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Analysis and Dissolution of Potential Contradictions in Thematic Lessons from a Cultural-Historical Activity Theory Perspective

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Abstract: Thematic learning features sustain exploratory activities focused on a selected theme, which contributes to developing students' core competencies, including problem solving, collaboration, and communication skills, supporting sustainable development in learning. For teachers and students, this emerging pedagogical approach may encounter various contradictions in the classroom activity system. Grounded in a cultural-historical activity theory perspective, this study examined the potential contradictions in an activity system in the context of thematic learning, as well as possible strategies for dissolving those potential contradictions and their effectiveness through a case study. Interviews with an experienced teacher were conducted to acquire information about potential contradictions, and the interview results suggested that these contradictions were perceived as potentially occurring between the students, tools, and objects. According to that information, solutions were proposed to dissolve those potential contradictions, including setting moderate learning objects, creating a sustainable exploratory learning environment, scaffolding necessary knowledge, and situating students in an interactive, communicative, and cooperative classroom environment. Finally, the dissolution of those contradictions was examined through classroom observations, with the results showing that students were able to apply their knowledge accurately and fluently to solve the problems. The contribution of this study involves adopting cultural-historical activity theory as guidance to dissolve the contradictions in classroom teaching, which expands the application of this theory compared to previous studies that focused more on the contradictions between researchers and teachers during a lesson study. This innovative application will inspire teachers to adopt this theory routinely to improve teaching, including designing instructional sequences and question chains that support students' sustained exploration.

Keywords: potential contradictions; cultural-historical activity theory; thematic lesson; lesson study; planar mosaics

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1. Introduction

To develop students' core competencies for sustainable development, including problem-solving, collaborative and communicative skills, etc., various countries are exploring innovative approaches to teaching and learning [1,2], with China being no exception [3–5]. Thematic learning (TL) is an innovation that seeks to provide students with an atmosphere of sustained, meaningful, and engaging exploration centered upon a theme to realize knowledge learning and competence development [6]. As evidenced by China's burgeoning iterations of education policy and curriculum standards, TL is being recognized as a promising innovative approach in teaching and learning. For example, the Mathematics Curriculum Standards for Compulsory Education (2022 Edition, hereafter the Curriculum Standard-2022), promulgated in April 2022, officially introduced this concept, and teachers in primary and secondary schools are encouraged to adopt TL to support students in engaging in meaningful and sustained exploration around a certain theme and accumulating experience through observation, conjecture, experiment, reasoning, collaboration, and communication, thus realizing their sustainable development in mathematics learning [7].

Lesson study (LS) has been noticed by educators in different cultures when designing and implementing innovative or emerging pedagogical approaches due to its working mode being cyclical, progressive, constantly refined, and iterative, and the advantage of collaborative participation of multiple groups, including researchers and teachers, as well as assembling collective intelligence (e.g., [8–11]). Notably, existing LSs seem to have concentrated more on teachers' sustainable development during such collaborative research activities (e.g., [12–15]), with less attention paid to student learning in the implemented lessons [16].

Considering the rapidly iterating educational ideas in China in recent years, TL is a novel approach to learning and teaching for both teachers and students and, therefore, they inevitably encounter various conflicts or contradictions in TL. Grounded in a cultural-historical activity theory (CHAT) perspective [17], this study examined the potential contradictions in an activity system that includes students, tools, and objects in a TL context, as well as the possible strategies for dissolving these potential contradictions and their effectiveness. This study not only offers possible solutions to the challenges that this emerging pedagogical approach of TL poses to students and teachers but also expands the concerns of existing LSs by focusing on student learning in the implemented lessons.

2. Literature Review and Framework

2.1. TL

TL describes a teaching and learning approach in which students independently and collaboratively engage in sustained, meaningful exploration around a specific theme to acquire knowledge [6]. TL is usually considered to be situated within the realm of contextual learning, as the learned contents are embedded in themes that serve as the learning environment, thus advocating that learning is realized in a meaningful environment in which students are engaged in activities [18,19]. TL has a clear relationship with lessons in the regular classroom as it concerns extended learning building on the fundamental knowledge students already possess, that is, engaging in meaningful explorations based on this acquired knowledge as a starting point [20], which is in line with the educational ideas promoted in China's recently promulgated the Curriculum Standard-2022 [7]. It is encouraging that TL is considered an effective teaching and learning approach as it promotes interaction, communication, and collaboration between teachers and students in the classroom, provides opportunities for students to make connections between subjects and life, and facilitates students to develop experiences of exploration in studying a particular theme [18,21]. Empirical research also confirmed that students were able to make meaningful connections between knowledge and life and experienced increased enjoyment of learning in a TL classroom environment [20].

However, according to previous studies, frontline teachers encounter various difficulties in implementing TL. For instance, teachers tend to favor traditional teaching approaches due to the absence of positive changes in their perception of students' roles [18]. Again, the TL plans designed by teachers are less innovative, lack attention to students' involvement, and the teaching process is not adapted to students' learning requirements [6]. It is also worth noting that the available research on TL seems to focus less on themes from the field of mathematics, with most of the literature concerning other subjects [18].

2.2. LS

LS is usually considered a vehicle for teachers' sustainable professional development, first popularized in Asia [22], which is characterized by its embeddedness in regular teaching, commitment to promoting teacher development and improving student learning, and emphasis on multiple collaborations [8,23]. In terms of working mode, LS consists of four stages: (a) study, where learning objects are set for students based on their learning status; (b) plan, where the community collaborates to design a research lesson to achieve the learning objects; (c) teach, where community members teach this lesson in the classroom as well as record the lesson; and (d) reflect, where the research lesson is reflected on and improved based on the classroom records [8]. Although some researchers nuanced these stages, a consensus has been reached concerning the critical characteristics of LS, including the emphasis on collaboration among groups [15] and evidence-based lesson analysis [24]. Empirical studies have shown that LS can enhance teachers' understanding of teaching and learning, improve their ability to design teaching, and facilitate their practice of curriculum ideas [9,25–28], as evidenced by the fact that its significance in promoting teachers' sustainable professional development has been extensively recognized at the international level.

Along with rapidly iterating curriculum standards and emerging pedagogical ideas, coupled with closer interactions, discussions, and seminars between university researchers and frontline teachers, various LSs are taking place in regular teaching in Chinese elementary and secondary schools (e.g., [15,29]. Chinese pedagogical research activities are dedicated to producing high-quality, exquisite lessons (jingpin lessons, in Chinese), and thus LS often involves an iterative cycle of study, design, formulation, reflection, and refinement [11,29] and usually requires intellectual support from academic and practical experts to build such exquisite lessons [13]. This LS paradigm, dedicated to promoting teachers' sustainable professional development, has been proven to build teachers' willingness and capacity to continuously improve their teaching design, refine their teaching practices [26], and positively impact the enhancement of K-12 school instruction [15,30]. However, to our knowledge, fewer LSs seem to have paid attention to student learning in the lessons implemented [8,16], and even fewer studies have focused on student learning in a TL classroom environment.

2.3. CHAT and Its Revealed Contradictions

LS requires an environment of collaborative research, wherein community members participate in research activities together and contribute their respective intelligence; thus, activity theory has been adopted to analyze these activities in LS (e.g., [31]). Developed into the third generation, CHAT has evolved into a methodology that depicts the activity system and structural relationships among its elements [17]. Specifically, CHAT describes an activity system with six core elements (see Figure 1), namely: (a) Subject(s), referring to the people in the activity system who pursue a common goal, such as students and prospective teachers in this study. (b) Objects refer to the 'problem space' to which the activities are addressed and which, through the impact of tools, can be transformed into outcomes [32]. In the context of this study, the outcome can be considered as the learning outcomes of the students and the enhancement of pedagogical ideas, skills, and experiences acquired by the prospective teachers. (c) Tools refer to the mediators that the subjects need to rely on in the process of reaching the outcome, in this study they can be considered as the teaching measures (including the didactical sequence and involved activities), the resources to support student learning, etc., for a TL. (d) Rules, in the context of this study, refer to the classroom and teaching rules, etc., agreed upon and shared by the subjects. (e) Community refers to a larger context in which the activity takes place, and in this study is the larger group that includes the researcher and schoolteacher. (f) Division of labor, which refers to the distribution of the subject's role in the activity, and in this study the respective efforts of the students and prospective teachers in the learning and teaching process.

The relationships between these elements are sometimes contradictory [17], and "contradictions can be viewed as an opportunity for development or learning" [33] (p. 132), so the process of dissolving these contradictions is also the one that facilitates the adjustment and development of an activity system, thus allowing it to function smoothly [12,31]. Recently, CHAT has been adopted to analyze contradictions in the classroom environment, thus allowing those different aspects of contradictions to be explicitly depicted [34]. Particularly, existing studies in mathematics education have also preliminarily analyzed the contradictions as well as their solutions, based on CHAT, in a partnership system that involves both mathematics education researchers and practitioners [12,35]. Engeström [36] divides these contradictions into four levels, as follows: (a) Primary contradictions, which refer to contradictions between the subjects, which for this study are those between the students and prospective teachers (i.e., the PC in the figure). (b) Secondary contradictions, which refer to the contradictions between different elements of an activity system, such as the contradictions between subjects and objects, and between subject and tool (e.g., SC-1 in the figure). (c) Tertiary contradictions, which refer to the contradictions between old and new systems, such as the contradictions between the TL-based activity system and the classroom's existing conventional teaching activity system (i.e., TC in the figure). (d) Quaternary contradictions refer to the contradictions between the central activity system and extended activity system, which for this study could be the contradictions between the activity system featuring TL and the examination pressure and educational atmosphere of the school (i.e., QC in the figure).

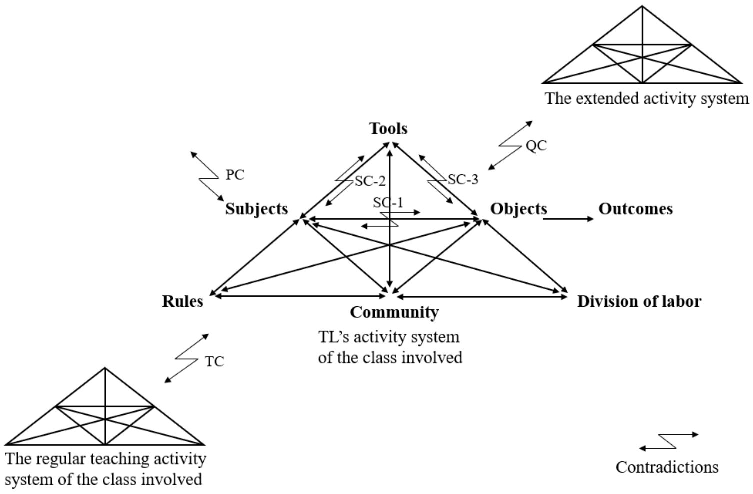


Figure 1. The six core elements and their relationships in an activity system as depicted in CHAT (Source: Cited from the literature [12], but elaborated with another approach).

2.4. This Study

As a teaching approach newly introduced in China's latest mathematics curriculum standard, TL is alien to both teachers and students, which implies that both subjects in the classroom may encounter certain contradictions. In a larger project, the LS approach was adopted, and CHAT was cited as a framework to analyze the contradictions encountered by prospective teachers and students in a TL activity system (TLAS) and their dissolution processes in a holistic way. However, to make the study more focused and considering the manuscript's length, we concentrated on students as the subject, which seems to have received less attention in existing studies. Specifically, we posed the following three interlocking questions.

1. What are the potential contradictions in a TLAS according to CHAT?
2. What measures can be adopted to dissolve those potential contradictions?
3. How are those contradictions dissolved as the lesson proceeds?

3. Research Design

3.1. Setting and Participants

This study was part of a larger project conducted by the researcher in collaboration with a middle school (medium size, moderate level) in Shanghai, which involved multiple participants, including the researcher (who engaged in the overall project, didactical plan design, interviews with teachers, and classroom observations), three prospective teachers (who participated in the didactical plan design, delivering lessons, etc.), a school teacher (i.e., Ms. Wang, as the interviewee of the researcher), and students (who were involved in a TL). This study was concerned with the contradictions and their dissolution in a TL classroom and focused on analyzing the students as a subject in the community. This means that the other subject, namely the prospective teachers, as well as their expected outcomes from this TL, that is, acquiring knowledge and experience of TL, mathematical task design, instructional sequence design, etc., were not the concerns of this study. In this study, we selected Planar Mosaics (PM), an extended piece of learning material in the seventh grade textbook, arranged in the chapter on translation, rotation, and folding, as the theme.

The three prospective teachers (PT-Xin, PT-Tang, and PT-Zhang) were all postgraduate students in mathematics education who had majored in mathematics with excellent academic performance at the undergraduate level and whose future career choice was to be a middle school or high school mathematics teacher. These prospective teachers had undergone a semester-long educational internship at both the undergraduate and postgraduate levels, and two of them had experienced one year of middle school teaching before entering graduate school. They possessed the subject knowledge, theoretical pedagogical knowledge, and limited practical knowledge required to properly teach middle school mathematics. Ms. Wang is a mathematics teacher at this school and also serves as a classroom manager. She holds a master's degree, has 9 years of teaching experience, and holds a Level 1 teaching certificate. She proactively pursues innovation in teaching, is familiar with curriculum standards, has chaired district-level educational projects, and is in charge of STEM tutoring for the school's students. The students who participated in this TL were in the seventh grade and came from Ms. Wang's class, totaling 34 members, and their academic performance was above the middle level of the school.

3.2. Procedure

Four main stages were included in this study, as follows. First, the researcher introduced CHAT to Ms. Wang in detail, and to help her better understand the theory, the researcher used vivid language expressions and illustrated it with concrete examples. Second, the researcher introduced Ms. Wang to the previously Conceived Didactical Sequence, a plan designed by the researcher and prospective teachers, and invited her to predict, from her personal experience, the potential contradictions that the TLAS might exhibit when implementing the TL. Third, the researcher and prospective teachers designed a detailed teaching plan by incorporating Ms. Wang's viewpoints and invited her to evaluate the feasibility of it and propose modifications, which eventually resulted in the Final Didactical Plan after three rounds of revisions. In this Final Didactical Plan, we divided the TL into four sessions, detailing the learning objects, didactical sequence, and activities involved in each one. Lastly, the prospective teachers conducted four sessions on the theme of PM from November to December 2021. These sessions were conducted once a week, with the fourth one used for students' presentations of their explorations, as well as for an award ceremony. It is noted that after each session, the researcher and prospective teachers discussed and fine-tuned the teaching process design for the next session based on the student's classroom performance.

3.3. Data Collection and Analysis

Considering that this study was concerned with potential contradictions and their dissolution in a TLAS, we collected data from two sources. One was the collection of evidence about potential contradictions, which was acquired by interviewing Ms. Wang, who has rich teaching experience and was familiar with the classroom situation involved in this study. To begin the interview, the researcher introduced CHAT to Ms. Wang in detail until she confirmed that she understood the theory, and then we invited her to predict the potential contradictions against this theory. In addition, the researcher and prospective teachers further discussed Ms. Wang's prediction to form a final judgment about the potential contradictions. In this process, CHAT was again adopted to guide us in dissecting the potential contradictions in the TLAS. The other was the collection of contradictions in authentic classrooms and evidence that these contradictions were dissolved, which were analyzed by capturing episodes such as discussions and interactions between teachers and students through classroom videos, borrowing from existing studies capturing data from classrooms [33]. Considering the length, the performance of the students' presentations in Session 4 was not specifically analyzed.

4. Research Process and Results

4.1. Analysis of Potential Contradictions

The researcher first introduced the purpose, content, period, and Conceived Didactical Sequence of this TL to Ms. Wang and invited her to comprehensively evaluate the contradictions students might encounter during their participation in the forthcoming TL based on these pieces of information as well as her knowledge of textbooks, students, and teaching. Concerning the Conceived Didactical Sequence, the researcher's preliminary plan was to divide those series of sessions into eight main sections: (a) introduce the concept of PM by observing life phenomena and artworks; (b) explore constructing PM patterns using a regular polygon as the basic figure (hereafter referred to as single PM); (c) explore constructing PM patterns using two regular polygons with equal side lengths as the basic figures (hereafter referred to as a combined PM); (d) discover that any triangle and any quadrilateral can be used as the basic figure to construct a single PM; (e) explore constructing a combined PM using a regular pentagon and special rhombus with two 36° angles as the basic figures; (f) observe and enjoy the single PM patterns constructed by using some non-regular pentagons as the basic figure; (g) students working collaboratively in groups design combined PM patterns using two regular polygons as the basic figures and mathematically explain them; and (h) students present their exploration process and findings. In addition, the researcher explained to Ms. Wang that in this TL, the interactions between students and teachers and among students would be emphasized, and various teaching aids (e.g., regular polygon cards) would be provided to give students the opportunity to play mathematics games and visual teaching would be performed via the interactive electronic whiteboard using GeoGebra software.

4.1.1. Primary Contradictions

Contradictions might occur between the two subjects, the prospective teachers and students. When asked about the possible primary contradictions, Ms. Wang mentioned:

> *I don't know if they have any previous experience in teaching, and if they can successfully establish a positive interaction with students in a short period; after all, such TL is more open and lively compared to regular teaching. In addition, I was concerned that students would not take these lessons seriously when facing these new faces*.

But then added:

> *Nonetheless, I think they are capable of dealing with these difficulties; after all, they are graduate students in mathematics education and have better knowledge than I do about the theory of mathematics education, the TL mentioned in the curriculum standard, and the knowledge about PM. In addition, they are young and creative, and I feel confident that they are competent in teaching such novel sessions*.

The researcher responded that all three prospective teachers have teaching credentials in either middle school or high school and usually have many opportunities to visit various schools along with the researcher to observe classroom teaching. The researcher further informed Ms. Wang that the three prospective teachers would adequately prepare lesson plans, slides, and possible materials and techniques involved, and refine every detail through repeated rehearsals to conduct the upcoming TL in its best state. Ms. Wang expressed her approval and confidence in the responses the researchers and prospective teachers would take to address these potential contradictions.

4.1.2. Secondary Contradictions

Ms. Wang highlighted this level of contradictions and considered them to be the ones that would determine the effectiveness of this TL and most urgently needed to be addressed.

> *I think that the establishment of positive interaction between the prospective teachers and the students that I just mentioned would not yet be particularly challenging since I believe that they (i.e., the prospective teachers) are psychologically prepared and also, I believe, they have the experience and ability to work with students. However, as teachers, these graduate students need to learn about students' learning status to design an achievable goal for them accordingly. In this process, how much do these graduate students know about the textbooks? How much do they know about the students? Do they have a clear idea of the lesson plan to achieve the goals? I'm not so sure about that, so I think those difficulties and challenges are probably the ones that need the most attention.*

Contradictions between subject and objects (SC-1). The subject of concern in this study was the students, for whom the objects were the experience of a TL, and the expectation of these objects (i.e., the outcomes) was to stretch the in-class knowledge, explore the rules of PM mathematically, and develop model concepts.

Ms. Wang briefly introduced the students' learning status from her teaching experience and accordingly presented her predicted contradictions between the subject and objects (including the outcomes).

> *Your Conceived Didactical Sequence for the TL is fun and will be engaging for the students. The students in this class are currently in seventh grade, and they are very active, and curious and enjoy hands-on activities and teamwork in the classroom. They have already learned preparatory knowledge such as the concepts of triangles, quadrilaterals, and polygons, and they have recently learned about translations, rotations, and folding before your PM-themed lessons, all of which can prepare them for this TL. In addition, students have many experiences with PM in their lives. So, the students probably won't have much difficulty understanding this concept. However, when exploring the single PM and combined PM, inevitably, the formula for the sum of the interior angles of polygons (including triangles) will be involved, whereas they have not learned this formula so far. So, I think students may encounter difficulties just after stage (b), which presents a challenge to your lesson plan.*

After Ms. Wang's elaboration, the researcher pointed out that the students had learned about linear equations with two unknowns in sixth grade and invited her to comment on whether the students might encounter difficulties in using them to solve problems. Ms. Wang stated:

> *Yes, students have learned about linear equations with two unknowns in the second semester of sixth grade and about finding integer solutions to a linear equation with two unknowns, so I think this is unlikely to be a difficult area for them. The biggest difficulty lies in the exploration of the formulas for the sum of the interior angles of triangles and the sum of the interior angles of polygons, and I think students need to be allowed to explore and discover.*

Contradictions between subject and tools (SC-2). With the subject and objects (including the outcomes) identified, the tools in this study included the teacher's teaching sequence, task design, and so forth. The researcher reintroduced the elements of these tools to Ms. Wang, who, based on the Conceived Didactical Sequence described by the researcher, predicted the possible contradictions among the elements according to her knowledge of the students' learning status and teaching experience.

> *I think your emphasis on interaction in the classroom is appealing to students, and the group work and presentations are also attractive. As far as I know, students enjoy a lively classroom, and personally, I love to provide students with opportunities to interact and*

discuss in my daily teaching. If there are any contradictions to be figured out, I think it might appear in the teaching design.

Ms. Wang preferred to detect potential contradictions in the teaching design. To obtain more specific information, the researcher invited her to elaborate on those contradictions specifically in terms of didactical sequence and task design. She mentioned:

According to your provided Conceived Didactical Sequence, I think the current approach is clear and logical. You have followed a teaching process from special to general, starting with the exploration of single PMs of such common geometric figures as regular triangles and squares, and gradually transitioning to exploring single PMs with special geometric figures such as regular pentagons and regular hexagons, a process of exploration accessible to the students. Also, it's good that you followed a logical progression from single PM to combined PM. For the fact that both an arbitrary triangle and an arbitrary quadrilateral can be used as the basic figure to construct single PMs, I am not sure if they are easily acceptable, so I suggest you invest more time in the class. In addition, I would recommend that you leave out the consideration of constructing PMs using regular pentagons and a special rhombus; after all, this is not well aligned with the previous theme (i.e., constructing combined PMs with two regular polygons as the basic figures), as well as it would increase students' learning burden. Lastly, as students have just finished the chapter on translation, rotation, and folding, for which PM is an extended learning material, I think you might consider introducing relevant elements involved in a life or in artwork to establish a connection between this chapter's knowledge and PM.

When consulted about the potential contradictions between the task design and the subject, Ms. Wang said:

Probably still lies in exploring the formula for the sum of the interior angles of polygons and applying this formula to analyze which figures are available as the basic one to construct PMs, which I think is challenging for students.

Ms. Wang provided a well-grounded analysis of the potential contradictions between the subject and tools (with a focus on didactical sequence and task design), as described above. The researcher and prospective teachers scrutinized the Conceived Didactical Sequence against these viewpoints, consistently agreed with the shortcomings Ms. Wang raised, and suggested another possible contradiction that constructing single PMs with some special non-orthogonal pentagons as the basic figure would be challenging, hence a brief introduction in the classroom would be appropriate.

Contradictions between tools and objects (SC-3). According to the Conceived Didactical Sequence, Ms. Wang said:

Compared to the (potential) contradictions between the subject and the tools, I think the (potential) ones between the tools and the objects are not as prominent.

The researcher explained:

Probably because we conceived the tools concerning the objects (including the outcomes), which ensured a theoretically coherent relationship between the tools and the objects. In other words, we designed the lessons based on the objects, so they appear to be harmonious. But still, we would appreciate some advice from you.

Ms. Wang commented:

Yes, your intention to develop students' comprehensive skills and modeling concepts is fine in terms of the teaching design alone. However, returning to my previous opinion, would the students be able to cope with the challenging tasks involved (i.e., the contradictions with the tools)? Let's talk back to the relationship between your design and your objects. It's good that your design runs through a theme from the very beginning to end, with logic and progression. I'm not sure if you place a particular emphasis on mathematical modeling in the classroom, but I don't think this is a good idea. Because what the curriculum *standard requires of middle school students is modeling concepts, not something like the rigorous mathematical modeling at the college level*.

The researcher concurred with Ms. Wang's observation that the potential contradictions between the tools and objects probably lie between the teaching design (including the didactical sequence and tasks) and the objects of developing students' modeling concepts, which implies that excessive demands cannot be placed on seventh graders. Therefore, it is important to ensure that the objects of the teaching design are geared toward model concepts rather than rigorous mathematical modeling.

Other contradictions. Ms. Wang did not elaborate much on the potential contradictions among the other elements. The researcher considered that, first, the school, teachers, and students all expressed a willingness and interest to participate in this research project. In particular, this school was willing to provide enough time to support the implementation of this project and offered to organize a competition and provide prizes for each group during the student presentation session (i.e., Session 4), which implied sound support of the rules for this TL. Second, although the researcher and Ms. Wang would not be involved in classroom teaching directly, they would remain deeply involved in the stages of teaching design, implementation, and post-class communication, and would keep close attention to the teaching process and students' learning status, which implied that the contradictions between the subject and community could be excluded. Third, the prospective teachers would repeatedly refine the teaching plan and teach following the ideas advocated by the new curriculum standard, with adequate respect for students' central role in learning to explicitly define the division of labor between teachers and students in the classroom. To sum up, this study focused on the potential contradictions among the subject, tools, and objects.

4.1.3. Tertiary and Quaternary Contradictions

According to Ms. Wang, she considered the potential contradictions in the TLAS to be mainly the secondary ones, as described above. Regarding the potential tertiary and quaternary contradictions, Ms. Wang thought they would not probably be prominent.

If I understood correctly, the tertiary contradictions are about the conflicts between the TL you will be conducting and the regular teaching. I don't think there are any conflicts because the current curriculum standard also advocates TL as well, and our school has already introduced similar open-ended, exploratory programs like that in the after-school service courses, where students enjoyed them.

When asked whether adequate time would be guaranteed to complete this TL, Ms. Wang said:

No problem, because this is our partnership project, the weekly after-school service hours can be used to carry out it, and this TL will exactly enrich the after-school service courses.

When asked about potential quaternary contradictions, that is, possible ones between the TLAS and its external activity system, Ms. Wang said:

I think there is probably not a problem because all the classes are teaching at the same pace and your sessions for this class will not affect the pace. . . . . . I feel that if we work well on this project, other classes will also be interested in joining your project.

4.1.4. The Overall Picture of the Contradictions Focused upon in this Study

According to Ms. Wang's judgment, the contradictions in this TLAS were mainly secondary ones. Moreover, the contradictions between the subject and objects, subject and tools, and tools and objects deserved special attention. Given this, we focused on analyzing the partial but key contradictions in this TLAS (see Figure 2).

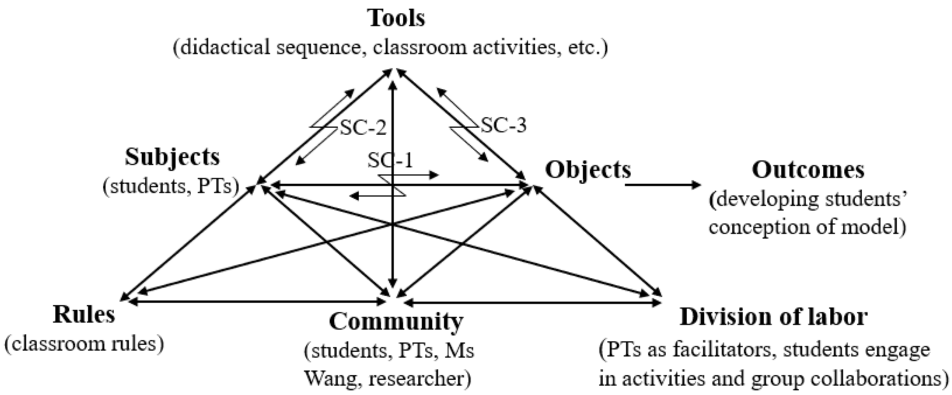


Figure 2. The key potential contradictions involved in the TLAS focused on in this study.

4.2. Pedagogical Considerations for Those Contradictions

Incorporating Ms. Wang's viewpoints, we adapted the Conceived Didactical Sequence and designed a detailed teaching plan. This plan included learning objects (including the overall objects and those for each session), the detailed didactical sequence, and the involved classroom activities. Following three rounds of discussion and revision, the Final Didactical Plan was developed.

4.2.1. Setting Moderate Learning Objects

Complying with the curriculum standard, teaching rigorous mathematical modeling at the middle school level is impractical, thus we positioned the objects of this TL as developing students' modeling concepts only, meaning establishing the concept of expressing and explaining real-world phenomena with mathematical models (e.g., formulas). With this in mind, the students were asked to write a research report after three sessions and to present it to the class in Session 4.

> *Hi everyone, after the three vivid and interesting sessions about PM, we have learned about its concept, explored its principles, conditions, etc., and experienced its beauty. Now, let's work together to apply this acquired knowledge!*
>
> *There are two kinds of regular polygons (regular m-sides and regular n-sides, respectively) with a number of x and y. Please construct a mathematical model of combined PM using them as the basic figures, draw a pattern of PM based on your model, and find an example of this pattern in real life (either a photo you shot or material obtained from the Internet), and finally form a research report*.

In addition, we considered the appropriateness when designing the concrete objects of each session. For example, constructing combined PMs using three regular polygons as the basic figures was not involved in Session 2, and constructing single PMs using certain non-regular pentagons as the basic figure was not described in depth in Session 3.

By setting moderate learning objects, the gap between the subject and objects would become appropriate and accessible, which might dissolve the potential contradictions between the subject and objects (i.e., SC-1).

4.2.2. Creating Sustainable Exploratory Learning Environments Featuring Progressive Didactical Sequence and Coherent Activities

Considering students' acceptability, we followed the principle of moving from simple to complex and from special to general when designing the didactical sequence (see Table 1 for the didactical sequence and activities involved). For instance, students learned the single PM first, and then the combined PM; they learned to construct the PM with a regular polygon as the basic figure first, and then with a general figure. In addition, we followed the principle of moving from external manipulation to internal mathematical thinking and then to mathematical application. For instance, in Session 1, students were first offered opportunities to assemble cards to explore and discuss whether a single PM could be constructed using a regular triangle, square, etc. as the basic figure (i.e., A-4). Then, groups were asked to communicate and analyze the conditions for constructing a single PM using a regular polygon as the basic figure using a mathematical model (i.e., A-5), and to mathematically explain which of the various regular polygons could be used as the basic figure to construct a single PM (i.e., A-6). Lastly, groups worked together to draw a pattern for constructing a single PM using a regular polygon as the basic figure (i.e., A-7). In Session 2, a similar progressive didactical sequence was also followed (e.g., A-10→A-11→A-12→A-13→A-14→A-15).

Such a progressive didactical sequence allowed the tools to conform to the student's mathematical learning, which would ensure a fluid learning process. Thus, such a design could dissolve the potential contradictions between the subject and tools (i.e., SC-2).

Table 1. The didactical sequence and activities involved.

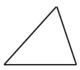
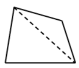
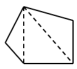
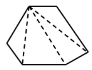
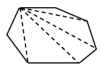
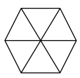
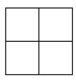
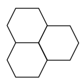
Session	The Didactical Sequence and Activities Involved
1	A-1: Observe floors, artwork, etc., and summarize the concept and types of PM (including single PM and combined PM). A-2: Students provide other examples of applications of PM and analyze how it is formed by basic shapes. A-3: Students study PM with a mathematical perspective and are guided to start their exploration by constructing a single PM using a regular polygon as the basic figure. A-4: Students explore and discuss whether a single PM can be constructed using a regular triangle, square, regular pentagon, and regular hexagon as basic figures by playing games. A-5: Through group communication, students analyze the conditions for constructing a single PM using a regular polygon as the basic figure with a mathematical model (Model 1). A-6: Through group communication, students mathematically explain which regular polygons can be used as basic figures to construct a single PM. A-7: Through group collaboration, students draw a pattern of a single PM constructed using a certain regular polygon as the basic figure.
2	A-8: Students recall the concept of PM and the mathematical model of constructing a single PM using a regular polygon as the basic figure. A-9: Students are guided to move on to constructing a combined PM using two regular polygons as basic figures. A-10: Through games, students explore whether a regular triangle and square can be used as basic figures to construct a combined PM. A-11: Through group communication, students mathematically explain why a combined PM can be constructed using a regular triangle and square as basic figures. A-12: By seeking integer solutions to linear equations with two unknowns, students find all possible cases for constructing a combined PM using a regular triangle and square as basic figures. A-13: Through group communication, students build the mathematical model (Model 2) for constructing a combined PM using two regular polygons as basic figures. A-14: Through group collaboration, students explore all possible cases of constructing a combined PM using a regular triangle and regular hexagon as basic figures. A-15: Students try to construct a combined PM using a regular octagon and another regular polygon as basic figures.
3	A-16: Students are guided to move to the general case to consider whether only regular polygons can be used as basic figures to construct single PMs. A-17: Through games and collaboration, students discover that any triangle and any quadrilateral can be used as basic figures to construct single PMs (which can be viewed as Model 3). A-18: The teacher shows the patterns of constructing single PMs using certain non-regular pentagons as basic figures to arouse students' impressions of the charm of PMs. A-19: Students observe irregular figures, and the teacher guides them to construct a PM using translations, rotations, and folding, as well as to draw beautiful patterns. A-20: Students systematically review the contents of these three sessions and are guided to summarize the exploration process of explaining problems in life with mathematical models.
4	A-21: Group presentations of research reports (the teacher provides a template for the research report; the report takes one week to complete collaboratively).

4.2.3. Scaffolding Necessary Knowledge for Students to Model Mathematically

Considering that the purpose of this TL was to help students develop the model concept, which requires learning certain supportive knowledge, it was necessary to provide them with the knowledge scaffolds. For example, when analyzing the conditions for constructing single PMs using a regular polygon as the basic figure through a model (i.e., A-5 in Session 1), students were required to know that this means each interior angle of those regular polygons is divisible by 360, and how to calculate those interior angles. The former is a necessary conceptual scaffold, while the latter is a necessary procedural one [37]. In response, we planned to use visual representations to guide students in exploring the formula for the sum of the interior angles of polygons and to guide them in applying it to analyze problems related to PM. In the classroom, we guided students to experience the process of observation and induction to conclude this formula and then to calculate each interior angle of different regular polygons, as well as to determine whether a single PM can be constructed using those regular polygons as the basic figure (see the designed worksheet in Table 2). As another instance, when enumerating all possible combinations in constructing a combined PM using a regular triangle and square as the basic figures (i.e., A-12 in Session 2), we guided students to transform this problem into one related to a linear equation with two unknowns (which is a conceptual scaffold) and guided them to recall what they had learned in the second semester of Grade 6 of finding integer solutions to a linear equation with two unknowns (which is a procedural scaffold).

By providing students with the necessary knowledge scaffolds, the potential contradictions between the tools and objects in the TLAS (i.e., SC-3) were expected to be dissolved.

Table 2. The worksheet for A-5 and A-6 in Session 1.

No. of Sides	**3**	**4**	**5**	**6**	**7**	. . .	**N**
Q2						. . .	
Q3	180°	360°	540°	720°	900°	. . .	(n−2)180°
Q4	60°	90°	108°	120°	900°/7	. . .	(n−2)180°/n
Q5	yes	yes	no	yes	no	. . .	yes
Q7	6	4	-	3	-	-	-
Q8			-		-	-	-

Question Chain.
Q1: What is the sum of the interior angles of a triangle?
Q2: How can the problem of the sum of the interior angles of polygons be transformed into the problem of the sum of the interior angles of triangles?
Q3: How to calculate the sum of the interior angles of a polygon?
Q4: If this polygon is a regular one, what is the degree of its every interior angle?
Q5: How do you determine if you can construct a single PM using this regular polygon as the basic figure? Why?
Q6: Why can't a regular polygon with more than six sides be used as the basic figure to construct a single PM?
Q7: If a certain regular polygon can be used as the basic figure to construct a single PM, how many such regular polygons are needed at least?
Q8: If a certain regular polygon can be used as the basic figure to construct a single PM, what does the pattern of this PM look like?

4.2.4. Situating Students in an Interactive, Communicative, and Cooperative Classroom Environment

Following the constructivist approach, students had adequate opportunities for independent thinking, group communication, collaborative problem solving, and interaction with the teachers in this TL. Teachers motivated students to actively think and explore through such measures as eliciting questions and formative assessments. It should be mentioned that the school provided resources such as the venue, facilities, and prizes to support the presentation activities in Session 4 and invited professors from universities and teachers from the school as judges to interact with the groups. In such a relaxed, open, democratic, and harmonious classroom atmosphere with an emphasis on interaction, the potential contradictions concerning rules, division of labor, and other elements were probably dissolved.

4.3. The Observed Dissolution of Those Contradictions

This study focused on examining the dissolution of the secondary contradictions and students achieving the objects. Due to the space limitation, only partial episodes are presented here.

4.3.1. Episode 1

This episode is an excerpt from Session 1 and involved the three activities A-1, A-2, and A-3.

> *PT-Xin: Guys, let's look at the floor under our feet first, then at the five pictures on the screen (the first one is the ceiling of the classroom, the second is a nice mural, the third is an artwork by Maurits Cornelis Escher, the fourth is a designed pattern for the floor of a shopping mall, and the fifth is a pattern decorating a wall). What characteristics do they have in common? Please communicate and discuss within each group and share your findings later.*
>
> . . .. . .
>
> *Students: All these patterns are pieced together.*
>
> *PT-Xin: With what?*
>
> *Student: They are formed by joining several smaller figures together at one point.*
>
> *PT-Xin: What are the characteristics of these smaller figures?*
>
> *Student: They are the same.*
>
> *PT-Xin: So, is the whole pattern made up of one or several figures of the same size and shape?*
>
> *Student: Yes.*
>
> *PT-Xin: So, if we look again, what do the figures look like at those joints?*
>
> *Student: The joints are seamless.*
>
> *PT-Xin: Good. These figures are pieced together without slits or overlaps, which we call a PM. Could any student define it based on these characteristics?*
>
> *Student: A pattern that is pieced together with the same figure without either slits or overlaps is called a PM.*
>
> *PT-Xin: Good, please sit down. Then a PM can be defined as a pattern made by piecing together one, or several, basic figures without either overlaps or slits. Do you know any other examples of PMs in our lives?*
>
> *Student: The floor, the ceiling, or putting desks together to form a conference table.*
>
> *Student: And honeycomb.*
>
> *PT-Xin: Good, everyone seems to find math from life well. Here, please look at these three patterns on the board again (see Figure 3). Which of them is the PM?*

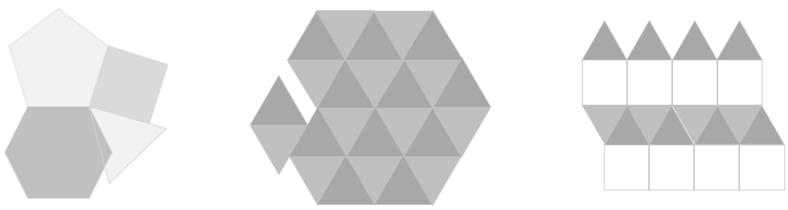


Figure 3. The three patterns presented to students.

> *Student: The first is not, because there is an overlap; the second is not, because there is a gap, and the third is.*

PT-Xin: Good. When judging whether a pattern is a PM or not, we can draw on its three characteristics. Let's observe the previous five patterns and the third one here. What are the basic figures that construct these PMs? How can the whole pattern be viewed as constructed from this or these basic figures? Relate this to what we have learned earlier about translation, rotation, and folding.

(Through communication among students, they developed further knowledge about the nature of PM)

PT-Xin: Well. We can find that PM can be considered as getting from a basic figure or several basic figures by translation, rotation, and/or folding, which is the nature of PM. So, can all figures be used as basic figures to construct a PM, and if not, which ones can? Here, we start from the simplest case and consider using one figure as the basic one to construct a PM, and, following the principle of moving from the particular to the general, we first choose the case of a regular polygon to explore.

. . .. . .

In the textbook, PM is placed as an extension learning material at the end of the chapter on translation, rotation, and folding, but the underlying ideas of PM are not explained from a mathematical perspective. In this case, students may not be able to perceive the applications of translation, rotation, and folding, nor develop a better understanding of PM simply by reading the material and observing the artworks involved. However, for students, PM is an unfamiliar concept and they have no idea how to explore the underlying mathematical model in PM, which implies that there may be contradictions between the students and objects. Considering these issues, students were guided to summarize the concept of PM from phenomena and examples in daily life and artwork, to discover connections between PM and previously learned knowledge, as well as to adopt a working mode from simple to complex and from particular to general to start their exploration. The potential contradictions between the subject and objects were visibly dissolved, judging from the student's responses in the classroom and communication between the teacher and students.

4.3.2. Episode 2

This episode is an excerpt from Session 2 and involved the activity A-12.

PT-Tang: Through the hands-on game earlier, we find that one can use a regular triangle and a square as the basic figures to construct a PM. For example, three regular triangles and two squares are available. Can you explain the mathematical rationale for that?

Student: This is because when piecing them together, the angles of the three regular triangles add up to 180 degrees and the angles of the two squares add up to 180 degrees.

PT-Tang: Excellent, that satisfies the definition of PM. We just discovered this through a hands-on game, and now we've confirmed it mathematically. In other words, $60 \times 3 + 90 \times 2$ *is exactly equal to 360. I noticed you all constructed very creative patterns, and altogether two patterns of PMs were provided (see Figure 4). Next, I have a question, can we only use three regular triangles and two squares to construct a PM, are there any other ways? Can it only be three regular triangles and two squares? Can it be four regular triangles and three squares? Can it be two regular triangles and two squares? Can you mathematically solve this problem?*

Student: Build an equation.

PT-Tang: Okay, how do we do it?

Student: Suppose the number of the regular triangles is x *and the number of the squares is* y*. Then* $60x + 90y = 360$*.*

PT-Tang: Is that correct? Can this equation be solved? Have you ever learned this equation?

Student: Yes, a linear equation with two unknowns.

PT-Tang: How do we solve this equation?

Student: First simplify it to get 2x + 3y = 12, then we can get the solutions.

PT-Tang: I feel that this equation has countless sets of solutions. For example, if x equals -1, y equals 14/3. So, what exactly should we seek for a solution to this equation?

Student: Positive integers.

PT-Tang: Excellent, why should we seek positive integer solutions?

Student: Because non-positive integers don't work.

PT-Tang: Why don't they work?

Student: The number of figures cannot be zero, because if x is zero then there is only one figure. And of course, it cannot ever be a negative number.

PT-Tang: Good, now we are clear that we have to find positive integer solutions to this equation (60x + 90y) = 360, right? How do we find it?

Students: Just try them one by one.

PT-Tang: Where can we start trying?

Student: Start with 1, but simplify it first, we can simplify it to 2x + 3y = 12.

PT-Tang: With x equal to 1, can we calculate the solution for y? What is it?

Student: 10/3

PT-Tang: Is it fine if y = 10/3?

Student: No.

PT-Tang: Next, it's time to use 2 regular triangles, right? Let's see how many squares it needs.

Student: 8/3.

. . .. . .

PT-Tang: x = 6, y = 0, which are both integers, does this set of solutions work?

Student: No.

PT-Tang: Why not?

Student: They cannot be zero.

PT-Tang: Yes. Since we are constructing a combined PM with two regular polygons, neither of them can be zero in number. Do we need to continue to try further down?

Student: No.

PT-Tang: Why not? Because if x is equal to 7, y must be a negative number, right? So, we've just found all the positive integers for this equation. How many cases have we found that are available?

Students: Only the third case.

PT-Tang: OK, that means we found a general way to solve the problem of constructing a PM using two regular polygons as the basic figure. We just need to know the degree of each interior angle of each regular polygon, and we can suppose the number of them separately. Can anyone build a generalized equation?

Student: $\alpha x + \beta y = 360$.

PT-Tang: What solutions should we find about α *and* β*?*

Student: The positive integer solutions.

. . .. . .

In addition to the formula for the sum of the interior angles of polygons, linear equations with two unknowns is also supportive knowledge affecting students' ability to solve the problem of constructing a combined PM with two basic figures. Without this supportive knowledge, the planned guidance for students to explore the mathematical model underlying the combined PM problem in Session 2 would not be achieved. This implies that some contradictions may occur between the tools and objects, making it necessary to provide students with knowledge scaffolds in teaching.

With these in mind, the teacher posed a series of eliciting questions to inspire students to explore based on what they had already learned after they provided two solutions for constructing a combined PM using a regular triangle and square as the basic figure. From the responses in the classroom and communication with the teacher, the students were able to apply their knowledge related to linear equations with two unknowns accurately and fluently to solve the problems, indicating a visible dissolution of the potential contradictions between the tools and objects.

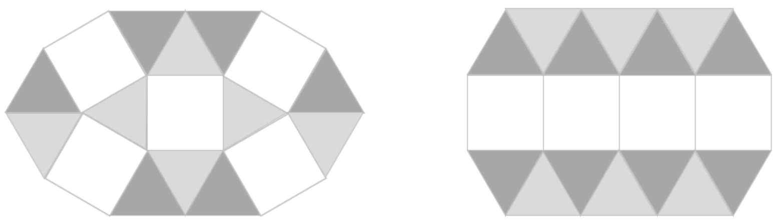


Figure 4. The two solutions provided by students for a combined PM constructed with three regular triangles and two squares.

5. Discussion

In this study, we obtained perspectives and evidence of potential contradictions by interviewing teachers who had rich experience with students, textbooks, and teaching, and examined these contradictions in a system based on a CHAT perspective. According to the information collected, the potential contradictions in this TLAS were mainly secondary ones.

When focusing on the secondary contradictions, Ms. Wang preferred to consider those occurring mainly between two of the three elements: subject, objects, and tools. Her particular concern about the contradictions among these three elements may be related to her role as a frontline teacher. For her and her peers, the core mission of classroom teaching is to help students better achieve their learning objectives through teachers' elaborated lessons, and Chinese teachers concern themselves with the analysis of learning objects and students' learning status when designing lesson plans [38]. As such, Chinese teachers are skilled at, or like to focus on, reflecting on the logic, rationality, and fluency of their teaching in terms of the relationship among the three elements mentioned above.

Specifically, regarding the contradictions between the subject and objects, Ms. Wang considered the exploration of the formula for the sum of the interior angles of polygons as an important factor that influenced students to reach the objects, as this was supportive knowledge to ensure that this TL was performed smoothly. Considering the close relationship among the various pieces of knowledge within the mathematical knowledge system [39], the influence of supportive knowledge on the learning of subsequent knowledge cannot be ignored. Regarding the contradictions between the subject and tools, Ms. Wang first excluded the problem of incompatibility between the teaching approach and students, saying that the teacher–student interaction, student–student interaction, and cooperation and communication advocated by TL are in line with the student's interests and their daily learning. From September 2021 to August 2022, the first author participated in The University Teachers Enter Primary and Secondary Schools Project, where he engaged in in-depth teaching and learning research activities and observed classrooms in the school involved in this study and found that this school placed great emphasis on teaching innovation and encouraged teachers to follow the latest curriculum standard ideas to develop lively forms of teaching and learning activities. This supported Ms. Wang's judgment that the teaching approach could be adapted to student learning. Second, Ms. Wang centered the contradictions on the didactical sequence and task design. While agreeing with our Conceived Didactical Sequence, she recommended forgoing exploring the problem of constructing combined PMs with a regular pentagon and special rhombus as the basic figure as well as the in-depth treatment of constructing single PMs with a non-regular pentagon as the basic figure, pointing out that these might increase students' learning burden. In addition, she suggested that we develop art-related materials at the end of these series of sessions to realize a substantive connection between this TL and the content of the chapter in which it was situated. These considerations reflected the teacher's concern for student acceptability when designing lessons and supported the pedagogical logic of Chinese teachers' emphasis on extending the application of knowledge from the content of the textbook [40]. Regarding the contradictions between the tools and objects, Ms. Wang considered that one cannot analyze them simply based on a teacher's perspective. If the didactical process and involved tasks are designed only against the objects, "they will certainly look harmonious". Ms. Wang's viewpoint greatly inspired us, that is, when considering the contradictions between the tools and objects, one should also take the subjects' acceptability into account, rather than pursuing a theoretical consistency between the tools and objects while neglecting the students' learning status. Ms. Wang's suggestions for our Conceived Didactical Sequence were constructive and meaningful.

We cannot claim with any certainty that tertiary and quaternary contradictions did not exist; however, we fully respect Ms. Wang's judgment as an experienced teacher. We considered two possible reasons to support her judgment: First, China has been introducing new mathematics curriculum standards since 2001 (the current Curriculum Standard-2022 is the second revision of the initial version), and although the concept of TL was not explicitly introduced at that time, the emerging concepts of contextualized teaching, problem solving, collaboration and communication, and teacher–student interaction were officially introduced [41]. As curriculum reform proceeds and the activities of teachers in learning the curriculum standards become routinized, teachers in Shanghai, an educationally developed city, can actively embrace the pedagogical innovations advocated by the latest curriculum standard [42]. As a result, teachers positively regarded attempts to implement pedagogical innovations in their daily teaching and were able to proactively address the challenges that these pedagogical innovations might raise. The school involved also showed a positive attitude towards this project, especially by providing a favorable venue and exquisite prizes for the student presentations in Session 4. An additional piece of direct evidence was that Ms. Wang conducted a district-level educational project on pedagogy innovation featuring project-based learning as early as 2017, which also showed her and her school's pursuit of pedagogical innovation, which may explain why the tertiary and quaternary contradictions were not particularly prominent in this TLAS. Second, in recent years, to alleviate students' strenuous practice at out-of-school tutoring institutions and to enrich their learning after school, Chinese primary and secondary schools have thoroughly implemented after-school service courses [43], in which two sessions are extended after daily classes finish (usually around 4:00 pm). The current TL coincided with the after-school service course idea, which was probably the other reason why tertiary and quaternary contradictions were not particularly prominent in this TLAS.

Following the analysis of potential contradictions, we designed targeted lessons to dissolve them. Specifically, we set appropriate learning objects, designed progressive teaching processes and exploratory activities, provided necessary knowledge scaffolds, and created an interactive and communicative learning atmosphere to purposefully dissolve the contradictions among the subject, objects, and tools. Fortunately, from the information we gathered from the classroom, the student's involvement was good, the learning process was smooth, and the communication and interaction were motivated. Through this elaborate TL, students were able to develop a comprehensive understanding of the concept of PM and the mathematical models involved, as well as the relationship between mathematics and life as well as art. It was shown that our measures to address the potential contradictions yielded positive effects. It should be reiterated that in examining the potential contradictions within this TLAS, we relied primarily on the perspectives of the teacher who taught this class, and in proposing measures to dissolve those potential contradictions, we were adequately faithful to the perspectives acquired from the teacher's interviews, as detailed earlier. We considered and analyzed the primary, secondary, tertiary, and quaternary contradictions, but focused specifically on certain critical pairs within the secondary ones, that is, potential contradictions between the students, tools, and objects. The proposed measures to address those critical contradictions were necessary and proven to be productive in facilitating student learning, but other aspects relating to the learning environment deserve attention as well. For example, although measures to create a relaxed, open, democratic, and harmonious classroom atmosphere for students were proposed, and student–student and student–teacher interactions were highlighted, the positive role of student agency as well as the educational infrastructure in the TLAS also deserve sounder attention.

In an earlier study also on PM-themed TL [21], the researcher highlighted examining the influence of teachers' pedagogic styles on their design of TL and their utilization of thematic contexts. However, less attention appears to have been dedicated to examining students' possible learning profiles in TL, which include the incompatibility, challenges, etc., that they may encounter. To address this issue, the present study explored the potential contradictions that may occur across multiple elements of the TLAS from a CHAT perspective when implementing PM-themed TL in the classroom and adopted tailored measures to dissolve those contradictions; thus, this new work is an extension of that earlier research. The innovation of this study lies in applying CHAT as theoretical guidance to predict the potential contradictions in a classroom activity system, propose feasible solutions to dissolve those potential contradictions, and finally analyze whether those potential contradictions are dissolved through classroom observation. The theoretical contribution of this study involves adopting CHAT as guidance to dissolve the contradictions in classroom teaching, which expands the application of the theory compared to previous studies that focused more on the contradictions between the researchers and teachers during a lesson study. The practical contribution of this study is that we adopted CHAT as guidance to comprehensively analyze the contradictions that may occur in the classroom so that corresponding preparations can be purposely made before the lesson. This innovative approach inspires teachers to adopt this theory routinely in their teaching to improve instruction, including the design of teaching sequences and tasks.

6. Conclusions

In this study, we asked three interlocking questions, that is: (a) What are the potential contradictions in a TLAS? (b) What measures can be adopted to dissolve those potential contradictions? (c) How are those contradictions dissolved as the lesson proceeds? To answer the first question, we analyzed the potential contradictions in a TLAS from a CHAT perspective. According to the predictions of teachers who were experienced in teaching, these contradictions were more likely to occur as secondary ones, and they were probably going to appear between two of the three elements: students, tools, and objects. Specifically, the potential contradictions between the subject (i.e., students) and objects mainly lay in the exploration of the formulas for the sum of the interior angles of triangles and the sum of the interior angles of polygons. The potential contradictions between the subject and tools mainly resided in the didactical sequence and task design; in addition, another contradiction might have been the challenge of constructing single PMs with some special non-orthogonal pentagons as the basic figure. The potential contradictions between the tools and objects mainly lay between the teaching design (including the didactical sequence and tasks) and the object of developing students' modeling concepts. To answer the second question, with these predicted potential contradictions as guidance, the researcher made adequate consideration of the learning objects, didactical sequence and activities involved, knowledge scaffolds, and classroom atmosphere, and designed lesson plans dedicated to dissolving these contradictions. To answer the third question, students' performance in the classroom was captured and analyzed, and the results revealed that those potential contradictions were effectively dissolved. The most important implication of this study for teaching and learning is that before designing a lesson, especially an emerging, unfamiliar, and challenging one, a thorough analysis of the potential contradictions in the activity system can be conducted with CHAT to acquire a well-founded judgment, and the learning objects, didactical sequence, and classroom activities can be tailored based on the analysis and judgment.

7. Limitations

We acknowledge that there are several limitations to this study. First, if those not directly involved in classroom activities (i.e., the researcher and Ms. Wang) are not considered, the subjects of the TLAS include the prospective teachers in addition to the students. The current study was part of a larger project that also aimed to explore the potential contradictions encountered by prospective teachers in their involvement in designing and implementing lessons and the dissolution process. However, we focused only on the students as the subject in this study. Second, the findings derived from this study are not representative due to differences in macro factors such as cultural background and educational system, as well as differences in school climate, grade level, teaching content, and subject involved. Nevertheless, this study focused on Curriculum Standard-2022′s latest proposed TL and modeling concepts, which are ongoing concerns in the international mathematics education field [6,44]; therefore, the methodology and findings of this study are still enlightening. Third, to dissolve the potential contradictions in the TLAS, the present study incorporated Ms. Wang's perspectives with more attention placed on the instructional efforts in terms of setting learning objects, tailoring didactical sequences and activities, delivering knowledge scaffolds, and creating classroom atmosphere. The classroom environment is a complex system, and although we concentrated on those possible major contradictions, other possible aspects were not exhaustively considered.

With these considerations, first, future research may focus on teachers, especially prospective teachers, as a subject, to examine the difficulties that may be encountered in the design and implementation of teaching and learning activities and the process of their dissolution. Second, comparative studies could be conducted between countries and between disciplines to examine differences in potential contradictions in the TLAS and whether the process of dissolving these contradictions varies by cultural background, discipline, and other factors. Third, concerning the dissolving of potential contradictions, it would be beneficial to acquire instructive information from an expanded pool of sources, such as an expanded group of interviews, and accordingly formulate more comprehensive and systematic instructional programs.

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Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki and approved by the Ethics Committee of Shanghai Normal University (protocol code 2023004 and date of approval, 6 March 2023).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: The data presented in this study are available upon request from the corresponding author.

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