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A Reliability-Based Traffic Equilibrium Model with Boundedly Rational Travelers Considering Acceptable Arrival Thresholds

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Abstract: This paper examines the effects of boundedly rational decision characteristics on travelers' route choice behavior. The concept of boundedly rational confidence level (BRCL) is redefined, which is the probability that a trip arrives between the acceptable earliest arrival time and the acceptable latest arrival time on the shortest travel time budget (TTB). Mathematically, the acceptable boundedly rational arrival thresholds are proposed. Then, a reliability-based boundedly rational traffic equilibrium model (R-BRTE) considering both travel time reliability and acceptable arrival thresholds is developed. Moreover, the equivalent variational inequality problem and uniqueness of solution on the proposed model are proved. A route-based solution algorithm is used to solve the proposed R-BRTE model. Numerical results present the important decision ideas of the proposed model. The results demonstrate that travelers' bounded rationality has a great impact on their route choice behavior and network performance.

Keywords: traffic equilibrium model; travel time budget; travel time reliability; boundedly rational confidence level; acceptable arrival threshold

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1. Introduction

The traffic network equilibrium modeling is a significant decision reference in traffic demand management and supply planning. It contributes to seeking an optimal or sub-optimal flow distribution pattern for sustainable transportation development to balance traffic network pressure and alleviate traffic congestion. The classical user equilibrium (UE) principle has usually been used to model network traffic flow distribution [1,2]. It implies that travelers are perfectly rational, whose travel decision only depends on the expected travel cost and always choose the route with the minimum travel cost. Moreover, it fails to incorporate travel time uncertainty and its impacts on travelers' route choice behavior. In addition, the majority of studies also demonstrate that travelers are boundedly rational but not perfectly rational in the route choice decision process [3–6].

Many studies have already proved that travel time reliability is an essential travel decision cost for travelers [7–9]. In order to model the effects of travel time reliability on travelers' route choice behavior, various traffic equilibrium models considering travel time reliability have been developed. Lo et al. (2006) defined the concept of travel time budget and developed the within-budget time reliability (WBTR) [10]. The TTB is defined as the sum of the mean travel time and a safety margin to arrive on time. In order to incorporate travel time reliability and unreliability simultaneously, Chen and Zhou (2010) developed the mean-excess traffic equilibrium (METE) model, in which travelers always choose the route with minimum unreliability impacts and ensure reliability [11]. By introducing the perception errors, the stochastic mean-excess traffic equilibrium (SMETE) model was formulated and derived by Chen et al. (2011) [12]. Shen et al. developed a reliability-based user equilibrium model to model network traffic assignment at signalized intersections, in which link travel time and wait time at intersections are assumed to follow normal distributions [13]. Fayyaz et al. (2021) introduced an incentive compatible driving simulator experiment to make participants experience the travel time of the used route and pay the cost with respect to a tolled road. Their experiment results indicate that travelers pay more attention to travel time reliability compared to travel time [14]. Furthermore, prospect theory-based and regret theory-based route choice models with travel time reliability were also developed to describe travelers' route choice behavior and network performance [15–21].

In general, the route travel time is always assumed to follow a continuous normal distribution or log-normal distribution. However, this assumption leaves the route travel time boundary out of consideration. Xu et al. (2017) investigated the impacts of link speed limits on the route choice behavior of travelers [22]. A truncated travel time probability density function was proposed and derived, in which the minimum travel time is the link length divided by the speed limit when imposing a speed limit. Zhao et al. (2018) regarded the route free-flow travel time as the lower bound of route travel time and developed the truncated travel time budget model [23]. Yan et al. (2015) explored the effects of speed limits on network equilibrium state and performance [24]. The study showed that imposing speed limits can reduce the mean and variance of total travel time, but the total TTB would increase.

Recently, bounded rationality has become increasingly of interest because it can model and predict travel behavior more realistically. Mahmassani and Chang (1987) introduced the concept of bounded rationality to the classical bottleneck model [25]. They derived and developed the trip timing decision model considering boundedly rational travelers. Lou et al. (2010) proposed the definition of the acceptable path, which is the difference between route travel cost and the least-cost path no longer than a pre-specified threshold value [26]. Moreover, the representation of boundedly rational user equilibrium (BRUE) was presented. Di et al. (2013) determined the solution set of the BRUE model and developed its equivalent nonlinear complementarity problem (NCP) [27]. Cantillo et al. (2006 and 2007) analyzed the survey data on travel modes using a discrete choice model with an inertia threshold [28,29]. The results indicated that the transportation benefit would be overestimated if the inertia decision is neglected. Sun et al. (2017) proposed the definition of BRCL, which is the probability that a trip arrives within the shortest travel time budget plus the acceptable travel time difference [30]. They developed a reliability-based boundedly rational user equilibrium (R-BRUE) model. Li et al. (2018) relaxed the perfect rationality assumption and developed a bounded rational binary logit (BRBL) model to simulate the day-to-day network traffic flow evolution and explored the impacts of bounded rationality on the network's day-to-day evolution [31]. Ramirez et al. (2021) designed a route choice experiment to help to understand how travelers process travel time. The results revealed that travelers do not always choose the shortest route and confirmed the hypothesis of bounded rationality [32].

This paper considers both travel time reliability and travelers' bounded rationality during their route choice process. Here, the concept of BRCL is redefined, which is the probability that a trip arrives between the acceptable earliest arrival time and acceptable latest arrival time on the shortest travel time budget. Subsequently, the R-BRTE model is developed to model route choice behavior and network performance with boundedly rational travelers.

The remainder of this paper is organized as follows. First, the truncated travel time budget model is introduced. Then, we redefine the concept of BRCL, and the R-BRTE model is proposed. Its equivalent variational inequality problem and uniqueness of solution on the model are proved. A route-based algorithm is designed to solve the R-BRTE model and determine the network traffic distribution pattern. Finally, numerical examples are conducted to illustrate the important decision ideas of the proposed model, and some conclusions are provided.

2. TTB with Route Travel Time Boundary

Consider a stochastic transportation network $G(N, A)$, which is composed of N nodes and A directed links. In the transportation network, W denotes the set of all origin-destination (OD) pairs. The set of all routes within the OD pair $w \in W$ is denoted by R_w. t_a^0 is the free-flow travel time of the link $a \in A$, and the traffic capacity of the link $a \in A$ is expressed as C_a. Let f_r^w denote the traffic flow associated with the route $r \in R_w$, $w \in W$. $\mathbf{f} = (f_r^w, r \in R_w, w \in W)^T$ is the route flow vector.

This paper uses the Bureau of Public Roads (BPR) function as a link performance function $t_a(v_a)$.

$$t_a(v_a) = t_a^0\left[1 + \beta\left(\frac{v_a}{C_a}\right)^n\right] \tag{1}$$

where β and n are two deterministic parameters of the BPR performance function, respectively. Assume that the link capacity C_a is stochastic due to weather conditions, traffic incidents, traffic management and control, and so on. It follows a uniform distribution $C_a \sim (\phi_a c_a, c_a)$, where c_a is the design traffic capacity of the link $a \in A$ and ϕ_a denotes the capacity degradable coefficient $\phi_a \in [0, 1)$. Since link capacity C_a is a random variable, the link travel time $t_a(v_a)$ is also a random variable.

We assume link capacity distributions are independent; thus, it can be deduced that route travel time T_r follows a normal distribution based on the central limit theorem. Let $E(T_r)$ be the mean of route $r \in R_w$ travel time and $\sigma(T_r)$ the standard deviation of route $r \in R_w$ travel time.

$$E(T_r) = \sum_{a \in A} \delta_{ar}^w E(t_a), \forall r \in R_w, w \in W \tag{2}$$

$$\sigma(T_r) = \sum_{a \in A} \delta_{ar}^w \sigma(t_a), \forall r \in R_w, w \in W \tag{3}$$

where route travel time T_r follows a normal distribution $N(E(T_r), \sigma(T_r))$. $E(t_a)$ is the mean of the link $a \in A$ travel time and $\sigma(t_a)$ is the standard deviation of the link $a \in A$ travel time. δ_{ar}^w is a binary variable, which equals 1 if link $a \in A$ is on route r, and 0 otherwise.

According to the derivation of Lo and Tung (2003), the mean $E(T_r)$ and standard deviation $\sigma(T_r)$ of route travel time T_r can be written as follows [33].

$$E(T_r) = \sum_{a \in A}\left\{\delta_{ar}^w \cdot \left[t_a^0 + \beta t_a^0 v_a^n \frac{1 - \phi_a^{1-n}}{c_a^n (1 - \phi_a)(1 - n)}\right]\right\} \tag{4}$$

$$\sigma(T_r) = \sqrt{\sum_{a \in A}\left[\delta_{ar}^w \cdot \beta^2 (t_a^0)^2 v_a^{2n}\left\{\frac{1 - \phi_a^{1-2n}}{c_a^{2n}(1 - \phi_a)(1 - 2n)} - \left[\frac{1 - \phi_a^{1-n}}{c_a^n (1 - \phi_a)(1 - n)}\right]^2\right\}\right]} \tag{5}$$

2.1. Truncated Route Travel Distribution

It is well known that route free-flow travel time is the shortest travel time of a specific route in a transportation network. Specifically, route travel time must be bounded, and it has a lower bound at least. Here, we assume that the route free-flow travel time is a determinate constant and transportation network uncertainty is only resulted from link capacity degradation. As mentioned above, route travel time follows a normal distribution. However, this distribution fails to take route travel time boundary into consideration. It implies route travel time is close to negative infinity. Route travel time cannot be one less than zero. In order to incorporate the effects of the lower bound on route travel time, Zhao et al. (2018) derived the truncated probability density function (PDF) of route travel time [23]. The relationships and differences between TTB models with and without route travel time boundaries were illustrated and proved. In this section, we introduce the truncated probability density function (PDF) and derive the TTB under truncated travel time distribution.

The free-flow travel time is considered as the lower bound of route travel time. More specifically, route travel time cannot be less than its free-flow travel time, and the probability of route travel time being less than the free-flow travel time should be zero. Mathematically, the truncated PDF with the lower bound can be expressed as

$$f(T|\mu,\sigma,\bar{t}) = \begin{cases} \frac{f(T)}{1-F(\bar{t})}, & T > \bar{t} \\ 0\ , & others \end{cases} \tag{6}$$

where, $f(\cdot)$ and $F(\cdot)$ are the PDF and cumulative distribution function (CDF) of the original normal distribution associated with route travel time T. $\bar{t}$ is the route free-flow travel time and $F(\bar{t}) = \Pr(T \leq \bar{t})$. μ and σ are the mean and standard deviation of route travel time, respectively.

For ease of elaboration, a single-route network with a mean of 20 and a standard deviation of 5 is used. Assume that the free-flow travel time is 15. The probability density functions (PDFs) of route travel time with and without lower boundaries are shown in Figure 1. The x-axis shows the change in route travel time. The solid blue line is the PDF curve without the lower boundary. The green dotted line represents the PDF curve with the free-flow travel time as a truncation of the original distribution. Figure 1 shows that the original distribution is truncated and becomes taller due to the appearance of a fixed denominator. Moreover, the probability is zero when route travel time with the lower boundary is less than free-flow travel time. The new PDF still satisfies the conservation property of a valid PDF.

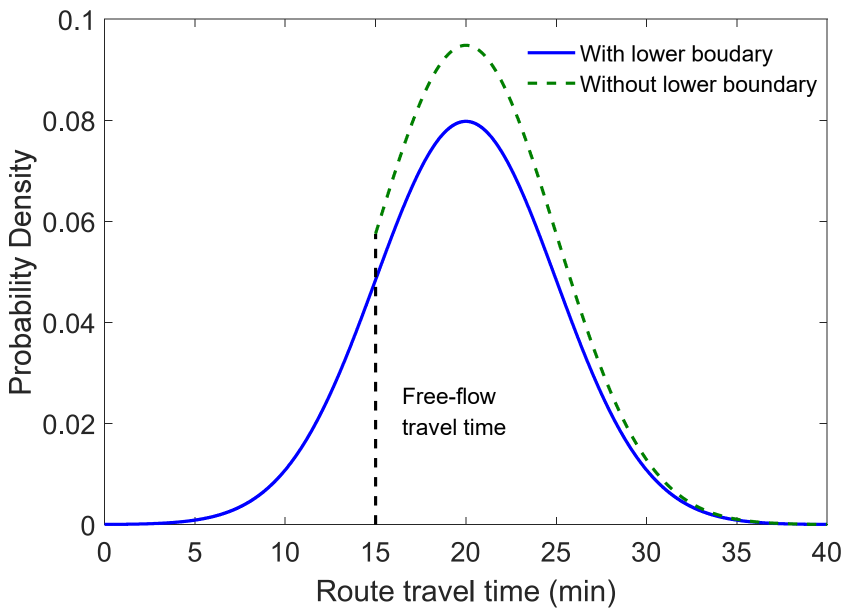


Figure 1. PDFs of route travel time with and without lower boundary.

The mean $E(T|T > \bar{t})$ and standard deviation $d(T|T > \bar{t})$ of truncated travel time distribution can be determined as follows. The detailed derivations can be seen in Reference [20].

$$\begin{aligned} E(T|T > \bar{t}) &= \frac{\int_{\bar{t}}^{+\infty} T(1/\sqrt{2\pi}\sigma)\exp(-(T-\mu)^2/2\sigma^2)dT}{\Phi(-(\bar{t}-\mu)/\sigma)} \\ &= \mu + \frac{(\sigma/\sqrt{2\pi})\exp(-(\bar{t}-\mu)^2/2\sigma^2)}{\Phi((\mu-\bar{t})/\sigma)} \end{aligned} \tag{7}$$

It is obvious that $E(T|T > \bar{t}) > \mu$ from Equation (7). It indicates that the mean of truncated normal distribution associated with route travel time moves right and becomes greater than the original.

The secondary moment $E(T^2|T > \bar{t})$ of the truncated normal distribution can be derived as.

$$\begin{aligned} E(T^2|T > \bar{t}) &= \frac{\int_{\bar{t}}^{+\infty} T^2(1/\sqrt{2\pi}\sigma)\exp(-(T-\mu)^2/2\sigma^2)dT}{\Phi(-(\bar{t}-\mu)/\sigma)} \\ &= \mu^2 + \sigma^2 + \frac{\sigma(\bar{t}+\mu)\exp(-(\bar{t}-\mu)^2/2\sigma^2)}{\sqrt{2\pi}\Phi((\mu-\bar{t})/\sigma)} \end{aligned} \tag{8}$$

Hence, the variance $D(T|T > \bar{t})$ and standard deviation $d(T|T > \bar{t})$ of the truncated normal distribution are

$$D(T|T > \bar{t}) = E(T^2|T > \bar{t}) - [E(T|T > \bar{t})]^2 \tag{9}$$

$$d(T|T > \bar{t}) = \sqrt{D(T|T > \bar{t})} \tag{10}$$

2.2. TTB with Lower Boundary

Assume ρ^w is a predefined confidence level, which refers to the probability that travelers can arrive on time within the TTB b_r^w. Hence, the within-budget travel time reliability when considering route travel time boundary is

$$P(T_r^w \le b_r^w | T_r^w > \bar{t}_r^w) = \rho^w, \forall r \in R_w, w \in W \tag{11}$$

where, $\bar{t}_r^w$ is the free-flow travel time of the route $r \in R_w$ between the OD pair w.

Mathematically, Equation (11) can be alternatively written as

$$\frac{P\left\{\bar{t}_r^w < T_r^w \le b_r^w\right\}}{P\left\{T_r^w > \bar{t}_r^w\right\}} = \rho^w, \forall r \in R_w, w \in W \tag{12}$$

Then, Equation (13) can be derived as follows

$$F(b_r^w) = \rho^w(1 - F(\bar{t}_r^w)) + F(\bar{t}_r^w), \forall r \in R_w, w \in W \tag{13}$$

Let $\xi_r^w = \rho^w(1 - F(\bar{t}_r^w)) + F(\bar{t}_r^w), \forall r \in R_w, w \in W$, we have

$$b_r^w = F^{-1}(\xi_r^w), \forall r \in R_w, w \in W \tag{14}$$

where $F^{-1}(\cdot)$ denotes the inverse function of the CDF of normal distribution.

The difference between the TTB with a lower boundary and the one without a lower boundary can be compared. Let $\bar{b}_r^w$ denote the TTB without a lower boundary of route travel time.

$$F(\bar{b}_r^w) = \rho^w, \forall r \in R_w, w \in W \tag{15}$$

The reliability relationship can be described as follows

$$\begin{aligned} & F(b_r^w) - F(\bar{b}_r^w) \\ & = \rho^w(1 - F(\bar{t}_r^w)) + F(\bar{t}_r^w) - \rho^w \\ & = F(\bar{t}_r^w) - \rho^w F(\bar{t}_r^w) \\ & = (1 - \rho^w)F(\bar{t}_r^w) > 0 \end{aligned} \tag{16}$$

Obviously, $F(b_r^w) > F(\bar{b}_r^w)$ It implies that the TTR, when considering the lower boundary of route travel time, becomes greater than one without a lower boundary if the predefined confidence levels are the same. Meanwhile, because the CDF of the normal distribution is monotonically increasing, $b_r^w > \bar{b}_r^w$—t demonstrates that travelers make a longer budget time under the same confidence level to reduce the possibility of arriving late compared to the original distribution. The travel cost of travelers will be underestimated when ignoring the effects of route travel time boundary. It can result in unpredictable network traffic distribution consequences and provide biased policy guidance.

3. R-BRTE Model

This section first provides a new definition of BRCL. The boundedly rational thresholds associated with the acceptable earliest arrival and latest arrival are estimated. Then the R-BRTE model is developed, and its equivalent variational inequality problem and uniqueness of solution on the model are proved.

3.1. Definition of BRCL

Sun et al. (2017) initially presented the definition of the BRCL, in which they assume that an acceptable travel time is within the shortest TTB plus the boundedly rational threshold [30]. The boundedly rational threshold is actually the acceptable latest arrival time of travelers. However, boundedly rational travelers can also tolerate early arrival to some extent. Based on the above, this paper simultaneously incorporates both the acceptable earliest arrival time and the acceptable latest arrival time on the shortest truncated travel time budget.

Definition 1 (Acceptable travel time). *The acceptable travel time is the time difference between the acceptable earliest arrival time and the acceptable latest arrival time, in which the acceptable earliest arrival time equals the shortest TTB minus the acceptable earliest arrival threshold, and the acceptable latest arrival time is the shortest TTB plus the acceptable latest arrival threshold.*

Hence, the acceptable earliest arrival time T_e^{arrive} and acceptable latest arrival time T_l^{arrive} can be written as

$$T_e^{arrive} = \min_r b_r^w - \varepsilon_e^w(\mathbf{b}), \forall r \in R_w, w \in W \tag{17}$$

$$T_l^{arrive} = \min_r b_r^w + \varepsilon_l^w(\mathbf{b}), \forall r \in R_w, w \in W \tag{18}$$

where, $\varepsilon_e^w(\mathbf{b})$ and $\varepsilon_l^w(\mathbf{b})$ are the acceptable early arrival and late arrival thresholds, respectively. Obviously, the acceptable travel time of travelers $T_r^{acceptable}$ lies between the acceptable earliest arrival time T_e^{arrive} and the acceptable latest arrival time T_l^{arrive}.

$$T_r^{acceptable} \in [T_e^{arrive}, T_l^{arrive}], \forall r \in R_w, w \in W \tag{19}$$

Definition 2 (BRCL). *The BRCL κ_r^w on the route of OD pair with respect to the acceptable arrival time $T_r^{acceptable}$ is the probability that a trip can arrive between the acceptable earliest arrival time T_e^{arrive} and the acceptable latest arrival time T_l^{arrive} .*

$$\kappa_r^w = P\left\{T_e^{arrive} \le T_r^w \le T_l^{arrive}\right\}, \forall r \in R_w, w \in W \tag{20}$$

According to the derivations in Section 2, it is apparent that the TTB of a specific route in a transportation network can be estimated by a predefined confidence level ρ. Due to the existence of introduced thresholds, the acceptable travel time may be more or less than the shortest TTB in all routes. Therefore, the BRCL not only considers travel time reliability and unreliability but also incorporates travelers' bounded rationality.

Figure 2 provides a detailed description of the deterministic process of BRCL. Here assumes that there are two parallel routes in the same OD pair. Their truncated PDFs and CDFs are known. Given a predefined on-time arrival probability ρ, then the travel time budgets using each route can be determined, and they are b_1 and b_2. From Figure 2, the shortest TTB of the transportation network is b_1. The acceptable early arrival and late arrival thresholds are assumed to be ε_e and ε_l, respectively. Thus, the acceptable earliest arrival time T_e^{arrive} and acceptable latest arrival time T_l^{arrive} can be derived according to

Equations (17) and (18). Obviously, the vertical dimension of intersection points of the T_e^{arrive}, T_l^{arrive}, and CDF curves is the BRCL of a specific route.

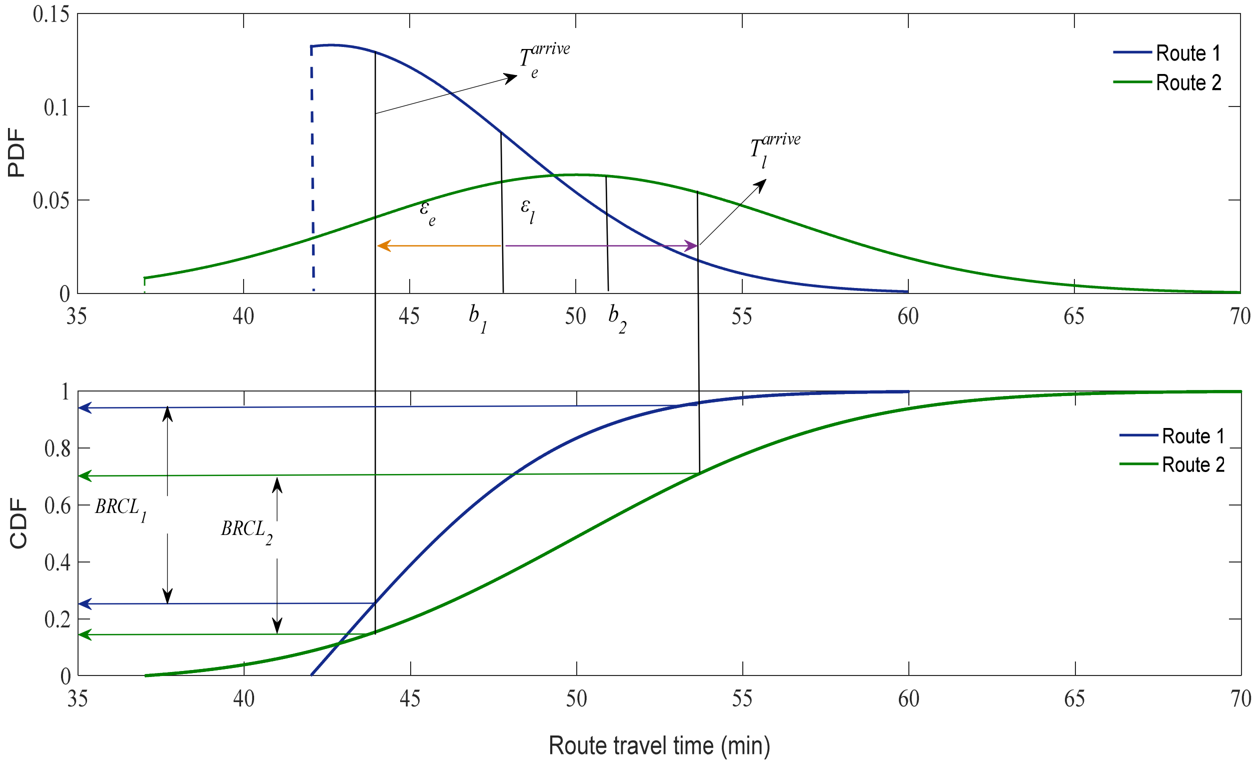


Figure 2. Illustration associated with BRCL.

3.2. Threshold Estimation

It is supposed that the acceptable early arrival and late arrival thresholds are both endogenous variables only associated with the travel time budget of travelers. In general, travelers are more sensitive to late arrival than early arrival, and the acceptable travel time variation is also limited. Thus, the threshold estimation functions can be written as

$$\varepsilon_e(\mathbf{b}) = \varepsilon_e^{\max}\left(1 - \frac{1}{\exp\left(0.1 \cdot \varphi_e \cdot \min_r b_r^w\right)}\right), \forall r \in R_w, w \in W \tag{21}$$

$$\varepsilon_l(\mathbf{b}) = \varepsilon_l^{\max}\left(1 - \frac{1}{\exp\left(0.1 \cdot \varphi_l \cdot \min_r b_r^w\right)}\right), \forall r \in R_w, w \in W \tag{22}$$

where, $\varepsilon_e^{\max}$ and $\varepsilon_l^{\max}$ denote the maximum values of acceptable early arrival and late arrival thresholds, respectively. φ_e and φ_l are the tolerance level parameters with respect to the early arrival and late arrival of travelers, and $\varphi_e > \varphi_l$.

Figure 3 describes the effects of tolerance parameters on acceptable early arrival and late arrival thresholds. It can be found that travelers can accept greater threshold variation with the increase of the tolerance parameter. Meanwhile, the greater shortest TTB results in a more significant acceptable threshold. It can also be inferred that the acceptable early arrival threshold is larger than the acceptable late arrival threshold of travelers.

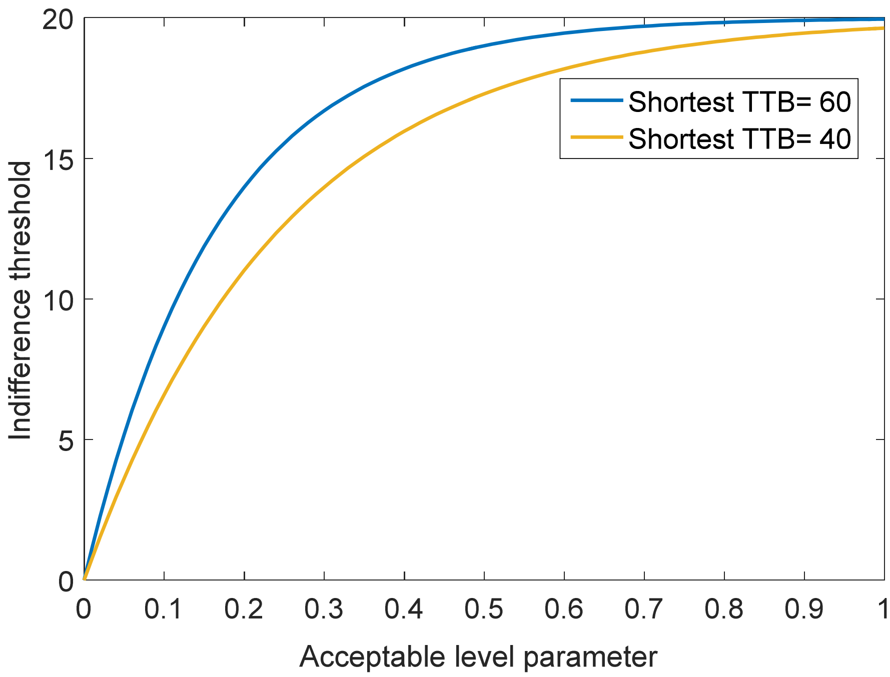


Figure 3. Effects of tolerance parameters on boundedly rational thresholds.

3.3. Equilibrium Model and VI Formulation

Assume that the route choice behavior of travelers follows the random unity maximization (RUM) principle. Travelers are always inclined to choose the routes with the greatest perceived BRCL. The perceived BRCL κ_r^w on the route $r \in R_w$ is

$$\kappa_r^w = b_r^w + \varepsilon_r^w, r \in R_w, w \in W \tag{23}$$

where b_r^w can be calculated by Equation (14). The random term ε_r^w is assumed to be identically and independently distributed (i.i.d.) Gumbel variate. The route choice probability p_r^w on the route $r \in R_w$ based on the RUM principle can be written as

$$p_r^w = \frac{\exp(\theta \kappa_r^w)}{\sum\limits_{r \in R_w} \exp(\theta \kappa_r^w)}, r \in R_w, w \in W \tag{24}$$

where θ is a dispersion parameter measuring the variability in perceived BRCL. A small θ value implies a greater perceived error in comparison with the shortest route. A higher θ value implies a smaller perceived error. The equilibrium flow pattern of the R-BRTE model can be obtained as follows

$$f_r^w = d_w \cdot p_r^w = d_w \cdot \frac{\exp(\theta \kappa_r^w)}{\sum\limits_{r \in R_w} \exp(\theta \kappa_r^w)}, r \in R_w, w \in W \tag{25}$$

Here, d_w is the travel demand for the OD pair w.

The R-BRTE model can be formulated equivalently as the following VI problem

$$\sum_{w \in W} \sum_{r \in R_w} (\kappa_r^{w*} - \frac{1}{\theta} \ln f_r^{w*})(f_r^w - f_r^{w*}) \geq 0, r \in R_w, w \in W \tag{26}$$

Theorems 1 and 2 provide the equivalence proof of the VI problem (26) and the proposed R-BRTE model, and the solution uniqueness proof of the VI problem (26).

Theorem 1. *The VI problem (26) is equivalent to the equilibrium pattern of the R-BRTE model.*

Proof. According to the KKT condition of Equation (26), we have

$$\kappa_r^w - \frac{1}{\theta}\ln f_r^w - \psi_r^w = 0, \forall r \in R_w, w \in W \tag{27}$$

$$\sum_{r \in R_w} f_r^w = d_w \tag{28}$$

where, ψ_r^w is the Lagrangian multiplier.

It can be derived based on Equation (27), such as

$$f_r^w = \exp[\theta(\kappa_r^w - \psi_r^w)], \forall r \in R_w, w \in W \tag{29}$$

Combine Equations (28) and (29). ψ_r^w can be derived as

$$\psi_r^w = -\frac{1}{\theta}\ln\frac{d_w}{\sum\limits_{r \in R_w}\exp(\theta\kappa_r^w)}, \forall r \in R_w, w \in W \tag{30}$$

Then, substitute Equation (30) into Equation (27), thus it can be obtained

$$\frac{f_r^w}{d_w} = \frac{\exp(\theta\kappa_r^w)}{\sum\limits_{r \in R_w}\exp(\theta\kappa_r^w)}, r \in R_w, w \in W \tag{31}$$

Obviously, Equation (31) is the equilibrium condition of the R-BRTE model. Thus, the VI problem is equivalent to the equilibrium pattern of the R-BRTE model. □

Theorem 2. *The VI problem (26) has one solution at least.*

Proof. Because the feasible region of the VI problem is nonempty and compact convexity. The mean value and standard deviation of route travel time are continuously associated with the flow $\mathbf{f}$. Thus, the TTB is also continuous with respect to $\mathbf{f}$. Therefore, it can be inferred that the BRCL and boundedly rational thresholds are continuous functions. In accordance with the fixed point theorem, the VI problem (26) has one solution at least. Moreover, the solution uniqueness of the VI problem can be ensured because the strict monotonicity of the BRCL is uncertain. □

4. Solution Algorithm

This paper employs the MSA algorithm to solve the proposed model. The algorithm steps are expressed as follows.

Step 1 (Initialization): Set the initial iteration number $n = 0$; the iteration termination error e is 0.001. Generate randomly the initial feasible flow $f_r^{w(0)}$ satisfying OD demand d_w.

Step 2 (Cost calculation): Calculate the mean and standard deviation of route travel time, and determine the TTB under truncated normal distribution according to Equation (5). Then calculate the BRCL κ_r^w and boundedly rational threshold $\varepsilon(\mathbf{b})$ based on Equations (9)–(11).

Step 3 (flow assignment): Set the auxiliary route flow $y_r^{w(n)}$, which can be yielded by Equations (13) and (14).

$$y_r^{w(n)} = d_w\frac{\exp(\theta\kappa_r^{w(n)})}{\sum\limits_{r \in R_w}\exp(\theta\kappa_r^{w(n)})}, \forall r \in R_w, w \in W \tag{32}$$

Step 4 (Update): The route flow $\mathbf{f}^{(n)}$ can be updated by the following

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + \frac{1}{n}(\mathbf{y}^{(n)} - \mathbf{f}^{(n)}) \tag{33}$$

Step 5 (Checking the convergence): If the iteration number n exceeds the maximum iteration number N or Equation (34) is satisfied,

$$\frac{\sqrt{\sum\limits_{r\in R_w}\left(f_r^{w(n+1)}-f_r^{w(n)}\right)}}{\sum\limits_{r\in R_w} f_r^{w(n)}} \leq 0.001 \tag{34}$$

5. Numerical Example

In this section, a nine-grid transportation network and the Nguyen–Dupuis transportation network are used to present the characteristics of the R-BRTE model. The nine-grid transportation network example is to demonstrate route choice cost and travelers' route choice behavior under bounded rationality. The Nguyen–Dupuis transportation network is introduced to model network performance.

5.1. A Nine-Grid Transportation Network

There are two OD pairs, six routes, and twelve links in the nine-grid network. The characteristic values of the link parameters, which include free-flow time, link capacity degradable coefficient, and link capacity, are presented in Figure 4. Two deterministic parameters β and n of the BPR performance function are 0.15 and 4 in Equation (1). The relationships between the routes and links of the grid transportation network are shown in Table 1. The travel demands of OD pair 1-6 and 7-6 are 1500 veh/h and 2000 veh/h, respectively. Without loss of generality, travelers are assumed to be homogeneous, with an identical expected on-time arrival probability ρ is 0.7. The dispersion parameter θ is 0.5. The tolerance level parameters of early arrival and late arrival are 0.6 and 0.4, respectively.

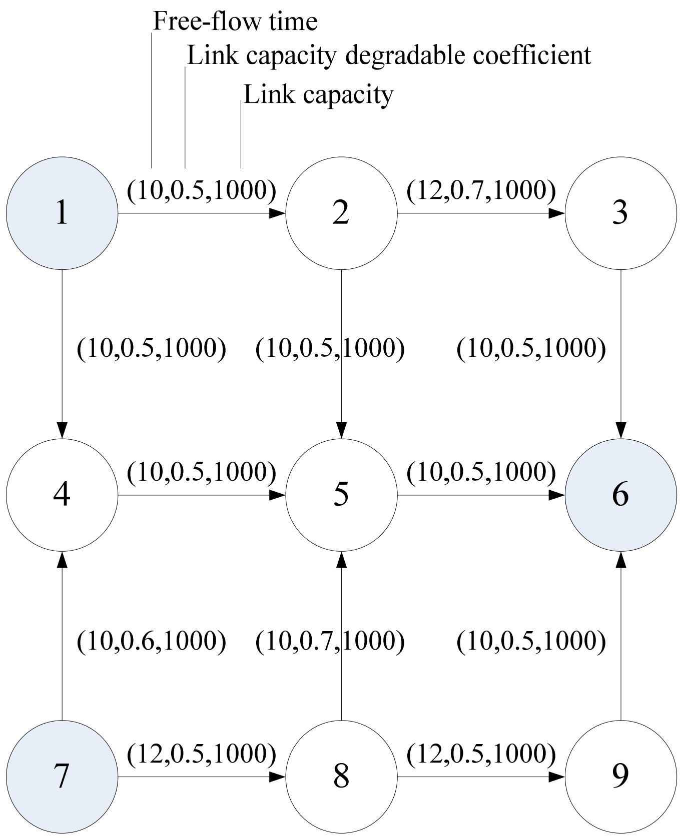


Figure 4. A nine-grid transportation network.

Table 1. The relationships between routes and links of the nine-grid transportation network.

OD Pair	Route #	Associated Link
1-6	1 2 3	1-2-3-6 1-2-5-6 1-5-5-6
7-6	4 5 6	7-8-9-6 7-8-5-6 7-4-5-6

The equilibrium results of the proposed model are presented in Table 2, Figures 5 and 6. From those tables and figures, we can see that the routes with greater standard deviation would result in more TTB. It implies that travelers would prepare more TTB to avoid travel time variation. Route 1 has the greatest BRCL in OD pair 1-6, and route 6 has the greatest BRCL in OD pair 7-6. Therefore, most travelers choose these two routes. It also suggests that the routes with minimum expected travel time or shortest TTB are not always chosen for boundedly rational travelers. They may be more inclined to use these routes, which can ensure arrival within their acceptable travel time.

Table 2. The equilibrium results of the R-BRTE model.

OD	Route	Route Flow (veh/h)	Shortest TTB (min)	Boundedly Rational Threshold		BRCL
				Early Arrival Threshold (min)	Early Arrival Threshold (min)	
1-6	1 2 3	566.32 456.35 477.33	48.99	14.21	8.59	0.88 0.45 0.54
7-6	4 5 6	597.28 621.97 780.74	56.38	14.49	8.95	0.40 0.48 0.94

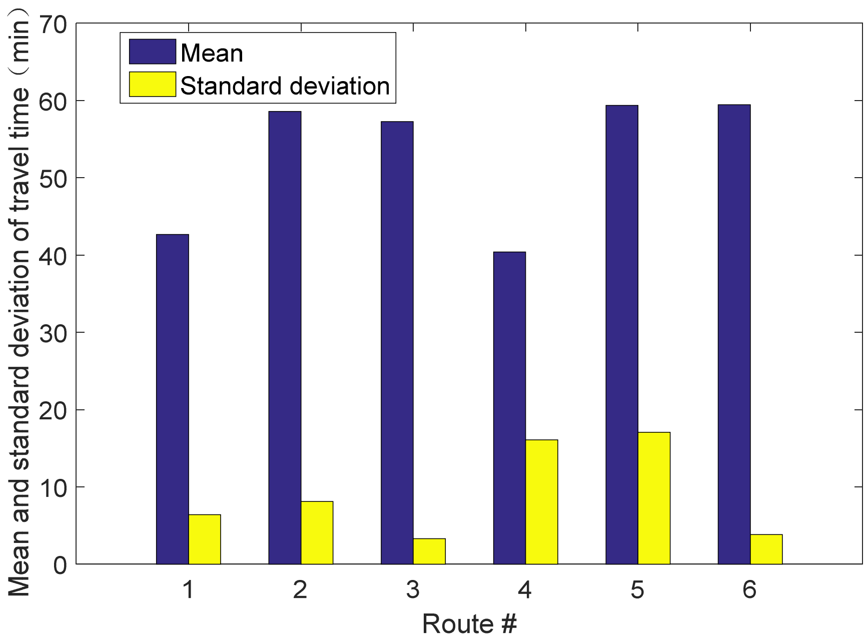


Figure 5. The mean and standard deviation of the route travel time at equilibrium.

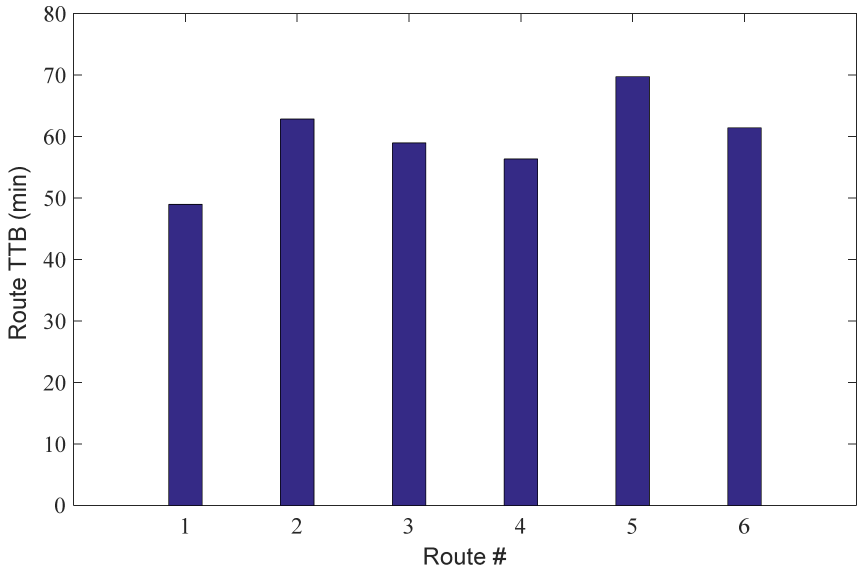


Figure 6. The TTBs of the used route at equilibrium.

From Table 2, we know that the shortest TTBs of both OD pairs are 48.99 min and 56.38 min. OD pair 7-6 has a greater TTB than OD pair 1-6. As a result, the acceptable early and late thresholds are also greater for travelers in OD pair 7-6. From the above results, we can see that the arrival probability for travelers using route 1 between 48.99 − 14.21 min and 48.99 + 8.59 min is 0.88. The arrival probability for travelers using route 6 between 56.38 − 14.49 min and 56.38 + 8.95 min is 0.94. It is because most travelers choose severally route 1 and route 6 in both OD pairs.

Figures 7 and 8 depict the effects of predefined confidence levels or expected on-time arrival probability ρ on the shortest TTB and boundedly rational thresholds. As the expected on-time arrival probability ρ increases, the shortest TTBs of both OD pairs become greater gradually, as shown in Figure 7. The shortest TTB of OD pair 7-6 is greater than that of OD pair 1-6. These results indicate that travelers with high requirements for arriving on time would make a longer budget time to avoid risk. In addition, the shortest TTBs are different because the alternative route sets have differences. Figure 8 describes the relationships between the expected on-time arrival probability ρ and boundedly rational thresholds. The acceptable early arrival and late arrival thresholds become greater as the expected on-time arrival probability ρ increases. This is because the boundedly rational thresholds are monotonically increasing with respect to the shortest TTB. The acceptable early arrival threshold is greater than the acceptable late arrival threshold for each OD pair due to travelers being more sensitive to late arrival.

It provides that the BRCL changes trend when the acceptable early arrival and late arrival level parameters have a difference in Figure 9. Here, Route 1 for OD pair 1-6 is regarded as an example to describe these impacts. It can be found that the BRCL increases when the acceptable arrival level becomes greater at equilibrium. Specifically, travelers' BRCL dramatically changes when the acceptable early arrival level parameter increases. It implies that the BRCL is more sensitive to the acceptable early arrival level than the acceptable late arrival level. These results also declare that the BRCL is closely associated with the acceptable arrival for boundedly rational travelers.

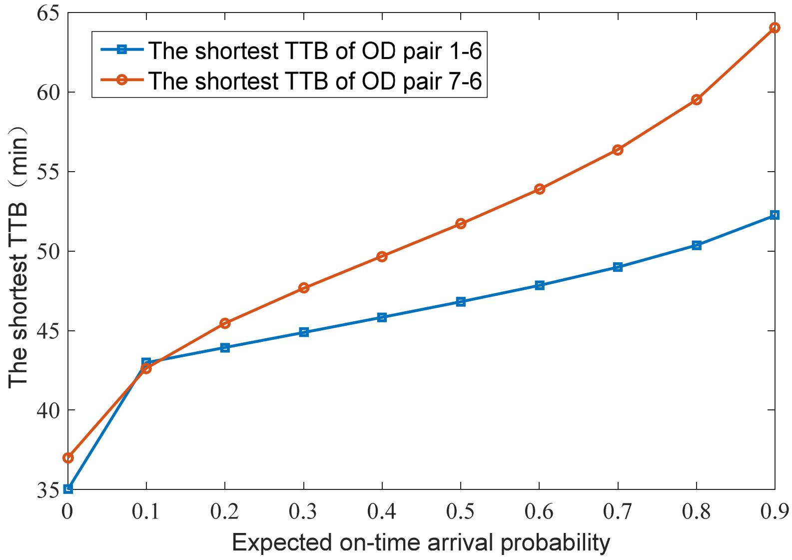


Figure 7. Effects of the travelers' expected on-time arrival on the shortest budget time.

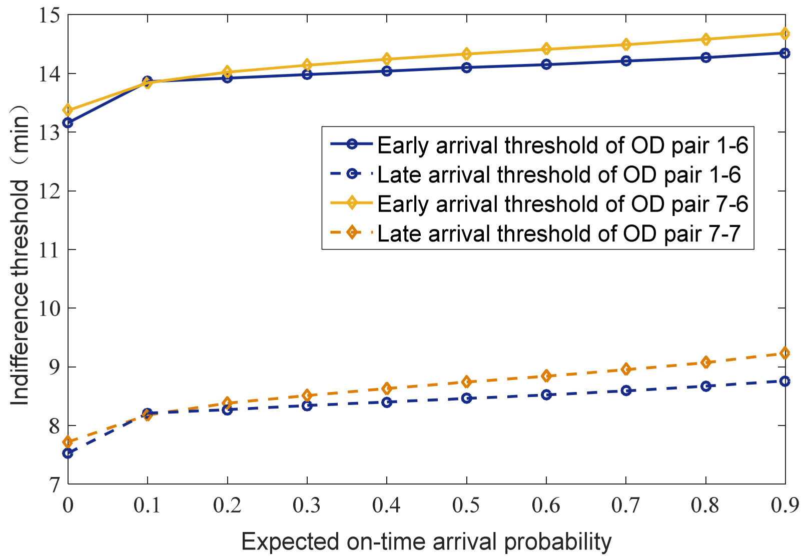


Figure 8. Effects of travelers' expected on-time arrival on boundedly rational thresholds.

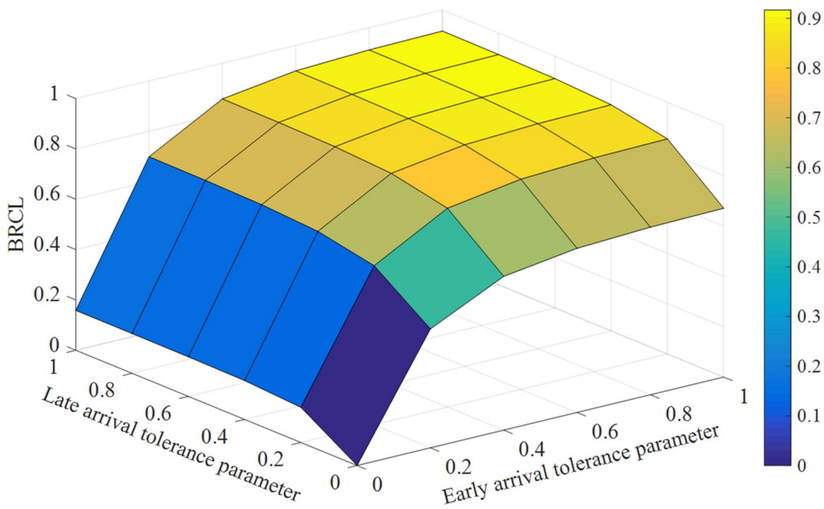


Figure 9. Joint effects of early and late tolerance parameter on BRCL.

5.2. Nguyen–Dupuis Transportation Network

The Nguyen–Dupuis transportation network topology and its characteristics are depicted in Figure 10, consisting of four OD pairs. The network consists of four OD pairs, thirteen nodes, and nineteen links. Two deterministic parameters β and n of the BPR performance function are 0.15 and 4 in Equation (1). The travel demands of OD pair 1-2 and 1-3 are 600 veh/h and 500 veh/h, and those of OD pair 4-2 and 4-3 are 800 veh/h and 400 veh/h. All link capacity degradable coefficients in the transportation network are assumed to be 0.4. The free-flow travel time and link capacity of each link are presented in Figure 10. Similarly, travelers are assumed to be homogeneous, with an identical expected on-time arrival probability ρ of 0.7. The dispersion parameter θ is 0.5. The tolerance level parameters of early arrival and late arrival are 0.6 and 0.4, respectively.

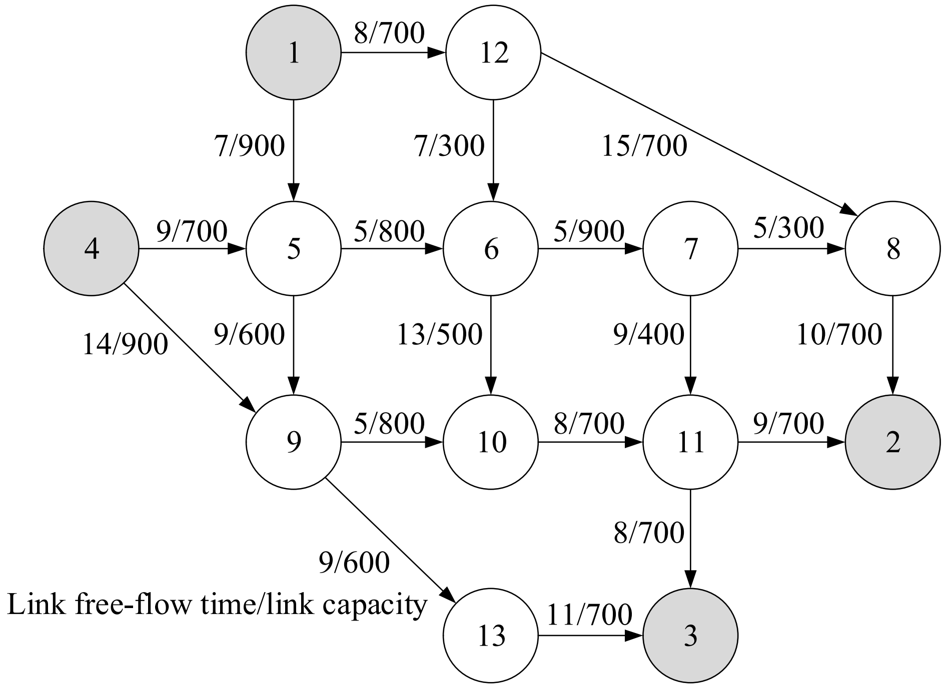


Figure 10. Nguyen–Dupuis transportation network and link parameters.

This section uses the Nguyen–Dupuis transportation network to examine the network performance under bounded rational decisions, which includes total mean total travel time (TMTT), the total standard deviation of route travel time (TSD), and total travel time budget (TTTB).

Mathematically, the TMTT, TSD, and TTTB can be derived as follows.

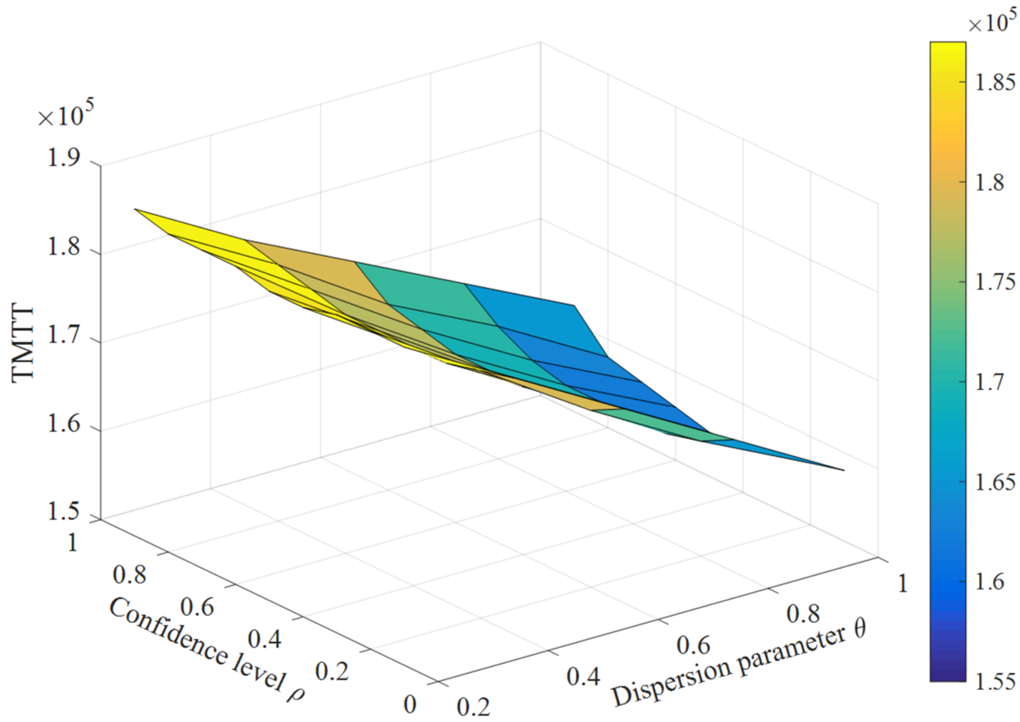
$$\text{TMTT} = \sum_{w \in W} \sum_{r \in R_w} E(T_r^w) \tag{35}$$

$$\text{TSD} = \sum_{w \in W} \sum_{r \in R_w} \sigma(T_r^w) \tag{36}$$

$$\text{TTTB} = \sum_{w \in W} \sum_{r \in R_w} b_r^w \tag{37}$$

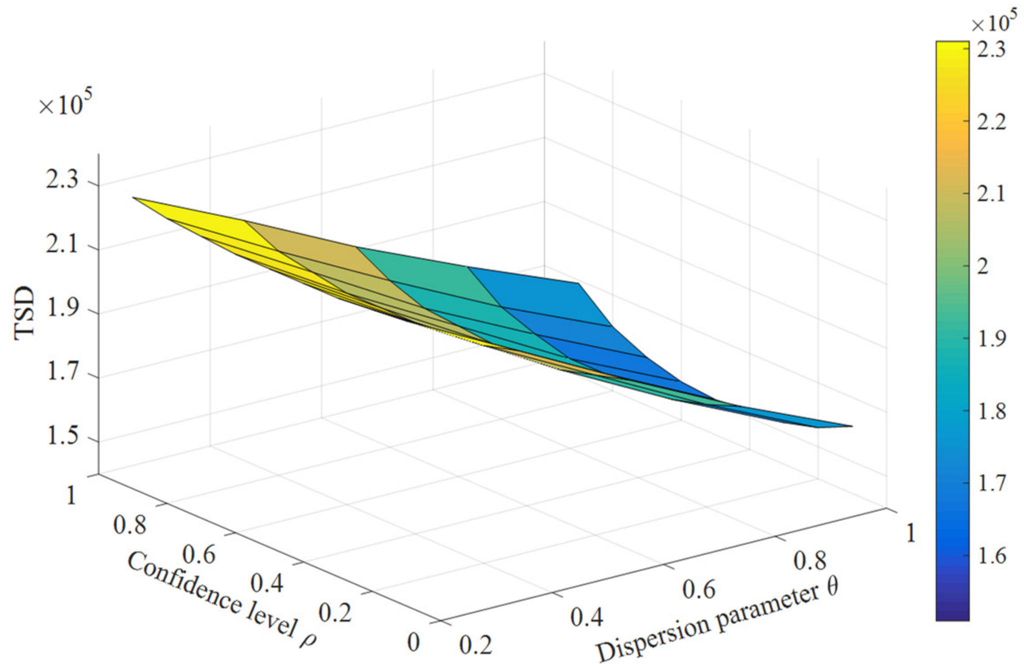
First, we simulate the effects of different combinations of confidence level ρ and dispersion parameter θ. With the loss of generality, the confidence level from 0.1 to 0.95 and the dispersion parameter from 0.2 to 1 are tested. The network performance of the equilibrium state is shown in Figure 11a–c. It is clear that the TMTT and TSD decrease as the dispersion parameter θ increases. As expected, the TTTB also reduces due to the trend of TMTT and TSD. The TTTB has a lower rate at a lower confidence level ρ than that at a higher confidence level ρ. Compared to the confidence level, TMTT and TSD are more sensitive to the dispersion level. It is also reasonable because the mean and standard deviation of route travel time are irrelevant to the confidence level according to Equations (7) and (10). The TTTB increases when the confidence level ρ increases, as shown in Figure 11c. The confidence level would significantly affect the route's TTB. These imply that the perceived error and confidence level of travelers have a significant impact on travelers' route choice behavior and network performance.

Second, the effects of various combinations of the early arrival tolerance parameter φ_e and the late arrival tolerance parameter φ_l are examined. Both level parameters are set from 0 to 1. The network performance of the equilibrium state is shown in Figure 12a–c. It can be found that the TMTT reduces as an early arrival tolerance parameter φ_e. It increases when the late arrival tolerance parameter φ_l increases. Their effects on TSD and TTTB are similar to that on TMTT. In addition, we can observe that the early arrival tolerance parameter φ_e has a greater impact on the network performance than the late arrival tolerance parameter φ_l. Moreover, the greater tolerance parameter φ_e and φ_l has an insignificant influence.

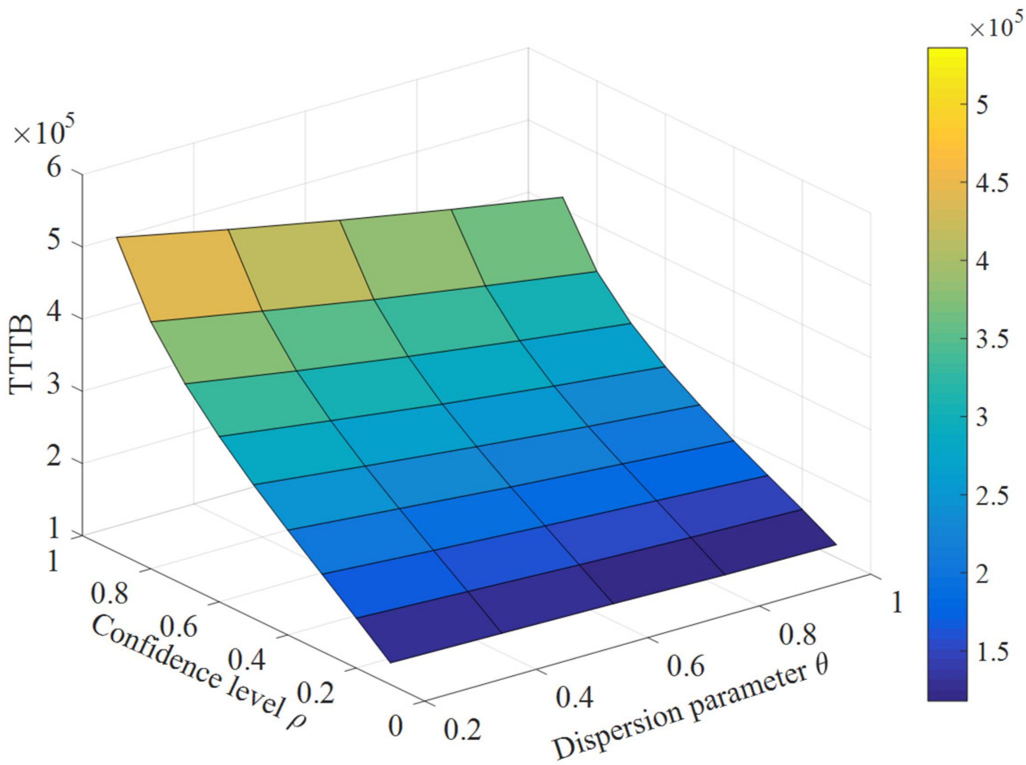


(**a**)

Figure 11. *Cont.*



(**b**)



(**c**)

Figure 11. Joint effects of dispersion parameter and confidence level on. TMTT (**a**), TSD (**b**) and TTTB (**c**).

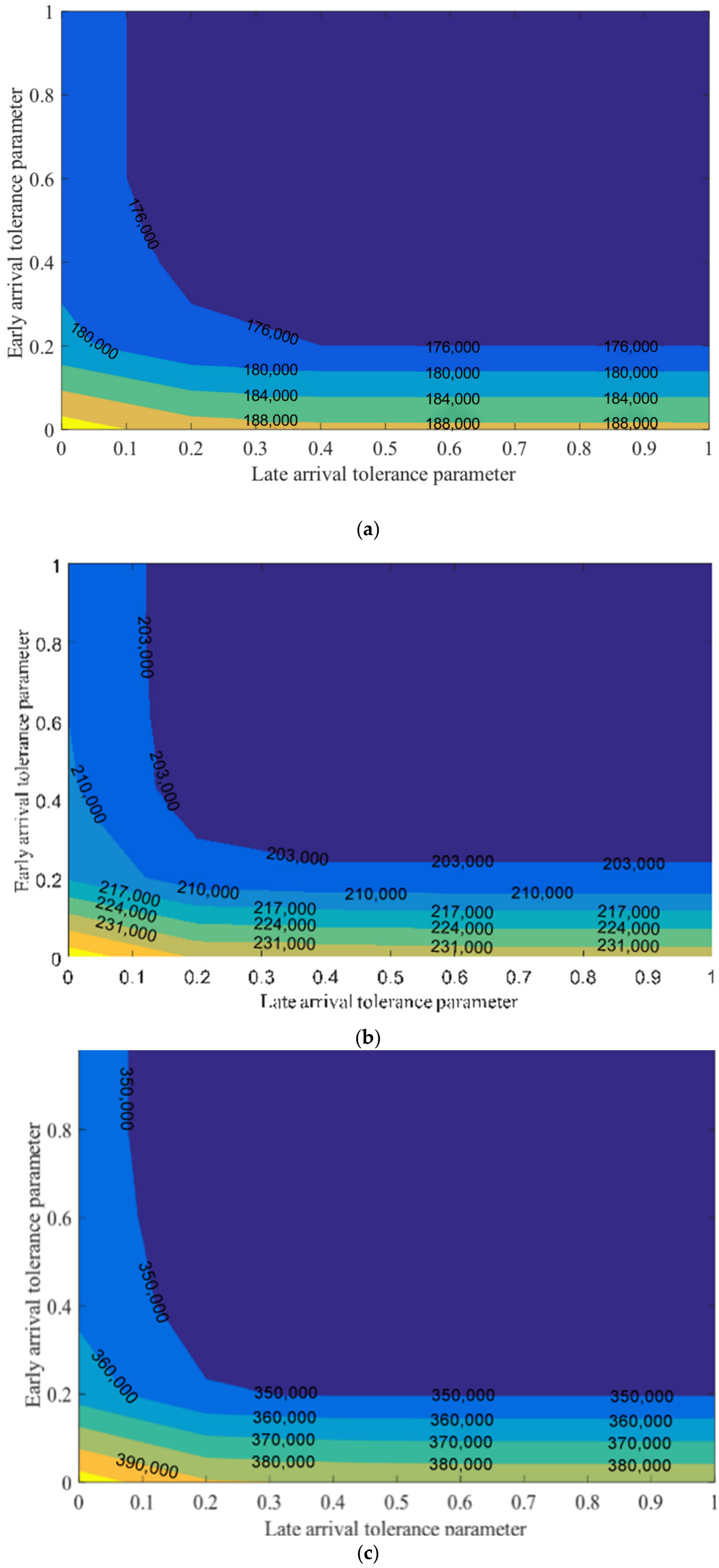


Figure 12. Joint effects of early and late arrival tolerance parameters on the TMTT (**a**), TSD (**b**), and TTTB (**c**).

6. Conclusions

This paper redefined the concept of BRCL. It was regarded as the probability that a trip would arrive between the acceptable earliest arrival time and the acceptable latest arrival time with respect to the shortest TTB. Then the R-BRTE model with boundedly rational travelers was proposed. Mathematically, boundedly rational thresholds were estimated. The equivalent variational inequality problem and uniqueness of solution on the proposed model are proved. A route-based solution algorithm was used to solve the equivalent VI problem of the proposed model. A numerical example was designed to model the route choice behavior of boundedly rational travelers and network performance. The results indicated that travelers would reserve a longer TTB to deal with travel time variation and avoid arriving late when they used a route with a higher standard deviation. The acceptable early level has a greater impact on BRCL, and BRCL is more sensitive to the variation of the acceptable early level parameter. Travelers are more inclined to use these routes to ensure arrival within their acceptable travel time. It should be noted that the proposed model can be further extended to model its effects on travel mode choice and travel time choice and etc. It can also be introduced and incorporated into dynamic assignment simulations to model the travel behavior of boundedly rational travelers and dynamic network performance.

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