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Development of Performance Measurement Models for Two-Lane Roads under Vehicular Platooning Using Conjugate Bayesian Analysis

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Abstract: Vehicular platooning is one of the most challenging issues affecting the level of service (LOS) of two-lane roads. This phenomenon has been involved with variables governing performance measures. Thus, to improve the quality of these roads and predict a comprehensive model for future plans under this phenomenon, the present study aimed to evaluate the effect of vehicular platooning variables on performance measures and then identify the critical headways of vehicular platooning associated with the vehicle-gap-acceptance behavior. Multiple linear regression (MLR) and Bayesian linear regression (BLR) models were used to develop performance measurement models that are based on conjugate Bayesian analysis. The vehicular platooning was formed in the threshold of a time headway of 2.4 s. According to a comparative evaluation of the developed models, the best predictive model was found between the traffic flow and the number of followers per capacity (NFPC). In addition, the BLR model showed a higher accuracy rate in predicting NFPC compared with the MLR model due to low errors and high prediction performance. Thus, NFPC was introduced as a surrogate performance measure, which had a premier capability to predict the LOS for unsaturated and saturated traffic conditions compared with the two performance measures from the Highway Capacity Manual (2010), including percent time spent following and average travel speed.

Keywords: two-lane roads; vehicular platooning; performance measures; level of service; Bayesian approach

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1. Introduction

Two-way two-lane rural roads play important roles in road transportation facilities. They serve two primary purposes of transportation and accessibility thanks to their uninterrupted traffic-flow facilities and no limitation on the movement of vehicles along the roads. According to the Iranian Road Maintenance and Transportation Organization [1], two-way two-lane rural roads account for approximately 30% of the network roads in Iran. Owing to the complex interaction of vehicles in one direction and the opposite direction, measuring traffic performance on two-lane roads is a challenging issue for traffic engineers and managers. Further, because two-lane roads are the essential roads in rural districts, platooning formation from the high percentage of heavy vehicles, the higher likelihood of overtaking and lane changes is widespread in comparison with other urban roads [2–5]. On urban roads, owing to the high speed and the high number of lanes, the formation of vehicular platooning is relatively lower than that on rural roads, such as two-lane roads [6]. Given the complexity of this phenomenon, traffic engineers frequently look for methods and measures to define the traffic capacity and quality of traffic flows for two-lane roads. According to some studies [7–11], vehicular platooning significantly affects the quality of the operational performance of two-lane roads, including the average travel speed (ATS) and capacity, leading to a reduction in the level of service (LOS). When platoons form in two-lane roads, variables that contribute to performance measures affect the quality of traffic flow and, accordingly, decrease operational performance [3,12–16]. Consequently, reducing the quality of the traffic flow and operational performance under the effect of vehicular platooning has a negative impact on the safety and risk of drivers, playing a role in collisions [17–21]. The Highway Capacity Manual (HCM) [22] has defined two performance measures for determining the LOS for two-lane roads, including percent time spent following (PTSF) and ATS. Platooning is defined when the time headway between follower and leader vehicles is 3 s [22,23]. However, other studies have shown that PTSF under various traffic-flow conditions is inconsistent with the 3 s rule [23–25]. Al-Kaisy and Durbin [2] introduced the ATS measure as the average percentage of travel time for the platoon of vehicles at a speed rate below the average. Owing to limitations in performance measures (PTSF and ATS), several studies have been conducted, and they have resulted in the introduction of some additional performance measures, including follower density (FD), the ATS of passenger cars (ATS_{pc}), the percentage of vehicles impeded (PI), and other measures [2,25–28].

Thus, introducing a surrogate measure as an alternative could help traffic engineers to better measure traffic performance and the rate of the traffic stream. Further, the improvement of models for the performance measures leads to an accurate estimation of the performance and thereby the LOS of two-lane roads. In addition, strategies and plans based on the development of two-lane roads according to necessity would be competitive and allow the implementation of road management organizations and road traffic controllers. In this regard, the present study investigates the effect of vehicular platooning characteristics on performance measures for two-lane roads, followed by identifying the critical headways of platooning by accounting for the vehicle-gap-acceptance behavior. Four case studies on two-lane roads in the Gilan province of Iran were used in this study. This province has attractive sightseeing areas. Most of the roads leading to these areas are accessed by two-lane roads. Additionally, the industrial areas in the province are accessed by two-lane roads. These case studies were selected on the basis of the high demands of their traffic, their heavy traffic flow, and thereby their more-frequent vehicular platooning.

Next, the Pearson correlation is applied to examine the relationships between vehicular platooning and performance measures. Depending on the normal conjugate function, a Bayesian approach or multiple linear regression (MLR) models are proposed for the BLR model in order to develop performance measures for two-lane roads as a function of vehicular platooning. Further, a comparative evaluation of Bayesian linear regression (BLR) and MLR models is considered on the basis of prediction performance criteria and statistical criteria, including the p-value, F-value, and R^2 value, for finding the most-influential vehicular platooning variable on performance measures. Thereafter, the LOS is evaluated among the measures for identifying the optimal performance measure. Finally, in accordance with the optimal measure, the classification results of two-lane roads are compared with the HCM [22].

The remaining sections of this study are explained as follows. Section 2 reviews the studies relevant to the examination of the platooning phenomenon and its effect on traffic performance by using platooning variables and performance measures from two-lane roads. Thereafter, it discusses the research gap in the previous studies to highlight the paper's overarching idea and its applied methods. Section 3 describes the research methodology for two-lane roads and denotes the platooning variables and performance measures by using the empirical data from different sites in Iran. Further, the Pearson correlation is evaluated as the initial investigation of the relationship between variables and measures in this section. Thereafter, the MLR and BLR models are applied to develop performance measurement models and select a surrogate measure as the optimal performance measure under the effect of vehicular platooning. Accordingly, a comparison evaluation is provided throughout the proposed models. Section 4 presents the results and a discussion of the proposed measures and developed models. Finally, Section 5 concludes the research findings.

2. Literature Review

Numerous studies related to the evaluation of the effect of vehicular platooning on the quality of traffic flow and traffic measures in two-lane roads are described, as follows. Kim and Elefteriadou [29] investigated the effect of heavy vehicles (%HV) on the performance measures of two-lane roads using ATS and capacity. The results indicated that capacity decreased by 40% when ATS was reduced. Penmetsa et al. [30] proposed a measure for the LOS classification by using the number of followers as a proportion of capacity (NFPC). The results revealed the superiority of this measure over the PTSF measure in the HCM [22] to evaluate the quality of traffic flow and traffic performance on Indian roads. Jrew et al. [31] used performance measures such as ATS, free-flow speed (FFS), and the PTSF to evaluate the LOS of two-lane roads in Jordan. They found that an increase in ATS and FFS led to an improvement in the LOS.

Some studies have also investigated the effect of vehicle type, ATS, FFS, and PTSF on the LOS of two-lane roads. The results revealed that capacity improved as ATS and FFS increased, while PTSF was accompanied by a decrease in capacity and the LOS [32,33]. Bessa and Setti [34] introduced PTSF and ATS as the main performance measures of two-lane roads affecting the LOS of two-lane roads. Moreover, Gaur and Mirchandani [35] proposed vehicular platooning in two-lane roads by using traffic flow, follower density (FD), number of platoons, and time headway distribution for measuring traffic performance. Arasan and Kashani [36] also identified platoon size as the most effective vehicular platooning variable on two-lane-road performance measures, including ATS and percentage of followers (PF). Some studies have emphasized the role of vehicle-gap acceptance, along with the traffic flow of two-lane roads, on platoon size by using time headway as a threshold value for identifying vehicular platooning [37–41]. Moreover, some other studies have indicated that platooning depends on time headway. Time headway for platooning ranges from 3 to 7 s, significantly affecting performance measures such as ATS, FD, and platoon speed [2,42,43]. Yang et al. [44] found that %HV and platoon size were negatively related to traffic flow on Dutch highways.

Some studies have also reported that under the emergence of time headways and %HV, vehicular platooning negatively affected the performance measures of two-lane roads, including ATS and FFS [45–47]. Other studies have proposed that platoon size and %HV are the main vehicular platooning variables influencing the performance measures of two-lane roads, measures such as ATS, FFS, FD, and PF [6,48]. Hashim and Abdel-Wahed [49] introduced FD as the surrogate performance measure among ATS, FFS, %HV, ATS of passenger cars (ATSpc), ATS_{PC} as a percentage of the free-flow speed of passenger cars (ATSpc/FFSpc), and ATS as a percentage of free-flow speed (ATS/FFS) under vehicular platooning for evaluating the LOS of two-lane roads in Egypt. They indicated that traffic flow has the greatest effect on FD. Moreover, some studies have investigated the effect of vehicular platooning variables (i.e., traffic flow and time headway) on ATS in two-lane rural roads. They showed that time headway was strongly correlated with ATS rather than with traffic flow [50,51]. Likewise, Moreno [52] introduced %HV as the variable most common to cause platooning compared with other variables (i.e., traffic flow, time headway, and opposing flow), impacting traffic performance variables such as the ATS and FD of two-lane roads in Spain. Al-Zerjawi et al. [53] found a strong relationship between some platooning variables, such as flow, time headway, the number of overtaking (NO) vehicles, and performance measures such as the ATS and PF of two-lane roads in Al-Mishkhab, Iraq. Furthermore, Ahmed and Easa [54] developed performance measurement models that were based on the PTSF of two-lane roads by means of the threshold of time headway. Kim [55] observed that the %HV is the most-influential vehicular platooning variable compared with platoon size and NO on performance measures such as ATS, FD, and PTSF. Previous studies have demonstrated that the most effective vehicular platooning variables on the performance measures include the %HV and platoon size; other performance measures include ATS, FD, and platoon speed [33,56–61].

Jin et al. [62], in a study focused on evaluating the impact of vehicle platooning on highway congestion, used a fluid-queuing approach. They showed that as the number of platoons increases, it leads to more congestion in traffic flow. Mena-Oreja et al. [63] also investigated the platooning maneuvers on traffic flow under mixed traffic conditions. They concluded that mixed traffic flow has an influencing effect on increasing the number of platoons. Kita and Yamada [64] introduced a vehicle velocity control approach that accounted for platoon merging. They showed that their model could successfully control the speed of drivers with a combination of platooning and merging. Zhu et al. [10] developed a dynamic model that was based on the formation of vehicular platooning on two-lane roads. Their results showed that an increase in the percentage of heavy vehicles led to more platooning in traffic flow. Moreover, Zhu et al. [10] concluded that a heterogeneous traffic flow, compared with a homogenous traffic flow, has a direct effect on the emergence of platoons. Al-Kaisy [4] reported that two-lane roads have more potential for vehicular platooning compared with other roads owing to the traffic composition of vehicles, overtaking maneuvers, and the speed beneficence of cars in comparison with heavy vehicles. Mauro et al. [65] developed a statistical model to evaluate the platoons of vehicles on two-lane roads. They indicated that the platoons of vehicles play important roles in traffic-flow characteristics. Further, developed statistical models provide a better performance prediction of the quality of two-lane roads than conventional statistical models do.

Therefore, a review of the previous studies relevant to the effect of vehicular platooning on the performance measures of two-lane roads, as shown in Table 1, revealed that no recent study has comprehensively evaluated the effect of vehicular platooning characteristics on the quality of traffic flow and traffic performance (i.e., time headway, platoon size, %HV, and opposing flow) and performance measures (i.e., ATS, platoon speed, NO, FD, PI, PF, NFPC, ATSpc, ATSpc/FFSpc, and ATS/FFS). The present study evaluated vehicular platooning under the critical thresholds of time headway by using vehicle-gap-acceptance behavior and the relationships between platooning variables influencing performance measures. Thereafter, the MLR and BLR models, based on regression and conjugate Bayesian analysis, respectively, were taken into consideration to develop performance measurement models. Accordingly, the optimal surrogate measures were selected on the basis of statistical criteria. Finally, the surrogate measure was compared with the HCM [22] regarding the effect of vehicular platooning on the LOS of two-lane roads.

Table 1. Summary of literature review.

Author (Year)	Subject	Platooning Variable and Performance Measure	Conclusion
Gaur and Mirchandani [35]	A method for real-time recognition of vehicle platoons	Traffic flow, FD, number of platoons, and time headway	Vehicular platooning as the most-influential phenomenon on performance measures
Arasan and Kashani [36]	Investigating the most effective vehicular platooning variables on performance measures	Platoon size, ATS, and PF	Platoon size as the most effective vehicular platooning variable on performance measures
Al-Kaisy and Karjala [6]	Evaluating indicators of performance on two-lane rural highways	Platoon size, heavy vehicle, ATS, FFS, FD, and FP	Identification of performance measures under the effect of vehicular platooning
Kim and Elefteriadou [29]	Evaluating the effect of %HV on the capacity of two-lane roads	HV, ATS, flow, capacity	Reduction in capacity under the effect of heavy vehicle
Hashim and Abdel-Wahed [49]	Investigating performance measures for rural two-lane roads in Egypt	Flow, ATS, FD, and PF	Follower density as the surrogate performance measure under vehicular platooning

Table 1. *Cont.*

Author (Year)	Subject	Platooning Variable and Performance Measure	Conclusion
Nadimi et al. [45]	Time headway analysis using vehicle types affecting on performance measures	Time headways, heavy vehicles, ATS, and FFS	Time headways and heavy vehicles with a negative effect on performance measures
Rossi et al. [50]	Flow-rate effects and the relationship between vehicular platooning and traffic characteristics in two-lane roads	Flow, time headway, and ATS	Time headway with a strong correlation with ATS instead of flow
Penmetsa et al. [30]	Evaluation of LOS under vehicular platooning	Flow, NF, NFPC, and PTSF	NFPC as the best platooning indicator compared with PTSF in the HCM (2010)
Jrew et al. [31]	Analysis and improvement of the LOS in two-lane roads under vehicular platooning	ATS, FFS, and PTSF	An increase in ATS and FFS and a reduction in PTSF
Boora et al. [33]	A study of performance measures in two-lane roads	ATS, FFS, vehicle type, and PTSF	Improvement of traffic performance in two-lane roads
Bessa and Setti [34]	Identifying the most effective performance measures in two-lane roads	PTSF and ATS	PTSF as the main performance measure of two-lane roads affecting the LOS
Al-Kaisy et al. [43]	An empirical analysis of vehicle time headways on platooning formation	Time headway, platoon size, platoon speed, ATS, and FD	Time headway between 3 and 7 s for forming vehicular platooning
Zhang et al. [41]	Examination of vehicle-gap acceptance on the formation of vehicular platooning	Time headway, platoon size, flow, and gap acceptance	Platoon size by vehicle-gap acceptance and the critical headway
Yang et al. [44]	Evaluating the impacts of heavy vehicles platooning on Dutch highways	Heavy vehicles, platoon size, and flow	Heavy vehicles and platoon size have a negative relationship with traffic flow
Moreno [52]	Identifying platooning variables on performance measure in two-lane roads in Spain	Flow, time headway, opposing flow, %HV, ATS, and FD	%HV as the main platooning variable affecting performance measures
Al-Zerjawi et al. [53]	Traffic characteristics of two-lane roads in Iraq	Flow, ATS, and NO	A strong relationship between platooning variables and performance measures
Ahmed and Easa [54]	Development of performance measurement models under vehicular platooning in two-lane highways	Flow, ATS, and PTSF	PTSF as the main performance measurement as compared with the HCM (2010), with low error in prediction
Kim [29]	Controlling heavy vehicle platoons according to platooning characteristics	Platoon size, %HV, NO, ATS, FD, and PTSF	%HV as the main influencing platooning variable on performance measures in comparison with others
Jain et al. [61]	Evaluating the most effective vehicular platooning variables on performance measures	ATS, PTSF, FD, HV (%), flow, platoon size, and platoon speed	Performance measures contributing to the improvement of traffic performance

Notes: ATS: average travel speed; NO: number of overtaking (vehicles); FD: follower density; PF: percentage of followers; HV: percentage of a heavy vehicles; NFPC: number of followers per capacity; PTSF: percent time spent following; LOS: level of service; HCM: Highway Capacity Manual; FFS: free-flow speed.

3. Research Method

In the present study, first, vehicular platooning was evaluated by using vehicle-gap-acceptance behavior to determine the threshold of time headway for measuring platooning variables. Thereafter, the effects of vehicular platooning variables on the quality of traffic performance on two-lane roads were investigated. In this step, the platooning variables were extracted from the video recordings. Straight and longitudinal traps of 30 m were selected, one per site at four sites in Iran. The weather conditions were daylight, clear, and sunny, and the pavement had a suitable condition. In addition, during the videography, we controlled the effects of longitudinal slope, intersections, and horizontal alignments in the straight sections of the road. After collecting empirical data, the videography analysis method was performed to extract field data at 30 frames per second by using videography software.

Second, the relationships between platooning variables and performance measures were examined using the Pearson correlation to determine the initial relationships between these variables. After examining the relationships, the MLR and BLR models, based on the regression approach and the conjugate Bayesian analysis, respectively, were provided for the development of performance measures for the variables and measures in order to determine the most-influential variables on measures according to statistical criteria such as R^2 value, *p*-value, significant F-value, and errors. Thereafter, a surrogate measure associated with the most effective variables was introduced by using the developed models and by comparing it with the HCM [22]. Thus, in the present study, the following performance measures were investigated:

- ATS: this measure is the best-known performance measure for evaluating road users' perceptions of the quality of traffic flow on two-lane roads [22].
- ATS_{PC}: this measure is more important than ATS on two-lane roads because passenger cars have a greater variety of ATSs than heavy vehicles do under different traffic conditions [6,66].
- ATS/FFS: this measure shows the average speed reduction from interactions with other vehicles. Reducing this measure is accompanied by a decrease in the LOS. Because the FFS of roads changes under various traffic conditions, including FFS in the ATS could control the reduction rate of the LOS in two-lane roads [6].
- ATS_{PC}/FFS_{PC}: this measure is defined similarly to ATS/FFS, but it refers only to passenger cars because of the sensitive interaction of their speeds under various traffic conditions compared with heavy vehicles, especially under high traffic flow [26].
- PF: this measure denotes the percentage of vehicles with short headways in the traffic flow. This measure is obtained on the basis of the headway, in which the HCM considers 3 s [22] as the time headway for estimating PF. However, for other roads, the time headway should be modified.
- FD: this measure represents the number of followers in a directional traffic flow over a given unit of length, which is defined as 1 km or 1 mile. It is important to consider the degree of congestion through PF and the density of two-lane roads [22,25]. This measure is obtained by using density (D) and PF according to Equations (1) and (2), as follows:

$$D = \frac{Q}{ATS}. \tag{1}$$

$$FD = D \times PF. \tag{2}$$

where Q is traffic flow (veh/h), ATS is the average travel speed (km/h), FD is the follower density (veh/km), D is the density (veh/km), and PF is the percentage of followers (%).

- Percent impeded (*PI*): this measure represents the percentage of vehicles impeded in the vehicular platoon's traffic flow [2,67]. PI is calculated by using Equation (3), as follows:

$$PI = P_P \times P_i. \tag{3}$$

where P_P is the probability of a vehicle in the vehicular platoon based on the time headway considered for platoon and P_i is the probability that a vehicle will be impeded in the platoon.

- NO: this measure is the most popular measure for evaluating the LOS of two-lane roads under platooning, reflecting the freedom of maneuverability [7,68,69].
- Platoon speed: one of the characteristics of vehicular platooning in a two-lane road under the formation of the platoon of vehicles is platoon speed in the direction of traffic flow, based on the following vehicle headway and slow-moving vehicles [70]. Vehicles in the platoon have a speed less than the ATS of the traffic flow [2].
- NFPC: this measure is introduced as a criterion for evaluating the effect of vehicular platooning and potential followers in platoon size and the degree of congestion as NFPC [30]. Thus, Equation (4) is proposed as a function of NF and capacity in two-lane roads, as follows:

$$NFPC = \frac{NF}{\max(f(Q))}. \tag{4}$$

where $NFPC$ is a number of followers per capacity in two-lane roads, NF is the number of followers (veh/h), and $\max(f(Q))$ is the maximum traffic flow (veh/h).

3.1. Case Study

In the present study, vehicular platooning variables and performance measures were examined as the empirical data from four sites of two-lane roads in the Gilan and Mazandaran provinces of Iran, including Rasht-Somesara, Fuman-Saravan, Rasht-Jirdeh, and Kiasar-Sari roads. During the collection of the field data, traffic flow varied, and the traffic included mainly drivers with light or heavy vehicles. In addition, the speed limit was 90 km/h, and the width of each road lane was 3.65 m. Figure 1 represents the composition of the traffic flow by the vehicle type passing on the two-lane roads. As shown in Figure 1, Kiasar-Sari's maximum percentage of passenger cars is about 96%, and the minimum percentage of heavy vehicles is approximately 4%. However, Rasht-Somesara's maximum percentage of heavy vehicles was about 18%, and the minimum percentage of passenger cars was nearly 82%.

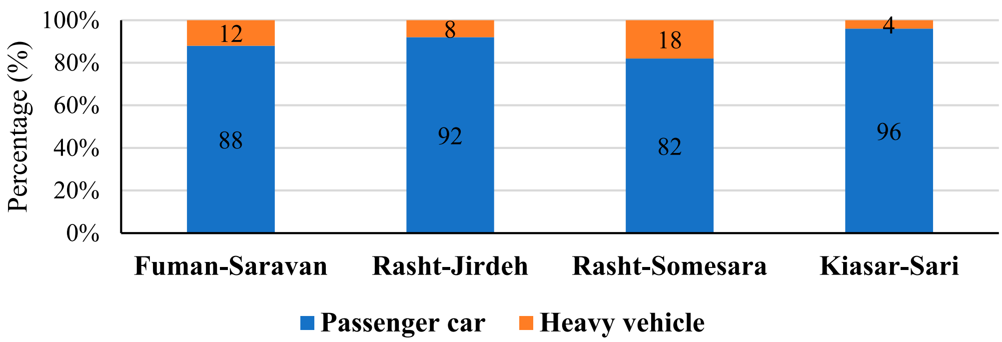


Figure 1. Percentage of vehicle types on two-lane roads.

3.2. Data Collection

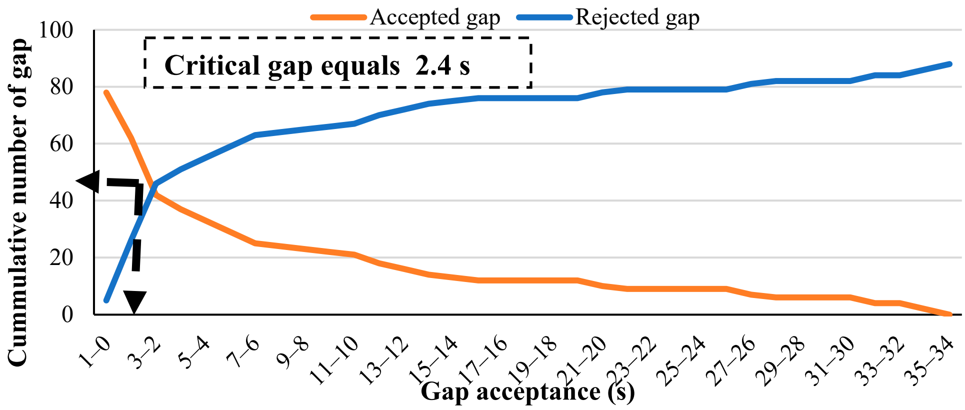
In the present study, the empirical data were collected from four sites of two-lane roads using 5 min intervals over a total of 8 h. Thus, the sample size was considered as 96 for all roads. For modeling purposes and to obtain a comprehensive model that is based on the proposed model, a summary of all data sets after the initial analysis is shown in Table 2. All data sets consist of statistical information, including minimum, maximum, mean, standard deviation, and variance for platooning variables such as flow, opposing flow, time headway, %HV, and platoon size. Further, Table 2 represents the statistical information for describing performance measures such as ATS, ATSpc, ATSpc/FFSpc, ATS/FFS, NO, PF, FD, PI, platoon speed, and NFPC. The coefficient-of-variation (CV) values are highest for platoon size, flow, and opposing flow, indicating higher dispersion. This implies that these performance measures such as FD, NO, and NFPC are more sensitive to variables with higher CV. Thus, it is essential to investigate the effect of vehicular platooning on performance and the relationships between the variables and measures.

Table 2. Total statistics of platooning variables and performance measures in all two-lane roads.

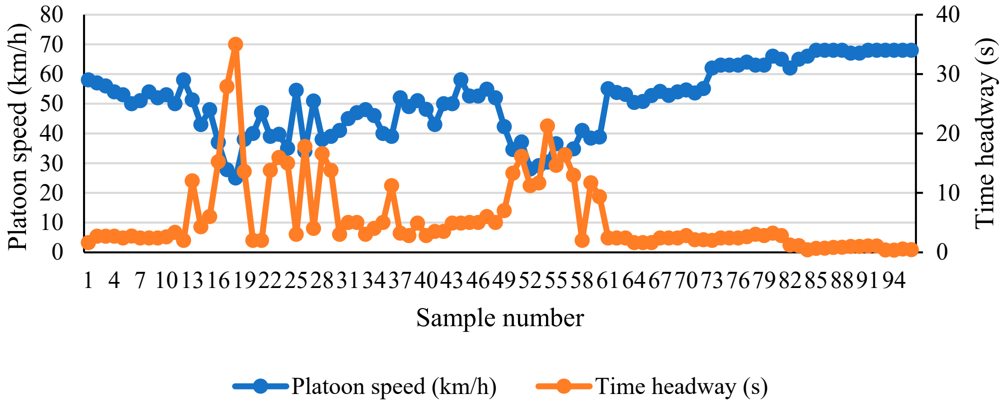
Variables		Mean	Std. Deviation	Variance	Minimum	Maximum	CV
Platooning Variables	Flow (veh/h)	825.96	534.74	273,889.0	70.00	1810.00	0.65
	Opposing Flow (veh/h)	579.17	374.27	140,079.0	50.00	1268.00	0.64
	Time Headway (s)	5.34	2.60	73.60	0.40	35.00	0.49
	%HV (%)	11.00	5.88	34.54	3.00	25.00	0.53
	Platoon Size (veh/h)	85.76	56.34	7.96	0.00	225.00	0.66
Performance Measures	ATS (km/h)	66.87	10.75	110.50	36.00	81.00	0.16
	ATSpc (km/h)	84.90	12.97	168.29	46.08	99.63	0.15
	ATSpc/FFSpc	0.97	0.14	0.020	0.82	1.40	0.14
	ATS/FFS	0.76	0.11	0.012	0.64	1.09	0.14
	PF (%)	13.27	6.93	48.02	5.00	33.00	0.52
	FD (veh/km)	7.89	5.19	27.12	1.17	20.75	0.66
	PI (%)	0.20	0.08	0.006	0.02	0.31	0.40
	NFPC	0.35	0.20	0.040	0.05	0.86	0.57
	NO (veh/h)	33.38	21.45	445.20	0.00	96.00	0.64
	Platoon Speed (km/h)	51.00	11.41	130.20	25.00	68.00	0.22

Notes: std. deviation: standard deviation; ATS: average travel speed; ATS_{PC}: average travel speed of passenger cars; ATS/FFS: average travel speed as a percentage of free-flow speed; ATSpc/FFSpc: ATS_{PC} as a percentage of free-flow speed of passenger cars; PI: percentage impeded; NO: number of overtaking (vehicles); FD: follower density; PF: percentage of followers; %HV: percentage of heavy vehicles; NFPC: number of followers per capacity; CV: coefficient of variation.

Figure 2 indicates the results from the examination of time headway after including gap-acceptance behavior and platoon speed. Figure 2a illustrates a 1 s interval of time headway for estimating the critical value as the threshold of vehicular platooning according to the vehicle-gap-acceptance behavior and the intersection of the accepted and rejected gaps from the selected four sites of two-lane roads. According to Figure 2a, the accepted and rejected gaps intersect in a time headway of 2.4 s. Further, Figure 2b demonstrates the relationship between the average platoon speed and the time headway of drivers regarding the formation of vehicular platooning on the studied roads. According to Figure 2b, it can be inferred that as the platoon speed of vehicles increases, this leads to a reduction in time headway, and the gap acceptance of drivers reaches the critical point. Further, as shown in Figure 2b, most of the platoons formed have a time headway longer than 2.4 s. In time headways shorter than 2.4 s, platoon size based on the field data decreases. Thus, an increase in the platoon speed causes a reduction in time headway, and the number of platoons decreases. Further, as seen in Figure 2b, the platoon size of vehicles has its maximum value as a platoon speed between 20 to 40 km/h.



(**a**)



(**b**)

Figure 2. Examination of time headway, including gap-acceptance behavior and platoon speed: (**a**) 1-s time interval of time headway for vehicle-gap-acceptance behavior and (**b**) relation between platoon speed and time headway.

3.3. MLR Model

Linear regression models are one of the most widely used methods in statistical analysis, and researchers have applied these methods in various applied sciences and engineering fields. Different linear regression models are recommended depending on the number of independent and dependent variables [71]. The MLR model is an appropriate statistical tool to test whether there is an influence from independent variables on a dependent variable and predict the value of the dependent variable, where the dependent variable y consists of n observations and k independent variables $(x_1, x_2, \ldots, x_j)$ [72]. Before applying the MLR model, the Pearson correlation is used to determine whether there is statistical evidence for a relationship between the independent and dependent variables [73]. Consider the following regression model with various explanatory variables and coefficients by using Equations (5) and (6), as follows [74]:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_j X_{ij} + \varepsilon_i. \tag{5}$$

$$Y = X\beta + \varepsilon. \tag{6}$$

Thus, the MLR model in the form of a matrix is written as follows:

$$Y = \begin{bmatrix} y_1 \\ y_1 \\ \vdots \\ y_n \end{bmatrix};\ X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1j} \\ 1 & x_{21} & \cdots & x_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nj} \end{bmatrix};\ \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_j \end{bmatrix};\ \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}. \tag{7}$$

where Y is the vector of observed random variables, which is a random normal vector by dimension n; X is the independent observation matrix, which consists of $n \times (k+1)$ dimension and is named the design matrix; β is the vector of dimension $(j+1)$ of the coefficient in the regression; and ε is the random error vector of dimension n of the ith observation for $i = 1, 2, \ldots, n$.

In Equation (7), β could be obtained by using Equation (8), based on the ordinary least square (OLS) method, which is as expressed:

$$\hat{\beta} = (X'X)^{-1} X'Y. \tag{8}$$

where X' is the transpose of matrix X.

3.4. BLR Model Using Conjugate Prior Distribution

Recently, conjugate Bayesian analysis has been used as a predictive model of output values in statistics and other engineering sciences [73]. This model, thanks to its utilizing Bayesian theory, is better able to sufficiently predict output data compared with other statistical models, such as multiple regression models [75–77]. Other advantages of using conjugate Bayesian analysis compared with statistical models include having lower errors and more-reliable accuracy [78]. Thus, the BLR model is one of the most common regression techniques because the Bayesian approach uses the parameter estimation method. The Bayesian approach is based on the prior function, likelihood distribution function, and posterior distribution function. This approach is a new method among statistical methods, playing a modifier role in the improvement of MLR models because the BLR model is formulated on the basis of the MLR model [79–82]. The main difference between the classical methods and the Bayesian approach is that the parameter β is an unknown parameter. In classical methods, β is estimated via the probability distribution $f(x, \beta)$, while in the Bayesian approach, β is considered as a random variable with probability distribution $g_2(\beta)$ as the function of the likelihood function $L(x, \beta)$ and prior distribution $g_1(\beta)$. Thus, in the Bayesian approach, the posterior distribution $g_2(\beta)$ is obtained by multiplying the prior distribution by the likelihood function, leading to accurate parameter estimation β [83–85]. In this approach, the parameter β is taken into account with random variables, including $\beta_1, \beta_2, \ldots, \beta_n$ and the probability function. The probability function indicates all the information on and experiences from the parameters. Thereafter, the prior information is added to the observed sample information. Because this approach is unbiased, more samples lead to further average values of the samples and thus accurate estimation of parameter β [86,87]. The steps involved in the BLR model are summarized in Figure A1 in Appendix A. Consider the following regression model given in Equation (9):

$$Y = X\,\beta + \varepsilon \tag{9}$$

where Y is the vector of observed random variables, which is a random normal vector by dimension n; X is the independent observation matrix, which consists of $n \times (k+1)$ dimension and is named the design matrix; β is the vector of dimension $(j+1)$ of the coefficient in the regression; and ε is the random error vector of dimension n. Thus, the BLR model, based on the dependent variable and error ε, is written as follows:

$$Y \sim n(X\beta, \sigma^2 I_n) \text{and } \varepsilon \sim n(0, \sigma^2 I_n) \tag{10}$$

where σ^2 is the variance and I_n is the identity matrix. Therefore, the maximum likelihood function, based on the sample information and normal distribution, is represented in Equation (11), as follows:

$$L(\beta, \sigma | y, x) = \frac{1}{(2\pi)^{\frac{n}{2}} \sigma^n} \exp\left\{ -\frac{1}{2\sigma^2} \left[(Y - X\beta)'(Y - X\beta) \right] \right\} \tag{11}$$

For simplicity, Equation (11), in terms of its dependent and independent variables, and Equations (12) and (13) are expressed as follows:

$$(Y - X\beta)'(Y - X\beta) = Y'Y - Y'X\beta - X'\beta'Y + \beta'X'X\beta. \tag{12}$$

$$(Y - X\beta)'(Y - X\beta) = Y'Y - 2X'\beta'Y + \beta'X'X\beta. \tag{13}$$

which assume that $b_0 = Z^{-1}X'XY$ and $Z = X'X$.

Thereafter, by substituting the sum of the squared estimate of errors (SSE) into Equations (12) and (13), Equation (14) is obtained, as follows:

$$(Y - X\beta)'(Y - X\beta) = \left[SSE(b_0) + (\beta - b_0)'Z(\beta - b_0)\right]. \tag{14}$$

Thus, the maximum likelihood function of Equation (11) is rewritten in Equation (15), as follows:

$$L(\beta, \sigma|y, x) = \frac{1}{(2\pi)^{\frac{n}{2}}\sigma^n} \exp\left\{-\frac{1}{2\sigma^2}\left[SSE(b_0) + (\beta - b_0)'Z(\beta - b_0)\right]\right\}. \tag{15}$$

By deleting the constants and the $SSE(b_0)$, which has the parameters β and the information σ^2, Equation (16) is expressed, as follows:

$$L(\beta) \propto \exp\left\{-\frac{1}{2\sigma^2}\left[(\beta - b_0)'Z(\beta - b_0)\right]\right\}. \tag{16}$$

The normal distribution of the parameter β is called the prior distribution, as $g_1(\beta)$, which is estimated by Equation (17), as follows:

$$g_1(\beta) \propto \exp\left\{-\frac{1}{2\sigma^2}\left[(\beta - \beta_0)'\Omega^{-1}(\beta - \beta_0)\right]\right\}. \tag{17}$$

where Ω is the precision matrix X that is estimated by $\Omega = \frac{X'X}{S^2}$, where X' and S^2 are the transpose of matrix X and an estimator of σ^2, respectively. Thus, the posterior distribution $g_2(\beta|y)$ is obtained by using Bayes's theory and merging the prior distribution function with the maximum likelihood function $L(\beta)$ according to Equation (16) and the prior distribution $g_1(\beta)$ by using Equation (17), as denoted by $g_2(\beta|y) \propto g_1(\beta).L(\beta)$ in Equation (18).

$$g_2(\beta|y) \propto \exp\left\{-\frac{1}{2\sigma^2}\left[(\beta - b_0)'Z(b - \beta_0) + (\beta - \beta_0)'\Omega^{-1}(\beta - \beta_0)\right]\right\}. \tag{18}$$

Equation (18) is simplified by using assumptions such as $c^* = (b_0 - \beta_0)'Z\Omega^*\Omega^{-1}(b_0 + \beta_0)$, $b^* = \Omega^*\left(Zb_0 + \Omega^{-1}\beta_0\right)$, and $\Omega^* = \left(Z + \Omega^{-1}\right)^{-1}$. Hence, the posterior distribution is reformulated according to Equation (18) and written in Equation (19), as follows:

$$g_2(\beta|y) \propto \exp\left\{-\frac{1}{2\sigma^2}\left[(\beta - b^*)'\Omega^{*-1}(\beta - b^*) + c^*\right]\right\}. \tag{19}$$

Equation (19) indicates the posterior distribution as a normal distribution of a mean b^*, a variance σ^2, and covariance matrix $\sigma^2\Omega^*$, where β is $N\left(b^*, \sigma^2\Omega^*\right)$. In Equations (17) and (19), the prior and posterior distributions have a normal distribution. Therefore, the prior distribution is named the conjugate prior distribution, which verifies the prior probability distribution. Accordingly, the posterior distribution is applied to estimate the regression model parameters. The following Bayes estimators are related to the parameters of the regression model β from Equations (20)–(22), which are given as follows:

$$\Omega^* = \left(Z + \Omega^{-1}\right)^{-1},\ b^*_{bayes} = \Omega^*\left(Zb_0 + \Omega^{-1}\beta_0\right). \tag{20}$$

$$b^*_{bayes} = \left(Z + \Omega^{-1}\right)^{-1} Z b_0 + \left(Z + \Omega^{-1}\right)^{-1} \Omega^{-1} \beta_0. \quad (21)$$

Thus, simplifying Equation (21) to calculate the estimated regression model leads to the formation of Equation (22), which is written as follows:

$$b^*_{bayes} = (H_1 b_0) + (H_2 \beta_0). \quad (22)$$

where $H_1 = \left(Z + \Omega^{-1}\right)^{-1} Z$ and $H_2 = \left(Z + \Omega^{-1}\right)^{-1} \Omega^{-1}$. In Equation (22), the regression parameters are obtained to calculate the maximum likelihood weighted by the weights matrix and the prior distribution average [86].

3.5. Comparison of Prediction Performance of Models

The prediction performance for the MLR and BLR models, using performance measures under vehicular platooning, is compared with the mean absolute percentage error (MAPE). The purpose is to select the best predictive model in terms of acceptable error and accuracy by using Equation (23), which is expressed as follows [30,80]:

$$MAPE = \frac{1}{N} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\%. \quad (23)$$

where y_i is the observed dependent variable based on the field data and $\hat{y}_i$ denotes the predicted value for the developed models based on the performance measures.

4. Results and Discussion

After examining the platooning variables and performance measures from the empirical data, the initial investigation of the relationships between these two variables using the Pearson correlation is represented. Thereafter, the results of the development of the MLR and BLR models for predicting performance measures concerning platooning variables are obtained. Regarding the developed MLR and BLR models, the relationships between dependent and independent variables are examined using statistical criteria such as the R^2, F-value, *p*-value, and *t*-value, along with errors in prediction performance, to select the most-influential platooning variable on the performance measures. Thus, according to the best fit model with low error and high prediction accuracy, a surrogate measure is proposed as a performance measure to evaluate the LOS in two-lane roads. The results obtained from the comparison evaluation in the present study with the HCM [22] are shown. The obtained results and findings are described in the following section.

4.1. Pearson Correlation and MLR Model

After examining the platooning variables and performance measures in Table 2, a Pearson correlation was performed, and the results are displayed in Table 3. According to Table 3, it can be inferred that traffic flow has the highest correlation with NFPC, ATS, platoon speed, FD, PI, PF, NO, and ATS_{PC}, in that order. Opposing flow has a strong correlation with NFPC, PF, ATS, FD, and PI, in that order. Further, examining the relationships between %HV and performance measures revealed that %HV has a high correlation with FD, ATS, PF, and PI, in that order. In addition, platoon size is significantly related to platoon speed, PI, ATS, and ATS_{pc}, in that order. Time headway also has a significant correlation with ATS, NFPC, ATS_{pc}, and FD, in that order.

After applying Equations (5)–(8) for the MLR model, the results were obtained, and they are shown in Table 4. According to Table 4 and statistical criteria such as the R^2, F-value, *p*-value, and absolute value of the *t*-value, traffic flow significantly influences the performance measures, including NFPC, ATS, platoon speed, FD, PI, PF, NO, and ATS_{PC}. Moreover, among the performance measures in Table 4, flow is identified as the most-influential variable on NFPC thanks to its higher absolute value of coefficient, *t*-value,

p-value (less than 0.05), and R^2 for the best predictive model. However, other platooning variables indicate a weak correlation with the performance measures in MLR models.

Table 3. Pearson correlation between platooning variables and performance measures.

Performance Measures	Platooning Variables				
	Flow (veh/h)	Opposing Flow (veh/h)	%HV	Platoon Size (veh/h)	Time Headway (s)
ATS (km/h)	−0.80 *	−0.66 *	−0.70 *	−0.63 *	−0.72 *
ATS/FFS	−0.09	−0.09	−0.10	−0.15	−0.32
ATS_{PC} (km/h)	−0.48 *	−0.27	−0.19	−0.56 *	−0.60 *
ATS_{PC}/FFS_{PC}	−0.07	−0.05	−0.08	−0.09	−0.08
FD (veh/km)	0.71 *	0.62 *	0.75 *	0.45	0.58 *
PI (%)	0.62 *	0.58 *	0.56 *	0.70 *	0.43
PF (%)	0.56 *	0.70 *	0.63 *	0.40	0.40
NO (veh/h)	0.52 *	0.36	0.20	0.37	−0.39
NFPC	0.84 *	0.75 *	0.38	0.39	0.67 *
Platoon Speed (km/h)	−0.76 *	−0.41	−0.50	−0.75 *	−0.33

Notes: ATS: average travel speed; $ATS_{PC:}$ average travel speed of passenger cars; ATS/FFS: average travel speed as a percentage of free-flow speed; ATSpc/FFSpc: ATS_{PC} as a percentage of the free-flow speed of passenger cars; PI: percentage impeded; NO: number of overtaking (vehicles); FD: follower density; PF: percentage of followers; %HV: percentage of a heavy vehicles; NFPC: number of followers per capacity. * The correlation is significant at the 5% level (2 tailed). * r denotes the Pearson correlation. If r = 0, it is completely irrelevant; if $0 < r < 0.3$, it is incompletely relevant; if $0.3 < r < 0.5$, it has low relevance; if $0.5 < r < 0.8$, it has high relevance. However, if $0.8 < r < 1$, it is a significant correlation. For $r = 1$, there is a high correlation between the two variables.

Table 4. Regression results of the performance measures.

Variable	Statistical Analysis	Performance Measures									
		ATS (km/h)	ATSpc (km/h)	ATSpc/FFSpc	ATS/FFS	PF (%)	FD (veh/km)	PI (%)	NFPC	Platoon Speed (km/h)	NO (veh/h)
Constant	Coefficient	81.84	109.08	0.83	0.65	−1.27	0.25	0.21	−0.005	60.26	−3.21
	t (p-value)	33.93 (0.00)	53.74 (0.00)	23.47 (0.001)	23.64 (0.00)	−0.98 (0.00)	0.27 (0.04)	12.85 (0.00)	−0.20 (0.001)	27.33 (0.00)	−1.35 (0.02)
Flow (veh/h)	Coefficient	−15.72	−2.54	−70.97	−69.27	1.22	4.34	1.78	0.53	−49.54	1.19
	t (p-value)	−4.28 (0.002)	−0.46 (0.001)	−10.30 (0.092)	−7.78 (0.081)	0.59 (0.04)	2.78 (0.00)	0.85 (0.003)	7.76 (0.004)	−3.34 (0.01)	0.50 (0.01)
Opposing Flow (veh/)	Coefficient	−0.03	−0.023	−0.001	−0.003	0.016	0.008	0.001	0.001	−0.042	0.069
	t (p-value)	−3.91 (0.003)	−4.61 (0.06)	−3.89 (0.073)	−3.85 (0.082)	4.63 (0.003)	3.18 (0.02)	1.95 (0.004)	3.71 (0.01)	−7.07 (0.06)	10.73 (0.09)
%HV	Coefficient	−0.65	−0.37	−0.016	−0.013	0.34	0.34	0.004	0.004	−0.28	0.034
	t (p-value)	−3.04 (0.03)	−2.03 (0.44)	−5.17 (0.081)	−5.17 (0.10)	2.92 (0.004)	4.18 (0.001)	2.84 (0.006)	1.96 (0.07)	−1.41 (0.059)	1.17 (0.11)
Platoon Size (veh/h)	Coefficient	−1.09	−1.22	−0.025	−0.02	0.13	0.10	0.013	0.004	−3.19	1.43
	t (p-value)	−1.59 (0.01)	−0.86 (0.03)	−2.45 (0.32)	−2.47 (0.42)	0.34 (0.15)	0.383 (0.087)	2.80 (0.003)	0.59 (0.08)	−5.03 (0.02)	2.01 (0.13)

Table 4. *Cont.*

Variable	Statistical Analysis	Performance Measures									
		ATS (km/h)	ATSpc (km/h)	ATSpc/FFSpc	ATS/FFS	PF (%)	FD (veh/km)	PI (%)	NFPC	Platoon Speed (km/h)	NO (veh/h)
Time Head-way (s)	Coefficient	−0.40	−0.27	−0.01	−0.008	0.25	0.009	0.005	0.003	−0.081	−0.29
	t (*p*-value)	−3.49 (0.00)	−2.34 (0.007)	−6.08 (0.45)	−6.08 (0.50)	3.98 (0.21)	0.21 (0.031)	6.09 (0.07)	2.92 (0.021)	−0.77 (0.10)	−2.54 (0.21)
SSR		1983.67	9095.07	0.91	0.55	0.51	3343.82	0.39	3.24	6040.12	30,075.99
SSE		420.67	5886.96	0.85	0.46	0.19	1026.39	0.12	0.38	1421.36	12,436.42
SST		2404.34	14,982.03	1.76	1.01	0.70	4370.21	0.51	3.62	7461.48	42,512.41
R^2		0.83	0.61	0.52	0.54	0.73	0.77	0.76	0.90	0.81	0.71
F (*p*-value)		100.69 (0.00)	60.45 (0.00)	35.97 (0.00)	56.24 (0.00)	79.22 (0.00)	90.86 (0.00)	87.86 (0.00)	189.44 (0.00)	94.21 (0.00)	66.07 (0.00)

Notes: ATS: average travel speed; ATS_{PC}: average travel speed of passenger cars; ATS/FFS: average travel speed as a percentage of free-flow speed; ATSpc/FFSpc: ATS_{PC} as a percentage of free-flow speed of passenger cars; PI: percentage impeded; NO: number of overtaking (vehicles); FD: follower density; PF: percentage of followers; %HV: percentage of heavy vehicles; NFPC: number of followers per capacity. Values given in parentheses are the *t*-statistic values of the coefficients. Sum of squares regression (SSR); sum of squares error (residual) (SSE); sum of squares total (SST).

4.2. BLR Model

The platooning variables were applied to predict performance measures in two-lane roads. The results were obtained for each performance measure as a function of platooning variables in the BLR model by using Equations (9)–(22). The present study obtained the prior and posterior distributions by using the Markov chain Monte Carlo (MCMC) method for 600 iterations. Further, the noninformative independent normal prior distributions with variance and the Gamma inverse distribution have been used for regression coefficients and parameter σ^2, respectively (Table 5). The highest posterior density (HPD) intervals for all parameters have been determined at a significance level of 0.05 in all models. Owing to the constraint of space, only the characteristics of prior and posterior distributions were provided in Tables 5 and 6, as well as in Figure 3, which was used to estimate the parameters of vehicular platooning affecting ATS on the basis of the trace plot of the MCMC chains. A similar scheme is followed for estimating parameters in other performance measures, the results of which are presented in Table 7. Thus, as it can be observed, the results based on the Monte Carlo standard error (MCSE), Geweke's test, and the *p*-value suggest that the generated chains did not converge for all parameters of the investigated model at any significant level. The results in Table 7 indicate that the relationship between platooning variables and NFPC is stronger than the relationship between other platooning variables and performance measures regarding the higher R^2 value and the *p*-value among the proposed BLR models. Further, flow significantly affects the performance measures, compared with other variables, thanks to a higher coefficient and a *p*-value of less than 0.05.

The following general vector of parameter β for prior distribution is estimated according to the noninformative independent normal prior distributions of each platooning variable concerning ATS, which are obtained in Equation (24), as follows:

$$\beta_0{}' = (75.34 \ -10.99 \ -0.09 \ -0.23 \ -0.61 \ -1.23)'. \tag{24}$$

The precision matrix Ω is followed by Equation (25):

$$\Omega = \exp(+5)\begin{pmatrix} 0.0013 & 0.0378 & 0.0532 & 0.0771 & 0.0037 & 0.0643 \\ 0.0102 & 0.8512 & 1.3220 & 2.1080 & 0.6800 & 0.0489 \\ 0.0678 & 1.9710 & 6.5081 & 7.5501 & 0.6170 & 2.0130 \\ 0.0410 & 0.8830 & 7.2810 & 8.4600 & 0.5981 & 2.617 \\ 0.0026 & 0.0594 & 0.4406 & 0.6901 & 0.0047 & 0.0855 \\ 0.0112 & 0.3950 & 1.5980 & 2.5124 & 0.0051 & 0.8059 \end{pmatrix}. \tag{25}$$

Thus, Z, which is estimated according to Equation (26), is written as follows:

$$Z = \exp(+5)\begin{pmatrix} 0.0020 & 0.0214 & 0.0841 & 0.0971 & 0.0043 & 0.0351 \\ 0.0235 & 0.9816 & 1.6319 & 2.3210 & 0.8760 & 0.0767 \\ 0.0863 & 1.4535 & 7.7646 & 8.6541 & 0.7640 & 2.1580 \\ 0.0753 & 0.6741 & 8.6531 & 9.6280 & 0.6530 & 2.7657 \\ 0.0033 & 0.0673 & 0.5421 & 0.8076 & 0.0053 & 0.0951 \\ 0.0233 & 0.4216 & 2.2670 & 2.7678 & 0.0071 & 0.8719 \end{pmatrix}. \tag{26}$$

To obtain the BLR model on the basis of using Equation (22), H_1 and H_2 are applied as the weights matrix for the independent matrix and the prior distribution, respectively, which are calculated by Equations (27) and (28), as follows:

$$H_1 = \left(Z + \Omega^{-1}\right)^{-1} Z = \begin{pmatrix} 0.0014 & 0.0015 & -0.0046 & 0.0075 & 0.0049 & 0.0128 \\ -0.0037 & 0.0018 & -0.0023 & 0.0014 & -0.0078 & 0.0269 \\ -0.0064 & -0.0038 & 0.0047 & -0.0043 & 0.0642 & -0.0050 \\ -0.0057 & 0.0045 & 0.0033 & 0.0082 & -0.0534 & 0.0087 \\ 0.0038 & 0.0077 & -0.0026 & 0.0079 & 0.0155 & 0.0954 \\ 0.0036 & -0.0019 & 0.0072 & -0.8100 & 0.0137 & 0.2739 \end{pmatrix}. \tag{27}$$

$$H_2 = \left(Z + \Omega^{-1}\right)^{-1} \Omega^{-1} = \begin{pmatrix} -0.0011 & -0.0012 & 0.0043 & -0.0070 & -0.0043 & -0.0012 \\ 0.0035 & -0.0016 & 0.0019 & -0.0010 & 0.0760 & -0.0025 \\ 0.0060 & 0.0031 & -0.0042 & 0.0040 & -0.0610 & 0.0480 \\ 0.0046 & -0.0041 & -0.0029 & -0.0074 & 0.0481 & -0.0076 \\ -0.0033 & -0.0067 & 0.0019 & -0.0066 & -0.0143 & -0.0851 \\ -0.0030 & 0.0016 & -0.0065 & 0.7478 & -0.0127 & -0.2019 \end{pmatrix}. \tag{28}$$

Thus, the parameters are estimated and denoted by $b_0{}'$ for the platooning variables, as expressed in Equation (29):

$$b_0{}' = (+59.85\ -5.76\ -0.12\ -0.40\ -0.15\ -0.37)'. \tag{29}$$

Thereafter, by applying Equation (22), the Bayes estimator for the parameters of the BLR model is obtained from Equation (30):

$$b^*{}_{bayes}{}' = (73.64\ -9.76\ -0.04\ -0.25\ -0.55\ -0.87)'. \tag{30}$$

Therefore, the final BLR model is obtained by Equation (31), and shown in Table 7, as follows:

$$\hat{Y}_{bayes} = 73.64 - 9.76X_1 - 0.04X_2 - 0.25X_3 - 0.55X_4 - 0.87X_5. \tag{31}$$

4.3. Analysis of the Most-Influential Platooning Variable on Performance Measures

For an analysis of the effect of platooning variables and performance measures in MLR and BLR models, see Tables 4 and 7 and Figure 4, which depict the results of the highest relationships. As displayed in Figure 4, traffic flow has a strong relationship with NFPC, ATS, platoon speed, and FD, in that order. This means that an increase in traffic flow leads to a decrease in ATS and platoon speed and an increase in NFPC and FD. Further, according to the results of Figure 4, the most-influential platooning variable on performance measure regarding R^2 coefficient is the relationship between flow and NFPC, compared with other variables, as verified by the results in Tables 4 and 7. Thus, the best predictive model for representing the surrogate performance measure is the relationship between flow and NFPC in the MLR and BLR models.

Table 5. The prior and posterior distributions.

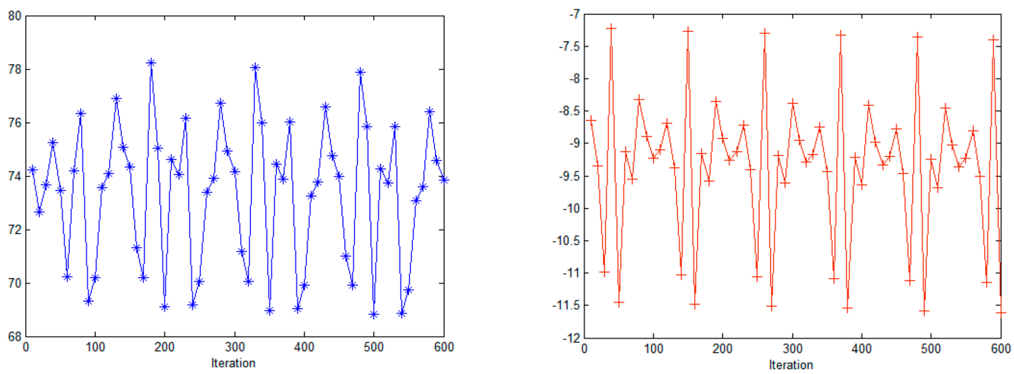
Variable	Prior Distributions		Posterior Distributions			
					HPD	
	Mean	Std. Deviation	Mean	Std. Deviation	Minimum	Maximum
Intercept	75.34	2.7	73.64	2.64	68.80	78.22
Flow (veh/h)	−10.99	1.29	−9.76	1.16	−11.61	−7.23
Opposing Flow (veh/h)	−0.09	0.08	−0.04	0.04	−0.13	0.03
%HV (%)	−0.23	0.009	−0.25	0.013	−0.26	−0.12
Platoon Size (veh/h)	−0.61	0.012	−0.55	0.014	−0.93	−0.19
Time Headway (s)	−1.23	0.015	−0.87	0.012	−1.32	−0.46

Notes: HPD: highest posterior density (HPD); σ^2: the square of standard deviation.

Table 6. Geweke convergence diagnostics and MCSE.

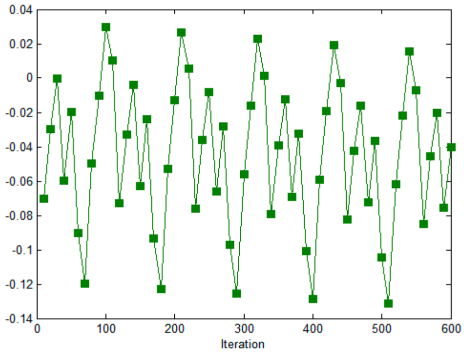
Variable	Geweke Diagnostics		MCSE
	z	*p*-Value	
Intercept	1.37	0.102	0.032
Flow (veh/h)	0.46	0.089	0.016
Opposing Flow (veh/h)	0.12	0.076	0.013
%HV (%)	−0.57	0.065	0.033
Platoon Size (veh/h)	−0.24	0.059	0.028
Time Headway (s)	−0.13	0.067	0.018

Notes: MCSE: Monte Carlo standard error; %HV: percentage of a heavy vehicles; Geweke's test is a statistical test for estimating the convergence of Markov chain Monte Carlo simulations [88].

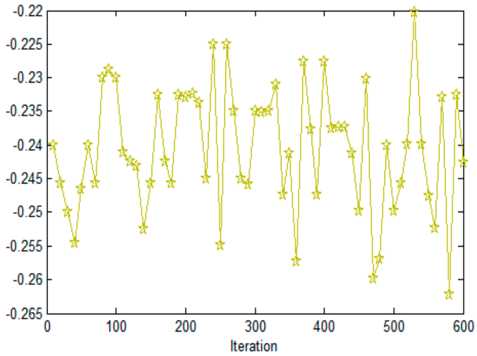


(**a**) (**b**)

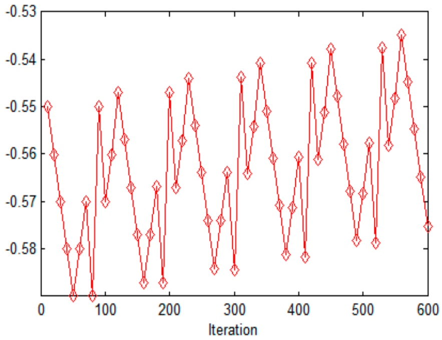
Figure 3. *Cont.*



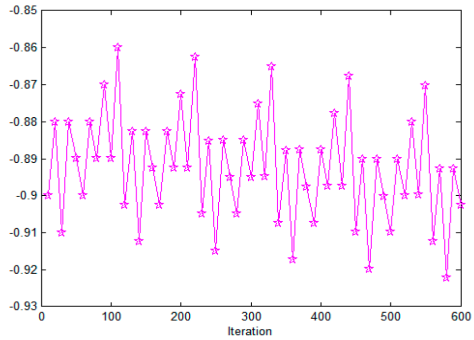
(**c**)



(**d**)

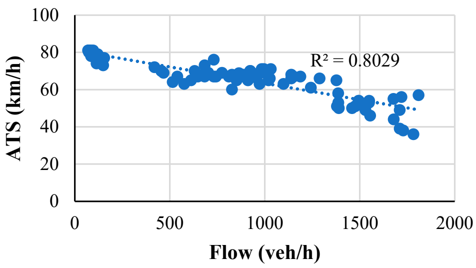


(**e**)

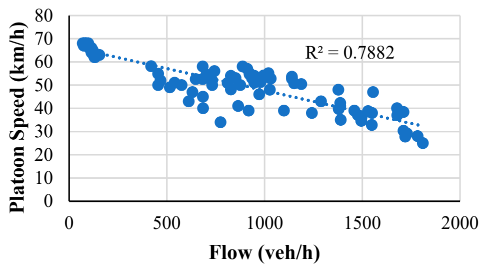


(**f**)

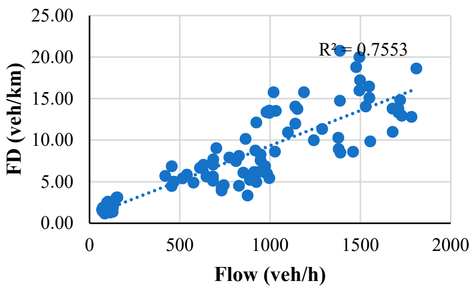
Figure 3. Trace plot of the Markov chain of ATS regarding the platooning variables. (**a**) Trace of intercept, (**b**) trace of flow (**c**) trace of opposing flow, (**d**) trace of %HV, (**e**) trace of platoon size, (**f**) trace of time headway. Notes: ATS: average travel speed; %HV: percentage of heavy vehicles.



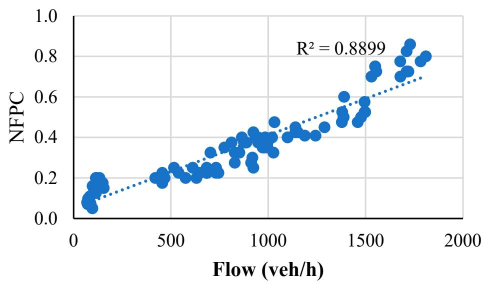
(**a**)



(**b**)



(**c**)



(**d**)

Figure 4. Relationship between traffic flow, platoon speed, FD, ATS, and NFPC: (**a**) Relationship between flow and ATS, (**b**) relationship between flow and platoon speed, (**c**) relationship between flow and FD, (**d**) relationship between flow and NFPC. Notes: ATS: average travel speed; FD: follower density; NFPC: number of followers per capacity.

Table 7. Coefficient for variables affecting performance measures.

Platooning Variables	Performance Measures																			
	ATS (km/h)		**ATSpc**		**ATSpc/FFSpc**		**ATS/FFS**		**PF (%)**		**FD (veh/km)**		**PI (%)**		**NFPC**		**NO (veh/h)**		**Platoon Speed (km/h)**	
	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**	β	***p*-Value**
Constant	73.64	0.001	98.65	0.003	0.39	0.002	0.41	0.001	−2.60	0.004	0.82	0.004	0.14	0.002	−0.009	0.001	−6.02	0.00	48.90	0.003
Flow (veh/h)	−9.76	0.003	−2.07	0.002	−77.08	0.058	−49.87	0.084	2.98	0.003	4.30	0.0003	2.44	0.002	0.67	0.001	3.02	0.003	−5.43	0.002
Opposing Flow (veh/h)	−0.04	0.002	−0.01	0.083	−0.003	0.091	−0.002	0.13	0.05	0.002	0.032	0.0005	0.003	0.00	0.005	0.013	0.080	0.083	−0.030	0.077
%HV (%)	−0.25	0.002	−0.57	0.51	−0.023	0.09	−0.016	0.07	0.14	0.001	0.54	0.004	0.005	0.003	0.007	0.073	0.48	0.091	−0.40	0.071
Platoon Size (veh/h)	−0.55	0.00	−0.35	0.002	−0.03	0.22	−0.011	0.31	0.18	0.10	0.39	0.091	0.72	0.007	0.006	0.12	0.10	0.08	−4.08	0.001
Time Headway (s)	−0.87	0.00	−0.18	0.002	−0.018	0.053	−0.060	0.43	0.34	0.18	0.10	0.003	0.002	0.09	0.004	0.002	−0.030	0.17	−0.040	0.07
SSR	2690.33		10,001.05		0.98		0.78		0.65		3989.06		0.70		6.78		38,022.68		8732.12	
SSE	410.29		5476.12		0.86		0.52		0.22		1043.23		0.20		0.54		14,110.20		1721.08	
SST	3100.62		15,477.17		1.84		1.30		0.87		5032.29		0.90		7.32		52,132.88		10,453.20	
R^2	0.87		0.65		0.53		0.60		0.75		0.80		0.78		0.93		0.73		0.84	
F (*p*-value)	183.06 (0.00)		63.55 (0.00)		43.80 (0.00)		58.71 (0.00)		81.20 (0.00)		130.51 (0.00)		96.22 (0.00)		225.08 (0.00)		70.49 (0.00)		159.20 (0.00)	

Notes: average travel speed (ATS); average travel speed of passenger cars (ATS_{PC}); average travel speed as a percentage of free-flow speed (ATS/FFS); ATS_{PC} as a percentage of free-flow speed of passenger cars (ATSpc/FFSpc); percentage impeded (PI); number of overtaking (NO) vehicles; follower density (FD); percentage of followers (PF); percentage of heavy vehicles (%HV); number of followers per capacity (NFPC). Sum of squares regression (SSR); sum of squares error (residual) (SSE); sum of squares total (SST).

Further, for the validation of and a comparative evaluation of the proposed models regarding the best fit for platooning variables and performance measures, the results of the predicted performance measures are examined via empirical data, and each measure's error (MAPE) under platooning variables is presented in Figure 5. As shown in Figure 5, the BLR model has worse error prediction performance compared with the MLR model. Further, the lowest error is related to NFPC in each model, suggesting that selecting this measure is an optimal choice as a surrogate performance measure among the performance measures, compared with other measures. Moreover, regarding the relationship between flow and NFPC in both models, the results of the best fit models and errors are obtained for MAPE according to Equation (23) (Table 8 and Figure 6). According to Table 8, the BLR model is the best predictive model for evaluating NFPC under traffic flow. Further, it can be inferred that the BLR model can predict NFPC as the most-influential performance measure to classify the LOS of two-lane roads by using statistical analysis and overall error values (Figure 6). Therefore, using BLR, based on the conjugate Bayesian analysis, is proposed as a predictive tool for determining the effect of platooning variables on performance measures thanks to its fewer errors and high prediction performance compared with the MLR model. The results indicate that conjugate Bayesian analysis offers better prediction performance in comparison with the MLP model. These results are the same as the results of other studies, which showed that the BLR model is better than MLR in other engineering fields [30,76,77].

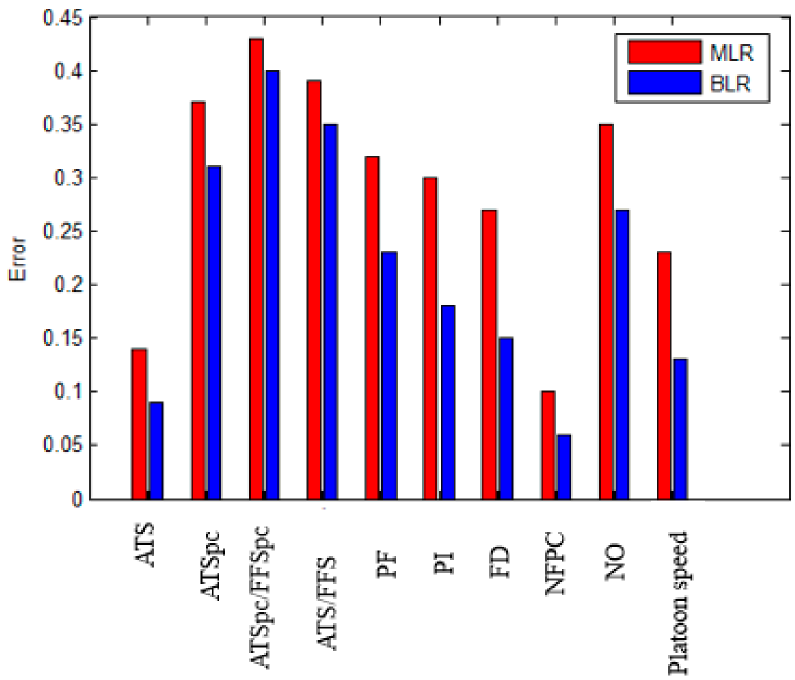


Figure 5. The error values of the proposed models. Notes: ATS: average travel speed; ATS_{PC}: average travel speed of passenger cars; ATS/FFS: average travel speed as a percentage of free-flow speed; ATSpc/FFSpc: ATS_{PC} as a percentage of free-flow speed of passenger cars; PI: percentage impeded; NO: number of overtaking (vehicles); FD: follower density; PF: percentage of followers; %HV: percentage of heavy vehicles; NFPC: number of followers per capacity; MLR: multiple linear regression; BLR: Bayesian linear regression.

Table 8. Comparison of the best fit of the proposed models.

Model	Best Fit	R^2 Value	MAPE
BLR	$NFPC = 0.0003Flow + 4.02 \times 10^{-8}Flow^2$	0.93	0.09
MLR	$NFPC = 0.0002Flow + 9 \times 10^{-8}Flow^2$	0.70	0.20

Notes: MLR: multiple linear regression; BLR: Bayesian linear regression; MAD: mean average deviation; RMSE: root mean square error; MAPE: mean absolute percentage error.

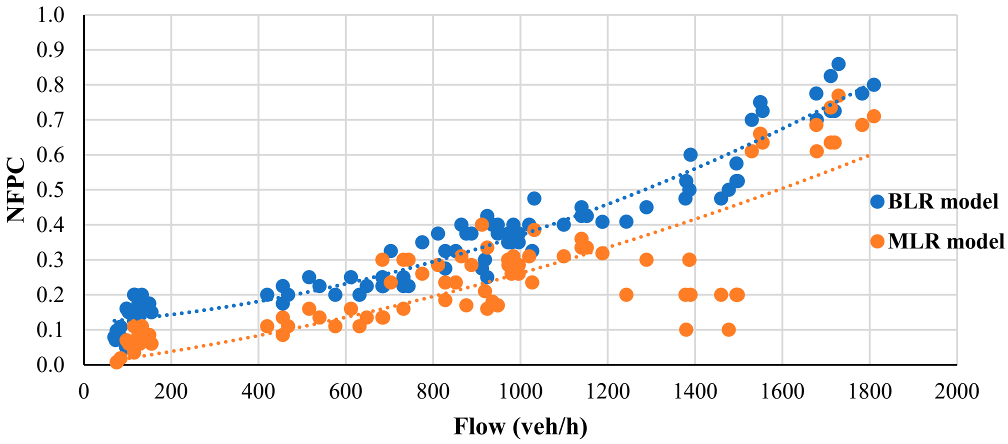


Figure 6. Relationships between flow and NFPC. Notes: MLR: multiple linear regression; BLR: Bayesian linear regression; NFPC: number of followers per capacity.

4.4. Evaluation of the LOS by Using the Preferred Performance Measure

To evaluate the LOS in two-lane roads, the surrogate measure is selected as the preferred performance measure for the proposed models and the average values. As displayed in Figures 4–6, the proposed surrogate performance measure is selected as NFPC thanks to its higher statistical criteria compared with those of other performance measures. Thus, the results of the proposed classification for the LOS are selected according to the BLR model (Table 9). This measure can classify LOS A and LOS B as between 0.20 and 0.40, respectively. Further, according to this measure, LOS C to LOS D ranges from 0.40 to 0.80. However, the HCM [22] determines the ATS to be greater than 88 km/h and PTSF to be less than 35% for LOS A. Moreover, LOS C to LOS E is classified by using an ATS between 64 and 80 km/h and a PTSF between 50% and 80%.

Table 9. Classification of the LOS, based on the present study and the HCM [22].

LOS	The HCM [22]		This Study
	ATS (km/h)	PTSF (%)	NFPC
A	>88	≤35	0.20≤
B	>80–88	>35–50	>0.20–0.40
C	>72–80	>50–65	>0.40–0.60
D	>64–72	>65–80	>0.60–0.80
E	≤64	>80	>0.80

Notes: LOS: level of service; ATS: average travel speed; PTSF: percent time spent following; NFPC: number of followers per capacity; HCM: Highway Capacity Manual.

Likewise, Table 10 illustrates the comparison between capacity and the LOS with the HCM [22] for the selected four sites of the two-lane roads on the basis of the NFPC. As demonstrated in Table 10, the HCM [22] classifies the LOS for the studied roads from C to E by using the ATS and PTSF, respectively, while in the present study, the LOS is classified from B to D according to NFPC measure. Thus, according to the surrogate measure in the present study, the LOS for the Rasht-Jirdeh and Kiasar-Sari roads is classified as B. However, regarding the HCM [22], the LOS for the Rasht-Jirdeh, Kiasar-Sari, and Fuman-Saravan roads is classified as C and D. Further, for the Fuman-Saravan and Rasht-Somesara roads with high traffic flow, the LOS is classified as C and D, respectively, on the basis of the NFPC. Accordingly, the proposed surrogate measure can effectively predict the LOS for the unsaturated and saturated conditions in two-lane roads when compared to the HCM [22]. Therefore, it can be concluded that for class I roads, the NFPC could act as a surrogate performance measure to better predict the LOS of two-lane roads under unsaturated and saturated conditions compared to using the two measures of the HCM [22], namely PTSF and ATS.

Table 10. Comparison between capacity and the LOS with the HCM [22] for two-lane roads.

Two-Lane Roads	NFPC	LOS	
		This Study	The HCM (2010) [22]
Fuman-Saravan	0.48	C	D
Rasht-Jirdeh	0.32	B	C
Rasht-Somesara	0.70	D	E
Kiasar-Sari	0.23	B	C

Notes: LOS: level of service; ATS: average travel speed; PTSF: percent time spent following; NFPC: number of followers per capacity; HCM: Highway Capacity Manual.

4.5. Policy Implications

The results of the present study show that vehicular platooning is an important phenomenon on two-lane roads. This traffic phenomenon involves traffic-flow characteristics on these roads. Regarding the importance of these roads for rural areas, evaluating traffic-flow characteristics and developing performance measures on two-lane roads will help road and transportation organizations to improve the capacity of these roads under high traffic demands in the future. Thus, to facilitate and increase the satisfaction of users on two-lane roads, the first step is to accurately evaluate traffic-flow characteristics on the basis of the developed performance measures.

The quality of two-lane roads should be examined on the basis of the obtained NFPC in the present study rather than ATS and PTSF because the NFPC could efficiently be related to the capacity of drivers and the number of vehicles following others. However, other studies have focused on the assessment of the quality of two-lane roads by using ATS and PTSF, and these studies did not mention the relation to the capacity of roads [22,48,89]. Because the complexity of the LOS of two-lane roads at near free-flow speed and under congested traffic-flow conditions increases, the measurement of the LOS under these conditions might not provide an accurate LOS to traffic engineers, given that the obtained results on performance measures such as ATS and PTSF in the present study showed that these measures do not provide exact predictions.

Furthermore, to stabilize the traffic flow on two-lane roads, it is necessary to connect the formation of platoons to capacity, as obtained from these variables in the present study, thanks to the developed performance measure models. These models, owing to their including the formation of platoons of vehicles and their effects on traffic-flow characteristics, are reliable and inform optimal policy recommendations for future traffic environments involving two-lane roads. Further, the criterion for the formation of platooning vehicles regarding time headway and the gap acceptance of drivers helps to count the number of followers in the queues on two-lane roads. Thus, to obtain the optimal policy regarding the driver satisfaction on two-lane roads under traffic conditions, the present study recommends the formation of platooning vehicles at intervals higher than 2.4 s, which decreases as the speed of drivers increases. Additionally, NFPC, compared with ATS and PTSF, could sufficiently work under various traffic conditions.

Moreover, new models must account for BLR, compared with conventional models such as MLR, to offer better performance at predicting the LOS for two-lane roads. Therefore, regarding the present threshold of forming platooning vehicles (2.4 s), the proposed performance measures (NFPC), and the BLR model, policymakers could enhance the traffic environment for drivers on two-lane roads.

5. Conclusions

Two-lane rural roads play important roles in urban transportation and accessibility. Vehicular platooning has been a challenging issue for engineers aiming to propose an optimal performance measure for evaluating the traffic performance in two-lane roads. Therefore, the present study aimed first to assess the effect of vehicular platooning variables on performance measures at four selected two-lane roads in Iran. The vehicular platooning variables included traffic flow, time headway, platoon size, %HV, and opposing flow. Performance measures included ATS, FD, PI, PF, NFPC, ATSpc, ATSpc/FFSpc, ATS/FFS, NO, and platoon speed. Further, this study identified the critical headways for the formation of vehicular platooning on the basis of vehicle-gap-acceptance behavior. The MLR and BLR models were applied to develop performance measurement models and find the best fit model for the surrogate measure and statistical criteria. The obtained results are described as follows:

1. According to the vehicle-gap-acceptance behavior, it was found that headways less than 2.4 s were identified as the thresholds for forming platooning in two-lane roads.
2. The results of the Pearson correlation indicated that the traffic flow has the highest correlation with NFPC, ATS, platoon speed, FD, PI, PF, NO, and ATS_{PC}, in that order. Moreover, the opposing flow strongly correlated with NFPC, PF, ATS, FD, and PI, in that order. Further, by examining the relationship between the %HV and performance measures, it can be concluded that the %HV had a high correlation with FD, ATS, PF, and PI, in that order. The relationships between platoon size and platoon speed, PI, ATS, and ATSpc are significant, in that order. Time headway was also significantly correlated with ATS, NFPC, ATSpc, and FD, in that order.
3. The results of the developed MLR and BLR models indicated that BLR could predict NFPC as the most-influential performance measure to classify the LOS of two-lane roads on the basis of accuracy and error values.
4. The comparison evaluation from the proposed surrogate measures with performance measures recommended in the HCM [22] indicated that the NFPC could be a surrogate performance measure for better predicting the LOS under unsaturated and saturated conditions compared to the two measures of the HCM [22], namely PTSF and ATS, under vehicular platooning.

The results of the present study could be useful for traffic engineers and road organizations aiming to estimate the quality of two-lane roads on the basis of the proposed performance measure. Further, the criterion for the formation of vehicular platooning regarding time headway and the gap acceptance of drivers on two-lane roads could help traffic engineers to accurately investigate the beginning of the formation of platooning for calculating queues and platoon size. Another application of the present study is the development of new models for evaluating the quality of two-lane roads in terms of the LOS and vehicular platooning and comparing them with conventional models, such as regression models. Other macroscopic performance models can be developed on the basis of the proposed surrogate performance measure in the present study to calibrate the models according to the traffic conditions of other sites. From a road safety aspect, this research could improve the quality of two-lane roads for the improvement of the driver's gap-acceptance behavior, which is involved in the proposed surrogate performance measure of two-lane roads for preventing the occurrence of collisions that are due to the formation of platoons and due to overtaking maneuvers.

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Appendix A

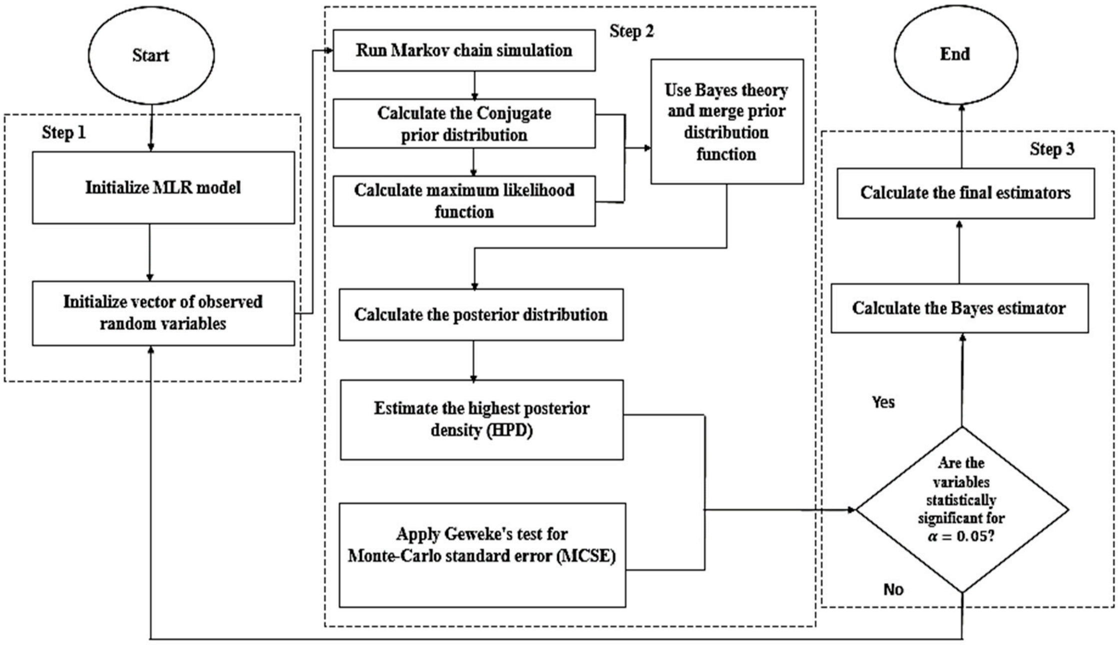

Figure A1. Flowchart of the BLR model. Notes: BLR: Bayesian linear regression; MLR: multiple linear regression.

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