sustainability

MDPI

Article

Optimizing Multi-Vehicle Demand-Responsive Bus Dispatching: A Real-Time Reservation-Based Approach

Xuemei Zhou [1,2,*], Guohui Wei [1], Yunbo Zhang [1], Qianlin Wang [1] and Huanwu Guo [1]

1 College of Transportation Engineering, Tongji University, Shanghai Key Laboratory of Rail Infrastructure Durability and System Safety, Key Laboratory of Road and Traffic Engineering of the State Ministry of Education, 4800 Caoan Highway, Shanghai 201804, China
2 Jiangsu Province Collaborative Innovation Center of Modern Urban Traffic Technologies, Southeast University, Dongnandaxue Road #2, Nanjing 211189, China
* Correspondence: zhouxm@tongji.edu.cn

Abstract: The demand-responsive public transport system with multi-vehicles has the potential to efficiently meet real-time and high-volume transportation needs through effective scheduling. This paper focuses on studying the real-time vehicle scheduling problem, which involves dispatching and controlling different model vehicles uniformly based on generated vehicle number tasks at a given point in time. By considering the immediacy of real-time itinerary tasks, this paper optimizes the vehicle scheduling problem at a single time point. The objective function is to minimize the total operating cost of the system while satisfying constraints such as passenger capacity and vehicle transfer time. To achieve this, a vehicle scheduling optimization model is constructed, and a solution approach is proposed by integrating bipartite graph optimal matching theory and the Kuhn–Munkres algorithm. The effectiveness of the proposed approach is demonstrated by comparing it with a traditional greedy algorithm using the same calculation example. The results show that the optimization method has higher solution efficiency and can generate a scheduling scheme that effectively reduces operating costs, improves transportation efficiency, and optimizes the operation organization process for demand-responsive buses.

Keywords: demand-responsive public transport system; vehicle dispatching; bipartite graph; optimal matching; Kuhn–Munkres algorithm

Citation: Zhou, X.; Wei, G.; Zhang, Y.; Wang, Q.; Guo, H. Optimizing Multi-Vehicle Demand-Responsive Bus Dispatching: A Real-Time Reservation-Based Approach. *Sustainability* **2023**, *15*, 5909. https://doi.org/10.3390/su15075909

Academic Editor: Fabio Carlucci

Received: 26 February 2023
Revised: 14 March 2023
Accepted: 16 March 2023
Published: 29 March 2023

1. Introduction

With the rapid urbanization process and growing traffic demand, public transportation is becoming an essential mode of travel to alleviate urban traffic congestion. Public transportation is a low-consumption and efficient travel mode that can improve resource utilization and promote the sustainable development of transportation. China has recognized the importance of prioritizing the development of urban public transportation and encouraging green public transportation. The "Outline for Building a Powerful Transportation Country" issued by China in 2019 clearly states that priority should be given to the development of urban public transportation, and green public transportation should be encouraged and guided. At the same time, development of the economy and the improvement of quality of life have prompted residents to pay more attention to the level of travel services and look forward to safe, fast, and comfortable travel. The "National Comprehensive Three-dimensional Transportation Network Planning Outline" pointed out that the demand for high-quality, diversified, and personalized traffic is constantly increasing, and it is necessary to build traffic that satisfies people and continuously enhances people's sense of gain, happiness, and security. However, traditional public transportation methods, such as conventional buses, are facing challenges due to overcrowding, long waiting times, and low ride comfort. In recent years, the passenger flow of public transport in most cities in China has shown a continuous decline. Taking Beijing and Shanghai as examples [1], their bus passenger traffic dropped by 59.05 and 51.44% respectively during the 10 years from 2011 to 2020. On the other hand, although rail transit is highly punctual and has a large transportation capacity, its system is still in the construction period. The overall system is not perfect and requires a long construction period. In this context, the rapid development of demand-responsive public transport provides passengers with a variety of travel services [2]. Demand-responsive buses set routes according to online information platforms, mobile apps, and other reservation needs, with one seat per person, flexible time and routes, and can provide passengers with fast, convenient, and comfortable travel services. Compared with conventional public transport, demand-responsive public transport is more reliable and more competitive, which helps to improve the efficiency of road resource use, alleviate urban traffic congestion, and reduce traffic pollution [3]. This paper will explore strategies for improving demand-responsive public transport, with a focus on high efficiency, high quality, and excellent service.

Reasonable operation scheduling is the basic requirement and guarantee for the orderly operation of a public transport system. The operation and planning process of public transportation is a complex and systematic decision-making problem, which is generally decomposed into multiple sub-problems, including bus route planning and road network design, service time and departure interval setting, timetable preparation, vehicle scheduling, and driving staff scheduling [4–7]. Whether the scheduling plan is scientific or not is related to whether the operation task can be completed. A good and reliable plan will have a positive impact on many aspects such as enterprise operating cost control, bus operation resource utilization, personnel work efficiency management, and passenger satisfaction evaluation. As an important part of public transport dispatching, vehicle dispatching refers to the rational planning and dispatching of vehicles based on public transport pre-dispatching work [8]. It is of great significance to ensure the smooth progress of the transportation plan, make full use of resources such as saving vehicles, and reduce the operating costs of public transport companies. With regard to demand-responsive public transport, although its operation mode is different from traditional public transport, its operation planning link still exists. Therefore, the content of vehicle scheduling and driver scheduling in traditional research has a certain reference value for research of the demand-oriented bus system.

This paper aims to propose a new demand-responsive bus mode based on passenger reservation, which combines the traditional demand-responsive bus mode with the travel reservation mode of taxis to respond to the needs of public passenger flow and provide high-quality service. It can generate an operation plan dynamically according to the reservation needs of passengers (starting station, destination station, number of people), and give passengers reservation feedback, while operating between multiple fixed stations on the line. Compared with general demand-responsive buses, it does not require detours to pick up and drop off passengers, significantly reducing travel time and saving passenger travel time costs, thereby improving passenger travel service quality. Compared with taxis, it can reduce passenger travel expenses and operating costs.

However, unlike other demand-responsive buses, how to coordinate and dispatch vehicles among multiple stations based on the vehicle status and the determined departure demand to improve resource utilization efficiency has become a complex issue that affects demand response, making research on this issue even more important. To address this research problem, this paper establishes a real-time vehicle scheduling model based on matching theory, and designs a corresponding solution algorithm. The model aims to minimize the total operating cost while considering the coordination and cooperation of multi-model vehicles, and achieve global optimal matching between the departure demand and the available vehicles at a certain departure time point in the system. The research results enrich and improve existing bus scheduling optimization theory and method, providing theoretical support for urban bus operation managers to formulate reasonable demand-responsive operation plans and guide the future intelligent development of public transport.

2. Literature Review

2.1. Summary of Research on Flexible Public Transport System

There are currently three main models of public transport systems in countries around the world, including the traditional Fixed-Route Transit System, the Customized Public Transport System, and the Flexible Public Transport System [9–11]. The fixed-route bus system is widely used in major cities in China. It is characterized by low cost, large usage, and simple operation mode, and is suitable for high-density travel environments. The customized bus system is more inclined to serve commuter passenger flow with a large amount of travel. This kind of service system has relatively fixed routes and is suitable for areas with high travel density and relatively fixed travel demand, but there are certain uncertainties in operating costs and income [12]. The flexible public transport system has the characteristics of the above two modes. It not only has a certain degree of flexibility, and can provide passengers with personalized and convenient services, but also can have the advantages of the low cost of fixed-route buses through a reasonable dispatching organization model [13].

The idea of a flexible public transport system was proposed 40 years ago. At present, flexible bus systems have been established in some areas in North America, Europe, and other countries, and the research on its dispatching system has also achieved certain results. Domestic research results mainly include two sub-branches of flexible public transport systems: Flexible-Route Transit System and Demand-Responsive transit system.

2.1.1. Flexible-Route Transit System

The concept of a flexible-route bus was first proposed by Daganzo [14] in 1984, and Cristian et al. [15] proposed a conceptual design of a flexible-route bus system from any point to any point based on real-time personalized travel demand and preliminary feasibility simulation results, and through analysis, it is believed that the system is more competitive than traditional public transport.

Domestic research on this type of flexible public transport system started relatively late, and the understanding of some concepts varies among different researchers. Wang [16] pointed out in his research that there are differences between variable route bus systems and demand-responsive bus systems. Demand-responsive public transport can be considered as the static stage of variable-line public transport, which mainly solves the commuting passenger flow of passengers. The stations are usually arranged at the attraction points of large passenger flow. However, flexible-route buses do not have high requirements for passenger flow, and this type of bus system is mainly distributed in areas with low travel density and scattered areas.

The current research on flexible-route bus dispatching is mainly divided into two aspects: static dispatching and dynamic dispatching. In the research of variable route bus systems [17–19], commonly used algorithms mainly include genetic algorithms, simulated annealing algorithms, and so on. Since the vehicle needs to determine its driving route in advance, it needs to decide whether to go directly and which station to stop at based on the existing reservation data. Therefore, the variable bus scheduling problem can be divided into static scheduling problems, and the operations research model is used to solve them.

2.1.2. Demand-Responsive Transit System

North America and Europe have nearly 40 years of operating history in the application of demand-responsive public transportation. The demand-responsive public transport system in the United States does not have a fixed operating route and stops at specified stations to provide boarding and landing services according to the needs of users. Europe's "Advanced Public Transport Operation System (SAMPO)" pursues high-level transport services based on the travel needs of different passenger groups. Germany, Poland, and other countries have relatively complete demand-responsive bus systems. Schofer et al. [20] pointed out that the application scenarios of this type of public transport are divided into two categories: one is the service provided for areas with low travel density and sparse distribution, and the second is the special service provided for special groups such as the elderly and the disabled.

According to studies, several problems need to be solved in the operation and development of a demand-responsive public transit system: vehicle scheduling and system operation, cost and price research, and popularization of related policies and regulations [21]. Regarding the scope of application of demand-responsive public transport systems, Gorev [22] believed that when passenger flow in some areas is relatively scarce, traditional public transport cannot provide efficient and high-quality services, while a demand-responsive public transport system is flexible and highly mobile. Its characteristics make it more suitable for travel areas with low travel density such as urban suburbs.

In the research on demand-responsive public transport and some flexible public transport system scheduling problems, researchers usually use dynamic scheduling models to describe the problem. In the process of real-time scheduling, the goal of scheduling is generally to reduce the impact on existing passengers and update the vehicle's driving path in time. Existing research on dynamic scheduling problems [23–25] often use plug-in heuristic algorithms to solve them.

2.2. A Review of Research on Multi-Vehicle Scheduling Problems

Existing studies on vehicle scheduling are mostly focused on the logistics industry. With the development of intelligent public transport systems and internet technology, the operating routes and stops of public transport vehicles are no longer restricted like traditional public transport. The needs of customers become personalized and flexible, and even point-to-point direct services can be realized. On this basis, transportation companies do not have to allocate large-capacity buses but can configure vehicles of different models according to the actual situation of operation, and deploy vehicles according to actual needs, to achieve the purpose of saving vehicle operating costs and purchase costs. Such problems are collectively referred to as multi-vehicle scheduling problems. In the context of flexible public transport, the problem of vehicle scheduling can be studied by referring to the research experience of logistics, railway transportation, and other industries, as well as related issues in traditional public transport and demand-responsive public transport.

2.2.1. Static Multi-Model Vehicle Scheduling

Different from the single-vehicle scheduling problem, in the multi-vehicle scheduling problem, the constraints increase, and the difficulty of solving the problem also increases. Yao [26] proposed a method for the scheduling problem of multi-model electric vehicles in public transportation, which fully considered the impact of charging power and discharge depth on the total cost and used a genetic algorithm to solve it. Total annual dispatch costs were reduced by nearly 16%. Bie [27] realized in the process of researching the logistics distribution vehicle scheduling scheme that the traditional genetic algorithm easily to fall into a local optimum when solving the multi-model vehicle routing problem, and there is a premature situation, so the genetic algorithm is improved to improve optimal efficiency.

2.2.2. Dynamic Multi-Model Vehicle Scheduling

Most of the existing research at home and abroad is based on static inputs; that is, in the process of path planning and vehicle allocation, all information such as passenger location and request service time are known and do not change over time. Under such conditions, the formulated vehicle routes are also consistent, so it is called a static multi-vehicle scheduling problem. Hu [28] established a two-stage model for the multi-model vehicle scheduling problem including dynamic demand and used the hybrid quantum evolutionary algorithm to solve the model, which met the real-time requirements of vehicle scheduling.

2.2.3. Multi-Depot and Multi-Model Vehicle Scheduling

Some researchers have added the constraints of multi-depots based on multi-vehicles, that is, multi-depots, and multi-model vehicle scheduling problems, and the solution process is more complicated. Guedes [29] used the cat swarm algorithm to solve the model in the closed multi-depot multi-model vehicle scheduling problem, and proved the superiority of the algorithm. Although the solution to the closed-form problem has been achieved, he pointed out that the solution to the open-form multi-depot and multi-model vehicle routing problem in which vehicles do not need to return to the original depot each time remains to be explored.

2.3. Research Summary

Generally speaking, although a lot of research and progress has been made in the research of vehicle dispatching systems, some of which involve road traffic and demand-responsive public transport systems, the research is not systematic and the content is scattered. To sum up, several points can be improved: (1) Most of the existing studies make ideal assumptions about the factors in the model and do not take into account the uncertainties in the actual traffic operation; (2) Most of the research related to the multi-vehicle VRP problem is based on the logistics distribution problem, and many studies on the multi-vehicle scheduling problem of actual road vehicles and public transportation are still in the stage of theoretical assumptions.

Given the above problems, several research directions of flexible bus scheduling are clarified: (1) Connect the flexible bus system with the corresponding vehicle scheduling model, learn from the research experience of the demand-responsive bus and taxi industry on the VRP problem, and improve existing models or establish new models; (2) To clarify the defects of existing models, improve the objective function and constraints, so that the model is closer to the reality; (3) According to the actual traffic status and road network conditions in China, from the aspects of planning and design, operation management, evaluation, and decision-making, etc., a flexible bus dispatching system suitable for China's road traffic conditions is studied.

3. Materials and Methods

3.1. Problem Description

3.1.1. Real-Time Scheduling Process of Demand-Responsive Buses

In the demand-response bus mode based on real-time reservations, the departure time and available stops of the vehicle are given. As shown in Figure 1, passengers choose the appropriate time, starting station, and final destination according to their own needs, and the system is based on passenger travel demand information. Orders are generated. After the reservation deadline, the demand-responsive bus system will first screen out the passenger orders that meet the service requirements according to its capacity conditions, and determine the tasks of each vehicle based on the scheduling algorithm. After the vehicle task is determined, according to the vehicle location distribution status and vehicle availability provided by the big data center, each vehicle task is assigned to the appropriate vehicle, the matching of each vehicle and each vehicle task is completed before departure, and combined into each departure time. The reasonable vehicle-task arrangement finally forms the overall vehicle scheduling scheme.

3.1.2. Problem Description Based on Weighted Bipartite Graph

Real-time vehicle scheduling is the process of dispatching and uniformly controlling vehicles of different models in the system based on generated vehicle tasks at a certain point in time. In the reservation-based demand-responsive bus real-time vehicle dispatching scenario shows in Figure 2, at every interval of a minimum time granularity, its pre-dispatch system will make statistics on passenger reservation orders and dynamically generate all vehicle tasks at the next departure time, including origin and destination stations, driving routes, required vehicle type, and other information. As far as the operation platform is concerned, only by selecting passenger vehicles reasonably to perform the tasks can the system's operation benefits be maximized.

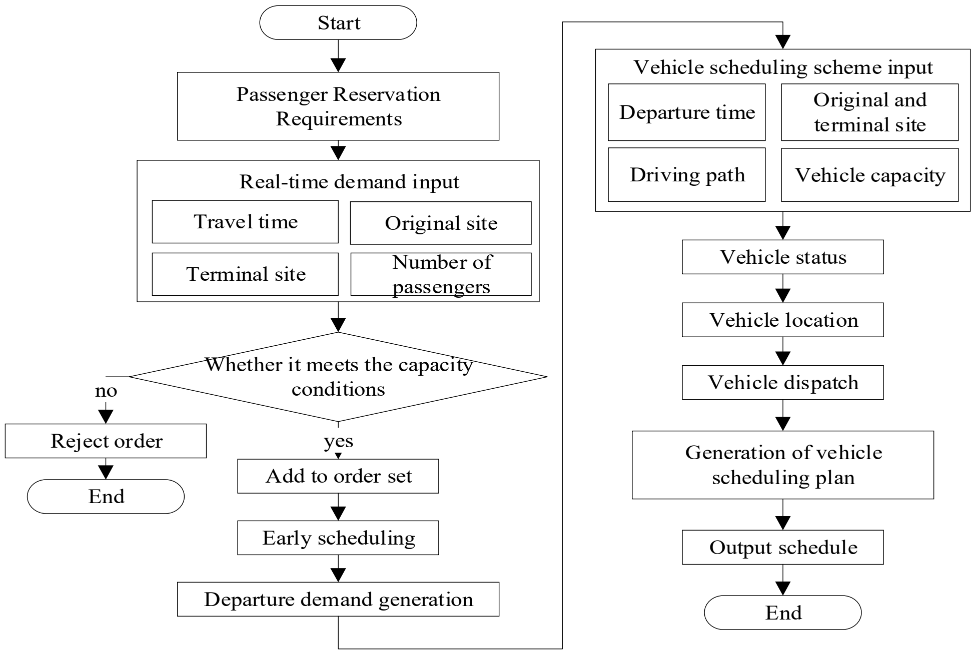

Figure 1. Real-time scheduling process.

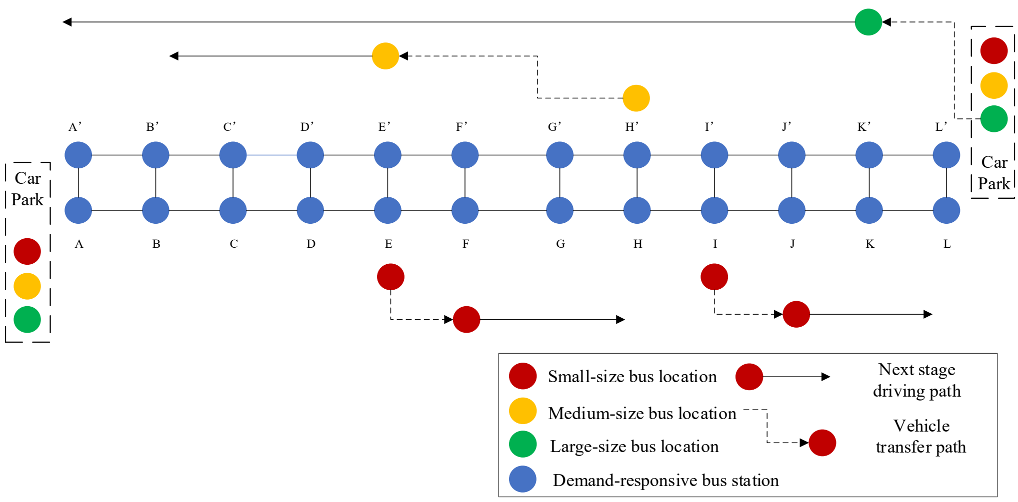

Figure 2. Schematic diagram of real-time vehicle scheduling problem.

It can be seen that the real-time vehicle scheduling problem is essentially a one-to-one matching problem between vehicles and departure demand; that is, based on satisfying various constraints, one-to-one matching between available vehicles staying in various places and departure demand points, to ensure the smooth execution of the transportation task, and finally minimize the total cost of the matching scheme. Therefore, this practical problem can be abstractly transformed into a bipartite graph-matching problem. As shown in Figure 3, where the circle represents the available vehicles, and the triangle represents the departure demand point. The value of each edge is the weight of the edge. The size should be taken as the cost that the vehicles connected at both ends of the edge need to consume after completing the corresponding vehicle task. The set of solid edges in the figure represents a match, that is, a possible vehicle scheduling scheme.

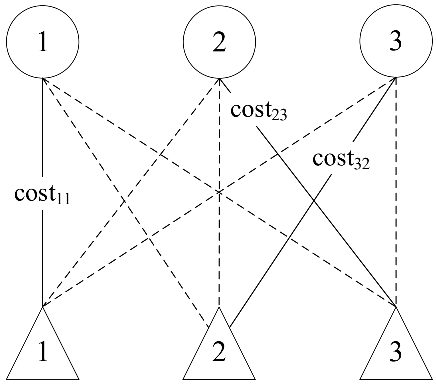

Figure 3. Bipartite graph transformation of graph real-time vehicle scheduling problem.

3.2. Assumptions and Parameter Descriptions

3.2.1. Assumptions

This section mainly discusses the problem of how to reasonably match the task of a vehicle at a certain time and the nearby transport capacity. According to the characteristics of this problem, an optimization model can be established based on matching theory by considering factors such as the origin and destination of each vehicle task and different vehicle types. To better describe the model, this paper makes the following assumptions:

(1) Before vehicle dispatching, the task of the next departure time is known, including the departure time, departure station, final arrival station, and required vehicle type;
(2) The vehicles are all dispatched by the dispatch center, and the model, location information, and passenger status of each vehicle are known;
(3) Different types of vehicles have the same basic conditions except for the different passenger capacities;
(4) The driving distance between each station is known;
(5) To ensure the service level of the system, it is necessary to ensure that the number of vehicles of each model in the system is sufficient;
(6) After the previous task of the vehicle is over, it stays at the terminal of the previous task and waits for dispatch;
(7) During the actual operation, the vehicle is allowed to turn around;
(8) The impact on vehicle dispatch due to emergencies and other reasons will not be considered for the time being.

3.2.2. Parameter Descriptions

According to the above assumptions, L is a set of sites and K is a set of vehicle types. Consider a scenario consisting of departure demands M and available vehicles N. At the departure time point T, the dispatch center aggregates the travel reservation demand of passengers at this moment, a total of trips are generated, and the trip set M is recorded as $V = \{V_1, V_2, \cdots, V_m, \cdots, V_M\}$, among them $M \geq 2$. And for T any vehicle task that needs to be dispatched at any time $V_m = \left\{l_m^o, l_m^d, k_m\right\}$, where l_m^o, l_m^d corresponding the starting station and the terminal station of the vehicle task respectively, k_m indicates the vehicle type required by the vehicle task.

In terms of vehicle resources, T is the set of vehicles available in the system at the departure time is $C = \{C_1, C_2, \cdots, C_n, \cdots, C_N\}$, among them $N \geq 2$. For any available vehicle C_n, there is $C_n = \{l_n, k_n\}$, where l_n it indicates the initial station position of the vehicle before transfer, and k_n represents the information of the corresponding vehicle type. On the basis of determining the demand and available resource set, and considering the operating cost, the matching optimization of vehicles and trips can be carried out. The parameter symbols and specific descriptions used in the model building process are shown in Table 1 below 0.

Table 1. Symbol explanation table.

Symbol	Symbol Description
T	Determine the departure time
V_m	The task of the vehicle numbered m
C_n	Idle vehicle numbered n
L	Collection of sites
l_m^o	The starting site of the vehicle task m
l_m^d	Terminal of vehicle number m
l_n	Initial station where the idle vehicle n is located
s_{mn}	The distance between the starting station of the vehicle task m and the initial station of the vehicle n
s_m^{od}	The starting and ending distance of the vehicle task m
t_{mn}	The vehicle n to move to the starting station of the vehicle task m when it is *empty*
v	Average vehicle speed
K	Vehicle types, $K = \{1,2,3\}$ respectively representing small-sized bus, Medium-sized bus, and large-sized bus
k_m	The vehicle type required for $k_m \in K$ the task m,
k_n	Vehicle n, $k_n \in K$
a_k	K -type vehicle unit distance running cost of an empty vehicle, that is, the cost of empty driving
b_k	K -type vehicle unit distance carrying passenger operating cost
α	Maximum transfer time allowed
x_{mn}	Decision variable, whether the vehicle task m matches the vehicle n

3.3. Mathematical Model Building

The globally optimized vehicle scheduling scheme is characterized by the minimum operating cost of the system, which includes both the passenger-carrying cost of the vehicle to perform the task, and the cost of empty driving when the vehicle is transferred between different vehicle tasks. The objective function of the model is shown in Function (1):

$$\min Z_r = \sum_{m \in M} \sum_{n \in N} (a_{k_n} s_{mn} + b_{k_n} s_m^{od}) x_{mn} \tag{1}$$

$$\sum_{m \in M} x_{mn} \le 1 \ \forall n \in N \tag{2}$$

$$\sum_{n \in N} x_{mn} = 1 \ \forall m \in M \tag{3}$$

$$(k_n - k_m) x_{mn} \ge 0 \ \forall m \in M, \ \forall n \in N \tag{4}$$

$$t_{mn} x_{mn} \le \alpha \ \forall m \in M, \forall n \in N \tag{5}$$

$$x_{mn} = \begin{cases} 1 \ n \text{ match } m \\ 0 \ \text{ otherwise} \end{cases} \tag{6}$$

In the model, the objective Function (1) represents a certain departure time point, and the total operating cost of the system is the smallest. The total cost consists of two parts. The first half represents the cost of empty driving during the vehicle transfer process, and the second half represents the vehicle. The cost of carrying passengers to complete the task of the trip. Equation (2) means that any available vehicle can only be matched with one trip task or no matching trip task. Equation (3) means that any trip task can and can only be performed by one vehicle. Equation (4) indicates that the vehicle type must meet the requirements of the vehicle type required for the mission. Due to the different capacity of different types of vehicles, large vehicles can replace small vehicles to perform vehicle tasks, while small vehicles cannot replace large vehicles. Equation (5) indicates that the time for any vehicle to transfer from the initial station position to the starting station of the next vehicle task cannot exceed the maximum allowable transfer time, otherwise it cannot meet the punctual departure requirements. Equation (6) indicates that the decision variable x_{mn} is a 0–1 variable. When x_{mn} = 1, it means that the vehicle task m *matches* the vehicle n, that is, the vehicle task m is executed by the vehicle n; when x_{mn} = 0, it means the vehicle task m does not match with vehicle n. The vehicle task number m is not performed by vehicle n.

At the same time, the distance parameter in the model is determined by the location of the stations, and the distance between the stations can be obtained by knowing the locations of the two stations; that is, the distance between the stations is a function of the location of the stations, so:

$$s_m^{od} = F(l_m^o, l_m^d) \tag{7}$$

$$s_{mn} = F(l_m^d, l_n) \tag{8}$$

The distance parameter in this model can use the actual distance data between stations, which can effectively guarantee the accuracy. In addition, the time parameter t_{mn} in the model represents the time required for the vehicle n to move to the starting station of the vehicle task m without empty driving. Its value is calculated by using the driving distance and the average speed of the vehicle, that is:

$$t_{mn} = \frac{s_{mn}}{v} \tag{9}$$

3.4. Principle of Optimal Matching and Kuhn–Munkres Algorithm

3.4.1. Overview of the Optimal Matching Problem

The matching problem is one of the common basic problems in human society, which can be used to describe the problems of selecting representatives for committees, the allocation of personnel and work tasks, etc. The matching problem in graph theory is also called the system of distinct representatives in the field of combinatorics. It can be described in the language of sets as: given a set family $A = \{A_1, A_2, A_3, \cdots, A_n\}$, if in the collection A_i one element is selected from each $a_i \in A_i$ so that $(a_1, a_2, a_3, \cdots, a_n)$, it is different from each other. The different elements n can be called $a = \{a_1, a_2, a_3, \cdots, a_n\}$ and is a different representative system of A.

But in most cases, modeling the matching problem as a bipartite graph is easier to analyze and solve than using a non-bipartite graph. Therefore, many matching problems in practical applications are solved by converting them into bipartite graphs. Therefore, in order to facilitate the understanding of bipartite graphs and optimal matching theory, some basic concepts involved will be introduced in this article. Figure 4 is a schematic diagram of a simple bipartite graph, where (a,b) can be called a bipartite graph.

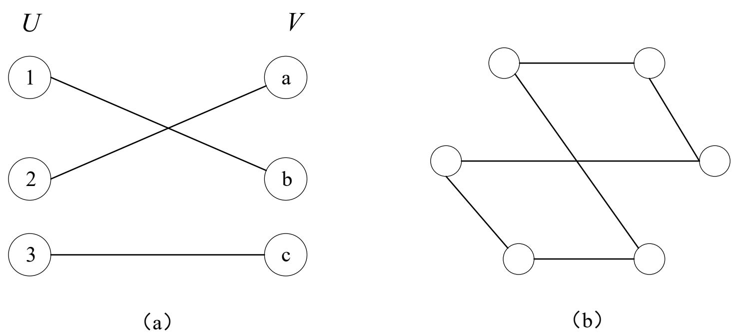

Figure 4. Bipartite graph graphical representation. (**a**,**b**) bipartite graph.

If the edge set in a bipartite graph is used to represent the connection relationship between two vertex sets and has a representative value, the value is called the weight of the edge. This kind of bipartite graph is a weighted bipartite graph. The weight of an edge is generally a real number, which can be positive or negative. When the weight is positive, it is generally used to represent the distance between two nodes in practical problems or the maximum capacity that a line can bear. If every pair of vertices in a bipartite graph can and can only be connected by a unique edge, then the graph is called a complete bipartite graph, also known as a complete bipartite graph.

3.4.2. Optimal Matching Theory

Matching is one of the important concepts in a bipartite graph, also known as an independent edge set. It is a subset of the edge set of a bipartite graph. The edges in this subset all have certain common characteristics. This feature is the difference between any two edges in the set. There is no common vertex between them. Under this feature, each edge will connect and match two vertices belonging to different vertex subsets, and the number of edges connected from each vertex is at most one. Therefore, matching can be defined as, in a bipartite graph $G = (U, V, E)$. If there is a subgraph $M \subseteq G$, and any two sides in M are not associated with the same vertex, it is called M is a matching of G, as shown by the solid line in Figure 5a is a match. It defines matching points, unmatched points, matching edges, and unmatched 0 edges. For matching, the following core concepts are often involved:

(1) Maximum matching

The matching when the number of matching edges of the subgraph is the largest is called the maximum matching, as shown in Figure 5b is a maximum matching of the bipartite graph, and the matching number of the graph refers to the number of edges in the maximum matching. The maximum matching characteristics are as follows:

a. It has the largest number of matching edges;
b. There may be many different matching methods, but the maximum number of matching sides is the same and certain;
c. It cannot have more than half the number of edges than the number of vertices in the graph.

The reason why the maximum matching has such characteristics is determined by the definition of matching. From the definition of matching, the probability that two edges connecting vertices are exactly the same is 0 in the matching. This situation will only occur when two edges are adjacent. There is a concept of maximal matching. It means that if M is a match of G, and any addition of an unmatched edge in it will cause the new edge subset to no longer be a match, that M is called a maximal matching. The maximum matching must also be the maximal matching, but the maximal matching is not necessarily the maximum matching.

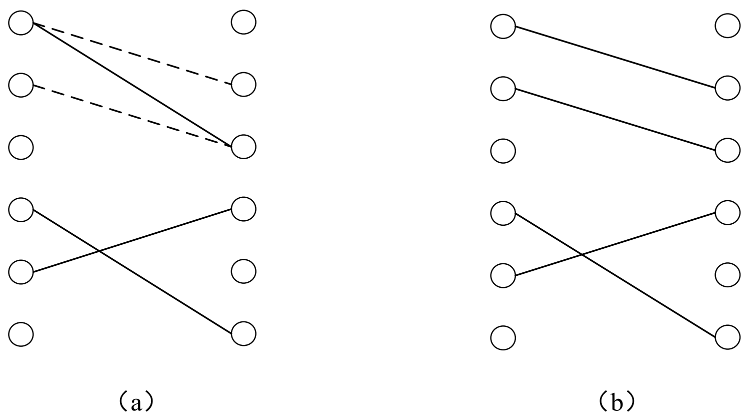

Figure 5. (**a**,**b**) Graph matching and maximum matching.

(2) Weighted maximum matching

Weighted maximum matching, also known as optimal matching, means that in a weighted bipartite graph, the edge i weight is set l_i to find a match that maximizes M the weight and value of $\sum l_i$ all matching edges $i \in M$. Similarly, if you need to solve the minimum weight matching, you only need to invert all the original weight values. Generally, when U the V number of vertices in the set is the same, the optimal matching will also be a perfect matching, that is, each vertex is a matching point. If the number of vertices in the two vertex sets is not equal, the conversion can be completed by filling points and adding zero-valued edges.

(3) Alternating path

Alternating path is a path which takes an unmatched point in the bipartite graph as the starting point, and passing through non-matching edges, matching edges, non-matching edges, etc. in turn. The solid points in Figure 6 are matching points, and the hollow points are unmatching points. The solid line is the matching edge. The dotted line is the unmatching edge. The path *d-4-c-1* can be called an alternating path.

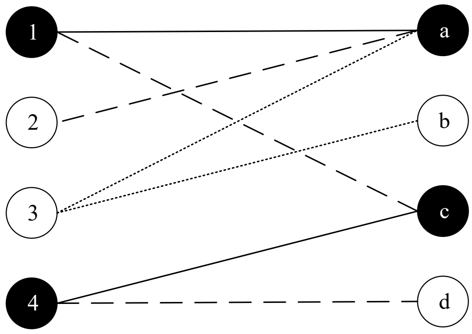

Figure 6. Graph path.

(4) Augmenting path

In a bipartite graph $G = (U, V, E)$, If M is a match of G, and a path P connecting two unmatched vertices which are respectively located in the subset U and V, and matching edges and unmatched edges appear alternately in this path, P is called an augmenting path of M. It can be seen from its definition that the path length of the augmenting path P must be an odd number, and the number of unmatched edges must be more than the number of matched edges. In simple terms, an augmented path is a path starting from an unmatched point, taking an alternate path and passing through another unmatched point. As shown in Figure 6, path *d-4-c-1-a-2* is a path augmenting path. The augmentation operation is a common operation used to solve the maximum matching problem of a bipartite graph. In set theory, this operation is also called a symmetric difference operation, and the symbol is expressed as $M \oplus P = (M \cup P) - (M \cap P)$. If and only if there is no augmenting path among them, M is the maximum matching of G.

3.4.3. Basic Principles of the Kuhn–Munkres Algorithm

The KM algorithm takes the Hungarian algorithm as the starting point. It introduces concepts such as feasible top marks and equal subgraphs to obtain the optimal match. The core idea of the algorithm is to repeatedly update the feasible top mark value on the equal subgraph in a loop to increase the number of feasible edges and expand the number of matches until a complete match can be obtained in the updated equal subgraph, then the optimal match of the graph can be obtained.

For any weighted bipartite graph $G = (U, V, E)$, the basic flow of the KM algorithm is as follows:

Step 1: Initialize the feasible top mark. Use $l(u_i)$ and $l(v_j)$ represent the top mark value of the vertex set U and V, and w_{ij} is the weight of the edge e_{ij} connecting the point u_i and v_j. During the execution of the algorithm, it is required that $l(u_i) + l(v_j) \geq w_{ij}$ for any edge e_{ij}. Initial value of $l(u_i)$ is the maximum weight of the associated edge of u_i, and $l(v_j) = 0$.

Step 2: Determine the equal subgraph G_l according to the top mark value, and use the Hungarian algorithm to find G_l the maximum matching of M' the equal subgraph G_l. If it is M' a complete match, it M' is the optimal match, and the calculation ends, otherwise go to step 3.

Step 3: Calculate the update amount d and update the top mark value, expand the feasible edge, and return to step 2.

$$d = \min_{u_i \in S, v_j \notin B} \{ l(u_i) + l(v_j) - w_{ij} \} \tag{10}$$

where S is the set of vertices that have been visited on the alternating path U, and B is the set of vertices that have been visited on the alternating path V.

3.4.4. Solution Process

In this paper, the real-time vehicle scheduling model is transformed into the optimal matching of a bipartite graph, and the solution is based on the Kuhn–Munkres algorithm. The specific process of the optimization algorithm is as follows:

Step 1: The algorithm starts and parameters are initialized.

To establish the task information table for vehicles, use the vehicle information table and the distance table between stations, and set the relevant parameter values.

Step 2: Calculate and generate the matching judgment matrix between the trip task and the available vehicles.

According to constraints such as vehicle model restrictions and maximum transfer time limits allowed in the model, it is judged whether any task can be matched with a vehicle, and a 0–1 matrix is generated. The value is 1 when the task and the vehicle can be matched, and the value is 0 when they cannot be matched.

Step 3: Calculate and generate task-vehicle matching weighted matrix.

According to the model objective function, the cost value required for the matchable task and vehicle matching is calculated, and used as the matching weight between two points to form a matching weighted matrix.

Step 4: Kuhn–Munkres algorithm for optimal matching.

Take the opposite number for the matching weight, and use the KM algorithm to solve the optimal matching.

Step 5: Output the optimal matching scheme and the minimum weight.

The optimal matching is output to form the optimal vehicle scheduling scheme, and the weighted sum is reversed again to determine the minimum cost of vehicle scheduling.

4. Case Studies

4.1. Introduction to Calculation Examples

The Shanghai-Yihai Highway (Chenxiang Road-Yecheng Road Section) demonstration line has a total length of 8.2 km, and the entire line adopts a point-to-point direct operation mode. Passengers initiate a demand reservation online from the app or small program, select the departure time and travel start and end stations, collect the reservation request information through the passenger service system, and send the reservation request to the background dispatching system after unified arrangement. The dispatching system generates a departure plan the driving route is determined, and the corresponding vehicles are dispatched to complete the transportation task on time.

There are a total of 12 demand-responsive bus stops on the demonstration line. According to the existing conditions of the line, both ends of the test line are set as depots, that is, the surrounding area of Zhaoxian Road of Huyi Highway and Chenxiang Road of Huyi Highway.

In this paper, based on the information displayed on the electronic map, the actual distance between each station is obtained. Taking Zhaoxian Road of Huyi Highway as the first station, the distance between each station and the first station is shown in Table 2.

Demonstration online demand-responsive buses run point-to-point directly between stations according to passenger reservation needs, without stopping in between, using bus-only lanes and supplemented by signal green waves when driving, with an average operating speed of 60 km/h. Three types of vehicles are temporarily used during the operation of the line. By consulting relevant information and combining with the actual situation, considering vehicle depreciation, the cost of each vehicle type is set as shown in Table 3.

Table 2. Station distance.

Station Number	Station Name	Coordinates (m)
01	Zhaoxian Road, Huyi Highway	0
02	Hongde Road, Huyi Highway	587
03	Baiyin Road, Huyi Highway	1117
04	Shigang	1683
05	Yining Road, Huyi Highway	2215
06	Huyi Highway Hope Road	2696
07	Shuangdan Road, Huyi Highway	3336
08	Malu Town	3896
09	Huyi Highway Baoan Highway	4540
10	Yagang Road, Huyi Highway	5393
11	Daqiaotou	7093
12	Chenxiang Road, Huyi Highway	7745

Table 3. Vehicle data.

Vehicle	Number of Seats	Vehicle Type	Fixed Cost	Passenger Operating Cost	Empty Operation Cost
2020 SAIC Roewe Ei5	5 seats	Small-sized bus	36 CNY/vehicle	1.35 CNY/km	1.30 CNY/km
2021 Maxus EV90	15 seats	Medium-sized bus	40 CNY/vehicle	1.9 CNY/km	1.85 CNY/km
Yancheng Brand HYK6700YBEV	23 seats	Large-sized bus	55 CNY/vehicle	2.1 CNY/km	2.05 CNY/km

4.2. Generation of Vehicle Tasks Based on Real-Time Demand

The relevant model parameters are set based on the basic data of the demonstration line stations and the information of the operating vehicles. Since the operational data of the demonstration line is not yet complete, the available vehicles and the number of task data in the calculation example are randomly generated within a reasonable range. Considering the status quo of line operation, 30 vehicle tasks at a certain departure time were randomly generated in the test, and the maximum allowable transfer time was set at 5 min. The randomly generated vehicle task information is shown in Table 4.

Table 4. Route based on real-time demand.

Task Number	Starting Station	Terminal Station	Required Vehicle
1	Hongde Road, Huyi Highway	Malu Town	Large-sized bus
2	Baiyin Road, Huyi Highway	Shigang	Small-sized bus
3	Zhaoxian Road, Huyi Highway	Shuangdan Road, Huyi Highway	Medium-sized bus
4	Yagang Road, Huyi Highway	Huyi Highway Baoan Highway	Large-sized bus
5	Shuangdan Road, Huyi Highway	Yagang Road, Huyi Highway	Medium-sized bus
6	Chenxiang Road, Huyi Highway	Baiyin Road, Huyi Highway	Medium-sized bus
7	Malu Town	Baiyin Road, Huyi Highway	Large-sized bus
8	Baiyin Road, Huyi Highway	Daqiaotou	Large-sized bus
9	Zhaoxian Road, Huyi Highway	Baiyin Road, Huyi Highway	Small-sized bus
10	Yagang Road, Huyi Highway	Huyi Highway Hope Road	Large-sized bus
11	Huyi Highway Hope Road	Hongde Road, Huyi Highway	Small-sized bus
12	Shigang	Huyi Highway Baoan Highway	Small-sized bus
13	Zhaoxian Road, Huyi Highway	Huyi Highway Baoan Highway	Small-sized bus
14	Malu Town	Shigang	Medium-sized bus
15	Shuangdan Road, Huyi Highway	Zhaoxian Road, Huyi Highway	Medium-sized bus
16	Hongde Road, Huyi Highway	Yagang Road, Huyi Highway	Medium-sized bus
17	Daqiaotou	Daqiaotou	Small-sized bus
18	Daqiaotou	Yining Road, Huyi Highway	Large-sized bus

Table 4. *Cont.*

Task Number	Starting Station	Terminal Station	Required Vehicle
19	Huyi Highway Hope Road	Zhaoxian Road, Huyi Highway	Large-sized bus
20	Huyi Highway Hope Road	Shuangdan Road, Huyi Highway	Large-sized bus
21	Baiyin Road, Huyi Highway	Daqiaotou	Large-sized bus
22	Zhaoxian Road, Huyi Highway	Baiyin Road, Huyi Highway	Small-sized bus
23	Huyi Highway Baoan Highway	Hongde Road, Huyi Highway	Medium-sized bus
24	Shigang	Malu Town	Small-sized bus
25	Baiyin Road, Huyi Highway	Huyi Highway Baoan Highway	Small-sized bus
26	Huyi Highway Hope Road	Baiyin Road, Huyi Highway	Medium-sized bus
27	Huyi Highway Baoan Highway	Huyi Highway Hope Road	Medium-sized bus
28	Shigang	Hongde Road, Huyi Highway	Large-sized bus
29	Daqiaotou	Hongde Road, Huyi Highway	Medium-sized bus
30	Shuangdan Road, Huyi Highway	Shuangdan Road, Huyi Highway	Small-sized bus

50 operating vehicles are randomly generated according to a certain ratio, and the initial positions of empty vehicles are randomly determined. The quantity information of each model is shown in Table 5.

Table 5. The number of vehicles.

Vehicle Type	Quantity
Small-sized bus	13
Medium-sized bus	20
Large-sized bus	17

4.3. Result Analysis

Figure 7 shows the task-vehicle bipartite graph matching scheme obtained by using the KM algorithm in the real-time vehicle scheduling scenario. The upper point C_n in the figure represents each available vehicle, the lower point V_m represents different vehicle tasks, and the red line in the middle indicates that the task-vehicle is successfully matched. The vehicle undertakes the corresponding vehicle task. The running time of the algorithm is 1.339 s, which meets the requirements of real-time vehicle scheduling in reality. At this departure time, the total operating cost of the vehicle scheduling scheme is 177.67 CNY.

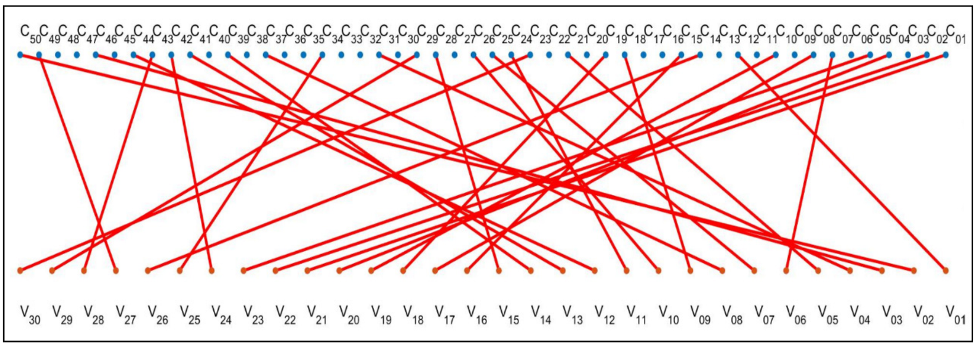

Figure 7. Graph of KM algorithm bipartite graph matching scheme.

At present, the demonstration line adopts a relatively common vehicle scheduling idea based on a greedy strategy and does not consider vehicle model substitution. The advantage of this idea is that it is simple in logic, easy to implement, and fast in matching speed. It can give relevant feedback in time when there are a large number of vehicle tasks. Table 6 shows the specific comparison results of the matching schemes under the KM algorithm and the greedy strategy.

Table 6. Comparison of matching schemes.

Task Number	KM Algorithm Matching Scheme		Greedy Algorithm Matching Scheme	
	Vehicle Number	Type of Vehicle	Vehicle Number	Type of Vehicle
1	12	Large-sized bus	12	Large-sized bus
2	50	Large-sized bus	34	Small-sized bus
3	46	Medium-sized bus	2	Medium-sized bus
4	31	Large-sized bus	26	Large-sized bus
5	21	Large-sized bus	39	Medium-sized bus
6	7	Medium-sized bus	7	Medium-sized bus
7	25	Large-sized bus	19	Large-sized bus
8	37	Large-sized bus	1	Large-sized bus
9	18	Medium-sized bus	41	Small-sized bus
10	26	Large-sized bus	31	Large-sized bus
11	24	Small-sized bus	42	Small-sized bus
12	44	Small-sized bus	44	Small-sized bus
13	41	Small-sized bus	23	Small-sized bus
14	39	Medium-sized bus	49	Medium-sized bus
15	28	Medium-sized bus	14	Medium-sized bus
16	15	Medium-sized bus	15	Medium-sized bus
17	8	Small-sized bus	8	Small-sized bus
18	19	Large-sized bus	25	Large-sized bus
19	10	Large-sized bus	4	Large-sized bus
20	4	Large-sized bus	10	Large-sized bus
21	1	Large-sized bus	37	Large-sized bus
22	2	Medium-sized bus	24	Small-sized bus
23	5	Medium-sized bus	5	Medium-sized bus
24	42	Small-sized bus	33	Small-sized bus
25	34	Small-sized bus	13	Small-sized bus
26	14	Medium-sized bus	28	Medium-sized bus
27	49	Medium-sized bus	45	Medium-sized bus
28	43	Large-sized bus	43	Large-sized bus
29	29	Medium-sized bus	29	Medium-sized bus
30	23	Small-sized bus	17	Small-sized bus
Total operating cost (CNY)	177.67		195.34	

Comparing the two results in the table, it can be seen that the method proposed in this paper can reduce the operating cost of a single departure from 195 yuan to 178 yuan under the same calculation example. This is because, firstly, the strategy of the greedy algorithm is to seek a global feasible solution from a local optimal solution. However, when the number of tasks is large and the number of vehicles is small, it is difficult to achieve a global optimum, and even a global solution with poor quality may appear. Secondly, compared with the algorithm in this paper, the greedy algorithm does not take into account the vehicles staying at the intermediate station. In the actual scheduling process, a certain vehicle may stay at the station for a long time without being called, which can increase the number of actual vehicles in the system and waste vehicle resources. Thirdly, when looking for a matching vehicle, only the model that matches the passenger capacity is considered, and the situation where a model with a large passenger capacity replaces a model with a small passenger capacity is not considered. This replacement may reduce the number of operating vehicles required and save the transfer cost generated in the process of vehicle transfer, thereby reducing the total cost of the system.

Moreover, under existing thinking, the replacement of small models by large models is not considered when looking for matching vehicles. However, such replacement can fully mobilize available vehicle resources and reduce the cost of empty driving in real-time scheduling scenarios. According to the results of the calculation example, considering that the demonstration line will depart every 10 min in the future, and the daily operating time is more than 8 h, the single-day operating cost can save nearly 1000 yuan, indicating significant benefits. Therefore, the model and algorithm adopted in this paper can effectively reduce system operating costs, achieve global optimal vehicle scheduling, and improve the utilization of global resources in real-time vehicle scheduling scenarios.

5. Conclusions

Demand-responsive transit systems offer a promising solution to improve urban transport efficiency and reduce costs, congestion, and pollution. This paper's research results mainly include four aspects. Firstly, this research reviews and summarizes demand-responsive transit system and vehicle scheduling problem research, explains the operation characteristics and development advantages of the system, and outlines the scope of the vehicle scheduling problem. Secondly, this paper addresses the real-time vehicle scheduling mode where the system stops passenger reservations before departure, and the complete passenger demand can only be determined before departure. To address this, the vehicle scheduling problem is transformed into an optimal matching problem of a bipartite graph. An integer programming model is then established, considering multi-vehicle coordination and vehicle scheduling time constraints with a minimum total operating cost. Thirdly, the Kuhn–Munkres algorithm is designed to solve the mathematical model, and global optimization of the real-time vehicle scheduling scheme is realized. Fourthly, this paper validates the proposed model by testing it on a typical line of demand-responsive public transport, the Huyi Highway Demonstration Line in Shanghai, using data from lines, stations, and demands. Through a two-step algorithm, the proposed model is validated and shown to be advanced in optimizing vehicle scheduling and reducing operating costs.

This paper has achieved certain research results in the field of customized bus scheduling methods based on passenger reservations. However, future research can be improved and perfected further in the following aspects:

(1) During the actual scheduling process, vehicles have limited cruising range, and there may be situations such as charging and refueling halfway that could impact the formulation of the vehicle scheduling plan. These factors should be considered in the follow-up research process to make the model more reliable and comprehensive.

(2) Future research can investigate the impact of external factors such as weather, traffic, and accidents on vehicle scheduling and optimize the scheduling plan accordingly.

(3) Another potential avenue for future research is to consider the vehicle recycling process and reduce the transition time between tasks, ultimately realizing the optimization of vehicle scheduling within a specific period.

Author Contributions: Conceptualization, X.Z. and G.W.; methodology, G.W.; software, G.W.; validation, X.Z., G.W.; formal analysis, Y.Z.; investigation, Q.W., H.G.; resources, H.G.; data curation, X.Z.; writing—original draft preparation, X.Z., G.W.; writing—review and editing, G.W.; visualization, Y.Z.; supervision, X.Z.; project administration, X.Z.; funding acquisition, X.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Natural Science Foundation of China (No. 61873190).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Acknowledgments: The research of the first author Xuemei Zhou is supported by the Natural Science Foundation of China (No.61873190).

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

References

1. Nie, Q.; Zhang, L.; Tong, Z.; Hubacek, K. Strategies for applying carbon trading to the new energy vehicle market in China: An improved evolutionary game analysis for the bus industry. *Energy* **2022**, *259*, 124904. [CrossRef]
2. Huang, D.; Tong, W.; Wang, L.; Yang, X. An analytical model for the many-to-one demand responsive transit systems. *Sustainability* **2019**, *12*, 298. [CrossRef]
3. Saxena, N.; Rashidi, T.; Rey, D. Determining the market uptake of demand responsive transport enabled public transport service. *Sustainability* **2020**, *12*, 4914. [CrossRef]
4. Wang, Y.; Zhang, D.; Hu, L.; Yang, Y.; Lee, L.H. A data-driven and optimal bus scheduling model with time-dependent traffic and demand. *IEEE Trans. Intell. Transp. Syst.* **2017**, *18*, 2443–2452. [CrossRef]
5. Perumal, S.S.G.; Lusby, R.M.; Larsen, J. Electric bus planning & scheduling: A review of related problems and methodologies. *Eur. J. Oper. Res.* **2022**, *301*, 395–413.
6. Zhou, X.; Xi, J.; Guan, Z.; Ji, X. Dispatching Design for Customized Bus of Hybrid Vehicles Based on Reservation Data. *J. Adv. Transp.* **2021**, *2021*, 8868291. [CrossRef]
7. Dakic, I.; Yang, K.; Menendez, M.; Chow, J.Y.J. On the design of an optimal flexible bus dispatching system with modular bus units: Using the three-dimensional macroscopic fundamental diagram. *Transp. Res. Part B Methodol.* **2021**, *148*, 38–59. [CrossRef]
8. Sadrani, M.; Tirachini, A.; Antoniou, C. Vehicle dispatching plan for minimizing passenger waiting time in a corridor with buses of different sizes: Model formulation and solution approaches. *Eur. J. Oper. Res.* **2022**, *299*, 263–282. [CrossRef]
9. Yan, X.; Zhao, X.; Han, Y.; Van Hentenryck, P.; Dillahunt, T. Mobility-on-demand versus fixed-route transit systems: An evaluation of traveler preferences in low-income communities. *Transp. Res. Part A Policy Pract.* **2021**, *148*, 481–495. [CrossRef]
10. Liu, T.; Ceder, A.A. Analysis of a new public-transport-service concept: Customized bus in China. *Transp. Policy* **2015**, *39*, 63–76. [CrossRef]
11. Li, J.; He, Z.; Zhong, J. The Multi-Type Demands Oriented Framework for Flex-Route Transit Design. *Sustainability* **2022**, *14*, 9727. [CrossRef]
12. Guo, R.; Guan, W.; Zhang, W. Route design problem of customized buses: Mixed integer programming model and case study. *J. Transp. Eng. Part A Syst.* **2018**, *144*, 04018069. [CrossRef]
13. Zhang, J.; Li, W.; Wang, G.; Yu, J. Feasibility Study of Transferring Shared Bicycle Users with Commuting Demand to Flex-Route Transit—A Case Study of Nanjing City, China. *Sustainability* **2021**, *13*, 6067. [CrossRef]
14. Daganzo, C.F. Checkpoint dial-a-ride systems. *Transp. Res. Part B* **1984**, *18*, 315–327. [CrossRef]
15. Cortés, C.E.; Jayakrishnan, R. Design and Operational Concepts of High-Coverage Point-to-Point Transit System. *Transp. Res. Record* **2002**, *1783*, 178–187. [CrossRef]
16. Wang, H.; Li, J.; Wang, P.; Teng, J.; Loo, B.P.Y. Adaptability analysis methods of demand responsive transit: A review and future directions. *Transp. Rev.* **2023**, 1–22. [CrossRef]
17. Kim, M.E.; Schonfeld, P. Maximizing net benefits for conventional and flexible bus services. *Transp. Res. Part A* **2015**, *80*, 116–133. [CrossRef]
18. Fang, Y.; Hu, X.; Wu, L.; Miao, Y. *A Real-Time Scheduling Method for a Variable-Route Bus in a Community[C]//Advances in Intelligent Decision Technologies: Proceedings of the Second KES International Symposium IDT 2010*; Springer: Berlin/Heidelberg, Germany, 2010; pp. 239–247.
19. Kim, M.E.; Levy, J.; Schonfeld, P. Optimal zone sizes and headways for flexible-route bus services. *Transp. Res. Part B Methodol.* **2019**, *130*, 67–81.
20. Schofer, J.L.; Nelson, B.L.; Eash, R.; Daskin, M.; Wan, Y.; Yan, J.; Medgyesy, L. Resource Requirements for Demand-Responsive Transportation Services. In *Transportation Research Board*; 2003; Available online: https://books.google.co.kr/books?hl=zh-CN&lr=&id=RG9wnNBKCy4C&oi=fnd&pg=PA1&dq=Resource+Requirements+for+Demand-Responsive+Transportation+Services&ots=vFhed40Vrg&sig=BH2eRBLY3xfUA514V41LRlfWuC4&redir_esc=y#v=onepage&q=Resource%20Requirements%20for%20Demand-Responsive%20Transportation%20Services&f=false (accessed on 12 February 2023).
21. Qiu, F.; Li, W.; Zhang, J. A dynamic station strategy to improve the performance of flex-route transit services. *Transp. Res. Part C* **2014**, *48*, 229–240. [CrossRef]
22. Gorev, A.; Popova, O.; Solodkij, A. Demand-responsive transit systems in areas with low transport demand of "smart city". *Transp. Res. Procedia* **2020**, *50*, 160–166. [CrossRef]
23. Mulley, C.; Nelson, J.; Teal, R.; Wright, S.; Daniels, R. Barriers to implementing flexible transport services: An international comparison of the experiences in Australia, Europe and USA. *Res. Transp. Bus. Manag.* **2012**, *3*, 3–11. [CrossRef]
24. Errico, F.; Crainic, T.G.; Malucelli, F.; Nonato, M. A survey on planning semi-flexible transit systems: Methodological issues and a unifying framework. *Transp. Res. Part C* **2013**, *36*, 324–338. [CrossRef]
25. Diana, M.; Dessouky, M.M.; Xia, N. A model for the fleet sizing of demand responsive transportation services with time windows. *Transp. Res. Part B Methodol.* **2006**, *40*, 651–666. [CrossRef]
26. Yao, E.; Liu, T.; Lu, T.; Yang, Y. Optimization of electric vehicle scheduling with multiple vehicle types in public transport. *Sustain. Cities Soc.* **2020**, *52*, 101862. [CrossRef]
27. Bie, Y.; Tang, R.; Wang, L. Bus scheduling of overlapping routes with multi-vehicle types based on passenger OD data. *IEEE Access* **2019**, *8*, 1406–1415. [CrossRef]

28. Hu, B.; Fu, Y.; Feng, S. Integrated optimization of multi-vehicle-type timetabling and scheduling to accommodate periodic passenger flow. In *Computer-Aided Civil and Infrastructure Engineering*; Wiley: Online, 2023.
29. Guedes, P.C.; Borenstein, D. Real-time multi-depot vehicle type rescheduling problem. *Transp. Res. Part B Methodol.* **2018**, *108*, 217–234. [CrossRef]