sustainability

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The Impact of Online Anti-Counterfeiting on Channel Structure and Pricing Decisions

Weiting Wang [1], Yi Liao [1] and Wenjing Shen [2,*]

[1] School of Business Administration, Southwestern University of Finance and Economics, Chengdu 610000, China; weitingswufe@foxmail.com (W.W.)
[2] Decision Sciences Department, LeBow College of Business, Drexel University, Philadelphia, PA 19104, USA
* Correspondence: ws84@drexel.edu

Abstract: Counterfeiting is an important challenge in maintaining the security and sustainability of supply chains. This paper examines a supply chain consisting of a luxury goods manufacturer (and a retailer) in the presence of counterfeit goods. Inspired by the reality that both manufacturers and retailers have incentives to implement anti-counterfeiting, this paper combines the psychological impact of anti-counterfeiting efforts on consumers and discusses the impact of anti-counterfeiting efforts on pricing and profits. We find that: (1) anti-counterfeiting has a positive impact on the selling price of brand products and the firms' profits. However, the impact on wholesale prices varies depending on who implements the anti-counterfeiting strategy. (2) Only when the quality of brand products is higher than the threshold, is the firm willing to input anti-counterfeiting efforts. Manufacturers in a reselling structure are more motivated to fight counterfeits. (3) Implementing anti-counterfeiting in a direct selling structure is the most effective strategy for manufacturers. Under a reselling structure, it is more beneficial for manufacturers to have the retailer input anti-counterfeiting efforts. Our study provides insights into the reasons why some manufacturers establish internal anti-counterfeiting teams under the direct selling structure, while others incentivize retailers to invest in anti-counterfeiting.

Keywords: game theory; online anti-counterfeit; supply chain; consumer behavior

Citation: Wang, W.; Liao, Y.; Shen, W. The Impact of Online Anti-Counterfeiting on Channel Structure and Pricing Decisions. *Sustainability* **2023**, *15*, 8253. https://doi.org/10.3390/su15108253

Academic Editor: Shaojian Qu

Received: 23 April 2023
Revised: 14 May 2023
Accepted: 15 May 2023
Published: 18 May 2023

Correction Statement: This article has been republished with a minor change. The change does not affect the scientific content of the article and further details are available within the backmatter of the website version of this article.

1. Introduction

Counterfeiting represents a significant and growing challenge to supply chain sustainability, with negative impacts on both business and the environment. On the one hand, counterfeit products can erode trust in the brand supply chain and reduce demand for authentic products, ultimately harming the economic sustainability of the supply chain [1]. On the other hand, counterfeit goods are often manufactured with cheaper, lower-quality materials. This can result in increased waste and pollution, which can harm the long-term sustainability of the supply chain [2]. In 2019, counterfeit goods accounted for approximately 3.3% of world trade and more than $500 billion in value per year [3]. Counterfeiting disproportionately affects luxury markets, such as LV and Chanel, particularly in the online channel. Over 50% of counterfeit luxury goods detected at the EU border originate from e-commerce, and more than 75% of these goods are from China [4].

To mitigate the adverse effects of counterfeiting on supply chain sustainability, luxury companies rely on their in-house anti-counterfeiting teams. For example, Richemont employs approximately 30 full-time staff dedicated to combating counterfeiting, while LVMH has twice that number. Versace allocates 2% of its annual operating expenses (€140 million) to its internal anti-counterfeiting team. Although some retail platforms have established procedures to assist manufacturers in removing counterfeit goods, such as Alibaba's "Integrity Mechanism Program" or Amazon's "Project Zero", they are also directly taking up the fight against counterfeiting. For example, since 2010, Alibaba has employed more than 2000 full-time staff and an additional 2700 volunteers to combat counterfeiting. Recently, through big data analysis and modeling, Alibaba has been able to detect and address counterfeit-related behavior for luxury brands. Similarly, Amazon uses machine learning to automatically scan its stores for suspicious items [5].

Many scholars have focused on the impact of fakes on supply chains and how companies design anti-counterfeiting strategies, especially in terms of pricing [6,7], quality [8,9], anti-counterfeiting investment [10,11], channel structure [12,13], blockchain applications [14–16], etc. However, few scholars have discussed which companies, retailers or manufacturers are better suited to invest in anti-counterfeiting efforts based on the framework of game theory. In fact, Olsen and Granzin [17] have demonstrated that anti-fraud efforts are more effective through dealer/retailer networks and suggest that manufacturers should incentivize retailers to work together to combat counterfeiting. Our study complements the literature on anti-counterfeiting strategies. Notably, this study focuses on anti-counterfeiting activities in the luxury industry, which are rarely mentioned in the existing anti-counterfeiting literature based on game theory. Luxury goods are the main area of counterfeiting, but the counterfeiting problem is different from that of general products (such as car parts, sports apparel). Due to the particularity of luxury goods, anti-counterfeiting in the luxury market will have two important impacts on consumers. On the one hand, consumers who purchase luxury goods are willing to pay a higher price and place a greater emphasis on the uniqueness of authentic goods [18]. Therefore, anti-counterfeiting measures can enhance the exclusivity of luxury brands and increase consumers' willingness to pay [19,20]. On the other hand, consumers who buy counterfeits have a lower willingness to pay. Generally, consumers perceive the purchase of fake goods as immoral, and the moral aversion generated by anti-counterfeiting measures can discourage consumers from buying counterfeit goods [21]. However, few scholars have introduced the uniqueness and morality of luxury consumption into anti-counterfeiting models. Therefore, considering the lack of existing literature focusing on luxury goods and studying the choice of anti-counterfeiting enterprises, we seek to address the following research questions: (1) what are the thresholds for firms to carry out the fight against counterfeiting? (2) What is the impact of anti-counterfeiting efforts on the supply chain compared to the absence of efforts? (3) What channel structure and anti-counterfeiting strategies should manufacturers select?

To study these questions, we develop a single-period game to examine how manufacturers choose channel structures and anti-counterfeiting strategies (including anti-counterfeiting firms, selling prices, wholesale prices and anti-counterfeit efforts). The manufacturer has two channel options-direct selling and reselling. In the direct-sales scenario, the manufacturer combats counterfeit themselves, which we denote as CM. In the reselling scenario, the manufacturer can allow the retailer to input anti-counterfeiting efforts, which we represent as DM or DR, depending on whether the manufacturer or the retailer performs the anti-counterfeiting activities.

Our study provides several key insights. First, firms are willing to engage in anti-counterfeiting activities only when the product quality is above a certain threshold. The tolerance threshold for the CM scenario is the same as that of DR but higher than that of DM. Second, under wholesale price contracts, manufacturers seeking to maximize profits tend to choose direct selling over reselling. In the reselling scenario, it is more advantageous for manufacturers to implement anti-counterfeiting activities through the retailer. Third, if the manufacturer changes the wholesale price contract to a revenue-sharing or cost-sharing contract, anti-counterfeiting by the retailer is not always the best strategy. Finally, the two consumer effects of anti-counterfeiting, the utility premium for buying authentic products and the utility loss for buying counterfeit products, have facilitation effects on the selling price of genuine products, anti-counterfeit effort, manufacturer's profit and retailer's profit. The magnitude of the facilitation effect depends on the unit anti-counterfeit cost.

The remainder of this paper proceeds as follows. Section 2 briefly reviews the relevant literature. Section 3 describes the basic assumptions and models. We present equilibrium solutions in Section 4 and compare them in Section 5. Section 6 contains a discussion of several extensions. Section 7 concludes the paper by discussing management implications and future research directions.

2. Literature Review

Based on the influence of anti-counterfeiting activities on luxury consumer psychology, this study studies which channel structure and which firms are more suitable for anti-counterfeiting activities, as well as pricing. We conduct a review of the existing literature on supply chain anti-counterfeiting strategies and luxury consumer psychology.

2.1. Anti-Counterfeiting in the Supply Chain

Fake products have emerged as competitors to genuine products in the market, leading to the need for research on how brand companies can crack down on counterfeit products. Firstly, under the framework of game theory, most scholars discuss anti-counterfeiting strategies from pricing and quality determination. Cho et al. [6] have proposed different counterfeit strategies based on the type of counterfeit products. They suggest improving quality and lowering prices to combat non-deceptive counterfeit products, while lowering quality and raising prices can be an effective strategy against deceptive counterfeit products. Qian et al. [8] have divided product quality into searchable quality (e.g., appearance) and experiential quality (e.g., function). They recommend investing in searchable quality when there are many counterfeit products, and investing in experiential quality when there are fewer counterfeit products. Zhang et al. [22] found that manufacturers in monopolistic environments tend to improve product quality rather than reduce counterfeit products directly, while small brand companies in competitive environments are more inclined to take a direct attack approach. Cui [9] has studied the effect of OEM investment strategies in quality improvement on preventing imitation and encroachment by contract manufacturers. Gao et al. [7] found that non-deceptive counterfeit products can reduce the price of branded products, benefiting consumers and improving their welfare.

Secondly, the threat of counterfeiting is a major concern for enterprises, and several studies have explored the ways in which businesses can deal with this problem from the perspective of channel structure. Qian [23] conducted an empirical study using macro-level panel data and found that vertical integration and other strategies can effectively reduce counterfeiting. Zhang and Zhang [12] investigated the optimal supply chain structure in the presence of counterfeit products and concluded that when reselling channels are infiltrated by counterfeit products, branded businesses may need to restructure their product reselling to rely on reliable channels such as certified manufacturer-owned stores to ensure 100% authenticity. Ghamat et al. [24] found that it is not always optimal for manufacturers to vertically integrate their supply chains, even when the investment in vertical integration is zero. Manufacturers may prefer to outsource production of their products rather than sign intellectual property agreements. Bian et al. [13] analyzed the strategic implications of vertical integration for nondeceptive counterfeiting and found that, for non-deceptive products, branded companies could benefit from consolidation, but that it might also benefit counterfeiters. However, vertical integration always improves consumer and social welfare.

A limited body of literature explores the impact of counterfeiting on sustainability with respect to social welfare, economics, intellectual property and other related fields. Tsai and Chiou [25] suggested that strict government enforcement against counterfeiting may have varying effects on benefits, which could either increase or decrease. Furthermore, the benefits of strict enforcement are not necessarily superior to those of non-enforcement when fraudulent activities occur. Yao [26] utilized China as an example to examine the economic ramifications of counterfeiting within the context of market equilibrium and failure. They believe that counterfeiting as a market behavior will continue as long as the satisfaction of consumption and the exorbitant profits of production endure. Based on the intellectual property theoretical model, imitation poses a threat to innovators as it deprives them of temporary monopoly gains, ultimately resulting in reduced incentives for innovation [27,28]. Fear of imitation can also dissuade companies from investing in new technologies, partnerships and innovation, ultimately impeding their growth and competitiveness [29,30]. However, Chen et al. [31] explored the impact of infringement and counterfeiting on firms' innovation performance. They found that firms operating in industries with a high risk of IP infringement or copycat behavior by competitors significantly increased the number of patents and citations, particularly financially robust and risk-taking firms.

2.2. Consumer Psychology in Luxury

Since this paper introduces into the utility function the perception of luxury uniqueness of consumers who buy genuine goods and the moral aversion of consumers who buy fake goods, we review the two demand influencing factors of emotion and morality.

The impact of emotions on luxury consumption has been a topic of great interest. Luxury goods are primarily associated with satisfying psychological and social needs, such as uniqueness, self-expression and social signaling [32]. Zhan and He [33] examined three psychological traits that make Chinese consumers unique compared to their global peers: value consciousness (VC), susceptibility to normative influence (SNI) and the need for uniqueness (NFU). Srisomthavil and Assarut [34] found that a proliferation of counterfeit luxury brands cannot be viewed in the same way as authentic luxury brand proliferation, which tends to have negative impacts on other brand values apart from uniqueness value. Kastanakis and Balabanis's study [35] showed that the self-concept of consumers' interdependence is the basis of bandwagon luxury consumption. This relationship is mediated by consumers' tendency to seek status, sensitivity to normative influences, and need for uniqueness.

As an important factor affecting the purchase of counterfeit products, moral issue has been considered by many scholars. Consumers tend to believe that buying counterfeit products is unethical [36,37]. de Lucio and Valero [38] analyzed the impact of collective and individual moral judgments on access to counterfeit goods. The results showed that, for both acquaintances and consumers, more severe moral judgment reduced purchase intention and actual purchase when buying fake and shoddy goods. Martinez and Jaeger [39] explored the impact of moral feelings, moral awareness and moral judgment on the consumption of counterfeit goods. Wilcox et al. [40] found that only when the consumers' attitude towards luxury brands has the function of value expression will the consumers' moral beliefs on counterfeit consumption affect their preference for counterfeit brands. Elsantil and Bedair [41] investigated counterfeiting in the Arab world and found that consumers' unethical beliefs and perceived risk had a negative impact on their willingness to buy counterfeit goods, while identity consumption had a positive impact on their willingness to buy counterfeit goods.

2.3. Literature Gaps

In the first literature stream, anti-counterfeiting in the supply chain, these literatures discussed in Section 2.1 primarily investigate how companies make decisions regarding price, quality and reselling channels to tackle counterfeit competition. However, little attention has been paid to identifying which enterprise is better suited to implement anti-counterfeiting measures in a decentralized supply chain, where there is only one manufacturer and one retailer involved. In reality, both manufacturers and retailers can take part in anti-counterfeiting efforts, but their strategies may differ. Thus, there is a gap in research to explain why some manufacturers choose to establish their own anti-counterfeiting teams while others partner with retailers. Furthermore, this study offers an innovative contribution by examining the impact of anti-counterfeiting efforts on consumers' utility in these activities. Additionally, there is a dearth of studies on anti-counterfeiting efforts in the luxury industry, and this study incorporates the unique characteristics of luxury anti-counterfeiting measures into the model.

In the second literature stream—consumer psychology in luxury, although studies have shown that the uniqueness of luxury goods is a factor in consumers' preference for authentic goods, while buying counterfeit goods may create ethical issues, these studies have mainly focused on psychology and marketing. Few studies have explored these issues in the context of enterprise operation and supply chain management. This study aims to examine the relationship between market demand and corporate profits by investigating the impact of anti-counterfeiting efforts on pricing and profits, while considering the intensity of uniqueness and moral aversion from a qualitative perspective.

In contrast to the existing literature, this paper makes significant contributions to the field of supply chain anti-counterfeiting in the following ways. Firstly, while previous research has overlooked the impact of anti-counterfeiting efforts on consumers' willingness to pay, this paper incorporates uniqueness perception and moral aversion to investigate the effects of anti-counterfeiting on pricing. This is an important feature that cannot be ignored in luxury consumption. Secondly, whereas most scholars have focused on offline anti-counterfeiting that involves the direct removal of counterfeit products, this article assumes that anti-counterfeiting will not result in a reduction in the fakes' market share. Since it is difficult for firms to seize fake inventory, counterfeiters can rapidly establish new online stores to sell fake products. This assumption better aligns with the reality of online counterfeiting but has received limited attention in previous research. Thirdly, we focus on the influence of channel structure and who the anti-counterfeiting firm on supply chain's decisions is, while most studies usually only assume that only one firm in the supply chain implements anti-counterfeiting activities.

3. The Model

Our study aims to investigate the optimal channel structure and anti-counterfeiting strategy that luxury goods manufacturers can adopt to combat counterfeit products in the online marketplace. To accomplish this, we employ a one-period game to address the research questions. In the direct selling scenario, only the manufacturer is capable of undertaking anti-counterfeiting activities, whereas both the manufacturer and the retailer can engage in such activities in the reselling scenario. We use LVMH as an example to illustrate two possible anti-counterfeiting strategies: the establishment of an internal anti-counterfeiting team to identify fakes or partnering with Alibaba to combat counterfeits. We designate direct selling, anti-counterfeiting by the manufacturer, and anti-counterfeiting by the retailer in the reselling channel as CM, DM and DR, respectively (See Figure 1). Normalizing the luxury market size to 1, each consumer may purchase one genuine product, one counterfeit product, or none. By examining how luxury manufacturers can combat counterfeits in the online marketplace by adopting different channel structures and anti-counterfeiting firms, this study contributes significantly to the literature.

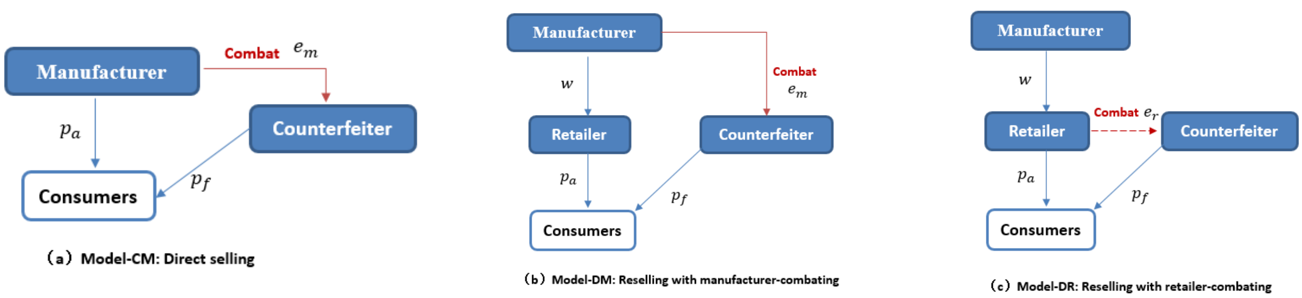


Figure 1. Description of anti-counterfeiting scenarios.

Product: This study aims to investigate the channel structure and anti-counterfeiting strategy adopted by a luxury goods manufacturer to combat fake products in the online marketplace. Prior research on this topic has been conducted by [6,20,22,42]. In this study, we assume that the true quality of the genuine product is q_a, while the quality of the counterfeit is always lower than q_a and can be described as $q_f = \beta q_a \triangleq \beta q$, where $\beta \in (0,\ 1)$ is an exogenous parameter that represents the quality similarity between the fake and genuine products. The retail prices of the genuine and counterfeit products are denoted by p_a and p_f, respectively.

Consumer utility: We assume that consumers can observe real anti-counterfeiting efforts. Due to the particularity of luxury products, online anti-counterfeiting activities have two effects on consumers' willingness to pay. First of all, consumers who buy authentic goods obtain extra utility from anti-counterfeiting activities because they pay more attention to the uniqueness of luxury goods [18]. The greater the anti-counterfeiting efforts, the fewer counterfeit products in the market, thus increasing the level of uniqueness of luxury goods [18]. Secondly, consumers who buy fake goods lose some utility from anti-counterfeiting activities. As is known to all, consumers tend to perceive luxury counterfeiting as immoral, described as consumers develop moral disgust for counterfeits [43]. Online anti-counterfeiting activities reinforce consumers' moral aversion to fakes [44] and further reduce consumers' willingness to pay for counterfeit products [43,45]. At present, these two effects have not been introduced into the anti-counterfeiting game model, so by referring to some models about consumers' green preference [46,47], we use $k_1 e_i$ and $k_2 e_i$ to represent the unique perception and moral aversion associated with anti-counterfeiting efforts, respectively. $k_1 \in (0,\ 1)$ represents the perception coefficient of a genuine product's uniqueness and $k_2 \in (0,\ 1)$ represents the moral disgust coefficient. e_i ($i = m$ or r) represents anti-counterfeiting carried out by manufacturer or retailer, respectively [13].

Adopting the customer utility model [48], we consider that a customer's willingness to pay for unit product quality θ is heterogeneous and uniformly distributed between 0 and 1. When fakes exist in the luxury market, the consumer has three possible utilities: (1) he gets utility $u_a = \theta q - p_a + k_1 e_i$ when buying the genuine product; (2) he gets utility $u_f = \theta\beta q - p_f - k_2 e_i$ when purchasing luxury fakes; (3) the utility is 0 when he buys nothing. According to the customer utility model [48], to avoid unrealistic situations, this study only focused on the coexistence of genuine and fake goods in the luxury market, in which $q > \frac{p_a - p_f - (k_1 + k_2) e_i}{1 - \beta}$. Therefore, the demand functions of consumers who buy genuine products and those who buy counterfeits are respectively,

$$D_a = 1 - \frac{p_a - p_f - (k_1 + k_2)e_i}{q(1-\beta)}, D_f = \frac{p_a - p_f - (k_1 + k_2)e_i}{q(1-\beta)} - \frac{p_f + k_2 e_i}{q\beta}$$

Cost: We assume that the anti-counterfeit efforts can be monetarily characterized. Previous literature by Krishnan, Kapuscinski and Butz [49] advocates the use of a quadratic cost function, denoted as $C(e_i) = \frac{1}{2} c {e_i}^2$, to model the anti-counterfeit cost of a firm, where c represents the unit anti-counterfeit cost. The costs incurred by firms in the fight against counterfeits may include employee salaries and operational expenses related to data systems that detect fake products. We assume that the production costs (fixed or marginal costs) for the authentic product and the counterfeit are equal, and both are normalized to zero [14,15,20]. Table 1 provides an overview of the main notations employed in this paper.

Table 1. Notations.

Parameters	Description
c	Unit anti-counterfeit cost
q	The quality of genuine products
β	The quality similarity between fake and genuine goods
k_1	The sensitivity coefficient of consumers to the uniqueness of genuine products
k_2	The sensitivity coefficient of consumers to the moral disgust from counterfeits
Variables	**Description**
e_m(or e_r)	Anti-counterfeit efforts put in by manufacturer (or the retailer)
p_a(or p_f)	The selling price of genuine (or counterfeit) products
w	The wholesale price

Sequence: In practice, luxury manufacturers play a Stackelberg leader in the supply chain, directly influencing retail and counterfeiting decisions [8,42]. Specifically, in the reselling structure, the retailer decides its sale price based on the wholesale price, while the counterfeiter must observe the sale price of the genuine product before determining the price of the fake product, because the price of the fake product is unlikely to be higher than the real one. Following some relative research [12,15,20], the game sequence of our study is shown in Figure 2.

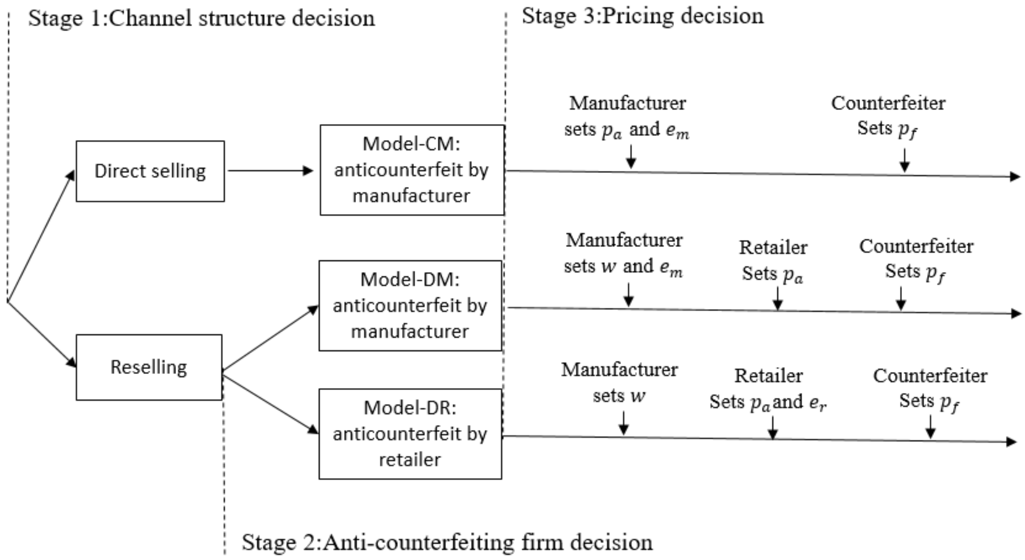


Figure 2. Sequence of Events.

Stage 1. Channel structure decision: the manufacturer decides whether to adopt direct selling or reselling channels.

Stage 2. Anti-counterfeiting firm decision: in the reselling structure, the manufacturer decides whether to implement anti-counterfeiting activities by itself or by the retailer.

Stage 3. Pricing decision: (1) in the direct selling structure, the manufacturer decides the price of the genuine product and the anti-counterfeiting efforts, and then the counterfeiter decides the price of the fake product. (2) In the reselling structure, when the manufacturer is the anti-counterfeiting firm, he first decides the wholesale price and anti-counterfeiting efforts, then the retailer decides the sale price of the genuine product, and finally the counterfeiter decides the price of the fake product. (3) In the reselling structure, when the retailer is the anti-counterfeiting firm, the manufacturer first decides the wholesale price, then the retailer decides the sale price of the genuine product and the anti-counterfeiting efforts, and finally the counterfeiter decides the price of the fake product.

4. Equilibrium Analysis

For the equilibrium analysis, we use the subgame perfect equilibrium concept. The game is solved by backward induction. Proofs of the paper are provided in the Appendix A. Superscript * implies an equilibrium solution throughout the paper.

4.1. Base Model: Without Anti-Counterfeiting

To compare the impact of anti-counterfeiting on supply chains, we first discuss two different supply chain structures without anti-counterfeiting: a reselling structure represented by D and a direct selling structure represented by C. In the direct selling setting, the manufacturer determines the optimal price p_a^C and then the counterfeiter determines the counterfeit price p_f^C. In the reselling setting, the manufacturer determines the wholesale price w^D, the retailer then determines the retail price of the genuine product p_a^D, and finally, the counterfeiter decides the price of counterfeits p_f^D. It is easy to find the equilibrium solutions without anti-counterfeit: under direct selling scenario, $p_a^{C*} = \frac{q(1-\beta)}{2-\beta}$, $p_f^{C*} = \frac{q(1-\beta)\beta}{2(2-\beta)}$ and $\pi_m^{C*} = \frac{q(1-\beta)}{2(2-\beta)}$; under reselling scenario, $w^{D*} = \frac{q(1-\beta)}{2-\beta}$, $p_a^{D*} = \frac{3q(1-\beta)}{2(2-\beta)}$, $p_f^{D*} = \frac{3q(1-\beta)^2}{4(\beta^2-3\beta+2)}$, $\pi_m^{D*} = \frac{q(1-\beta)}{4(2-\beta)}$ and $\pi_r^{D*} = \frac{q(1-\beta)}{8(2-\beta)}$.

4.2. Model CM: Anti-Counterfeiting by Manufacturer in a Direct Selling Structure

In this scenario, the anti-counterfeit firm is the manufacturer. The manufacturer decides the price of the genuine product and the anti-counterfeiting effort, and then the counterfeiter decides the fake price. They follow the following optimization questions to maximize profits.

$$\max_{(p_a, e_m)} \pi_m = D_a p_a - \frac{1}{2} c {e_m}^2$$

$$\max_{p_f} \pi_f = D_f p_f$$

In a given scenario, the anti-counterfeit firm is willing to carry out anti-counterfeiting activities only when the profit with anti-counterfeiting is greater than that without anti-counterfeiting. Let $\hat{q}^{CM} = \frac{(2k_1+k_2-\beta k_1)^2}{4c(\beta-1)(\beta-2)}$. When $q \geq \hat{q}^{CM}$, the manufacturer is willing to fight counterfeit products and there exists a unique subgame perfect equilibrium solution. By substituting the demand function, we obtain the equilibrium solution of this scenario as follows:

$$e_m^{CM*} = \frac{2q(1-\beta)(2k_1+k_2-\beta k_1)}{4cq(\beta^2-3\beta+2) - (2k_1+k_2-\beta k_1)^2}, p_a^{CM*} = \frac{4q^2 c(\beta-1)^2}{4cq(\beta^2-3\beta+2) - (2k_1+k_2-\beta k_1)^2},$$

$$p_f^{CM*} = \frac{q(1-\beta)\left[2k_1k_2 + {k_2}^2 + \beta {k_1}^2(2-\beta) + 2\beta qc(\beta-1)\right]}{(2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)},\ \pi_m^{CM*} = \frac{2q^2 c(\beta-1)^2}{4cq(\beta^2-3\beta+2) - (2k_1+k_2-\beta k_1)^2}$$

4.3. Model DM: Anti-Counterfeiting by Manufacturer in a Reselling Structure

In this scenario, the anti-counterfeit firm is the manufacturer. First, the manufacturer chooses the optimal w under the wholesale price contract and e_m. The retailer decides the retail price p_a. At last, the counterfeiter observes p_a and decides on the counterfeit selling price p_f. The manufacturer, retailer and counterfeiter follow the following optimization questions to maximize profits.

$$\max_{(w, e_m)} \pi_m = D_a w - \tfrac{1}{2} c {e_m}^2$$
$$\max_{p_a} \pi_r = D_a (p_a - w)$$
$$\max_{p_f} \pi_f = D_f p_f$$

Let $\hat{q}^{DM} = \frac{(2k_1+k_2-\beta k_1)^2}{8c(\beta-1)(\beta-2)}$. The condition for the manufacturer to be willing to participate in anti-counterfeit is $q \geq \hat{q}^{DM}$. By substituting the demand function, we obtain the equilibrium solution of this scenario as follows:

$$w^{DM*} = \frac{-8q^2 c(\beta-1)^2}{(2k_1+k_2-\beta k_1)^2 - 8cq(\beta^2-3\beta+2)},\ e_m^{DM*} = \frac{2q(\beta-1)(2k_1+k_2-\beta k_1)}{(2k_1+k_2-\beta k_1)^2 - 8cq(\beta^2-3\beta+2)},$$
$$p_f^{DM*} = \frac{q(1-\beta)\left[2k_1k_2 + {k_2}^2 + \beta {k_1}^2(2-\beta) + 6\beta qc(\beta-1)\right]}{(2k_1+k_2-\beta k_1)^2 - 8cq(\beta^2-3\beta+2)},\ p_a^{DM*} = \frac{12q^2 c(\beta-1)^2}{8cq(\beta^2-3\beta+2) - (2k_1+k_2-\beta k_1)^2},$$
$$\pi_m^{DM*} = \frac{-2q^2 c(\beta-1)^2}{(2k_1+k_2-\beta k_1)^2 - 8cq(\beta^2-3\beta+2)},\ \pi_r^{DM*} = \frac{8q^2 c^2(\beta-1)^3 q(\beta-2)}{\left((2k_1+k_2-\beta k_1)^2 - 8cq(\beta^2-3\beta+2)\right)^2}.$$

4.4. Model DR: Anti-Counterfeiting by Retailer in a Reselling Structure

In this scenario, the anti-counterfeit firm is the retailer. First, the manufacturer decides the optimal wholesale price w, and the retailer decides the retail price p_a and the anti-counterfeit effort e_r. Given the price of the genuine product, the counterfeiter decides the price of the fake product. The manufacturer, retailer and counterfeiter follow the following optimization questions to maximize profits.

$$\max_{w} \pi_m = D_a w$$
$$\max_{(p_a, e_r)} \pi_r = D_a(p_a - w) - \tfrac{1}{2} c {e_r}^2$$
$$\max_{p_f} \pi_f = D_f p_f$$

Let $\hat{q}^{DR} = \frac{(2k_1+k_2-\beta k_1)^2}{4c(\beta-1)(\beta-2)}$. The condition for the retailer to be willing to participate in anti-counterfeit is $q \geq \hat{q}^{DR}$. By substituting the demand function, we obtain the equilibrium solution of this scenario as follows:

$$w^{DR*} = \frac{q(\beta-1)}{\beta-2},\ p_f^{DR*} = \frac{q(\beta-1)\left[\beta {k_1}^2(\beta-2)^2 - 3\beta c q\left(\beta^2-3\beta+2\right) + 2k_1k_2 + {k_2}^2 - \beta k_1 k_2(\beta-1)\right]}{(\beta-2)\left((2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)\right)},$$
$$e_r^{DR*} = \frac{q(\beta-1)(2k_1+k_2-\beta k_1)}{(2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)},\ p_a^{DR*} = \frac{q(\beta-1)\left((2k_1+k_2-\beta k_1)^2 - 6cq\left(\beta^2-3\beta+2\right)\right)}{(\beta-2)\left((2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)\right)},$$
$$\pi_m^{DR*} = \frac{-cq^2(\beta-1)^2}{(2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)},\ \pi_r^{DR*} = \frac{-cq^2(\beta-1)^2}{2\left((2k_1+k_2-\beta k_1)^2 - 4cq(\beta^2-3\beta+2)\right)}.$$

Lemma 1. *There are quality thresholds for a firm to implement anti-counterfeiting activities. When the quality of the fake is more similar to that of the genuine product, the quality thresholds are higher, i.e.,* $\frac{\partial \hat{q}^{CM}}{\partial \beta} > 0$, $\frac{\partial \hat{q}^{DM}}{\partial \beta} > 0$ *and* $\frac{\partial \hat{q}^{DR}}{\partial \beta} > 0$.

Lemma 1 shows that the firms will implement anti-counterfeit only if the quality of the genuine product is high enough. This is driven by two facts. First, when $q_a = q$ is relatively low, the difference between q_a and q_f is not large enough for a given β. Because the impact of a small quality gap on the firm is negligible, the firm will not carry out anti-counterfeit. Second, the higher p_a and lower q_a result in a disadvantage in market share for genuine products. The firm may not be able to generate enough revenue to cover the effort cost of the anti-counterfeit. Further, lemma 1 also shows that as counterfeiting technology improves (i.e., β increases), the firm raises the bottom line of their tolerance for counterfeit products. As the quality gap between genuine and counterfeit products narrows, counterfeit products gain greater market share by price advantage. At this point, the firm must carry out anti-counterfeit at a higher quality to allow the threshold to remain unchanged.

Lemma 2. *Compared with scenarios without anti-counterfeiting, anti-counterfeit can increase* p_a, π_m, π_r *and reduces* p_f. *In a reselling structure, anti-counterfeit by the manufacturer raises* w, *while anti-counterfeit by the retailer does not change* w.

According to Lemma 2, regardless of whether the anti-counterfeiting firm is the manufacturer or retailer, it is always advantageous to combat counterfeit products since anti-counterfeit helps expand the market demand for genuine products. Moreover, the counterfeiter must lower prices to attract consumers, and the manufacturer's increased market power enables them to raise the selling price of genuine products to offset the anti-counterfeit cost. This conclusion is in contrast to the findings of Cho et al. [6], who argued that prices should be lowered to combat non-deceptive counterfeits, but anti-counterfeiting efforts provide room to increase selling prices. However, anti-counterfeit can exacerbate double marginalization, leading to higher prices for consumers. In the DM model, the retailer is willing to pay a higher wholesale price to capture greater market demand from anti-counterfeit by the manufacturer and increase the selling price to alleviate the cost burden. To avoid free-riding behavior by retailers, the manufacturer sets a higher wholesale price. Conversely, in the DR model, if the manufacturer increases wholesale prices, retailers will spend less on anti-counterfeit. Therefore, the manufacturer keeps the wholesale price constant while the retailer raises the selling price of genuine products to offset the combating expense.

5. Comparison among CM, DM, DR

To compare the equilibrium solutions of different models, we assume $q \geq \frac{(2k_1+k_2-\beta k_1)^2}{4c(\beta-1)(\beta-2)}$. Before the comparison, we first examine the differences in the firms' enthusiasm for anti-counterfeiting actions in the three combating scenarios.

Lemma 3. *Compared to CM and DR, the manufacturer in a reselling structure is keener to combat counterfeits, i.e.,* $\hat{q}^{DM} < \hat{q}^{DR} = \hat{q}^{CM}$.

Yi et al. [20] found that retailers in DR mode are more enthusiastic about fighting counterfeiting than manufacturers in DM mode (i.e., $\hat{q}^{DR} < \hat{q}^{DM}$), but because the models adopted are different, our conclusions are contrary to theirs. Lemma 3 implies that the manufacturer in the DM model has the lowest tolerance for counterfeits, making it the most actively engaged in online anti-counterfeiting activities. This is because, in comparison to a reselling structure without anti-counterfeit, the manufacturer in the DR model does not decrease the wholesale price to incentivize retailers, resulting in the retailer bearing the entire anti-counterfeit cost. Therefore, when faced with a given market environment (k_1, k_2, c, β), the minimum quality level at which the retailer is willing to implement anti-counterfeit is the same as that of the manufacturer in the CM model. However, in the DM model, the manufacturer shifts a portion of the anti-counterfeit cost to the retailer by increasing the wholesale price. As a result, the lower unit anti-counterfeit cost allows the manufacturer to carry out anti-counterfeit at a lower quality level and still generate positive profit.

Proposition 1. *(1) Comparing the optimal anti-counterfeit efforts, we have* $e_m^{CM*} > e_r^{DR*} > e_m^{DM*}$. *(2) Comparing the optimal wholesale prices, we have* $w^{DM*} > w^{DR*}$. *(3) Comparing the optimal genuine prices, we have that when the unit anti-counterfeit cost is at a high level,* ${p_a}^{CM*} < {p_a}^{DM*} < {p_a}^{DR*}$*; when the unit anti-counterfeit cost is at a low level,* ${p_a}^{CM*} > {p_a}^{DR*} > {p_a}^{DM*}$. *(4) Comparing the optimal counterfeit's selling prices, we have* $p_f^{CM*} < p_f^{DR*} < p_f^{DM*}$.

Proposition 1 demonstrates that although the manufacturer under DM is more passionate about fighting counterfeit goods, it exerts the least amount of effort in anti-counterfeiting compared to the other two scenarios. We elucidate the discrepancy in effort investment by combining the wholesale price and the genuine selling price. In a reselling structure, the retailer is more likely to invest more resources in anti-counterfeit to expand the market demand than the manufacturer. A constant wholesale price and a higher selling price for genuine products also encourage the retailer to increase their anti-counterfeit efforts since anti-counterfeit increases marginal revenue. Conversely, the manufacturer, who does not have to pay wholesale prices, can exert greater effort than the retailer due to the low cost of anti-counterfeit.

Proposition 1 highlights that the comparison results of selling prices among CM, DR and DM rely on the unit anti-counterfeit cost. In a reselling structure, ${p_a}^{DR*}$ is always greater than ${p_a}^{DM*}$ due to the larger market share of genuine products in DR, allowing for an increase in the selling price. However, although the manufacturer invests the most anti-counterfeit effort under CM, the selling price may not be the highest. Higher unit anti-counterfeit costs exacerbate double marginalization, causing selling prices under direct selling to be lower than those under reselling. When the unit anti-counterfeit cost is low, double marginalization's effect on the selling price in the reselling channel is negligible. In such cases, the larger market demand for genuine products under CM benefits the manufacturer in setting an advantageous price. Moreover, more aggressive combat leads to a higher market share of seized counterfeiters. The counterfeiter can only offer a low price to increase their attractiveness in a small market. Hence, the selling price of counterfeit products under direct selling is the most appealing to consumers.

Proposition 2. *(1) Comparing the optimal manufacturer's profits, we have $\pi_m^{CM*} > \pi_m^{DR*} > \pi_m^{DM*}$. (2) Comparing the optimal retailer's profits, we have $\pi_r^{DM*} < \pi_r^{DR*}$.*

First of all, Proposition 2 confirms that direct selling is the optimal channel structure. This conclusion is similar to the research conclusions of Qian [23], Zhang and Zhang [12], Bian et al. [13], which also affirmed that the vertical integration of the supply chain structure is the way for enterprises to deal with the problem of counterfeiting. Secondly, Proposition 2 also addresses a crucial question regarding anti-counterfeit strategies in a reselling structure, identifying the most suitable firm to carry out such activities. Specifically, the retailer is deemed to be the optimal choice for conducting anti-counterfeit efforts in this setting, as indicated in Proposition 2 (1), which reveals that both the manufacturer and the retailer enjoy higher profits under the DR scenario compared to the DM scenario. Additionally, Lemma 2 highlights that, while the manufacturer under DM can increase wholesale prices and shift some of the anti-counterfeit costs to the retailer, the manufacturer under DR can reap the benefits of anti-counterfeit without incurring any costs. Thus, the free ride option is more rational for the manufacturer. This conclusion is consistent with the research conclusion of Yi et al. [20], who also suggested that manufacturers always prefer retailers to implement anti-counterfeiting.

Lemma 4. *In all scenarios, we have $\frac{\partial p_a^S}{\partial k_j} > 0$, $\frac{\partial \pi_i^S}{\partial k_j} > 0$, $\frac{\partial e_i^S}{\partial k_j} > 0$, $\frac{\partial p_f^S}{\partial k_j} < 0$, $\frac{\partial w^{DM}}{\partial k_j} > 0$, $\frac{\partial w^{DR}}{\partial k_j} = 0$ where $j = 1$ or 2.*

Lemma 4 demonstrates that firms are more willing to invest in anti-counterfeit activities as the sensitivity coefficients of the uniqueness of genuine products and the moral disgust towards counterfeits become stronger, resulting in higher profits for both the manufacturer and the retailer under anti-counterfeit. As k_1 (or k_2) increases, the demand for genuine products expands, motivating firms to invest more in anti-counterfeit activities, i.e., $\frac{\partial e_i^S}{\partial k_j} > 0$. Nevertheless, an increase in k_1 (or k_2) also reduces consumer demand for counterfeit goods, which forces counterfeiters to decrease their prices to counteract the negative effects of anti-counterfeit, i.e., $\frac{\partial p_f^S}{\partial k_j} < 0$. According to Lemma 2, as k_1 (or k_2) increases, the firm faces greater anti-counterfeit effort and higher market share, allowing the brand supply chain to increase the degree of double marginalization to compensate for anti-counterfeit costs. Interestingly, under the DR scenario, the manufacturer does not have an incentive to increase the wholesale price, even though a higher k_1 (or k_2) can enable the retailer to earn greater profits. When the manufacturer raises the wholesale price, the reduced anti-counterfeit effort has a more significant impact than the increase in the wholesale price.

Lemma 5. *The effect of k_1 (or k_2) on the selling price of genuine products in different models is as follows. Let $\hat{c}_1 = \frac{(2k_1+k_2-\beta k_1)^2(\sqrt{3}-1)}{4q(\beta^2-3\beta+2)}$, $\hat{c}_2 = \frac{(2k_1+k_2-\beta k_1)^2(4-\sqrt{6})}{8q(\beta^2-3\beta+2)}$ and $\hat{c}_3 = \frac{(2k_1+k_2-\beta k_1)^2(\sqrt{6}+4)}{8q(\beta^2-3\beta+2)}$. When $0 < c < \hat{c}_1$, $\frac{\partial p_a^{DR*}}{\partial k_j} < \frac{\partial p_a^{CM*}}{\partial k_j} < \frac{\partial p_a^{DM*}}{\partial k_j}$. When $\hat{c}_1 \le c < \hat{c}_2$, $\frac{\partial p_a^{DR*}}{\partial k_j} < \frac{\partial p_a^{DM*}}{\partial k_j} \le \frac{\partial p_a^{CM*}}{\partial k_j}$. When $\hat{c}_2 \le c < \hat{c}_3$, $\frac{\partial p_a^{DM*}}{\partial k_j} \le \frac{\partial p_a^{DR*}}{\partial k_j} < \frac{\partial p_a^{CM*}}{\partial k_j}$. When $\hat{c}_3 \le c$, $\frac{\partial p_a^{DR*}}{\partial k_j} \le \frac{\partial p_a^{DM*}}{\partial k_j} < \frac{\partial p_a^{CM*}}{\partial k_j}$.*

Lemma 5 outlines the circumstances under which the two sensitivities have the strongest impact on the selling price of genuine products, based on the unit anti-counterfeit cost. When the unit anti-counterfeit cost is low, the sensitivity parameter k_1 (or k_2) has the most substantial effect on the selling price under a direct manufacturer (DM) supply chain. As the unit anti-counterfeit cost increases, the effect of k_1 (or k_2) on the selling price is most pronounced under a direct selling structure. This is because a direct selling structure allows the manufacturer to set a high price to offset the high anti-counterfeit costs. In contrast, a reselling structure sees the retailer setting prices based on wholesale prices and market demand. Furthermore, the increase in selling price resulting from k_1 (or k_2) is greatest in DM when c is too low or too high, and the marginal gain increases significantly in a reselling structure when c is below the maximum threshold of unit anti-counterfeit cost. These findings suggest that retailers can increase selling prices to cover anti-counterfeit costs when such costs are not particularly high. However, when unit anti-counterfeit costs are too high, excessive marginal gains may hinder market expansion.

Lemma 6. *The effect of k_1 and k_2 on the anti-counterfeit effort in different models is as follows. Let $\hat{c}_4 = \frac{(2k_1+k_2-\beta k_1)^2(\sqrt{33}-3)}{16q(\beta^2-3\beta+2)}$ and $\hat{c}_5 = \frac{(2k_1+k_2-\beta k_1)^2(\sqrt{33}+3)}{48q(\beta^2-3\beta+2)}$. When $0 < c < \hat{c}_4$, $\frac{\partial e_r^{DR*}}{\partial k_j} < \frac{\partial e_m^{CM*}}{\partial k_j} < \frac{\partial e_m^{DM*}}{\partial k_j}$. When $\hat{c}_4 \leq c < \hat{c}_5$, $\frac{\partial e_r^{DR*}}{\partial k_j} < \frac{\partial e_m^{DM*}}{\partial k_j} \leq \frac{\partial e_m^{CM*}}{\partial k_j}$. When $\hat{c}_5 \leq c$, $\frac{\partial e_m^{DM*}}{\partial k_j} \leq \frac{\partial e_r^{DR*}}{\partial k_j} < \frac{\partial e_m^{CM*}}{\partial k_j}$.*

Lemma 6 reveals that the effectiveness of the two sensitivities in facilitating anti-counterfeit efforts also varies depending on the unit anti-counterfeit cost. Specifically, the contribution of k_1 (or k_2) to anti-counterfeit effort is consistently the most substantial under a direct selling structure when the unit anti-counterfeit cost is not particularly low. This is because a direct selling structure eliminates the double marginalization problem and allows the firm to implement optimal anti-counterfeit efforts. In contrast, in a reselling manufacturer (DM) supply chain, the increase in anti-counterfeit effort is more significant when the unit anti-counterfeit cost is not too high, whereas it is more substantial under a reselling structure when the unit anti-counterfeit cost is at a higher level. This is because, when the anti-counterfeit effort is low, the new market demand is insufficient to cover the higher combat cost. Therefore, the retailer under DR will increase their marginal effort more with an increase in k_1 (or k_2) when the anti-counterfeit cost per unit is high.

Proposition 3. *The effect of k_1 and k_2 on the manufacturer's profit in different models is as follows. When $0 < c < \frac{(2k_1+k_2-\beta k_1)^2\sqrt{2}}{8q(\beta^2-3\beta+2)}$, $\frac{\partial \pi_m^{CM*}}{\partial k_j} > \frac{\partial \pi_m^{DM*}}{\partial k_j} > \frac{\partial \pi_m^{DR*}}{\partial k_j}$. When $\frac{(2k_1+k_2-\beta k_1)^2\sqrt{2}}{8q(\beta^2-3\beta+2)} \leq c$, $\frac{\partial \pi_m^{DM*}}{\partial k_j} \leq \frac{\partial \pi_m^{DR*}}{\partial k_j} < \frac{\partial \pi_m^{CM*}}{\partial k_j}$. The effect of k_1 and k_2 on the retailer's profit in different models is $\frac{\partial \pi_r^{DM*}}{\partial k_j} > \frac{\partial \pi_r^{DR*}}{\partial k_j}$.*

Proposition 3 indicates that the impact of uniqueness sensitivity and moral disgust on the manufacturer's profit through facilitation is most significant under a direct selling structure. This suggests that the manufacturer can fully benefit from combating counterfeit products when consumers perceive uniqueness and moral disgust. In a reselling manufacturer (DM) supply chain, the manufacturer's profit increases more significantly when the unit anti-counterfeit cost (c) is low, whereas in a reselling structure, the manufacturer's profit grows more if case c is high. This can be explained by considering the effect of k_1 (or k_2) on anti-counterfeit efforts. As the marginal effort increases, the market demand grows faster, leading to higher marginal profits for the free-riding manufacturer from the anti-counterfeit. Proposition 3 also reveals that the impact of k_1 (or k_2) on the retailer's profit is more significant under DM, possibly because in a DR supply chain, the increase in k_1 (or k_2) on retailer gains needs to compensate for the additional anti-counterfeit costs, thereby weakening the facilitation effect on the retailer's profit.

6. Extensions

This section introduces three extensions to the basic models and investigates them through numerical experiments. Section 6.1 assumes that in reselling structures, the manufacturer and retailer sign a revenue-sharing contract. Section 6.2 assumes that the manufacturer and retailer have a cost-sharing contract. Finally, Section 6.3 assumes that the market's consumers are divided into normal consumers and brand loyalists.

6.1. Revenue-Sharing Contract

Due to double marginalization, the manufacturer's profit suffers in a reselling structure, so a revenue-sharing contract is used for supply chain coordination. The manufacturer shares the retailer's sales revenue with a ratio $x_1 \in (0,\ 1)$, and the retailer receives only $1 - x_1$ of its own sales revenue. Therefore, the manufacturer's profit is $\pi_m = D_a w - \frac{1}{2} c e_m{}^2 \mathrm{V} + D_a x_1 p_a$, while the retailer's profit is $\pi_r = D_a[(1 - x_1)p_a - w] - \frac{1}{2} c e_r{}^2 \mathrm{V}$. $V \in \{0,\ 1\}$ is a binary variable. If the firm implements anti-counterfeiting activities, the value of V in its profit function is 1, and vice versa, V = 0. Taking the DMR model as an example, since the manufacturer is the anti-counterfeiting firm, V = 1 in the manufacturer's profit function and V = 0 in the retailer's profit function. We denote the manufacturer combat and retailer combat under the revenue-sharing contract by DMR and DRR. We assume that the sharing ratios are in the range such that both the manufacturer and the retailer are willing to accept the contract, i.e., $\pi_r^{DMR*} \geq \pi_r^{DM*}$, $\pi_r^{DRR*} \geq \pi_r^{DR*}$, $\pi_m^{DMR*} \geq \pi_m^{DM*}$ and $\pi_m^{DRR*} \geq \pi_m^{DR*}$. The profits of the manufacturer and the retailer at equilibrium are shown below.

$$\pi_r^{DMR*} = \frac{8q^3c^2(\beta-1)^3(\beta-2)(x_1-1)}{\left((2k_1+k_2-\beta k_1)^2+4cq(x_1-2)(\beta^2-3\beta+2)\right)^2},\quad \pi_m^{DMR*} = \frac{-2q^2c(\beta-1)^2}{(2k_1+k_2-\beta k_1)^2+4cq(x_1-2)(\beta^2-3\beta+2)},$$

$$\pi_r^{DRR*} = \frac{-q^2c(\beta-1)^2(x_1-1)\left[(x_1-1)(2k_1+k_2-\beta k_1)^2+4cq(\beta^2-3\beta+2)\right]}{2\left((x_1-1)(2k_1+k_2-\beta k_1)^2-2cq(x_1-2)(\beta^2-3\beta+2)\right)^2},$$

$$\pi_m^{DRR*} = \frac{q^2c(\beta-1)^2}{(x_1-1)(2k_1+k_2-\beta k_1)^2-2cq(x_1-2)(\beta^2-3\beta+2)}.$$

Proposition 4. *When the sharing ratio is below 0.5, it is optimal for the retailer to implement anti-counterfeit, while when the sharing ratio is above 0.5, it is optimal for the manufacturer to do so.*

Figure 3a,b demonstrate that a revenue-sharing contract can increase the manufacturer's profit, regardless of whether the manufacturer or the retailer combats counterfeit products. However, the optimal decision to combat counterfeits may vary depending on the sharing ratio. Numerical experiments reveal that if the sharing ratio is less than 0.5, the manufacturer earns more profit under a reselling structure with a retailer-led anti-counterfeit approach (i.e., DRR). Conversely, if the sharing ratio is greater than 0.5, the manufacturer will conduct anti-counterfeit activities instead of the retailer. The optimal anti-counterfeiting strategy is shown to depend on the sharing ratio, as explained by the integration of Figure 3c,d. Under the DRR strategy, when the sharing ratio is low, low wholesale price is unfavorable to manufacturers, but it encourages retailers to implement more anti-counterfeiting efforts, which makes the demand increase offset the disadvantages of low wholesale price. However, when the sharing ratio is high, the incentive for retailers from low wholesale prices is not enough to offset the increase in profits, which inhibits the incentive for retailers to improve their anti-counterfeiting efforts. Obviously, manufacturers under a high sharing rate have more incentive to implement anti-counterfeiting activities, and also charge a higher wholesale price.

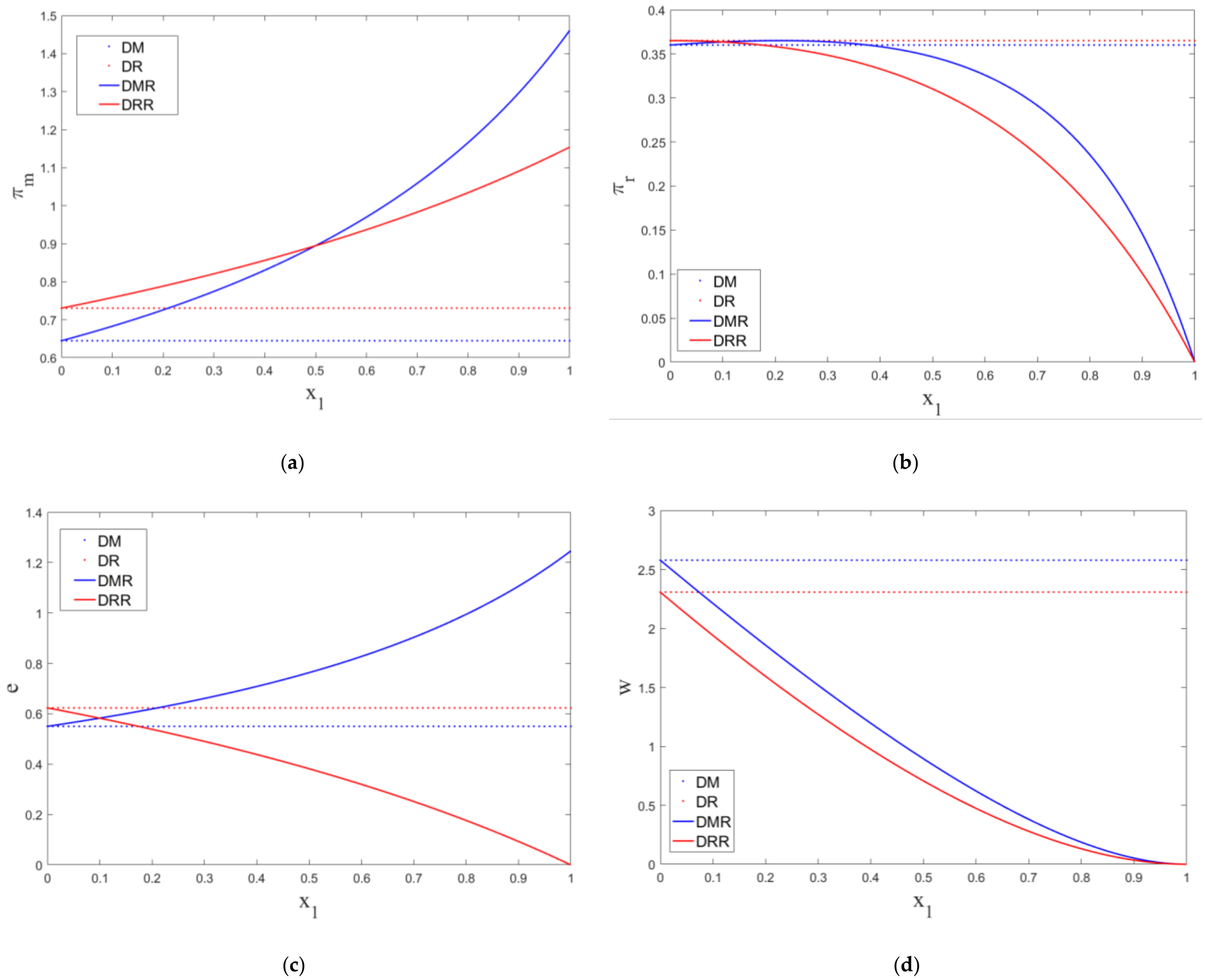


Figure 3. Impact of revenue-sharing ratios on manufacturer's profit in (**a**), retailer's profit in (**b**), anti-counterfeit effort in (**c**) and wholesale prices in (**d**): $k_1 = 0.6$, $k_2 = 0.5$, $\beta = 0.7$, $q = 10$ and $c = 0.5$.

6.2. Cost-Sharing Contract

To promote combating firms to invest more efforts to expand their markets, retailers enter into an anti-counterfeit cost-sharing contract with manufacturers, i.e., firms that do not participate in combating bear the anti-counterfeit cost in the proportion $x_2 \in (0, 1)$. In DM, manufacturer profit is $\pi_m = D_a w - (1 - x_2)\frac{1}{2}ce_m{}^2$, while retailer profit is $\pi_r = D_a(p_a - w) - x_2\frac{1}{2}ce_m{}^2$. In DR, manufacturer profit is $\pi_m = D_a w - x_2\frac{1}{2}ce_r{}^2$, while retailer profit is $\pi_r = D_a(p_a - w) - (1 - x_2)\frac{1}{2}ce_r{}^2$. In equilibrium, we denote the manufacturers combat and the retailers combat under the cost-sharing contract by DMC and DRC, respectively. We assume that the sharing ratios are in the range such that both manufacturers and retailers are willing to accept the contract, i.e., $\pi_r^{DMC*} \geq \pi_r^{DM*}$, $\pi_r^{DRC*} \geq \pi_r^{DR*}$, $\pi_m^{DMC*} \geq \pi_m^{DM*}$ and $\pi_m^{DRC*} \geq \pi_m^{DR*}$. The profits of the manufacturer and the retailer at equilibrium are shown below.

$$\pi_m^{DMC*} = \frac{2cq^2(\beta - 1)^2(x_2 - 1)}{(2k_1 + k_2 - \beta k_1)^2 + 8cq(x_2 - 1)(\beta^2 - 3\beta + 2)}, \ \pi_m^{DRC*} = \frac{2cq^2(\beta - 1)^2(x_2 - 1)^2}{(x_2 - 2)(2k_1 + k_2 - \beta k_1)^2 + 8cq(x_2 - 1)^2(\beta^2 - 3\beta + 2)}$$

$$\pi_r^{DMC*} = \frac{2cq^2(\beta-1)^2\left[x_2(2k_1+k_2-\beta k_1)^2+4cq(x_2-1)^2\left(\beta^2-3\beta+2\right)\right]}{\left[(2k_1+k_2-\beta k_1)^2+8cq(x_2-1)(\beta^2-3\beta+2)\right]^2}$$

$$\pi_r^{DRC*} = \frac{2cq^2(\beta-1)^2(x_2-1)^3\left[(2k_1+k_2-\beta k_1)^2+4cq(x_2-1)\left(\beta^2-3\beta+2\right)\right]}{\left[(x_2-2)(2k_1+k_2-\beta k_1)^2+8cq(x_2-1)^2(\beta^2-3\beta+2)\right]^2}$$

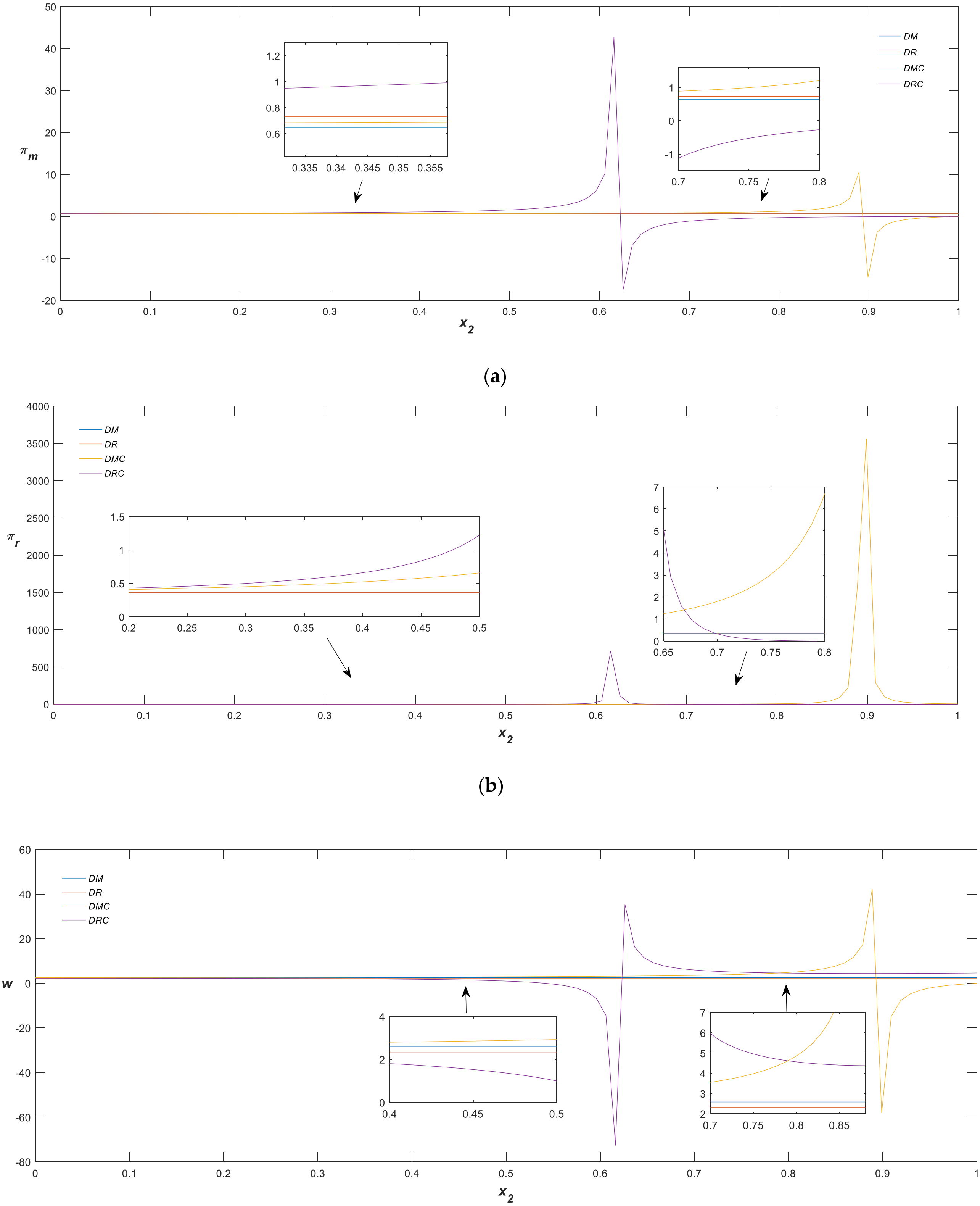
Proposition 5. *When the sharing ratio is low (e.g., below 0.5), the optimal anti-counterfeiting strategy is combated by the retailer. When the sharing ratio is high, the manufacturer prefers to implement anticounterfeiting by itself. However, when the sharing ratio is very high (e.g., above 0.9), the retailer and the manufacturer cannot agree on a cost coordination contract.*

Proposition 5 offers insightful findings. Firstly, the acceptance range of sharing ratios significantly differs between DMC and DRC, and for a given ratio, the probability of acceptance is higher in DMC. As demonstrated in Figure 4a,b, both the manufacturer and retailer readily accept the cost-sharing contract in DRC and DMC when the ratio is low, whereas it is only accepted by both firms under DMC when the ratio is too high. The reason for this lies in the fact that in DRC, the manufacturer incurs a higher combat cost than the product sales revenue when facing a high x_2. However, under DMC, while the retailer shares most of the manufacturer's combat costs, they can increase the selling price to offset the additional combat cost.

Secondly, cost-sharing contracts increase the profits of the manufacturer and retailer, and may change the optimal decision, which depends on x_2. This can be explained by the change in wholesale prices illustrated in Figure 4c. When x_2 is low, the manufacturer under DRC not only bears part of the anti-counterfeit cost but also reduces the wholesale price further to incentivize the retailer to invest more in anti-counterfeit efforts. In contrast, the manufacturer's increase in wholesale price in DMC exacerbates double marginalization, which is not conducive to expanding the market for consumers' demand for genuine products. Thus, the manufacturer allows the retailer to complete anti-counterfeit in the face of lower x_2. However, when x_2 is high, the cost-sharing contract is only effective in DMC. The manufacturer bears a small portion of the combat cost and profits from the higher wholesale price. Despite the pressure on the retailer to pay high wholesale prices, the retailer still benefits from the rapidly increasing consumer demand.

6.3. A Consumer Market with Loyalists

In this section, we assume that a fraction $\eta \in (0, 1)$ of the customers are brand loyalists who only buy genuine products and have the perceived uniqueness of genuine products, while the rest compare both products and purchase the one with higher utility. The demand functions for genuine and counterfeit products are $D_a = \eta\left(1-\frac{p_a-k_1e_i}{q}\right)+(1-\eta)\left(1-\frac{p_a-p_f-(k_1+k_2)e_i}{q(1-\beta)}\right)$, $D_f = (1-\eta)\left(\frac{p_a-p_f-(k_1+k_2)e_i}{q(1-\beta)}-\frac{p_f+k_2e_i}{q\beta}\right)$. In the new market including loyal consumers, we represent the strategies of manufacturer combat and retailer combat in the reselling structure by DML and DRL. The manufacturer profit and retailer profit at equilibrium are obtained as follows.



(**a**)

(**b**)

(**c**)

Figure 4. Impact of cost−sharing ratios on manufacturer's profit in (**a**), retailer's profit in (**b**) and wholesale price in (**c**): $k_1 = 0.6$, $k_2 = 0.5$, $\beta = 0.7$, $q = 10$ and $c = 0.5$.

$$\pi_m^{DML*} = \frac{2cq^2(\beta-1)^2}{8cq(\beta-1)(\beta+\eta\beta-2)-(\eta k_2-k_2-2k_1+\beta k_1+\eta k_1)^2},$$

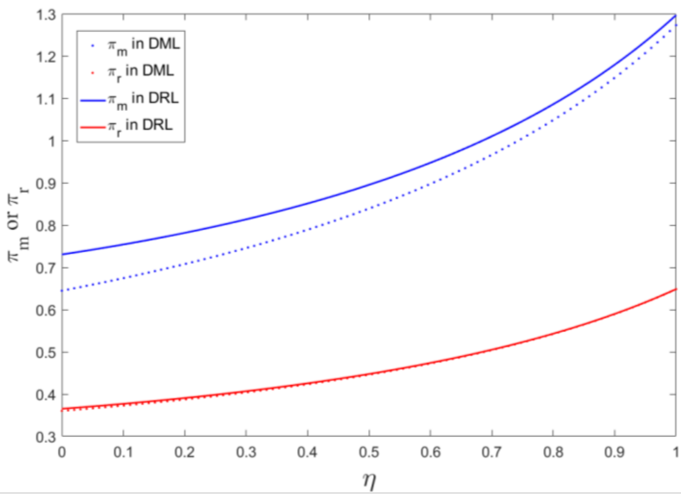
$$\pi_r^{DML*} = \frac{8c^2q^3(\beta-1)^3(\beta+\eta\beta-2)}{\left[(\eta k_2-k_2-2k_1+\beta k_1+\eta k_1)^2-8cq(\beta-1)(\beta+\eta\beta-2)\right]^2},$$

$$\pi_m^{DRL*} = \frac{2cq^2(\beta-1)^2}{4cq(\beta-1)(\beta+\eta\beta-2)-(\eta k_2-k_2-2k_1+\beta k_1+\eta k_1)^2},$$

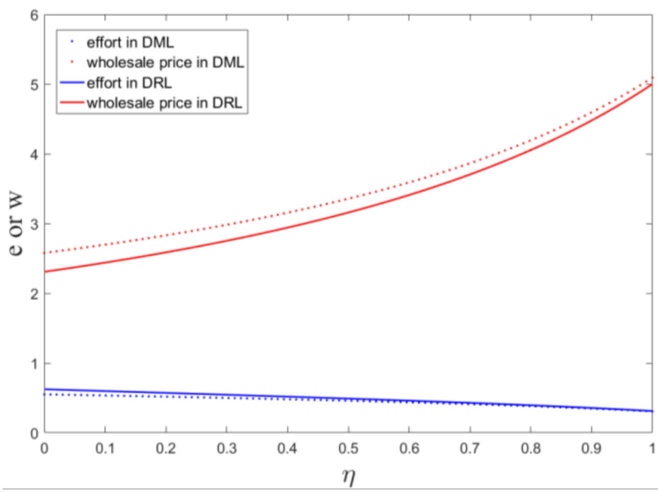
$$\pi_r^{DRL*} = \frac{2cq^2(\beta-1)^2}{2\left[4cq(\beta-1)(\beta+\eta\beta-2)-(\eta k_2-k_2-2k_1+\beta k_1+\eta k_1)^2\right]}$$

Proposition 6. *Combating by the retailer is still the optimal strategy in a market that contains both loyalists and normal consumers. As the proportion of loyalists increases, the gap between manufacturer profits in DRL and DML continues to narrow.*

As shown in Figure 5a, both the manufacturer and retailer earn higher profits under reselling with retailer-combating (DRL). Compared to reselling with manufacturer-combating (DML), the retailer has an advantage in both wholesale prices and anti-counterfeit efforts. The lower wholesale price and higher anti-counterfeiting efforts in the DRL strategy led to a wider profit differential when the proportion of loyalists was low. Additionally, as the number of brand loyalists in the market increases, the profit advantage under DRL gradually diminishes. When the market is almost saturated with loyalists (e.g., $\eta > 0.9$), the difference between anti-counterfeit performed by the retailer or by the manufacturer becomes insignificant. This can be explained by combining the wholesale price with the anti-counterfeit effort. When η is low, the advantage of DRL in wholesale price and anti-counterfeit effort is very clear. However, as η increases, anti-counterfeit efforts decline and fall more rapidly in DRL, while wholesale prices continue to rise and have a larger marginal increment in DRL. This leads to a narrowing gap between DRL and DML in anti-counterfeit effort and wholesale price. Moreover, double marginalization is more severe under DML, as shown in Figure 5b.



(**a**)



(**b**)

Figure 5. Impact of loyalist share on manufacturer's profit and retailer's profit in (**a**), anti-counterfeit effort and wholesale price in (**b**): $k_1 = 0.6$, $k_2 = 0.5$, $\beta = 0.7$, $q = 10$ and $c = 0.5$.

7. Managerial Discussion and Conclusions

The spread of online counterfeiting has posed a serious challenge to the sustainability of luxury supply chains. To improve sustainability, firms often establish in-house anti-counterfeiting teams. Moreover, retail platforms have also started to participate in anti-counterfeiting activities. If both the manufacturer and retailer can implement anti-counterfeiting, it is a key question which firm to implement anti-counterfeiting is most beneficial. In order to solve this problem, this study incorporates the impact of anti-counterfeiting on consumer behavior to examine the optimal channel structure and anti-counterfeiting strategies, including the choice of anti-counterfeiting firm, the level of anti-counterfeiting effort and the pricing decisions. This study reveals several novel insights that have both practical and theoretical implications for the luxury market.

First, our findings show that the direct selling structure is better than the reselling structure. This implies that most luxury manufacturers have an incentive to change their reselling strategy to a direct selling strategy, which has no double marginal effect and can fully benefit from anti-counterfeiting activities. Further, it is more beneficial for manufacturers to implement anti-counterfeiting activities by retailers under the reselling structure. This indicates that manufacturers can reduce wholesale prices to motivate retailers to increase anti-counterfeiting investment. Since the increase in demand due to anti-counterfeiting efforts is higher than the decrease in demand due to selling prices, both manufacturers and retailers can benefit from it.

Second, we find that consumer sensitivities to the uniqueness of genuine products and moral disgust to counterfeit products have facilitation effects on selling prices, anticounterfeit efforts, manufacturer profits and retailer profits, while they have a dampening effect on selling prices of counterfeits. For wholesale prices, an increase in k_1 (or k_2) will increase wholesale prices under DM, but has no effect on wholesale prices under DR. We further compare the extent to which the supply chain is affected by k_1 (or k_2) in different combating environments. In a reselling structure, the facilitation effects on anticounterfeit efforts and manufacturer's profit are the strongest under retailer combating when the unit combat cost is high; the facilitation effects on anticounterfeit effort and manufacturer's profit under manufacturer combating are the strongest when unit combat cost is low. The strength of a facilitation effect is directly correlated with the profitability of a given investment. When it comes to anti-counterfeiting activities, the unit cost of implementing measures varies greatly. For instance, the unit combat cost of blockchain-based solutions tends to be relatively high compared to the unit combat cost of employing anti-counterfeit personnel. As such, manufacturers may opt for anti-counterfeiting firms based on the unit combat cost in the short term. However, in the long run, retailers are regarded as the most effective anti-counterfeiting firms under a reselling structure.

We further extend the reselling models and explore it by numerical experiments. Our study demonstrates that both profit-sharing and cost-sharing contracts significantly enhance the profits of manufacturers and retailers when the sharing ratio is low. In a practical setting with a low sharing ratio, both the manufacturer and the retailer are incentivized to negotiate and agree upon a coordination contract. However, when the sharing ratio is high, the retailer's profit margin is adversely impacted, rendering the negotiation of a coordination contract challenging. Our findings also reveal that, irrespective of whether a revenue-sharing or cost-sharing contract is utilized, it is more advantageous for manufacturers to adopt anti-counterfeiting measures when the sharing ratio is low. Conversely, when the sharing ratio is high, it is advisable for manufacturers to independently invest in anti-counterfeiting initiatives. This finding highlights the importance of manufacturers choosing an anti-counterfeiting enterprise based on the sharing ratio, rather than solely relying on the retailer to implement such measures, particularly when a coordination contract is in place. We also explore the case where there are loyalists in the consumer market and find that retailer combats remain the optimal strategy, but the retailer's advantage of achieving anti-counterfeiting over the manufacturer shrinks as loyalty rates increase. This suggests that when the number of loyal consumers is small, manufacturers should take full advantage of retailers' anti-counterfeiting practices, which is necessary in terms of incremental profits.

The findings of this study can explain why some luxury firms (such as LV) opt for the direct selling structure and establish in-house anti-counterfeiting teams, while others (such as Rolex) use reselling channels and incentivize retailers to implement anti-counterfeiting. Since luxury exclusivity and moral aversion have significant impacts on consumer purchases, it is essential for manufacturers to consider them when devising anti-counterfeiting strategies.

This article combines the consumer psychology of luxury goods and luxury fakes to discuss the anti-counterfeiting strategies in the supply chain. However, here are some limitations. First, we assume that the production costs of both fake and genuine products are equal to zero. Although this assumption has been used by some articles published in UTD 24 journals, we did not discuss the scenario of unequal production costs in the extended literature. Secondly, this study focuses on non-deceptive fakes of luxury goods, but in reality, there are still some consumers who buy deceptive luxury goods (that is, they think they are buying genuine products, but in fact they are counterfeit products). If this paper can discuss the anti-counterfeiting strategies of deceptive fakes, the conclusion of the paper will be richer. Finally, the demand function used in this study is certain, and it will be more realistic if it is changed to uncertain demand.

There are two key directions for future research. First, we must investigate how luxury manufacturers can best select and deploy blockchain-based anti-counterfeiting strategies. With both manufacturers and retailers now leveraging blockchain services, such as LVMH Group's Aura Blockchain Consortium and JD.com and Tmall's blockchain traceability services, we need to assess whether luxury manufacturers should develop their own blockchain systems or partner with downstream retailers. By modeling and studying this scenario, we can offer valuable insights to industry stakeholders. Second, the emerging trend of luxury goods manufacturers entering the secondary market merits attention. While manufacturers previously viewed the secondary market as a competitor, some are now selling second-hand goods themselves. For example, Rolex's "The Rolex Certified Pre-Owned Programme" certifies and sells recycled Rolex watches in authorized retail stores. This development raises important questions about the circumstances under which luxury goods manufacturers will enter the secondary market. Through modeling and analysis, we can provide a nuanced understanding of this trend and its implications for the luxury industry.

Author Contributions: Conceptualization, W.S.; methodology, W.W.; writing—original draft preparation, W.W.; writing—review and editing, Y.L.; supervision, W.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Just to make the expression simpler, we set $H_1 = (2k_1 + k_2 - \beta k_1)^2$, $H_2 = q(\beta^2 - 3\beta + 2)$, $\beta \in (0,\ 1)$, $k_1 \in (0,\ 1)$ and $k_2 \in (0,\ 1)$.

Appendix A.1. Proof of Section 4

Proof of Section 4.2. This is a backward induction approach, where we first discuss how the counterfeiter makes a price for fakes and then analyzes the equilibrium problem of the manufacturer. First, take the second derivative of the counterfeiter's profit with respect to the fake's price to obtain $\frac{\partial^2 \pi_f^{CM}}{\partial {p_f}^2} = \frac{2}{\beta q(\beta - 1)} < 0$. From the second derivative, we can see that ${p_f}^{CM*}$, which makes $\frac{\partial \pi_f^{CM}}{\partial p_f} = 0$ valid, is the optimal fake price. Here, ${p_f}^{CM*} = \left(\beta {p_a}^{CM*} - {e_m}^{CM*}k_2 - \beta {e_m}^{CM*}k_1\right)/2$. We substitute ${p_f}^{CM*}$ into the manufacturer's profit function, calculate the Hessian matrix of π_m^{CM} with respect to ${p_a}^{CM}$ and ${e_m}^{CM}$, and obtain

$$H_m^{CM} = \begin{vmatrix} \frac{\partial^2 \pi_m^{CM}}{\partial {p_a}^2} & \frac{\partial^2 \pi_m^{CM}}{\partial p_a \partial e_m} \\ \frac{\partial^2 \pi_m^{CM}}{\partial e_m \partial p_a} & \frac{\partial^2 \pi_m^{CM}}{\partial {e_m}^2} \end{vmatrix} = \begin{vmatrix} \frac{2-\beta}{q(\beta-1)} & -\frac{\sqrt{H_1}}{2q(\beta-1)} \\ -\frac{\sqrt{H_1}}{2q(\beta-1)} & -c \end{vmatrix}$$

It is easy to prove that the first-order and second-order principal subexpressions of the Hesse matrix are, respectively, $H_{m1}^{CM} = \frac{2-\beta}{q(\beta-1)} < 0$ and $H_{m2}^{CM} = \frac{4cH_2 + 4k_1^2(\beta-1) - (k_2-\beta k_1)^2 - 4k_1k_2}{4q^2(\beta-1)^2}$. When $q > \frac{H_1}{(4c(\beta-1)(\beta-2))}$, $H_{m2}^{CM} > 0$. At this time, π_m^{CM} is a joint concave function of ${p_a}^{CM}$ and ${e_m}^{CM}$, which means that its first derivative solution is optimal. By combining $\frac{\partial \pi_m^{CM}}{\partial p_a} = 0$ and $\frac{\partial \pi_m^{CM}}{\partial e_m} = 0$, the optimal selling price of genuine products is $p_a^{CM*} = \frac{4q^2c(\beta-1)^2}{4cH_2 - H_1}$ and the optimal combating effort is $e_m^{CM*} = \frac{2q(\beta-1)\sqrt{H_1}}{H_1 - 4cH_2}$. We put ${p_a}^{CM*}$ and ${e_m}^{CM*}$ in ${p_f}^{CM*}$, and obtain $p_f^{CM*} = \frac{q(1-\beta)\left[2k_1k_2 + {k_2}^2 + \beta {k_1}^2(2-\beta) + 2\beta qc(\beta-1)\right]}{H_1 - 4cH_2}$. According to the above optimal solution, we can solve that the genuine manufacturer's profit π_m^{CM*} is equal to $\frac{-2q^2c(\beta-1)^2}{H_1 - 4cH_2}$. □

Proof of Section 4.3. According to the backward induction method, we first solve the price decision of the counterfeiter, then solve the equilibrium of the retailer, and finally find the equilibrium of the manufacturer. First, the price decision faced by the counterfeiter is the same as in the CM scenario, so ${p_f}^{DM*} = \frac{\left(\beta {p_a}^{DM*} - {e_m}^{DM*}k_2 - \beta {e_m}^{DM*}k_1\right)}{2}$. Then, we put ${p_f}^{DM*}$ into the retailer's profit function, and take the second derivative of π_r^{DM} with respect to ${p_a}^{DM}$, and obtain $\frac{\partial^2 \pi_r^{DM}}{\partial {p_a}^2} = \frac{2-\beta}{q(\beta-1)} < 0$. According to $\frac{\partial \pi_r^{DM}}{\partial p_a} = 0$, we obtain ${p_a}^{DM*} = \frac{2\left(q + w^{DM*} - \beta q\right) + e_m^{DM*}(2k_1 + k_2 - \beta k_1) - \beta w^{DM*}}{2(2-\beta)}$. Finally, both ${p_f}^{DM*}$ and ${p_a}^{DM*}$ are put into the manufacturer's profit function, and the Hessian matrix of π_m^{DM} with respect to w^{DM} and e_m^{DM} is calculated.

$$H_m^{DM} = \begin{vmatrix} \frac{\partial^2 \pi_m^{DM}}{\partial w^2} & \frac{\partial^2 \pi_m^{DM}}{\partial w \partial e_m} \\ \frac{\partial^2 \pi_m^{DM}}{\partial e_m \partial w} & \frac{\partial^2 \pi_m^{DM}}{\partial {e_m}^2} \end{vmatrix} = \begin{vmatrix} \frac{2-\beta}{2q(\beta-1)} & -\frac{\sqrt{H_1}}{4q(\beta-1)} \\ -\frac{\sqrt{H_1}}{4q(\beta-1)} & -c \end{vmatrix}$$

It is easy to prove that the first-order and second-order principal subforums of the matrix are, respectively, $H_{m1}^{DM} = \frac{2-\beta}{q(\beta-1)} < 0$ and $H_{m2}^{DM} = \frac{4cH_2 + 4k_1^2(\beta-1) - (k_2-\beta k_1)^2 - 4k_1k_2}{4q^2(\beta-1)^2}$. When $q > \frac{H_1}{8c(\beta-1)(\beta-2)}$, $H_{m2}^{DM} > 0$. In this case, π_m^{DM} with respect to w^{DM} and e_m^{DM} are jointly concave functions, and the first-order derivative solution is optimal. By combining $\frac{\partial \pi_m^{DM}}{\partial w} = 0$ and $\frac{\partial \pi_m^{DM}}{\partial e_m} = 0$, the optimal wholesale prices is $w^{DM*} = \frac{-8q^2c(\beta-1)^2}{H_1 - 8cH_2}$ and the optimal combating effort is $e_m^{DM*} = \frac{2q(\beta-1)\sqrt{H_1}}{H_1 - 8cH_2}$. We put w^{DM*} and e_m^{DM*} into π_m^{CM*} and π_r^{CM*}. Both the manufacturer's profit and the retailer's profit at equilibrium are shown in the manuscript. □

Proof of Section 4.4. The sequence of the solution process is the same as in the DM scenario. First, the price decision faced by the counterfeiter is the same as in the CM scenario, so

${p_f}^{DR*} = \frac{\left(\beta {p_a}^{DR*} - {e_r}^{DR*} k_2 - \beta {e_r}^{DR*} k_1\right)}{2}$. Then, we put ${p_f}^{DR*}$ into the retailer's profit function, calculate the Hessian matrix of π_r^{DR} with respect to ${p_a}^{DR}$ and e_r^{DR}, and get

$$H_r^{DR} = \begin{vmatrix} \frac{\partial^2 \pi_r^{DR}}{\partial {p_a}^2} & \frac{\partial^2 \pi_r^{DR}}{\partial p_a \partial e_r} \\ \frac{\partial^2 \pi_r^{DR}}{\partial e_r \partial p_a} & \frac{\partial^2 \pi_r^{DR}}{\partial {e_r}^2} \end{vmatrix} = \begin{vmatrix} \frac{2-\beta}{q(\beta-1)} & -\frac{\sqrt{H_1}}{2q(\beta-1)} \\ -\frac{\sqrt{H_1}}{2q(\beta-1)} & -c \end{vmatrix}$$

It is easy to prove that the first-order principal sub formula of the matrix is less than 0, and the second-order principal sub formula $H_{r2}^{DR} = -\frac{H_1-4cH_2}{4q^2(\beta-1)^2}$. When $q > \frac{H_1}{4c(\beta-1)(\beta-2)}$, $H_{r2}^{DR} > 0$. By combining $\frac{\partial \pi_r^{DR}}{\partial p_a} = 0$ and $\frac{\partial \pi_r^{DR}}{\partial e_r} = 0$, we obtain the optimal selling price $p_a^{DR*} = \frac{wH_1+2cq(\beta-1)(2q+2w-2\beta q-\beta w)}{H_1-4cH_2}$ and combating effort $e_r^{DR*} = \frac{\sqrt{H_1}(2q-2w-2\beta q+\beta w)}{4cH_2-H_1}$. Finally, we put ${p_a}^{DR*}$ and e_r^{DR*} into the manufacturer's profit function and find the second order derivative of π_m^{DR} with respect to w^{DR}, i.e., $\frac{\partial^2 \pi_r^{DR}}{\partial w^2} = \frac{2c(\beta-2)^2}{H_1-4cH_2} < 0$. Let $\frac{\partial \pi_r^{DR}}{\partial w} = 0$, and solve for $w^{DR*} = q\frac{\beta-1}{\beta-2}$. We put w^{DR*} into the previously obtained p_a^{DR*}, p_f^{DR*} and e_r^{DR*}, and calculate π_m^{DR*} and π_r^{DR*}. The results are shown in the text.

According to the above analysis, in order to ensure that various expressions in CM model have certain economic significance, the exogenous quality of genuine products is required to meet $q > \frac{H_1}{4c(\beta-1)(\beta-2)}$. Similarly, the quality in the DM scenario needs to satisfy the condition $q > \frac{H_1}{8c(\beta-1)(\beta-2)}$, and the quality in the DR scenario needs to satisfy the condition $q > \frac{H_1}{4c(\beta-1)(\beta-2)}$. To subsequently facilitate the comparison of the equilibrium solutions of the different models, I need to let the exogenous quality satisfy all three models at the same time. Therefore, we assume that the exogenous mass satisfies the condition $q > \frac{H_1}{4c(\beta-1)(\beta-2)}$. $\square$

Appendix A.2. Proof of Section 5

Proof of Proposition 1. First, we compare ${p_a}^{CM*}$ with ${p_a}^{DM*}$, i.e., ${p_a}^{CM*} - {p_a}^{DM*} = \frac{12q^2c(\beta-1)^2}{12cH_2-3H_1} - \frac{12q^2c(\beta-1)^2}{8cH_2-H_1}$. Let $12cH_2 - 3H_1 = 8cH_2 - H_1$, and obtain $c = \frac{H_1}{H_2}$, denoted as c_1. Therefore, when $0 < c < c_1$, ${p_a}^{CM*} > {p_a}^{DM*}$. When $c \geq c_1$, ${p_a}^{CM*} \leq {p_a}^{DM*}$.

Second, we compare ${p_a}^{CM*}$ with ${p_a}^{DR*}$, i.e., ${p_a}^{CM*} - {p_a}^{DR*} = \frac{q(1-\beta)[H_2(4-6c)+H_1]}{(2-\beta)(4cH_2-H_1)}$. We can find that the result of the comparison depends on $H_2(4-6c)+H_1$. Let $H_2(4-6c)+H_1 = 0$, and obtain $c = \frac{H_1+4H_2}{6H_2}$, denoted as c_2. Therefore, when $0 < c < c_2$, ${p_a}^{CM*} > {p_a}^{DR*}$. When $c \geq c_2$, ${p_a}^{CM*} \leq {p_a}^{DR*}$.

Third, we compare ${p_a}^{DM*}$ and ${p_a}^{DR*}$, i.e., ${p_a}^{DM*} - {p_a}^{DR*} = \frac{q(1-\beta)[12H_2(4cH_2-H_1)-(6cH_2-H_1)(8cH_2-H_1)]}{(8cH_2-H_1)(4cH_2-H_1)(2-\beta)}$. We can find that the result of the comparison depends on $12H_2(4cH_2-H_1)-(6cH_2-H_1)(8cH_2-H_1)$, denoted as y_1. We find $\frac{\partial^2 y_1}{\partial c^2} = -96q^2(\beta-1)^2(\beta-2)^2 < 0$, which means y_1 is a concave function on c. Let $y_1 = 0$, and obtain $c_3 = \frac{48q(1-\beta)-\sqrt{Q_1}+7{k_2}^2+7{k_1}^2(4-3\beta)+14k_1k_2(2-\beta)}{48H_2}$ and $c_4 = \frac{48q(1-\beta)+\sqrt{Q_1}+7{k_2}^2+7{k_1}^2(4-3\beta)+14k_1k_2(2-\beta)}{48H_2}$, where $Q_1 = \beta^4\left(k_1^4 - 240k_2^2q + 576q^2\right) - 4\beta^3\left(2k_1^4 + k_2k_1^3 - 420k_1^2q - 120qk_2k_1 + 864q^2\right) + 6\beta^2\left(4k_1^4 + 4k_1^3k_2 + k_1^2k_2^2 - 720k_1^2q - 400k_1k_2q - 40k_2^2q + 1248q^2\right) - 4\beta\left(8k_1^4 + 12k_1^3k_2 + 6k_1^2k_2^2 - 1200k_1^2q + k_1k_2^3 - 960\,k_1k_2q - 180k_2^2q + 1728q^2\right) + (2k_1+k_2)^4 - 96q\left(20{k_1}^2 + 20k_1k_2 + 5k_2^2 - 24q\right)$ and $c_3 - c_4 < 0$. When $c = 0$, $y_1 = -H_1^2 - 12H_1H_2 < 0$, which means $0 < c_3 < c_4$. Therefore, when $0 < c < c_3$ or $c > c_4$, ${p_a}^{DM*} < {p_a}^{DR*}$. When $c_3 \leq c \leq c_4$, ${p_a}^{DM*} \geq {p_a}^{DR*}$.

Based on the above discussion we find that the expression for the boundary point is rather complex, but what can be determined here is that ${p_a}^{CM*} < {p_a}^{DM*} < {p_a}^{DR*}$ when $c > \max\{c_1, c_2, c_4\}$, while ${p_a}^{CM*} > {p_a}^{DR*} > {p_a}^{DM*}$ while $c < \min\{c_1, c_2, c_3\}$.

Now we compare efforts. It is obvious to find $e_m^{CM*} < e_r^{DR*}$ and $e_m^{CM*} > e_m^{DM*}$. $e_m^{DM*} - e_r^{DR*} = \frac{2q\sqrt{H_1}(1-\beta)}{8cH_2-H_1} - \frac{2q\sqrt{H_1}(1-\beta)}{8cH_2-2H_1} < 0$, so $e_m^{DM*} < e_r^{DR*}$. Therefore, $e_m^{CM*} > e_r^{DR*} > e_m^{DM*}$.

Continue, we compare counterfeit's selling price. Because $p_f^{CM*} - p_f^{DR*} = \frac{q(\beta-1)^2\left[cq\beta(\beta-2)+k_1k_2(\beta-2)-k_2^2\right]}{(2-\beta)(4cH_2-H_1)} < 0$, $p_f^{CM*} < p_f^{DR*}$. Further, we make the difference to obtain $p_f^{DM*} - p_f^{DR*} = \frac{q(\beta-1)^2H_1^2\left[(cq\beta+k_1k_2)(2-\beta)+k_2^2\right]}{(2-\beta)(4cH_2-H_1)(8cH_2-H_1)} > 0$. Therefore, $p_f^{CM*} < p_f^{DR*} < p_f^{DM*}$.

Continue, we compare wholesale prices. Because $w^{DM*} - w^{DR*} = \frac{q(1-\beta)H_1}{(H_1-8cH_2)(\beta-2)} > 0$, $w^{DM*} > w^{DR*}$. □

Proof of Proposition 2. We compare retailer's profits. According to $\pi_r^{DM*} - \pi_r^{DR*} = \frac{cq^2H_1^2(\beta-1)^2}{2(H_1-4cH_2)(8cH_2-H_1)^2} < 0$, $\pi_r^{DM*} < \pi_r^{DR*}$. Continue, we compare manufacturer's profits. According to $\pi_m^{CM*} - \pi_m^{DR*} = \frac{q^2(\beta-1)^2}{4cH_2-H_1} > 0$ and $\pi_m^{DM*} - \pi_m^{DR*} = \frac{-cq^2(\beta-1)^2H_1}{(4cH_2-H_1)(8cH_2-H_1)} < 0$, we find $\pi_m^{CM*} > \pi_m^{DR*} > \pi_m^{DM*}$. □

Proof of Lemma 5. At first, we study the Influence degree of k_1 or k_2 on p_a.

$$\frac{\partial p_a^{CM*}}{\partial k_1} = \frac{8cq^2(\beta-1)^2(2-\beta)\sqrt{H_1}}{(H_1-4cH_2)^2},\ \frac{\partial p_a^{DM*}}{\partial k_1} = \frac{24cq^2(\beta-1)^2(2-\beta)\sqrt{H_1}}{(H_1-8cH_2)^2},\ \frac{\partial p_a^{DR*}}{\partial k_1} = \frac{4cq(1-\beta)H_2\sqrt{H_1}}{(H_1-4cH_2)^2}$$

$$\frac{\partial p_a^{CM*}}{\partial k_2} = \frac{4cq^2(\beta-1)^2 2\sqrt{H_1}}{(H_1-4cH_2)^2},\ \frac{\partial p_a^{DM*}}{\partial k_2} = \frac{12cq^2(\beta-1)^2 2\sqrt{H_1}}{(H_1-8cH_2)^2},\ \frac{\partial p_a^{DR*}}{\partial k_1} = \frac{4cq^2(1-\beta)^2\sqrt{H_1}}{(H_1-4cH_2)^2}$$

Obviously, we can observe that $\frac{\partial p_a}{\partial k_1}$ and $\frac{\partial p_a}{\partial k_2}$ are greater than zero in CM, DM and DR. Firstly, we compare the impact of k_1 on p_a in different scenarios. Because $\frac{\partial p_a^{CM*}}{\partial k_1} - \frac{\partial p_a^{DR*}}{\partial k_1} = \frac{4cq^2(\beta-1)^2(2-\beta)\sqrt{H_1}}{(H_1-4cH_2)^2} > 0$, $\frac{\partial p_a^{CM*}}{\partial k_1} > \frac{\partial p_a^{DR*}}{\partial k_1}$. Next we compare $\frac{\partial p_a^{CM*}}{\partial k_1}$ and $\frac{\partial p_a^{DM*}}{\partial k_1}$, and obtain $\frac{\partial p_a^{CM*}}{\partial k_1} - \frac{\partial p_a^{DM*}}{\partial k_1} = \frac{16cq(1-\beta)H_2\sqrt{H_1}Q_2}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$ where $Q_2 = \beta^4\left(8c^2q^2+4ck_1^2q-k_1^4\right) - 4\beta^3\left(12c^2q^2+7ck_1^2q+2ck_1k_2q-2k_1^4-k_2k_1^3\right) + 2\beta^2(52c^2q^2+36ck_1^2q+20ck_1k_2q+2cqk_2^2-12k_2k_1^3-3k_1^2k_2^2) - 4\beta(6\beta k_1^4-k_1k_2^3-12k_1^3k_2-8k_1^4+24c^2q^2-6k_1^2k_2^2+20ck_1^2q+3ck_2^2q+16ck_1k_2q) + 8cq\left(4k_1^2+4k_1k_2+k_2^2-4cq\right) - (2k_1+k_2)^4$. According to $\frac{\partial^2 Q_2}{\partial c^2} = 16H_2^2 > 0$, Q_2 is convex with respect to c. Let $Q_2 = 0$ and obtain $c_5 = \frac{H_1\left(\sqrt{3}+1\right)}{-4H_2}, c_6 = \frac{H_1\left(\sqrt{3}-1\right)}{4H_2}$. When $c = 0$, $Q_2 = -H_1^2 < 0$ which means $c_5 < 0 < c_6$. Therefore, when $0 < c < c_6$, $\frac{\partial p_a^{CM*}}{\partial k_1} < \frac{\partial p_a^{DM*}}{\partial k_1}$. When $c \geq c_6$, $\frac{\partial p_a^{CM*}}{\partial k_1} \geq \frac{\partial p_a^{DM*}}{\partial k_1}$. At last we compare $\frac{\partial p_a^{DM*}}{\partial k_1}$ and $\frac{\partial p_a^{DR*}}{\partial k_1}$ and obtain $\frac{\partial p_a^{DM*}}{\partial k_1} - \frac{\partial p_a^{DR*}}{\partial k_1} = \frac{4cq(1-\beta)H_2\sqrt{H_1}Q_3}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$ where $Q_3 = \beta^4\left(32c^2q^2-32cqk_1^2+5k_1^4\right) - 4\beta^3\left(48c^2q^2-56cqk_1^2-16cqk_1k_2+10k_1^4+5k_2k_1^3\right) + 2\beta^2(208c^2q^2-288cqk_1^2-160cqk_1k_2-16ck_2^2q+60k_1^4+60k_1^3k_2+15k_1^2k_2^2) - 4\beta(96c^2q^2+(2k_1+k_2)(20k_1^3+20k_1^2k_2+5k_1k_2^2-80cqk_1-24cqk_2)) - 64cq\left(4k_1^2+4k_1k_2+k_2^2-2cq\right) + 5(2k_1+k_2)^4$. According to $\frac{\partial^2 Q_3}{\partial c^2} = 64H_2^2 > 0$, Q_3 is convex with respect to c Let $Q_3 = 0$ and obtain $c_7 = \frac{H_1\left(\sqrt{6}+4\right)}{8H_2}, c_8 = \frac{H_1\left(4-\sqrt{6}\right)}{8H_2}$. It is obvious to see $0 < c_8 < c_7$. When $0 < c < c_8$ or $c > c_7$, $\frac{\partial p_a^{DR*}}{\partial k_1} < \frac{\partial p_a^{DM*}}{\partial k_1}$. When $c_7 \leq c \leq c_8$, $\frac{\partial p_a^{DR*}}{\partial k_1} \geq \frac{\partial p_a^{DM*}}{\partial k_1}$.

Next, we will compare the influence of k_2 on p_a in different models. Obviously, we can observe $\frac{\partial p_a^{CM*}}{\partial k_2} - \frac{\partial p_a^{DR*}}{\partial k_2} > 0$ directly. According to $\frac{\partial p_a^{CM*}}{\partial k_2} - \frac{\partial p_a^{DM*}}{\partial k_2} = \frac{4cq^2(1-\beta)^2\sqrt{H_1}Q_2}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, the comparison depends on Q_2. Therefore, the analysis result here is the same as $\frac{\partial p_a^{CM*}}{\partial k_1} - \frac{\partial p_a^{DM*}}{\partial k_1}$. Because $\frac{\partial p_a^{DM*}}{\partial k_2} - \frac{\partial p_a^{DR*}}{\partial k_2} = \frac{4cq^2(1-\beta)^2\sqrt{H_1}Q_3}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, the comparison depends on Q_2. Therefore, the analysis result here is the same as $\frac{\partial p_a^{DM*}}{\partial k_1} - \frac{\partial p_a^{DR*}}{\partial k_1}$.

From what has been discussed above, when $0 < c < c_6$, $\frac{\partial p_a^{DR*}}{\partial k_1} < \frac{\partial p_a^{CM*}}{\partial k_1} < \frac{\partial p_a^{DM*}}{\partial k_1}$ and $\frac{\partial p_a^{DR*}}{\partial k_2} < \frac{\partial p_a^{CM*}}{\partial k_2} < \frac{\partial p_a^{DM*}}{\partial k_2}$. When $c_6 \le c < c_8$, $\frac{\partial p_a^{DR*}}{\partial k_1} < \frac{\partial p_a^{DM*}}{\partial k_1} \le \frac{\partial p_a^{CM*}}{\partial k_1}$ and $\frac{\partial p_a^{DR*}}{\partial k_2} < \frac{\partial p_a^{DM*}}{\partial k_2} \le \frac{\partial p_a^{CM*}}{\partial k_2}$. When $c_8 \le c < c_7$, $\frac{\partial p_a^{DM*}}{\partial k_1} \le \frac{\partial p_a^{DR*}}{\partial k_1} < \frac{\partial p_a^{CM*}}{\partial k_1}$ and $\frac{\partial p_a^{DM*}}{\partial k_2} \le \frac{\partial p_a^{DR*}}{\partial k_2} < \frac{\partial p_a^{CM*}}{\partial k_2}$. When $c_7 \le c$, $\frac{\partial p_a^{DR*}}{\partial k_1} \le \frac{\partial p_a^{DM*}}{\partial k_1} < \frac{\partial p_a^{CM*}}{\partial k_1}$ and $\frac{\partial p_a^{DR*}}{\partial k_2} \le \frac{\partial p_a^{DM*}}{\partial k_2} < \frac{\partial p_a^{CM*}}{\partial k_2}$. □

Proof of Lemma 6. Next, we study the Influence degree of k_1 or k_2 on e.

$$\frac{\partial e_m^{CM*}}{\partial k_1} = \frac{2H_2(H_1+4cH_2)}{(H_1-4cH_2)^2},\ \frac{\partial e_m^{DM*}}{\partial k_1} = \frac{2H_2(H_1+8cH_2)}{(H_1-8cH_2)^2},\ \frac{\partial e_r^{DR*}}{\partial k_1} = \frac{H_2(H_1+4cH_2)}{(H_1-4cH_2)^2}$$
$$\frac{\partial e_m^{CM*}}{\partial k_2} = \frac{2q(1-\beta)(H_1+4cH_2)}{(H_1-4cH_2)^2},\ \frac{\partial e_m^{DM*}}{\partial k_2} = \frac{2q(1-\beta)(H_1+8cH_2)}{(H_1-8cH_2)^2},\ \frac{\partial e_r^{DR*}}{\partial k_1} = \frac{q(1-\beta)(H_1+4cH_2)}{(H_1-4cH_2)^2}$$

Obviously, we can observe $\frac{\partial e}{\partial k_1}$ and $\frac{\partial e}{\partial k_2}$ in CM, DM, DR.

Firstly, we compare the influence degree of k_1 and e under different models. We can observe $\frac{\partial e_m^{CM*}}{\partial k_1} - \frac{\partial e_r^{DR*}}{\partial k_1} > 0$.We can calculate that $\frac{\partial e_m^{CM*}}{\partial k_1} - \frac{\partial e_m^{DM*}}{\partial k_1} = \frac{8cH_2{}^2Q_4}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$ where $\mathrm{Q_4} = \beta^4\left(32c^2q^2 + 12cqk_1^2 - 3k_1^4\right) - 12\beta^3\left(16c^2q^2 + 7cqk_1^2 + 2cqk_1k_2 - 2k_1^4 - k_2k_1^3\right) + 2\beta^2\left(208c^2q^2 + 108cqk_1^2 + 60cqk_1k_2 + 6cqk_2^2 - 36k_1^4 - 36k_2k_1^3 - 9k_1^2k_2^2\right) + 12\beta(-32c^2q^2 - 20cqk_1^2 - 16cqk_1k_2 - 3cqk_2^2 + 8k_1^4 + 12k_2k_1^3 + 6k_1^2k_2^2 + k_1k_2^3) + 8cq(12k_1^2 + 12k_1k_2 + 3k_2^2 + 16cq) - 3(2k_1 + k_2)^4$. Because $\frac{\partial^2 Q_4}{\partial c^2} = 64H_2^2 > 0$, Q_4 is convex with respect to c. Let $Q_4 = 0$ and obtain $c_9 = \frac{H_1(\sqrt{33}+3)}{-16H_2}$ and $c_{10} = \frac{H_1(\sqrt{33}-3)}{16H_2}$. Therefore, when $0 < c < c_{10}$, $\frac{\partial e_m^{CM*}}{\partial k_1} < \frac{\partial e_m^{DM*}}{\partial k_1}$. When $c \ge c_{10}$, $\frac{\partial e_m^{CM*}}{\partial k_1} \ge \frac{\partial e_m^{DM*}}{\partial k_1}$. $\frac{\partial e_m^{DM*}}{\partial k_1} - \frac{\partial e_r^{DR*}}{\partial k_1} = \frac{H_2H_1Q_5}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, where $Q_5 = \beta^4\left(-96c^2q^2 + 12cqk_1^2 + k_1^4\right) - 4\beta^3\left(-144c^2q^2 + 21cqk_1^2 + 6cqk_1k_2 + 2k_1^4 + k_2k_1^3\right) + 6\beta^2\left(-208c^2q^2 + 36cqk_1^2 + 20cqk_1k_2 + 2cqk_2^2 + 4k_1^4 + 4k_2k_1^3 + k_1^2k_2^2\right) - 4\beta(-288c^2q^2 + 60cqk_1^2 + 48cqk_1k_2 + 9cqk_2^2 + 8k_1^4 + 12k_2k_1^3 + 6k_1^2k_2^2 + k_1k_2^3) + 24cq\left(4k_1^2 + 4k_1k_2 + k_2^2 - 16cq\right) + (2k_1 + k_2)^4$. Because $\frac{\partial^2 Q_5}{\partial c^2} = -192H_2^2 < 0$, Q_5 is a concave function of c. Let $Q_5 = 0$ and obtain $c_{11} = \frac{H_1(\sqrt{33}+3)}{48H_2}, c_{12} = \frac{H_1(\sqrt{33}-3)}{-48H_2}$. Therefore, $\frac{\partial e_r^{DR*}}{\partial k_1} < \frac{\partial e_m^{DM*}}{\partial k_1}$ when $0 < c < c_{11}$, while $\frac{\partial e_r^{DR*}}{\partial k_1} \ge \frac{\partial e_m^{DM*}}{\partial k_1}$ when $c \ge c_{11}$.

Next, we compare the effect of k_2 on the attack effort under different models. We can observe $\frac{\partial e_m^{CM*}}{\partial k_2} - \frac{\partial e_r^{DR*}}{\partial k_2} > 0$ directly. $\frac{\partial e_m^{CM*}}{\partial k_2} - \frac{\partial e_m^{DM*}}{\partial k_2} = \frac{8cq^2(1-\beta)Q_4}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$; therefore, the comparison depends on Q_4. The analysis result here is the same as $\frac{\partial e_m^{CM*}}{\partial k_1} - \frac{\partial e_m^{DM*}}{\partial k_1}$. According to $\frac{\partial e_m^{DM*}}{\partial k_2} - \frac{\partial e_r^{DR*}}{\partial k_2} = \frac{(1-\beta)qH_1Q_5}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, the comparison depends on Q_5. The analysis result here is the same as $\frac{\partial e_m^{DM*}}{\partial k_1} - \frac{\partial e_r^{DR*}}{\partial k_2}$.

From what has been discussed above, when $0 < c < c_{10}$, $\frac{\partial e_r^{DR*}}{\partial k_1} < \frac{\partial e_m^{CM*}}{\partial k_1} < \frac{\partial e_m^{DM*}}{\partial k_1}$ and $\frac{\partial e_r^{DR*}}{\partial k_2} < \frac{\partial e_m^{CM*}}{\partial k_2} < \frac{\partial e_m^{DM*}}{\partial k_2}$; when $c_{10} \le c < c_{11}$, $\frac{\partial e_r^{DR*}}{\partial k_1} < \frac{\partial e_m^{DM*}}{\partial k_1} \le \frac{\partial e_m^{CM*}}{\partial k_1}$ and $\frac{\partial e_r^{DR*}}{\partial k_2} < \frac{\partial e_m^{DM*}}{\partial k_2} \le \frac{\partial e_m^{CM*}}{\partial k_2}$; when $c_{11} \le c$, $\frac{\partial e_m^{DM*}}{\partial k_1} \le \frac{\partial e_r^{DR*}}{\partial k_1} < \frac{\partial e_m^{CM*}}{\partial k_1}$ and $\frac{\partial e_m^{DM*}}{\partial k_2} \le \frac{\partial e_r^{DR*}}{\partial k_2} < \frac{\partial e_m^{CM*}}{\partial k_2}$. □

Proof of Proposition 3.

$$\frac{\partial \pi_m^{CM*}}{\partial k_1} = \frac{4cqH_2\sqrt{H_1}(1-\beta)}{(H_1-4cH_2)^2},\ \frac{\partial \pi_m^{DM*}}{\partial k_1} = \frac{4cqH_2\sqrt{H_1}(1-\beta)}{(H_1-8cH_2)^2},\ \frac{\partial \pi_m^{DR*}}{\partial k_1} = \frac{2cqH_2\sqrt{H_1}(1-\beta)}{(H_1-4cH_2)^2}$$
$$\frac{\partial \pi_m^{CM*}}{\partial k_2} = \frac{4cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-4cH_2)^2},\ \frac{\partial \pi_m^{DM*}}{\partial k_2} = \frac{4cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-8cH_2)^2},\ \frac{\partial \pi_m^{DR*}}{\partial k_2} = \frac{4cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-4cH_2)^2}$$

Obviously, we can observe $\frac{\partial \pi_m}{\partial k_1}$ and $\frac{\partial \pi_m}{\partial k_2}$ are greater than zero in CM and DM, DR.

Firstly, we compare the influence degree of k_1 on manufacturer's profit under different models. Obviously, we can find out $\frac{\partial \pi_m^{CM*}}{\partial k_1} - \frac{\partial \pi_m^{DM*}}{\partial k_1} > 0$ and $\frac{\partial \pi_m^{CM*}}{\partial k_1} - \frac{\partial \pi_m^{DR*}}{\partial k_1} > 0$ directly.

$\frac{\partial \pi_m^{DM*}}{\partial k_1} - \frac{\partial \pi_m^{DR*}}{\partial k_1} = \frac{2cq(1-\beta)H_2\sqrt{H_1}Q_6}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, where $Q_6 = \beta^4\left(-32c^2q^2 + k_1^4\right) - 4\beta^3(-48c^2q^2 + 2k_1^4 + k_1^3k_2) + 2\beta^2\left(-208c^2q^2 + 12k_1^4 + 12k_1^3k_2 + 12k_1^2k_2^2\right) - 4\beta(-96c^2q^2 + 8k_1^4 + 12k_1^3k_2 + 6k_1^2k_2^2 + 6k_2^3k_1) - 128c^2q^2 + (2k_1 + k_2)^4$. According to $\frac{\partial^2 Q_6}{\partial c^2} = -64H_2^2 < 0$, Q_6 is a concave function of c. Let $Q_6 = 0$ and obtain $c_{12} = \frac{H_1\sqrt{2}}{-8H_2}, c_{13} = \frac{H_1\sqrt{2}}{8H_2}$. Therefore, when $0 < c < c_{13}$, $\frac{\partial \pi_m^{DM*}}{\partial k_1} > \frac{\partial \pi_m^{DR*}}{\partial k_1}$. When $c_{13} \le c$, $\frac{\partial \pi_m^{DM*}}{\partial k_1} \le \frac{\partial \pi_m^{DR*}}{\partial k_1}$.

Next, we discuss the influence degree of k_2 on manufacturer's profit under different models. Obviously, we can find out $\frac{\partial \pi_m^{CM*}}{\partial k_2} - \frac{\partial \pi_m^{DM*}}{\partial k_2} > 0$ and $\frac{\partial \pi_m^{CM*}}{\partial k_2} - \frac{\partial \pi_m^{DR*}}{\partial k_2} > 0$ directly. Because $\frac{\partial \pi_m^{DM*}}{\partial k_2} - \frac{\partial \pi_m^{DR*}}{\partial k_2} = \frac{2cq^2(1-\beta)^2\sqrt{H_1}Q_6}{(H_1-8cH_2)^2(H_1-4cH_2)^2}$, the comparison depends on Q_6. Therefore, the analysis result is the same as $\frac{\partial \pi_m^{DM*}}{\partial k_1} - \frac{\partial \pi_m^{DR*}}{\partial k_1}$.

From what has been discussed above, $\frac{\partial \pi_m^{CM*}}{\partial k_1} > \frac{\partial \pi_m^{DM*}}{\partial k_1} > \frac{\partial \pi_m^{DR*}}{\partial k_1}$ when $0 < c < c_{13}$, while $\frac{\partial \pi_m^{DM*}}{\partial k_1} \le \frac{\partial \pi_m^{DR*}}{\partial k_1} < \frac{\partial \pi_m^{CM*}}{\partial k_1}$ when $c_{13} \le c$.

$$\frac{\partial \pi_r^{DM*}}{\partial k_1} = \frac{4cqH_2\sqrt{H_1}(1-\beta)}{(H_1-4cH_2)^2}, \quad \frac{\partial \pi_r^{DR*}}{\partial k_1} = \frac{4cqH_2\sqrt{H_1}(1-\beta)}{(H_1-8cH_2)^2}$$

$$\frac{\partial \pi_m^{DM*}}{\partial k_2} = \frac{4cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-4cH_2)^2}, \quad \frac{\partial \pi_m^{DR*}}{\partial k_2} = \frac{4cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-8cH_2)^2}$$

Obviously, we can observe $\frac{\partial \pi_r}{\partial k_1}$ and $\frac{\partial \pi_r}{\partial k_2}$ are greater than zero in CM and DM, DR. We can observe $\frac{\partial \pi_r^{DM*}}{\partial k_1} - \frac{\partial \pi_r^{DR*}}{\partial k_1} > 0$ and $\frac{\partial \pi_m^{DM*}}{\partial k_2} - \frac{\partial \pi_m^{DR*}}{\partial k_2} > 0$ directly. Therefore, no matter what the value of c is, there is always $\frac{\partial \pi_r^{DM*}}{\partial k_1} > \frac{\partial \pi_r^{DR*}}{\partial k_1}$ and $\frac{\partial \pi_m^{DM*}}{\partial k_2} > \frac{\partial \pi_m^{DR*}}{\partial k_2}$. $\square$

Proof of Lemma 4. In w and p_f

$$\frac{\partial w^{DM*}}{\partial k_1} = \frac{16cqH_2\sqrt{H_1}(1-\beta)}{(H_1-8cH_2)^2}, \quad \frac{\partial w^{DM*}}{\partial k_2} = \frac{16cq^2\sqrt{H_1}(1-\beta)^2}{(H_1-8cH_2)^2}.$$

$$\frac{\partial w^{DR*}}{\partial k_1} = \frac{\partial w^{DR*}}{\partial k_2} = 0.$$

In DR, wholesale price has nothing to do with k_1 and k_2. Whereas in DM, $\frac{\partial w^{DM*}}{\partial k_1}$ and $\frac{\partial w^{DM*}}{\partial k_2}$ are both greater than 0.

$$\frac{\partial p_f^{CM*}}{\partial k_1} = \frac{-2q(\beta-1)^2\left[\beta^2\left(k_1^2k_2+2\beta cqk_1+2cqk_2\right)-8cq(\beta-1)(k_2+\beta k_1)+2k_1k_2(2k_1(1-\beta)+k_2(2-\beta))\right]}{(H_1-4cH_2)^2}$$
$$\frac{\partial p_f^{CM*}}{\partial k_2} = \frac{2q(\beta-1)^2\left[k_1H_1-2cq\left(4k_1+4k_2-4\beta k_1-3\beta k_2+\beta^2k_1\right)\right]}{(H_1-4cH_2)^2}$$
$$\frac{\partial p_f^{DM*}}{\partial k_1} = \frac{-2q(\beta-1)^2[k_2H_1-2cq(\beta-2)(-k_2(3\beta-4)-\beta k_1(\beta-2))\ ]}{(H_1-8cH_2)^2}$$
$$\frac{\partial p_f^{DM*}}{\partial k_2} = \frac{2q(\beta-1)^2\left[\ k_1H_1-2cq\left(8k_1+3k_1\beta^2-10\beta k_1+8k_2-7\beta k_2\right)\right]}{(H_1-8cH_2)^2}$$
$$\frac{\partial p_f^{DR*}}{\partial k_1} = \frac{-q(\beta-1)^2\left[2cq(\beta-2)^2(k_2+\beta k_1)+k_2H_1\ \right]}{(H_1-4cH_2)^2}$$
$$\frac{\partial p_f^{DR*}}{\partial k_2} = \frac{q(\beta-1)^2\left[k_1H_1-2cq\left(4k_1+4k_2-4\beta k_1-3\beta k_2+\beta^2k_1\right)\ \right]}{(H_1-4cH_2)^2}$$

Because $0 < \beta < 1$, $\frac{\partial p_f^{CM*}}{\partial k_1} < 0$, $\frac{\partial p_f^{DM*}}{\partial k_1} < 0$ and $\frac{\partial p_f^{DR*}}{\partial k_1} < 0$. in terms of $\frac{\partial p_f^{CM*}}{\partial k_2}$, we can simplify $4k_1 + 4k_2 - 4\beta k_1 - 3\beta k_2 + \beta^2 k_1$ to be $k_1(\beta-2)^2 + k_2(4-3\beta)$. We find a relationship $k_1H_1 - 2cq\left(4k_1 + 4k_2 - 4\beta k_1 - 3\beta k_2 + \beta^2 k_1\right) < k_1\left[H_1 - 2cq(\beta-2)^2\right]$. Because $q > \frac{H_1}{4c(\beta-1)(\beta-2)}$, $k_1\left[H_1 - 2cq(\beta-2)^2\right] < H_1\frac{\beta}{2(\beta-1)} < 0$. According to the above analysis, we can draw the conclusion $\frac{\partial p_f^{CM*}}{\partial k_2} < 0$. In terms of $\frac{\partial p_f^{DM*}}{\partial k_2}$, we can simplify $8k_1 + 3k_1\beta^2 - 10\beta k_1 + 8k_2 - 7\beta k_2$ to be $k_1(\beta-2)(3\beta-4) + k_2(8-7\beta)$. We find a relationship $2q(\beta-1)^2\left[k_1 H_1 - 2cq\left(8k_1 + 3k_1\beta^2 - 10\beta k_1 + 8k_2 - 7\beta k_2\right)\right] < k_1[H_1 - 2cq(\beta-2)(3\beta-4)]$. Because the quality satisfies the condition $q > \frac{H_1}{8c(\beta-1)(\beta-2)}$ in DM and $k_1[H_1 - 2cq(\beta-2)(3\beta-4)] < k_1 H_1 \frac{\beta}{4(\beta-1)} < 0$, $\frac{\partial p_f^{DM*}}{\partial k_2} < 0$. Since $\frac{\partial p_f^{DR*}}{\partial k_2}$ has the same relationship with 0 as $\frac{\partial p_f^{CM*}}{\partial k_2}$ has with 0, therefore $\frac{\partial p_f^{DR*}}{\partial k_2} < 0$. □

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