

Article Roundabout Trajectory Planning: Integrating Human Driving Models for Autonomous Vehicles

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Abstract: This research investigates the utilization of human driving models in autonomous vehicles, particularly in scenarios with minimal or no interactions with other vehicles. Human driving models provide valuable insights into driver behavior and play a crucial role in shaping the behavior of autonomous vehicles, enhancing their performance and user experience. The primary focus of this study is the creation of a planning model for autonomous vehicles when navigating roundabouts in the absence of traffic. This model seeks to emulate human driving behavior, ensuring predictability, safety, the optimization of traffic flow, and adaptation to various roundabout geometries. To achieve this, the research introduces a trajectory model that takes into account geometric attributes and speed variations within roundabouts. The model is calibrated using empirical data and generalizes parameters through statistical regression methodologies. In particular, speed profile modeling is evaluated for its consistency in creating plans that faithfully replicate human driving behavior in roundabouts. While the study presents a promising approach, it acknowledges limitations related to the model's reliance on geometric attributes and its inability to account for external factors like weather conditions. This research underscores the importance of bridging the gap between theoretical research and practical application, with the aim of enhancing safety and the overall user experience in real-world driving scenarios.

Keywords: autonomous vehicles; human driving models; roundabouts; speed profiles; traffic-free planning

1. Introduction

In 2019, an estimated 1.35 million people died as a result of road accidents, which corresponds to around 3700 deaths per day. Road accidents are the ninth leading cause of death worldwide, ranking first among people aged 15 to 29. The geographical distribution shows that 93% of road-related deaths occur in low- and middle-income countries, due to factors such as inadequate road infrastructure, the use of unsafe vehicles and a lack of road traffic laws [1,2].

In Italy, 165,889 road accidents with injuries occurred in 2022, an increase of 9.2% compared to the previous year. The number of victims amounted to 3159, an increase of 9.9% [3]. These results are in line with those of 2019, the last year before the outbreak of the COVID-19 pandemic, with the road accident landscape in 2022 being the first in which global traffic conditions influenced by the pandemic are considered to be almost inconsequential.

The main risk factors for road accidents include excessive speed, driving under the influence of alcohol or drugs, distracted driving and the non-use of safety devices such as seat belts and helmets. Autonomous vehicles (AVs) have the potential to improve road safety in several ways [4,5]:

Reducing human error: AVs are not prone to the same human errors that can lead to accidents, such as distraction, fatigue and drunk driving;



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- Improve environmental awareness: AVs can use sensors to recognize road conditions in real time, including friction conditions;
- Adjusting speed and trajectory more efficiently: AVs can use friction data to adjust speed and trajectory more efficiently, reducing the risk of accidents.

In terms of the safety aspects associated with the risk of accidents due to human error, autonomous vehicles can be trained to recognize and react to dangerous situations using models derived from human behavior, much like a human would. This assertion is supported by ongoing research in the field of autonomous driving. Engineers and researchers are developing methods to train autonomous vehicles using driving data collected from human-driven vehicles. These data can contain information about how people interact with traffic, react to dangerous situations and make driving decisions [6].

Autonomous vehicles can benefit from human driving models in various ways, whether they need to interact with other vehicles in complex situations or if interactions with other vehicles are rare or limited [7,8]. Here's how these models can be useful in both cases:

- (1) Interaction with other vehicles: In contexts where autonomous vehicles have to interact with other vehicles on the roads, human driving models can provide valuable insights into driver behavior and driving dynamics. By studying and learning from human driver data, autonomous vehicles can learn how to behave in complex situations such as standard intersections, roundabouts, or curves. For example, human driving models can provide information on trajectory choices, appropriate speeds in certain situations, and common driving habits. This information can be used by autonomous vehicles to make safer and more predictable decisions during interactions with other vehicles on the road.
- (2) Limited interaction scenarios: Even when interactions with other vehicles are rare or limited, human driving models can be useful for autonomous vehicles. For example, in autonomous driving situations in rural areas or areas with low traffic density, vehicles encounter fewer or no vehicles. Nevertheless, human driving models can provide information on how to handle certain road elements with conditional geometry (curves, roundabouts, highway ramps, etc.) or traffic signs or adverse weather conditions. Additionally, human driving models can be used to provide a more comfortable and familiar driving experience for passengers. For instance, if a human driver prefers gradual acceleration or gentle braking in certain situations, the autonomous vehicle can learn such habits and replicate them to provide a more human-like driving experience.

In this context, speed profiles obtained through naturalistic observations can be extremely useful for the human driving models employed in the learning phases of autonomous vehicles [9,10]. Here's why:

- (a) Real-world data: Naturalistic speed profiles are based on real data collected from vehicles in real driving conditions. These data represent the actual behavior of human drivers in real-world situations, allowing the human driving models to learn from authentic experiences. This helps make the models more accurate and adaptable to various road situations.
- (b) Contextual variation: Naturalistic speed profiles capture the variations in speed in different driving situations and contexts. This includes information about average speeds, maximum speeds, and typical decelerations/accelerations in certain areas or types of roads. Learning from these variations allows the driving models to guide autonomous vehicles to behave more realistically and consistently with human drivers in different scenarios, improving safety and efficiency in the autonomous driving system.
- (c) Consideration of individual preferences: Naturalistic speed profiles can also reflect individual driver preferences regarding speed and driving style. These preferences can be learned and taken into account by the human driving models during the learning process of autonomous vehicles. This enables autonomous vehicles to adapt

to the preferences of human drivers or passengers, providing a more familiar and personalized driving experience.

(d) Performance enhancement: Using naturalistic speed profiles can contribute to overall performance improvements in autonomous vehicles. For example, they can be used to fine-tune control algorithms, improve trajectory planning, or optimize acceleration and deceleration strategies. Integrating real driver data into the human driving models helps autonomous vehicles learn from realistic driving examples and develop more effective driving strategies.

Specifically with respect to the approach of autonomous vehicles to roundabouts, the collection of speed profiles obtained through naturalistic surveys may be particularly useful for the following reasons [11–14]:

- These profiles help autonomous vehicles understand appropriate speeds, acceleration, and deceleration required for safe driving.
- Contextualized speed profiles help autonomous vehicles make more informed decisions based on specific road contexts. For example, they enable speed adjustment based on the presence of other vehicles in roundabouts or traffic conditions.
- Naturalistic speed profiles allow autonomous vehicles to adapt to different roundabout geometries, and to understand and respect the laws of physics governing the dynamic behavior of vehicles. This includes managing centripetal force, optimizing tire friction and grip, and maintaining stability during curves.
- Naturalistic speed profiles can be used for validation and testing of autonomous driving systems. They allow comparing the behavior of autonomous vehicles with known human speed profiles to evaluate the effectiveness of the autonomous system and identify any necessary improvements.

This study proposes a model for planning the crossing of a single-lane roundabout, based simultaneously on the specification of the geometric curve and on the generation of the speed plane. The aim is to achieve a "human-like" planning, i.e., a planning based on the imitation of the human driving behavior within the limits of a safe driving mode.

It should be noted that the crossing trajectory considered is the fastest trajectory in a roundabout, i.e., the crossing trajectory without significant interference from other user categories (vehicles, pedestrians, bicyclists, etc.). Therefore, traffic-free design results in a crossing trajectory that considers only the geometric elements that make up a roundabout (diameter, entry radius, exit radius, deflection angle, lane width) and ignores other road users and other obstacles. Therefore, the crossing trajectory planning model must be able to generate a dynamically feasible trajectory based on aspects of human driving behavior that does not involve traffic impacts.

It is believed that this type of modeling is useful for at least the following four aspects:

- Predictable behavior: The fastest crossing trajectory with no interaction with other road users represents a predictable and safe behavior model that has been solidified by human driver experience. Validation and testing of autonomous vehicles on this route allows them to learn and adopt behaviors that humans recognize as effective.
- Safety: crossing roundabouts quickly and efficiently can contribute to road safety. Modeling autonomous vehicles on the fastest crossing route can verify that the autonomous system can maintain an appropriate speed and perform the proper maneuvers to safely traverse the roundabout, avoiding slowdowns and potential hazards.
- Optimizing traffic flow: the correct behavior of autonomous vehicles when passing through single-lane roundabouts can help optimize traffic flow. If autonomous vehicles follow the fastest crossing trajectory without interacting with other road users, they can help reduce roundabout crossing times and improve traffic flow.
- Adaptation to road conditions: Naturalistic speed profiles based on the fastest crossing trajectory can vary depending on the geometric characteristics of roundabouts. The model to be proposed allows autonomous vehicles to adapt to different roundabout configurations, such as diameter, entry radius, exit radius, deflection angle, and lane

width. In this way, autonomous vehicles can learn the appropriate behavior and be able to negotiate single-lane roundabouts safely and efficiently, regardless of the specific geometric specifications.

In order to better understand how the authors proceeded to achieve the above goal and what their methodological and experimental contribution was, the framework of the analysis proposed in this paper is described below (Figure 1).



Figure 1. Framework of the proposed analysis.

The framework describes the systematic approach intended to be proposed in the context of navigation of autonomous vehicles on single-lane roundabouts. The basic input is derived from "Roundabout Data" obtained from a set of integrated sensors in the autonomous vehicle. The subsequent phases of the framework include the sequential development of the "Path Model" and the "Speed Model". The result of the "Path Model", referred to as the "Reference Path", serves as input for the subsequent "Speed Model". Both models culminate in specific optimization processes. The "Reference Speed Model" goes through a tripartite procedural sequence that includes "Calibration", "Learning", and "Evaluation". This process is explained in the last segment of the framework. Importantly, it is conducted on the basis of empirical data obtained through an experimental investigation centered on the collection of trajectory and speed data derived from a sample set of five single-lane urban roundabouts.

To summarize, the framework describes a cascading process that begins with the autonomous vehicle gathering information about the geometric and functional characteristics of the existing roundabouts. This process culminates in the autonomous vehicle learning speed profiles that are modeled on human driving behavior. This is a key element not only for improving the performance of autonomous vehicles, but also for significantly improving their operational capabilities and the overall experience of users using this technology.

Ultimately, the original contribution of the present study can essentially be summarized in the implementation of the methodological process that led to the determination of the following two models, developed and calibrated in relation to the driving of crossing trajectories on single-lane roundabouts by autonomous vehicles: (1) the Path Model; (2) the Speed Model. Furthermore, thanks to the support provided by the experimental investigation described in Section 5, it was possible to analytically characterize any possible "Reference Speed Profile" based on human behaviors that, although typical of the Italian context in which the investigation was carried out, nevertheless constitute a reference knowledge base to extend the proposed type of analysis to other study contexts.

This work has been structured as follows: Section 2 outlines the review of the relevant literature. Section 3 introduces suitable path and speed models aimed at capturing human driving behavior. Section 4 elaborates on the calibration process of the speed model. Section 5 details the experimental investigations. Sections 6 and 7 are dedicated to presenting the outcomes of the learning process, summarizing the study, and offering recommendations for future research.

2. Literature Review

In recent decades, the focus of trajectory planning for autonomous vehicles has primarily been on the identification of an optimal and safe spatial maneuver. Subsequently, a rule-based velocity assignment is used to create a target trajectory that is evaluated for collision avoidance [15,16].

Some studies have looked at the effects of road geometry on vehicle movement in different road scenarios. For example, winding roads require a reduction in vehicle speed to reduce the discomfort resulting from increasing lateral accelerations. In a study, a geometry-based methodology for speed planning was proposed. This involved creating a reference path for the autonomous vehicle by merging a smooth, peak-reduced curve with a parameterized speed model derived from human driving data [17].

A combined approach that integrates behavior planning and trajectory planning was also presented. In this method, trajectories are grouped based on topological properties in the spatio-temporal domain to generate different high-level maneuver patterns [18].

For road intersections, a temporal speed planning approach was formulated by capturing behavioral patterns of human drivers in a simulated experiment. Speed profiles from intersection scenarios were extracted to generate temporal behavioral plans by applying the k-means clustering technique [19].

Further studies have attempted to understand human driving behavior and strategies to support decision making in complex traffic scenarios. Considering that experienced human drivers exhibit adaptive longitudinal speed behavior and develop strategies to effectively navigate complicated traffic scenarios, incorporating human-inspired longitudinal speed control is a promising avenue for autonomous vehicle applications. This approach offers two advantages: first, it increases the naturalness of autonomous driving [20] and facilitates seamless integration into environments with other semi-autonomous and human-driven vehicles; second, it improves the overall driving experience, especially in scenarios with frequent stop-start movements or roads with pronounced curves [21]. In one particular study, a risk-aware decision-making approach was used to select human-like longitudinal behavior profiles for navigation in a roundabout scenario. First, speed profiles were created based on patterns derived from human driving behavior and then adapted to the dynamic characteristics of the scenario. This work brings two innovations: first, the generation of naturalistic profiles for human-like navigation, and second, a risk-aware, multi-criteria decision-making approach that considers driving comfort and performance in addition to safety. A comparative analysis with human driving data from experimental studies showed encouraging advantages [22].

Developing an advanced driver assistance system also means learning from human behavior to increase driving safety [23–25]. Entering roundabouts smoothly is a challenge even for human drivers [26–29]. Several studies have proposed different approaches for defining new decision models based on imitation learning to provide recommendations for entering a roundabout.

A work has highlighted the capabilities of an Adaptive Tactical Behavior Planner (ATBP) for autonomous vehicles, demonstrating its ability to emulate human-like movement behavior while navigating roundabouts. This is achieved through a sophisticated combination of naturalistic behavior planning and tactical decision algorithms [30].

In another study, a comprehensive multi-camera approach to image processing is presented for roundabouts, employing different grid sizes to enhance accuracy and safe-guard autonomous vehicles during roundabout entry. The utilization of multiple cameras allows the system to replicate the visual perception of an actual driver when approaching a roundabout, thereby facilitating human-like decision-making processes [31].

Another noteworthy contribution involves the introduction of an innovative strategy aimed at generating diverse speed profiles for a set of path candidates. The objective is to facilitate a merging maneuver on roundabouts based on the current traffic conditions. The autonomous driving system, as outlined in this paper, underwent rigorous evaluation in real-world conditions, demonstrating its adeptness at navigating roundabouts with narrow gaps while prioritizing both comfort and safety [32].

Drawing insights from driving data, a study proposes the application of numerical optimization techniques to minimize travel time and enhance comfort through meticulous motion planning and speed profiling. The investigation also delves into the analysis of driving risks in roundabouts, seeking to influence the behavior of autonomous vehicles for improved driving comfort and overall road safety [33]. Furthermore, a machine learning model is trained to discern safe vehicle movements and potential exits, complemented by the development of an optimal control method aimed at minimizing travel time and enhancing energy efficiency. This approach takes into careful consideration the constraints associated with collision avoidance in roundabouts [34].

Addressing the coordination of autonomous vehicles in roundabouts, researchers have developed sophisticated control strategies integrating artificial intelligence (AI) approaches and models to ensure safe traffic flow. Various algorithms, including support vector machine, linear regression, and deep learning, are rigorously compared for their effectiveness in predicting vehicle speed and steering angles in roundabouts with different geometries. Simultaneously, action rules for autonomous vehicles to execute maneuvers in roundabouts are established [35]. Another piece of research explores the use of algorithms that predict vehicle movements, combining dynamic Bayesian networks and sequential neural network models [36]. Moreover, the adversarial multi-agent reinforcement learning method is applied to coordinate the passage of autonomous vehicles through roundabouts, considering behaviors analogous to those of human drivers. This approach proves instrumental in improving travel time and average vehicle speed [37]. Additionally, a fuzzy behavior-based roundabout coordination algorithm is developed to calculate speed profiles for diverse vehicles, aiming to achieve more comfortable driving profiles and reduce congestion [38].

Numerous research works leverage Model Predictive Control (MPC) strategies grounded in analytical calculations of driving time and the design of speed profiles, showcasing their efficacy in ensuring the secure operation of autonomous vehicles in roundabouts. The control designs incorporate various constraints such as speed limits, acceleration limits, and maximum cornering speeds to uphold safety standards [39]. Another study proposes a method addressing the roundabout merging problem, incorporating a target trajectory generated by Bezier curves in conjunction with the MPC method [40]. Furthermore, a study introduces a controller for trajectory tracking within roundabouts. Given the choice of exits, the MPC tracking controller is employed to assess the impact of weight parameters and target speed on the performance of the tracking controller [41].

In another study, a controller for roundabout trajectory tracking was presented. Given the choice of exits, the MPC tracking controller is used to test the effects of weight parameters and target speed on tracking controller performance [41].

3. Model Design

In this section, models are proposed that adapt to human maneuver data and efficiently generate both the geometric curve for crossing a single-lane roundabouts and the speed plan. In order to efficiently achieve these two objectives, the path and speed models are developed and explained independently.

3.1. Path Model

The autonomous vehicle must always be able to detect the presence of the roundabout on the road using sensors such as cameras, LiDAR, or radar. This allows the system to detect the position and shape of the roundabout to adapt to the predefined path. The following steps must be followed to model the path:

(1) A priori definition of the path: It is necessary to define in advance the path that the autonomous vehicle must follow to cross the roundabout. The path can be mapped based on appropriate assumptions, e.g., assuming that the vehicle must follow a curved path within the lane, maintaining a constant distance from the inner edges

of the circulatory roadway. To achieve this goal, this study uses the fastest path for crossing a single-lane roundabout, as described in accordance with guidance in NCHRP Report 672 [42].

- (2) Waypoints generation: The predefined path can be represented by a series of control points (waypoints) that indicate the ideal position of the vehicle along the path. These waypoints can be generated manually or by algorithms that take into account the geometry of the roundabout and the formulated assumptions. The waypoints should be arranged to ensure smooth and safe navigation through the roundabout.
- (3) Least squares optimization: Using the Levenberg–Marquardt algorithm, it is possible to optimize the through path based on the least squares formula. In this case, the objective is to minimize the difference between the desired path (represented by the waypoints) and the actual path of the autonomous vehicle. The Levenberg–Marquardt algorithm iteratively updates the model parameters to approach the optimal solution.

Operationally, the three steps described above can be translated into the following procedural process:

Path definition: The path for crossing the roundabout is determined following the NCHRP model of path [42]. This model is based on the concept of the "fastest through route", which is determined by the geometry of the roundabout and sets the negotiation speed for each movement—entry, circulation around the central island and exit. This path represents the smoothest and flattest trajectory possible for a single vehicle, assuming there is no other traffic and the lane markings are disregarded.

Figure 2 illustrates the construction of the fastest vehicle path in a single lane roundabout.



Figure 2. Construction of the fastest through path according to the NCHRP model. In this example: a = 1.0 m, b = 1.5 m, c = 1.5 m.

The fastest path for through traffic consists of a sequence of counter turns, namely a right curve followed by a left curve and another right curve. When describing this path, it is important to include tangents between the successive curves to account for the time it takes a driver to turn the steering wheel. In particular, three critical radii completely define the fastest passage path:

- Entry path radius (R₁): this is the minimum radius on the fastest through path before reaching the entry line;
- Radius of the circulation path (R₂): this is the minimum radius on the fastest through path as the vehicle circulates around the center island;
- Exit path radius (R₃): this is the minimum radius on the fastest transit path as the vehicle leaves the island.

In the context of a vehicle, it is assumed to be 2 m wide and maintain a minimum distance of 0.5 m from the centerline of the roadway or a concrete curb, following a painted edge line. Consequently, the centerline of the vehicle is marked with certain distances to various geometric features as follows:

• 1.0 m from a painted edge line;

- 1.5 m from a concrete curb;
- 1.5 m from a roadway centerline.
- Parametric representation of the path: To describe the path through the roundabout, a parametric representation is used (Figure 3). In polar coordinates, where r is the radial distance from the center of the roundabout and θ is the angle relative to the horizontal axis, the path is expressed as

$$r(\theta) = r_0 + a \cdot \theta^2$$

where r_0 represents the initial distance from the roundabout, a governs the curvature of the path, and θ varies from the entry angle (θ_{entry}) to the exit angle (θ_{exit}).

Least squares formulation: The objective is to minimize the error between the calculated path (p_{calculated}) and the desired path (p_{desired}). The objective function is the sum of squared differences:

$$Objective = \sum_{i=1}^{N} (p_{calculated}(\theta_i) - p_{desired}(\theta_i))^2$$

where N is the number of sampled points along the through path.



Figure 3. Parametric representation of the fastest through path.

Levenberg–Marquardt Algorithm: The Levenberg–Marquardt algorithm is employed to minimize the objective. During iteration k, the model parameters (r₀ and a) are updated using the Jacobian matrix of partial gradients. The updates are given by

$$\Delta r_{0}^{(k)} = \left(J_{calculated}^{(k)T} \cdot J_{calculated}^{(k)} + \lambda^{(k)} \cdot diag \left(J_{calculated}^{(k)T} \cdot J_{calculated}^{(k)}\right)\right)^{-1} \cdot J_{calculated}^{(k)T} \cdot \Delta r_{calculated}^{(k)} \cdot \Delta r_{ca$$

$$\Delta a^{(k)} = \left(J^{(k)T}_{calculated} \cdot J^{(k)}_{calculated} + \lambda^{(k)} \cdot diag \left(J^{(k)T}_{calculated} \cdot J^{(k)}_{calculated}\right)\right)^{-1} \cdot J^{(k)T}_{calculated} \cdot \Delta a^{(k)}$$

where $J^{(k)}_{calculated}$ is the Jacobian matrix in iteration k, $\lambda^{(k)}$ is the regularization parameter, and Δr and Δa are the parameter changes in iteration k.

- Iteration and convergence: The Levenberg–Marquardt algorithm continues iteratively until acceptable convergence is achieved or a maximum number of iterations is reached. In each iteration, parameters are updated according to the algorithm's formulas, and the objective is gradually reduced.
- Path Model output: After completing the optimization with the Levenberg–Marquardt algorithm, the optimal values of parameters r₀ and a which define the fastest through path of the roundabout are obtained. These parameters constitute the optimized path for the autonomous vehicle. The final output of the path model consists of these

optimal values, enabling the vehicle to safely and efficiently cross the roundabout while adhering to the NCHRP model of the path [42].

3.2. Speed Model

The uniform reference path generated for crossing a single-lane roundabout represents the main input for generating the specific speed model for that path.

A dual-phase speed model has been formulated with the aim of accurately replicating human driving patterns. In the initial phase, data concerning the geometric attributes of the reference path are harnessed to craft a foundational speed profile specific to the through maneuver, particularly tailored for the scenario of a single-lane roundabout.

In particular, the most appropriate speed profile in response to the geometry of the path must take into account the sequence of three critical radii that characterizes the path itself (Figure 2). In these cases, in order to model the driving behavior, it is necessary to take into account that the decelerations at entry, due to the effect of the critical radius R_1 , occur from a distance from the entry that varies according to the caution adopted by the human drivers, and that the accelerations required to approach the circulation radius (R_2) and the exit radius (R_3) are also very different according to the behavior of the drivers when approaching these maneuvers.

To model this guidance pattern, the various path features are first marked to position the speed profile by referring to the NCHRP template described in Section 3.1. In particular, the proposed M^{Roundabout} speed model is based on the speed profile shown in the lower part of Figure 4, whose construction requires the following steps:

- (1) The entire path is divided into three main "Turning Regions": TR₁ starts at the point before entering the roundabout, from which the vehicle decelerates, and ends in the middle of the section passed under acceleration, between the first and second circular arcs of the path; TR₂ starts at the end of the previous region and ends in the middle of the section that is passed under acceleration, between the second and third circular arcs of the path; TR₃ starts at the end of the previous region and ends at the point where the user varies their speed (accelerates) after passing the last section with constant curvature of the crossing path.
- (2) For each of the "Turning Regions", the longitudinal distance L_{si} is defined (i = 1, 2, 3) between the starting point of the region and the point where the speed value s_i is reached (with i = 1, 2, 3), which on average remains constant along the maneuver radius within the region itself.
- (3) For each of the "Turning Regions", the longitudinal distanc Δ_{Si} (i = 1, 2, 3) is defined between the central point of the region and the point where the travel starts at constant speed s_i (i = 1, 2, 3). Thus, $\Delta_{Si} = TR_i/2 - L_{Si}$.
- (4) In addition:
 - s₀ = characteristic speed of the road before and after the roundabout. It could also be indicated by vertical signs, and for a particular design there may be other relationships. For example, a curve before the entrance (with radius R₀) can determine the speed that can be reached at the entrance. An entry coming from a parking lot may have a much lower speed than an entry coming from a high-speed rural road, even with the same entry geometry [42]. Therefore, the speed s₀ may be a constraint that the autonomous vehicle must learn.
 - d₀₁ = deceleration from speed s₀ to speed s₁ typical of the circumference of radius R₁ within region TR₁.
 - a_{12} = acceleration from speed s_1 to speed s_2 typical of the circumference of radius R_2 within region TR₂.
 - a₂₃ = acceleration from speed s₂ to speed s₃ typical of the circumference of radius R₃ within region TR₃.

Finally, the formulation of the speed profile $\{s_i\}$ proposed to describe the crossing trajectory of a single-lane roundabout in parametric form:

$$\{s_i\} = M^{roundabout}(\{r(\theta)\}, \mathbf{Q})$$

where

- $\{r(\theta)\}$ describes the entire through path;
- $\mathbf{Q} = [s_0, s_1, s_2, s_3, d_{01}, a_{12}, a_{23}, \Delta_{S1}, \Delta_{S2}, \Delta_{S3}]^T$ defines the shape of the speed profile.



Figure 4. Speed model (M^{Roundabout}) to describe the speed profile during the crossing maneuver of a single-lane roundabout.

It is important to observe that the suggested speed model, denoted as M^{Roundabout}, has the potential to generate speed profiles featuring non-smooth transition points (indicated by a high value of longitudinal jerk) that connect linear segments. In order to enhance the smoothness of the speed profile, the subsequent phase involves an iterative process where the numerically estimated jerk is constrained until the maximum jerk value is reduced below a predefined threshold (jerk_{lon}).

$$|\ddot{s}| \leq \text{jerk}_{\text{lon}}$$

4. Model Calibration

The presented model necessitates the calibration of numerous parameters using data obtained from human driving. Specifically, the path model is tasked with optimizing two criteria associated with the geometric attributes of the path through a roundabout: smoothness and the seamless connection between the three successive turning regions within the fastest crossing path. It is assumed that a typical human driving pattern inherently seeks to optimize these two aspects. Consequently, the adjustment of the path model (followed by subsequent learning) becomes unnecessary in such circumstances. As a result, this section is primarily dedicated to ascertaining the parameters for the M^{Roundabout} speed model.

The parameters for the M^{Roundabout} model, denoted as **Q**, can be determined through an optimization process aimed at minimizing the least square error between the M^{Roundabout} model and the human driving data:

$$\mathbf{Q} = \underset{\mathbf{Q}}{\operatorname{argmin}} \left\| \left\{ s_{i}^{\text{human}} \right\} - M^{\text{Roundabout}}(\{r(\theta)\}, \mathbf{Q}) \right\|$$

In this equation, $\{s_i^{human}\}$ represents the dataset comprising human driving speeds along the fastest through path. The parameter s_i (where i = 1, 2, 3) in **Q** can be readily derived by examining the human driving data for fitting purposes. The previous equation is then employed to determine the remaining six parameters in **Q**.

The "argmin" function used in the equation signifies the argument that minimizes the subsequent expression. In this context, it identifies the set of parameters \mathbf{Q} (denoted as \mathbf{Q}^{\uparrow}) that results in the smallest least square error when comparing the M^{Roundabout} model's predictions with the observed human driving data. This optimization process effectively tunes the parameters of the M^{Roundabout} model to align it as closely as possible with real-world human driving behavior along the specified path.

In Figure 5, an illustrative through maneuver in a roundabout, utilizing the aforementioned optimization routine, is presented. The human driving data concerning the roundabout crossing maneuver illustrated in the figure were obtained from the dataset collected as a result of the experimental study described in Section 5 of this manuscript. It is evident that the profiles generated by the model align quite closely with the actual speed data collected. Notably, the incorporation of jerk smoothing further enhances the precision and faithfulness of this alignment, as evidenced by the distinctive blue curve. This underscores the effective adaptation of the proposed speed model (M^{Roundabout}) to human driving behavior, even though it cannot fully replicate it.



Figure 5. Example of fitting a speed profile by the proposed model (M^{Roundabout}).

5. Experimental Investigation

To substantiate the claims made in this paper, an experimental investigation was conducted. The purpose of this investigation is twofold: (1) to explain the results of parameter learning for the proposed speed model (M^{Roundabout}); (2) to evaluate the M^{Roundabout} by comparing the results of the model with actual human driving data.

5.1. Selection of Roundabouts

The roundabouts of the experimental investigation are located in Italy in the district of San Giovanni Galermo (northwest of Catania) and in the municipality of Mascalucia (which is part of the metropolitan city of Catania and about 12 km from it). In the area of S. Giovanni Galermo, 3 of the 5 roundabouts are located (respectively named "Roundabout n.1", "Roundabout n.2" and "Roundabout n.3"). These roundabouts are arranged along the same route in a rather homogeneous territorial context. The other two roundabouts ("Roundabout n.4" and "Roundabout n.5"), located in the municipality of Mascalucia, are also arranged one after the other and are located along the Provincial Road 10 (SP10), the so-called "Via Alcide de Gasperi", which is used by many inhabitants of the neighboring municipalities.

The geometrical characteristics of the roundabouts are shown in Table 1. In detail, the following parameters are given: the number of legs, diameter, and width of the circulatory roadway. In addition, the widths of the entry leg and exit leg in Table 1 refer to the legs of interest for the crossing paths analyzed in this study.

Roundabout	Through Path	Number of Legs	Diameter (m)	Circulatory Roadway Width (m)	Entry Width (m)	Exit Width (m)
1	Leg A–Leg B	3	40	6.70	3.70	4.10
2	Leg C–Leg A	3	30	5.50	3.40	3.70
3	Leg B–Leg D	- 4	40	7.50	3.70	3.90
	Leg D–Leg B				3.60	4.50
4	Leg B–Leg D	- 4	35/33	7.00	4.30	4.70
	Leg D–Leg B				4.30	4.70
5	Leg B–Leg D	- 4	35	8.00	4.20	5.10
	Leg D–Leg B				4.40	6.60

Table 1. Geometrical characteristics of the roundabouts subject to the experimental investigation.

Figure 6 shows the aerial photographs of the five roundabouts subject to the experimental investigation.



Figure 6. Aerial photographs of the 5 roundabouts subject to the experimental investigation.

5.2. Data Collection

The experimental study on crossing maneuvers in roundabouts was conducted with a sample of 15 drivers between the ages of 23 and 62 (7 men and 8 women).

The drivers were recruited by the University of Catania via an announcement posted on the Department of Civil Engineering and Architecture's website. The advertisement contained details about the study and a questionnaire for participant recruitment. To be eligible, drivers had to be between twenty-one and sixty-five years old and have held a valid driver's license for at least three years. Before participating in the experiment, participants gave their informed consent. They were assured that all data collected would be treated confidentially and used exclusively for research purposes.

Participants were explicitly informed that their driving skills would not be assessed and that the sole focus of the study was to collect data on distances traveled, which included trajectories and speeds. The study complied with ethical guidelines and followed the principles of the Declaration of Helsinki. The protocol was approved by the DISS— Center for Road Safety of the University of Parma, as evidenced by the decision of the Steering Committee (Protocol 211117/2021 of 24 February 2021).

The 15 test drivers performed the planned routes in summer and at low-traffic times, i.e., in the time windows between 10:00 and 11:00 a.m. and between 3:00 and 4:00 p.m., so that the conditions for performing the maneuvers were always little affected by interactions with other vehicles. Survey data (position, direction and curvature of the path, longitudinal speed and accelerations) were collected using a tracking system based on the differential GPS placed in the center of the rear axle of the vehicle and used from time to time by the driver involved in the test. It should be noted that each driver performed the test using their own vehicle so that their behavior was as natural as possible. The surveys, conducted in the time windows indicated above, spanned several days until the complete database of all 15 drivers was available. Each driver performed all the crossing maneuvers indicated in Table 2. In cases where the maneuvers were affected by other vehicles or external events, drivers were asked to repeat the maneuver. A total of 9 trajectories per driver were validly recorded, for a total of 135. In detail

D 11 (Thursday Dath	Number of Trajectories Acquired		
Koundabout	I nrough Path	For Learning	For Evaluation	
1	Leg A–Leg B	15	1	
2	Leg C–Leg A	15	2	
2	Leg B–Leg D	15	2	
3	Leg D–Leg B	15	1	
4	Leg B–Leg D	15	3	
4	Leg D–Leg B	15	1	
	Leg B–Leg D	15	2	
5	Leg D–Leg B	15	3	
	Total	120	15	

Table 2. Main data from the experimental investigation.

Each test driver performed all 8 planned crossing maneuvers. The resulting 120 trajectories were considered for the parameter learning phase of the model (see Section 6.1).

Each test driver was asked to repeat one maneuver from those already performed. In this way, the parameters for an additional 15 trajectories were acquired to be used for the evaluation phase of the model (see Section 6.2). These additional maneuvers are adequately specified in Table 2.

Figure 7 shows the speed profiles described by each of the 15 test drivers involved in the experiment for each of the through paths considered. The diagrams for each through path are divided into two parts. The upper part contains the experimentally determined speed profiles, each represented with a different color corresponding to one of the 15 test

drivers. The lower part represents the average profiles and the profiles determined by adding and subtracting the standard deviations (σ): in particular, the average profiles is depicted in black, while the profiles determined by adding and subtracting the standard deviations (σ) are represented in red and green for the average profiles with + σ and average profiles – σ , respectively. All curves representing the speed variation show the characteristic trend of speeds approaching roundabouts, which is characterized by a minimum point at the entry section.

However, it is evident that the speed reduction varies depending on the geometrical characteristics of the roundabouts and their influence on the deflection of the trajectory: For example, in roundabouts n.3 and n.4, where trajectories experience noticeable deflection, minimum speeds are about 20 km/h lower than in the first sections (i.e., the sections furthest from the roundabout entrance). In contrast, the curves in the diagram in Figure 7 for roundabout n.5 are flatter (less pronounced deflections), and the minimum speed values are about 15–17 km/h lower than in the sections furthest from the entrance.



Figure 7. Speed profiles obtained by experimental investigation and representative curves of average speed values and corresponding standard deviations.

6. Results and Discussion

6.1. Learning Results and Discussion

The proposed speed model incorporates parameters such as s_1 , s_2 , s_3 , d_{01} , a_{12} , a_{23} , Δ_{S1} , Δ_{S2} , and Δ_{S3} , all contingent upon the reference path that underpins the model, as elucidated in the paper's initial section. The objective of this study is to discern and unveil the fundamental attributes intrinsic to the human driving model. This facilitates the adjustment of the theoretical speed model, referred to as $M^{Roundabout}$, to accommodate the fluctuations in human behavior observed during navigation through the three turning regions delineated within the path model.

Consequently, it becomes imperative to identify the nine parameters within the M^{Roundabout} speed model that can be tailored to align with the driving conditions that result from the approaches taken by human drivers when maneuvering through various crossing paths. To achieve this, a statistical regression methodology was employed, wherein the nine parameters for model adaptation, denoted by an asterisk, were statistically derived using data from the experimental study outlined in prior sections.

Symbolically, this can be represented as

$$\mathbf{Q}^{*} = \begin{bmatrix} s_{1}^{*}, \, s_{2}^{*}, \, s_{3}^{*}, \, d_{01}^{*}, \, a_{12}^{*}, \, a_{23}^{*}, \, \Delta_{s1}^{*}, \, \Delta_{s2}^{*}, \, \Delta_{s3}^{*} \end{bmatrix}^{T}$$

From an operational point of view, the learning took place as follows:

- For each of the 120 crossing paths experimentally performed by the test drivers, the three characteristic radii of the path (R₁, R₂ e R₃) that characterize the curvature of the three turning regions (TR₁, TR₂ e TR₃) were evaluated, again as a function of the geometric characteristics of the roundabouts;
- (2) With respect to each of the trajectories obtained through the experimental study, the average speed values (s₁, s₂, s₃) corresponding to the radii that make up the turning regions were also obtained;
- (3) For each trajectory, the average values of the deceleration (d₀₁) characteristic of the first turning region and of the accelerations (a₁₂ and a₁₃) for the two following turning regions were calculated;
- (4) The distances Δ_{S1} , Δ_{S2} and Δ_{S3} were also determined starting from the data of the trajectories obtained experimentally, after determining for each trajectory the extent of each turning region and the transition points between the sections covered with deceleration/acceleration and those covered with constant speed;
- (5) The nine parameters of the speed model evaluated experimentally for each of the three curve regions were plotted in different scatter plots as a function of the values of the characteristic radii;
- (6) Linear regressions were used to learn the correlations between the parameters of interest and, consequently, to explain the formulations describing the variability of the statistically learned parameters.

Figure 8 shows, for each of the three turning regions, the scatter plots on the nine parameters assessed after the experimental tests and the corresponding regression lines with the associated R^2 determination coefficients.

Below are the various representative formulations of the linear regressions obtained for the statistically learned parameters:

$$\begin{split} s_1^*(R_1) &= 2.619 \cdot R_1 - 40.65 \\ s_2^*(R_2) &= 1.202 \cdot R_2 - 7.488 \\ s_3^*(R_3) &= 0.824 \cdot R_3 + 3.586 \\ d_{01}^*(R_1) &= -0.0928 \cdot R_1 + 3.227 \end{split}$$

$$\begin{split} a_{12}^*(R_2) &= 0.0386 \cdot R_2 - 0.571 \\ a_{23}^*(R_3) &= 0.0299 \cdot R_3 - 0.596 \\ \Delta_{s1}^*(R_1) &= -0.1193 \cdot R_1 + 10.284 \\ \Delta_{s2}^*(R_2) &= -0.1059 \cdot R_2 + 15.882 \\ \Delta_{s3}^*(R_3) &= -0.0968 \cdot R_3 + 20.371 \end{split}$$

The analyzed parameters showed correlations ranging from very strong to very weak.



Figure 8. Scatter plot of the speed model M^{Roundabout} parameters. From subfigure (**a**–**i**), blue symbols are the scattered parameter values after model fitting. Black lines are the results after linear regression.

The closer their coefficients of determination (\mathbb{R}^2) are to 1, the stronger the linearity can be concluded. Upon examination of scatter plots for each parameter (Figure 8), certain discernible human driving patterns emerged, which can be statistically elucidated:

Strong linearity in s_1 , s_2 , and s_3 : The analysis showed a strong linear correlation between the speeds s_1 , s_2 , and s_3 and the radii of curvature characterizing the three turning regions. In other words, as logically expected, as the radius of curvature decreased, the speed of execution of the maneuver to also decrease [28,29]. Specifically, it was observed that the highest degree of linearity was evident at speed s_1 when the radius values R_1 fell within the range of 25 m to 28 m (with a variation of $\Delta R_1 = 3$ m). This corresponds to a speed change ranging from 23 km/h to 40 km/h (with a variation of $\Delta_{S1} = 17$ km/h). With respect to speed s_2 , the strongest linearity occured for values of R_2 between 34 m and 44 m ($\Delta R_2 = 10$ m), where the speed varied between 27 km/h and 46 km/h ($\Delta_{S2} = 19$ km/h). When examining speed s_3 , it becomes evident that no pronounced linearity exists within the spectrum of radius values R_3 (with a total variation in ΔR_3 equivalent to 21 m). As a result, the speed variance in the third turning region spanned the complete range between 25 km/h and 51 km/h (with a total variation of Δ_{S3} equal to 26 km/h). Thus, it is noteworthy that the Δ_S and ΔR intervals, which are linked to the growing significance of the correlations between radii and speed, exhibited substantial expansion as the final turning region was approached. This means that road users felt more influenced on the first approach and therefore adopted a more cautious behavior. Conversely, drivers approaching the last part of the trajectory (the exit part) felt less constrained and therefore tended to adopt variable speeds within a very wide range.

- ➤ Low linearity in d_{01} , a_{12} , and a_{23} : For the M^{Roundabout} model, there was a low level of linearity observed in the parameters d_{01} , a_{12} , and a_{23} . Moreover, the values of the coefficients of determination R_2 for all three correlations were almost identical in the scattering diagrams (d, e, and f) shown in Figure 8. A closer analysis of the three diagrams also shows that in none of the diagrams is there such a density of data that highlights a stronger linearity in one part of the diagram compared to other parts. This result suggests that accelerations/decelerations can vary significantly and may be influenced by factors that are hard to quantify, such as the driver's mood. This finding aligns with the argument presented in [43].
- → Weak linear correlation in Δ_{s1} , Δ_{s2} , and Δ_{s3} : The analysis showed a weak linear correlation between Δ_{s1} , Δ_{s2} , Δ_{s3} , and their respective reference radii. This indicates that as the radius of curvature of the crossing trajectory decreased drivers tended to behave more cautiously by slowing down earlier. Essentially, when faced with smaller radii, human drivers tend to exhibit more conservative driving behavior. This also means that drivers are less predictable when navigating through turning regions with small radii. These considerations were stronger in the case of turning maneuvers that took place in the first region. Indeed, Figure 7g shows a lower dispersion of the data and a higher R₂ coefficient compared to the behavioral situations described in Figure 7h,i. This confirms that all the correlations found describe more accurately the human behavior in the first approach phase to roundabouts, i.e., in the entry phase.

6.2. Speed Model Evaluation and Discussion

The evaluation of the learned model, known as $M^{Roundabout}$, involved the generation of speed plans and their comparison with the behavior observed in 15 human driving processes learned during the experimental study phase (as described in Section 5.2, the test drivers were used for further trajectory measurements, in addition to the 120 whose results were used for learning the parameters). Parameter **Q**, representing this model, was specifically tailored to capture the characteristics of navigation through roundabouts. The objective of this evaluation is to gauge the model's ability to replicate human driving behavior.

In Figure 9, the outcomes of the comparison between the speed plans generated by the M^{Roundabout} model and the driving data of test drivers No. 3, No. 10, No. 13, and No. 14 are presented. The remarkable alignment between the speed profiles generated by the model and the actual profiles is readily apparent. Notably, for drivers 3 and 10, the congruence between the speed profiles derived from the model and the real counterparts is nearly flawless. Conversely, for drivers No. 13 and No. 14, there are more substantial deviations between the two curves, even if the degree of overlap is still very considerable.

This study highlights an important aspect: despite the difficulties of achieving a perfect one-to-one match with individual human data points, traffic-free reference plans are successfully generated.

These reference plans are designed to capture the essential characteristics of human behavior during roundabout crossing maneuvers, albeit in a statistical sense. In other words, while the generated plans may not precisely replicate any specific human instance, they encompass the core features commonly observed in such situations.



Figure 9. Learned speed model evaluation results. In the subfigures from (**a**–**d**), some emblematic examples are shown for the comparison between four generated speed plans and as many human driving processes.

Furthermore, the use of the learned model M^{Roundabout} offers significant advantages in the context of autonomous vehicle planning. By incorporating statistical representations of human-like behavior into the planning algorithms, the resulting plans are characterized by fluidity and predictability. This means that autonomous vehicles can navigate roundabout crossings in a manner that not only aligns with common human patterns but also ensures a smooth and predictable driving experience.

In summary, while the generated plans may not have achieved perfect matches with random human data, the study demonstrates the ability to capture essential human driving characteristics statistically. This contributes to the development of autonomous vehicle planning systems that prioritize smooth and predictable driving maneuvers during roundabout crossings, ultimately enhancing both safety and the overall driving experience for passengers and other road users.

19 of 21

7. Conclusions

This research was an exploration into enhancing the user experience within the domain of autonomous driving. The primary objective is the development of a planning model that operates without the influence of traffic, aiming to mimic human driving behavior, especially when navigating roundabouts.

The central focus of this research primarily revolved around traffic-free planning, which entails a deep understanding of how individuals navigate roundabouts in isolation from other vehicles. To accomplish this, a trajectory model was introduced as a solution to accurately represent the path that a vehicle takes within a roundabout, taking into account variations in speed. The process involved the identification of the various parameters governing this trajectory model. This undertaking was structured as a least square optimization problem, aiming to derive parameter values that align optimally with empirical data observations. Subsequently, statistical regression methodologies were employed to generalize these parameters. This stage served to define the speed model (referred to as M^{Roundabout}) and assesses the sensitivity of each parameter to the inherent unpredictability that characterizes human driving behavior. The model's effectiveness was then assessed by subjecting it to a new dataset, with the goal of gauging its consistency in generating plans that closely emulate human driving behavior within roundabouts.

However, it is of paramount importance to acknowledge a substantial limitation inherent in this study. The model's reference speed is solely determined by the geometric attributes of the roundabout and does not account for external factors, such as weather conditions or the state of the road surface, which can significantly impact human driving behavior. As a result, the model's adaptability to real-world driving scenarios may have been subject to constraints.

In terms of future research directions, the authors intend to outline several procedural and operational initiatives. Foremost among these is a clear intention to transition from a theoretical research concept to practical implementation. This entails the integration of the model into the planning system of an operational autonomous vehicle to contribute to tangible advancements in autonomous driving technology. Additionally, there is a recognized need to explore the development of planning models that take into account the presence of traffic and are designed to seamlessly adapt to dynamic environments where other vehicles are concurrently in operation. This adaptability is particularly crucial in scenarios where effective vehicular interactions and decision making are fundamental aspects of a comprehensive and adaptable autonomous driving system.

These proposed intentions underscore the significance of bridging the divide between research endeavors and practical implementation. They emphasize the urgency of addressing the intricate complexities that characterize real-world driving scenarios, characterized by varying traffic dynamics and dynamic environments. The pursuit of these intentions aims to advance the ultimate objective of creating a resilient and versatile autonomous driving system that enhances safety and enriches the overall user experience.

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