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Mining Method Optimization of Difficult-to-Mine Complicated Orebody Using Pythagorean Fuzzy Sets and TOPSIS Method

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Abstract: In Suichang gold mine, the altered rock type gold deposits were cut by faults and joint fissures, leading to complex resource endowment characteristics, large changes in occurrence, a serious complex of ore vein branches and great difficulty in mining. In order to select a suitable mining method for such a difficult and complicated orebody, a multi-factor and multi-index comprehensive evaluation system involving benefits, costs, safety and other aspects was constructed by using the Pythagorean fuzzy sets and TOPSIS method. Taking Suichang gold mine as an example, the weighted aggregation evaluation matrix was constructed, the closeness index of the four mining schemes were 0.8436, 0.3370, 0.4296 and 0.4334, and the mechanized upward horizontal layering method was determined as the optimal scheme. This method overcame the fuzzy comparison of economic and technical indicators directly, but converted them into corresponding fuzzy numbers to obtain accurate closeness index for optimization. The application of this method not only ensured a safe, efficient and environment-friendly mining effect, but also provided a reference for the optimization of the mining scheme of the severely branched composite orebody.

Keywords: mining method optimization; difficult-to-mine complicated orebody; multiple attribute decision making; Pythagorean fuzzy sets and TOPSIS method; mechanized upward horizontal layering method

Citation: Li, S.; Huang, Q.; Hu, B.; Pan, J.; Chen, J.; Yang, J.; Zhou, X.; Wang, X.; Yu, H. Mining Method Optimization of Difficult-to-Mine Complicated Orebody Using Pythagorean Fuzzy Sets and TOPSIS Method. *Sustainability* **2023**, *15*, 3692. https://doi.org/10.3390/su15043692

Academic Editor: Jianjun Ma

Received: 4 January 2023
Revised: 8 February 2023
Accepted: 13 February 2023
Published: 16 February 2023

1. Introduction

In the design process of new mines and reconstruction and upgrading of mines, the design of mining methods is an important part and is the core of the whole design work, which directly determines the subsequent technical personnel allocation, production organization management and industrial supporting facilities [1–3]. Choosing the most appropriate mining method to mine the deposit is very important to the safety, economy and environmental protection of mining operations and affects the benefits and long-term development of mining companies [4].

For a continuous orebody with regular shape, the most appropriate mining method can be selected by comparing the advantages and disadvantages and the economic and technical indicators. However, taking the difficult-to-mine complicated orebody (DCO) (see Figure 1) as example, it was greatly affected by faults and joint fissures, leading to obvious branching and compounding phenomenon. Since it is difficult to determine the optimal plan through traditional methods for DCO, more reliance is placed on the experience of design decision-makers and limited data, through the comprehensive comparison of various indicators. This is typical of Multiple Attribute Decision Making (MADM).

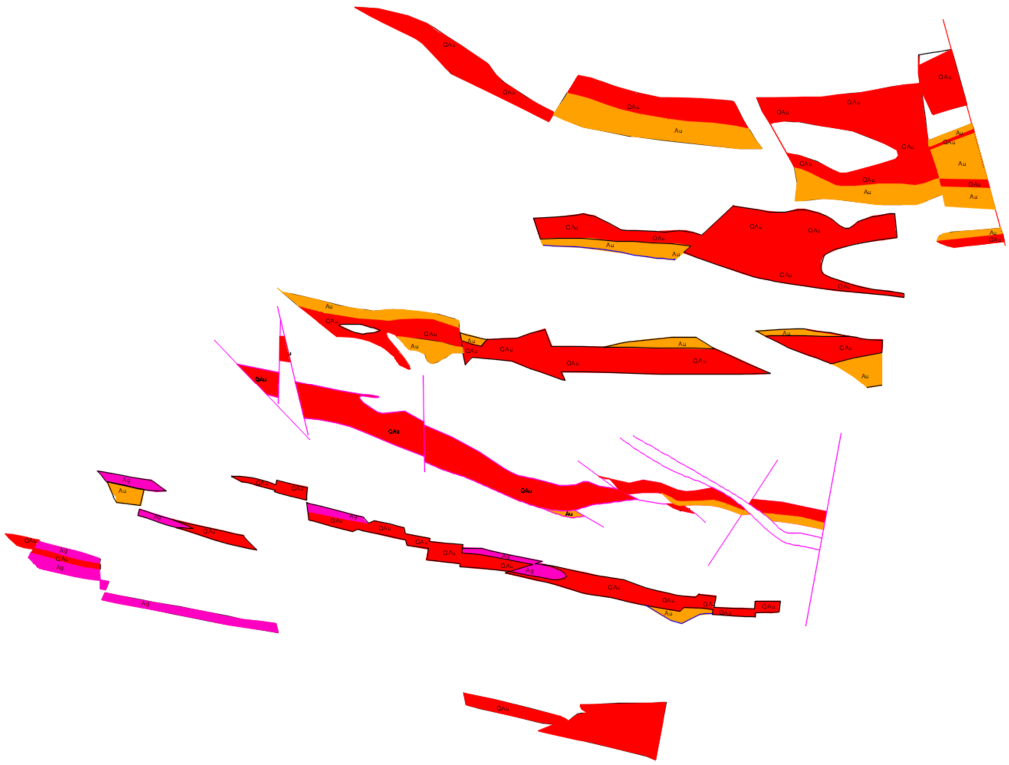

Figure 1. Plan of 140–260 m middle section of Suichang Gold Mine.

Multiple Attribute Decision Making refers to the decision making problem of selecting the best alternative or ranking under a condition of considering multiple indicators [5]. In order to solve such problems, researchers have developed a variety of multi criteria decision-making methods; typical representative methods include the Analytic Hierarchy Process [6], Entropy method [7], CRITIC method [8], TOPSIS method [9], GST (Grey Target Decision) method [10], DEA method [11], VIKOR method [12], Fuzzy comprehensive evaluation method [13], etc.

However, with increase in the complexity and scientific requirements of evaluation, a single decision-making method cannot guarantee optimal or accurate results [14,15]. Under this background, researchers have explored and developed mixed decision-making methods, for example, mixtures of Analytic Hierarchy Process and fuzzy, Entropy weight method and TOPSIS, Analytic Hierarchy Process, fuzzy set and VIKOR, fuzzy set and TOPSIS, etc.

In the field of mining engineering, many experts rely on these decision-making methods to optimize mining methods, for example, Karimnia [16] proposed the fuzzy analytical hierarchy process method, the most suitable method selected for Iran's Qapliq salt mine. Yavuz [17] used the AHP method and fuzzy multiple attribute decision-making method, respectively, in a lignite mine located in Istanbul, carried out sensitivity analysis of the two methods and concluded that the Room and pillar method with filling is the most appropriate method. Qinqiang Guo [18] used the mixed method of AHP to determine the index weight and TOPSIS to rank, selecting the most suitable mining method from the Soft Broken Complex Orebody, and achieved very good results, Iphar [19] is committed to developing a mobile application, integrating several decision-making methods, and the optimal mining method can be obtained by inputting the original parameters for reference by engineering researchers.

However, in view of the complexity of mining method selection, simple expert scoring cannot fully reflect the fuzzy information, and the cognitive differences between different experts are easy to cause distortion of results. In terms of sorting, different ranging methods may have different results. In this case, the development of fuzzy set theory provides a good idea for solving such problems [20]. Bajić [21] transformed the indicators into triangular fuzzy numbers, constructed a fuzzy decision matrix and a fuzzy performance matrix, used to select the optimal alternative, and verified this through sensitivity analysis. Memori [22] uses the TOPSIS method based on intuitionistic fuzzy sets, providing an accurate sustainable ranking of suppliers and a relevant solution for sustainable sourcing decisions that is validated through a real-world case study. Narayanamoorthy [23] selected the best scheme for the selection of industrial robots by using a combination of Interval-valued intuitionistic hesitant fuzzy entropy and VIKOR.

Pythagorean fuzzy sets (PFS) generalized by Yager [24] is a new method to deal with fuzzy problems. Its main contribution is to go beyond the limit that the sum of membership and no membership of fuzzy sets is less than 1 (see Figure 2). Compared with other fuzzy sets such as intuitionistic fuzzy sets, it can more fully and accurately represent uncertain information [25]. In view of this, this paper introduces a TOPSIS method based on PFS [26]. The framework of this PFS–TOPSIS method is illustrated (see Figure 3), which is used for mining method decision-making for the Suichang Gold Mine in Zhejiang Province, China, and has achieved good results in actual production.

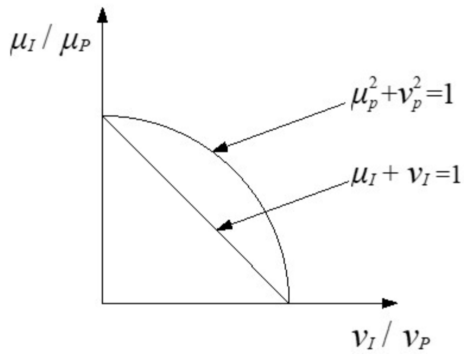

Figure 2. Spatial comparison between PFS and other fuzzy sets.

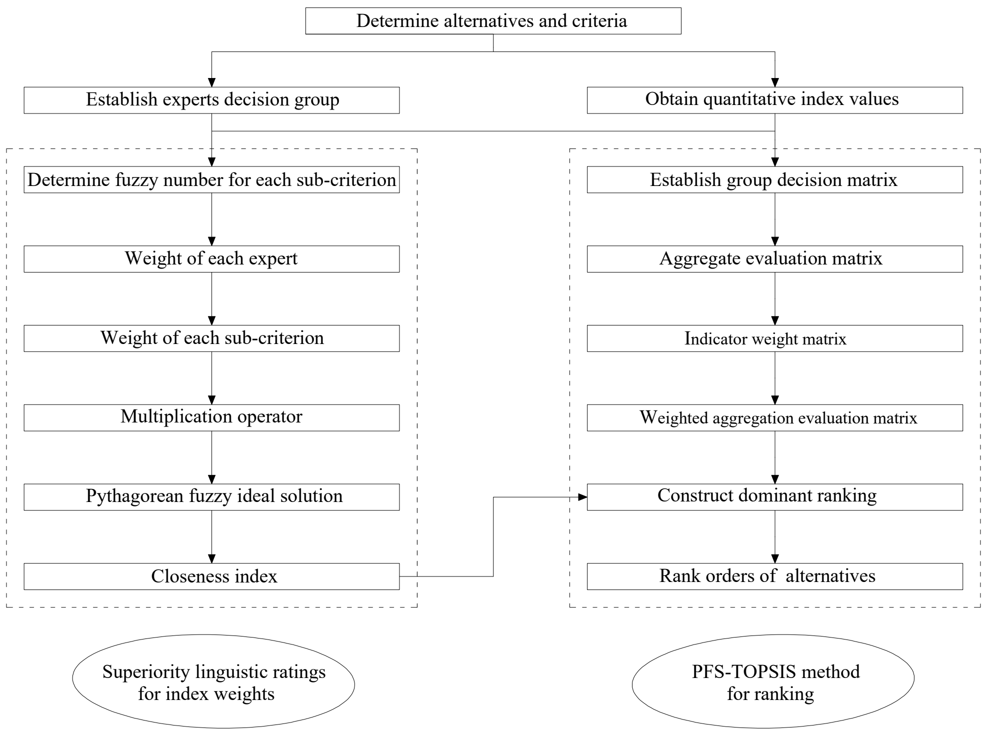

Figure 3. Framework of PFS-TOPSIS method.

2. Methods Introduction

2.1. Introduction of PFS Method

Definition 1. *Let X be a universe of discourse [27,28]. The PFS ξ on X is given by Equation (1).*

$$\xi = \{ [x, \mu_\xi(x), \nu_\xi(x) | x \epsilon X] \} \tag{1}$$

where the functions $\mu_\xi(x)$:X $\rightarrow$ [0, 1] and $\nu_\xi(x)$: X $\rightarrow$ [0, 1] define the degree of membership and the degree of non-membership of the element $x \in X$ to the set ξ, respectively, with the condition that $0 \leq (\mu_\xi(x))^2 + (\nu_\xi(x))^2 \leq 1, \forall x \in X$. $\pi_\xi(x) = \sqrt{1 - (\mu_\xi(x))^2 - (\nu_\xi(x))^2}$ is called the degree of indeterminacy of element $x \in X$. For convenience, they are called ($\mu_\xi(x)$, $\nu_\xi(x)$) and a Pythagorean fuzzy number (PFN) denoted by $\xi = (\mu_{\xi,}\nu_{\xi,})$ [29].

Definition 2. *For the collection $\xi_i = (\mu_{\xi,}\nu_{\xi,})$ (i = 1,2, ... n) of the of PFNs with the weight vector $w = (w_1\ w_2 \ldots \ldots w_n)$ of ξ_i(i = 1,2, ... n) such that $\sum_{i=1}^{n} w_i = 1$, the Pythagorean fuzzy weighted averaging (PFWA) operator and the Pythagorean fuzzy weighted geometric (PFWG) operator can be defined as in Equations (2) and (3), respectively [30].*

$$\begin{aligned} PFWA_w(\xi_1, \xi_2, \cdots, \xi_n) &= w_1\xi_1 \oplus w_2\xi_2 \oplus \cdots \oplus w_n\xi_n \\ &= \left(\sqrt{1 - \prod_{i=1}^{n} \left(1 - (\mu_{\xi_i})^2\right)^{w_i}}, \prod_{i=1}^{n} (\nu_{\xi_i})^{w_i} \right), \end{aligned} \tag{2}$$

$$\begin{aligned} PFWG_w(\xi_1, \xi_2, \cdots, \xi_n) &= w_1\xi_1 \otimes w_2\xi_2 \otimes \cdots \otimes w_n\xi_n \\ &= \left(\prod_{i=1}^{n} (\mu_{\xi_i})^{w_i}, \sqrt{1 - \prod_{i=1}^{n} \left(1 - (\upsilon_{\xi_i})^2\right)^{w_i}} \right). \end{aligned} \tag{3}$$

Definition 3. *Let A and B be PFSs of X = {x_1, x_2, . . . x_n} [27]. Then, the sum of A and B is defined as Equation (4).*

$$\begin{aligned} A \oplus B = \quad & \{ \langle x, \sqrt{(\mu_A(x))^2 + (\mu_B(x))^2 - (\mu_A(x))^2 (\mu_B(x))^2}, \\ & \nu_A(x)\nu_B(x) \rangle \mid x \in X \}, \end{aligned} \tag{4}$$

The product of A and B is defined as Equation (5).

$$\begin{aligned} A \otimes B = \quad & \{ \langle x, \mu_A(x)\mu_B(x), \\ & \sqrt{(\nu_A(x))^2 + (\nu_B(x))^2 - (\nu_A(x))^2 (\nu_B(x))^2} \rangle \mid x \in X \}. \end{aligned} \tag{5}$$

2.2. Introduction of TOPSIS Method

The TOPSIS method was developed by Hwang [31], first put forward in 1981, and is a method of ranking according to the closeness of a limited number of evaluation objects to the ideal target. After years of development, dozens of derivative methods have been created by combining various mathematical theories, and have been widely used in various fields such as economic, management, engineering, medicine, etc. Its core ideas and steps are as follows [32]:

(1) Quantification of evaluation indicators, converting natural language into numbers, and ensuring a certain distinction between good and bad, D is the evaluation objective and X is the evaluation index. The characteristic matrix is defined as Equation (6).

$$D = \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \ldots & x_{in} \\ \vdots & & \vdots & & \vdots \\ x_{m1} & \ldots & x_{mj} & \ldots & x_{mn} \end{bmatrix} = \begin{bmatrix} D_1(x_1) \\ \vdots \\ D_i(x_j) \\ \vdots \\ D_m(x_n) \end{bmatrix} = \left[X_1(x_1), \ldots X_j(x_i), \ldots, X_n(x_m)\right]. \quad (6)$$

(2) Normalize the characteristic matrix, obtain the normalized vector r_{ij} and establish the normalized matrix about the normalized vector. This is defined as Equation (7).

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \quad (7)$$

(3) Normalize the value v_{ij} by calculating the weight; weight normalization matrix is defined as Equation (8).

$$\nu_{ij} = w_j r_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \quad (8)$$

(4) Determine ideal solution A^+ and anti-ideal solution A^-; in the ideal solution and anti-ideal solution, J_1 is the optimal value of profitability index set expressed on the i index; J_2 is the worst value of the i index of the loss index set. A^+ and A^- are defined as Equations (9) and (10).

$$A^+ = \left(max_i \nu_{ij} \middle| j \in J_1\right), \left(min_i \nu_{ij} \middle| j \in J_1\right), \middle| i = 1, 2, \ldots, m = \nu_1^+, \nu_2^+, \ldots, \nu_j^+, \ldots, \nu_n^+. \quad (9)$$

$$A^- = \left(min_i \nu_{ij} \middle| j \in J_1\right), \left(max_i \nu_{ij} \middle| j \in J_1\right), \middle| i = 1, 2, \ldots, m = \nu_1^-, \nu_2^-, \ldots, \nu_j^-, \ldots, \nu_n^-. \quad (10)$$

(5) Calculate the distance S^+ from the target to the ideal solution A^+ and the distance S^- from the target to the ideal solution A^-. The distances are defined as Equation (11).

$$S^+ = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - \nu_j^+\right)^2}, S^- = \sqrt{\sum_{j=1}^{n} \left(V_{ij} - \nu_j^-\right)^2}, i = 1, 2, \ldots, m. \quad (11)$$

(6) Calculate the closeness index of the ideal solution. It is defined as Equation (12).

$$C_i^+ = \frac{S_i^-}{\left(S_i^+ + S_i^-\right)}, i = 1, 2, \ldots, m. \quad (12)$$

(7) Ranking according to the size of the ideal pasting progress.

2.3. Distance Measures and Similarity Measures for PFS

Distance measure for PFSs is a term that describes the difference between PFS. Let A and B be PFSs of $X = \{x_1, x_2, \ldots x_n\}$ with three parameters $\mu(x)$, $\nu(x)$ and $\pi(x)$. Here, some distance measures (DM) are presented for PFSs.

The normalized Hamming distance is defined as Equation (13).

$$d_{PFS}(A, B)_{nH} = \frac{1}{2n} \sum_{i=1}^{n} \left(\mid \mu_A{}^2(x_i) - \mu_B{}^2(x_i) \mid + \mid \nu_A{}^2(x_i) - \nu_B{}^2(x_i) \mid + \mid \pi_A{}^2(x_i) - \pi_B{}^2(x_i) \mid \right) \quad (13)$$

The normalized Euclidean distance is defined as Equation (14).

$$d_{PFS}(A, B)_{nE} = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} \left(\left(\mu_A{}^2(x_i) - \mu_B{}^2(x_i)\right)^2 + \left(\nu_A{}^2(x_i) - \nu_B{}^2(x_i)\right)^2 + \left(\pi_A{}^2(x_i) - \pi_B{}^2(x_i)\right)^2 \right)} \quad (14)$$

The normalized Hausdorff distance is defined as Equation (15).

$$d_{PFS}(A,B)_{nHd} = \frac{1}{n}\sum_{i=1}^{n} \max\left[\mid {\mu_A}^2(x_i) - {\mu_B}^2(x_i) \mid, \mid {\nu_A}^2(x_i) - {\nu_B}^2(x_i) \mid\right] \tag{15}$$

For convenience, the above is called formula d_1, d_2 and d_3, and d_4 is defined as Equation (16).

$$d_4(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mid {\mu_A}^2(x_i) - {\mu_B}^2(x_i) \mid + \mid {\nu_A}^2(x_i) - {\nu_B}^2(x_i) \mid}{{\mu_A}^2(x_i) + {\mu_B}^2(x_i) + {\nu_A}^2(x_i) + {\nu_B}^2(x_i)} \tag{16}$$

Like measure distance, similarity distance is also an important parameter between fuzzy sets. Let f be a monotone decreasing function. Then, the similarity measure between PFSs A and B can be defined as Equation (17).

$$s(A,B) = \frac{f(d(A,B)) - f(1)}{f(0) - f(1)} \tag{17}$$

By defining f, different similarity measures are obtained. Here, some simple methods are introduced. When $f(x) = 1 - x$, the similarity measure is defined as Equation (18).

$$s(A,B) = 1 - d(A,B) \tag{18}$$

When $f(x) = 1/(1 + x)$, the similarity measure is defined as Equation (19).

$$s(A,B) = \frac{1 - d(A,B)}{1 + d(A,B)} \tag{19}$$

2.4. PFS–TOPSIS Method for MADM

The Pythagorean fuzzy set has a broader value space than the traditional fuzzy set and can represent uncertain information in more detail. In addition, with better adaptability, the combination with other MADM methods has achieved many successful cases. TOPSIS is a classic evaluation or ranking method. Based on this, this section introduced a TOPSIS method based on the PFS. The detailed procedure is presented in the following:

Step 1: First, determine alternatives, criteria and experts, and also determine the corresponding transformation relationship between natural language and fuzzy numbers.

Step 2: Establish a group decision matrix scored by experts R = $(x_{(k)\ ij})_{l\times m}$, which can be defined as Equation (20).

$$x_{ij}^{(k)} = \left\{ \left[\mu_{A_i}(C_j)^1, \nu_{A_i}(C_j)^1, \pi_{A_i}(C_j)^1\right] \ldots \left[\mu_{A_i}(C_j)^k, \nu_{A_i}(C_j)^k, \pi_{A_i}(C_j)^k\right] \ldots \left[\mu_{A_i}(C_j)^n, \nu_{A_i}(C_j)^n, \pi_{A_i}(C_j)^n\right] \right\}. \tag{20}$$

This represents PFS formed by n experts' evaluation of a certain index of a certain scheme. For convenience, $(\mu_{A_i}(C_j)^k, \nu_{A_i}(C_j)^k, \pi_{A_i}(C_j)^k)$ is represented by $(\mu_{ij}^k, \nu_{ij}^k, \pi_{ij}^k)$. Therefore, the group decision matrix is obtained as Equation (21).

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_l \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} ((\mu_{11}^1,\nu_{11}^1,\pi_{11}^1)\cdots(\mu_{11}^n,\nu_{11}^n,\pi_{11}^n)) & ((\mu_{12}^1,\nu_{12}^1,\pi_{12}^1)\cdots(\mu_{12}^n,\nu_{12}^n,\pi_{12}^n)) & \cdots & ((\mu_{1m}^1,\nu_{1m}^1,\pi_{1m}^1)\cdots(\mu_{1m}^n,\nu_{1m}^n,\pi_{1m}^n)) \\ ((\mu_{21}^1,\nu_{21}^1,\pi_{21}^1)\cdots(\mu_{21}^n,\nu_{21}^n,\pi_{21}^n)) & ((\mu_{22}^1,\nu_{22}^1,\pi_{22}^1)\cdots(\mu_{22}^n,\nu_{22}^n,\pi_{22}^n)) & \cdots & ((\mu_{2m}^1,\nu_{2m}^1,\pi_{2m}^1)\cdots(\mu_{2m}^n,\nu_{2m}^n,\pi_{2m}^n)) \\ \vdots & \vdots & \ddots & \vdots \\ ((\mu_{l1}^1,\nu_{l1}^1,\pi_{l1}^1)\cdots(\mu_{l1}^n,\nu_{l1}^n,\pi_{l1}^n)) & ((\mu_{l2}^1,\nu_{l2}^1,\pi_{l2}^1)\cdots(\mu_{l2}^n,\nu_{l2}^n,\pi_{l2}^n)) & \cdots & ((\mu_{lm}^1,\nu_{lm}^1,\pi_{lm}^1)\cdots(\mu_{lm}^n,\nu_{lm}^n,\pi_{lm}^n)) \end{bmatrix} \end{array} \tag{21}$$

Step 3: Since the knowledge level, experience and focus of each expert are different, the importance of each expert is different, so the weight σ_k of each expert should be determined according to certain standards. At the same time, for the evaluation of the same indicator of the same scheme, the individual opinions of all experts need to be aggregated into a general evaluation view, i.e., transforming a PFS $x_{(k)_{ij}}$ into a Pythagorean fuzzy number x_{ij} = $(\mu_{A_i}(C_j)$, $\nu_{A_i}(C_j)$, $\pi_{A_i}(C_j))$. For convenience, this is expressed as x_{ij} = $(\mu_{ij}\ \nu_{ij}\ \pi_{ij})$; this transformation process is realized through the Python fuzzy aggregated averaging (PFWA) operator, which can be defined as Equation (22).

$$\begin{aligned} x_{ij} &= PFWA_\sigma\left(x_{ij}^1, x_{ij}^2, \cdots, x_{ij}^k, \cdots, x_{ij}^n\right) \\ &= \sigma_1 x_{ij}^1 \oplus \sigma_2 x_{ij}^2 \oplus \cdots \oplus \sigma_k x_{ij}^k \oplus \cdots \oplus \sigma_n x_{ij}^n \\ &= \left(\sqrt{1-\prod_{k=1}^{n}\left(1-\left(\mu_{ij}^k\right)^2\right)^{\sigma_k}}, \prod_{k=1}^{n}\left(\nu_{ij}^k\right)^{\sigma_k},\right. \\ &\quad \left.\sqrt{\prod_{k=1}^{n}\left(1-\left(\mu_{ij}^k\right)^2\right)^{\sigma_k} - \left(\prod_{k=1}^{n}\left(\nu_{ij}^k\right)^{\sigma_k}\right)^2}\right). \end{aligned} \tag{22}$$

At the same time, the aggregation evaluation matrix can be obtained as Equation (23).

$$R_A = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_l \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} (\mu_{11}, \nu_{11}, \pi_{11}) & (\mu_{12} \cdot \nu_{12}, \pi_{12}) & \cdots & (\mu_{1m}, \nu_{1m}, \pi_{1m}) \\ (\mu_{21}, \nu_{21}, \pi_{21}) & (\mu_{22}, \nu_{22}, \pi_{22}) & \cdots & (\mu_{2m}, \nu_{2m}, \pi_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{l1}, \nu_{l1}, \pi_{l1}) & (\mu_{l2}, \nu_{l2}, \pi_{l2}) & \cdots & (\mu_{lm}, \nu_{lm}, \pi_{lm}) \end{bmatrix} \end{array} \tag{23}$$

Step 4: All criteria may not have equal importance, so it is necessary to assign weight to indicators. This step is also determined by experts. Let $w_j^k = [\mu_j^k\ \nu_j^k\ \pi_j^k]$ be a PFN, which is used to indicate the evaluation and scoring of the j indicator by the k expert. Different experts' evaluation and scoring of an indicator also need to be aggregated into a Pythagorean fuzzy number $w_j = (\mu_j\ \nu_j\ \pi_j)$. This process is also implemented through the Python fuzzy aggregated averaging (PFWA) operator as in Equation (24).

$$\begin{aligned} w_j &= PFWA_\sigma\left(w_j^1, w_j^2, \cdots, w_j^k, \cdots w_j^n\right) \\ &= \sigma_1 w_j^1 \oplus \sigma_2 w_j^2 \oplus \cdots \oplus \sigma_k w_j^k \oplus \cdots \oplus \sigma_n w_j^n \\ &= \left(\sqrt{1-\prod_{k=1}^{n}\left(1-\left(\mu_j^k\right)^2\right)^{\sigma_k}}, \prod_{k=1}^{n}\left(\nu_j^k\right)^{\sigma_k},\right. \\ &\quad \left.\sqrt{\prod_{k=1}^{n}(1-(\mu_j^k)^2)^{\sigma_k} - \left(\prod_{k=1}^{n}(\nu_j^k)^{\sigma_k}\right)^2}\right) \end{aligned} \tag{24}$$

At the same time, construct a weight matrix for all indicators W, $W = (w_1\ w_2\ \ldots\ w_m)$.

Step 5: After the aggregation evaluation matrix and index weight matrix are obtained, the weighted aggregation Python fuzzy decision matrix (PFDM) can be obtained through the multiple operator $R_{WA} = (x_{Wij})_{l\times m}$, where $x_{Wij} = (\mu_{Wij}, \nu_{Wij}, \pi_{Wij})$, the multiplication operator, is as in Equation (25).

$$\begin{aligned} x_{Wij} &= x_{ij} \otimes w_j \\ &= \left(\mu_{ij}\cdot\mu_j, \sqrt{(\nu_{ij})^2 + (\nu_j)^2 - (\nu_{ij})^2\cdot(\nu_j)^2},\right. \\ &\quad \left.\sqrt{1-(\mu_{ij}\cdot\mu_j)^2 - (\nu_{ij})^2 - (\nu_j)^2 + (\nu_{ij})^2\cdot(\nu_j)^2}\right). \end{aligned} \tag{25}$$

The weighted aggregated PFDM can be constructed as in Equation (26).

$$R_{wA} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_l \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{bmatrix} (\mu_{W11}, \nu_{W11}, \pi_{W11}) & (\mu_{W12}, \nu_{W12}, \pi_{W12}) & \cdots & (\mu_{W1m}, \nu_{W1m}, \pi_{W1m}) \\ (\mu_{W21}, \nu_{W21}, \pi_{W21}) & (\mu_{W22}, \nu_{W22}, \pi_{W22}) & \cdots & (\mu_{W2m}, \nu_{W2m}, \pi_{W2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{Wl1}, \nu_{Wl1}, \pi_{Wl1}) & (\mu_{Wl2}, \nu_{Wl2}, \pi_{Wl2}) & \cdots & (\mu_{Wlm}, \nu_{Wlm}, \pi_{Wlm}) \end{bmatrix} \end{array} \tag{26}$$

Step 6: Let J_1 and J_2 be the collection of benefit-type criteria and cost-type criteria. The Pythagorean fuzzy positive ideal solution (PFPIS) A^+ and the Pythagorean fuzzy negative ideal solution (PFNIS) A^- are as in Equations (27)–(30).

$$A^+ = \{\langle C_j, \mu^+_{Wij}, \nu^+_{Wij} \rangle | C_j \in C, j = 1, 2, \cdots, m\}, \tag{27}$$

$$A^- = \{\langle C_j, \mu^-_{Wij}, \nu^-_{Wij} \rangle | C_j \in C, \ j = 1, 2, \cdots, m\}, \tag{28}$$

$$\mu^+_{Wij} = \begin{cases} \max\limits_{1 \le i \le l} \mu_{Wij} \ if \ C_j \in J_1 \\ \min\limits_{1 \le i \le l} \mu_{Wij} \ if \ C_j \in J_2 \end{cases}, \ \nu^+_{Wij} = \begin{cases} \min\limits_{1 \le i \le l} \nu_{Wij} \ if \ C_j \in J_1 \\ \max\limits_{1 \le i \le l} \nu_{Wij} \ if \ C_j \in J_2 \end{cases}, \tag{29}$$

$$\mu^-_{Wij} = \begin{cases} \min\limits_{1 \le i \le l} \mu_{Wij} \ if \ C_j \in J_1 \\ \max\limits_{1 \le i \le l} \mu_{Wij} \ if \ C_j \in J_2 \end{cases}, \ \nu^-_{Wij} = \begin{cases} \max\limits_{1 \le i \le l} \nu_{Wij} \ if \ C_j \in J_1 \\ \min\limits_{1 \le i \le l} \nu_{Wij} \ if \ C_j \in J_2 \end{cases} \tag{30}$$

Step 7: After obtaining PFPIS and PFNIS, the next step is to calculate the distance between each scheme and the optimal solution D (A_i, A^+) and the worst solution D (A_i, A^-). The normalized hamming distance formula is used. Then, the proximity between the alternatives and PFPIS is obtained and the calculation formula is as in Equation (31).

$$C(A_i) = \frac{D(A_i, A^-)}{D(A_i, A^+) + D(A_i, A^+)} \tag{31}$$

Step 8: According to the calculated closeness, rank each alternative from high to low and select the best one.

3. Results and Application

3.1. Background of Suichang Gold Mine

Suichang Gold Mine, the largest state-owned gold mining enterprise in Zhejiang Province (see Figure 4), the backbone of national gold system production and the first member of the Shanghai Gold Exchange, won the honorary titles of "National Green Mine", "National Excellent Mining Enterprise for Saving and Comprehensive Utilization of Mine Resources" and "National 4A Tourist Attraction" (see Figure 5). The mining rights area of Suichang Gold Mine is 2.3729 km^2 and the design production scale is 91,800 t/a. There are two gold and silver ore bodies in the main mining area, which are distributed in layers and veins, with an obvious branching compound phenomenon. The ore veins are 27~190 m long, with occurrence elevation of 125~317 m, dip angle of 35~85°, average thickness of 1~4 m and average grade of Au 15 g/t and Ag 400 g/t. The surrounding rock of the roof of the orebody in the middle section is relatively stable, while the roof of the orebody in the west section is controlled by the compressive torsional fracture, the surrounding rock is relatively broken and the joints are developed, often resulting in the collapse of the surrounding rock of the roof in the goaf [33].

At present, the mine faces the following three main technical problems.

(1) The stability of ore and rock in the altered zone is poor and mining technology is difficult. The endowment characteristics of altered rock type gold deposits are complex, the occurrence, grade and dip angle vary greatly and the ore veins intersect and branch seriously.
(2) The shrinkage method is not applicable to ore bodies with complex resource endowments such as large thickness changes and serious branching, the level of mechanized equipment is low, and the labor intensity of workers is high.
(3) The technology of replacing ore pillar with concrete is complex, with low labor efficiency and high cost.

Figure 4. Plan of the Suichang gold mine (accessed on 10 July 2022. https://www.fengyunditu.com/?ver=bd-wx-1604).

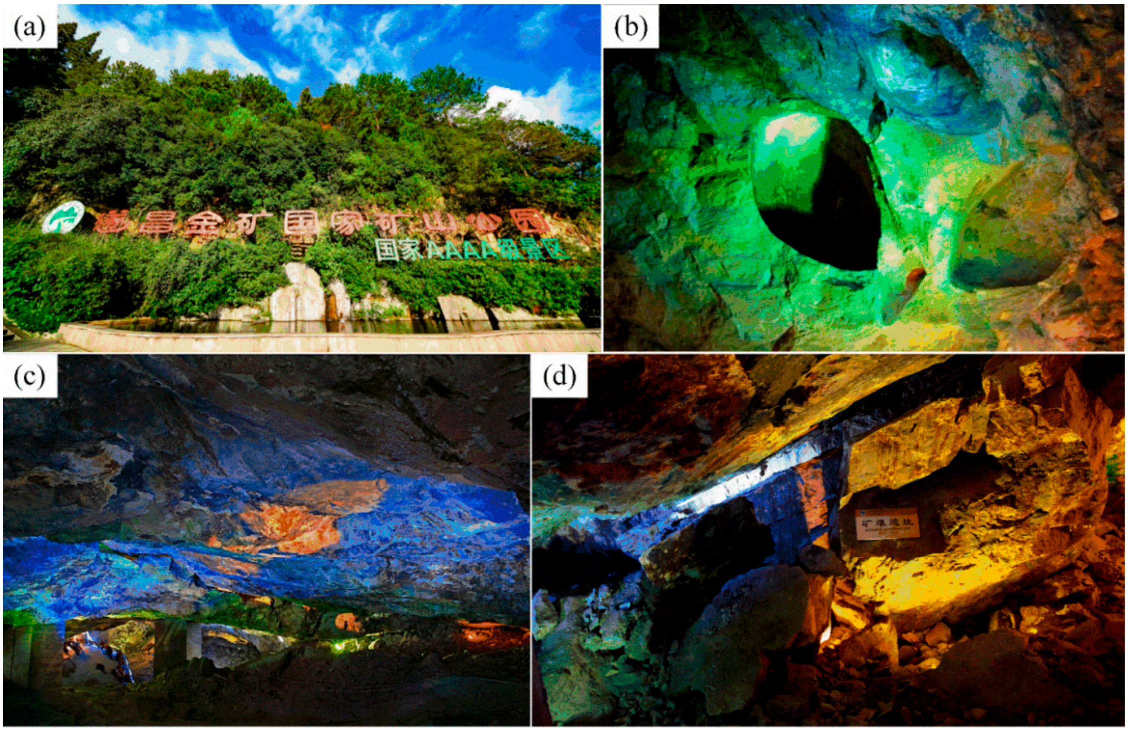

Figure 5. Construction of green mine at SuiChang gold mine: (**a**) exterior view of SuiChang National Mine Park; (**b**) the gold grottoes of the Dang Dynasty; (**c**) the gold grottoes of the Song Dynasty; (**d**) mining disaster size from the Ming Dynasty [29]. (cc by-sa 4.0).

3.2. Primary Selection of Mining Method

For the further development of the enterprise, Suichang Gold Mine decided to upgrade and reconstruct the mine, optimize mining methods, improve supporting facilities and working environment, and increase production capacity. After the preliminary analysis of technical conditions, investigation of engineering rock mechanics and investigation of mineability, four mining methods were preliminarily selected four mining methods. These are the mechanized upward horizontal layering method (MUH), general upward horizontal layering method (GUH), upward horizontal approach filling method (UHA) and shrinkage filling method (SFM).

MUH adopts trackless mechanized equipment such as drilling jumbos and scrapers for production, which can realize strong mining, strong extraction and strong filling, with large production capacity (see Figure 6).

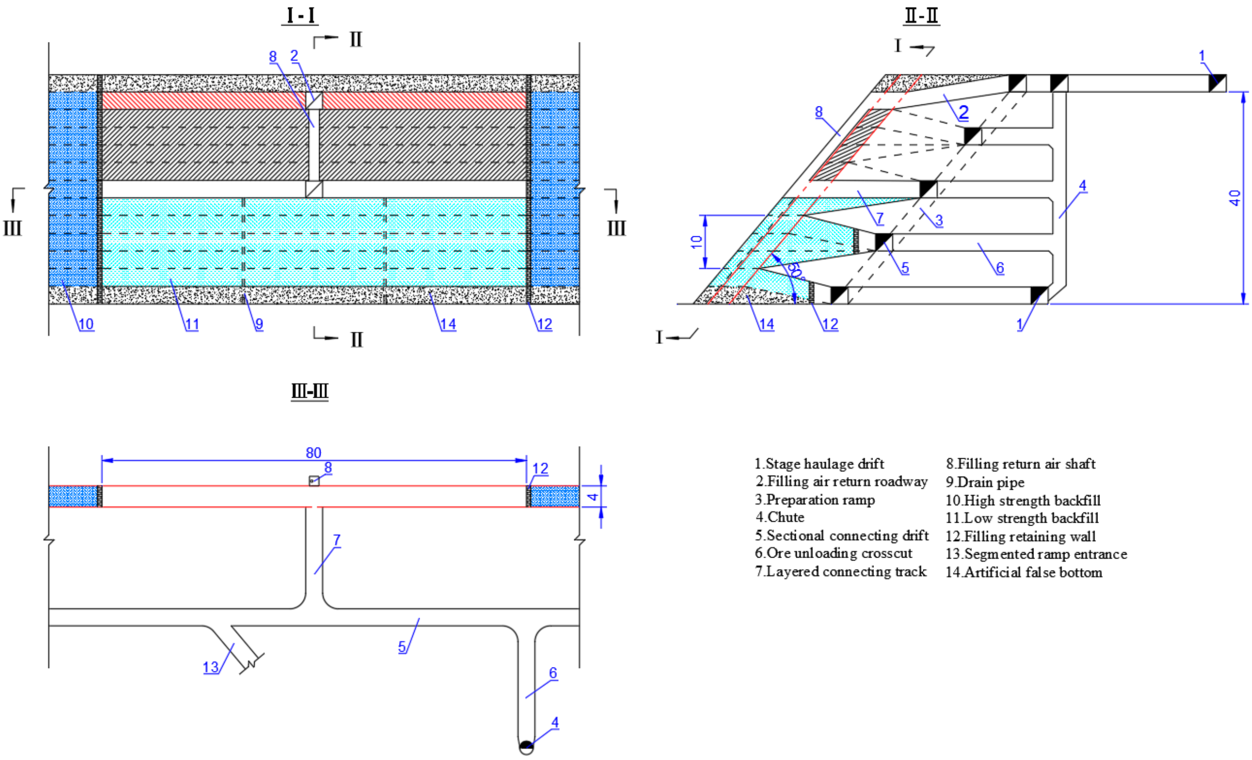

Figure 6. Mining method diagram of MUH.

GUH faces problems of small production capacity, low mechanization, high labor intensity, large amount of preparation work, large amount of reserved space pillar and bottom pillar and low ore drawing efficiency of its two electric harrows (see Figure 7).

UHA adopts trackless mechanized equipment such as the drilling jumbo and scraper, with large production capacity (see Figure 8).

SFM has low drilling efficiency, large amount of reserved space pillar and bottom pillar, difficulty of recovery, low recovery rate and large resource loss (see Figure 9).

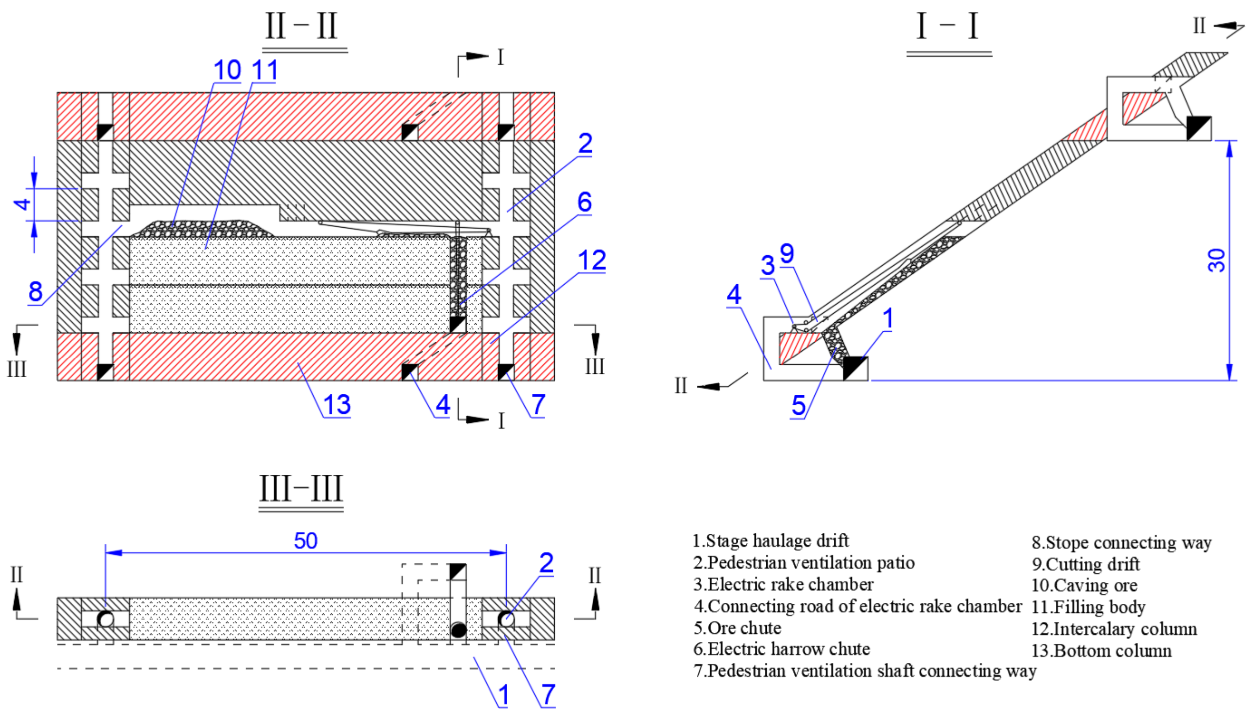

Figure 7. Mining method diagram of GUH.

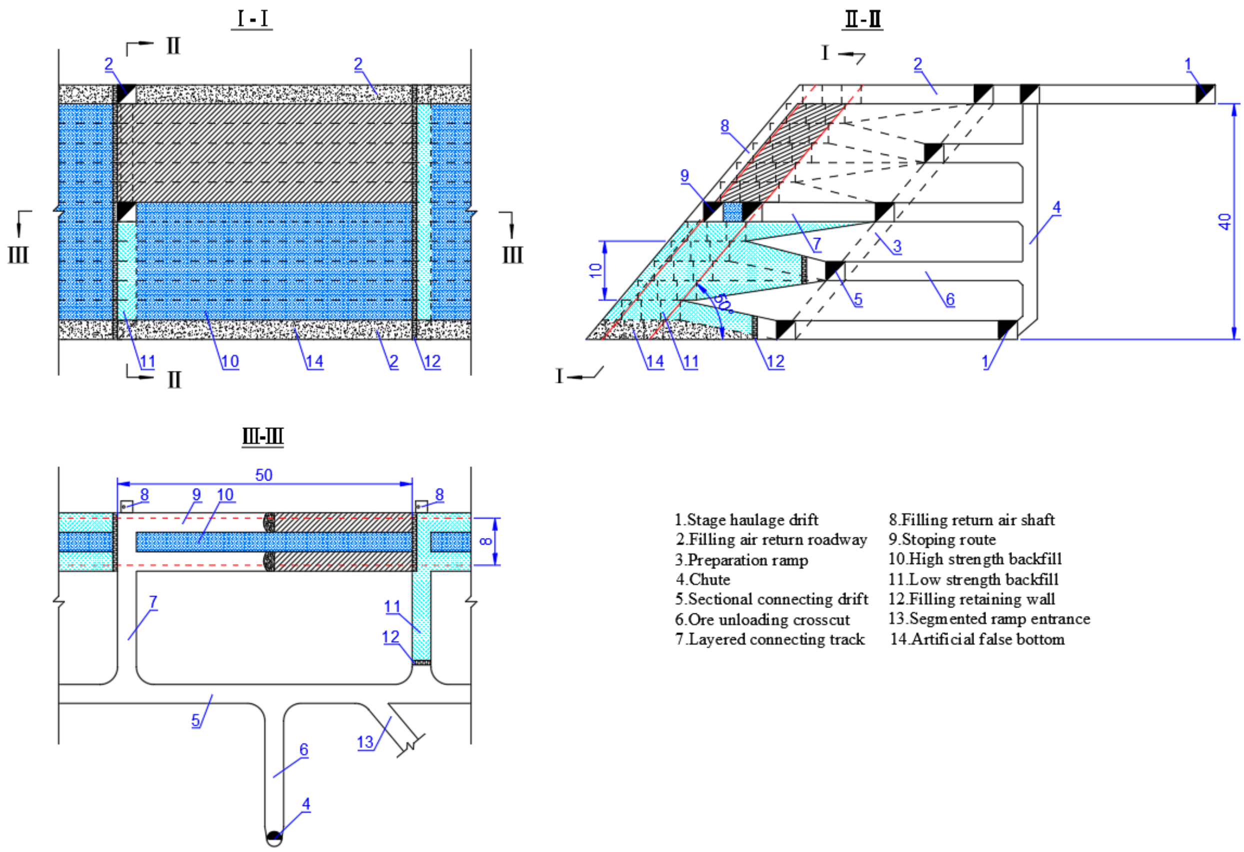

Figure 8. Mining method diagram of UHA.

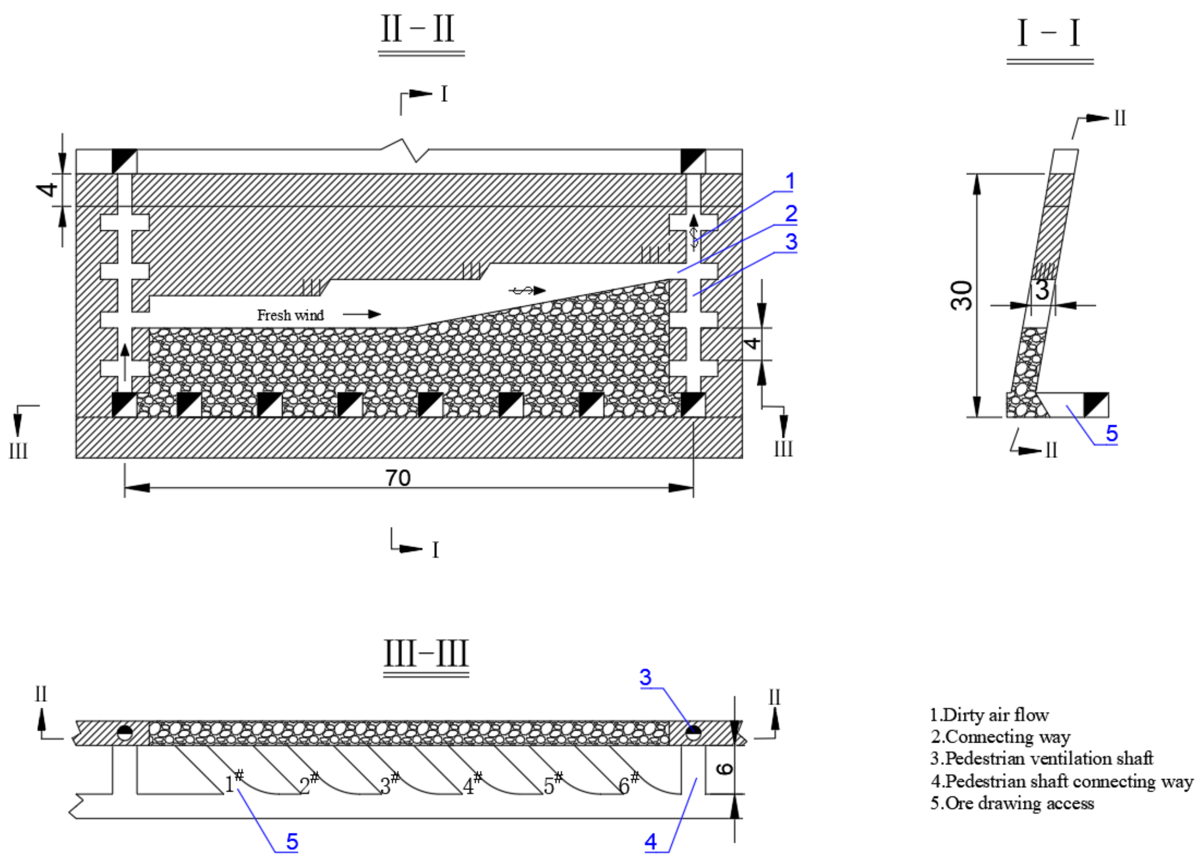

Figure 9. Mining method diagram of SFM.

3.3. Mining Method Optimization

The specific steps of the PFS–TOPSIS model for mining method optimization are as follows:

Step 1: In the optimization of mining methods, we selected three parties as scoring experts, namely the designer (E_1), the mining enterprise (E_2) and the operator (E_3). The four alternatives are MUHSM (A_1), GUHLM (A_2), UHAFM (A_3) and SFM (A_4). The indexes considered are ore recovery rate (C_1), stope production capacity (C_2), flexibility and adaptability (C_3), stope safety conditions (C_4), ore dilution rate (C_5), mining and cutting quantities (C_6), construction organization and labor intensity (C_7) and comprehensive total cost (C_8). Obviously, C_1–C_4 belongs to benefit index (J_1), and C_5–C_8 belongs to cost index (J_2). Next, the corresponding relationship between natural evaluation language and fuzzy number is defined. Table 1 defines the conversion criteria between the relative importance of indicators and PFN and Table 2 defines the conversion criteria between the relative superiority of the scheme and PFN.

Step 2: Three party experts will evaluate and score the superiority of the four schemes under each indicator, as shown in Table 3.

Table 1. The conversion criteria for the relative importance of indicators and PFN.

Linguistic Variables	PFNs
Very important (VI)	(0.90, 0.20, 0.39)
Important (I)	(0.75, 0.30, 0.59)
Medium (M)	(0.60, 0.50, 0.62)
Unimportant (U)	(0.45, 0.70, 0.55)
Very unimportant (VU)	(0.20, 0.90, 0.39)

Table 2. The conversion criteria for the relative superiority of the scheme and PFN.

Linguistic Variables	PFNs
Perfect (VI)	(1.00, 0.00, 0.00)
Very very good (VVG)	(0.90, 0.20, 0.39)
Very good (VG)	(0.80, 0.30, 0.52)
Good (G)	(0.70, 0.35, 0.62)
Medium (M)	(0.60, 0.50, 0.62)
Medium bad (MB)	(0.50, 0.60, 0.62)
Bad (B)	(0.40, 0.70, 0.59)
Very bad (VB)	(0.25, 0.80, 0.55)
Very very bad (VVB)	(0.10, 0.90, 0.42)

Table 3. Superiority evaluation result and fuzzy number.

Criteria	Alternatives	Expert		
		E_1	E_2	E_3
C_1	A_1	VG (0.80, 0.30, 0.52)	VG (0.80, 0.30, 0.52)	VVG (0.90, 0.20, 0.39)
	A_2	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)
	A_3	VVG (0.90, 0.20, 0.39)	VG (0.80, 0.30, 0.52)	VG (0.80, 0.30, 0.52)
	A_4	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)
C_2	A_1	VVG (0.90, 0.20, 0.39)	VG (0.80, 0.30, 0.52)	VG (0.80, 0.30, 0.52)
	A_2	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)
	A_3	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)
	A_4	VG (0.80, 0.30, 0.52)	G (0.70, 0.35, 0.62)	VG (0.80, 0.30, 0.52)
C_3	A_1	G (0.70, 0.35, 0.62)	G (0.70, 0.35, 0.62)	VG (0.80, 0.30, 0.52)
	A_2	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)
	A_3	VG (0.80, 0.30, 0.52)	VG (0.80, 0.30, 0.52)	G (0.70, 0.35, 0.62)
	A_4	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)
C_4	A_1	VG (0.80, 0.30, 0.52)	G (0.70, 0.35, 0.62)	VG (0.80, 0.30, 0.52)
	A_2	VG (0.80, 0.30, 0.52)	G (0.70, 0.35, 0.62)	G (0.70, 0.35, 0.62)
	A_3	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)
	A_4	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)
C_5	A_1	VB (0.25, 0.80, 0.55)	VB (0.25, 0.80, 0.55)	VB (0.25, 0.80, 0.55)
	A_2	M (0.60, 0.50, 0.62)	MB (0.50, 0.60, 0.62)	M (0.60, 0.50, 0.62)
	A_3	VB (0.25, 0.80, 0.55)	B (0.40, 0.70, 0.59)	VB (0.25, 0.80, 0.55)
	A_4	B (0.40, 0.70, 0.59)	VB (0.25, 0.80, 0.55)	B (0.40, 0.70, 0.59)
C_6	A_1	B (0.40, 0.70, 0.59)	MB (0.50, 0.60, 0.62)	MB (0.50, 0.60, 0.62)
	A_2	B (0.40, 0.70, 0.59)	B (0.40, 0.70, 0.59)	B (0.40, 0.70, 0.59)
	A_3	MB (0.50, 0.60, 0.62)	M (0.60, 0.50, 0.62)	MB (0.50, 0.60, 0.62)
	A_4	B (0.40, 0.70, 0.59)	VB (0.25, 0.80, 0.55)	B (0.40, 0.70, 0.59)
C_7	A_1	B (0.40, 0.70, 0.59)	B (0.40, 0.70, 0.59)	VB (0.25, 0.80, 0.55)
	A_2	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)	G (0.70, 0.35, 0.62)
	A_3	MB (0.50, 0.60, 0.62)	MB (0.50, 0.60, 0.62)	M (0.60, 0.50, 0.62)
	A_4	M (0.60, 0.50, 0.62)	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)
C_8	A_1	MB (0.50, 0.60, 0.62)	M (0.60, 0.50, 0.62)	MB (0.50, 0.60, 0.62)
	A_2	B (0.40, 0.70, 0.59)	B (0.40, 0.70, 0.59)	MB (0.50, 0.60, 0.62)
	A_3	M (0.60, 0.50, 0.62)	G (0.70, 0.35, 0.62)	M (0.60, 0.50, 0.62)
	A_4	B (0.40, 0.70, 0.59)	MB (0.50, 0.60, 0.62)	MB (0.50, 0.60, 0.62)

Step 3: Determine the importance of all experts, that is, assign a certain weight. Muhammad Akram proposed a method to determine the weight according to the three elements of PFN and established a corresponding relationship between natural language variables and weight. For convenience, this section directly refers to σ_1 = 0.3252, σ_2 = 0.3754 and σ_3 = 0.2994. The weighted aggregation of experts' scores is conducted through PFWA and the aggregation evaluation matrix is obtained, as shown in Table 4.

Table 4. Aggregate evaluation matrix.

	A_1	A_2	A_3	A_4
C_1	(0.838, 0.266, 0.476)	(0.667, 0.400, 0.629)	(0.841, 0.263, 0.473)	(0.642, 0.437, 0.630)
C_2	(0.841, 0.263, 0.473)	(0.642, 0.437, 0.630)	(0.637, 0.445, 0.629)	(0.768, 0.318, 0.556)
C_3	(0.735, 0.334, 0.590)	(0.667, 0.400, 0.629)	(0.775, 0.314, 0.548)	(0.600, 0.500, 0.624)
C_4	(0.768, 0.318, 0.556)	(0.738, 0.333, 0.587)	(0.637, 0.445, 0.629)	(0.637, 0.445, 0.629)
C_5	(0.250, 0.800, 0.545)	(0.566, 0.535, 0.627)	(0.317, 0.761, 0.566)	(0.353, 0.736, 0.578)
C_6	(0.471, 0.631, 0.616)	(0.400, 0.700, 0.591)	(0.542, 0.560, 0.627)	(0.353, 0.736, 0.578)
C_7	(0.363, 0.729, 0.580)	(0.672, 0.393, 0.628)	(0.534, 0.568, 0.626)	(0.634, 0.449, 0.630)
C_8	(0.542, 0.560, 0.627)	(0.434, 0.668, 0.604)	(0.642, 0.437, 0.630)	(0.471, 0.631, 0.616)

Step 4: Each expert will evaluate the superiority of various indicators, as shown in Table 5 for the indicator evaluation. The results will be weighted and aggregated into the indicator weight matrix as Equation (32).

$$W = \begin{bmatrix} (0.8695,\ 0.2258,\ 0.4393) \\ (0.8010,\ 0.3002,\ 0.5180) \\ (0.7140,\ 0.3496,\ 0.6066) \\ (0.9000,\ 0.2000,\ 0.3873) \\ (0.7709,\ 0.3545,\ 0.5292) \\ (0.8010,\ 0.3002,\ 0.5180) \\ (0.7838,\ 0.3137,\ 0.5360) \\ (0.7709,\ 0.3545,\ 0.5292) \end{bmatrix}^T \tag{32}$$

Table 5. Index evaluation.

Criteria	Experts		
	E_1	E_2	E_3
C_1	(0.90, 0.20, 0.39)	(0.90, 0.20, 0.39)	(0.75, 0.30, 0.59)
C_2	(0.75, 0.30, 0.59)	(0.90, 0.20, 0.39)	(0.60, 0.50, 0.62)
C_3	(0.75, 0.30, 0.59)	(0.75, 0.30, 0.59)	(0.60, 0.50, 0.62)
C_4	(0.90, 0.20, 0.39)	(0.90, 0.20, 0.39)	(0.90, 0.20, 0.39)
C_5	(0.60, 0.50, 0.62)	(0.90, 0.20, 0.39)	(0.60, 0.50, 0.62)
C_6	(0.75, 0.30, 0.59)	(0.90, 0.20, 0.39)	(0.60, 0.50, 0.62)
C_7	(0.60, 0.50, 0.62)	(0.75, 0.30, 0.59)	(0.90, 0.20, 0.39)
C_8	(0.60, 0.50, 0.62)	(0.90, 0.20, 0.39)	(0.60, 0.50, 0.62)

Step 5: After the aggregation evaluation matrix and index weight matrix are obtained, the weighted aggregation evaluation matrix can be obtained through the multiplication operator, as shown in Table 6.

Table 6. Weighted aggregate evaluation matrix.

	A_1	A_2	A_3	A_4
C_1	(0.729, 0.344, 0.592)	(0.580, 0.450, 0.679)	(0.731, 0.342, 0.590)	(0.558, 0.482, 0.676)
C_2	(0.674, 0.391, 0.627)	(0.514, 0.514, 0.687)	(0.510, 0.520, 0.685)	(0.615, 0.427, 0.663)
C_3	(0.525, 0.469, 0.710)	(0.476, 0.513, 0.714)	(0.553, 0.457, 0.697)	(0.428, 0.585, 0.689)
C_4	(0.691, 0.370, 0.621)	(0.664, 0.383, 0.642)	(0.573, 0.480, 0.664)	(0.573, 0.480, 0.664)
C_5	(0.193, 0.828, 0.526)	(0.436, 0.613, 0.659)	(0.244, 0.795, 0.555)	(0.272, 0.774, 0.572)
C_6	(0.377, 0.672, 0.637)	(0.320, 0.732, 0.601)	(0.434, 0.613, 0.660)	(0.283, 0.764, 0.580)
C_7	(0.285, 0.760, 0.584)	(0.527, 0.488, 0.696)	(0.419, 0.624, 0.660)	(0.497, 0.529, 0.688)
C_8	(0.418, 0.632, 0.653)	(0.335, 0.718, 0.610)	(0.495, 0.541, 0.680)	(0.363, 0.688, 0.628)

Step 6: The Pythagorean fuzzy positive ideal solution (PFPIS) A^+ and the Pythagorean fuzzy negative ideal solution (PFNIS) A^- are given as in Equations (33) and (34).

$$\begin{aligned} A^+ = \; & \{(0.731, 0.342, 0.590), (0.674, 0.391, 0.627), (0.553, 0.457, 0.697), \\ & (0.691, 0.370, 0.621), (0.193, 0.828, 0.526), (0.283, 0.764, 0.580), \\ & (0.285, 0.760, 0.584), (0.335, 0.718, 0.610)\} \end{aligned} \tag{33}$$

$$\begin{aligned} A^- = \; & \{(0.558, 0.482, 0.676), (0.510, 0.520, 0.685), (0.428, 0.585, 0.689), \\ & (0.573, 0.480, 0.664), (0.436, 0.613, 0.659), (0.434, 0.613, 0.660), \\ & (0.527, 0.488, 0.696), (0.495, 0.541, 0.680)\} \end{aligned} \tag{34}$$

Step 7: Calculate the distance between each scheme and the best solution and the worst solution and rank the schemes by calculating the closeness index to obtain the best scheme, as shown in Table 7.

Table 7. Optimal scheme ranking.

Alternatives	$D(A_i, A^+)$	$D(A_i, A^-)$	$C(A_i)$	Ranks
A_1	0.0352	0.1897	0.8436	1
A_2	0.1501	0.0763	0.3370	4
A_3	0.1270	0.0956	0.4296	3
A_4	0.1261	0.0964	0.4334	2

It can be seen from the above table that A_1 is the best choice, that is, the MUH is the best scheme along with mechanized mining (see Figure 10).

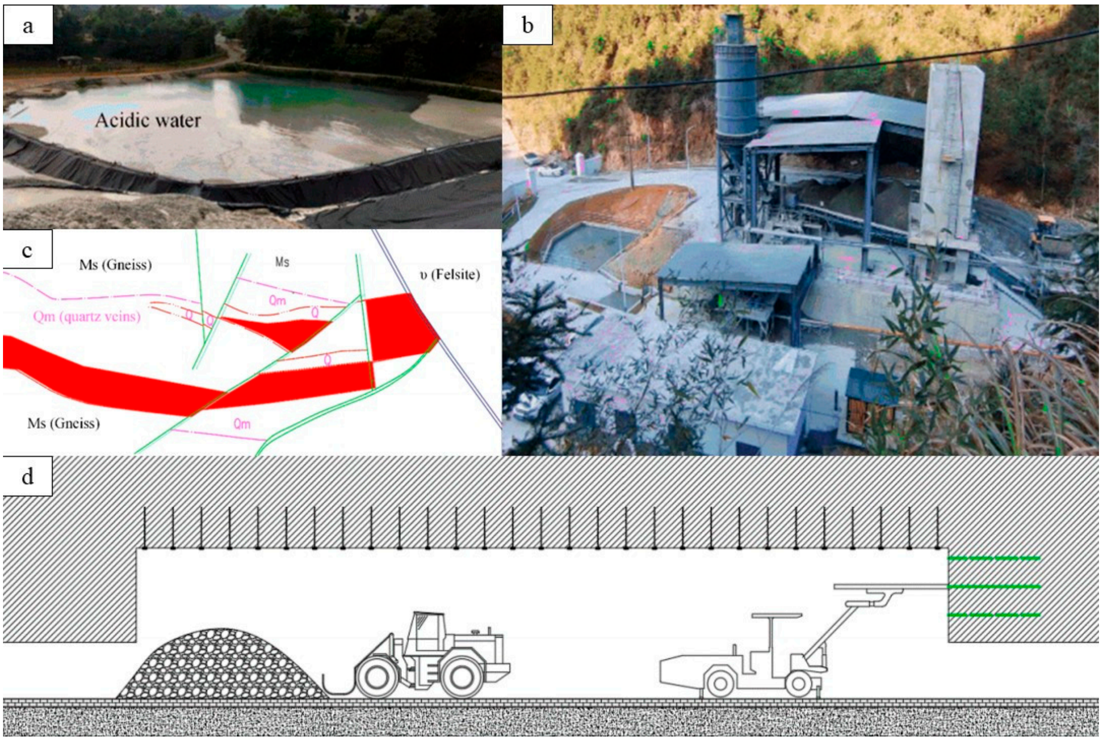

Figure 10. Mechanized mining in the Suichang gold mine: (**a**) filling material; (**b**) filling station; (**c**) resource survey; (**d**) mechanized mining [34]. (cc by-sa 4.0).

4. Discussion

The selection of mining method is very important and complex. In this paper, through a TOPSIS method based on PFS, the MUH is selected as the final scheme among the four mining methods suitable for Suichang Gold Mine. By combining the advantages of PFS which can fully represent fuzzy information with the advantages of TOPSIS ranking science, an ideal result is achieved.

However, due to the importance and particularity of mining method decision making, the above models and calculations cannot fully guarantee the scientific accuracy of the results. In fact, as TOPSIS methods that rank according to the proximity of good and bad solutions, the core influencing factors are the distance between the scheme and the positive and negative ideal solutions, as well as the ranking method. As mentioned above, there are many methods to measure the distance between PFS and each method has its own advantages and disadvantages and the most suitable application [35–38]. In terms of sorting, Hadi-Vencheh [39] believes that the traditional closeness index rank may not be able to produce an optimal alternative, being close to the PIS and far from the NIS. Consequently, they introduced the revised closeness index as Equation (35).

$$RC(A_i) = \frac{D(A_i, A^-)}{D_{\max}(A_i, A^-)} - \frac{D(A_i, A^+)}{D_{\min}(A_i, A^+)} \tag{35}$$

Mahanta [31] adopted a method of ranking by similarity in their article. It is defined in Equations (29) and (30).

$$S_r = \frac{S(D_+^i)}{S(D_+^i) + S(D_-^i)} \tag{36}$$

$$D_+^i = D(A_i, A^+), D_-^i = D(A_i, A^-), S(D) = (1 - D)/(1 + D) \tag{37}$$

In view of this, the stability of the above results is analyzed through the four distance measures and three ranking methods mentioned above (see Figures 11–13), as shown in Tables 8 and 9, and the data in Figure 12 have been normalized.

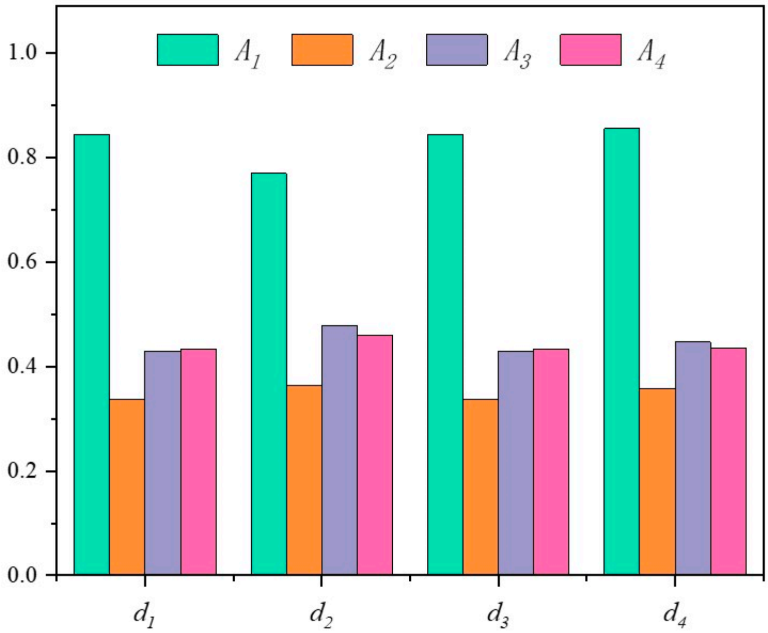

Figure 11. Closeness index rank based on different distance measures.

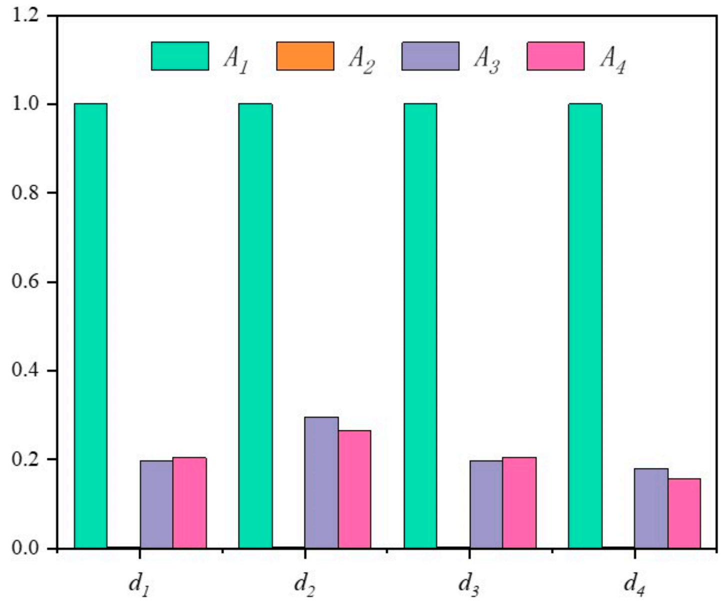

Figure 12. Revised closeness index rank based on different distance measures.

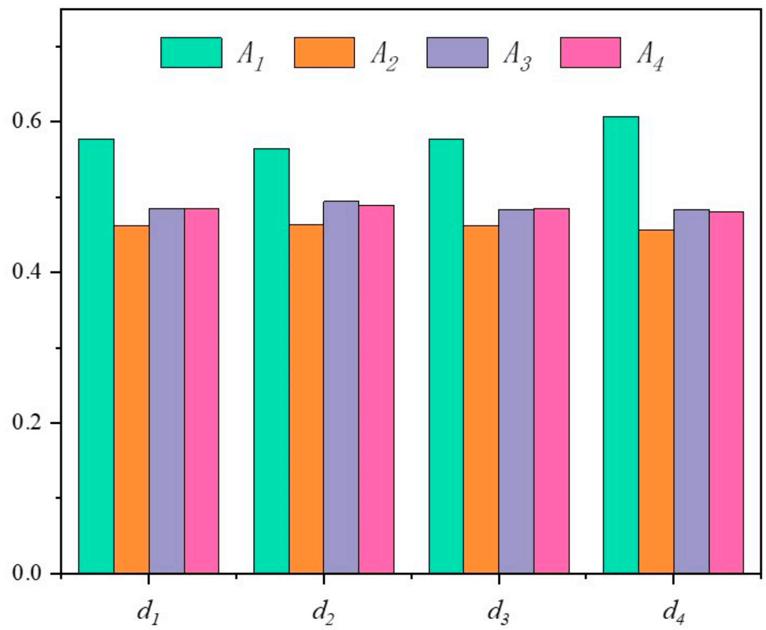

Figure 13. Relative similarity values rank based on different distance measures.

Table 8. Results of distance measures.

		A_1	A_2	A_3	A_4
d_1	$D(A,A^+)$	0.03517	0.15011	0.12696	0.12611
	$D(A,A^-)$	0.18968	0.07630	0.09560	0.09645
d_2	$D(A,A^+)$	0.05468	0.16732	0.13444	0.13656
	$D(A,A^-)$	0.18260	0.09535	0.12303	0.11595
d_3	$D(A,A^+)$	0.03516	0.15013	0.12699	0.12600
	$D(A,A^-)$	0.18969	0.07632	0.09548	0.09646
d_4	$D(A,A^+)$	0.04340	0.19304	0.16600	0.16947
	$D(A,A^-)$	0.25637	0.10775	0.13392	0.13089

Table 9. Results of ranking methods.

		A_1	A_2	A_3	A_4	Rank
$C(A_i)$	d_1	0.84359	0.33702	0.42955	0.43336	$A_1 > A_4 > A_3 > A_2$
	d_2	0.76956	0.36300	0.47783	0.45919	$A_1 > A_3 > A_4 > A_2$
	d_3	0.84361	0.33703	0.42920	0.43361	$A_1 > A_4 > A_3 > A_2$
	d_4	0.85523	0.35822	0.44652	0.43577	$A_1 > A_3 > A_4 > A_2$
$RC(A_i)$	d_1	0	−3.8661	−3.1061	−3.0775	$A_1 > A_4 > A_3 > A_2$
	d_2	0	−2.5379	−1.7851	−1.8625	$A_1 > A_3 > A_4 > A_2$
	d_3	0	−3.8669	−3.1078	−3.0747	$A_1 > A_4 > A_3 > A_2$
	d_4	0	−4.0280	−3.3029	−3.3947	$A_1 > A_3 > A_4 > A_2$
$S_r(A_i)$	d_1	0.57777	0.46267	0.48413	0.48499	$A_1 > A_4 > A_3 > A_2$
	d_2	0.56460	0.46343	0.49420	0.48953	$A_1 > A_3 > A_4 > A_2$
	d_3	0.57778	0.46267	0.48406	0.48505	$A_1 > A_4 > A_3 > A_2$
	d_4	0.60768	0.45645	0.48360	0.48027	$A_1 > A_3 > A_4 > A_2$

It can be seen from the analysis results that the best scheme obtained by changing the distance measures and ranking methods is still the MUH, so it can be considered that the results are accurate. In fact, in the actual production of Suichang Gold Mine, the application of this method has also achieved the ideal results of safety, efficiency and environmental protection.

Whether it can be considered that the model is universal and can be generalized in mining method optimization, the answer is obviously unknown. As is known, the factors that need to be considered in the optimization of mining methods are not completely fuzzy information, e.g., the recovery rate, cut ratio and other factors have certain empirical values. Therefore, the perfect solution is to build a corresponding transformation relationship between the exact value and the PFS and to build a mining method optimization model that combines fuzzy information with accurate data.

5. Conclusions

(1) Through the PFS–TOPSIS method, based on the selection of technical and economic mining methods, a comprehensive evaluation system with multiple factors and indicators was constructed and an accurate closeness index was obtained to optimize mining methods. This overcomes the uncertainty and unpredictability of the traditional optimization system and provides a reference for the mining of the difficult-to-mine complicated orebody.

(2) Taking Suichang Gold Mine as an example, according to the PFSTOPSIS method, a weighted aggregation evaluation matrix was constructed, and the closeness index of the four mining methods were calculated to be 0.8436, 0.3370, 0.4296 and 0.4334, respectively. The MUH has the highest closeness index, so this method was the best scheme.

(3) There were many ranging methods and ranking methods for PFS and only one method could not ensure the accuracy and scientific nature of the results. This paper mainly used the first ranging method, which was ranked by the traditional closeness index. Finally, it discussed the three methods of traditional closeness index, revised closeness index and relative similarity values for comprehensive ranking under the four distance measures. When using the first distance measure, the revised closeness index of the four mining methods was 0, −3.8661, −3.1061 and −3.0775, and the relative similarity values were 0.5777, 0.46267, 0.48413 and 0.48499. It was concluded that MUH was the best scheme, which not only verified the accuracy of the results, but also showed that PFS was applicable to the selection of mining methods.

Of course, there are still limitations to the mathematical approach used in this study, i.e., indicators with definite values need to be evaluated first in natural language and then converted to fuzzy sets. Even so, the method still performs well in problems with a large amount of fuzzy information, such as the selection of mining methods. Therefore, we advocate the application of such methods to more mines and encourage more and more researchers to test and optimize them in practice.

Author Contributions: Conceptualization, S.L.; resources, S.L.; writing—original draft preparation, Q.H.; writing—review and editing, Q.H., B.H. and J.P.; visualization, Q.H., B.H. and J.P.; supervision, J.C., J.Y. and. X.Z.; project administration, X.W. and H.Y.; funding acquisition, S.L. All authors have read and agreed to the published version of the manuscript.

Funding: Authors thank the financial support from the Natural Science Foundation of Hunan Province (Grant No. 2021JJ40745) and the National Natural Science Foundation of China (Grant No. 51804337).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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