*}+v_{k}^{o*}]\times[q_{ko}-q_{ko}^{*}]+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{n*}-d_{k}^{n*}(\rho_{k}^{n*},\rho_{k}^{r*},k_n)]\times[\rho_{k}^{n}-\rho_{k}^{n*}]\\&+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{r*}-d_{k}^{r*}(\rho_{k}^{n*},\rho_{k}^{r*},k_r)]\times[\rho_{k}^{r}-\rho_{k}^{r*}]+\sum_{k=1}^{K}[\sum_{j=1}^{J}(q_{jk}^{n*}+q_{jk}^{r*})-\sum_{o=1}^{O}q_{ko}^{*}]\times[v_{k}^{o}-v_{k}^{o*}]\geq 0,\\&\forall(Q^4,Q^5,Q^8,\rho^n,\rho^r,v^o)\in R^{2JK+KO+3K}.\end{aligned}\quad(28)$$

where v_k^o *is the Lagrange multiplier of constraint (28).*

4.2.3. 3PRs' Equilibrium Decisions

O competing 3PRs under the ORS recycle the used products and produce the remanufactured products. The 3PR O obtains the revenue $\sum_{i=1}^{I}\rho_{oi}q_{oi}$ from the transaction of remanufactured products with the I OEMs but needs to pay the recycling cost $\sum_{k=1}^{K}\rho_{ko}q_{ko}$ (ρ_{ko} is the endogenous price of the recycled products). Since only $\alpha\sum_{k=1}^{K}q_{ko}$ can be remanufactured, 3PR o has to pay various costs for the production and sales of products: production cost $f_o(q_{ko},\theta)$, transaction cost $c_{oi}(q_{oi})$, recovery transaction cost $c_{ko}(q_{ko},\theta)$, and waste product disposal cost $\eta(1-\alpha)\sum_{k=1}^{K}q_{ko}$. Thus, the maximum profit of 3PR o (denoted by π_o) is:

$$\max\pi_o=\sum_{i=1}^{I}\rho_{oi}q_{oi}-\sum_{k=1}^{K}\rho_{ko}q_{ko}-f_o(q_{ko},\theta)-\sum_{i=1}^{I}c_{oi}(q_{oi})-\sum_{k=1}^{K}c_{ko}(q_{ko},\theta)-\eta(1-\alpha)\sum_{k=1}^{K}q_{ko}\quad(29)$$

$$s.t\sum_{i=1}^{I}q_{oi}\leq\alpha\sum_{k=1}^{K}q_{ko}\quad(30)$$

$$q_{oi},q_{ko}\geq 0\ \forall i,k\quad(31)$$

Inequality (30) represents the sales volume constraint of 3PR *0*.

Proposition 4. *The profit function of 3PR* $o(\pi_o)$ *is a concave function of* q_{oi} *and* q_{ko}*.*

Proof. The proof is similar to that of Theorem 1. □

Theorem 6. *Under the ORS, all the 3PRs at the same level are competing non-cooperatively in the Nash manner [45]. Then from Proposition 4, the equilibrium decisions of all 3PRs can be determined by solving the following VI (32) [12]: Determine* $(Q^{7*},Q^{8*},\varpi_o^*)\in R_+^{OI+KO+O}$ *which satisfies:*

$$\begin{aligned}&\sum_{o=1}^{O}\sum_{i=1}^{I}[-\rho_{oi}^{*}+\frac{\partial c_{oi}(q_{oi}^{*})}{\partial q_{oi}}+\varpi_o^{*}]\times[q_{oi}-q_{oi}^{*}]\\&+\sum_{k=1}^{K}\sum_{o=1}^{O}[\rho_{ko}^{*}+\frac{\partial f_o(q_{ko}^{*},\theta)}{\partial q_{ko}}+\frac{\partial c_{ko}(q_{ko}^{*},\theta)}{\partial q_{ko}}+\eta(1-\alpha)-\alpha\varpi_o^{*}]\times[q_{ko}-q_{ko}^{*}]\\&+\sum_{o=1}^{O}[\alpha\sum_{k=1}^{K}q_{ko}^{*}-\sum_{i=1}^{I}q_{oi}^{*}]\times[\varpi_o-\varpi_o^{*}]\geq 0,\forall(Q^7,Q^8,\varpi_o)\in \mathrm{R}^{OI+KO+O}.\end{aligned}\quad(32)$$

where ϖ_o *is the Lagrange multiplier of constraint (30).*

4.2.4. CLSC Network Equilibrium in ORS

Similar to the discussion in Section 4.1.4, by adding VI (24), VI(28), and VI(32) and subtracting all the endogenous prices $\rho_{ij}^{n*}\rho_{ij}^{r*}\rho_{jk}^{n*}\rho_{jk}^{r*}\rho_{ko}^{*}$, and ρ_{oi}^{*} from the resulting inequality, we obtain that the equilibrium of the network for the ORS determines $(Q^{1*}, Q^{2*}, Q^{3*}, Q^{5*}, Q^{6*}, Q^{7*}, Q^{8*}, \rho^{n*}, \rho^{r*}, \lambda_1^{o*}, \lambda_2^{o*}, \mu_1^{o*}, \mu_2^{o*}, v^{o*}, \varpi^*) \in R^{I+2IJ+2JK+OI+KO+2K+2I+2J+K+O}$, which satisfies the following VI:

$$\begin{aligned}
&\sum_{i=1}^{I}[\frac{\partial f_i^n(q_i^{n*},\theta)}{\partial q_i^n}+\frac{\partial c_i(q_i^{n*},\theta)}{\partial q_i^n}+s_n+t_1^*e_{nn}-\lambda_{1i}^{o*}]\times[q_i^n-q_i^{n*}]\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n}+\frac{\partial c_j^n(q_{ij}^{n*})}{\partial q_{ij}^n}-\mu_{1j}^{o*}+\lambda_{1i}^{o*}]\times[q_{ij}^n-q_{ij}^{n*}]+\sum_{i=1}^{I}\sum_{j=1}^{J}[\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r}+\frac{\partial c_j^r(q_{ij}^{r*})}{\partial q_{ij}^r}-\mu_{2j}^{o*}+\lambda_{2i}^{o*}]\times[q_{ij}^r-q_{ij}^{r*}]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}[c_{jk}^n(q_{jk}^{n*})-\rho_k^{n*}-v_k^{o*}+\mu_{1j}^{o*}]\times[q_{jk}^n-q_{jk}^{n*}]+\sum_{j=1}^{J}\sum_{k=1}^{K}[c_{jk}^r(q_{jk}^{r*})-\rho_k^{r*}-v_k^{o*}+\mu_{2j}^{o*}]\times[q_{jk}^r-q_{jk}^{r*}]\\
&+\sum_{o=1}^{O}\sum_{i=1}^{I}[\frac{\partial c_{oi}(q_{oi}^*)}{\partial q_{oi}}+t_2^*e_{rr}-s_r-\lambda_{2i}^{o*}+\varpi_o]\times[q_{oi}-q_{oi}^*]\\
&+\sum_{k=1}^{K}\sum_{o=1}^{O}[\frac{\partial f_o(q_{ko}^*,\theta)}{\partial q_{ko}}+\frac{\partial c_{ko}(q_{ko}^*)}{\partial q_{ko}}+\eta(1-\alpha)-\alpha\varpi_o^*+\alpha_k(q_{ko}^*)+v_k^{o*}]\times[q_{ko}-q_{ko}^*]\\
&+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{n*}-d_k^{n*}(\rho_k^{n*},\rho_k^{r*},k_n)]\times[\rho_k^n-\rho_k^{n*}]+\sum_{k=1}^{K}[\sum_{j=1}^{J}q_{jk}^{r*}-d_k^{r*}(\rho_k^{n*},\rho_k^{r*},k_r)]\times[\rho_k^r-\rho_k^{r*}]\\
&+\sum_{i=1}^{I}[q_i^{n*}-\sum_{j=1}^{J}q_{ij}^{n*}]\times[\lambda_{1i}^o-\lambda_{1i}^{o*}]+\sum_{i=1}^{I}[\sum_{o=1}^{O}q_{oi}^*-\sum_{j=1}^{J}q_{ij}^{r*}]\times[\lambda_{2i}^o-\lambda_{2i}^{o*}]+\sum_{j=1}^{J}[\sum_{i=1}^{I}q_{ij}^{n*}-\sum_{k=1}^{K}q_{jk}^{n*}]\times[\mu_{1j}^o-\mu_{1j}^{o*}]\\
&+\sum_{j=1}^{J}[\sum_{i=1}^{I}q_{ij}^{r*}-\sum_{k=1}^{K}q_{jk}^{r*}]\times[\mu_{2j}^o-\mu_{2j}^{o*}]+\sum_{k=1}^{K}[\sum_{j=1}^{J}(q_{jk}^{n*}+q_{jk}^{r*})-\sum_{i=1}^{I}q_{ki}^*]\times[v_k^o-v_k^{o*}]\\
&+\sum_{o=1}^{O}[\alpha\sum_{k=1}^{K}q_{ko}^*-\sum_{i=1}^{I}q_{oi}^*]\times[\varpi_o-\varpi_o^*]\geq 0\\
&\forall(Q^1,Q^2,Q^3,Q^5,Q^6,Q^7,Q^8,\rho^n,\rho^r,\lambda_1^o,\lambda_2^o,\mu_1^o,\mu_2^o,v^o,\varpi)\in R^{I+2IJ+2JK+OI+KO+2K+2I+2J+K+O}.
\end{aligned} \tag{33}$$

We obtain the endogenous prices $\rho_{ij}^{n*}=\frac{\partial c_{ij}^n(q_{ij}^{n*})}{\partial q_{ij}^n}+\lambda_{1i}^{o*}$ and $\rho_{ij}^{r*}=\frac{\partial c_{ij}^r(q_{ij}^{r*})}{\partial q_{ij}^r}+\lambda_{2i}^{o*}$ from VI (24) [50]. We also obtain $\rho_{jk}^{n*}=\mu_{1j}^{o*}$ and $\rho_{jk}^{r*}=\mu_{2j}^{o*}$ from VI (10), $\rho_{ko}^*=\alpha_k(q_{ko}^*)+v_k^{o*}$ from VI (28), and $\rho_{oi}^*=\frac{\partial c_{oi}(q_{oi}^*)}{\partial q_{oi}}+\varpi_o^*$ from VI (32).

5. Numerical Analyses

In the previous section, a CLSC network equilibrium model under different remanufacturing strategies considering government regulations and OEMs' DFEs was established. So in this section, we use numerical examples to demonstrate the impact of government regulations and DFE on equilibrium decisions, profits, carbon emissions, carbon taxes, and OEMs' remanufacturing strategies. Then, the optimal regulatory policies of the governments, the optimal DFE input of the OEM, and the marginal conditions for the choice of OEMs' remanufacturing strategies are analyzed by solving numerical examples. By using the comparison analysis, we will show how the decision-makers should choose their remanufacturing strategies under different DFE levels and government regulations.

In the example, it is assumed that there are two decision makers at each layer of Figures 1 and 2, i.e., $I=2, J=2, K=2, O=2$, and the cost functions in Table 5 are modified according to [12] as follows (All these cost functions satisfy Hypothesis 3, the demand functions satisfy Hypothesis 2, and all the cost functions are differentiable and convex):

Table 5. The cost functions in the example.

The Members of Supply Chain	Cost Functions	
OEM i	$f_i^n(q_i^n,\theta) = (3-\theta)(q_i^n)^2 + 2q_1^n q_2^n + 2(1-\theta)q_i^n$	
	$f_i^r(q_{ki},\theta) = (2-\theta)(\alpha\sum_{k=1}^{2} q_{ki})^2 + \alpha^2\sum_{k=1}^{2} q_{k1}\sum_{k=1}^{2} q_{k2} + 2\alpha(1-\theta)\sum_{k=1}^{2} q_{ki}$	
	$c_i(q_i^n,\theta) = 360\theta^2 q_i^n$	$c_{ij}^n(q_{ij}^n) = 0.5(q_{ij}^n)^2 + 3.5q_{ij}^n$
	$c_{ij}^r(q_{ij}^r) = 0.5(q_{ij}^r)^2 + 3.5q_{ij}^r$	$c_{ki}(q_{ki}) = 0.2(q_{ki})^2 + q_{ki}$
retailer j	$c_j^n(q_{ij}^n) = 0.3\sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij}^n$	$c_j^r(q_{ij}^r) = 0.3\sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij}^r$
	$\bar{c}_j^n(q_{ij}^n,k_n) = 60k_n^2$	$\bar{c}_j^r(q_{ij}^r,k_r) = 60k_r^2$
Demand market k	$c_k^n(q_{jk}^n) = q_{jk}^n + 5$	$c_k^r(q_{jk}^r) = q_{jk}^r + 5$
	$\alpha_k(q_{ki}) = 2\sum_{k=1}^{K}\sum_{i=1}^{I} q_{ki} + 5$	$\alpha_k(q_{ko}) = 2\sum_{k=1}^{K}\sum_{o=1}^{O} q_{ko} + 5$
Demand functions	$d_k^n = 1200(1+k_n) - 2.5\rho_k^n - 0.5\sum_{c=1,c\neq k}^{K}\rho_c^n + 0.3\rho_k^r$	
	$d_k^r = 400(1+k_r) - 1.5\rho_k^r - 0.5\sum_{c=1,c\neq k}^{K}\rho_c^r + 0.3\rho_k^n$	
3PR o	$f_i^r(q_{ki},\theta) = (2-\theta)(\alpha\sum_{k=1}^{2} q_{ki})^2 + \alpha^2\sum_{k=1}^{2} q_{k1}\sum_{k=1}^{2} q_{k2} + 2\alpha(1-\theta)\sum_{k=1}^{2} q_{ki}$	
	$c_{oi}(q_{oi}) = 0.2(q_{oi})^2 + q_{oi}$	$c_{ko} = 1.5(1-\theta)(\sum_{k=1}^{K} q_{ko})^2$

We solve VI (19) and VI (33) by the projection algorithm [51] using MATLAB with the step size in each iteration equal to 0.01 and the stopping criterion equal to 10^{-4}. Then we obtain the equilibrium decisions under the IHRS and the ORS, together with the profits, the quantities of the carbon emissions, and the carbon taxes.

5.1. Impact of DFE on Remanufacturing Strategies

In this section, we first analyze the impact of OEMs' level of DFE on the decision-makers' profits, carbon emissions, and carbon taxes at equilibrium, and the OEMs' remanufacturing strategies at equilibrium. The governments' parameters are fixed as: $s_n = 30$, $s_r = 50$ [9]. According to [43], the other parameters in Table 2 are fixed as: $\alpha = 0.8$, $\eta = 2$, $k_n = 0.8$, $k_r = 0.6$, $e_{new} = 3$, $e_r = 1.5$, $t_1 = 40$, $t_2 = 30$, $v = 0.8$ (All these parameters satisfy Hypothesis 3).

To analyze the impact of different DFE levels on OEMs' remanufacturing strategy and the equilibrium decision of each decision maker, we change the parameter of the DFE level gradually from 0 to 0.9, using a step length of 0.1. We first solve VI (19) to obtain the equilibrium decisions under the IHRS, including the quantity of product and transaction, and the price of the product. By using these equilibrium decisions, we obtain the profits of OEMs from (1), the carbon emission from $e_{nn}q_i^n + e_{rr}\alpha\sum_{k=1}^{K} q_{ki}$ (LCA) [49], the carbon emission tax from $t_1 e_{nn} q_i^n + t_2 e_{rr}\alpha\sum_{k=1}^{K} q_{ki}$, and the profits of retailers under the IHRS from (6). In the same way, we solve VI (33) to obtain the equilibrium decisions under the ORS. Then, we obtain the profits of OEMs from (20), the carbon emission from $e_{nn}q_i^n + e_{rr}\sum_{o=1}^{O} q_{oi}$(LCA) [49], the carbon emission tax from $t_1 e_{nn} q_i^n + t_2 e_{rr}\sum_{o=1}^{O} q_{oi}$, the profits of retailers from (6), and the profits of 3PRs under the ORS from (29). By using these results, we conduct a comparative analysis to show how the decision-makers should choose the IHRS or the ORS under different DFE levels. The comparative analyses of the equilibrium decisions are given in Figures 3–11. The comparative analyses of profits are given in Figures 12–14. The comparative analyses of carbon taxes and carbon emissions are given in Figures 15 and 16.

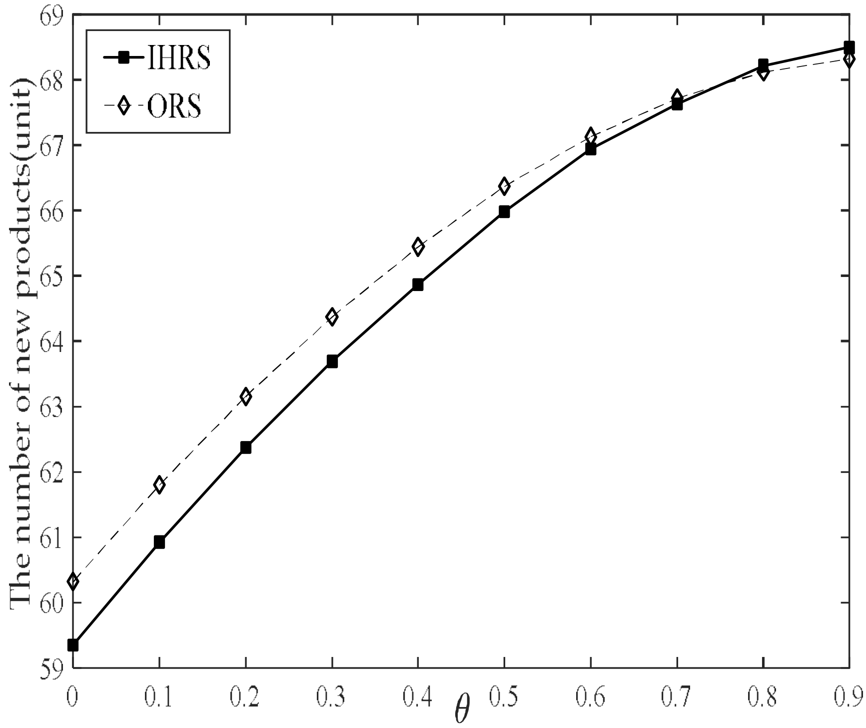

Figure 3. Changes in new products in Section 5.1.

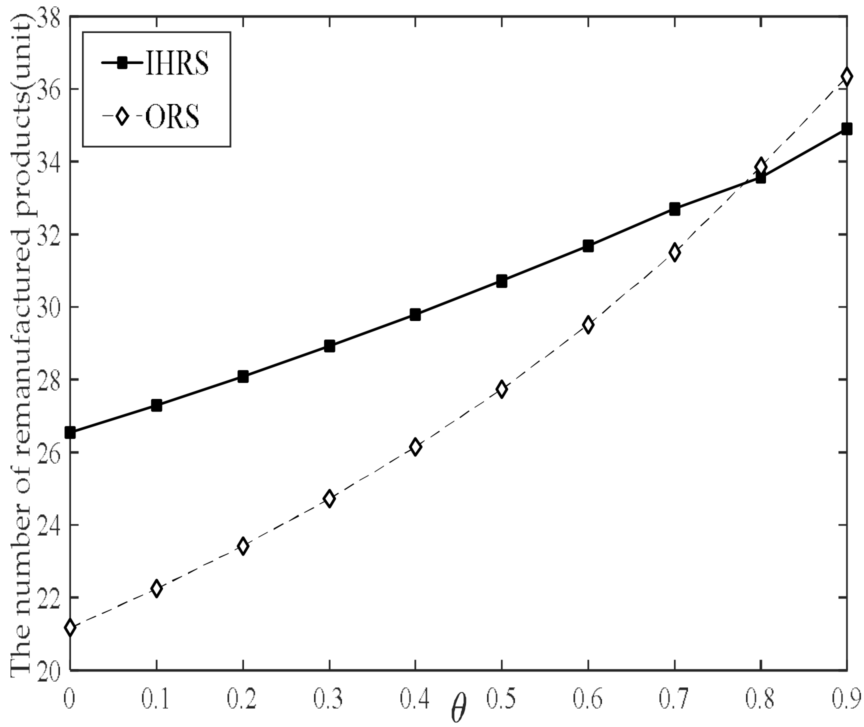

Figure 4. Changes in remanufactured products in Section 5.1.

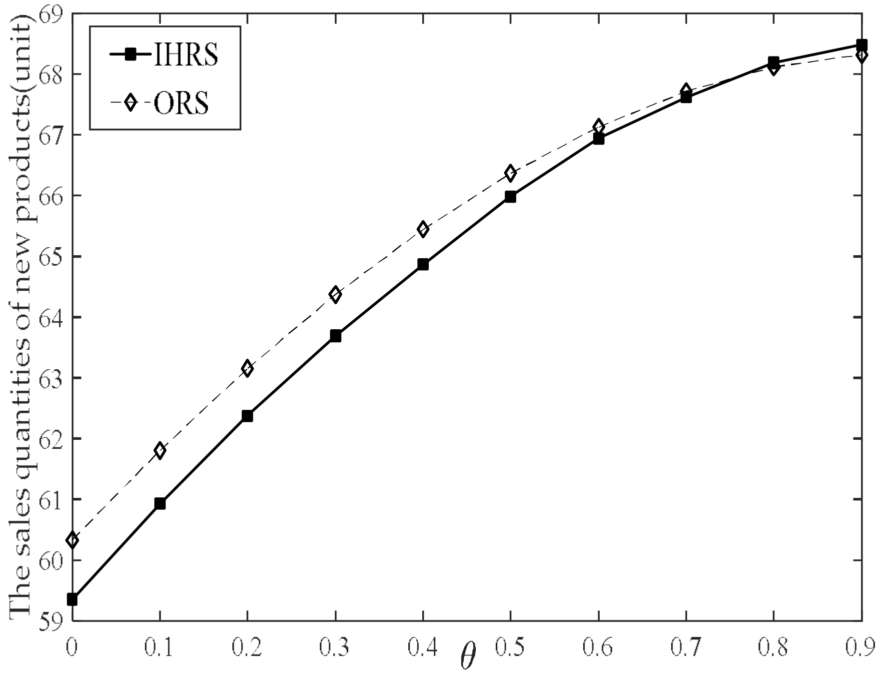

Figure 5. Changes in new product transactions in Section 5.1.

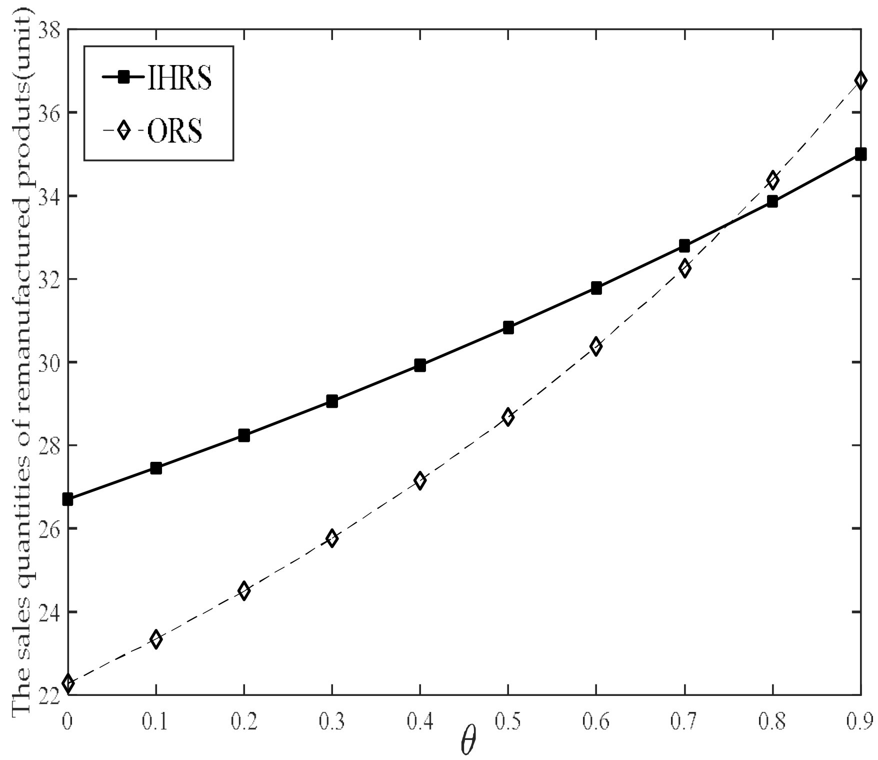

Figure 6. Changes in remanufactured product transactions in Section 5.1.

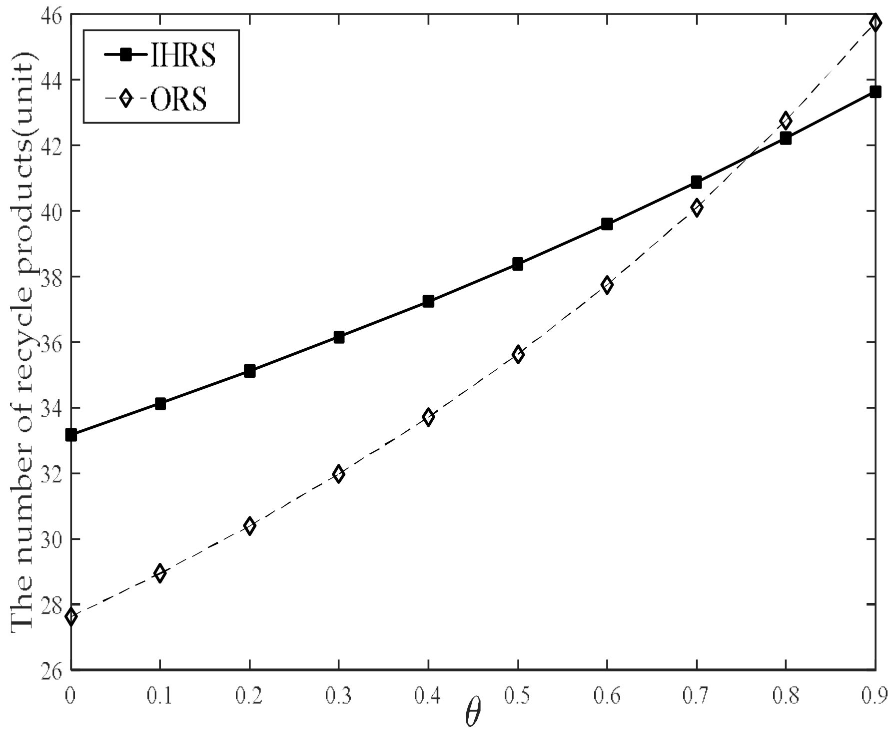

Figure 7. Change in recycled products in Section 5.1.

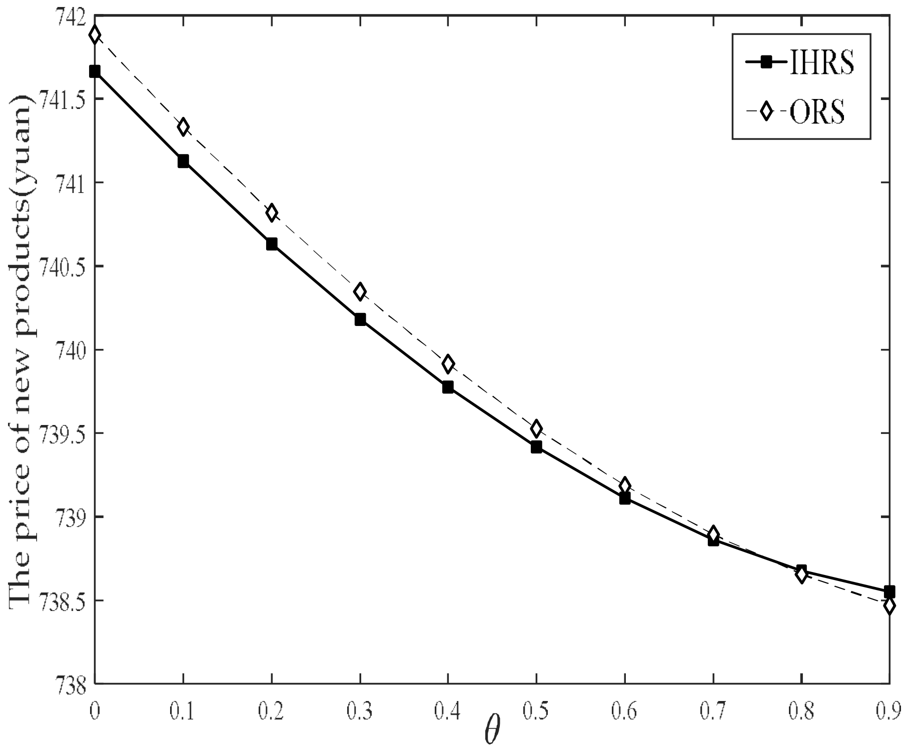

Figure 8. Changes in the prices of new products in Section 5.1.

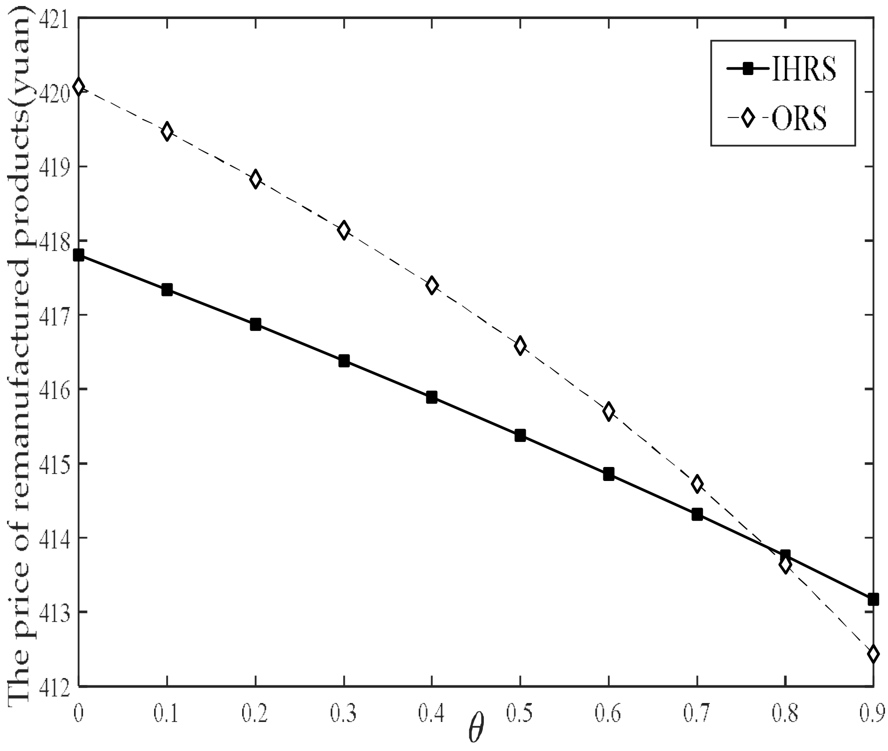

Figure 9. Changes in the prices of remanufactured products in Section 5.1.

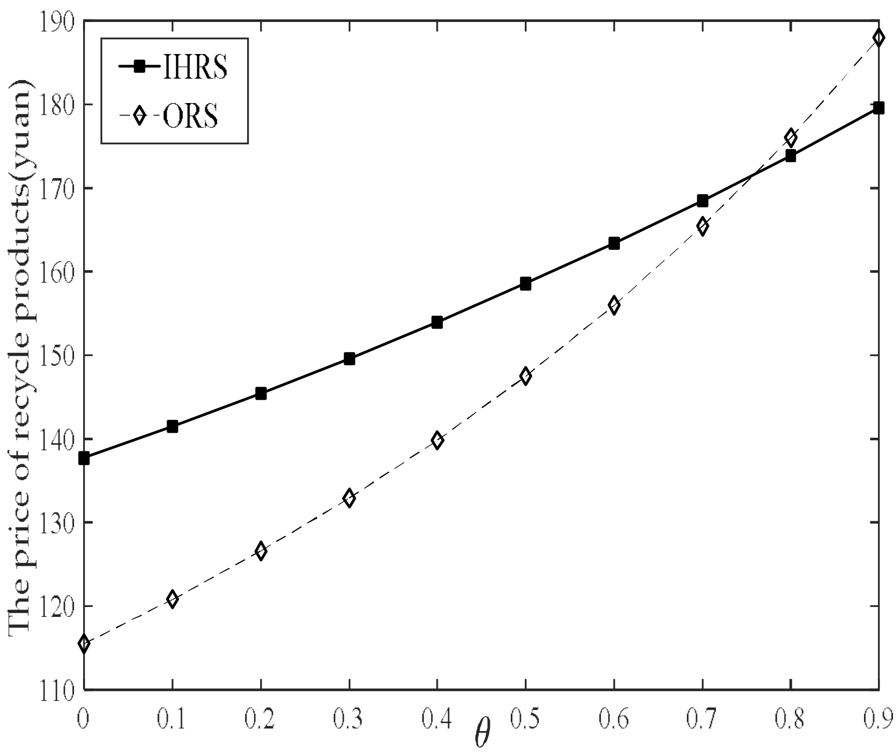

Figure 10. Changes in recycled products in Section 5.1.

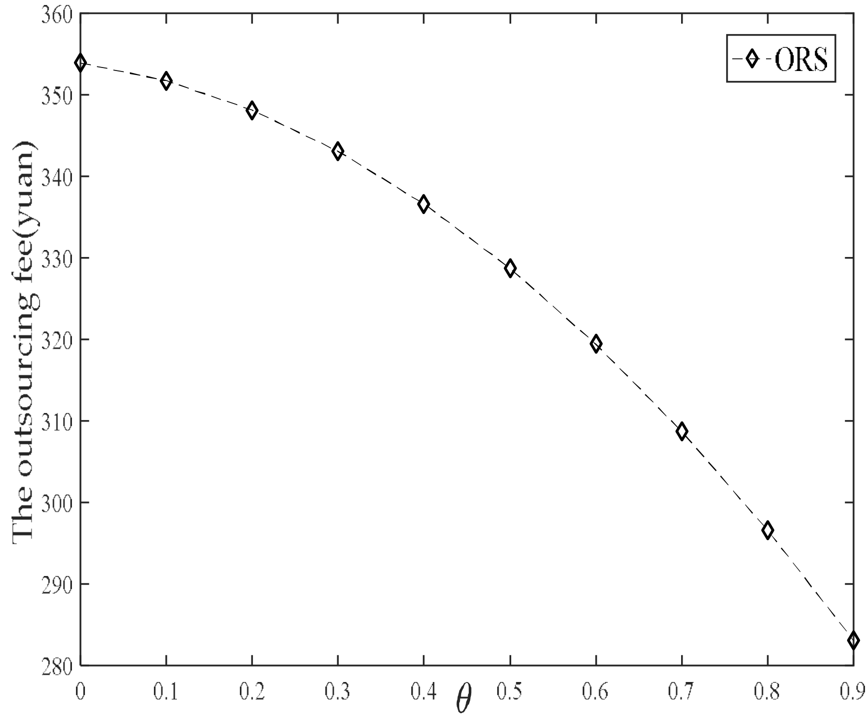

Figure 11. Changes in the outsourcing costs in Section 5.1.

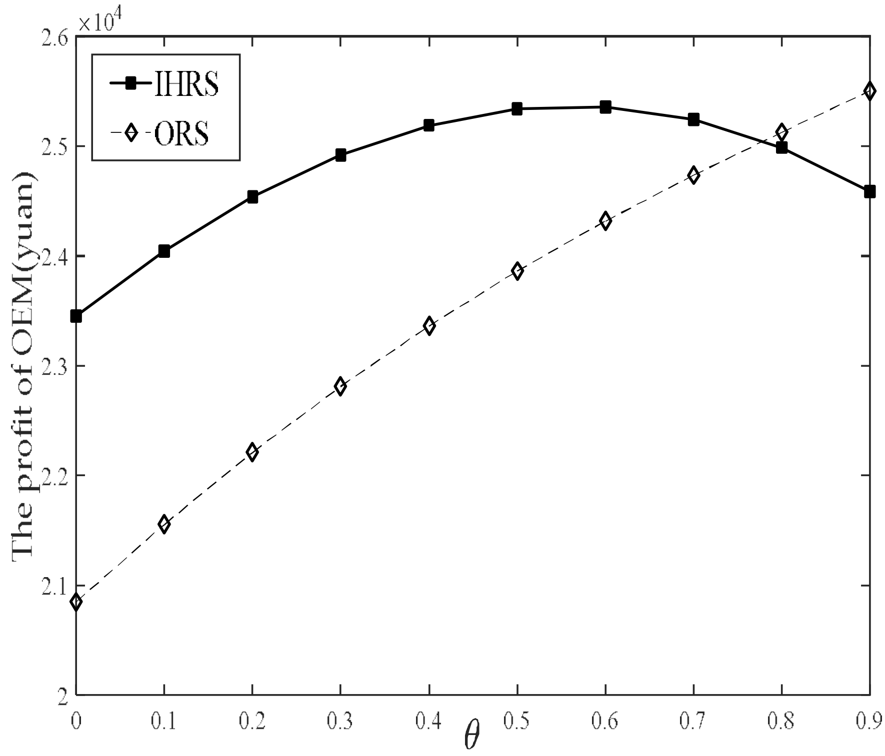

Figure 12. Changes in OEMs' profits in Section 5.1.

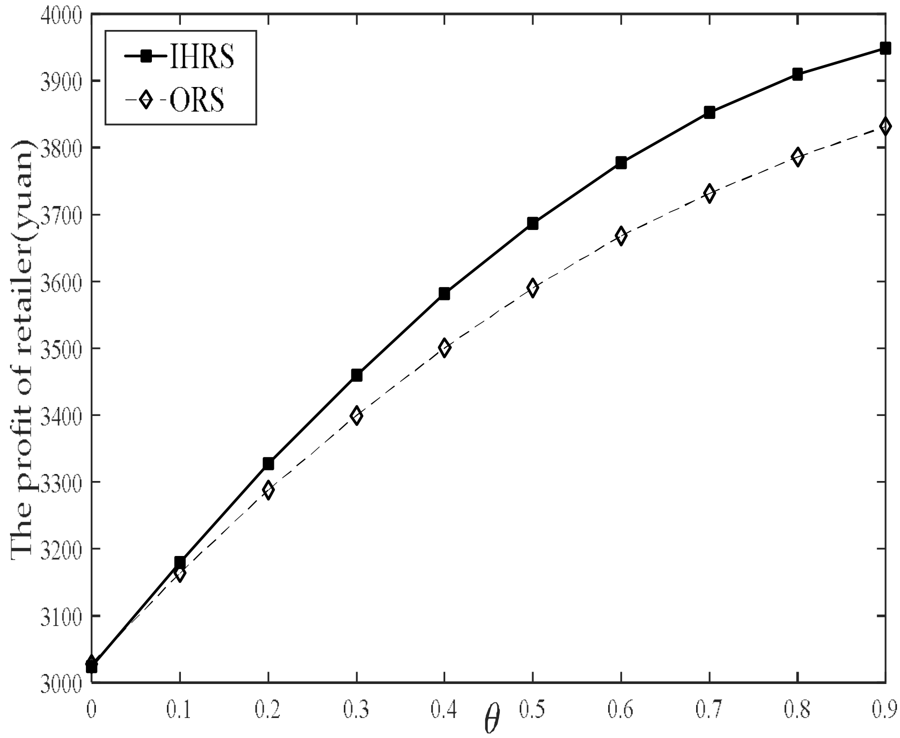

Figure 13. Changes in retailers' profits in Section 5.1.

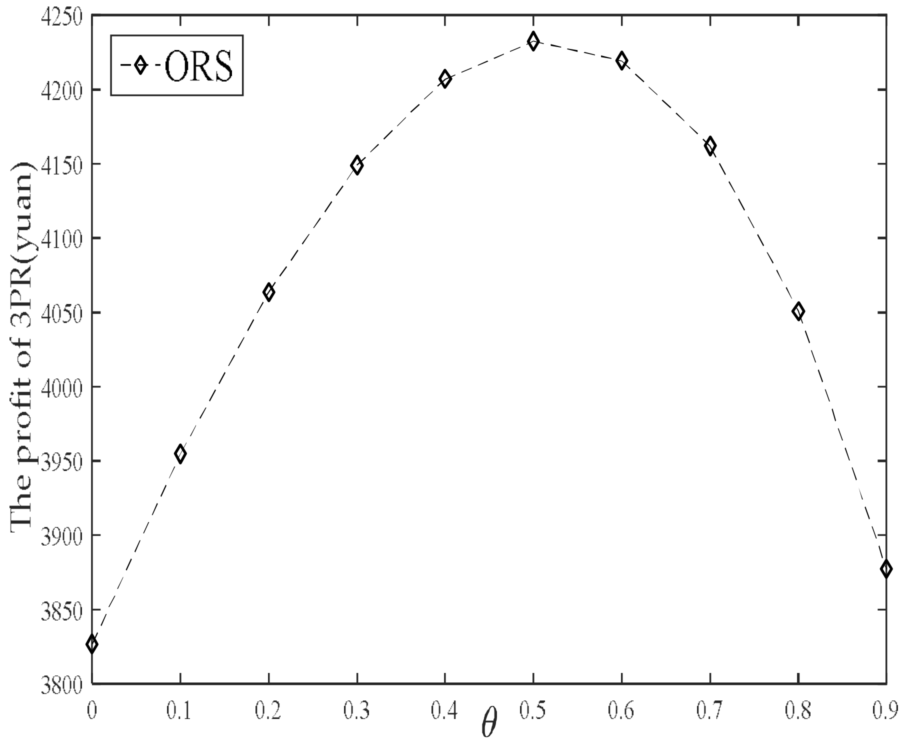

Figure 14. Changes in 3PRs' profits in Section 5.1.

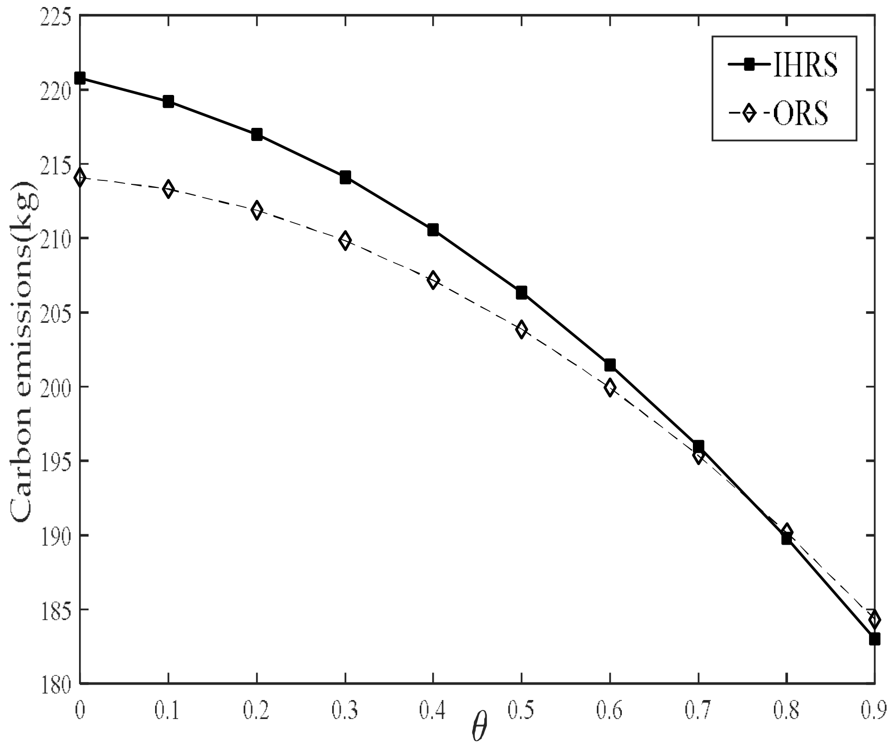

Figure 15. Changes in carbon emissions in Section 5.1.

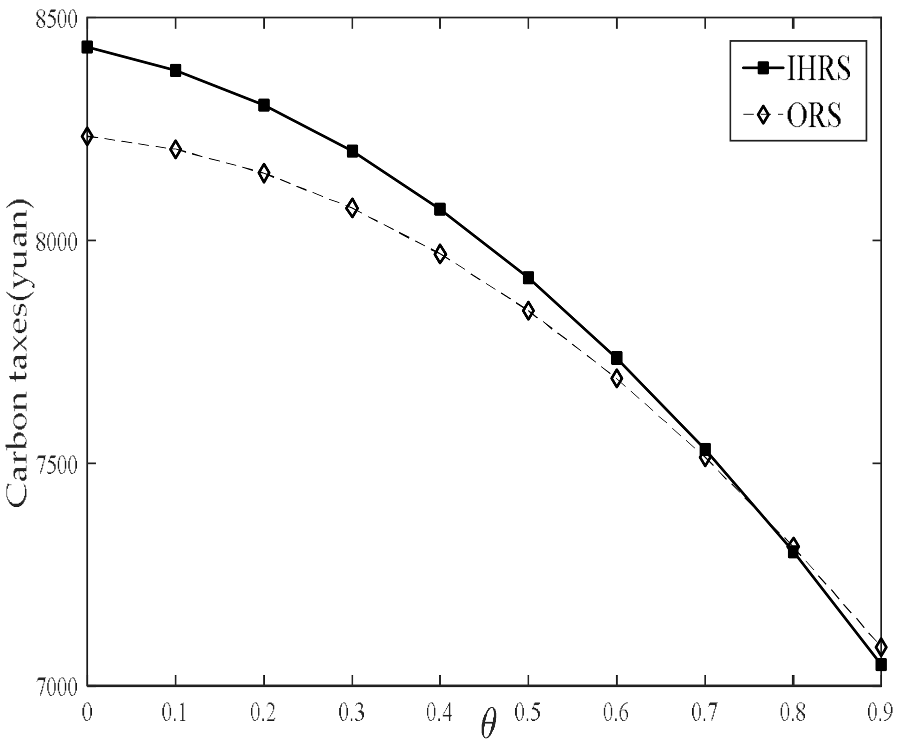

Figure 16. Changes in the OEMs' carbon taxes in Section 5.1.

(1) Figures 3–10 show that when the level of OEMs' DFE θ increases, OEM technology or material innovation leads to a decrease in the production cost of new products, thereby stimulating the increase in the production of new OEM products. On the other hand,

when θ increases, the recyclability of waste products is improved, which prompts OEMs or 3PRs to recycle more products, and the corresponding remanufactured products also increase. These results conform to the statement of Property 1. These results are consistent with real-life problems. For example, Canon and Hewlett-Packard use the design for the environment to promote their remanufacturing activities. Moreover, the reduction in costs leads to a decrease in the price of the OEMs' products; hence, the transactions in the demand markets also increase. Thus, the improvement of the DFE level leads to an increase in the transaction volumes in the demand markets. Thus, the demand markets have more preferences in keeping waste products, which increases the product recycling price.

(2) Figures 11–14 show the impact of the level of OEMs' DFE θ on the outsourcing costs as well as the economic benefits of OEMs and 3PRs under the IHRS. When $\theta \geq 0.6$, the profits of the OEMs under the IHRS decrease with the increase in θ. The reason is that when $\theta \geq 0.6$, OEMs under the IHRS need to pay excessive DFE costs, which greatly affect their own profits. On the other hand, the profits of the OEMs under the ORS are always increasing with respect to θ. The reason is that OEMs under the ORS can obtain dividends from their own DFEs for recycling and remanufacturing products, and hence they will correspondingly reduce the outsourcing cost of remanufactured products so that their own DFE costs can also be reduced to a certain extent. When $\theta \geq 0.8$, under the pressure of paying excessive costs, OEMs prefer using the ORS to the IRHS to obtain more profits. When $\theta \geq 0.6$, due to 3PRs' reduction in the outsourcing costs and the increase in prices of the remanufactured products in the ORS, the profit of the OEMs under ORS decrease.

(3) Figures 15 and 16 show that the increase in the value of θ has a significant effect on reducing the unit carbon emission of product production; although the product output increases, the carbon emission tends to decrease, and hence the carbon tax paid by OEMs decreases. Therefore, when the value of θ increases, the OEMs have more environmental benefits and hence have more capital to invest in DFE.

OEMs' choice of remanufacturing strategy in pursuit of profit maximization is shown in Figure 17. We know that OEMs choosing the ORS have less remanufactured products and income, but the corresponding polluting emissions under the ORS are less than those under the IHRS. Thus, from Figure 17 that when the environmental setting level is low, i.e., $\theta < 0.8$, 3PRs will choose the IHRS because they have less incentive to recycle and remanufacture products due to the high cost of remanufacturing. (Suppose that 3PRs choose the ORS when $\theta < 0.8$, they have fewer remanufactured products and less income, but the corresponding polluting emissions under the ORS are less than those under the IHRS.) When the level of DFE is high, i.e., $\theta \geq 0.8$, OEMs will choose the ORS because they can obtain higher returns to compensate for the costs of their own investments, although the carbon emissions generated under the ORS are higher than those under the IHRS, which harms the benefit of the environment.

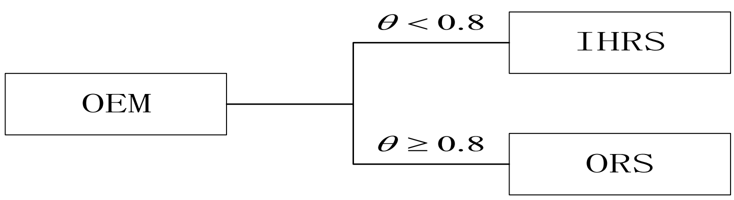

Figure 17. The effect of the DFE level on the remanufacturing strategies.

5.2. Impact of Government Regulations on Remanufacturing Strategies

In this section, we focus on the impact of government regulations on the decision-makers' profits, carbon emissions, carbon taxes, and OEMs' remanufacturing strategies at equilibrium. From the discussion in the conclusion of Section 5.1, we use low DFE level ($\theta = 0.3$) versus high DFE level ($\theta = 0.8$) to study the impact of the DFE level on the optimal decisions and OEMs' remanufacturing strategies at equilibrium. We vary the values of the new product processing fee s_n and the remanufacturing subsidy s_r from 0 to 220 by 20 and keep the values of the other parameters the same as those given in Section 5.1. By using the same method as Section 5.1, we obtain the equilibrium decisions, profits, carbon taxes, and carbon emissions for the two remanufacturing strategies. The comparative analyses of the equilibrium decisions are given in Figures 18–24. The comparative analyses of profits are given in Figures 25–27. The comparative analyses of carbon taxes and carbon emissions are given in Figures 28 and 29.

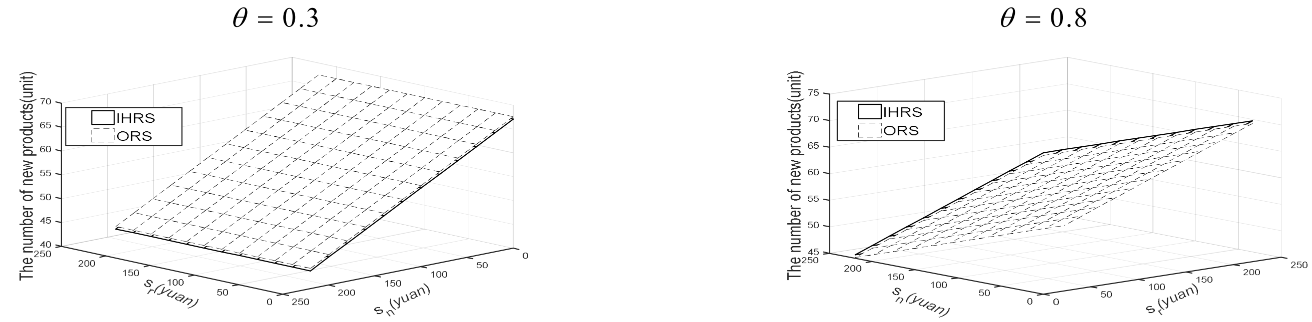

Figure 18. Changes in the new products in Section 5.2.

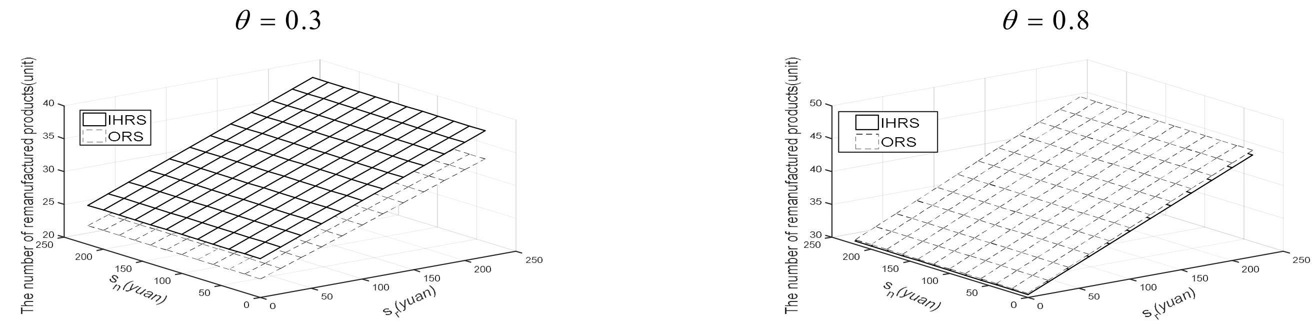

Figure 19. Changes in the remanufactured products in Section 5.2.

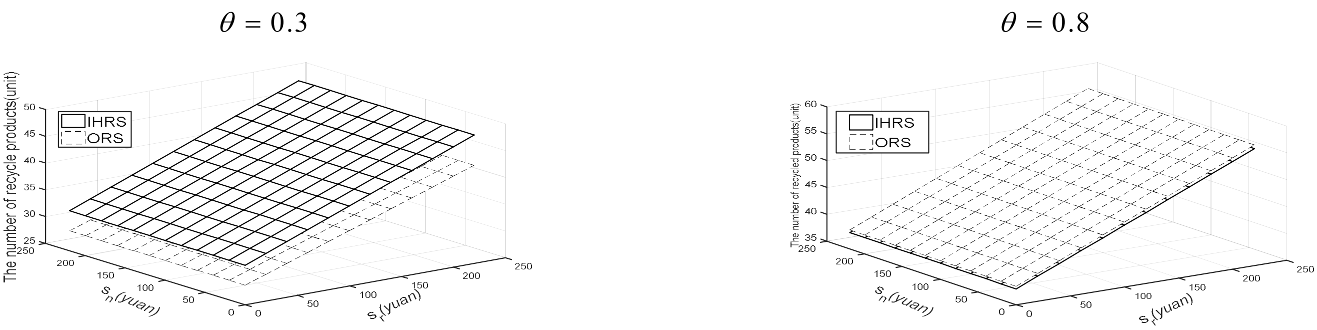

Figure 20. Changes in the recycled products in Section 5.2.

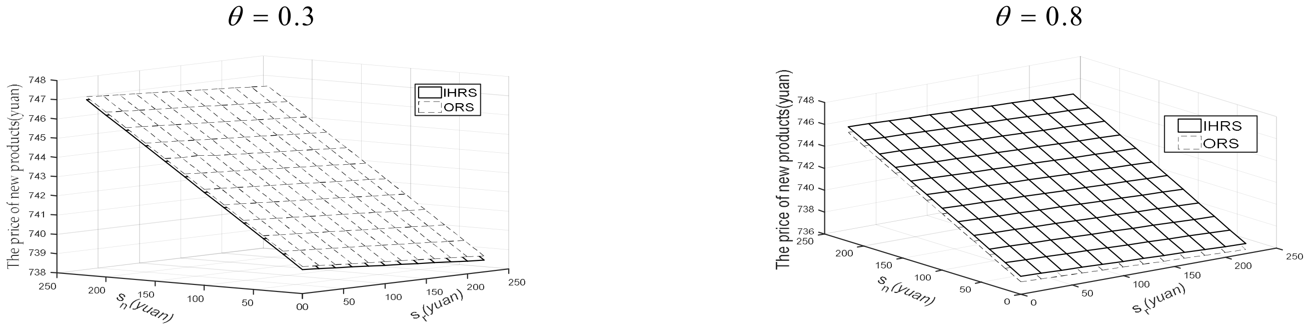

Figure 21. Changes in the prices of the new products in Section 5.2.

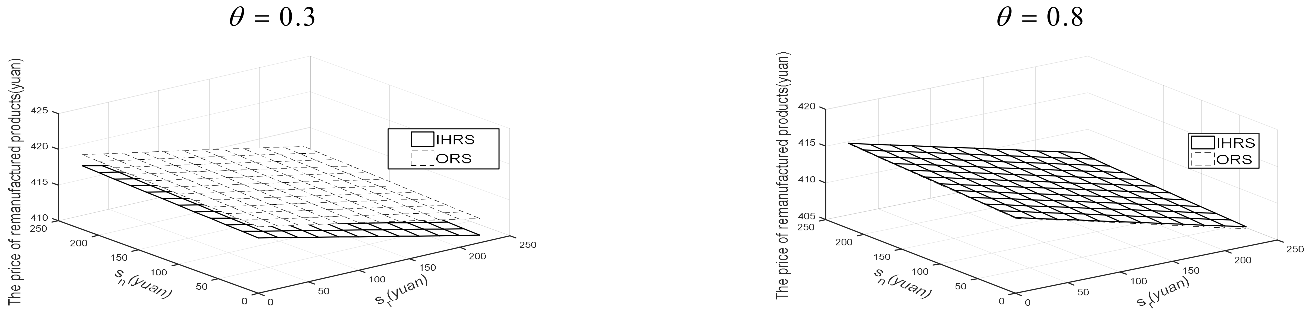

Figure 22. Changes in the prices of the remanufactured products in Section 5.2.

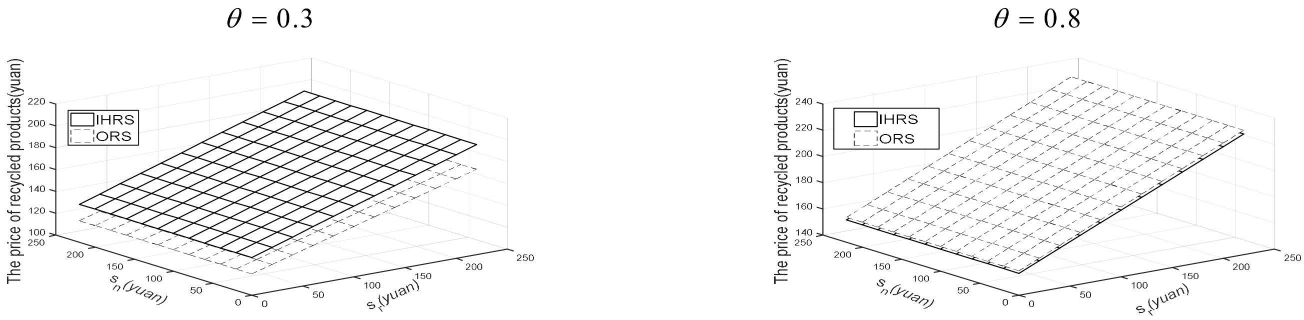

Figure 23. Changes in the prices of the recycled products in Section 5.2.

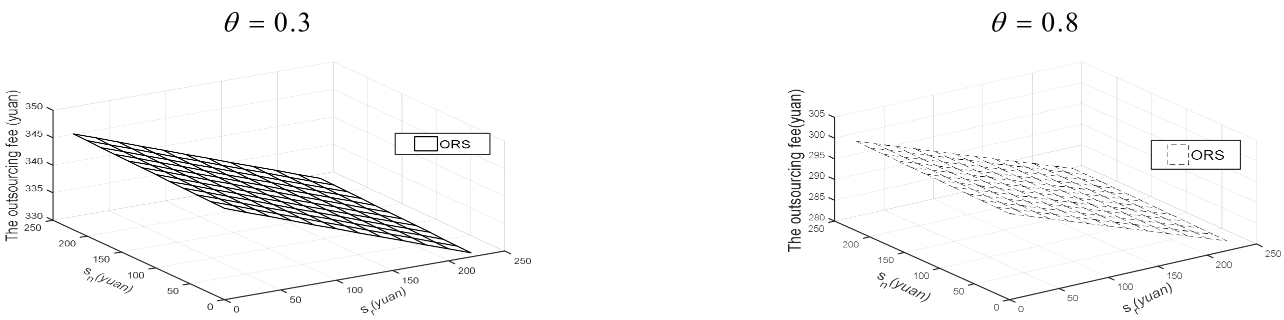

Figure 24. Changes in the outsourcing costs in Section 5.2.

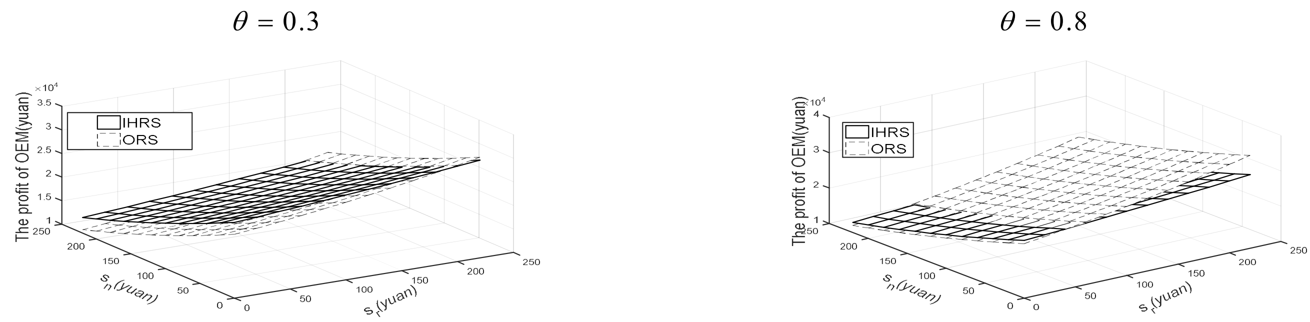

Figure 25. Changes in OEMs' profits in Section 5.2.

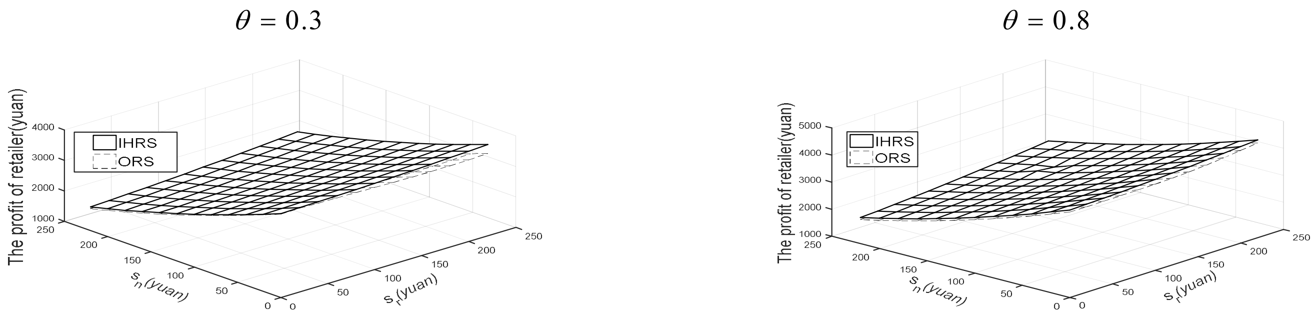

Figure 26. Changes in retailers' profits in Section 5.2.

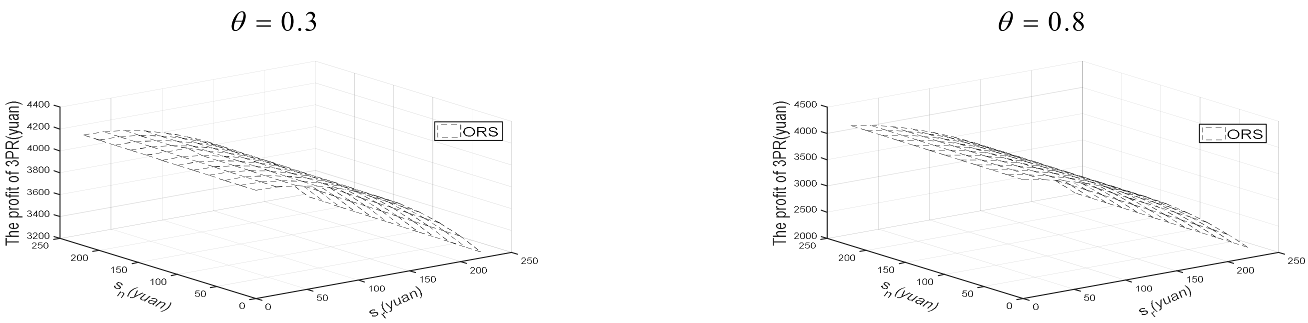

Figure 27. Changes in 3PRs' profits in Section 5.2.

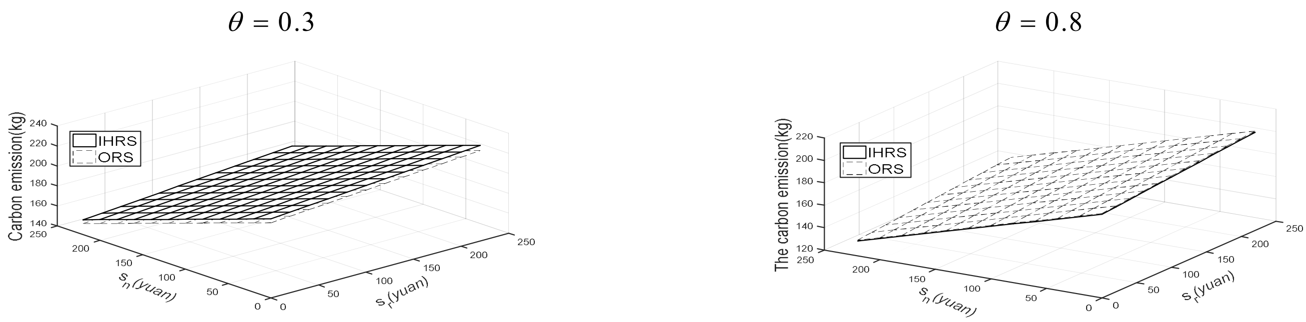

Figure 28. Changes in the amount of carbon emissions in Section 5.2.

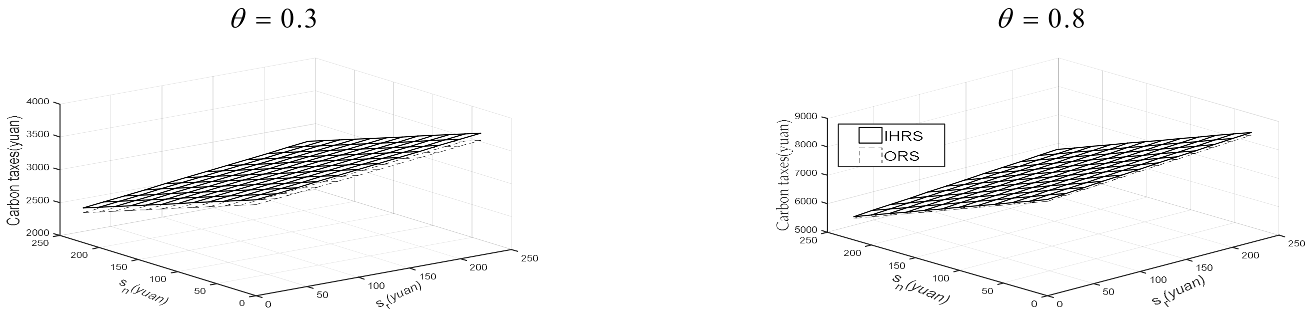

Figure 29. Changes in OEMs' carbon tax in Section 5.2.

(1) Figures 18–20 show that the number of OEMs' new products produced is negatively correlated with s_n; however, OEMs' and 3PRs' recycled products and remanufactured products produced are positively correlated with s_r, which is consistent with the statement of Property 2. These results are also consistent with real-life problems. For example, governments such as India and Canada promote remanufacturing activities through subsidies. Due to the imposition of the system of the remanufacturing subsidy and product handling fee by the governments, OEMs will be stimulated to decrease the production of new products to reduce cost expenditure, and at the same time, they will be encouraged to engage in remanufacturing activities to obtain subsidies as well as to save production materials the system of remanufacturing subsidy and new product handling fee and reduce carbon emissions.

(2) Figures 21–24 show that the price of the new products increases with the value of s_r but decreases with the value of s_n; however, the price of the remanufactured products changes in the opposite direction. Due to the imposition of new product processing fees, OEMs need to increase product sales prices to offset the payment of the processing fees. On the other hand, due to the run-on effect on the sales of products, to stimulate demand markets' interest in purchasing remanufactured products, the governments subsidize OEMs to reduce the price of remanufactured products so that the sales of remanufactured products increase.

(3) Figures 25–27 show that under the government regulations, OEMs' profits are negatively correlated with s_n and positively correlated with s_r, whereas 3PRs' profits are positively correlated with s_n and negatively correlated with s_r When the value of s_r increases (i.e., governments' new product unit processing fee increases), the environmental cost borne by OEMs' new production unit increases, which leads to the decrease in OEMs' production quantities of the new products, resulting in a decrease in OEMs' profits; however, the government's subsidies not only stimulate the production of OEMs' remanufactured products but also improve OEMs' profits. On the other hand, when the value of s_n increases, 3PRs' production quantities of the remanufactured products increase due to the following reasons: Under the ORS, when the value of s_n increases, OEMs will produce more remanufactured products to reduce the environmental costs. Thus, due to the increase in outsourcing costs, 3PRs will sell more remanufactured products. However, when the value of s_r increases, OEMs will have more incentive to sell remanufactured products, which leads to a decrease in the prices of the remanufactured products and OEMs' decrease in income obtained from the outsourcing cost. When the sale price of the 3PRs' remanufactured products substantially increases, the income of the 3PRs decreases.

(4) As shown in Figures 28 and 29, from the perspective of environmental benefits, the increase in the value of s_n has a significant effect on the reduction of carbon emissions; although the increase in the value of s_r is conducive to remanufacturing activities, it also increases production emissions, which is not good for environmental benefits.

OEMs' choices of remanufacturing strategy for maximum profit under the role of government remanufacturing subsidies are shown in Figures 30 and 31. Figures 30 and 31 show that from the perspective of OEMs' remanufacturing strategy choices, the impact of the increase in the value of s_n on OEMs' remanufacturing strategy choices is not significant. On the other hand, when the level of the DFE of the OEMs' products is small since the cost reduction of remanufacturing brought about by DFE is small, the increase in the value of s_r does not have an incentive effect on OEMs' recycling and remanufacturing. At the same time, when the level of the DFE is small, the outsourcing fee paid by the OEM is higher; hence the OEM is more inclined to choose the IHRS. Only when the value of s_r is higher (i.e., $s_r \geq 180$), the outsourcing cost will be reduced, and the OEM will choose ORS. However, when the DFE level of OEMs' products (s_r) is large, given that the OEMs themselves need to pay a high DFE cost and the price of remanufactured products is reduced, the boundary condition of OEMs' to choose the ORS becomes more easily satisfied, and the OEMs' pressure of paying high costs is alleviated by the reduction in the outsourcing costs.

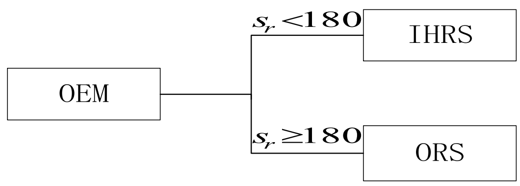

Figure 30. The effect of the government remanufacturing subsidies $(\theta = 0.3)$ on the remanufacturing strategies.

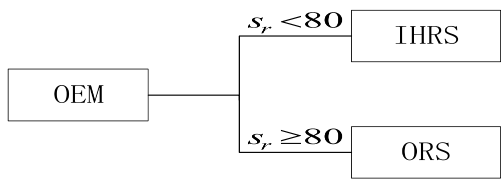

Figure 31. The effect of government remanufacturing subsidies $(\theta = 0.8)$ on the remanufacturing strategies.

6. Conclusions

6.1. Discussion of Results

In this paper, we study the government regulations and OEMs' DFE in the CLSC network under the IHRS and the ORS. We use VI to obtain the equilibrium decisions of the decision-makers, together with their respective profits, carbon taxes, and carbon emissions, in the CLSC model. By using the comparison analysis, we study how the decision-makers should choose their remanufacturing strategies under different DFE levels and government regulations. The results are as follows:

(1) Under two remanufacturing strategies, the production and trading volume of new products (remanufactured products), as well as the recycled products, are positively related to OEMs' level of DFE. It means that when the level of DFE increases, OEMs can recycle and remanufacture more products, which is consistent with [11]. The carbon taxes and carbon emissions are negatively correlated with the level of DFE. It means that when the level of DFE increases, the production emissions in the supply chain decrease, which is good for reducing environmental pollution.

(2) By comparing the OEMs' profits under the IHRS and ORS, when the level of DFE is low, OEM should choose IHRS; otherwise, OEM should choose ORS to get more profits. However, from the perspective of environmental benefits, OEMs should choose IHRS when the level of DFE is high to get fewer carbon emissions and carbon taxes.

(3) The number of OEMs' new products is negatively correlated with the new product handling fee. The numbers of recycled products and remanufactured products are positively correlated with the remanufacturing subsidies. The carbon taxes and carbon

emissions are positively correlated with the new product handling fee and negatively correlated with the remanufacturing subsidies, which is consistent with the viewpoint of [29].

(4) From the perspective of profit, OEMs' profits are negatively correlated with new product handling fees and positively correlated with remanufacturing subsidies, whereas the trend of 3PRs'profit is in the opposite direction. Thus, when the remanufacturing subsidies are low, OEM should choose IHRS; otherwise, OEM should choose ORS to get more profits.

(5) According to the comparison results of government regulations at different levels of DEF, when the government's subsidy for remanufactured products increases to a threshold, it will prompt OEM to adopt ORS.

6.2. Policy Implication

Using the results obtained in Section 6.1, we get the following policy implication:

(1) As far as OEMs are concerned, they should insist on investing in DFE to achieve the unity of economic benefits and environmental benefits. Moreover, OEMs must recognize the importance of DFE in establishing environmental friendliness so that they will actively assume social responsibility and establish a good corporate image to enhance their competitiveness in supply chain management. OEMs can share technology and costs by using ORS but should maximize the production cost savings from DFE to stimulate each other's willingness to cooperate and achieve a win-win situation.

(2) Governments can control OEMs' production emissions by setting up new product handling fees; however, this will be detrimental to OEMs, especially for companies with low levels of DFE, because it will decrease their incentive to invest in DFE. In view of this weakness, from the perspective of improving the level of the OEMs' DFEs, the government can implement different regularity policies according to OEMs' economic strength. More precisely, for OEMs with weak economic strength, governments can increase their remanufacturing subsidies to alleviate their pressure for paying high costs due to the implementation of regularity policies.

6.3. Limitations and Future Research

This paper mainly studies how two factors, i.e., government regulations and OEMs' DFE, can affect OEMs' remanufacturing strategies. However, in real-life situations, there are much more complicated factors, i.e., the degree of consumers' preferences for remanufacturing products and governments' subsidizing strategy for remanufacturers, that can affect OEMs' optimal remanufacturing strategies. Due to the limitation of our model, we are unable to consider these factors in this paper. Thus, our future research will be very challenging. We shall investigate the degree of consumers' approval of the remanufacturing products since consumers' preferences may also directly affect OEMs' choice of remanufacturing strategies. From the governments' point of view, they should choose an appropriate subsidizing strategy to help manufacturers and remanufacturers so that the problem involving low recycling rates and high remanufacturing costs due to remanufacturers' inexperience or lack of technical skill can be overcome. Thus, how consumers' preferences for remanufacturing products and the governments' subsidizing strategy can influence OEMs' remanufacturing strategies will be our future research area.

Author Contributions: This paper was completed by Y.Z., M.H. and K.-H.W.; Y.Z. and M.H. constructed the model and wrote the whole paper. K.-H.W. checked the draft and corrected the language. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (grant number 71901129).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: We would like to thank the anonymous reviewers for their constructive comments.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

From (1), the Hessian matrix of the profit function π_i is

$$\begin{bmatrix} -\frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^{n2}} - \frac{\partial^2 c_i(q_{ki},\theta)}{\partial q_{ki}^2} & 0 & 0 & 0 \\ 0 & -\frac{\partial^2 c_{ij}^n(q_{ij}^n)}{\partial q_{ij}^{n2}} & 0 & 0 \\ 0 & 0 & -\frac{\partial^2 c_{ij}^r(q_{ij}^r)}{\partial q_{ij}^{r2}} & 0 \\ 0 & 0 & 0 & -\frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki}^2} - \frac{\partial^2 c_{ki}(q_{ki})}{\partial q_{ki}^2} \end{bmatrix}. \quad (A1)$$

All the leading principal minors of odd order are positive, and all leading principal minors of even order are negative. Thus, the matrix is negative definite, and the proof of Proposition 1 is completed.

Appendix B

Multiplying (2) and (3) by the Lagrange multipliers λ_{1i} and λ_{2i} respectively, we obtain the Lagrangian function L_i as follows:

$$\begin{aligned} L_i = &-\sum_{j=1}^{J} (\rho_{ij}^n q_{ij}^n + \rho_{ij}^r q_{ij}^r) + \sum_{k=1}^{K} \rho_{ki} q_{ki} + f_i^n(q_i^n,\theta) + f_i^r(q_{ki},\theta) + c_i(q_i^n,\theta) + \sum_{j=1}^{J} c_{ij}^n(q_{ij}^n) + \sum_{j=1}^{J} c_{ij}^r(q_{ij}^r) + \sum_{k=1}^{K} c_{ki}(q_{ki}) \\ &+\eta(1-\alpha)q_{ki} - s_r\alpha \sum_{k=1}^{K} q_{ki} + s_n q_i^n + t_1 e_{nn} q_i^n + t_2 e_{rr} \alpha \sum_{k=1}^{K} q_{ki} + \lambda_{1i}(\sum_{j=1}^{J} q_{ij}^n - q_i^n) + \lambda_{2i}(\sum_{j=1}^{J} q_{ij}^r - \alpha \sum_{k=1}^{K} q_{ki}) \end{aligned} \quad (A2)$$

Thus, the partial derivatives of L_i with respect to q_{ij}^n, q_{ij}^r, q_i^n, q_{ki}, and λ_{1i} become

$$\frac{\partial L_i}{\partial q_{ij}^n} = -\rho_{ij}^n + \frac{\partial c_{ij}^n(q_{ij}^n)}{\partial q_{ij}^n} - \lambda_{1i}$$

$$\frac{\partial L_i}{\partial q_{ij}^r} = -\rho_{ij}^r + \frac{\partial c_{ij}^r(q_{ij}^r)}{\partial q_{ij}^r} - \lambda_{2i}$$

$$\frac{\partial L_i}{\partial q_i^n} = \frac{\partial f_i^n(q_i^n,\theta)}{\partial q_i^n} + \frac{\partial c_i(q_i^n,\theta)}{\partial q_i^n} + s_n + t_1 e_{nn} + \lambda_{1i}$$

$$\frac{\partial L_i}{\partial q_{ki}} = \rho_{ki} + \frac{\partial f_i^r(q_{ki},\theta)}{\partial q_{ki}} + \frac{\partial c_{ki}(q_{ki})}{\partial q_{ki}} + \eta(1-\alpha) - s_r\alpha + t_2\alpha e_{rr} + \alpha\lambda_{2i}$$

$$\frac{\partial L_i}{\partial \lambda_{1i}} = \sum_{j=1}^{J} q_{ij}^n - q_i^n, \ \frac{\partial L_i}{\partial \lambda_{2i}} = \sum_{j=1}^{J} q_{ij}^r - \alpha \sum_{k=1}^{K} q_{ki}$$

Thus, we get Equation (6), and Theorem 1 is proved.

Appendix C

From the proof of Theorem 1, the total derivatives of q_i^n, q_{ij}^n, q_{ki} and q_{ij}^r at equilibrium with respect to θ are

$$\frac{\partial q_i^n}{\partial \theta} = \frac{t_1 v - \frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^n \partial \theta} - \frac{\partial^2 c_i(q_i^n,\theta)}{\partial q_i^n \partial \theta}}{\frac{\partial^2 c_{ij}^n(q_{ij}^n)}{\partial q_{ij}^{n2}} + \frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^{n2}} + \frac{\partial^2 c_i(q_i^n,\theta)}{\partial q_i^{n2}}}, \quad \frac{\partial q_{ij}^n}{\partial \theta} = \frac{\partial q_i^n}{\partial \theta}$$

$$\frac{\partial q_{ki}}{\partial \theta} = \frac{t_2 \alpha v - \frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki} \partial \theta}}{\alpha^2 \frac{\partial^2 c_{ij}^r(q_{ij}^r)}{\partial q_{ij}^{r2}} + \frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki}^2} + \frac{\partial c_{ki}^2(q_{ki})}{\partial q_{ki}^2}}$$

From Hypothesis 3, when $t_1 \nu - \frac{\partial^2 f_i^n(q_i^n,\theta)}{\partial q_i^n \partial \theta} - \frac{\partial^2 c_i(q_i^n,\theta)}{\partial q_i^n \partial \theta} \geq 0$, we get $\frac{\partial q_i^n}{\partial \theta} \geq 0$; $\frac{\partial q_{ij}^n}{\partial \theta} \geq 0$. Thus, q_i^n and q_{ij}^n are increasing functions of the DFE level θ. Similarly, when $t_2 \alpha v - \frac{\partial^2 f_i^r(q_{ki},\theta)}{\partial q_{ki} \partial \theta} > 0$, we get $\frac{\partial q_{ki}}{\partial \theta} \geq 0$ and $\frac{\partial q_{ij}^r}{\partial \theta} \geq 0$. Thus, q_{ki} and q_{ij}^r are also increasing functions of the DFE level θ. Thus, we obtain Property 1.

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