

Article

Optimal Allocation of Distribution Static Synchronous Compensators in Distribution Networks Considering Various Load Models Using the Black Widow Optimization Algorithm

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Abstract: Incorporating Distribution Static Synchronous Compensator (DSTATCOM) units into the radial distribution network (RDN) represents a practical approach to providing reactive compensation, minimizing power loss, and enhancing voltage profile and stability. This research introduces a unique optimization technique called the Black Widow Optimization (BWO) algorithm for strategically placing DSTATCOM units within the RDN. The primary objective is to minimize power loss while simultaneously evaluating various techno-economic parameters such as the voltage profile index (VPI), voltage stability index (VSI), and annual cost savings. The analysis of optimal DSTATCOM allocation, employing the proposed BWO algorithm, encompasses different load models, including constant impedance (CZ), constant current (CI), constant power (CP), and composite (ZIP) models. These analyses consider three distinct scenarios: single and multiple DSTATCOM integration. To gauge the effectiveness of the proposed BWO technique, it is applied to the IEEE 33-bus and 69-bus RDNs as test cases. Simulation results confirm the efficiency of the proposed approach across all four load models. Notably, in the case of the constant power model, the percentage reduction in power loss is substantial, with a reduction of 34.79% for the IEEE 33-bus RDN and 36.09% for the IEEE 69-bus RDN compared to their respective base cases.

Keywords: distribution static synchronous compensator; radial distribution network; load models; black widow optimization; power loss



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1. Introduction

The distribution network represents the final stage of the electrical power system, responsible for delivering electrical energy to end-users in various sectors such as residential, commercial, agricultural, and industrial. Typically, distribution networks adopt a radial or weakly meshed configuration for ease of operation, as power flows unidirectionally from the substation to all parts of the network. The radial distribution network (RDN) offers several benefits, including cost-effectiveness, straightforward protection mechanisms, ease of voltage control, and power flow management [1]. However, the RDN faces significant challenges due to non-linear loads, high resistance-to-reactance ratios, and its structural and topological characteristics. These challenges manifest as huge power losses, voltage drops leading to reduced voltage profiles, low power factors, and poor voltage stability [2]. It has been estimated that about 13% of the total power generated is wasted as losses in the power system, and the distribution system accounts for about 70% of these losses [3]. These factors render the RDN unreliable, insecure, and costly to operate.

Several researchers have explored various approaches in response to the technical and economic challenges within the RDN. Common strategies include the allocation of

distributed generation (DG) [4], the reconfiguration of distribution networks [5], and the incorporation of reactive compensating devices like series voltage regulators [6], shunt capacitors [7], and Distribution Flexible Alternating Current Transmission (DFACTS) devices such as the Distribution Static Synchronous Compensator (DSTATCOM) [8]. DG allocation is considered the most efficient among these methods, particularly in reducing power loss. Nevertheless, the substantial investment and procurement costs associated with DG make alternatives like distribution network reconfiguration, shunt capacitors, and DSTATCOM allocation attractive options, as they offer significant efficiency improvements and cost-effectiveness [9]. While shunt capacitors inject reactive power into the distribution network, they do so in fixed steps without considering the dynamic nature of load variations over time [10]. DSTATCOM, a power electronic device, exhibits versatility by adjusting reactive power injection or absorption based on the changing load demand within the distribution network.

Obtaining the optimal allocation of DSTATCOM in the RDN is a non-linear, integer-based complex, and combinatorial problem that can be solved by any appropriate optimization technique [11]. Researchers have proposed various optimization techniques for the optimal allocation of DSTATCOM. An analytical approach was presented for DSTATCOM allocation in [12]. In [13], voltage stability was utilized to obtain the DSTATCOM allocation considering various load models. Ref. [11] has proposed a firefly algorithm (FA) to allocate DSTATCOM to enhance power quality. Ref. [14] proposed a power loss index (PLI) technique to solve the problem considering power loss and improvement of voltage profile as their objective function. A social spider optimization (SSO) algorithm has been proposed to tackle this problem, considering the uncertainty of the real and reactive loads [15]. Ref. [16] has investigated the efficacy of the harmony search algorithm to allocate DSTATCOM for the reduction of power loss optimally. Immune algorithm (IA) in [17] and particle swarm optimization (PSO) in [18] techniques have also been deployed for DSTATCOM allocation.

Cost-based optimal allocation of DSTATCOM was carried out using a differential evolution algorithm (DEA) [19]. Ref. [20] has presented an imperial competitive algorithm (ICA) for the optimal allocation of DSTATCOM. In [21], a combination of particle swarm optimization (PSO) and a general algebraic modeling system (GAMS) was used to obtain an optimal allocation of DSTATCOM under load variation and growth scenarios to improve the cost of energy savings. In [8], the whale optimization algorithm (WOA) was used for sizing, while the location of the DSTATCOM was obtained using the voltage stability index. A gravitational search algorithm (GSA) has been proposed for DSTATCOM allocation for power loss reduction, voltage profile improvement, and annual energy saving [22]. Ref. [23] has proposed a voltage stability index and bat algorithm to solve the problem for different load levels. In [24], a new voltage stability index was proposed for the siting of DSTATCOM, while the bat algorithm was utilized for its sizing.

Ref. [25] has proposed a cuckoo search algorithm (CSA) for optimal allocation of DSTATCOM, intending to minimize power loss considering industrial, residential, and commercial loads. A novel modified grey wolf optimization (MGWO) has been utilized for DSTATCOM allocation to reduce power loss and cost due to energy loss [26]. An improved bald eagle search (IBES) algorithm has been used to solve the problem of loss reduction and voltage profile improvement considering different types of load modeling [27]. Ref. [10] proposed a discrete-continuous codification of the Chu–Beasley Genetic Algorithm to address the DSTATCOM allocation considering the annual operating cost of the RDN. Ref. [28] has presented an improved bacterial foraging search algorithm (IBFA) for DSTATCOM allocation to minimize power loss and improve voltage profile and stability. Ref. [29] proposed the ant lion optimization (ALO) to solve the problem of DSTATCOM allocation based on the cost of energy purchased from the substation.

Authors in ref. [30] have performed the simultaneous allocation of DSTATCOM and photovoltaic DG in RDNs at different loading conditions using several hybrid optimization methods that combine the firefly algorithm (FA) with various acceleration coefficients PSO algorithms. The work considered a multi-objective function consisting of power loss,

short circuit, voltage deviation, net saving, and environmental pollution. In [31], a Light Search Algorithm (LSA) was proposed for the simultaneous placement of DSTATCOM and Distributed Generation (DG) in the RDNs for minimizing power loss and total voltage deviation (TVD), as well as maximizing the voltage stability index (VSI). Ref. [32] solved the DSTATCOM allocation problem alongside DG penetration using a student psychology-based optimization (SPBO) algorithm. A multi-objective framework was utilized in the work with a real power loss index, bus voltage variation index, voltage stability index, and system annual cost minimization index.

Ref. [33] has proposed the African Vultures Optimization Algorithm (AVOA) for optimal allocation of DSTATCOM, DG, and Electric Vehicle Charging Stations with the objective function of reducing the real power loss index and voltage deviation index while enhancing the voltage stability index. In [34], a new analytical method was proposed to allocate D-STATCOM using a new model optimally and to maximize profit for distribution network operators (DNOs). Ref. [35] addressed the problem regarding the optimal allocation of D-STATCOMs in RDNs via a stochastic mixed-integer convex (SMIC) model in the complex domain, intending to minimize the annual installation and operating costs. Ref. [36] presented an integrated approach of loss sensitivity factor and Dwarf Mongoose Optimization to decide the optimum allocation of DSTATCOM and DG for diminishing the loss of power, voltage profile improvement, and operation cost. The studies mentioned above in [10–36] have utilized various optimization techniques to achieve optimal sizing and allocation of DSTATCOM in RDN, yielding minimal power losses, significant operating cost reduction, and maximal voltage profile improvement. When tackling other problems, an optimization method might not be able to provide a global solution even while it performs well when solving a handful of problems. Therefore, the development of new and additional optimization approaches has been encouraged.

Distribution networks are critical infrastructures that are crucial for transporting electricity from substations to end customers in an efficient manner, and their design planning must be supported by accurate decision making. Thus, advanced optimization approaches aid in finding the ideal placement of DG units, FACTs, substations, power line routing, energy resource allocation, and electrical power supply within the network [37]. To construct a balanced and cost-effective network design, they can efficiently analyze load demand, equipment specifications, safety rules, and cost concerns. These algorithms assist utility companies and grid operators in improving power distribution reliability, minimizing energy losses, and improving the overall performance of radial electricity distribution networks, resulting in a more resilient and sustainable electrical infrastructure. Generally, advanced algorithms, such as hybrid heuristics and metaheuristics, adaptive algorithms, self-adaptive algorithms, island algorithms, polyploid algorithms, hyperheuristics, and so on, make it easier to scale and adapt, improve operational control, guarantee regulatory compliance, and support network operators in efficient decision making [38–40]. The optimization results using such robust and sophisticated algorithms guarantee consistency of results due to their ability to manage the trade-offs and convergence procedures better than most existing algorithms [41,42].

This paper, therefore, seeks to solve the optimal allocation of DSTATCOM using a novel black widow algorithm (BWO) to minimize the total power loss of the RDN. At the same time, other techno-economic indices like voltage stability index (VSI), voltage profile index (VPI), and annual cost savings (ACS) are calculated to evaluate the impact of the proposed method. The BWO was presented by Hayyolalam and Pourhaji Kazem in 2020 [43] and draws its inspiration from the lifecycle and bizarre mating behavior of black widow spiders. Compared to other meta-heuristic optimization techniques, its main advantage is that it requires fewer parameters to be tuned to obtain a global solution. The BWO algorithm is flexible and adaptable to solve any engineering and related optimization problem. This work considers different load model scenarios for both single and multiple allocations of DSTATCOM in the IEEE 33-bus and 69-bus RDN. The obtained results for

the proposed method were compared to existing techniques found in the open literature and they were found to be effective.

The remaining parts of the manuscripts are arranged as follows: the mathematical models of the DSTATCOM device and load types are presented in Section 2; and the objective function, constraints, and performance assessment indices are shown as the problem formulation in Section 3. The overview of the novel black widow optimizer (BWO) and its application for the allocation of DSTATCOM in the RDN is presented in Section 4. The result obtained from the simulation is discussed in Section 5, and the conclusion is presented in Section 6.

2. DSTATCOM and Load Models

This section delves into DSTATCOM mathematical modeling and the distribution network's loads [24,29].

2.1. Mathematical Modeling of DSTATCOM

The distribution line between buses 'A' and 'B' shows the assignment of D-STATCOM at bus 'B'. r_A represents the line resistance and reactance, $P_B + jQ_B$ denote load demands, v_A and v_B are the node voltages. Let $v'_B \angle \Delta'_B$, $v_A \angle \Delta_A$, $i_A \angle \delta_A$ and $i_{DST} \angle \left(\Delta'_B + \frac{\pi}{2}\right)$ symbolize the voltage of bus 'B' after the placement of DSTATCOM, the voltage of bus 'A', current flow in line after DSTATCOM allocation, and injected current by DSTATCOM, respectively.

From Figure 1 above,

$$v'_B \angle \Delta'_B = v_A \angle \Delta_A - (r_A + jx_A)i_A \angle \delta - (r_A + jx_A)i_{DST} \angle \left(\Delta'_B + \frac{\pi}{2}\right) \quad (1)$$

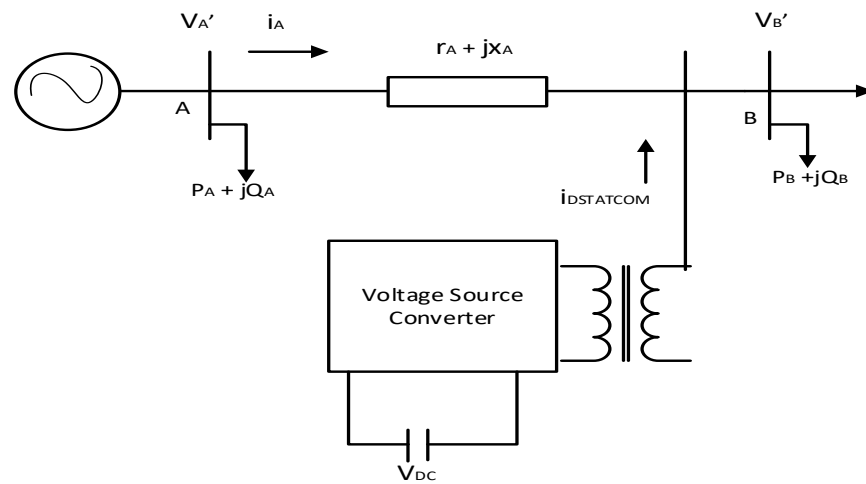


Figure 1. Distribution line with D-STATCOM installation at the receiving bus.

Separating the real and imaginary parts of Equation (1) yields

$$v'_B \cos \Delta'_B = \text{Re}(v_A \angle \Delta_A) - \text{Re}[(r_A + jx_A)i_A \angle \delta] + x_A i_{DST} \sin\left(\Delta'_B + \frac{\pi}{2}\right) - r_A i_{DST} \cos\left(\Delta'_B + \frac{\pi}{2}\right) \quad (2)$$

$$v'_B \sin \Delta'_B = \text{Im}(v_A \angle \Delta_A) - \text{Im}[(r_A + jx_A)i_A \angle \delta] - x_A i_{DST} \cos\left(\Delta'_B + \frac{\pi}{2}\right) - r_A i_{DST} \sin\left(\Delta'_B + \frac{\pi}{2}\right) \quad (3)$$

Let the following notations represent the real and imaginary parts of Equations (2) and (3):

$$K_1 = \text{Re}(v_A \angle \Delta_A) - \text{Re}[(r_A + jx_A)i_A \angle \delta]$$

$$K_2 = \text{Im}(v_A \underline{\Delta}_A) - \text{Im}[(r_A + jx_A)i_A \underline{\delta}]$$

$$\left. \begin{aligned} C_1 &= r_A \\ C_2 &= x_A \\ E &= v'_B \\ U &= i_{DST} \\ W &= \Delta'_B + \frac{\pi}{2} \end{aligned} \right\} \quad (4)$$

Equations (3) and (4) can be written as:

$$E \sin W = K_1 - C_1 U \cos W + C_2 U \sin W \quad (5)$$

$$-E \cos W = K_2 - C_1 U \sin W - C_2 U \cos W \quad (6)$$

where K_1 , K_2 , C_1 , and C_2 are taken as constants.

From Equations (5) and (6), the following is obtained as:

$$U = \frac{E \sin W - K_1}{C_2 \sin W - C_1 \cos W} \quad (7)$$

$$U = \frac{E \cos W + K_2}{C_1 \sin W + C_2 \cos W} \quad (8)$$

Equations (7) and (8) and taking $X = \sin W$ yields:

$$(Q_1^2 + Q_2^2)X^2 - (2Q_1EC_1)X + (E^2C_1^2 - Q_2^2) = 0 \quad (9)$$

where $Q_1 = K_1C_1 + K_2C_2$ and $Q_2 = K_2C_1 - K_1C_2$.

Solving Equation (9) using quadratic formula gives

$$X = \frac{2Q_1EC_1 \pm \sqrt{(2Q_1EC_2)^2 - 4(Q_1^2 + Q_2^2)(E^2C_1^2 - Q_2^2)}}{2(Q_1^2 + Q_2^2)} \quad (10)$$

But

$$\Delta'_B + \frac{\pi}{2} = W = \sin^{-1}X \quad (11)$$

The injected voltage, current, and reactive power at bus B are as follows:

$$v'_B = v'_B \boxed{\Delta'_B} \quad (12)$$

$$i_{DST} = i_{DST} \boxed{\left(\Delta'_B + \frac{\pi}{2}\right)} \quad (13)$$

$$jQ_{DST} = v'_B i'_{DST} \quad (14)$$

* signifies complex conjugate.

Equation (12) calculates the injected voltage at bus B, current via the DSTATCOM, and reactive power through the DSTATCOM Equation (14).

Equation (12) computes the voltage injected at bus B, the current passing through the DSTATCOM, and the reactive power handled by the DSTATCOM, as expressed in Equation (14). The DSTATCOM device is modeled as a negative Q-load by Equation (14) at the bus of the RDN in which it is connected for running the load flow. The BWO determines the size and bus location of the DSTATCOM.

2.2. Modeling of the Load Types

The loads in a distribution system are classified as [44]:

- (1) *Constant real and reactive power (CP) load*: The real and reactive power requirements remain consistent throughout the iterative process. Illustrations of constant power (CP) loads encompass devices like induction motors and air conditioners.
- (2) *Constant Impedance (CZ) load*: The load's impedance remains unvarying during the load flow iteration process. Such constant impedance loads include incandescent lighting, resistive water heating, cooking appliances, and similar devices.
- (3) *Constant current (CI) load*: The current magnitude remains consistent throughout the load flow procedure. Such situations include welding, smelting, electroplating, and similar applications.

In an actual radial distribution network (RDN), these three load categories are typically present, with the last category being a blend or combination known as ZIP load, representing a mixture of these loads. These load categories can be effectively represented through mathematical models called polynomial load (ZIP) models, as described in reference [25]:

$$P(V_n) = \left(k_0 + k_1 \left| \frac{V_a}{V_n} \right| + k_2 \left| \frac{V_a}{V_n} \right|^2 \right) P_n \quad (15)$$

$$Q(V_n) = \left(k_0 + k_1 \left| \frac{V_a}{V_n} \right| + k_2 \left| \frac{V_a}{V_n} \right|^2 \right) Q_n \quad (16)$$

where P_n represents the nominal real power and Q_n represents the nominal reactive power of the load, V_n stands for the nominal voltage of the load. Additionally, V_a denotes the actual voltage during load flow, while k_0 , k_1 , and k_2 are the unchanging constituents of the CP, CI, and CZ loads. The summation of these constants must equal one. The specific values for these constant components and the load model specifications are detailed in Table 1. In this research, we have opted for the values of $k_0 = 0.5$, $k_1 = 0.2$, and $k_2 = 0.3$ for the constant components in the ZIP load models as suggested by [25]. These different load models are considered for other cases in the study for optimization of the size and location of the DSTATCOM(s).

Table 1. Load models [44].

Load Type	Constant Component	Load Models
CP	$k_0 = 1, k_1 = k_2 = 0$	$P(V_n) = P_n, \quad Q(V_n) = Q_n$
CI	$k_2 = 0, k_0 = k_1 = 1$	$P(V_n) = \left \frac{V_a}{V_n} \right P_n, \quad Q(V_n) = \left \frac{V_a}{V_n} \right Q_n$
CZ	$k_1 = 1, k_0 = k_2 = 0$	$P(V_n) = \left \frac{V_a}{V_n} \right ^2 P_n, \quad Q(V_n) = \left \frac{V_a}{V_n} \right ^2 Q_n$
ZIP	k_0, k_1, k_2	$P(V_n) = \left(k_0 + k_1 \left \frac{V_a}{V_n} \right + k_2 \left \frac{V_a}{V_n} \right ^2 \right) P_n, \quad Q(V_n) = \left(k_0 + k_1 \left \frac{V_a}{V_n} \right + k_2 \left \frac{V_a}{V_n} \right ^2 \right) Q_n$

3. Problem Formulation

3.1. Objective Function

The primary objective of DSTATCOM allocation is to minimize the overall power loss within the network. Consequently, this serves as the fundamental objective function in this study. The total power in any radial distribution network (RDN) is calculated as the sum of losses across its various line sections. Top of Form

$$OF_{loss} = \sum_{i=1}^{n_b} |I_i|^2 r_i \quad (17)$$

Here, OF_{min} denotes the objective function of real power losses (RP_{loss}), n_b represents the total quantity of branches within the RDN, r_i indicates the resistance of the i th branch in

the RDN, and $|I_i|$ expresses the magnitude of current flowing through the i th branch of the RDN.

3.2. Constraints

The objective function is subject to the following constraints:

3.2.1. Power Flow Equations

The power flow equation is addressed in the optimization process by employing the direct load flow (DLF) technique [45]. These equations are presented as follows:

$$P_{gi} = P_{Di} + \sum_{j=1}^{n_b} |V_i| |V_j| [G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}] \quad (18)$$

$$Q_{gi} = Q_{Di} + \sum_{j=1}^{n_b} |V_i| |V_j| [G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}] \quad (19)$$

where V_i and V_j are the voltages of buses ' i ' and ' j ', respectively; P_{gi} and P_{Di} are the real power generated and power demanded at bus ' i '; Q_{gi} and Q_{Di} are the reactive power generated and demanded at bus ' i '; G_{ij} and B_{ij} are the conductance and susceptance of branch ' ij ' and θ_{ij} is the difference between the voltage angles of buses ' i ' and ' j '.

3.2.2. Reactive Power Generation Constraint of DSTATCOM

The magnitude of each of the installed DSTATCOM is constrained within the limits below:

$$Q_{DST(min)} \leq Q_{DST} \leq Q_{DST(max)} \quad (20)$$

where $Q_{DST(min)} = 100$ kVar and $Q_{DST(max)}$ is 75% of the total reactive power demand of the network [46].

3.2.3. Bus Voltage Limitation

The voltage must fall within the standard limits for RDN [46].

$$V_{i_{maxmin}} \quad (21)$$

where V_{min} is the minimum voltage ($V_{min} = 0.95$), V_{max} is the maximum voltage ($V_{max} = 1.05$), and V_i is the bus voltage.

3.2.4. DSTATCOM Penetration Limitation

The total reactive power provided by the installed DSTATCOM should not exceed the total reactive power required by the network [46].

$$\sum_{l=1}^m Q_{DST}(l) \leq \sum_{i=1}^N Q_d(i) \quad (22)$$

3.3. Performance Assessment Indices

To evaluate the suggested approach, specific metrics serve as benchmark parameters for comparing different scenarios and alternative methods found in the existing literature. The performance metrics encompass:

3.3.1. Power Loss and Percentage Power Loss Reduction Index

Power loss encompasses the cumulative losses across all the lines within the RDN, as specified in Equation (3). To ensure an equitable comparison with alternative techniques,

which might employ a baseline power loss distinct from the one applied in this study, we compute the percentage index for power loss reduction as detailed below:

$$\%PLRI = \frac{RP_{loss(before_DST)} - RP_{loss(after_DST)}}{RP_{loss(before_DST)}} \times 100\% \quad (23)$$

where $RP_{loss(after_DST)}$ is the power loss of the RDN after DSTATCOM placement.

3.3.2. Voltage Profile Index and Minimum Voltage

The voltage profile encompasses the voltage levels at all the buses within the RDN, which are determined through load flow analysis. Conversely, the term minimum voltage (V_{min}) denotes explicitly the voltage at the bus with the lowest magnitude. An index has been introduced to gauge the conformity between the observed and ideal voltage [47]. This index is denoted as VPI and is defined as follows:

$$VPI = \log_{10} \left(k \times \left| \frac{1}{V_{\mu} - 1} \right| \right) \quad (24)$$

V_{μ} and k can be determined as follows:

$$V_{\mu} = \frac{1}{N} \sum_{i=1}^N V_i \quad (25)$$

$$k = 1 - V_{\sigma} \quad (26)$$

$$V_{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^N (V_i - V_{\mu})^2} \quad (27)$$

where N is no of buses, V_i is the magnitude of the voltage at bus i , V_{μ} is the mean bus voltage and V_{σ} is the standard deviation of the bus voltage. For two scenarios, A and B, if $VPI_A > VPI_B$, scenario A provides a better voltage profile.

3.3.3. Voltage Stability Index (VSI) and Minimum VSI

When RDNs become overloaded and experience stress, they become susceptible to voltage instability and potential collapse. To pinpoint buses that are nearing the brink of collapse and may require compensation, the VSI is employed [28]. The formula for VSI is as follows:

$$VSI_{ij} = |V_i|^4 - 4[P_j r_{ij} + Q_j X_{ij}] |V_j|^2 - 4[P_j r_{ij} + Q_j X_{ij}] \quad (28)$$

where f and t denote the sending and receiving end buses, V , P , and Q represent voltage magnitude, real power, and reactive power. r and X symbolize the resistance and reactance existing between the sending and receiving buses. The value of the least VSI of the RDN farther from the collapse point is called the minimum VSI (VSI_{min}).

3.3.4. DSTATCOM Cost

The purchase or investment cost of DSTATCOM per year [23] can be obtained as follows:

$$DST_{cost_yr} = DST_{cost} \times \sum_{l=1}^m Q_{DST}(l) \times \frac{(1 + r_t)^y \times 1}{(1 + r_t)^y - 1} \quad (29)$$

where DST_{cost_yr} is the annual cost of DSTATCOM, DST_{cost} is the purchase cost of DSTATCOM, r_t is the asset rate of return, $\sum_{l=1}^m Q_{DST}(l)$ is the total capacity of installed DSTATCOMs, and y represents the lifespan of DSTATCOMs.

The annual cost savings represent the reduction in cost attributed to power losses achieved through compensation. This figure is calculated as the disparity between the cost

of power losses before compensation and the cost of power losses following compensation, considering the annual cost of DSTATCOM compensation.

$$ACS = C_P \times RP_{loss} \times t_y - [C_P \times RP_{loss} \times t + C_{TDP} \times DST_{cost_yr}] \quad (30)$$

where C_P is the cost of power losses, t_y hours per year, and time duration proportion.

4. Solution Technique

This section may be divided by subheadings. In this research, a contemporary evolutionary algorithm called the black widow optimizer (BWO) addresses the optimal allocation problem, wherein the objective function is centered around power loss. The BWO algorithm was selected because it maintains the balance between exploitation and exploration, delivering fast convergence speed and avoiding local optima problems.

4.1. Black Widow Optimizer

The black widow algorithm is inspired by the life cycle and peculiar mating behavior of black widow spiders [43]. In this behavior, the female spider consumes the male during or after mating and then transfers the eggs to her egg sac. After hatching, the offspring sometimes engage in cannibalism among siblings, and there is even the possibility of them consuming their mother. This process ultimately selects the fittest and most robust individuals. The essential steps of the black widow optimizer (BWO) are delineated as follows [37,43]:

(1) *Population Initialization*: The algorithm models the potential answer to a problem as a black widow spider contained in a N_{pop} population. The fitness of all the black widows in the population is also calculated.

(2) *Selection*: The number of reproductions N_r is calculated based on the procreation rate, P_r (i.e., $N_r = N_{pop} \times P_r$). The best N_r solutions in W are selected in descending order of fitness value, c_i and stored as W_{pop1} based on the fitness value.

(3) *Procreation*: The BWO constructs an array called alpha as long as the widow array with random numbers is contained, then offspring is produced by applying the following Equation (9) in which x_1 and x_2 are parents and y_1 and y_2 are offspring [43].

$$\left. \begin{aligned} y_1 &= \alpha \times x_1 + (1 - \alpha)x_2 \\ y_2 &= \alpha \times x_2 + (1 - \alpha)x_1 \end{aligned} \right\} \quad (31)$$

The BWO undergoes the reproduction process N_r times, ensuring that the selected integers are not duplicated. Subsequently, the offspring and their mother are included in an array, which is organized in ascending order of their respective fitness values. Lastly, the BWO incorporates the top-performing individuals from this newly generated population, considering their cannibalism rating.

(4) *Cannibalism*: There are three distinct types of cannibalism observed in this context. The first one involves a black widow consuming her mate either during or after mating, which is referred to as sexual cannibalism. This algorithm uses fitness scores to differentiate between male and female spiders. The second form of cannibalism is sibling cannibalism, where more robust spiders consume their weaker siblings. This algorithm resolves this by establishing a cannibalism rating based on the number of survivors. For evaluating the spiders, their fitness values are employed. The resulting population is then stored as W_{pop2} .

(5) *Mutation*: Determine the number of mutated offspring based on the mutation rate (m_r). A mutant child, denoted as m , is derived from the new population (W_{pop2}) by randomly selecting an individual (w_i) and modifying one of its chromosomes (variables). This process is repeated for a total of times. The resulting solution is then stored as W_{pop3} .

(6) *Update the population*: If the convergence condition is not satisfied, update the population by adding W_{pop2} and W_{pop3} ; otherwise, return to step 2.

4.2. Implementation of BWO for Optimal Allocation of DSTATCOM in RDN

The application of the BWO algorithm for SC allocation is shown in the flowchart in Figure 2. Based on the rules and steps of the BWO, the algorithm is implemented for DSTATCOM allocation as follows:

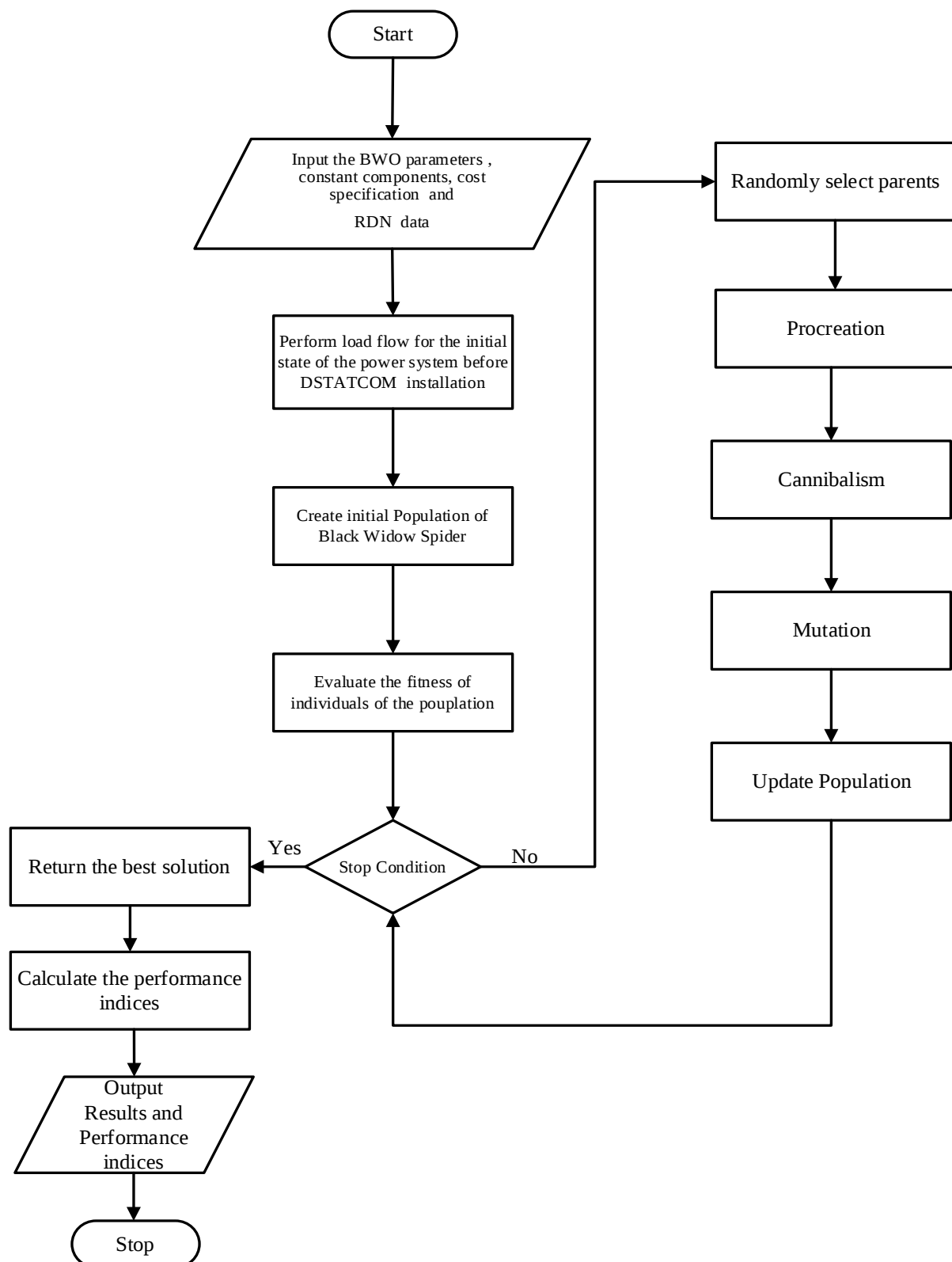


Figure 2. Flowchart for allocation of DSTATCOM.

Step 1—Input the line and load data of the RDN, constant components, BWO parameters, and the cost specifications. The BWO input parameters are the population size ($N = 40$), the dimension of the control variables (D) (D is either 2, 4, or 6 depending on the case being considered), the maximum iteration ($iter_{max} = 100$). By performing load flow, determine the power loss, bus voltages, VPI, VSI, and total annual cost for the base case of the RDN.

Step 2—Initialization: a population of search agents is represented by

$$W_{POP} = \begin{bmatrix} W_1 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} Lo.DST_1^1 & \cdots & Lo.DST_m^1 & Size.DST_1^1 & \cdots & Size.DST_m^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Lo.DST_1^n & \cdots & Lo.DST_m^n & Size.DST_1^n & \cdots & Size.DST_m^n \end{bmatrix} \quad (32)$$

A black widow, which is a solution in the population (W_{POP}), can be represented as:

$$W_i = [Lo.DST_1^i \quad \cdots \quad Lo.DST_m^i \quad Size.DST_1^i \quad \cdots \quad Size.DST_m^i] \quad (33)$$

The first part of the solution vector is the number of buses chosen for DSTATCOM allocation, and the second part is the DSTATCOMs (kVar) sizes at buses, respectively. At the same time, m and n are the number of DSTATCOMs and the population size, respectively, while n is the population size. In the BWO, each black widow W_i is regarded as a potential solution and randomly generated in the initialization as follows:

$$Lo.DST_i = round[Lo_{lb,D1}^i + rand(0,1) \times (Lo_{ub,D1}^i - Lo_{lb,D1}^i)] \quad (34)$$

$$Size.DST_i = round[Size_{lb,D2}^i + rand(0,1) \times (Size_{ub,D2}^i - Size_{lb,D2}^i)] \quad (35)$$

where $D_1 = 1, 2, \dots, m$ and $D_2 = 1, 2, \dots, m$. DSTATCOMs are located at any bus apart from the substation, referred to as the first or slack bus. Therefore, the lower bound (lb) and the upper bound (ub) of each DSTATCOM location are from bus 2 to the last bus of the distribution network and the size of the DSTATCOM is from minimum to maximum reactive power of DSTATCOM as specified in the constraint limits.

Based on the black widows' initial population, each black widow's objective function is calculated by performing load flow using the load flow approach [33].

Step 3—Selection.

Step 4—Procreation and cannibalism: In N_r times, two parents are selected from W_{pop1} and used to generate offspring according to Equation (31). The parent with a higher fitness value represents the mother, while the latter with a lower value is the father. The father is destroyed while the mother is moved to the next generation. Based on the cannibalism rate, eliminate some of the children (new solution) following the ascending order of the fitness value. Save the remaining solution into W_{pop2} .

Step 5—Mutation.

Step 6—Update the population: If the convergence condition is not satisfied, update the population using Equation (48); otherwise, return to step 3.

$$W_{pop} = W_{pop2} + W_{pop3} \quad (36)$$

Step 7—Return the solution with the best fitness value.

Step 8—Calculate the performance assessment indices for the best solution.

Step 9—Print the optimum output result and its performance indices.

Step 10—Stop.

5. Result and Discussion

The algorithms were implemented using the MATLAB simulation tool on a core i3 laptop clocked at 1.70 GHz. The proposed methodology was tested on the standard IEEE

33- and 69-bus RDNs. The detailed description of these networks, single line diagram, and their line and load data are found in [48]. The load flow approach in ref. [45] is adopted to perform the load flow analysis in this study. The BWO parameters utilized in the study were obtained from [43] as the procreation rate, $p_m = 0.6$; cannibalism rate, $c_r = 0.44$; mutation rate, $p_m = 0.4$; population, $N_{pop} = 40$; and maximum iteration, $iter_{max} = 100$. The cost specifications for calculating the ACS obtained from [23] are $DST_{cost} = 50$ \$/kVAr, $r_t = 0.1$, $y = 30$ years, $C_p = 0.06$ \$/kWh, and $t_y = 8760$ h. In this study, four different scenarios are considered depending on the load type:

- Scenario 1: optimal allocation of DSTATCOM considering constant current (CI) load model;
- Scenario 2: optimal allocation of DSTATCOM considering constant impedance (CZ) load model;
- Scenario 3: optimal allocation of DSTATCOM considering constant power (CP) load model;
- Scenario 4: optimal allocation of DSTATCOM considering composite or mix (ZIP) load model.

In each scenario, three different cases of single and multiple DSTATCOM installations were considered as follows:

- Case 1: base case before integration of DSTATCOM;
- Case 2: optimal allocation of one DSTATCOM;
- Case 3: optimal allocation of two DSTATCOMs;
- Case 4: optimal allocation of three DSTATCOMs.

5.1. IEEE 33-Bus RDN

The summary of the outcomes obtained for applying the proposed method to the IEEE 33-bus radial distribution network (RDN) in all four considered scenarios and cases is presented in Table 2.

Table 2. Summary of results for DSTATCOM allocation on the IEEE 33-bus RDN.

Scenario	Case	Q_{DST} (kVAr)	RP_{loss} (kW)	% PLRI	RQ_{loss} (kVAr)	VPI	V_{min} (p.u.)	VSI_{min} (p.u.)	ACS (\$)
CI Scenario	1	-----	176.62	----	117.60	1.2934	0.9194	0.7162	92,831.47
	2	1204 (30)	129.94	26.43	87.01	1.3945	0.9296	0.7484	18,142.25
	3	427 (12) 1032 (30)	123.95	29.82	82.60	1.4325	0.9383	0.7770	19,936.64
	4	369 (12) 579 (24) 967 (30)	120.87	31.56	80.72	1.4327	0.9375	0.7743	19,134.31
CZ Scenario	1	-----	156.86	----	104.25	1.2616	0.9245	0.7320	824,445.62
	2	1164 (30)	118.74	24.30	79.31	1.3752	0.9330	0.7594	13,862.10
	3	418 (12) 993 (30)	114.02	27.31	75.84	1.4198	0.9408	0.7700	15,032.82
	4	352 (12) 549 (24) 927 (30)	111.21	29.10	88.40	1.4264	0.9398	0.7757	14,298.00
CP Scenario	1	-----	202.67	----	135.24	1.2616	0.9131	0.6965	106,523.35
	2	1251 (30)	143.59	29.15	96.41	1.3751	0.9256	0.7359	24,410.14
	3	467 (12) 1058 (30)	135.74	33.02	90.61	1.4195	0.9362	0.7702	27,081.27
	4	311 (13) 352 (25) 1041 (30)	132.85	34.45	88.79	1.4110	0.9351	0.7667	27,649.46
ZIP Scenario	1	-----	170.59	----	113.53	1.2866	0.9209	0.7209	89,662.10
	2	1192 (30)	126.55	25.82	84.67	1.3884	0.9306	0.7517	16,818.38
	3	430 (12) 1018 (30)	120.94	29.10	80.55	1.4271	0.9392	0.7797	18,407.74
	4	345 (13) 525 (24) 996 (30)	117.86	30.91	78.67	1.4925	0.9403	0.7836	17,807.17

5.1.1. Scenario 1: DSTATCOM Allocation Considering Constant Current (CI) Load Model

Before the installation of DSTATCOM (case 1), the IEEE 33-bus RDN had power losses of 176.62 kW, reactive power losses of 117.60 kVAr, a voltage profile index (VPI) of 1.2934, a minimum voltage stability index (VSI) of 0.7162 p.u., and an annual operating cost of \$92,831.47. In case 2, the optimal DSTATCOM size was determined to be 1204 kVAr at bus 30, resulting in a power loss reduction of 26.43% (PLRI) compared to the base case. Other assessment indices such as reactive power loss, VPI, V_{min} , and VSI_{min} were measured at 87.01 kVAr, 1.3945, 0.9296, and 0.7484, respectively. The annual cost savings (ACS)

due to compensation amounted to \$18,142.25, equivalent to 19.54% of the operating cost of the base case. In case 3, optimal DSTATCOM sizes of 427 kVar and 1032 kVar were allocated to buses 12 and 30, respectively, resulting in a power loss of 123.95 kW and an ACS of \$19,936.31. For case 4, the optimal DSTATCOM sizes were 369 kVar, 579 kVar, and 967 kVar, positioned at buses 12, 24, and 30, respectively. The power loss reduction was 31.56% (PLRI) relative to the base case, and the ACS was \$19,134.31.

The values of all the assessment indices for the CI scenario are presented in the corresponding column of Table 2, demonstrating improvements in the parameters with an increasing number of optimally installed DSTATCOMs, except for ACS, where the installation of 2 DSTATCOMs yielded the highest cost savings. For this scenario, Figures 3a and 4a display the voltage profile and VSI for cases 1–4, showing significant and progressive enhancement with the addition of DSTATCOMs. Figure 5a illustrates the convergence characteristics for optimal cases 2–4, establishing that multiple DSTATCOMs allocations have more impact on the power loss reduction.

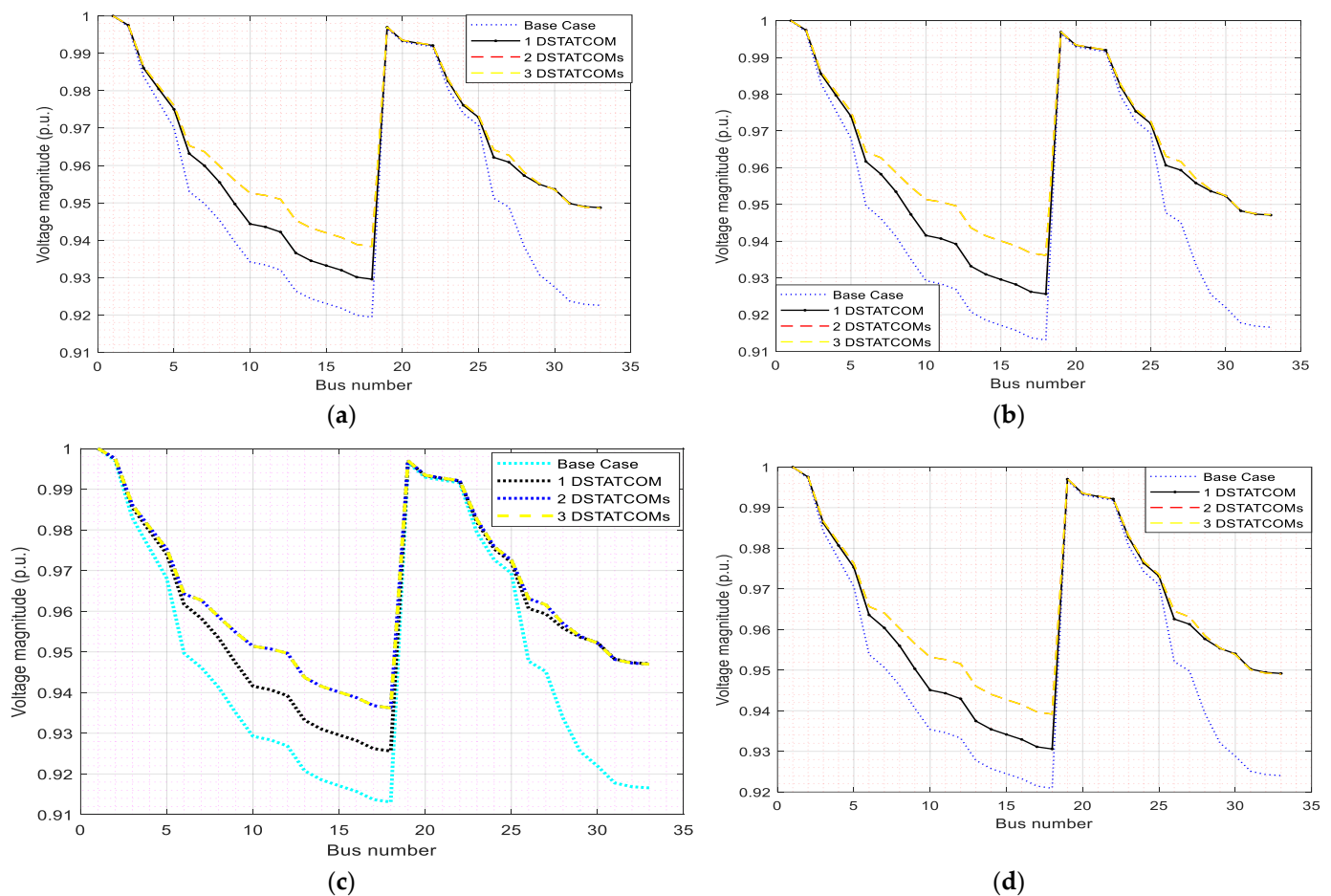


Figure 3. Voltage profile of (a) CZ; (b) CI; (c) CP; (d) ZIP scenarios for IEEE 33-bus RDN.

5.1.2. Scenario 2: DSTATCOM Allocation Considering Constant Impedance (CZ) Load Model

Before installation of DSTATCOM, the power loss, reactive power loss, VPI, minimum VSI, and annual operating cost are 156.86 kW, 104.25 kVar, 1.2616, 0.7320 p.u., and \$82,445.62. For case 2, the BWO obtained an optimal size of 1164 kVar DSTATCOM located at bus 30. The power loss is 118.74 kW, indicating a 24.30% PLRI compared to the base case, and the ACS is \$13,862.10. In case 3, the optimal DSTATCOM sizes are 418 kVar and 993 kVar placed at buses 12 and 30, respectively, with a power loss of 114.02 kW. This value equals 27.31% of power loss reduction compared to the base case. The annual

cost-saving outcome for case 3 is \$15,032.82. Optimal sizes of 381 kVar, 549 kVar, and 1041 kVar of DSTATCOM installed at buses 13, 24, and 30, respectively, were obtained for case 4, resulting in a power loss of 111.21 kW and an ACS of \$14,298.00. The power loss is equivalent to 29.10% PLRI of the base case.

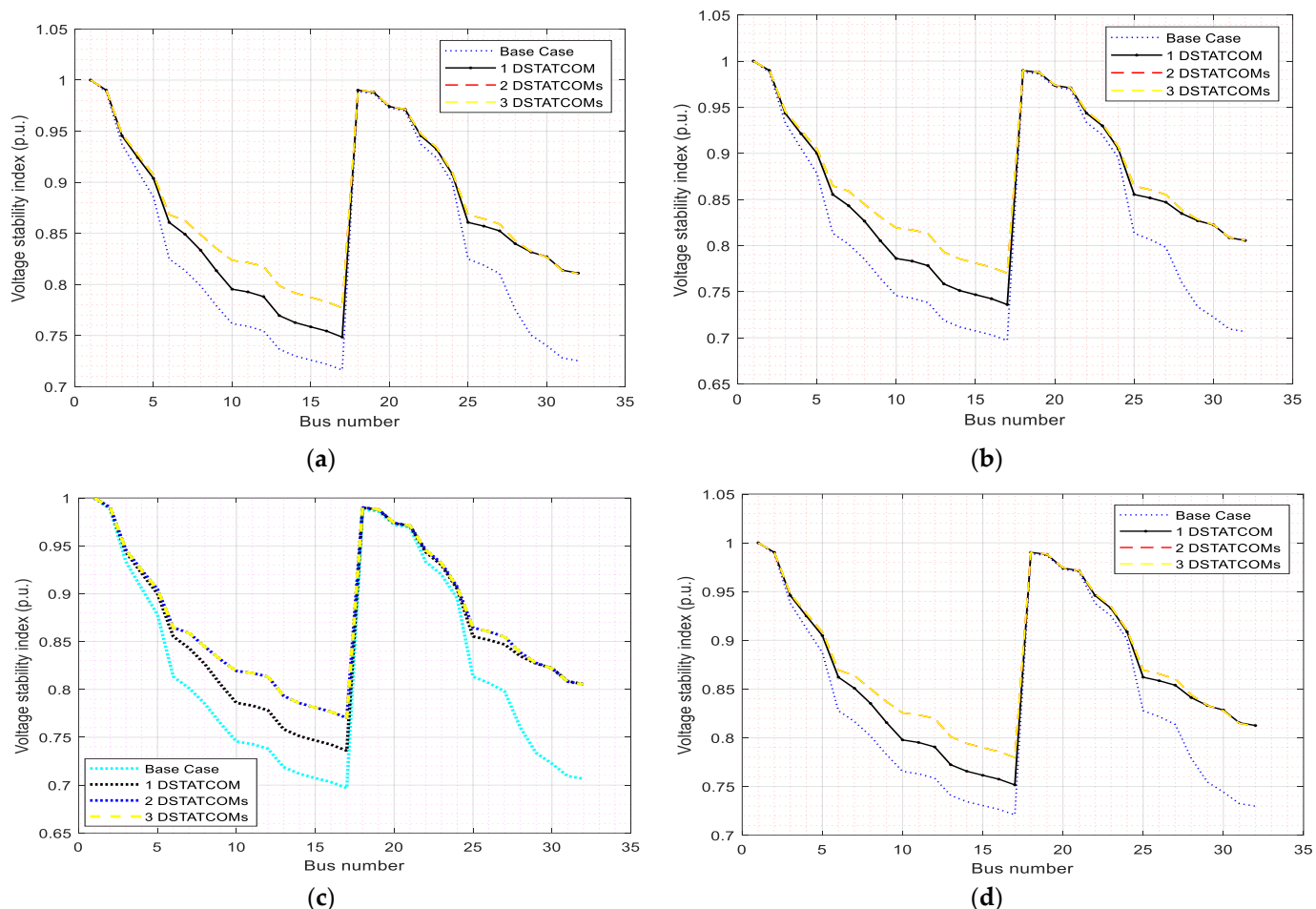


Figure 4. VSI of (a) CI; (b) CZ; (c) CP; (d) ZIP scenarios for IEEE 33-bus RDN.

The performance assessment indices in the CZ scenario column of Table 1 show increased improvement in the parameters with the number of DSTATCOM optimally allocated. The only exception is the ACS, where case 3 gave the highest value. For this scenario, the voltage profile and VSI for cases 1–4 are displayed in Figures 3b and 4b. It is clearly shown in the figures that there is significant and progressive improvement in the voltage profile and VSI of the distribution system with the number of DSTATCOMs. Figure 5b depicts the convergence characteristic for the optimization procedure in cases 2–4. The multiple DSTATCOM cases have lower power losses progressively compared to single allocation.

5.1.3. Scenario 3: DSTATCOM Allocation Considering Constant Power (CP) Load Model

The assessment indices for all cases are presented in the CP scenario column of Table 2. Case 2 achieved a power loss reduction of 29.15% and an ACS of \$24,410.14 after an optimal DSTATCOM size of 1251 kVar was allocated to bus 30 by the BWO algorithm. Case 3 involved optimal DSTATCOM sizes of 467 kVar and 1069 kVar located at buses 12 and 30, respectively, with a power loss reduction of 33.02% and an ACS of \$27,649.46. Case 4 resulted in a power loss reduction of 34.46% and an ACS of \$27,649.46 after the optimal allocation of three DSTATCOMs of sizes 311 kVar, 352 kVar, and 1041 kVar at buses 13, 25, and 30, respectively.

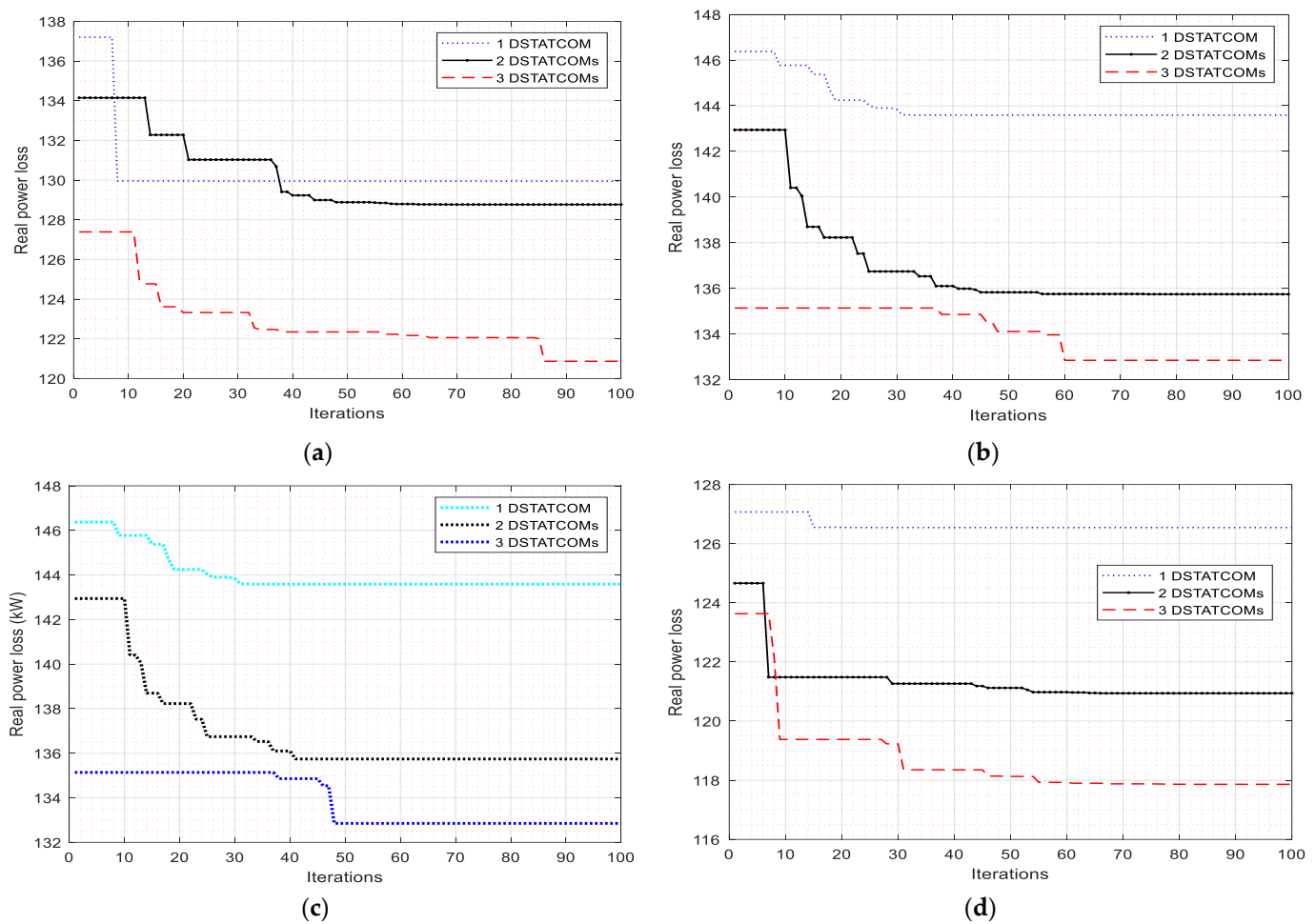


Figure 5. Convergence characteristics of (a) CI; (b) CZ; (c) CP; (d) ZIP for IEEE 33-bus RDN.

The performance assessment indices in the CP scenario column of Table 2 demonstrate increased parameter improvement with the number of optimally allocated DSTATCOMs. Figures 3c and 4c display the voltage profile and VSI for cases 1–4, while Figure 5c illustrates the convergence characteristics for cases 2–4 for scenario 3. A comparison with other existing techniques in the open literature, as provided in Table 3, shows that the BWO technique is more efficient for power loss reduction for all the cases considered.

Table 3. Comparison of results of various techniques of the CP scenario for IEEE 33-bus RDN.

Case	Technique	Q_{DST} (kVar)	RP_{loss} (kW)	RQ_{loss} (kVar)	% PLRI	V_{min} (p.u.)	VSI_{min} (p.u.)	ACS (\$)
Case 2	BAT [23]	1150 (30)	143.79	96.47	28.97	0.9244	0.7242	24,768
	IA [17]	962.49 (12)	171.26	115.26	15.24	0.9258	0.7266	11,680
	HSA [16]	1150 (30)	143.97	96.47	28.97	0.9236	-----	24,264
	PSO [18]	1380 (30)	144.17	-----	28.86	0.9268	0.7381	23,428.34
	IGWO [26]	1252.5 (30)	143.5	96	29.15	0.9300	-----	-----
	DEA [19]	1252.7 (30)	143.50	-----	-----	0.9256	-----	-----
	PSO + GAMS [21]	1105 (30)	144.37	-----	28.76	0.9247	0.7301	24,780.72
	BWO	1251 (30)	143.59	96.41	29.15	0.9256	0.7359	24,410.14
Case 3	BAT [23]	450 (10) 995 (30)	136.05	90.63	32.87	0.9356	0.7548	27,356.97
	PSO [18]	472 (12) 1062 (30)	135.75	-----	33.02	0.9364	0.7692	27,042.95
	PSO + GAMS [21]	384 (12) 952 (30)	136.71	-----	32.54	0.9638	0.7585	27,716.97
	ICA [20]	455 (10) 1058 (30)	140.24	93.67	30.31	0.9361	-----	-----
	BWO	467 (12) 1058 (30)	135.74	90.61	33.02	0.9362	0.7702	27,081.27
Case 4	CSA [25]	350 (14) 570 (24) 1010 (30)	138.45	-----	34.45	0.9304	0.7432	28,150
	BWO	311 (13) 352 (25) 1041 (30)	132.85	88.40	34.45	0.9379	0.7757	27,649.46

5.1.4. Scenario 4: DSTATCOM Allocation Considering Composite (ZIP) Load Model

The assessment indices for all cases are presented in the ZIP scenario column of Table 2. Case 2 achieved a power loss reduction of 25.82% and an ACS of \$16,818.38 after an optimal DSTATCOM size of 1192 kVar was allocated to bus 30 by the BWO algorithm. Case 3 involved optimal DSTATCOM sizes of 430 kVar and 1018 kVar located at buses 12 and 30, respectively, with a power loss reduction of 29.10% and an ACS of \$18,407.74. Case 4 resulted in a power loss reduction of 30.91% and an ACS of \$17,807.17 after the optimal allocation of three DSTATCOMs of sizes 345 kVar, 525 kVar, and 996 kVar at buses 13, 24, and 30, respectively.

The performance assessment indices in the ZIP scenario column of Table 2 demonstrate increased parameter improvement with the increase in optimally allocated DSTATCOMs. Figures 3d and 4d display the voltage profile and VSI for cases 1–4, while Figure 5d illustrates the convergence characteristics for cases 2–4 for Scenario 4. The case of three DSTATCOMs allocation in Figure 5d has the least power loss compared to the other two cases.

5.2. IEEE 69-Bus RDN

The summary of the results obtained for implementing the proposed BWO on the IEEE 69-bus RDN for all four considered scenarios and cases is displayed in Table 4.

Table 4. Summary of results for DSTATCOM allocation on the IEEE 69-bus RDN.

	Case	Q_{DST} (kVar)	RP_{loss} (kW)	% PLRI	RQ_{loss} (kVar)	VPI	V_{min} (p.u.)	VSI_{min} (p.u.)	ACS (\$)
CI Scenario	BC	-----	222.58	-----	101.06	1.5620	0.9094	0.6858	116,988.05
	1	1325 (61)	150.26	32.49	69.72	1.6508	0.9309	0.7526	30,976.17
	2	357 (17) 1274 (61)	145.16	34.78	67.66	1.6798	0.9312	0.7539	32,032.00
	3	306 (18) 1244 (61) 248 (67)	144.16	35.23	67.21	1.6921	0.9315	0.7545	31,139.93
CZ Scenario	BC	-----	220.42	-----	100.08	1.5667	0.9097	0.6865	115,852.75
	1	1321 (61)	148.67	32.55	69.02	1.6455	0.9310	0.7531	30,706.27
	2	553 (12) 1242 (61)	144.05	34.65	67.05	1.6761	0.9315	0.7548	30,626.57
	3	312 (12) 132 (25) 1189 (61)	143.51	34.89	66.96	1.6794	0.9315	0.7511	31,768.99
CP Scenario	BC	-----	224.96	-----	102.14	1.5567	0.9092	0.6850	118,238.98
	1	1330 (61)	152.01	32.43	70.48	1.6455	0.9307	0.7522	31,280.76
	2	361 (17) 1275 (61)	146.42	34.91	68.22	1.6761	0.9311	0.7535	32,594.12
	3	265 (17) 1252 (61) 229 (67)	143.87	36.04	67.10	1.6798	0.9314	0.7545	33,350.35
ZIP Scenario	BC	-----	221.94	-----	100.77	1.5609	0.9095	0.6860	116,651.66
	1	1324 (61)	149.79	32.07	69.51	1.6496	0.9309	0.7528	30,892.13
	2	357 (17) 1273 (61)	144.82	34.28	67.50	1.6789	0.9313	0.7540	31,879.62
	3	256 (18) 1235 (61) 345 (67)	143.75	35.23	67.02	1.7199	0.9315	0.7548	31,348.23

5.2.1. Scenario 1: DSTATCOM Allocation Considering Constant Current (CI) Load Model

The power loss, reactive power loss, voltage profile index (VPI), minimum VSI, and annual operating cost of the IEEE 69-bus RDN before installation of DSTATCOM (case 1) were 222.58 kW, 101.06 kVar, 1.5620, 0.6858 p.u., and \$116,988.05. For case 2, the optimal size of the DSTATCOM obtained was 1325 kVar at bus 61 with a power loss of 150.26 kW, corresponding to 32.49% power loss reduction (PLRI) compared to the base case. Other assessment indices such as reactive power loss, VPI, V_{min} , and VSI_{min} obtained are 87.01 kVar, 1.6508, 0.9309, and 0.7526, respectively. The annual cost saving (ACS) due compensation is \$30,976.17, equivalent to 26.54% of the operating cost of the base case. In case 3, the obtained optimal sizes are 357 kVar and 1276 kVar located at buses 17 and 61, respectively, with a power loss of 145.16 kW (34.78% PLR) and ACS of \$30,976.17. The optimal DSTATCOM sizes for case 4 are 306 kVar, 1244 kVar, and 248 kVar, located at buses 18, 61, and 67, respectively. The power loss is 144.16 kW, indicating a 35.23% PLRI relative to the base case, while the ACS is \$31,139.93.

The values of all the assessment indices are displayed in the CI scenario column of Table 4 and reveal improvement in the parameters with the increasing number of

DSTATCOM optimally installed in the network. The only exception is the ACS, where optimal installation of 2 DSTATCOM yielded the highest cost savings. The voltage profile and VSI for cases 1–4 are displayed in Figures 6a and 7a. It is vividly shown that there is significant and progressive improvement in the voltage profile and VSI of the distribution system with the number of DSTATCOMs. Figure 8a depicts the convergence characteristic for cases 2–4. The case of 3 DSTATCOMs allocation had the least power in the RDN.

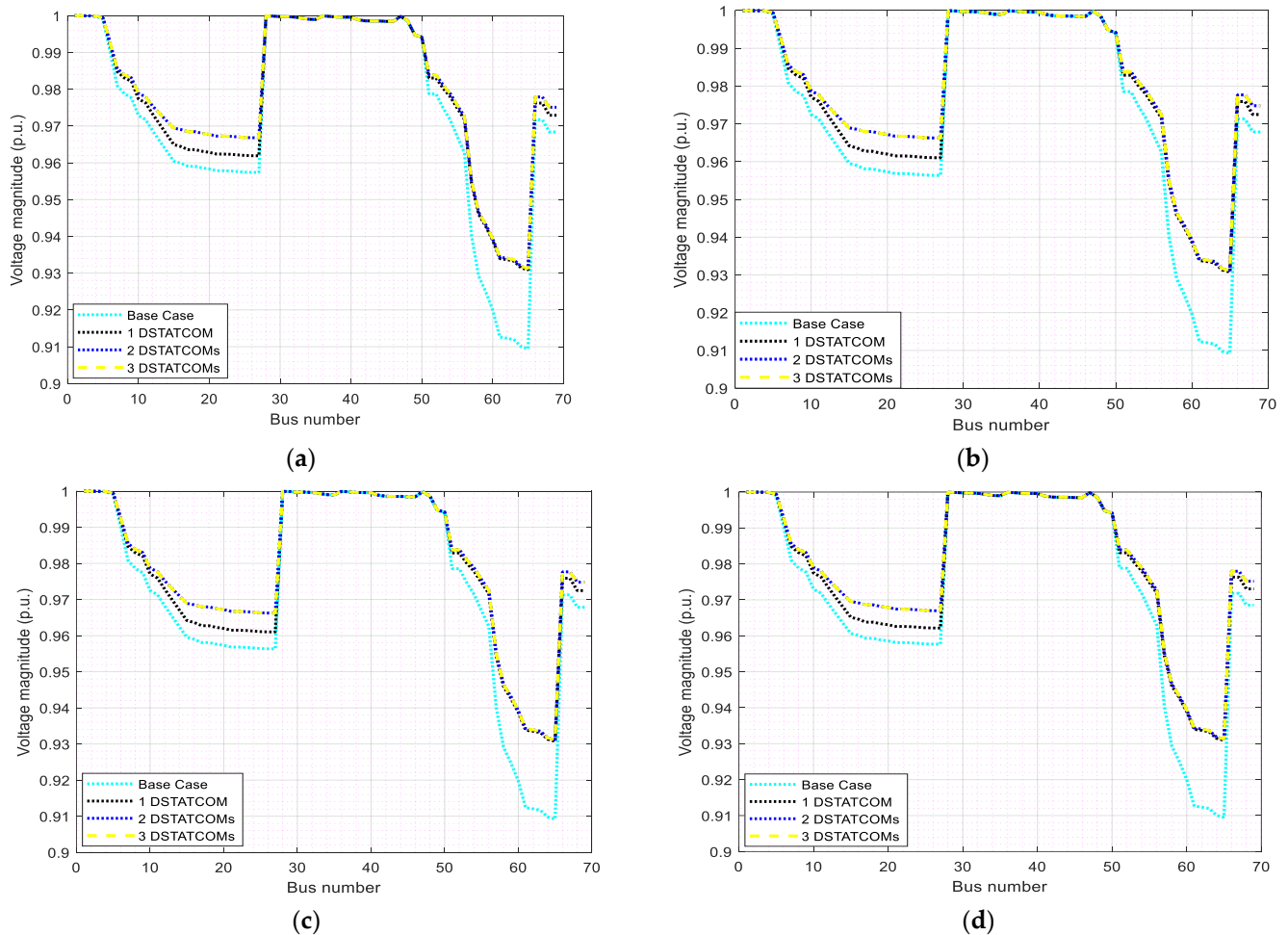


Figure 6. Voltage profile of (a) CI; (b) CZ; (c) CP; (d) ZIP scenarios for IEEE 69-bus RDN.

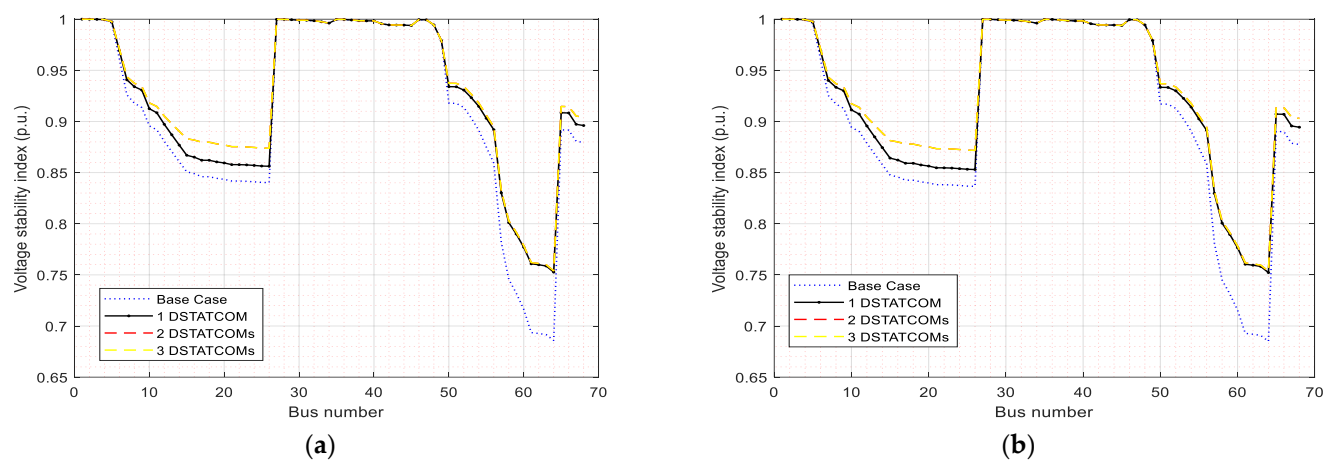


Figure 7. Cont.

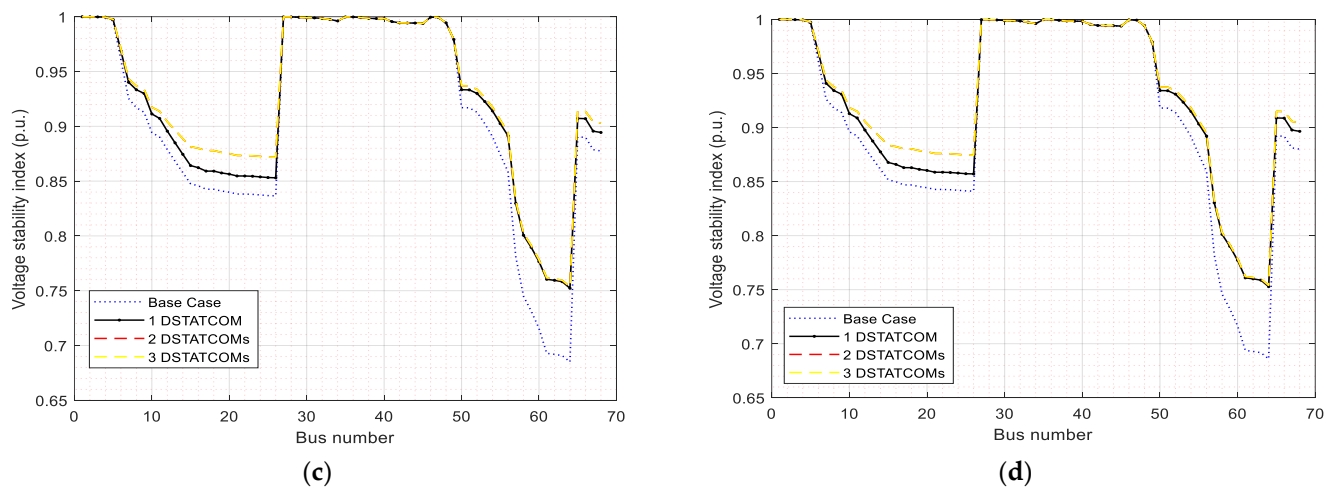


Figure 7. VSI of (a) CI; (b) CZ; (c) CP; (d) ZIP scenarios for IEEE 69-bus RDN.

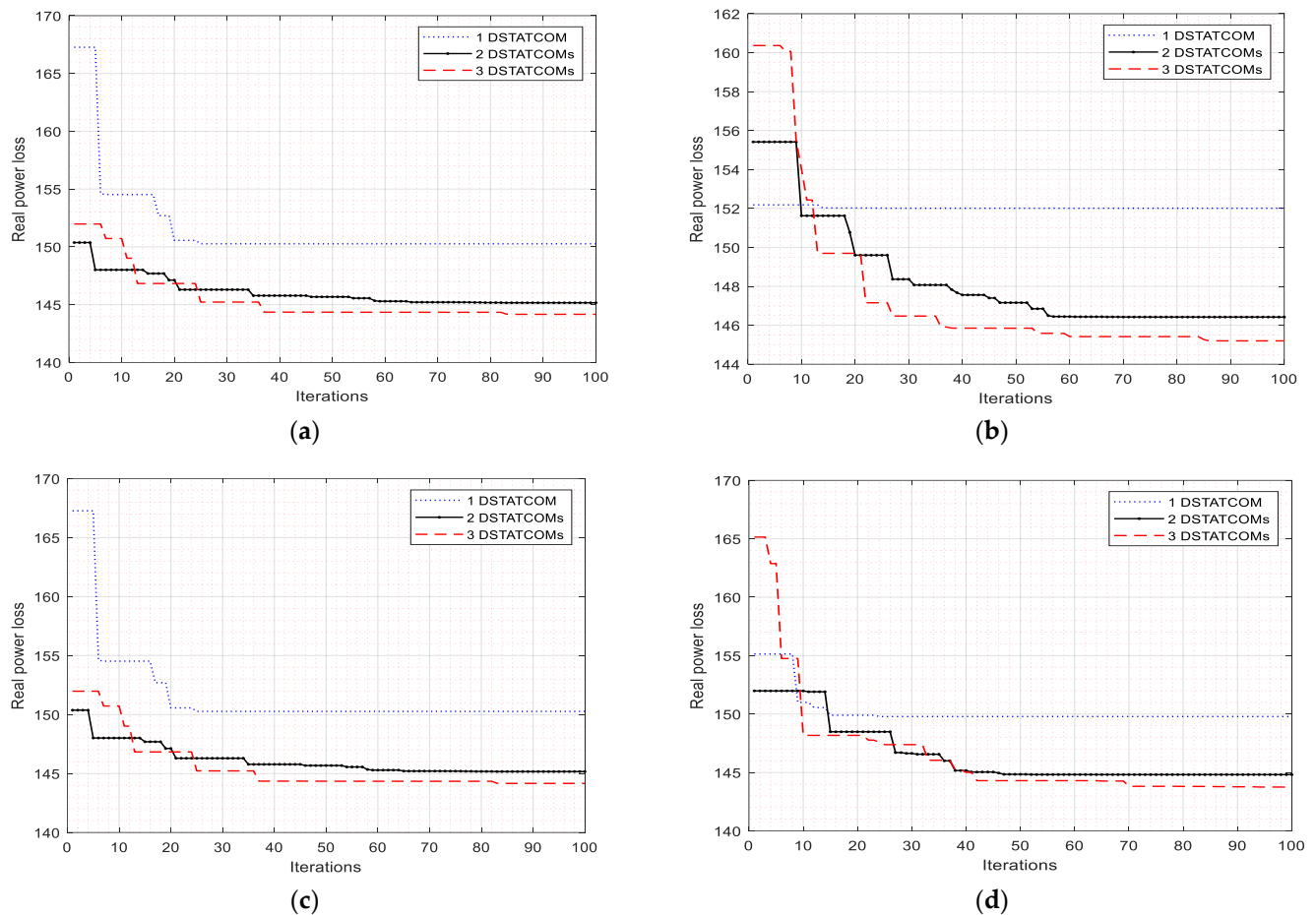


Figure 8. Convergence characteristics of (a) CI; (b) CZ; (c) CP; (d) ZIP for IEEE 69-bus RDN.

5.2.2. Scenario 2: DSTATCOM Allocation Considering Constant Impedance (CZ) Load Model

Before installation of DSTATCOM, the power loss, reactive power loss, VPI, minimum VSI, and annual operating cost are 220.42 kW, 100.08 kVar, 1.5667, 0.7531 p.u., and \$115,852.75. For case 2, the BWO obtained an optimal size of 1321 kVar DSTATCOM located at bus 61. The power loss is 148.67 kW, indicating a 32.43% PLRI compared to the base case, and the ACS is \$30,706.27. In case 3, the optimal DSTATCOM sizes are 553 kVar

and 1242 kVar placed at buses 12 and 61, respectively, with a power loss of 144.05 kW. This value equals 34.65% of power loss reduction compared to the base case. The annual cost-saving outcome for case 3 is \$30,626.57. Optimal sizes of 312 kVar, 132 kVar, and 1189 kVar of DSTATCOM installed at buses 12, 25, and 61, respectively, were obtained for case 4, resulting in a power loss of 143.51 kW and an ACS of \$31,768.99. The power loss is equivalent to 34.89% PLRI of the base case.

The performance assessment indices in the CZ scenario column of Table 4 show increased parameter improvement with the number of DSTATCOM optimally allocated. The only exception is the ACS, where case 3 gave the highest value. The voltage profile and VSI for cases 1–4 are displayed in Figures 6b and 7b. It is clearly shown in the figures that there is significant and progressive improvement in the voltage profile and VSI of the distribution system with the number of DSTATCOMs. Figure 8b depicts the convergence characteristic for cases 2–4.

5.2.3. Scenario 3: DSTATCOM Allocation Considering Constant Power (CP) Load Model

The values of the assessment indices of all the cases are given in the CP scenario column of Table 4. A power loss reduction of 32.43% and ACS of \$31,280.76 was obtained for case 2 after an optimal size of 1330 kVar of DSTATCOM was allocated to bus 61 by BWO. In case 3, the optimal DSTATCOM sizes are 361 kVar and 1275 kVar located at buses 17 and 61, respectively, with a power loss reduction of 32.43% and ACS of \$31,280.76. For case 4, the power loss reduction of 36.04% and an ACS of \$33,350.35 were obtained after the optimal allocation of three DSTATCOMs of sizes 365 kVar, 1252 kVar, and 229 kVar at buses 17, 61, and 67, respectively. The performance assessment indices in the CP scenario column of Table 4 show increased parameter improvement with the number of DSTATCOM optimally allocated. The voltage profile and VSI for cases 1–4 are displayed in Figures 6c and 7c, while Figure 8c depicts the convergence characteristic for cases 2–4. Comparison to other existing techniques in the open literature, as given in Table 5, shows that the BWO technique is more efficient in the percentage power loss reduction index in most cases.

Table 5. Comparison of results of various techniques of the CP scenario for IEEE 69-bus RDN.

Case	Technique	Q_{DST} (kVar)	RP_{loss} (kW)	RQ_{loss} (kVar)	% PLRI	V_{min} (p.u.)	VSI_{min} (p.u.)	ACS (\$)
1	BAT [23]	1150 (61)	153.26	71.26	31.9	0.9278	0.7356	31,573
	IA [17]	1704.42 (61)	157.5	72.40	30.0	0.9353	0.7561	26,438
	CSA [23]	1200 (61)	152.95	-----	32.02	0.9285	0.7444	31,588
	SSO [15]	1326 (61)	150.20	-----	-----	0.9309	0.7424	32,282
	PSO [18]	1330.5 (61)	152.14	-----	32.42	0.9307	0.7507	31,328
	IGWO [26]	1365.5 (61)	152.10	70.80	32.40	0.9390	-----	-----
	PSO+GAMS [21]	1203 (61)	152.78	-----	32.10	0.9288	0.7446	31,756
	WOA [8]	1300 (61)	152.09	-----	32.40	0.9389	0.7584	-----
	BWO	1330 (61)	152.01	70.48	32.44	0.9307	0.7522	31,280.76
2	BAT [23]	330 (15) 1220 (61)	146.73	68.43	34.79	0.9299	0.7418	32,923.72
	SSO [15]	350 (17) 1270 (61)	144.85	-----	-----	0.9313	-----	-----
	PSO [18]	355 (17) 1278.6 (61)	146.48	-----	34.94	0.9312	0.7522	32,689.49
	PSO+GAMS [21]	260.14 (17) 1162.87 (61)	147.45	-----	34.51	0.9291	0.7456	33,297.17
	WOA [8]	350 (17) 1250 (61)	146.34	-----	34.96	0.9417	0.7630	-----
	ICA [20]	375 (15) 1280 (61)	147.35	72.38	34.60	0.9324	-----	-----
	BWO	361 (17) 1275 (61)	146.42	68.22	34.91	0.9311	0.7535	32,594.12
3	CSA [25]	350 (11) 230 (18) 1170 (61)	145.34	-----	35.40	0.9301	0.7428	32,587
	LSA [31]	374 (11) 240 (18) 1217 (61)	145.16	-----	35.52	0.9307	0.7446	32,327.98
	PSO+GAMS [21]	30 (11) 252 (18) 1160 (61)	147.25	-----	34.50	0.9291	0.7456	33,304.24
	BWO	265 (17) 1252 (61) 229 (67)	143.87	67.10	36.06	0.9314	0.7545	33350.35

5.2.4. Scenario 4: DSTATCOM Allocation Considering Composite (ZIP) Load Model

The values of the assessment indices of all the cases are in the ZIP scenario column of Table 3. A power loss reduction of 32.07% and ACS of \$30,892.13 was obtained for case 2 after an optimal size of 1324 kVar of DSTATCOM was allocated to bus 61 by BWO. In

case 3, the optimal DSTATCOM sizes are 357 kVAr and 1273 kVAr located at buses 17 and 61, respectively, with a power loss reduction of 34.28% and ACS of \$31,879.62. For case 4, the power loss reduction of 36.04% and an ACS of \$31,348.23 were obtained after the optimal allocation of three DSTATCOMs of sizes 256 kVAr, 1235 kVAr, and 345 kVAr at buses 18, 61, and 67, respectively. The values of the performance assessment indices in the ZIP scenario column of Table 4 show increased parameter improvement with the increase in DSTATCOM optimally allocated and the BWO algorithm is shown to perform more efficiently compared to the other algorithms as shown in Table 5. The voltage profile and VSI for cases 1–4 are displayed in Figures 6c and 7c, while Figure 8c depicts the convergence characteristic for cases 2–4.

6. Conclusions

This study proposes a novel black widow algorithm for the optimal allocation of DSTATCOM in a radial distribution network, taking real power loss as the objective function under some constraints. The power loss reduction, voltage stability index (VSI), voltage profile index (VPI), and annual cost were evaluated. Analysis of the optimal DSTATCOM allocation was considered for four different load models, namely, constant impedance (CZ), constant current (CI), constant power (CP), and composite (ZIP) models. In addition, three cases of one, two, and three DSTATCOM allocations were considered under each load model scenario. The proposed BWO was implemented on the IEEE 33-bus and 69-bus RDN. From the results presented and discussed for all the scenarios and cases, the following can be deduced:

- (i) The constant power load model for the three DSTATCOM (case 4) integration yielded the highest power loss reduction of 34.45% and 36.06% for both the IEEE 33-bus and 69-bus RDN, respectively.
- (ii) The constant current and power models recorded the highest annual cost savings (ACS). However, the best ACS is not necessarily obtained in the case of three DSTATCOM allocations in all the load models considered. This means that increasing the number of DSTATCOMs optimally installed may not translate into an increase in the ACS. Hence, the ACS needs to be optimized to obtain the best outcome for any RDN.
- (iii) Multiple DSTATCOM allocations yielded better technical assessment indices compared to single DSTATCOM allocation.
- (iv) Comparison of the results of the CP model with that of related works in the literature establishes the superiority and efficiency of the proposed BWO.

In the future, the reconfiguration of the radial distribution network with the optimal integration of electric vehicles alongside other network enhancement options will be explored for the possibility of obtaining higher enhancement of the RDN's performance. Moreover, other optimization approaches including metaheuristic and classical techniques will be comparatively applied for performance validation.

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Nomenclature

DSTATCOM	Distribution Static Synchronous Compensator	r_A and x_A	line resistance and reactance
RDN	Radial Distribution Network	$P_A + jQ_A$	real and reactive loads at buses A and B
BWO	Black Widow Optimization	v_A and v_B	voltages at buses A and B
VPI	Voltage Profile Index	$i_{DST} \left(\Delta'_B + \frac{\pi}{2} \right)$	current injected by DSTATCOM
VSI	Voltage Stability Index	Q_{DST}	reactive power injected by DSTATCOM
CZ	Constant Impedance	P_n	nominal real power
CI	Constant Current	Q_n	nominal reactive power
CP	Constant Power	V_n	nominal voltage
ZIP	Composite Load	V_a	actual voltage during load flow
DG	Distributed Generation	k_0, k_1 and k_2	constituents of the CP, CI, and CZ loads
FA	Firefly Algorithm	RP_{loss}	total power loss
PLI	Power Loss Index	V_i and V_j	voltages of buses 'i' and 'j'
SSO	Social Spider Optimization	P_{gi} and P_{Di}	real power generated and power demanded at bus 'i'
IA	Immune Algorithm	Q_{gi} and Q_{Di}	reactive power generated and demanded at bus 'i'
PSO	Particle Swarm Optimization	G_{ij} and B_{ij}	conductance and susceptance of branch 'ij'
DEA	Differential Evolution Algorithm	θ_{ij}	difference of voltage angles of buses 'i' and 'j'.
ICA	Imperial Competitive Algorithm	V_{min} and V_{max}	minimum and maximum voltage
GAMS	General Algebraic Modeling System	%PLRI	percentage power loss reduction index
WOA	Whale Optimization Algorithm	$RP_{loss(after_DST)}$	power loss after DSTATCOM
GSA	Gravitational Search Algorithm	V_μ	mean bus voltage
CSA	Cuckoo Search Algorithm	V_σ	standard deviation of the bus voltage
MGWO	Modified Grey Wolf Optimization	f and t	sending and receiving buses
IBES	Improved Bald Eagle Search	DST_{cost_yr}	annual cost of DSTATCOM
IBFA	Improved Bacterial Foraging Search Algorithm	DST_{cost}	purchase cost of DSTATCOM
ALO	Ant Lion Optimization	r_i	asset rate of return
LSA	Light Search Algorithm	Y	lifespan of DSTATCOMs
TVD	Total Voltage Deviation	C_P	cost of power losses
SPBO	Student Psychology-based Optimization	t_y	hours per year
ACS	Annual Cost Savings	C_{TDP}	time duration proportion

References

1. Mokryani, G. Distribution network types and configurations. In *Future Distribution Networks: Planning, Operation, and Control* 2022; AIP Publishing LLC: Melville, NY, USA, 2022; p. 1.
2. Salimon, S.A.; Adepoju, G.A.; Adebayo, I.G.; Adewuyi, O.B. Comparative assessment of techno-economic and environmental benefits in optimal allocation of distributed generators in distribution networks. *Sci. Afr.* **2023**, *19*, e01546. [\[CrossRef\]](#)
3. Rao, R.S.; Narasimham, S.V.L.; Ramalingaraju, M. Optimal capacitor placement in a radial distribution system using plant growth simulation algorithm. *Int. J. Electr. Power Energy Syst.* **2011**, *33*, 1133–1139. [\[CrossRef\]](#)
4. Colmenar-Santos, A.; Reino-Rio, C.; Borge-Diez, D.; Collado-Fernández, E. Distributed generation: A review of factors that can contribute most to achieve a scenario of DG units embedded in the new distribution networks. *Renew. Sustain. Energy Rev.* **2016**, *59*, 1130–1148. [\[CrossRef\]](#)
5. Muruganatham, B.; Selvam, M.M.; Gnanadass, R.; Padhy, N.P. Energy loss reduction and load balancing through network reconfiguration in practical LV distribution feeder using GAMS. In Proceedings of the 7th International Conference on Power Systems (ICPS), Pune, India, 21–23 December 2017; IEEE: Piscataway, NJ, USA, 2017.
6. Franco, J.F.; Rider, M.J.; Lavorato, M.; Romero, R. A mixed-integer LP model for the optimal allocation of voltage regulators and capacitors in radial distribution systems. *Int. J. Electr. Power Energy Syst.* **2013**, *48*, 123–130. [\[CrossRef\]](#)
7. Salimon, S.A.; Adepoju, G.A.; Adebayo, I.G.; Ayanlade, S.O. Impact of shunt capacitor penetration level in radial distribution system considering techno-economic benefits. *Niger. J. Technol. Dev.* **2022**, *19*, 101–109. [\[CrossRef\]](#)
8. Balamurugan, P.; Yuvaraj, T.; Muthukannan, P. Optimal allocation of DSTATCOM in distribution network using whale optimization algorithm. *Eng. Technol. Appl. Sci. Res.* **2018**, *8*, 3445–3449. [\[CrossRef\]](#)
9. Montoya, O.D.; Gil-González, W.; Hernández, J.C. Efficient operative cost reduction in distribution grids considering the optimal placement and sizing of D-STATCOMs using a discrete-continuous VSA. *Appl. Sci.* **2021**, *11*, 2175. [\[CrossRef\]](#)
10. Castiblanco-Pérez, C.M.; Toro-Rodríguez, D.E.; Montoya, O.D.; Giral-Ramírez, D.A. Optimal Placement and sizing of D-STATCOM in radial and meshed distribution networks using a discrete-continuous version of the genetic algorithm. *Electronics* **2021**, *10*, 1452. [\[CrossRef\]](#)
11. Farhoodnea, M.; Mohamed, A.; Shareef, H.; Zayandehroodi, H. Optimum D-STATCOM placement using firefly algorithm for power quality enhancement. In Proceedings of the 2013 IEEE 7th International Power Engineering and Optimization Conference (PEOCO), Langkawi, Malaysia, 3–4 June 2013; pp. 98–102.
12. Hussain, S.S.; Subbaramiah, M. An analytical approach for optimal location of DSTATCOM in radial distribution system. In Proceedings of the 2013 International Conference on Energy Efficient Technologies for Sustainability, Nagercoil, India, 10–12 April 2013; pp. 1365–1369.

13. Jain, A.; Gupta, A.R.; Kumar, A. An efficient method for D-STATCOM placement in radial distribution system. In Proceedings of the 2014 IEEE 6th India International Conference on Power Electronics (IICPE), Kurukshetra, India, 8–10 December 2014; pp. 1–6.
14. Gupta, A.R.; Jain, A.; Kumar, A. Optimal D-STATCOM placement in radial distribution system based on power loss index approach. In Proceedings of the 2015 International Conference on Energy, Power and Environment: Towards Sustainable Growth (ICEPE), Shillong, India, 12–13 June 2015; pp. 1–5.
15. Abbasi, A.-R.; Khoramini, R.; Dehghan, B.; Abbasi, M.; Karimi, E. A new intelligent method for optimal allocation of D-STATCOM with uncertainty. *J. Intell. Fuzzy Syst.* **2015**, *29*, 1881–1888. [\[CrossRef\]](#)
16. Yuvaraj, T.; Devabalaji, K.; Ravi, K. Optimal placement and sizing of DSTATCOM using harmony search algorithm. *Energy Procedia* **2015**, *79*, 759–765. [\[CrossRef\]](#)
17. Taher, S.A.; Afsari, S.A. Optimal location and sizing of DSTATCOM in distribution systems by immune algorithm. *Int. J. Electr. Power Energy Syst.* **2014**, *60*, 34–44. [\[CrossRef\]](#)
18. Devi, S.; Geethanjali, M. Placement and sizing of D-STATCOM using particle swarm optimization. In *Power Electronics and Renewable Energy Systems*; Springer: New Delhi, India, 2015; pp. 941–951.
19. Sanam, J.; Ganguly, S.; Panda, A.K.; Hemanth, C. Optimization of energy loss cost of distribution networks with the optimal placement and sizing of DSTATCOM using differential evolution algorithm. *Arab. J. Sci. Eng.* **2017**, *42*, 2851–2865. [\[CrossRef\]](#)
20. Sedighzadeh, M.; Eisapour-Moarref, A. The imperialist competitive algorithm for optimal multi-objective location and sizing of DSTATCOM in distribution systems considering loads uncertainty. *INAE Lett.* **2017**, *2*, 83–95. [\[CrossRef\]](#)
21. Murty, V.V.; Kumar, A. Impact of D-STATCOM in distribution systems with load growth on stability margin enhancement and energy savings using PSO and GAMS. *Int. Trans. Electr. Energy Syst.* **2018**, *28*, e2624. [\[CrossRef\]](#)
22. Arya, A.K.; Kumar, A.; Chanana, S. Analysis of distribution system with D-STATCOM by gravitational search algorithm (GSA). *J. Inst. Eng. (India) Ser. B* **2019**, *100*, 207–215. [\[CrossRef\]](#)
23. Yuvaraj, T.; Ravi, K.; Devabalaji, K. DSTATCOM allocation in distribution networks considering load variations using bat algorithm. *Ain Shams Eng. J.* **2017**, *8*, 391–403. [\[CrossRef\]](#)
24. Okelola, M.O.; Salimon, S.A.; Adegbola, O.A.; Ogunwole, E.I.; Ayanlade, S.O.; Aderemi, B.A. Optimal Siting and Sizing of D-STATCOM in Distribution System using New Voltage Stability Index and Bat Algorithm. In Proceedings of the 2021 International Congress of Advanced Technology and Engineering (ICOTEN), Taiz, Yemen, 4–5 July 2021; pp. 1–5.
25. Rukmani, D.K.; Thangaraj, Y.; Subramaniam, U.; Ramachandran, S.; Madurai Elavarasan, R.; Das, N.; Baringo, L.; Imran Abdul Rasheed, M. A new approach to optimal location and sizing of DSTATCOM in radial distribution networks using bio-inspired cuckoo search algorithm. *Energies* **2020**, *13*, 4615. [\[CrossRef\]](#)
26. Mekhamer, S.F.; Shehata, R.H.; Abdelaziz, A.Y.; Al-Gabalawy, M.A. Enhancing radial distribution system performance by optimal placement of DSTATCOM. *Int. J. Electr. Comput. Eng.* **2020**, *10*, 2850. [\[CrossRef\]](#)
27. Rani, K.R.; Rani, P.S.; Chaitanya, N.; Janamala, V. Improved Bald Eagle Search for Optimal Allocation of D-STATCOM in Modern Electrical Distribution Networks with Emerging Loads. *J. Intell. Eng. Syst.* **2021**, *15*, 554–563.
28. Khan, B.; Redae, K.; Gidey, E.; Mahela, O.P.; Taha, I.B.; Hussien, M.G. Optimal integration of DSTATCOM using improved bacterial search algorithm for distribution network optimization. *Alex. Eng. J.* **2022**, *61*, 5539–5555. [\[CrossRef\]](#)
29. Salimon, S.A.; Lawal, Q.O.; Adebiyi, O.W.; Okelola, M.O. Cost-Benefit of Optimal Allocation of DSTATCOM in Distribution Networks Using Ant-Lion Optimization Algorithm. *Period. Polytech. Electr. Eng. Comput. Sci.* **2022**, *66*, 350–360. [\[CrossRef\]](#)
30. Zellagui, M.; Lasmari, A.; Settoul, S.; El-Sehiemy, R.A.; El-Bayeh, C.Z.; Chenni, R. Simultaneous allocation of photovoltaic DG and DSTATCOM for techno-economic and environmental benefits in electrical distribution systems at different loading conditions using novel hybrid optimization algorithms. *Int. Trans. Electr. Energy Syst.* **2021**, *31*, e12992. [\[CrossRef\]](#)
31. Thangaraj, Y.; Kuppan, R. Multi-objective simultaneous placement of DG and DSTATCOM using novel lightning search algorithm. *J. Appl. Res. Technol.* **2017**, *15*, 477–491. [\[CrossRef\]](#)
32. Dash, S.K.; Mishra, S.; Abdelaziz, A.Y.; Hong, J.; Geem, Z.W. Optimal planning of multitype DGs and D-STATCOMs in power distribution network using an efficient parameter free metaheuristic algorithm. *Energies* **2022**, *15*, 3433. [\[CrossRef\]](#)
33. Pratap, A.; Tiwari, P.; Maurya, R.; Singh, B. Minimisation of electric vehicle charging stations impact on radial distribution networks by optimal allocation of DSTATCOM and DG using African vulture optimisation algorithm. *Int. J. Ambient. Energy* **2022**, *43*, 8653–8672. [\[CrossRef\]](#)
34. Dash, S.K.; Mishra, S.; Rout, U.; Abdelaziz, A.Y. A New Analytical Approach to Optimally Allocate D-STATCOMs in Electric Distribution Networks. In Proceedings of the 2022 IEEE 2nd International Symposium on Sustainable Energy, Signal Processing and Cyber Security (iSSSC), Gunupur, Odisha, India, 15–17 December 2022; pp. 1–5.
35. Gil-González, W. Optimal Placement and Sizing of D-STATCOMs in Electrical Distribution Networks Using a Stochastic Mixed-Integer Convex Model. *Electronics* **2023**, *12*, 1565. [\[CrossRef\]](#)
36. Raj, A.F.; Saravanan, A.G. An optimization approach for optimal location & size of DSTATCOM and DG. *Appl. Energy* **2023**, *336*, 120797.
37. Santamaria-Henao, N.; Montoya, O.D.; Trujillo-Rodríguez, C.L. Optimal Siting and Sizing of FACTS in Distribution Networks Using the Black Widow Algorithm. *Algorithms* **2023**, *16*, 225. [\[CrossRef\]](#)
38. Dulebenets, M.A. An Adaptive Polyploid Memetic Algorithm for scheduling trucks at a cross-docking terminal. *Inf. Sci.* **2021**, *565*, 390–421. [\[CrossRef\]](#)

39. Pasha, J.; Nwodu, A.L.; Fathollahi-Fard, A.M.; Tian, G.; Li, Z.; Wang, H.; Dulebenets, M.A. Exact and metaheuristic algorithms for the vehicle routing problem with a factory-in-a-box in multi-objective settings. *Adv. Eng. Inform.* **2022**, *52*, 101623. [[CrossRef](#)]
40. Singh, P.; Pasha, J.; Moses, R.; Sobanjo, J.; Ozguven, E.E.; Dulebenets, M.A. Development of exact and heuristic optimization methods for safety improvement projects at level crossings under conflicting objectives. *Reliab. Eng. Syst. Saf.* **2022**, *220*, 108296. [[CrossRef](#)]
41. Chen, M.; Tan, Y. SF-FWA: A Self-Adaptive Fast Fireworks Algorithm for effective large-scale optimization. *Swarm Evol. Comput.* **2023**, *80*, 101314. [[CrossRef](#)]
42. Singh, E.; Pillay, N. A study of ant-based pheromone spaces for generation constructive hyper-heuristics. *Swarm Evol. Comput.* **2022**, *72*, 101095. [[CrossRef](#)]
43. Hayyolalam, V.; Kazem, A.A. Black widow optimization algorithm: A novel meta-heuristic approach for solving engineering optimization problems. *Eng. Appl. Artif. Intell.* **2020**, *87*, 103249. [[CrossRef](#)]
44. Das, J.C. *Power System Analysis: Short-Circuit Load Flow and Harmonics*; CRC Press: Boca Raton, FL, USA, 2002.
45. Teng, J.H. A direct approach for distribution system load flow solutions. *IEEE Trans. Power Deliv.* **2003**, *18*, 882–887. [[CrossRef](#)]
46. Ali, E.; Elazim, S.A.; Abdelaziz, A. Improved harmony algorithm and power loss index for optimal locations and sizing of capacitors in radial distribution systems. *Int. J. Electr. Power Energy Syst.* **2016**, *80*, 252–263. [[CrossRef](#)]
47. Ogunjuyigbe, A.; Ayodele, T.; Akinola, O. Impact of distributed generators on the power loss and voltage profile of sub-transmission network. *J. Electr. Syst. Inf. Technol.* **2016**, *3*, 94–107. [[CrossRef](#)]
48. Taher, S.A.; Karimi, M.H. Optimal reconfiguration and DG allocation in balanced and unbalanced distribution systems. *Ain Shams Eng. J.* **2014**, *5*, 735–749. [[CrossRef](#)]

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