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Simulation Study on Topology Characteristics and Cascading Failure of Hefei Subway Network

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Abstract: The structural characteristics and robustness of subway networks are important for improving the safety and efficiency of subway operations. Based on complex network theory, this study analyzed the structural characteristics of the Hefei subway network and evaluated its robustness after suffering from accidents. Specifically: (1) A model of the Hefei subway network was established using the space-L method, and its topological structural characteristics were quantitatively analyzed; (2) An improved cascading failure simulation model was established, and a node importance evaluation system was developed to identify the critical nodes in the Hefei subway network; (3) A simulation analysis was conducted to evaluate the robustness of the Hefei subway network under different scenarios. The results show that the Hefei subway network is different from a scale-free network and small-world network, and the structure was most severely damaged when facing attacks against critical nodes in the cascading failure scenario. Moreover, as the value of the parameter of the control node capacity in the cascade failure model varied, the degree of damage to the subway network also varied considerably. We believe that the results obtained from the study could provide a reference for the construction and planning of the subway network.

Keywords: Hefei subway network; complex network; cascading failure; robustness; space-L; simulation

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1. Introduction

With the economic development of cities and the growth in residents' travel demands, subway transportation has become a major transportation mode for daily travel in many cities due to the fact of its convenience and efficiency. At present, 47 cities in China have opened subways with more than 5400 stations in operation. In this context, a comprehensive and systematic analysis of the structural characteristics and complex features of an urban subway transportation network can promote the development of subway transportation towards being more efficient, safe, and convenient. Thus, many scholars have started to research urban subway transportation networks.

Some scholars have analyzed the complexity of urban subway network topology. Bao et al. [1] found that a composite transportation network consisting of a subway network and a bus network has a more stable structure and possesses stronger robustness in response to attacks. Xing et al. [2] analyzed the destructive resistance of the network structure by comparing the degree of damage to the urban subway network structure by random attacks and node-specific attacks. Shi et al. [3] found that after studying the evolution of the Shanghai rail transit network structure, the robustness of the network was improving year by year and provided new ideas for the identification of central nodes. Zhang et al. [4] emphasized the impact of passengers on the subway network structure in their study and used the Nanjing subway network to confirm that the vulnerability of a subway network increases with the increase in the outgoing passenger flow. Ding et al. [5] found that by analyzing the structure of subway systems in several cities around the world, the network of the average degree rises slightly with the increase in network size, but the complexity and connectivity of the network decreases accordingly. Ma et al. [6] analyzed the vulnerability change characteristics of a subway network under the condition of passenger flow interference, and they took the Xi'an subway network as the object for verification, which fully proved the influence of passenger flow characteristics and passenger demand on the vulnerability of the subway network. Zhu et al. [7] constructed an improved vulnerability assessment model of subway networks based on four factors that have an impact on a subway's operation and verified the application value of the model through practical cases. These studies provide a thorough analysis of the structure of urban subway networks, but there is a lack of studies that further integrate the study of subway network junction robustness with the study of subway network cascade failure phenomena. However, as the construction of an urban subway transportation network continues, it will lead to the increasing complexity of the subway network, where the failure of a subway station will cause a chain reaction of more node failures. Therefore, the concept of cascading failures began to be applied to the study of subway transportation networks. Using complex network theory, Cai et al. [8] carefully analyzed the structural characteristics of the Changsha subway network and conducted an in-depth study on the trend of robustness changes in the subway network under the cascading failure phenomenon. Xiong et al. [9] used the coupled map lattice model as a tool to simulate the propagation process of urban rail traffic congestion under different traffic states, thus providing an in-depth analysis of the structural characteristics of the subway network when the cascading failure phenomenon occurs. Fan et al. [10] combined a linear model with threshold values to simulate the cascading failure process of a subway network, verifying that the spatial and temporal characteristics of traffic flow can cause cascading failures in subway operations. Li et al. [11] studied the congestion propagation process of a subway network and conducted a comparative study of the differences in the results by selecting different initial failure sites for cascading failure simulation experiments. Wang et al. [12] developed an analysis of the congestion characteristics of urban subway networks and advocated for paying attention to the impact of cascade failure of network sites on network service performance degradation. Xie et al. [13] proposed a rail transit cascade failure improvement model and conducted a simulation to validate it with the Zhengzhou rail transit network as the research object. Yang et al. [14] established a cascade failure simulation model to simulate the effects of different attacks on the Shanghai rail transit network and analyzed the reliability and destructive resistance of the subway network. A comparative analysis of these research results shows that the research objects of the cascading failure phenomenon of urban subway networks were mostly Shanghai, Changsha, and Zhengzhou, while studies on the structural characteristics of the subway network in Hefei and the cascading failure phenomenon are fewer.

In summary, there are two points for improvement in the study of urban subway transportation networks: (1) Most of the existing studies focused on the topology and robustness of the subway network, but there were fewer studies on the cascade failure phenomenon caused by the failure of key stations; (2) The urban subway transportation networks were mainly studied in more economically developed cities, such as Beijing, Shanghai, and Guangzhou, while there were fewer studies on the subway network in Hefei, and there was a lack of studies comparing subway networks in different periods.

Therefore, in order to address these two points so that they can be improved, this paper took the Hefei subway network as the research object, analyzed in depth the complex characteristics of the Hefei subway network in different periods and based on this, established an improved cascading failure simulation model. These works highly combined the study of the structural characteristics of the Hefei subway network with the study of the cascading failure phenomenon of the subway network so as to systematically evaluate the robustness of the Hefei subway network and further find weak areas and stations that need to be strengthened. This study could provide a reference for the maintenance and planning of the Hefei subway network and also has reference value for subway construction in other cities.

2. Basic Structure Analysis of the Hefei Subway Network

2.1. Hefei Subway Topology Network Construction

The long-term plan for the Hefei subway's construction has 15 subway lines and one airport-specific line; the short-term plan contains 8 subway lines and one airport-specific line (Line S1). As of May 2022, Lines 1 to 4 and the southern section of Line 5 are already in operation. The paper combined the actual construction and planning objectives of the Hefei subway project with the subway network consisting of Lines 1 to 8 and S1 in Hefei, which contain 217 different subway stations, and a 217 $\times$ 217 adjacency matrix was constructed from this network.

To objectively map the geographical distribution of the stations and the distance between the station pairs, the space-L method was used to construct a topological map of the Hefei subway network when drawing the topological structure of the subway network. A traffic network model constructed using the space-L method can clearly reflect the connection relationship between sites, which provides convenience for the study of the topological characteristics of the traffic network and the analysis of network vulnerability [15–17]. As of now, the space-L method has been widely used in research on public transport network structures. Yi et al. [18] used the space-L method in the study of transportation networks and improved it to analyze the dynamic spatial evolution characteristics of network models with this method. Fei et al. [19] proposed an augmented space-L network model considering the difference in the bidirectional paths and transportation capacity of bus routes and used it to express the topology structure of the bus network. The topology map of the Hefei subway network constructed based on the space-L method is shown in Figure 1.

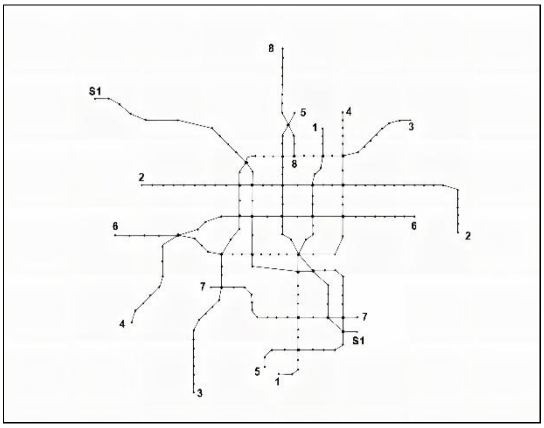


Figure 1. Hefei Subway Network Topology Diagram.

Modeling description of the topology diagram of the Hefei subway network:

(1) The data used in the topology diagram were sourced from the public scheme of the Hefei subway's construction planning and actual subway operation data, which are generally consistent with the actual future of the Hefei subway's construction in terms of the general development direction and construction scale. Although there may be some minor differences in the specific station planning, the impact on the analysis results was small.

(2) The nodes in the subway network are unique, and it was stipulated that an adjacency matrix with a value of 1 indicates the connected edge between station pairs in the line network, and, conversely, if there is no relationship between two nodes, it is indicated by 0.

(3) The difference between the upstream and downstream of a subway traffic line was not considered.

(4) The track train model, frequency, station traffic, facilities, and other factors were not considered.

(5) According to the Hefei subway's construction plan, the four stations within the interval from Qinglonggang station to Beiyanhu station of Line 4 will be classified within Line 6 after the passage of Line 6, and a topological map will be drawn according to the planning scheme after the passage of Line 6.

2.2. Statistics on the Basic Characteristic Parameters of the Hefei Subway Network

2.2.1. Indicators of the Subway Network's Characteristic Parameters

The subway network's characteristic parameters include indicators related to degree, betweenness, and connectivity and indicators related to the shortest path length, which are defined and calculated as described below.

(1) Degree-related indicators

This includes node degree, average degree, and cumulative degree. The node degree determines the scale of the node connected to the other nodes, and the higher the node degree, the greater the importance of the node.

The node degree is the most basic parameter to be studied to characterize the network structure, and is defined as follows:

$$k_i = \sum h_{ij} \tag{1}$$

In: k_i represents the degree of node i; h_{ij} is the element in the adjacency matrix H, if the node i and j are directly connected to the edge, the value is 1, otherwise the value is 0.

The average degree of a network is a positive global indicator, which can measure the density level of the node distribution of the entire network. It is defined as follows:

$$\langle k \rangle = \frac{\sum_{i=1}^{N} k_i}{N} \tag{2}$$

The cumulative degree represents the probability of the occurrence of a node that is not less than a certain node degree. It is defined as follows:

$$P_k(K > k) = \sum_{k'>k}^{\infty} p(k') \tag{3}$$

In: $p(k)$ is the degree distribution function, which describes the distribution probability of nodes with degree k.

(2) Betweenness

Betweenness can reflect the influence of the nodes in the network. The calculation formula is as follows:

$$BC_i = \sum_{\substack{t \neq j \neq i \\ t < j}} \frac{{N_d}^{tj}(i)}{{N_d}^{tj}} \tag{4}$$

In: ${N_d}^{tj}$ is the number of shortest paths between nodes t and j; ${N_d}^{tj}(i)$ is the number of all shortest paths between nodes i and j passing through node i.

(3) Connectivity

Network connectivity is a parameter that describes the overall characteristics of the network [20]. It can be used as a positive indicator to reflect the overall development scale of the network. The greater the degree of connectivity, the more edges the network has, and the more stable the network structure. The specific expression formula is as follows:

$$z = \frac{e^E}{e^E_{\max}} \tag{5}$$

In: e^E is the actual number of edges in the network; $e^E_{\max}$ is the maximum number of edges the network can theoretically have.

A subway network has plane characteristics [21], which will lead to new nodes at the intersection of different lines. Therefore, the maximum number of edges in the subway network is $3n-6$; so, the calculation formula of the connectivity of the subway network can be further obtained, as shown below:

$$z = \frac{e^E}{e^E_{\max}} = \frac{e^E}{3n-6} \tag{6}$$

In: n is the number of functioning nodes in the current network.

(4) Indicators related to the shortest path length

The d_{ij} is the shortest path length, which is the travel path with the lowest number of edges between nodes i and j; the network diameter is the largest term among all of the shortest paths; the average shortest path length is the average of the sum of all of the shortest paths in the network.

2.2.2. Relevant Parameters of the Hefei Subway Network

The adjacency matrix packets obtained from the realistic subway network mapping were imported into MATLAB software for data analysis, and the results are shown in Table 1 below.

Table 1. Indicators indicating the topology of the Hefei subway network.

Indicator	Value	Indicator	Value
Number of Nodes	217	Network Diameter	43
Number of network edges	237	Clustering coefficient	0.0025
Average Degree	2.1843	Maximum number of edges	645
Network Efficiency	0.1056	Connectedness	0.3674
Average Betweenness	0.0306	Average shortest path length	14.1436

Statistical results show that the Hefei subway network contains a total of 217 different stations, and more than 80% of the stations have a node degree of no more than two. This reflects that most nodes only maintain contact with neighboring nodes on the same line, and the overall structure of the network is sparse. The average node degree of the network is 2.1843, which means that each node in the network has two neighboring nodes on average. Among them, there are 17 nodes with a node degree of one, which are the first and last stations of Line 1 to Line 8 and S1; 172 nodes with a node degree of two, which are only passed by one line; 1 node with a node degree of three, which is the intersection of the last station of Line 8 and Line 3. Among the remaining 27 nodes, 26 stations are the transit stations of two lines with a node of degree four; one node is at the intersection of three lines, because it is at the intersection of three lines; thus, it has a node degree of six.

Meanwhile, the average betweenness and clustering coefficient level of the whole network is comparatively low, and the network connectivity is 0.3674, which reflects the small scale of the number of interchange stations in the Hefei subway network and the need to consume more time resources between nodes to form effective connectivity; from a planning perspective, the Hefei subway network still has much room for improvement.

The network diameter is 43; thus, there are two nodes in the network that need to pass through at least 43 stations for contact to occur. The average shortest path length of the network is 14.1436; so, the average two subway stations need to pass through at least 14 stations between them to maintain contact.

Figure 2 shows the specific situation of the node degree of the Hefei subway network, while Figure 3 tallies its probability distribution. The node degree of most nodes in the whole network is low, the percentage of nodes with a large node degree is very small, and the connection between network nodes shows a loose trend.

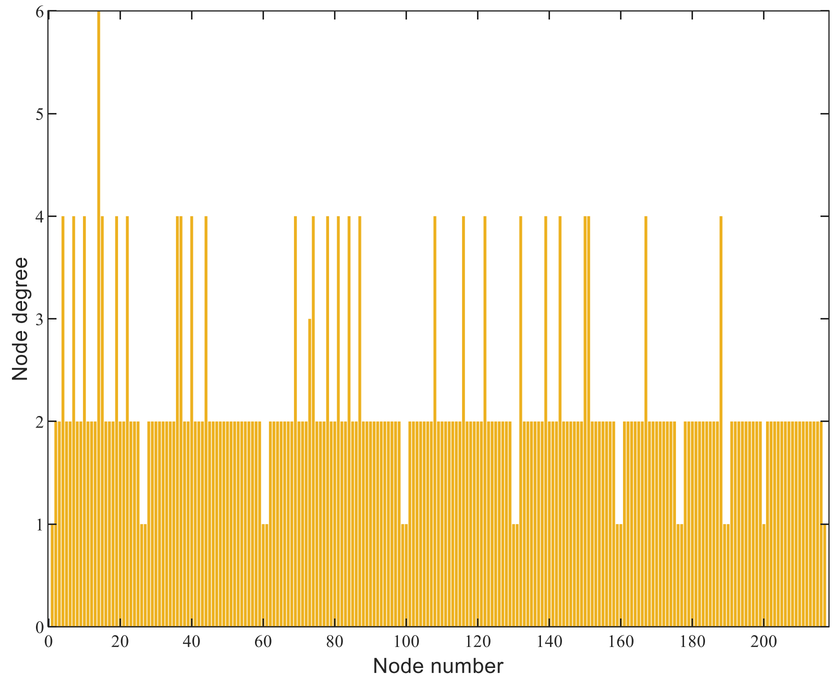


Figure 2. Node degree distribution.

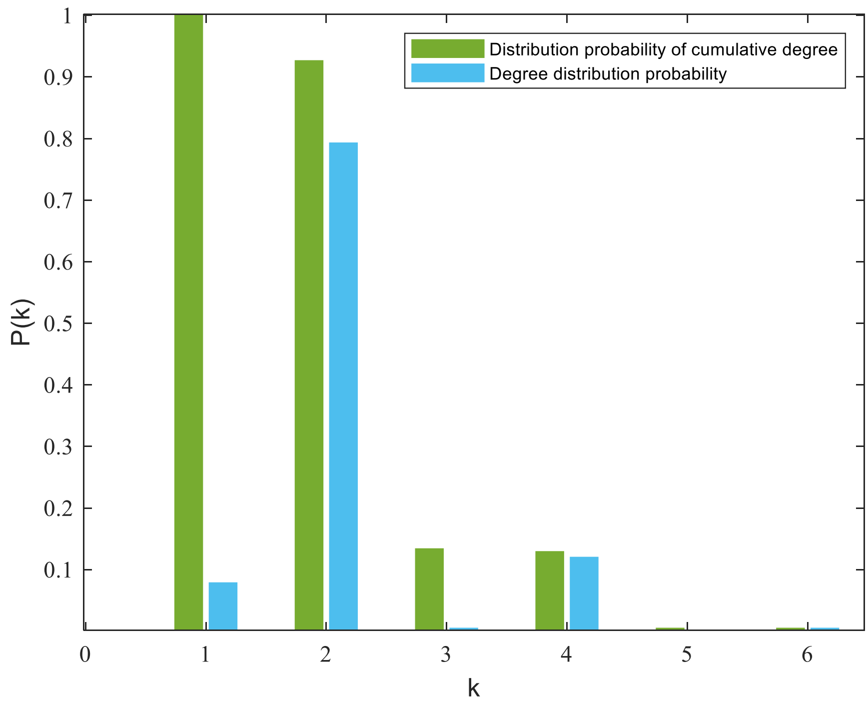


Figure 3. Probability distribution of node degree.

Figure 4 shows the probability distribution and cumulative probability distribution of the shortest path length of the nodes in the Hefei subway network, and Figure 5 is a schematic diagram of the results of fitting the Gaussian function to the probability distribution of the shortest path length of the nodes. It can be seen that 5.63% of the nodes have a shortest path length of 12, which has the largest distribution probability; 43.16% of the nodes have a shortest path length of no more than 14.1436; and more than 82.23% of the nodes have a shortest path length of 20 or less, which represents that most of the passengers have a short travel distance. In addition, although the network diameter reaches 43, the average shortest path length of the network is 14.1436. From the overall residents' travel situation, the station locations of the Hefei subway transportation network are reasonably set and can meet the travel demand of most passengers. The results of fitting the Gaussian function for the probability distribution on the shortest path length of the nodes in the Hefei subway network are satisfactory, with R^2 equal to 0.9976, and the fitted results are expressed as follows:

$$P(d) = 0.05655 \times e^{-((d-13.32)/10.37)^2} \tag{7}$$

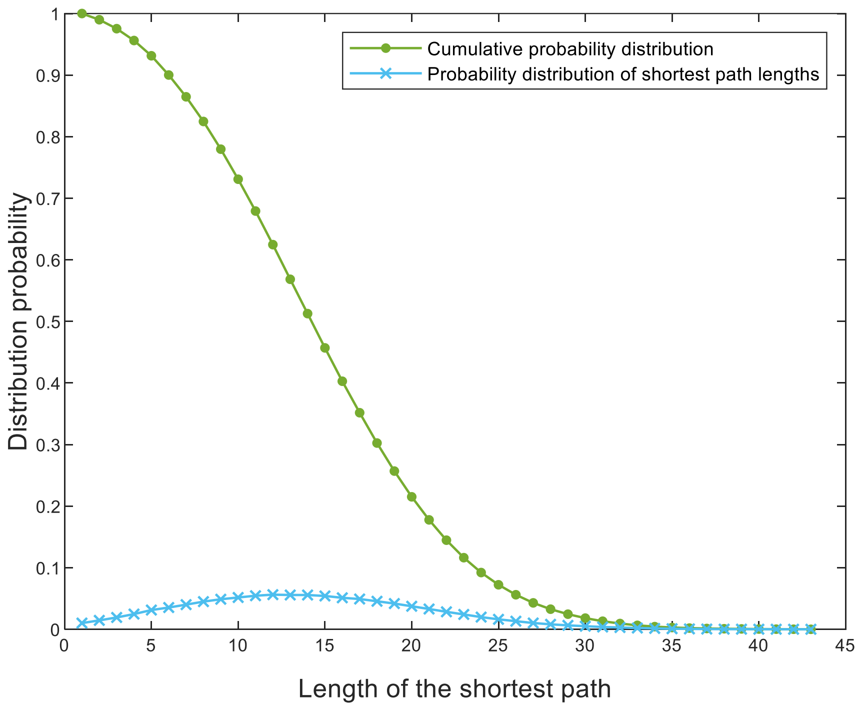


Figure 4. Probability distribution and cumulative probability distribution of the shortest path length.

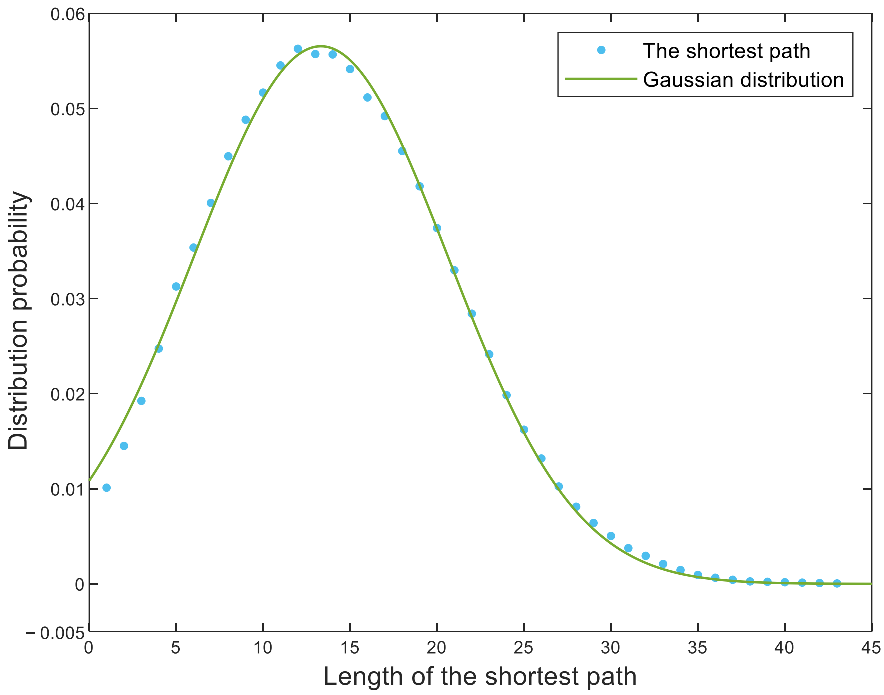


Figure 5. Fitting curve of probability distribution of shortest path length.

Figure 6 is a betweenness distribution diagram of the Hefei subway network's nodes. On this basis, Table 2 displays further statistics on the distribution probability of the node betweenness. The results show that more than 96% of the nodes have a betweennesses below 0.1, and only seven sites have a betweennesses above 0.1. The highest betweenness is 0.1309, and the corresponding node is Honggang Station. Most of the high betweenness nodes are transit sites, which are the necessary sites for many of the shortest paths, such as node 10, node 14, node 15, node 44, node 81, node 84, node 108, node 139, and node 167, which are all important transit stations in the subway network, and they are all passed by at least two different lines. In addition, the betweenness of 44.24% of the nodes in the network is lower than 0.02, and the betweenness of 17 nodes is 0; these nodes are distributed in the edge positions in the network structure and have a limited contribution to optimal path planning.

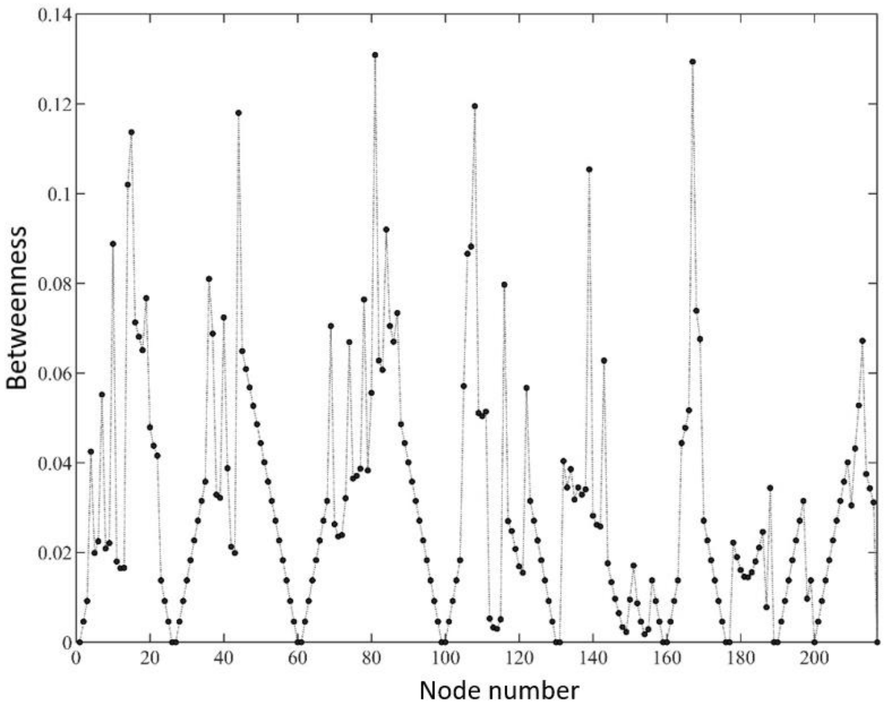


Figure 6. Distribution of betweenness.

Table 2. Distribution interval of node betweenness.

Betweenness Interval	Distribution Probability	Betweenness Interval	Distribution Probability
0.1–0.2	3.23%	0.04–0.06	11.98%
0.08–0.1	2.30%	0.02–0.04	28.57%
0.06–0.08	9.68%	0–0.02	44.24%

2.3. Statistics on the Changes of Indicators of Hefei Subway Network in Different Periods

Using MATLAB programming, the parameter data for the Hefei rail transit network over the ten-year construction and development period from 2016 to 2025 were obtained. The statistical results are shown in Table 3. With the continuous increase in the number of new lines in the subway network, the network coverage area is becoming larger and larger, and the overall changes to the average node degree and average shortest path length and connectivity degree of the network are also increasing. At the same time, due to the continuous increase in the network scale, the distance between the nodes becomes larger, so the average betweenness of the Hefei subway network and the overall change in network efficiency show a declining trend.

Table 3. Parameter values of Hefei subway network in different periods.

Year	Number of Nodes	Number of Edges	Average Node Degree	Network Efficiency	Average Betweenness	Clustering Coefficient	Average Shortest Path Length	Connectivity
2016	23	22	1.9130	0.2486	0.1667	0	8	0.3493
2017	46	45	1.9565	0.1677	0.1233	0	11.8493	0.3409
2019	77	77	2	0.1329	0.0829	0	13.4282	0.3422
2020	95	96	2.0211	0.1204	0.0743	0	14.8172	0.3441
2021	122	127	2.0820	0.1214	0.0429	0.0036	11.2208	0.3528
2022	136	143	2.1029	0.1220	0.0433	0	12.5995	0.3557
2023	155	160	2.0645	0.1060	0.0462	0	15.1479	0.3486
2024	182	192	2.1099	0.1018	0.0399	0.0024	15.3742	0.3556
2025	217	237	2.1843	0.1056	0.0306	0.0025	14.1436	0.3674

The newly added Line 4 in 2021 connects multiple subway lines, which increases the convenience of transfer between stations; so, the average shortest path length becomes smaller. Similarly, the planning for Subway Line 6 and Line S1 in 2025 will be further improved. With the convenience of transfer at the subway network sites, the average shortest path length declined again; this change has also led to an increase in network efficiency and average betweenness in 2021, 2022, and 2025; after the expansion of Lines 2 and 3 in 2023, many nodes that do not intersect with other lines will generate, resulting in a decrease in the average degree and connectivity of the network.

From 2016 to 2020, there were few operating lines in the Hefei subway network, and the average clustering coefficient was 0, and the subway planning from 2022 to 2023 is only to expand the existing lines. This leads to the sparse geographical distribution of the nodes of the subway network in Hefei. Therefore, during this period, the average clustering coefficient of the subway network is 0. After that, with the opening of Lines 7 and 8, the distribution of network nodes will be increasingly clustered.

3. Analysis of the Complexity of the Hefei Subway Network

Using MATLAB programming to analyze the complex characteristics of the subway traffic network in Hefei city.

(1) Analysis of the distribution pattern of the node degree

In a study on the structural characteristics of subway networks, some scholars have analyzed the regulation of the degree distribution of subway network nodes to verify the complex characteristics of subway networks. For example, Yang et al. [22] obtained the degree distribution formula of the nodes of the Beijing subway network by fitting, thus proving that the Beijing subway network has the characteristics of a scale-free network. Wang et al. [23] also found the scale-free network property of the Shanghai subway network by comparing the node degree distribution with the approximation of the power distribution.

The reason scholars conducted it this way is that the degree distribution of the nodes of the scale-free network is approximately close to the power distribution [24]. Therefore, in order to verify whether the Hefei subway network is a scale-free network, the cumulative degree distribution of the Hefei subway traffic network was fitted in this paper. The fitted curves are shown in Figure 7, and the fitted equations are shown below.

The formula for fitting the power law function is shown below:

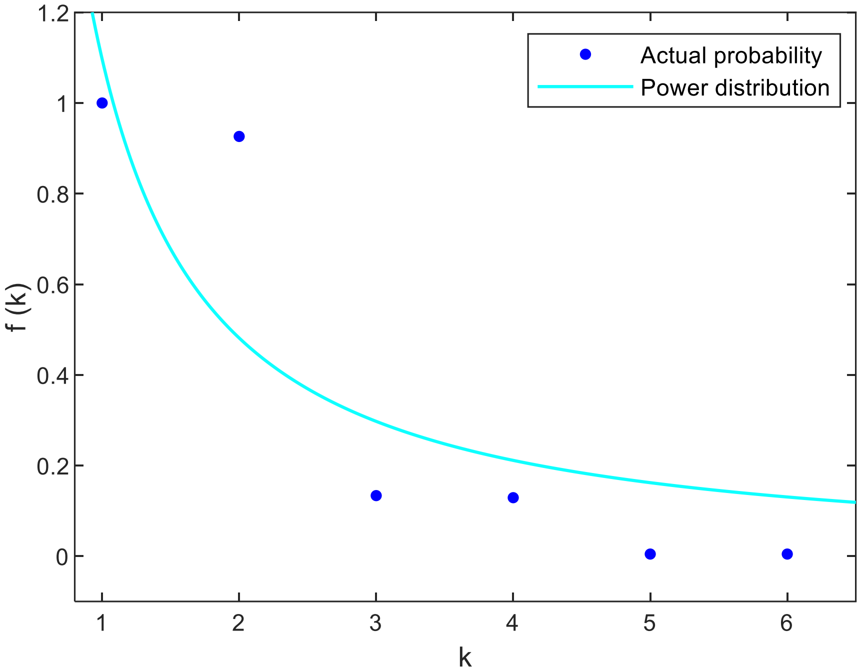
$$f(k) = 1.097 \times k - 1.188, R^2 = 0.7411 \tag{8}$$

The fitting formula for the exponential function is shown below:

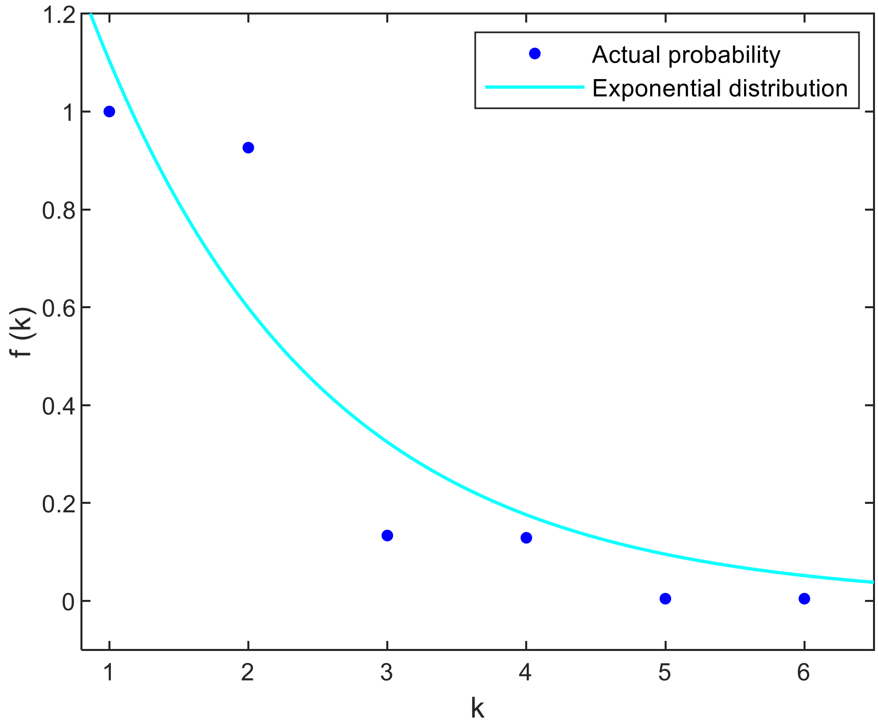
$$f(k) = 2.036 \times e^{-0.6119k}, R^2 = 0.8461 \tag{9}$$

The formula for fitting the Gaussian function is shown below:

$$f(k) = 1.216 \times e^{-((k-1.459)/1.037)^2}, R^2 = 0.9913 \quad (10)$$

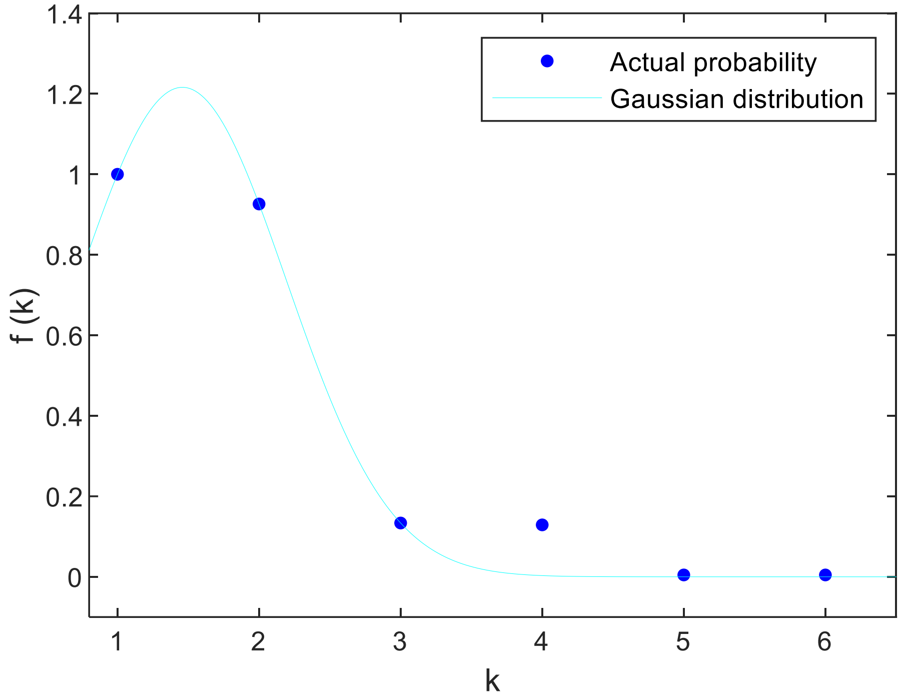


(**a**)



(**b**)

Figure 7. *Cont.*



(c)

Figure 7. Fitting curve of cumulative probability distribution of node degree. (**a**) Fitting curve for power law function. (**b**) Fitting curve of the exponential function. (**c**) Fitting curve of Gaussian function.

By comparing the three values of R^2, results of which are shown in Figure 7, it can be seen that the fitting effect of the power function of the degree distribution of the Hefei subway network is not ideal, while the fitting effect of the Gaussian function is the best, and the node degree value distribution has the characteristics of concentration, symmetry and uniformity, which is in line with a Gaussian distribution; therefore, it can be judged that the Hefei subway network does not have the characteristics of a scale-free network.

(2) Analysis of the small-world network characteristics

To verify whether the Hefei subway network has the characteristics of a small-world network, the clustering coefficients and the average shortest path lengths of the Hefei subway network and a random network of the same size were calculated separately (the same size means that the number of nodes, the number of edges, and the average degree of the two are equal), and the results are shown in Table 4.

Table 4. Comparison of Hefei subway network with a random network with the same size.

Indicators	Clustering Coefficient	Average Shortest Path Length
Hefei Subway Network	0.0025	14.1436
Random Network	0.0069	5.9350
Shanghai Subway Network	0.3667	15.2208
Shenzhen Subway Network	0.3678	12.0769

The results show that the clustering coefficient of the Hefei subway network only reaches 36.23% of the random network, but the average shortest path length is much longer than that of the random network. In terms of network structure, the nodes of the Hefei subway network are loosely distributed, and the connections between the nodes are not sufficiently tight. Since small-world networks are characterized by tight connections between nodes and shortest path lengths that are small [25,26], the Hefei subway transportation network does not have the characteristics of a small-world network. However, among the existing related studies on subway networks, Geng et al. [27] found that the Shenzhen subway network has obvious small-world characteristics, and Du et al. [28] found that the small-world characteristics of the Shanghai subway network are becoming increasingly significant with the development of the subway network. Therefore, as the demand by Hefei residents for travel convenience increases, the number of stations and the scale of the connected edges of the urban subway network will continue to increase, and the complexity of the network will further increase, so that the subway network in Hefei is bound to evolve towards a small-world network.

(3) Analysis of the degree of aggregation of the subway stations

The average clustering coefficient of the Hefei subway network is 0.0025, which reflects the loose distribution of stations. The six stations with non-zero clustering coefficients, all of which are important interchange stations in the subway network, are counted in Table 5. Among them, Hefei South Station, South Square Station, and Shengda Station are triangular in geographic location, and Yicheng Station, Provincial Administrative Center East Station, and Huifu Road Station are also triangular in geographic location, as shown in Figure 8. The triangular structure can improve the overall connectivity of the subway network and can reflect the development level of the urban rail transit network [29], the triangular structure of the Hefei subway network is less, and the vast majority of the stations have a clustering coefficient of 0. Therefore, the subway network has a low tolerance for operational failures, and once a node has a functional failure, it may cause a serious blow to the normal operation of the whole network.

Table 5. Clustering Coefficient Statistics.

Station Number	Station Name	Clustering Coefficient
14	Hefei South Station	0.0476
15	South Square Station	0.1000
143	Shengda Station	0.1000
150	Yicheng Station	0.1000
151	Provincial Administrative Center East Station	0.1000
188	Huifu Road Station	0.1000

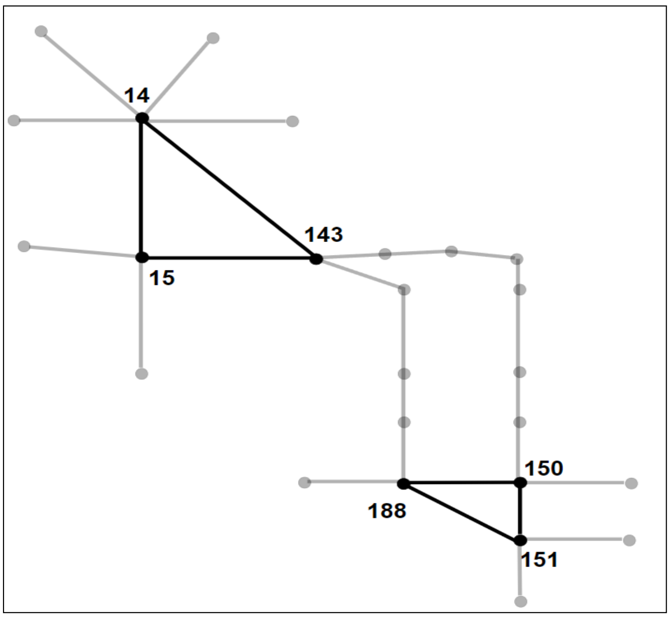


Figure 8. Geographical distribution of the triangular structure in the Hefei subway network.

(4) Node degree and betweenness relationship analysis

In order to explore the relationship between the degree and the betweenness of the nodes in the Hefei subway network, a fitting analysis was performed based on the data of the degree and betweenness of 217 nodes in the Hefei subway network, which is shown in Figure 9, and a fitting curve of the relationship between the degree and the average betweenness of all nodes with the same degree values was also obtained, which corresponds to the black diagonal line in Figure 9. The expression formula of the fitting curve is shown in Equation (11). From the equation of the expression of the fitted curve, this fit worked well, and there was a positive correlation between the degree of the nodes and the betweenness, where the higher the degree value, the higher the average value of betweenness. From the overall situation shown in Figure 9, some of the nodes in the Hefei subway network have lower degree values, but they have higher betweenness values because of the higher proportion of shortest paths passed on these nodes.

$$f(k) = 1922 \times k - 1793, R^2 = 0.9613 \tag{11}$$

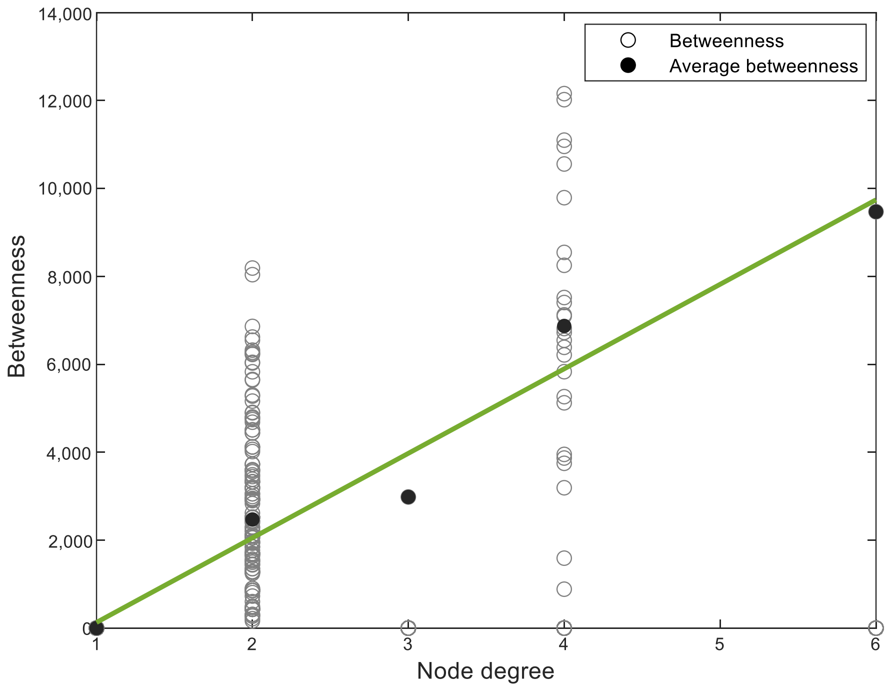


Figure 9. Relationship between Node degree and Betweenness.

(5) Analysis of the closeness of the nodes

Figure 10 reflects the distribution of the average closeness under different node degrees, and a function fitting is performed on the data. The closeness of a node refers to the reciprocal of the sum of the shortest path lengths from a node to other nodes. The higher the closeness, the closer the node is to the core area of the network. Among the 217 nodes in the Hefei subway network, only six nodes have a closeness not zero, namely, Hefei South Station, South Square Station, Shengda Station, Yicheng Station, Provincial Administrative Center East Station, and Huifu Road Station, as shown in Table 6. These six nodes are close to the central area of the subway network and are the necessary transit points for many shortest paths, while most of the remaining stations are close to the edge of the network structure, and the communication between node pairs lacks convenience; so, the closeness of the nodes is not high. A straight line with a slope of 0.0130 was obtained after fitting the data of the nodal degree and the closeness of the nodes in the Hefei subway network, which indicates that the two are positively correlated.

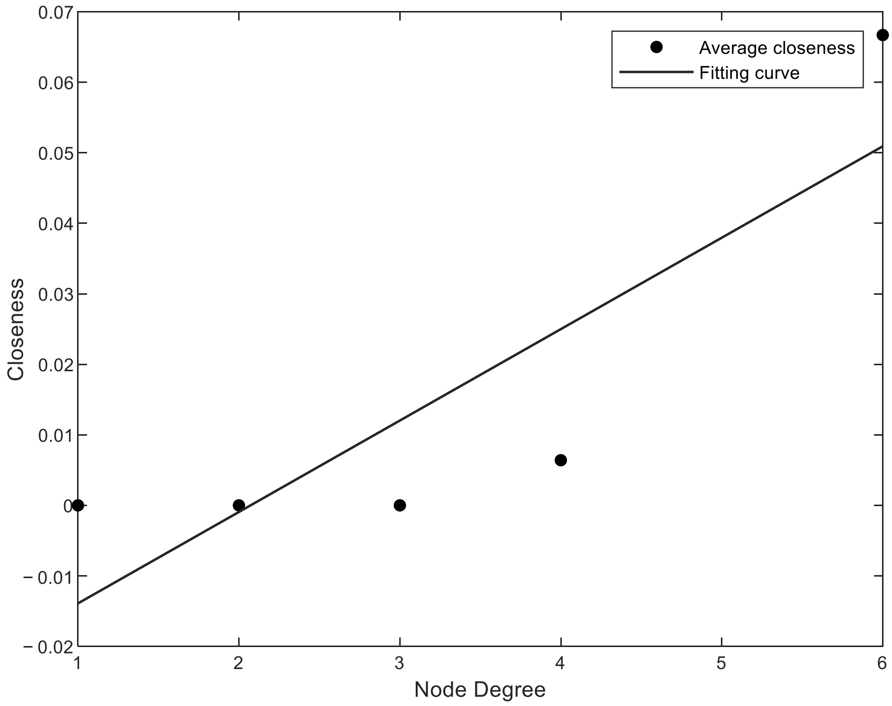


Figure 10. Relationship between Node Degree and Closeness.

Table 6. Relationship between node Degree and Closeness.

Station Name	Node Degree	Closeness	Average Closeness
Hefei South Station	6	0.0667	0.0667
South Square Station	4	0.1667	
Shengda Station	4	0.1667	
Yicheng Station	4	0.1667	0.0321
Provincial Administrative Center East Station	4	0.1667	
Huifu Road Station	4	0.1667	

Through the above analysis results, the node degree, betweenness, and closeness index of the Hefei subway network were positively correlated. The node degree reflects the importance of nodes from an individual point of view, and the betweenness and closeness reflect the importance of nodes from an overall point of view. This reflects that the importance of the nodes in the Hefei subway network from a microperspective is similar to that of the macroperspective.

4. Improved Cascade Failure Model for the Hefei Subway Network

The status of subway stations is divided into normal status and failure status. During the normal operation of a station, it may enter the failure state under the influence of many factors. In the failure state of a station, it cannot perform its transportation function, and the current traffic will migrate to an adjacent station. At the same time, an adjacent station may fail to function due to the large amount of traffic it receives in a short period of time. This is more likely to cause more stations to fail due to the internal surge and, thus, enter the failure state. In order to avoid serious damage to the subway network caused by the cascading failure phenomenon, it is necessary to establish an appropriate simulation model for research based on the actual situation.

4.1. Node Load Settings

4.1.1. Initial Load

There are many different ways to set the initial load of a node, but considering the impact of COVID-19 on passenger flow in the Hefei subway transportation network and the theoretical guidance based on the research results of relevant scholars [30,31], this paper defined the initial load of the node according to the betweenness and defined the initial load of node i as $G_i^{(t_0)}$, as shown below:

$$G_i{}^{(t_0)} = BC_i \tag{12}$$

4.1.2. Node Capacity

The planning of subway stations is affected by technical, topographical, demographic, and economic factors; so, the maximum carrying capacity of each station is different. The ML model proposes that there is a positive correlation between the load-bearing capacity of a node and its initial load [32]. According to this characteristic, the capacity of a node is recorded as C_i as a parameter to judge whether the node is in a normal state. If the actual passenger flow of the station is lower than C_i, the function of the station is normal; otherwise, the station is in an abnormal state. The specific definition formula of node capacity is as follows:

$$C_i = \partial G_i{}^{(t_0)} + G_i{}^{(t_0)} \tag{13}$$

In: ∂ $(\partial > 0)$ represents the node's ability to handle loads that exceed the initial load.

4.1.3. Ultimate Load

The load level of the node at the time of failure is defined as its ultimate load, which is the maximum passenger flow that the station can bear. When the actual passenger flow of a subway station exceeds the limit load, the station will stop running, and the function of the station is considered to be invalid at that moment.

The greater the difference between the ultimate load and C_i, the stronger the compressive capacity of the node. The specific definition is as follows:

$$C_{lt}(i) = \beta C_i \tag{14}$$

In: $\beta(\beta \geq 1)$ reflects the maximum capacity of the subway station for passenger flow.

4.2. Load Redistribution Model

If the function of a node in the subway network fails, its load will be actively transferred to other nodes, which is the load redistribution stage. At this stage, its adjacent nodes will be constrained by various factors when receiving traffic, such as objective environment, and subjective attitude of passengers. Therefore, θ is set as a constraint coefficient to control the proportion of load received by adjacent nodes. Assuming that node i fails and j is its adjacent node, the specific allocation model is expressed as follows:

$$G_j{}^{(t)} = G_j{}^{(t-1)} + \Delta G_j{}^{(t)} \tag{15}$$

$$\Delta G_j{}^{(t)} = \theta_j G_i = \frac{BC_j}{\sum\limits_{j \in \varphi} BC_j} G_i \tag{16}$$

In: $G_j{}^{(t)}$ is the traffic flow of node j at time t; $\Delta G_j{}^{(t)}$ is the newly added traffic flow of node j at time t; φ is the set of adjacent nodes of the node fails; θ_j is the constraint coefficient, representing the proportion of load that node j can receive.

A propagation function of the load of the failed node is established to determine whether the receiving node of the load will fail due to the fact of overloading as follows:

$$f_j^{(t)} = \begin{cases} 1\,, & G_j^{(t_0)} \le G_j^{(t)} < C_j \\ 0\,, & C_j < G_j^{(t)} < C_{It}(j) \\ -1\,, & G_j^{(t)} \ge C_{It}(j) \end{cases} \tag{17}$$

In: $f_j^{(t)} = 1$ represents the normal function of the node; $f_j^{(t)} = 0$ represents that the node is overloaded and has not failed; $f_j^{(t)} = -1$ represents that the node is in a failed state.

(1) When the actual load of the node is between the initial load and the node capacity, the node is defined as a normal state.
(2) When the actual load of the node has not yet reached the maximum limit but has exceeded the node capacity, it is defined that the node is in a state of overload but not failure. At this time, reasonable dredging of the load can speed up the return of the node to a normal state.
(3) When the actual load of the node exceeds the limit capacity, the node is defined as a failure state. At this time, the station will be interrupted, and all its traffic will be propagated along the edge of the network to adjacent nodes.

5. Impact Analysis of Cascading Failures

5.1. Attack Test

5.1.1. Attack Strategy

Attacks on a subway transportation network can generally be classified into two types. The first is to select random nodes to attack. For example, the site has random failures, such as a signal interruption, earthquake, flood, and machine aging. The second is a deliberate attack in which a specific node is selected for priority attack according to a certain rule, such as wars, fires, traffic accidents, and other failures caused by human factors. In order to obtain more comprehensive research results, this study used two attack methods to attack the Hefei subway network at the same time; on this basis, eight different attack modes can be further subdivided, which are random attacks in noncascading failure scenarios, attack mode conditioned by node degree in noncascading failure scenarios, betweenness-conditioned attack mode in noncascading failure scenarios, attack mode conditioned by node importance in noncascading failure scenarios, random attacks in cascading failure scenarios, attack mode conditioned by node degree in cascading failure scenarios, betweenness-conditioned attack mode in cascading failure scenarios, and attack mode conditioned by node importance in cascading failure scenarios. The specific process was realized using MATLAB programming.

5.1.2. Node Importance Evaluation System

(1) Evaluation indexes of importance

The importance of the Hefei subway network nodes was evaluated using three indicators, including node degree, betweenness, and node centrality. The definitions of node degree and betweenness are the same as above.

Node centrality is composed of the node degree centrality and node closeness. The larger its value, the closer the node is to the center in the network, and the more important it is to the connectivity of the network. Its expression formula is as follows:

$$U_i = E_i + O_i \tag{18}$$

$$E_i = \frac{k_i}{N-1} \tag{19}$$

$$O_i = \frac{1}{N-1} \sum_{\substack{j=1 \\ i \neq j}}^{N} d_{ij} \tag{20}$$

In: N is the number of nodes; U_i is the centrality of node i; O_i and E_i are the closeness and node degree centrality of node i, which reflect the connection difficulty and degree of connection between node i and other nodes, respectively.

(2) Calculation of the index weights

In order to obtain more objective and reasonable evaluation results, the coefficient of variation method and the entropy value method were used to calculate the index weights, respectively. According to the minimum information entropy theory, a coupled weight calculation method was introduced [33], and the coupled analysis of the calculation results of the two methods could obtain more accurate index weight data. The coupling weight calculation principle is as follows:

$$\text{minz} = \sum_{i}^{n} w_i (\ln \frac{w_i^2}{w_i' w_i''}) \tag{21}$$

$$s.t. \begin{cases} w_{\ i} > 0 \\ w_{\ 1} + w_{\ 2} \ + \cdots + w_{\ n} = 1 \end{cases} \tag{22}$$

In: z is the representation function of the minimum information entropy; w_i is the combined weight of the index i; w_i' and w_i'' are the weights of the index i under the coefficient of variation method and the entropy method, respectively.

(3) Node importance evaluation

The index data on 217 stations in the Hefei subway network were counted, and the weights of the three indicators were calculated. Due to the dimensional differences between the three indicators, the TOPSIS method was used to process the data [34], and the specific processing process was realized using MATLAB programming. Finally, the importance rankings of all nodes in the Hefei subway transportation network were obtained.

5.2. Robustness Analysis

The robustness of an urban subway network refers to the ability of a network to maintain normal functions when attacked. In this paper, the robustness of the Hefei subway network was quantitatively analyzed by three indicators: network efficiency, the proportion of maximal connected subgraphs, and the degree of connectivity. The definition of the connectivity index is the same as above.

(1) Network efficiency

E_R is the network efficiency, which is an important parameter to measure the current state of an urban subway network. The specific calculation formula is as follows:

$$E_R = \frac{\sum\limits_{i \neq j} {d_{ij}}^{-1}}{(N-1)N} \tag{23}$$

(2) The proportion of maximal connected subgraphs

V is the proportion of the maximal connected subgraphs, and its calculation formula is as follows:

$$V = N' \times \frac{1}{N} \tag{24}$$

In: N' represents the number of nodes in the maximal connected subgraph; N represents the number of nodes in the original network graph.

5.3. Analysis of the Results

A random attack and a deliberate attack were carried out on the Hefei subway traffic network. According to the different ways of selecting nodes, it was subdivided into four attack modes: select the attacked nodes according to the descending order of the node degree; select the attacked nodes according to the descending order of the betweenness; select the attacked nodes according to the descending order of the node importance node; randomly select the attacked node.

The changes to the robustness of the subway network in Hefei under the two scenarios of cascading failure and noncascading failure were compared and analyzed. Among them, ∂ and β were collectively referred to as nodal coefficients, which were taken as 0.5 and 1.6, respectively.

5.3.1. Analysis of the Index Changes

Figure 11 shows the changes in network efficiency when the Hefei subway network was under different attacks.

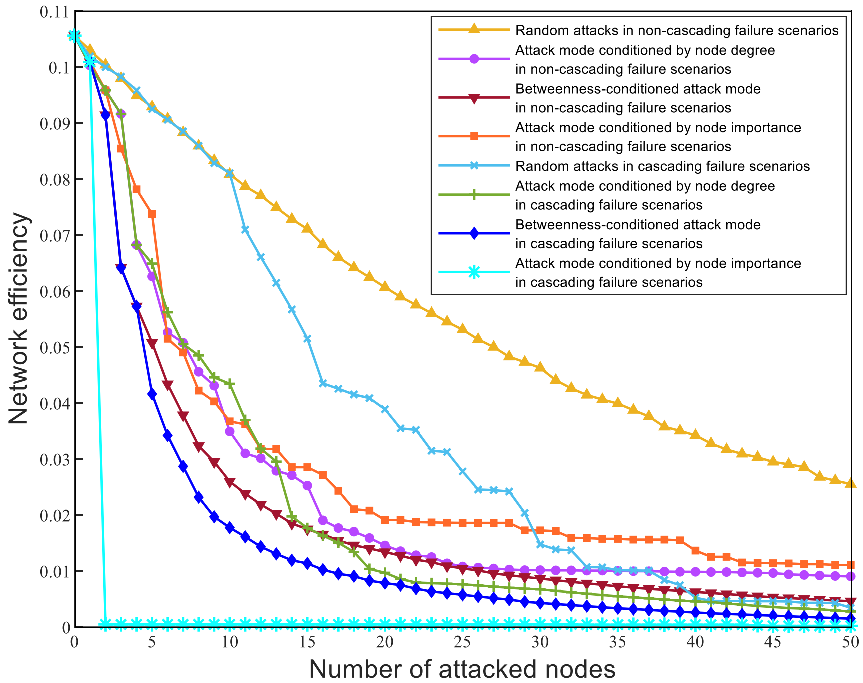


Figure 11. The changing trend of network efficiency.

In the noncascading failure scenario, 50 sites in the network were randomly selected for removal, and the decline trend in the network efficiency was the most stable among all attack results. After removing one site on average, the network efficiency was reduced by 1.52%, and the network efficiency after all 50 sites were removed was still 24.15% of the initial state.

In the cascade failure scenario, the change in the network efficiency was less than the difference in the noncascade failure until the removal of the ninth node, and then the decline became steeper. The reason may be that the ninth important node (i.e., Zilu Station) is an important transit point in the subway network, and after its failure, the load flows to the adjacent node (i.e., Tangxihe Park Station), while the tenth selected node was also Tangxihe Park Station, thus causing a local blockage in the network and a significant decrease in the network efficiency. In the subsequent process, important station failures occurred one after another, and further triggered a number of neighboring stations to fail one after another, resulting in the trend of an extremely rapid decrease in network efficiency. At the end of the whole random attack process, the decline in the subway network efficiency reached

96.69%, and the lowest value of the network efficiency was only 13.73% of the random attack in the noncascading failure state.

In the noncascading failure environment, both the attack mode by the node degree descending rule and the attack mode by the betweenness descending rule were more destructive than the attack mode by node importance descending rule. The decrease in network efficiency was 91.38%, 95.64%, and 89.49%, respectively. Because the betweenness, degree and node centrality were comprehensively considered in the calculation of the node importance, this attack mode was less destructive to the network in the case of noncascading failures without considering the load distribution.

When the subway network was in a cascading failure scenario, the nodes were selected to attack according to the descending order of node degrees. When the seventh node was attacked, the network efficiency was less than half of the initial state. However, in the mode of selecting the nodes to attack according to the descending order of the betweenness, the network efficiency dropped by 60.61% when only five nodes failed. As the number of failed nodes increased to 19, network efficiency in both modes began to decline steadily. At this time, the network efficiency of the two modes was 9.85% and 7.86% of the initial network efficiency, respectively, and the decline rate exceeded 90%. Obviously, these 19 stations are extremely important to the connectivity of the entire network, and they are the key nodes to ensure the normal operation of the subway network.

Among all of the attacks, the attacks on the nodes with high importance in the cascading failure scenario caused the most significant damage to the network. When the second most important node (i.e., Dongzhi Road Station) failed, the network efficiency dropped suddenly to only 0.45% of the initial state, and then the network efficiency declined steadily. After all of the 50 important nodes failed, the network efficiency was only 0.18% of the initial value. The reason is that after the failure of the Dongzhi Road Station, its load was distributed to adjacent nodes according to the rules, resulting in the failure of Huangshan Road Station and Dongliu Road Station, and then it continued to trigger a new round of load distribution. The final results show that the failure of Dongzhi Road Station caused a total of 179 subway network site failures, which greatly damaged the network structure. After that, the declining trend of network efficiency was close to a horizontal line.

Figure 12 shows the changes to the proportion of the maximal connected subgraphs when the Hefei subway network was under different attacks.

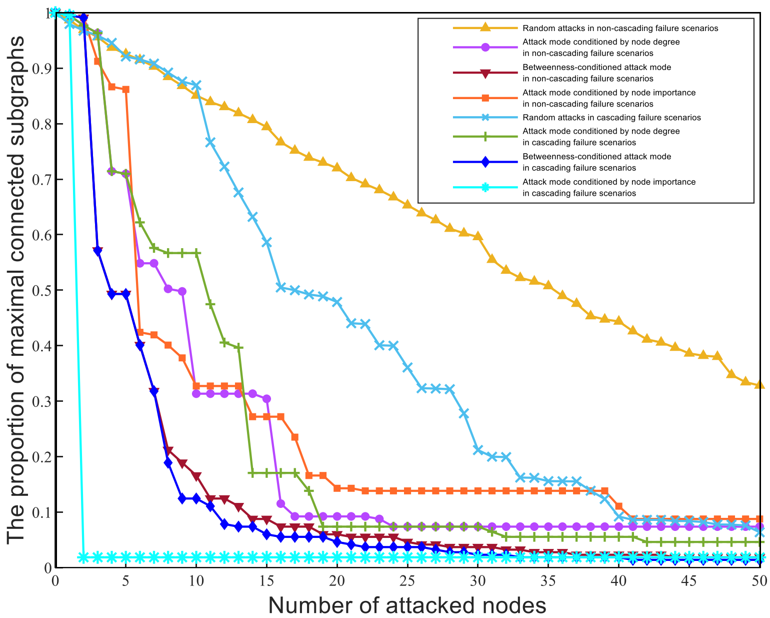


Figure 12. Proportional variation trend of maximal connected subgraphs.

When the network was in the state of noncascading failure, the decreasing trend of the proportion of the maximal connected subgraphs under random attack was relatively smooth. While in the cascading failure scenario, there was a sudden drop, and the overall performance of the curve was steeper.

In the mode of attacking nodes selected according to the descending order of the node degrees, the proportion of the maximal connected subgraphs of the Hefei subway network under different scenarios showed a stepwise decrease. At the end of the attack, the proportion of the maximal connected subgraphs of the network under cascade failure decreased to the original 4.61% and 7.37% under the noncascade failure.

In the mode of attack with the descending selection of nodes based on the node degree, the two decreasing curves of the proportion of the maximal connected subgraphs largely coincided until the failure of the eighth node (i.e., Library Station) and then deviated but to a lesser extent than the mode of attack with the descending selection of nodes based on the node degree. At the end of the attack, the proportion of the maximal connected subgraphs was 0.0138 and 0.0184 in the case of the cascade failure and noncascade failure, respectively, with a decrease of 86.9% and 82.6%.

When the nodes were selected for attack in a descending order of importance, the decreasing trend of the proportion of the maximal connected subgraphs differed greatly under the noncascade failure and cascade failure. In the Hefei subway network, the fifth important node (i.e., South Square Station) forms a kind of triangular structure together with two other important nodes. This structure can provide interchange service for a large number of passengers and improve the operational efficiency of the subway. Therefore, when in a cascade failure scenario, the failure of the South Square Station will greatly affect the operational efficiency of the subway network and, therefore, the proportion of the maximal connected subgraphs will suddenly appear to decrease. Under the cascade failure, the failure of the second important node (i.e., Dongzhi Road Station) triggered the cascade failure, which directly led to the significant destruction of the network structure and the formation of numerous isolated subgraphs, so that the proportion of the maximal connected subgraphs plummeted from 99.54% before the failure to 1.84% at this time, with the most significant decline and trend among all attack methods.

Figure 13 shows the change in the network connectivity when the important nodes of the Hefei subway network failed one after another (to enhance the visualization, the cyclic decrease in the connectivity in Figure 13a is enlarged ten times). When the network was in a cascade failure state, the network connectivity dropped abruptly after the failure of the first node (i.e., Honggang Station), and then it decreased slightly after the failure of the 33rd node, 44th node, and 45th node. The statistical results at this point show that the subway network had reduced to a total of 229 connected edges, and only four nodes remained connected, and the network connectivity dropped to 1.72% of the initial state, a reduction of 98.28%.

In the context of a noncascade failure, the connectivity did not plummet, and when all 50 nodes failed, the network connectivity dropped to 0.2016, a reduction of 0.1658 from the initial state, which is only 46.70% of that in the case of the cascade failure. In particular, the failure of the sixth important node (i.e., Hefei South Station) caused the greatest decrease in the connectivity of the subway network, with a value of 0.0078. Because Hefei South Station is an important transit station connecting three subway lines, its failure causes the subway network to break six edges at the same time.

In addition, some nodes become isolated nodes after the failure of the predecessor node; so, their failure does not reduce the existing edge size of the network and, therefore, the connectivity of the subway network does not change.

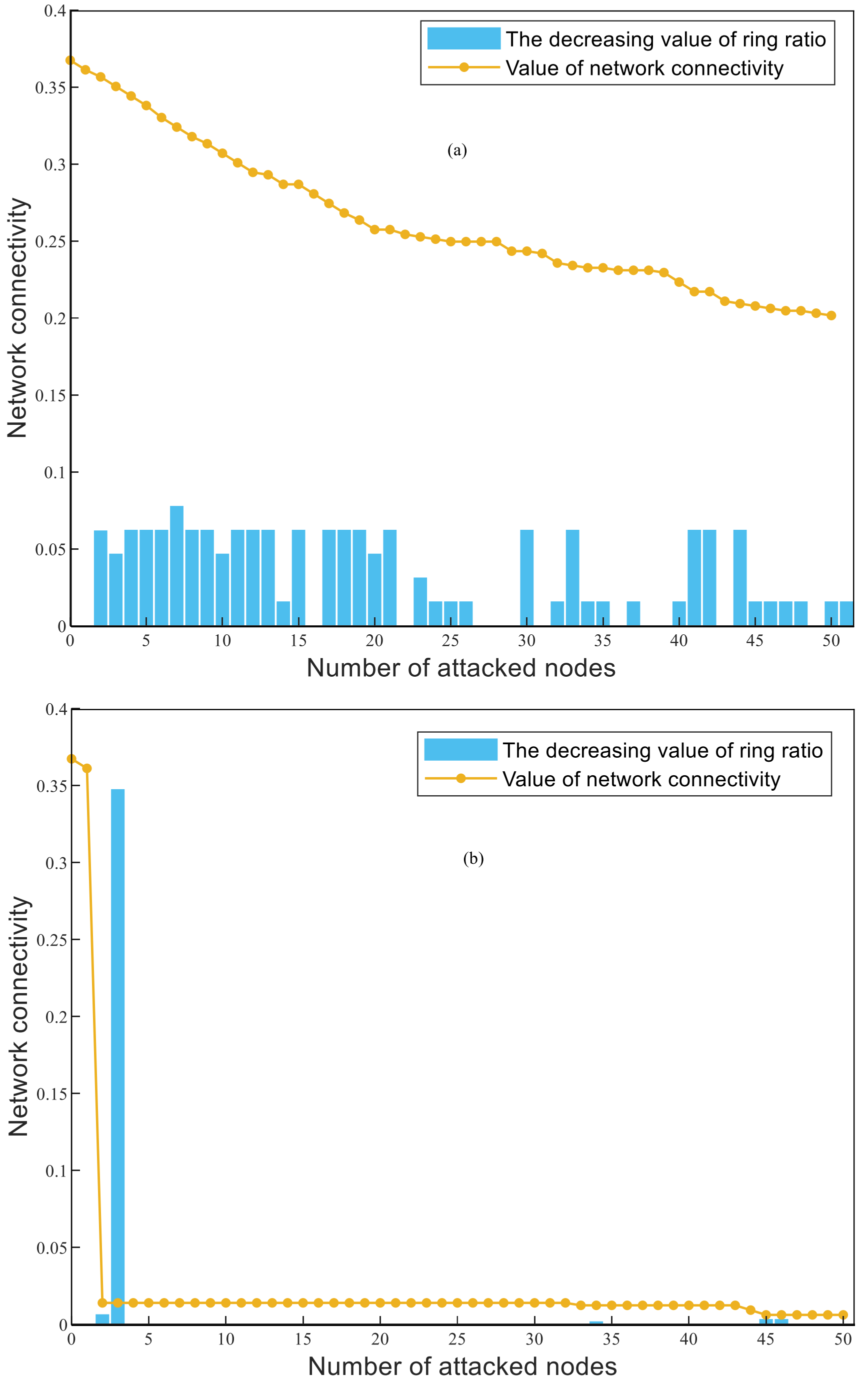


Figure 13. Schematic diagram of network connectivity change. (**a**) Noncascading failure scenario. (**b**) Cascading failure scenario.

5.3.2. Adjustment Analysis of the Nodal Coefficients

Figures 14 and 15 show the effects of the sequential failure of important nodes on the network structure for different ∂ and β, respectively. As can be seen from Figure 14, the change curve of the proportion of the maximal connected subgraphs in both noncascaded and cascaded failure scenarios was the smoothest at $\partial = 0.9$. Compared with $\partial = 0.1$, the minimal values of the proportion of the maximal connected subgraphs increased by 18.11% and 4.61%, respectively, which was because the increase in ∂ essentially represented the larger scale of passenger traffic that the subway stations can carry in the normal state; so, they will not enter the abnormal state after receiving traffic from other failed stations. The reason is that the increase in ∂ essentially means that the subway station can carry more passengers in the normal condition; therefore, it will not enter into an abnormal condition after receiving traffic from other failed stations.

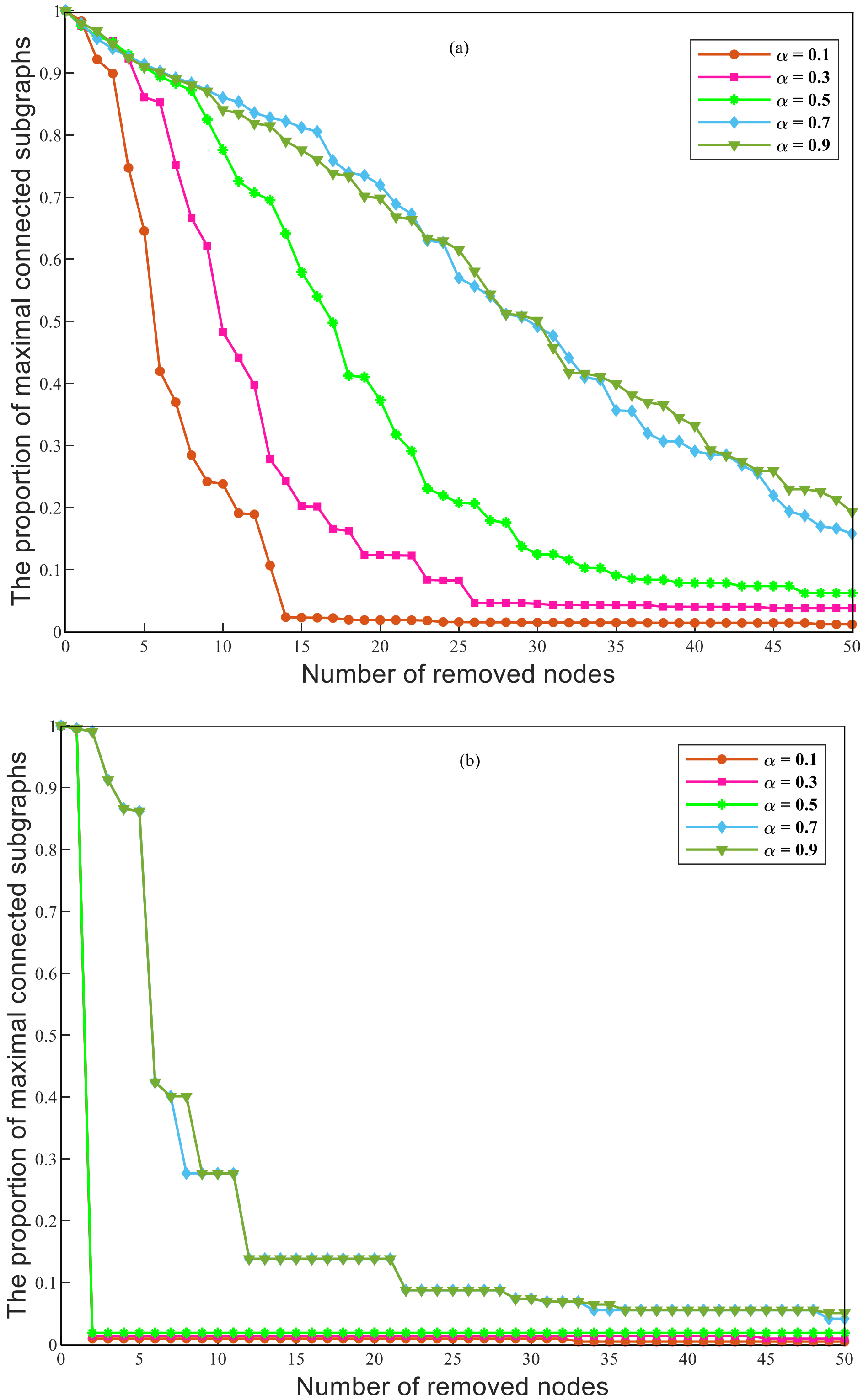


Figure 14. Trend of network function under different ∂. (**a**) Noncascade failure scenario. (**b**) Cascade failure scenario.

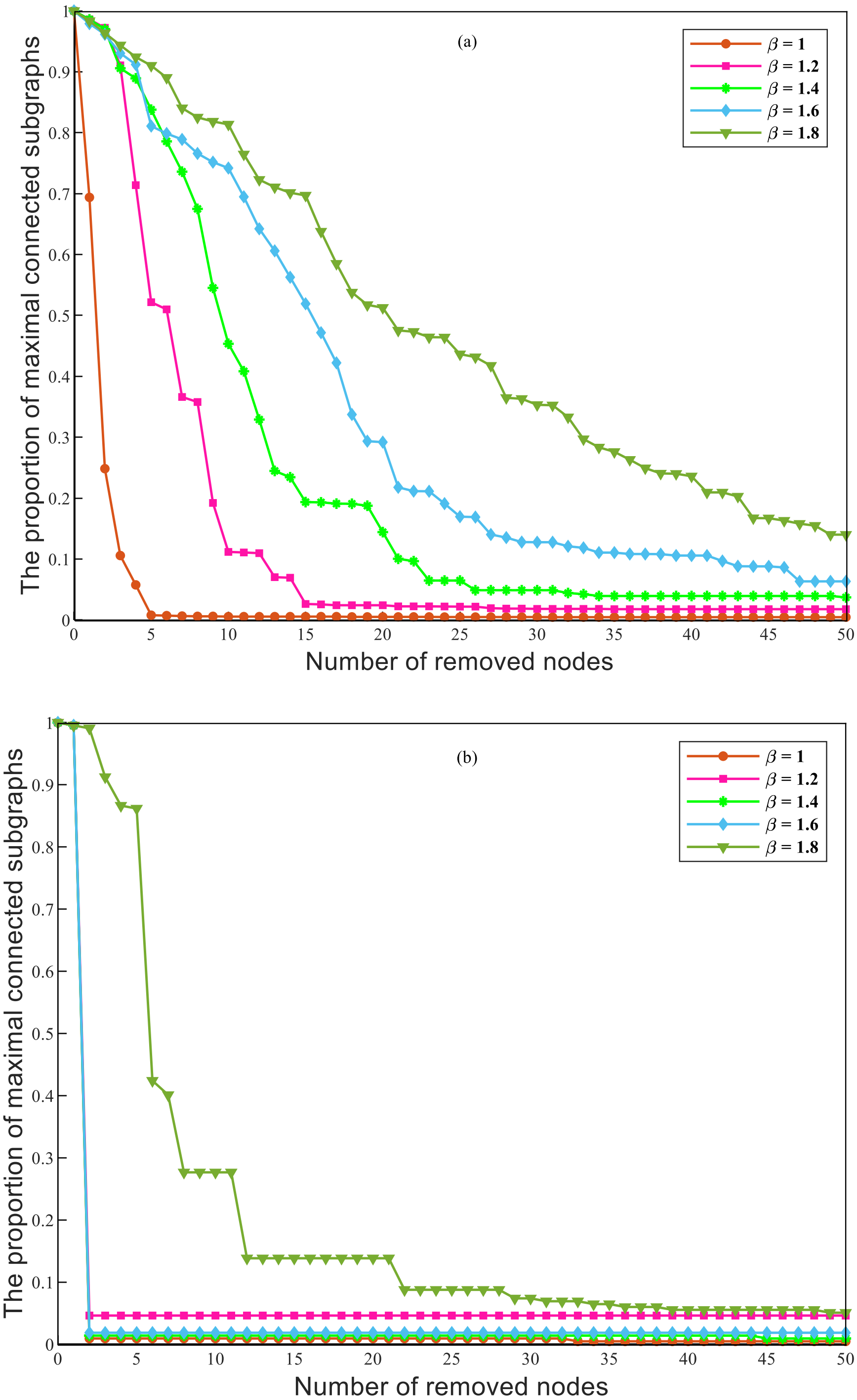


Figure 15. Trend of network function under different β. (**a**) Noncascade failure scenario. (**b**) Cascade failure scenario.

After further analysis of Figure 15, it is clear that increasing β can effectively avoid damage to the network structure from node failure when the site enters an overloaded but not failed state due to the fact of an excessive influx of traffic in a short period of time; when β was increased from 1 to 1.8, the decrease in the proportion of the greatly connected subgraphs was relatively reduced by 13.55% in the noncascaded failure scenario and by 4.61%.

From the analysis of the results, both ∂ and β will have an impact on the robustness of the subway network. The larger their values, the better the site's ability to handle traffic, and the stronger the site's defense, so the lower the chance of a cascade failure of the subway network. However, the cost of the site's construction will also increase as a result, which will also cause excess site capacity and other resource waste problems.

Therefore, the construction of stations needs to be based on the balance between the node capacity, limit capacity, and cost expenditure. In the construction of subway projects, the station capacity should be appropriately increased to ensure the station's ability to handle the sudden increase in passenger flow and to ensure the maximum efficiency of passenger transportation at the station. Especially for interchange stations and other subway stations prone to overload operation, reasonable expansion should be carried out after comprehensive consideration of station capacity, personnel management capacity, and self-regulation ability of the load so as to reduce the chance of cascade failure of the subway network and to enhance the robustness and operational safety of the subway network.

6. Conclusions

Based on complex network theory, MATLAB was used to analyze the basic characteristics of the topology of the Hefei subway network, and on this basis, a reasonable index system was established to identify the important stations of the Hefei subway network, and a node failure simulation was carried out in a well-constructed cascade failure model to further investigate the characteristics of the Hefei subway network under the cascade failure scenario. The results show that:

(1) The cumulative distribution of the node degrees of the Hefei subway network conforms to a Gaussian distribution, which is quite different from both the scale-free network and the random network and does not have the characteristics of a small-world network. However, as the planning work progresses, the subway network will gradually evolve towards the direction of a small-world network.

(2) The changes in network structure indicators at different periods of the Hefei subway network revealed that the network structure is developing towards complexity, and the aggregation of the nodes and the network connectivity are increasing. In addition, since a triangular structure formed by three nodes penetrating each other in the subway network can realize fast transfer of passengers, if the number of triangular structures is consciously increased in the planning and construction of the subway network, it will be beneficial to the stability of the subway network structure and the improvement of the transportation efficiency.

(3) The Hefei subway network is more resistant to random attacks than deliberate attacks; so, in the actual construction and maintenance, the capacity of the important nodes should be reasonably increased, and the passenger transportation capacity and load regulation capacity of key stations should be improved to prevent and reduce the chance of node failure and the cascade failure phenomenon.

This paper used a number of indicators to construct an evaluation model for the importance of subway stations. The top ten stations obtained by the model were Honggang Station, Dongzhi Road Station, Dongqili Station, Yaoduhe Road Station, South Square Station, Hefei South Station, the Third hospital Station, Library Station, Zhugang Station, and Xiqilitang Station. Combined with the actual operation of the Hefei subway, these stations are burdened with a huge task of passenger flow transportation. At the same time, from the perspective of the structure of the Hefei subway network, these stations are located at the intersection of multiple lines and are important transfer stations. From the results, this evaluation model not only absorbed the advantages of a single index but also reduced the one-sided effect of the single index evaluation. Therefore, the screening results were more in line with the actual situation of the Hefei subway project, which has a certain reference value and significance for the further construction of the Hefei subway project.

Meanwhile, β was introduced to improve the cascade failure simulation model, emphasizing that even if the stations are overloaded, they still have a certain load handling capacity, making the simulation process more in line with the actual operation process of the subway stations, and the final research results can provide more reasonable references for the overall planning, line adjustment, and station maintenance of the Hefei subway.

Additionally, the research did not consider the influence of train departure frequency and interchange information on the cascade failure simulation results, and the actual passenger flow of each station in the Hefei subway network on the subway operation was not fully reflected, which may deviate from the actual results. In the future, a more in-depth study on the characteristics of the urban subway network will be conducted to address the abovementioned contents.

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