

Article

Load Deformation Effect of CBD Ground Cluster in Zhengzhou City

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Abstract: With the increase of the number of urban cluster buildings, it is bound to cause land subsidence near cluster buildings. Based on Farrell's load deformation theory, this paper studies the surface deformation and geoid change caused by Zhengzhou CBD cluster buildings. The results show that the maximum horizontal displacement of the International Convention and Exhibition Center is 0.1 mm, the maximum vertical displacement is 0.33 mm, and the maximum geoid fluctuation is 17.4 mm; the maximum inclination of geoid is 1.89 s. In addition, the influence of independent single buildings on ground deformation is centered on the single building and diffuses outward; the influence of cluster load buildings on the ground deformation of a single building is nonlinear attenuation and superimposed with each other.

Keywords: load Love numbers; the load deformation; Zhengzhou CBD



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1. Introduction

Under the action of various factors, the shape of the earth is changing at all times. These factors include the force source generated by the change of the earth's internal structure and the change of surface load mass, as well as the tidal force of external celestial bodies [1–3]. Among them, solid tide and load tide have relatively complete theoretical models, and the deformation effect can be obtained by actual observation or theoretical simulation [4]. As the physical structure of the real earth's interior is very complex, people can only simulate the deformation effects of solid tide and load tide through various earth models, which is the tide simulation theory [5].

The theoretical models of earth deformation mainly include: (1) Linear model, establishing the relationship between environmental load changes in local areas and shape variables, and calculate the corresponding values. This model was first studied by Caputo [6,7] and Rabbel and Zschau [8], among whom, the research results of Rabbel W et al. were written into the IERS Code in 1996. (2) Spherical harmonic function model, that is, spherical harmonic expansion of force source and calculation of shape variables combined with load Love numbers. For example, Longman [9,10], Farrell [11], Guo [12], etc. calculated the load off number of the isotropic earth model by using different methods based on Farrell theory. In addition, Xi [13] and Wu et al. [14] gave the theoretical model of the variation of the earth's gravity field. (3) Load Green's function, that is, the load is convoluted with the load Green's function to obtain the load deformation. The essence of load Green's function is the same as that of spherical harmonic function, and its prototype is derived from the theory of Caputo M et al. Based on Longman's theory, the load Green's function for calculating surface displacement, ground tilt, gravity and strain was derived, and the numerical solution of 1066 Earth model was calculated [15]. In the past decade, more and more scholars have studied the application of Green's function. For example, Sun [16] studied Green's function of atmospheric gravity. Refs. [17,18] compared the differences

of the two Green's functions calculated in Fourier space. Zhang et al. [19] derived the relationship between ground displacement and crustal stress change by using Green's second internal integral formula, and discussed the calculation of earth stress field change by using the change rate of ground displacement. Shen et al. [20] compared the calculation accuracy of Green's function and spherical harmonic function for surface displacement under surface fluid mass load. Wang et al. [21] used spherical harmonic transformation and load response theory to calculate the surface deformation of the surrounding area when the surface water storage migrated greatly. Li et al. [22] studied the vertical deformation of land water load in China over 10 months by using radial load Love numbers.

In recent years, there have been more and more studies on the application of load deformation theory to engineering examples. Wu et al. [23] studied the load effect of gravity field and crustal deformation based on the theory of elasticity. Xing et al. [24] studied the relationship between the water storage of the Three Gorges Reservoir and the activity of Xiannushan Fault. Akatsuka et al. [25] studied the seasonal response of continental water load in southern Alaska to crustal deformation; Gladkikh et al. [26] systematically analyzed the influence of atmospheric load on crustal deformation in New Zealand; Sun et al. [27] used the digital crustal deformation and tide data of Hubei Province from 2003 to 2013, and the wavelet analysis method, to study the response of crustal deformation observation data to various influencing factors of atmospheric environment change (atmospheric pressure, temperature, rainfall, typhoon and solar eclipse activity); Wang et al. [28], based on SRTM, used the water storage load model of the Three Gorges Reservoir area's construction to study the response of surface gravity and deformation caused by water storage. The change of environmental load can cause the vertical displacement of tens of mm on the surface, and the influence on the horizontal displacement is 1/3 to 1/10 [29,30].

At present, load tides have become the main research field of scholars at home and abroad, and most of them focus on the changes in the geophysical field under the action of natural loads. Scholars involved include Zhang et al. [31]. Wang [32] took the Three Gorges Reservoir and cluster buildings as examples to study the geoid deformation caused by man-made loads. Based on previous studies, this paper is based on load tide theory, adopts load Green's function, and uses actual data of Zhengzhou CBD cluster load buildings to calculate and analyze the geoid undulation, vertical crustal deformation, and its effect on elevation caused by the load of the building group.

2. Research Methods and Data Sources

2.1. Research Method of Cluster Load Effect

For a given internal structure model of the Earth (including the distribution of elastic parameters, density, gravity, etc.), the tidal deformation equation of the elastic (or rheological) earth can be solved theoretically by using elastic mechanics or rheological methods, which is the solid tide theory [33]. The classical tidal theory of the Earth is derived from an elastic earth model with spherical cambium distribution, self-gravitation, and non-rotation. Its numerical solution was first obtained by Takenuchi and developed by Molodensky, Altermon, Longman, and others. For rigid earth, the theoretical values of geoid tide, gravity earth tide, and tilt earth tide (astronomical longitude and latitude earth tide) can be calculated [34–36]. Load tide is the deformation of the earth under the action of surface load, and changes the earth's gravity field, astronomical longitude, and latitude. Compared with the earth tides, the time function of the two tides are the same. The difference is that the earth tide is the direct effect, while the load tide is the indirect effect. The solution of earth tidal generating force is supported by relevant theories, while the solution of load tidal generating force relies more on the elastic structure of the earth crust. Since the Earth is not a real rigid body, in order to describe the elastic deformation of the Earth under the action of tidal forces, Love [37], in England, introduced two dimensionless numbers, h and k , in 1909. In 1912, T.Shida [38] of Japan introduced the third dimensionless, number L . These three numbers, collectively known as Love Numbers, are parameters that mark the internal characteristics of the Earth. They are the ratio of the tidal amount of the elastic

earth to the rigid earth and can be used to describe the elasticity of the earth. H is the ratio of the radial displacement of the elastic earth tidal deformation (true earth tidal height) to the gravity level tidal height (theoretical earth tidal height); k is the ratio of the additional gravitational potential generated by the tidal deformation part of the elastic earth to the tide-generating potential. L is the ratio of the horizontal displacement of the elastic earth's tidal deformation to the horizontal component of tidal generating force. The special feature of l is that it is the ratio of displacement to force, and the dimension is not uniform [34]. H_n is the ratio of the radial displacement of a point on the Earth under tidal level action to the radial displacement of the corresponding point when the earth is assumed to be in equilibrium tidal state. l_n is the ratio of the tangential displacement of a point on the Earth under the action of tidal level to that of the corresponding point under the assumption that the Earth is in equilibrium tidal state. k_n represents the ratio of the additional gravitational potential to tidal potential caused by the redistribution of density in the solid earth due to tidal deformation. Load Love number is related to the elasticity and density distribution of the earth's internal structure and material, and it is used to describe the deformation of the solid elastic earth under the load of space-time changes. There are three quantities, h_n' , l_n' , and k_n' , corresponding to Love numbers h_n , l_n , and k_n respectively.

Subsequently, many people have studied different structures in detail. In view of the accuracy of current experimental observation techniques and the accuracy of tidal load correction, the global theoretical Love number is considered to be a known parameter value. In 1962, I.M. Long Man of University of California first introduced load Love number and Green's function, and calculated load Love number from $n = 20$ to $n = 40$ using the Gutenberg Earth model in order to obtain load tide load Love number. However, due to the slow convergence rate, Farrell (1972) calculated the load Love number to 10,000 orders by changing the boundary conditions of the Love number. Since then, domestic and foreign scholars have found, on the basis of Longman and Farrell's theory of load deformation, that the load Love values with higher order and precision are calculated by numerical calculation method.

2.1.1. Ground Deformation Caused by a Single Building

For the convenience of calculation, each building is considered as a point of mass, with the center of mass as its geometric center. As shown in Figure 1, in spherical coordinate system, O is the center of the earth, $P(r_1)$ is the calculation point, and r_1 is the OP vector. $r_1 = (\gamma, \psi, \lambda)$, where r is the distance from point P to the center of the earth, ψ is codimension, and γ is longitude. $Q(r_2)$ is the centroid of the building, r_2 is OQ vector, $r_2 = (\gamma', \psi', \lambda')$, the mass of the building is m , s is the distance from point P to point Q , and the gravitational potential of point Q to point P is [39]:

$$v = \frac{Gm}{|r_2 - r_1|} = \frac{Gm}{s} \quad (1)$$

G is the universal gravitational constant. Expand $1/s$ according to Legendre polynomial to obtain:

$$v = Gm \sum_{n=0}^{\infty} \frac{|r_1|^n}{|r_2|^{n+1}} P_n(\cos\beta) \quad (2)$$

Since P and Q are on the surface of the earth, R is the average radius of the Earth:

$$|r_1| \approx |r_2| \approx R \quad (3)$$

So there are:

$$v = \frac{Gm}{R} \sum_{n=0}^{\infty} P_n(\cos\beta) \quad (4)$$

$$g = G \frac{M}{R^2} \quad (5)$$

Substitute the Equation (5) into the above equation to obtain:

$$v = \frac{mgR}{M} \sum_{n=0}^{\infty} P_n(\cos\beta) \quad (6)$$

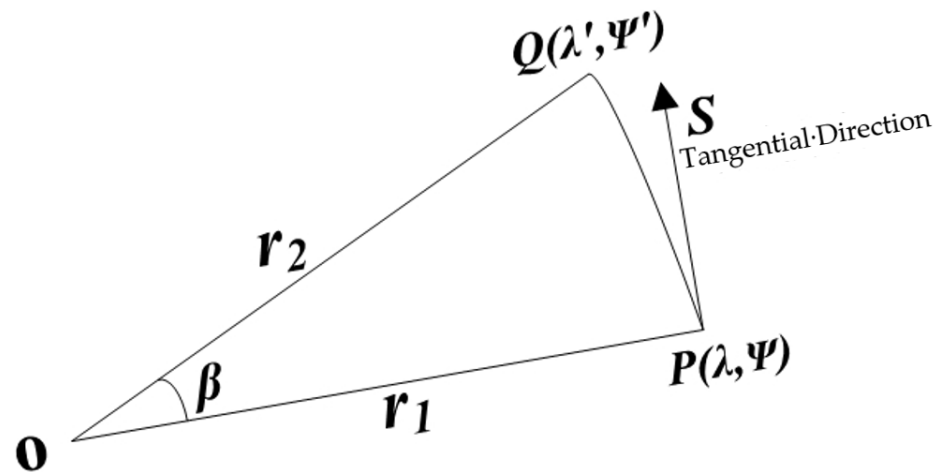


Figure 1. Schematic diagram of relation between calculation point, particle element, and geocentric.

According to the definition of load Love number, according to Equation (7), and the calculated load Love numbers h_n , l_n , and k_n , the vertical displacement and horizontal displacement of point P caused by particle load Q are [40]:

$$\begin{aligned} u &= \sum_{n=0}^{\infty} u_n = \sum u(r) P_n(\cos\theta) \\ v &= \sum_{n=0}^{\infty} v_n = \sum_{n=0}^{\infty} v(r) \frac{\partial}{\partial \theta} P_n(\cos\theta) \\ \varphi &= \sum_{n=0}^{\infty} \varphi_n = \sum_{n=0}^{\infty} Q(r) \frac{\partial}{\partial \theta} P_n(\cos\theta) \end{aligned} \quad (7)$$

$$\begin{cases} u_r = \frac{Rm}{M} \sum_{n=0}^{\infty} h'_n P_n(\cos\beta) \\ u_\theta = \frac{Rm}{M} \sum_{n=0}^{\infty} l'_n \frac{\partial p_n(\cos\beta)}{\partial \beta} \end{cases} \quad (8)$$

In the formula, the vertical displacement U_R is positive upward, and the horizontal displacement U_V is positive in the direction of increasing β Angle.

Gravity potential of undeformed Earth [41]:

$$\varphi(\theta) = \frac{R}{M} g_0(R) \sum_{n=0}^{\infty} (1 + k'_n) P_n(\cos\theta) \quad (9)$$

The gravity perturbation potential on the deformed earth's surface is:

$$\varphi(\theta) = \frac{R}{M} g_0(R) \sum_{n=0}^{\infty} (1 + k'_n - h'_n) P_n(\cos\theta) \quad (10)$$

According to the asymptotic property of load Love number, when n is large, the load numbers h'_n and nl'_n are constant. According to Farrell definition (11), the vertical

displacement of P caused by particle load Q can be written as a Kummer transform (12) by using the asymptotic value of h'_n .

$$\lim_{n \rightarrow \infty} \begin{Bmatrix} h'_n \\ nl'_n \\ nk'_n \end{Bmatrix} = \begin{Bmatrix} h'_\infty \\ l'_\infty \\ k'_\infty \end{Bmatrix} \quad (11)$$

$$u_r = \frac{Rmh'_\infty}{M} \sum_{n=0}^{\infty} P_n(\cos\beta) + \frac{Rm}{M} \sum_{n=0}^{\infty} (h'_n - h'_\infty) P_n(\cos\beta) \quad (12)$$

The sum of the first term in the formula can be obtained:

$$\sum_{n=0}^{\infty} P_n(\cos\beta) = \frac{1}{2} \sin^{-1} \frac{\beta}{2} \quad (13)$$

The second term of Equation (13) approaches zero after a finite number of terms.

$$n > N, h'_n - h'_\infty \quad (14)$$

$$u_r = \frac{Rmh'_\infty}{2M \sin \frac{\beta}{2}} + \frac{Rm}{M} \sum_{n=0}^N (h'_n - h'_\infty) P_n(\cos\beta) \approx \frac{Rmh'_\infty}{2M \sin \frac{\beta}{2}} \quad (15)$$

According to the RN factor method used to calculate the load Love number under the SNREI earth model calculated by Zhang et al. [39], it can be seen that when $n = 13,000$, the calculation accuracy reached the requirement.

Using the asymptotic property of nl'_n , the horizontal displacement of point P caused by particle load Q can be expressed as:

$$u_\theta = \frac{Rml'_\infty}{M} \sum_{n=1}^{\infty} \frac{1}{n} \frac{\partial P_n(\cos\beta)}{\partial \beta} + \frac{Rm}{M} \sum_{n=1}^{\infty} (nl'_n - l'_\infty) \frac{1}{n} \frac{\partial P_n(\cos\beta)}{\partial \beta} \quad (16)$$

The sum formula of the first term in the formula can be obtained according to the spherical harmonic function:

$$\sum_{n=1}^{\infty} \frac{1}{n} \frac{\partial P_n(\cos\beta)}{\partial \beta} = -\frac{\cos \frac{\beta}{2} (1 + 2 \sin \frac{\beta}{2})}{2 \sin \frac{\beta}{2} (1 + \sin \frac{\beta}{2})} \quad (17)$$

$$u_\theta = -\frac{Rml'_\infty}{M} \frac{\cos \frac{\beta}{2} (1 + 2 \sin \frac{\beta}{2})}{2 \sin \frac{\beta}{2} (1 + \sin \frac{\beta}{2})} + \frac{Rm}{M} \sum_{n=1}^N (nl'_n - l'_\infty) \frac{1}{n} \frac{\partial P_n(\cos\beta)}{\partial \beta} \approx -\frac{Rml'_\infty}{M} \frac{\cos \frac{\beta}{2} (1 + 2 \sin \frac{\beta}{2})}{2 \sin \frac{\beta}{2} (1 + \sin \frac{\beta}{2})} \quad (18)$$

The second term in Equation (18) tends to zero after a finite number of terms.

In the analysis of geoid variation caused by building load, the change of gravity level at ground point under surface load was first calculated by Green's function. The gravity variation of deformed surface depends on three factors: the direct attraction of particle load, the change of internal density field, and the deformation of earth surface. Combining these three factors, the gravity change function can be obtained as [42]:

$$\delta g = \frac{mg_0}{M} \sum_{n=0}^{\infty} [n + 2h'_n - (n + 1)k'_n] P_n(\cos\beta) \quad (19)$$

On the deformable ground, according to the progressive load Love number, h'_∞ is substituted for h'_n , and $(n + 1)k'_n$ substituted for $(n + 1)k'_\infty$, and the change of gravity at point P under load is obtained as follows:

$$\delta g = \frac{mg_0}{M} \sum_{n=0}^{\infty} (n + 2h'_{\infty} - k'_{\infty}) P_n(\cos\beta) + \frac{mg_0}{M} \sum_{n=0}^{\infty} [2(h'_n - h'_{\infty}) - (n + 1)k'_n + k'_{\infty}] P_n(\cos\beta) \quad (20)$$

According to the properties of spherical harmonic function,

$$\begin{cases} \sum_{n=0}^{\infty} P_n(\cos\beta) = \frac{1}{2} \sin^{-1} \frac{\beta}{2} \\ \sum_{n=0}^{\infty} n P_n(\cos\beta) = -\frac{1}{4} \sin^{-1} \frac{\beta}{2} \end{cases} \quad (21)$$

Thus, Equation (21) can be simplified as:

$$\delta g = \frac{mg_0}{4M \sin\left(\frac{\beta}{2}\right)} (-1 + 4h'_{\infty} - 2k'_{\infty}) + \frac{mg_0}{M} \sum_{n=0}^N [2(h'_n - h'_{\infty}) - (n + 1)k'_n + k'_{\infty}] P_n(\cos\beta) \quad (22)$$

The gravity perturbation potential on the deformed earth's surface is:

$$v'(\theta) = \frac{mR}{M} g_0(R) \sum_{n=0}^{\infty} (k'_n - h'_n) P_n(\cos\theta) \quad (23)$$

Therefore, the total displacement generated by point P is:

$$\phi = v + v' \quad (24)$$

The fluctuation of geoid is obtained according to Bruns formula [43]:

$$\delta h = \frac{\phi}{g} = \frac{Gm}{gR} \sum_{n=0}^{\infty} P_n(\cos\beta) (1 + k'_n - h'_n) \quad (25)$$

Substitute the Equation (5) into the above equation, then:

$$\delta h = \frac{\phi}{g} = \frac{mR}{M} \sum_{n=0}^{\infty} P_n(\cos\beta) (1 + k'_n - h'_n) \quad (26)$$

According to the load tide theory, the derivative of the gravitational potential to the horizontal direction is the horizontal component of the gravitational change, which is represented by the tilt change of the geoid, $\delta\Lambda$. It was previously determined that the sum of the gravitational potential generated by the building load Q on point P and the additional response potential of the ground is:

$$\phi = v + v' = \frac{Gm}{R} \sum_{n=0}^{\infty} (1 + k'_n - h'_n) P_n(\cos\beta) \quad (27)$$

$$\delta\Lambda = \frac{1}{gR} \frac{\partial\phi}{\partial\beta} \quad (28)$$

According to Equation (29), the total inclination of the geoid at point P is:

$$\delta\Lambda = \frac{Gm}{gR^2} \sum_{n=0}^{\infty} (1 + k'_n - h'_n) \frac{\partial P_n(\cos\beta)}{\partial\beta} \quad (29)$$

Substitute the Equation (5) into the above equation, then:

$$\delta\Lambda = \frac{m}{M} \sum_{n=0}^{\infty} (1 + k'_n - h'_n) \frac{\partial P_n(\cos\beta)}{\partial\beta} \quad (30)$$

2.1.2. Calculation Model of Ground Deformation and Geoid Change Caused by Cluster Load

It is necessary to calculate the total vertical and horizontal ground displacements of observation points caused by many buildings. Since the vertical displacements are in the same direction, the total ground vertical displacements of observation points caused by all buildings can be obtained by adding them together.

$$\delta u_r = \sum \delta u_r \quad (31)$$

Horizontal displacement directions are not identical. The building of the horizontal displacement observation point cannot be directly together. So, first of all, we choose east-west and north-south directions at our observation sites. Second, the horizontal displacement of the point caused by each building is decomposed into east-west and north-south directions. Third, the east-west and north-south components are added separately. Fourth, it is synthesized according to the principle of vector addition. Finally, the total horizontal displacement of the observation point is obtained.

Suppose the azimuth of PQ is A , then:

$$\begin{cases} u_\lambda = u_v \sin(A) \\ u_\phi = u_v \cos(A) \end{cases} \quad (32)$$

A can be solved by spherical triangle, as shown in Figure 2:

$$A = \arctg \frac{\cos \varphi' \sin(\lambda' - \lambda)}{\cos \varphi \sin \varphi' - \sin \varphi \cos \varphi' \cos(\lambda' - \lambda)} \quad (33)$$

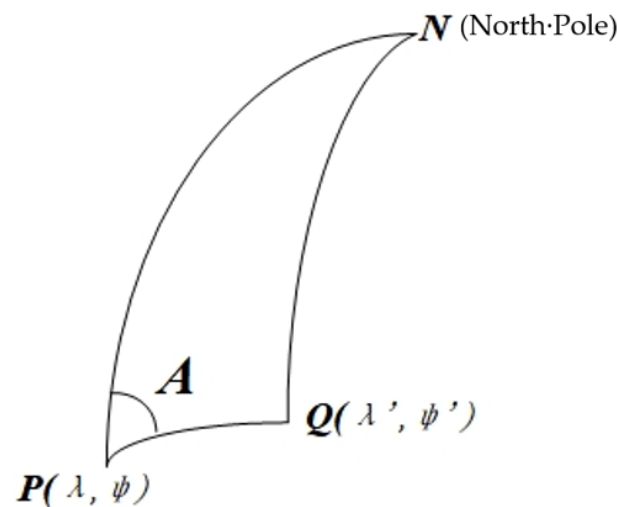


Figure 2. Calculates the spherical triangle of point, particle, and North Pole.

When the gaussian plane coordinates of two points are known, A can be directly obtained by calculating the azimuth angle from the coordinates of two points. The total horizontal displacement of observation points caused by the whole building group can be expressed as:

$$u = \sqrt{(\sum u_\gamma)^2 + (\sum u_\phi)^2} \quad (34)$$

The horizontal displacement angle can be calculated according to the displacement of east-west and north-south, namely:

$$\alpha = \arctg \frac{\sum \mu_{\lambda 1}}{\sum \mu_{\varphi 1}} + \begin{cases} 0 \\ \pi \\ 2\pi \end{cases} \quad (35)$$

The total displacement change of observation point caused by the urban architectural group is equal to the sum of the displacement change of observation point caused by each building. The geoid fluctuation caused is the sum of the geoid fluctuation caused by each building:

$$\delta h = \sum \delta h_i \quad (36)$$

The inclination of the tilt is:

$$\delta \Lambda = \sum \delta \Lambda_i \quad (37)$$

2.2. Data Source

According to the data provided by the Surveying and Mapping Institute of the Bureau of Geology and Mineral Resources of Henan Province, Zhengzhou Millennium Plaza Minsheng Bank is used as an example to calculate the impact of the independent single building on the ground and geoid. The height of the building is 180 m. The single-story building area is 1560 m², and the weight of the building is 112,300 tons. When analyzing the cluster load effect, the International Convention and Exhibition Center of Zhengzhou Millennium Plaza was used as the target to calculate the deformation of the ground and geoid under the load of 58 cluster buildings in Millennium Plaza (Figure 3).

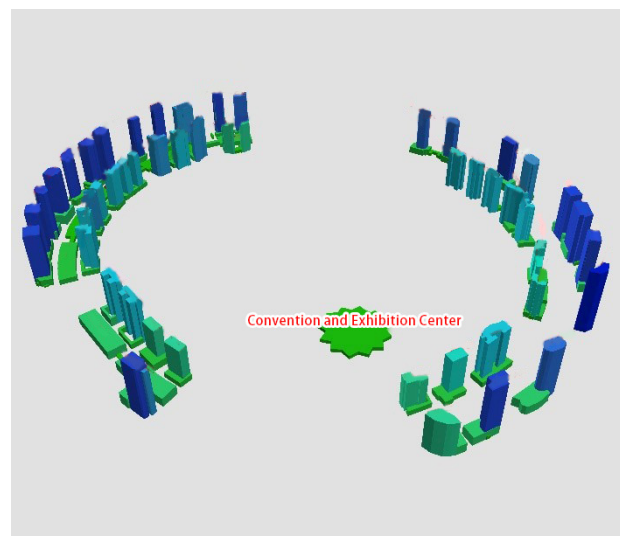


Figure 3. CBD cluster load distribution diagram.

2.3. Counting Process

According to the calculation formula in Section 2.1, Equations (15) and (18) were adopted for vertical displacement and horizontal displacement generated by single buildings on the ground. Equations (26) and (30) were adopted for geoid fluctuation and inclination. Equation (31) can calculate the vertical displacement produced by cluster load on Zhengzhou Millennium Square International Convention and Exhibition Center. Equation (34) can calculate the horizontal displacement produced by cluster load on Zhengzhou Millennium Square International Convention and Exhibition Center. Equation (36) can calculate the geoid fluctuation caused by cluster load on Zhengzhou Millennium Square International Convention and Exhibition Center. Equation (37) can calculate the geoid tilt caused by cluster load on the International Convention and Exhibition Center in Zhengzhou Millennium Square. It can be seen from the above formula that vertical displacement, horizontal displacement, fluctuation of geoid, and inclination of geoid are proportional to the earth

radius, R , building mass, M , and the corresponding load Love number. The earth mass, M , building, and the target body and the core of the angle β are inversely proportional. The included angle β can be converted by the latitude and longitude of the building and the target body, and is directly related to the distance between the building and the target body. Given the earth radius, R , and the earth mass, M , the corresponding load Love number was found by substituting the building mass, M , and the distance, R , between the building and the target body into the above formula. Tables 1 and 2 show the calculations.

Table 1. Ground deformation results caused by an independent single building.

Ground Distance to Minsheng Bank (m)	Horizontal Displacement (mm) $\times 10^{-4}$	Vertical Displacement (mm) $\times 10^{-4}$	Geoid Changes (mm) $\times 10^{-3}$	The Geoid Is Tilted (S) $\times 10^{-4}$
100	141.6865	−447.0230	419.3370	−87.5873
200	70.8424	−222.4988	414.9873	−116.1458
300	47.2252	−147.6574	407.8269	−171.5090
400	35.4161	−110.2408	397.9935	−223.6964
500	28.3288	−87.7892	385.6572	−271.7679
600	23.6037	−72.8193	371.0529	−314.8861
700	20.2275	−62.1289	354.4479	−352.3400
800	17.6945	−54.1110	336.1257	−383.5529
900	15.7233	−47.8757	316.4265	−408.0824
1000	14.1457	−42.8868	295.6743	−425.6757
1500	11.5801	−31.6680	198.8753	−350.4609
2000	7.0156	−20.4492	102.0762	−275.2461
2500	6.2314	−16.7157	87.8133	−154.8783
3000	4.6164	−12.9822	73.5504	−34.5105
4000	3.4172	−9.2603	63.4287	−15.5991
5000	2.6812	−7.0421	41.6534	−14.9756
6000	2.1707	−5.5758	33.2820	−6.2907
7000	1.8103	−4.5408	34.0216	−5.6269
8000	1.5438	−3.7775	26.5664	−5.1875

Table 2. Superposition effect of Zhengzhou CBD buildings on Zhengzhou International Convention and Exhibition Center.

The Serial Number	Building Name	Distance to the International Convention and Exhibition Center (m)	Horizontal Displacement ($\times 10^{-4}$ mm)	Vertical Displacement ($\times 10^{-4}$ mm)	Geoid Changes ($\times 10^{-3}$ mm)	The Amount of Geoid Inclination ($\times 10^{-4}$ s)
1	Dennis 1 World	569	31.45023793	−100.7346377	494.4622581	−407.0086996
2	CITIC Tower	500	22.28609813	−71.58994436	318.3766515	−230.0360243
3	Minsheng Bank Building	609	31.8830502	−101.9255062	530.6703983	−468.5733139
4	Xin'ao Building	532	20.45825586	−65.63559866	305.0021309	−234.4014766
5	Dennis 2 World	698	12.7534247	−40.60965778	233.9544765	−237.7211436
6	Five Elements Garden Tower A	598	25.4898459	−81.52411958	419.1448961	−363.4184917
7	Five Elements Garden Tower B	629	23.15702903	−73.97067496	393.368437	−358.7369287
8	Dennis 3 Worlds	905	3.796945281	−11.98191921	80.32582026	−106.7947076

Table 2. Cont.

The Serial Number	Building Name	Distance to the International Convention and Exhibition Center (m)	Horizontal Displacement ($\times 10^{-4}$ mm)	Vertical Displacement ($\times 10^{-4}$ mm)	Geoid Changes ($\times 10^{-3}$ mm)	The Amount of Geoid Inclination ($\times 10^{-4}$ s)
9	International Chamber of Commerce Building	934	27.43757742	−86.48290938	586.5287866	−804.8882877
10	Guolong Building	991	21.80839029	−68.56679636	476.5889363	−696.4885789
11	Five Elements Garden Block D	861	11.1266982	−35.18269975	229.4652653	−289.2871798
12	Five Elements Garden Block D	813	12.88057745	−40.81024067	258.5519355	−307.35943
13	Dennis 4 Worlds	995	4.520173596	−14.20899181	98.94288012	−145.2405721
14	Five Elements Garden Tower E	918	8.456283241	−26.67152532	179.7216709	−242.3527261
15	Weiye Caizhi Plaza	949	4.152928583	−13.08169576	89.27430762	−124.5442942
16	Expo Tower	1100	9.230506099	−28.95034766	199.2507399	−303.1017238
17	Liji Upper East International	1000	17.22282853	−54.12637633	377.7700455	−557.6359029
18	1 Level I upper east international	1100	4.736154894	−14.8543676	102.2351707	−155.5209105
19	New Mango Building	1100	13.75283459	−43.13407504	296.8702299	−451.6012261
20	Zhongyuan Guangfa Financial Building	1100	16.07894611	−50.42963786	347.081934	−527.983648
21	CNPC International Huijin Building	1200	9.38582782	−29.37137886	198.9695094	−312.995569
22	Longhu Building	977	9.961163531	−31.33876793	216.4653261	−311.4645876
23	Oriental International (South)	991	6.907265602	−21.7168286	150.9477005	−220.595447
24	Oriental International (North)	1000	6.596191806	−20.72992595	144.6826039	−213.5696449
25	Dennis 6 Worlds	1100	0.990395935	−3.106255099	21.37879772	−32.52158786
26	Greenland Century Mansion	1200	11.24028574	−35.17459484	238.2820334	−374.8374355
27	Greenland Summit World	1200	10.05456805	−31.4640896	213.1460869	−335.2965033
28	Road King Building	1200	9.850577681	−30.82573584	208.8217093	−328.493898
29	Dennis International Apartment (A)	1000	12.43225584	−39.07098983	272.6923657	−402.5280865
30	Dennis International Apartment (B)	1000	13.8868525	−43.64236707	304.597871	−449.6246085
31	Dennis International Apartment (C)	1000	11.9427605	−37.53264732	261.9556462	−386.6793437

Table 2. Cont.

The Serial Number	Building Name	Distance to the International Convention and Exhibition Center (m)	Horizontal Displacement ($\times 10^{-4}$ mm)	Vertical Displacement ($\times 10^{-4}$ mm)	Geoid Changes ($\times 10^{-3}$ mm)	The Amount of Geoid Inclination ($\times 10^{-4}$ s)
32	Dennis 7 Worlds	1100	4.727362032	−14.82678987	102.045367	−155.2321797
33	Vancouver Building	1200	7.944649097	−24.86145101	168.4180621	−264.9356042
34	Henan Agricultural Bank Building	1200	9.600793554	−30.04407818	203.526553	−320.1641772
35	World Trade Building (A)	1000	7.155134041	−22.48651998	156.9425898	−231.6669196
36	World Trade Building (B)	1000	9.057557916	−28.46528882	198.6708548	−293.2630653
37	Mobile building	254	11.81174111	−38.37932228	87.72484668	−31.95131788
38	Futures Building	385	36.80129836	−118.8447767	414.8423738	−229.8514987
39	Xin Po Building	276	43.48678596	−141.1473868	353.3399022	−139.9635576
40	Cathay Pacific Fortune Center	383	54.55038978	−176.1819911	611.3389495	−336.9206096
41	China Grain Store	310	64.28028134	−208.2647557	596.0240243	−265.4599272
42	United Center Building	427	42.52385922	−137.0691714	525.6722705	−323.2217623
43	Wanda Futures Building	441	10.87449407	−35.03222146	137.9250956	−87.60130898
44	Jingfeng International Center	551	33.03784179	−105.9075875	505.8941363	−402.8799715
45	Future International Building 26	495	14.28824269	−45.91093369	201.9208415	−144.3722277
46	Future International Building 25	525	14.71388877	−47.21970501	217.2709484	−164.770953
47	Wang Ding International	645	32.16422201	−102.673251	555.6776147	−519.8503882
48	Future International Building 22	673	8.522531917	−27.1709336	151.8549254	−148.4534779
49	People's Insurance Building	680	41.25931907	−131.4961481	741.0651138	−732.3875599
50	Future International Building 24	587	12.53806419	−40.12353748	202.6188634	−172.2813869
51	Kunwu Building	742	29.30217769	−93.13656382	556.386559	−601.2320672
52	Future International Building 23	623	15.57058579	−49.74938943	262.8634769	−237.4217711
53	Zhengdong Financial Building	821	25.72409535	−81.47709046	518.6101605	−622.5791084
54	Jincheng Sunshine Century (South)	708	11.31098792	−36.00178765	209.227494	−215.6517103

Table 2. Cont.

The Serial Number	Building Name	Distance to the International Convention and Exhibition Center (m)	Horizontal Displacement ($\times 10^{-4}$ mm)	Vertical Displacement ($\times 10^{-4}$ mm)	Geoid Changes ($\times 10^{-3}$ mm)	The Amount of Geoid Inclination ($\times 10^{-4}$ s)
55	Jincheng Sunshine Century (North)	774	15.85801396	−50.33253341	309.0925354	−349.0362695
56	Jincheng Sunshine Century (Main)	747	14.695301	−46.69876694	280.1470389	−304.8253995
57	Hailian Building	877	15.73914514	−49.73139059	327.6976662	−421.2945106
58	Zhengzhou Italy International Building	940	13.15189714	−41.44422365	281.7707075	−389.2237426
59	Gram Business Building	968	13.4870856	−42.4488975	292.050751	−416.0573946
Total			1040.10468	−3315.5718	17,420.14634	−18,903.85184

3. Result Analysis

3.1. Analysis of Ground Deformation and Stress Change Caused by Single Building

The horizontal displacement at 100 m distance from Minsheng Bank was the largest, at 0.01417 mm; the horizontal displacement at 8000 m was the smallest, at 0.026 mm (Table 1). The maximum change rate of horizontal displacement was 70.84%, which gradually decreased to 0.02%, within 500 m from the ground of Minsheng Bank. The displacement change rate was the most obvious, decreasing from 70.84% to 4.7%; in the range of 500–1000 m, the change rate decreased from 4.7% to 0.5%, and the trend slowed down significantly. After 1000 m, the values of the horizontal displacement change rate were all less than 0.5%, and the changes were not obvious (Figure 4).

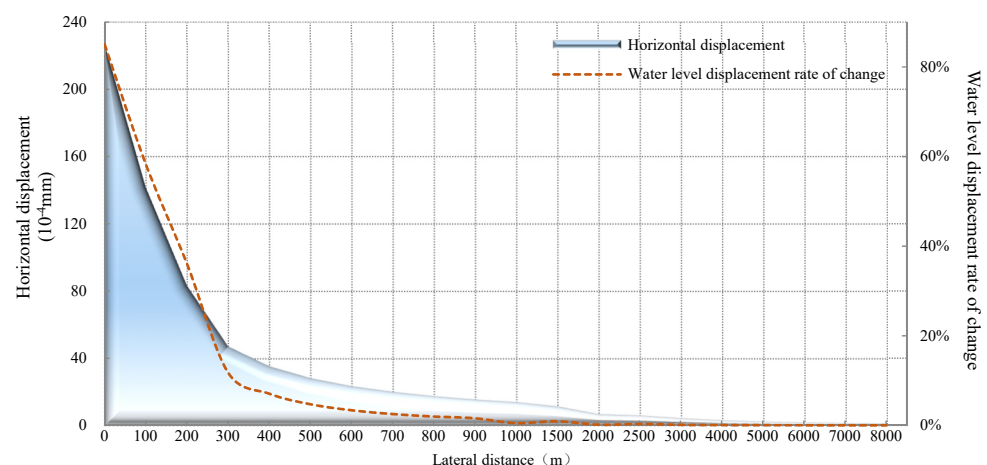


Figure 4. The amount and change rate of horizontal ground displacement caused by an independent single building.

The maximum vertical displacement at 100 m from the Minsheng Bank was 0.0447 mm, and the farthest vertical displacement at 8000 m was 0.000378 mm (Table 1). The maximum vertical displacement change rate of 225% was gradually reduced to 0.05%, and the distance from the ground of Minsheng Bank was within 500 m. The vertical displacement change rate was the most obvious, decreasing from 225% to about 15%; within the range of 500–1000 m, the vertical displacement's rate of change decreased from 15% to 2%, and the trend was obviously slowed; within the range of 1000–2000 m, the value of the rate of

vertical displacement decreased from 2% to 0.7%; after 2000 m, the rate of change of vertical displacement was less than 0.7%, and the change was not obvious (Figure 5).

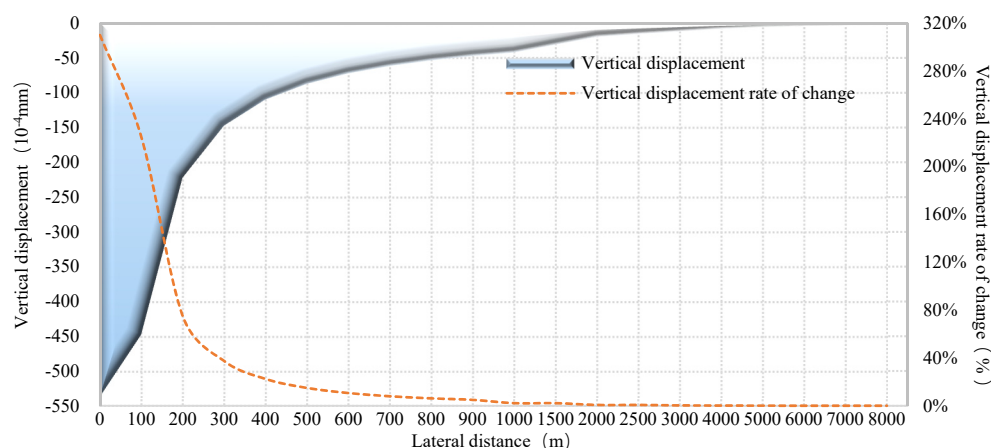


Figure 5. The amount of vertical displacement on the ground and the rate of change of vertical displacement caused by an independent single building.

The geoid fluctuates the most at the nearest 100 m from the Minsheng Bank, which was 0.47 mm, and the vertical displacement at the farthest 8000 m was 0.026 mm (Table 1). The maximum geoid undulation rate was gradually reduced to 0.21%, and the distance from the ground of Minsheng Bank was within 300 m. The geoid undulation rate was the most obvious, decreasing from 80% to about 12%; within the range of 300–1500 m, the vertical displacement's rate of change hovered around 15–20%, and the rate of change was not large, indicating that the change of the geoid no longer dropped sharply, but was relatively slow; within the range of 1500–3000 m, the fluctuation rate of the geoid increased again, indicating that the region's geoid undulations increased; the undulation rate of the geoid outside the 3000 m range was close to smooth (Figure 6).

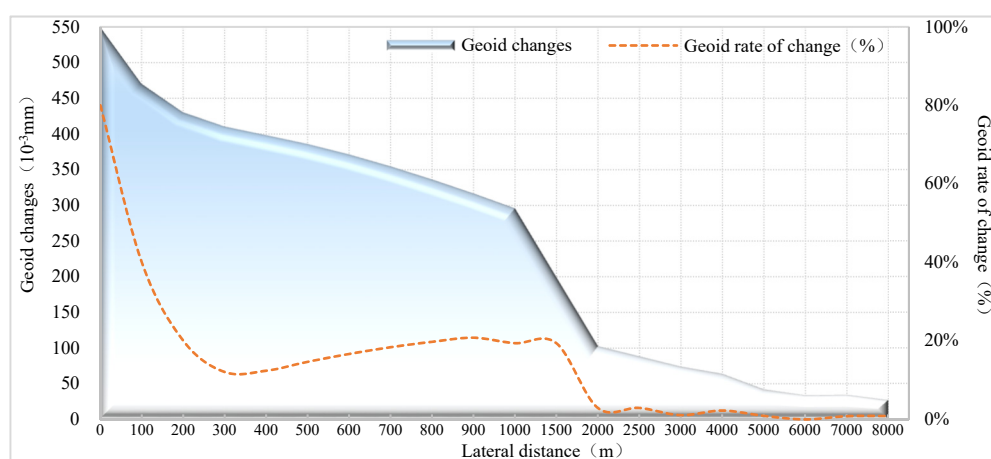


Figure 6. Geoid undulation and rate of geoid undulation caused by independent single building.

3.2. Analysis of Nearby Ground Deformation and Gravity Changes Caused by Cluster Loads

The maximum horizontal displacement caused by the CBD cluster building load on the center was the Zhongchuliang Building (Table 2), with a distance of 310 m from the ground of the center, a building height of 140 m, and a bottom area of 1955 m², resulting in a horizontal displacement of 0.0064 mm; the minimum horizontal displacement generated by the center was Dennis 6 Tiandi, which was 1100 m from the center of the ground, the height of the building was 12 m, and the bottom area was 1240 m². The amount of horizontal displacement caused was 0.0001 mm (Figures 7 and 8).

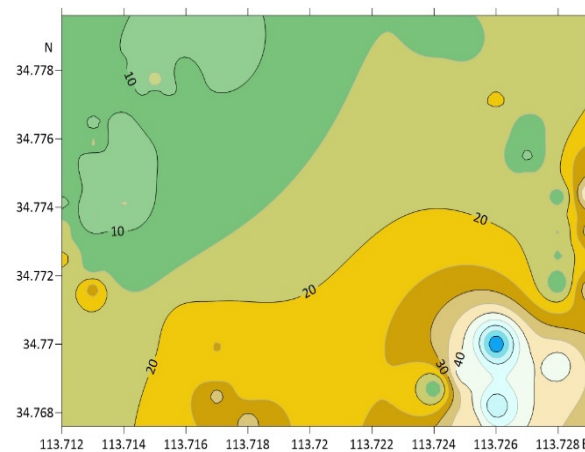


Figure 7. Ground horizontal displacement caused by cluster load.

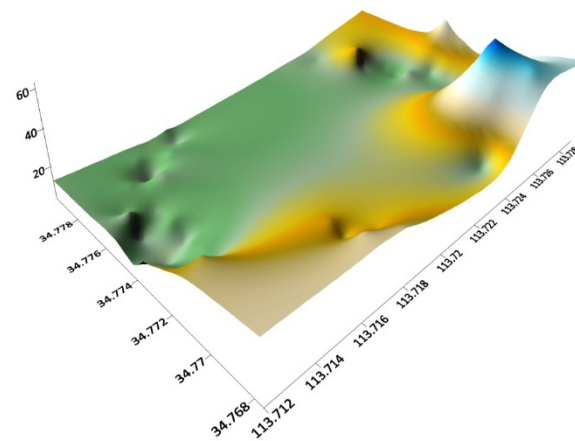


Figure 8. Three-dimensional diagram of ground horizontal displacement caused by cluster load.

The maximum vertical displacement caused by the CBD cluster building load on the center was the Zhongchuliang Building (Table 2), and the vertical displacement caused by it was -0.0208 mm; the minimum vertical displacement caused by the center was Dennis 6 days, and the vertical displacement was -0.0003 mm (Figures 9–11).

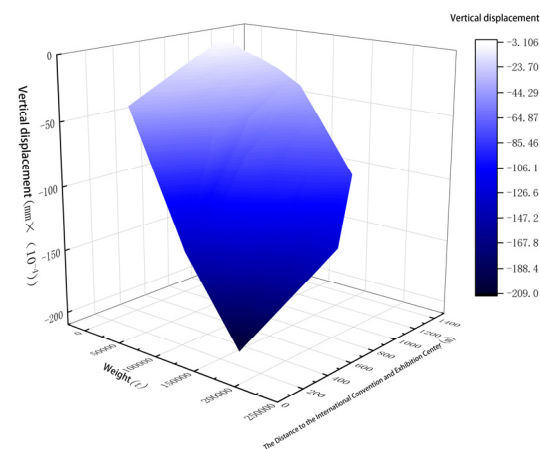


Figure 9. Three-dimensional diagram of vertical displacement caused by cluster load building.

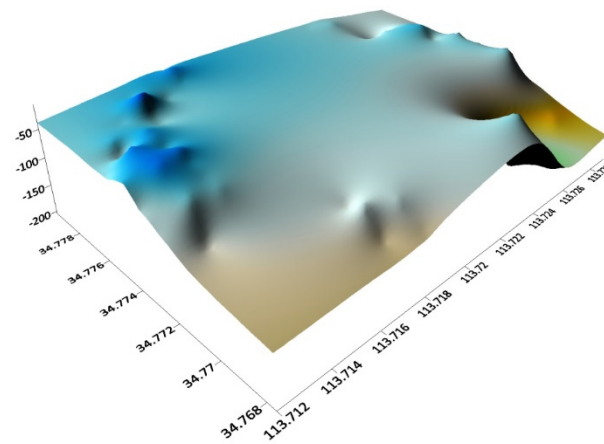


Figure 10. Three-dimensional topographic map of vertical displacement caused by cluster load building.

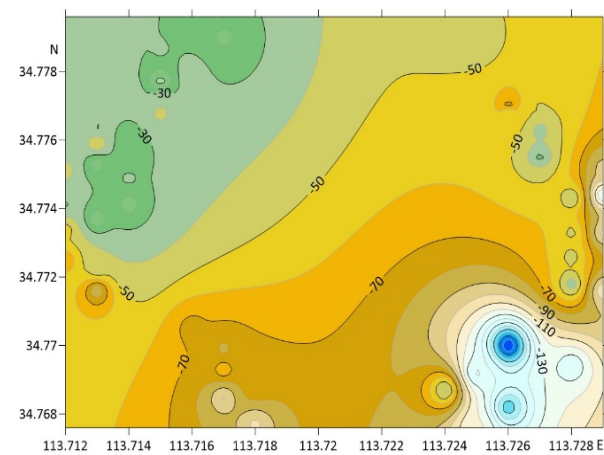


Figure 11. Vertical displacement caused by cluster load building.

The maximum geoid undulation of the CBD cluster building load to the center was PICC Building (Table 2), which was 680 m from the center of the ground, the height of the building was 186 m, and the bottom area was 2140 m², and the vertical displacement caused by it was 0.741 mm; the smallest geoid undulation produced by the center was Dennis 6 heaven and earth, and the vertical displacement caused by it was 0.021 mm (Figures 12 and 13).

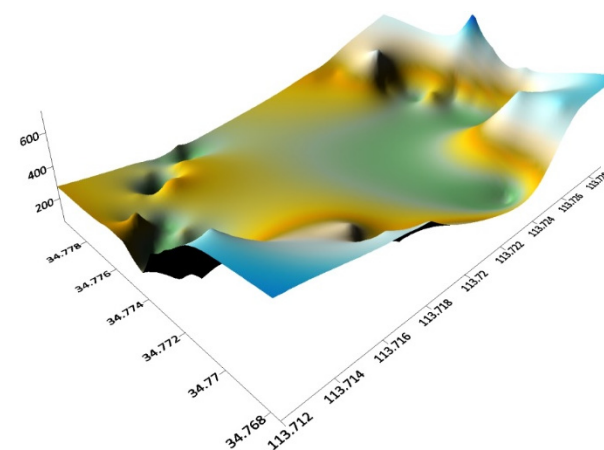


Figure 12. Three-dimensional topographic tilt undulation caused by cluster-loaded buildings (10⁻³ mm).

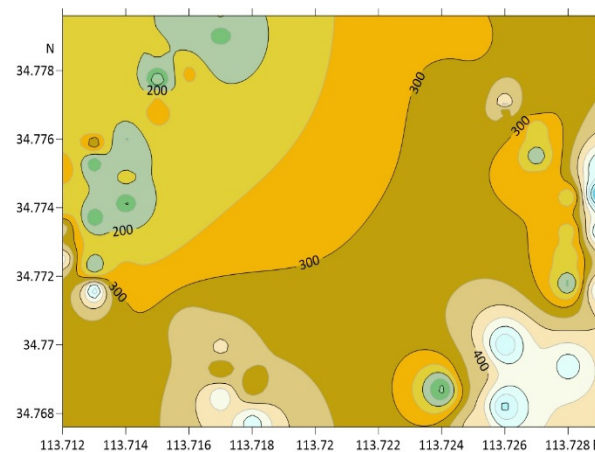


Figure 13. Geoid undulation caused by cluster-loaded buildings (10^{-3} mm).

The maximum geoid inclination displacement caused by the CBD cluster building load on the center was the International Chamber of Commerce Building (Table 2). The displacement amount was -0.084 s; the smallest amount of geoid tilt to the center was Dennis 6, and the amount of geoid tilt caused was -0.031 s (Figures 14 and 15).

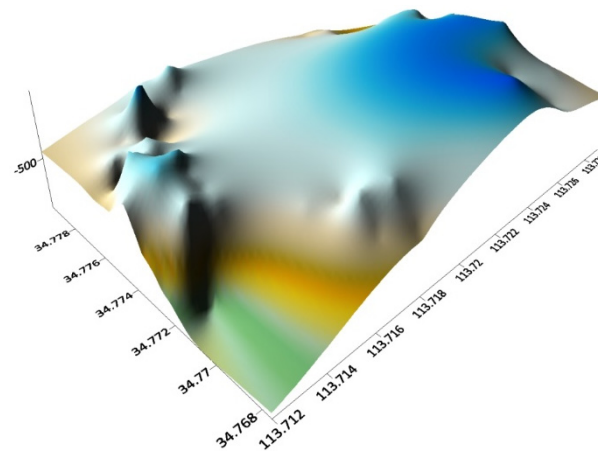


Figure 14. Three-dimensional topographic tilt caused by cluster-loaded buildings (10^{-4} mm).

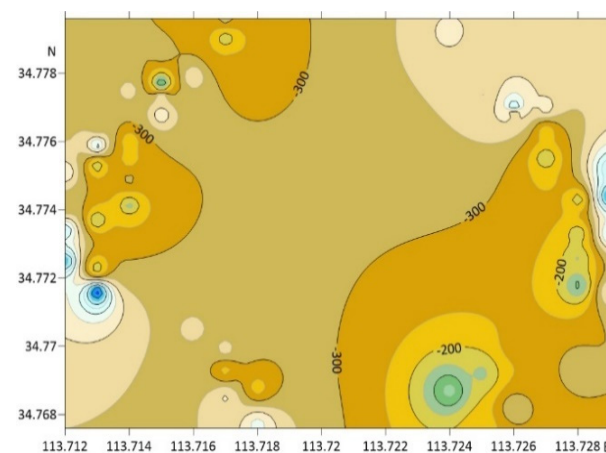


Figure 15. Geoid tilt caused by cluster-loaded buildings (10^{-4} mm).

The impact of the CBD cluster building load on the displacement of the International Convention and Exhibition Center was centered on the International Convention and Exhi-

bition Center, and the influence of each building was superimposed on each other to form a decreasing horizontal displacement belt (Table 2). For the International Convention and Exhibition Center, the cumulative horizontal displacement was 0.1027 mm, the cumulative vertical displacement was -0.3275 mm, the cumulative earth undulation was 17.217 mm, and the cumulative earth inclination was -1.8 s. Figure 6 is a three-dimensional diagram of the vertical displacement of the International Convention and Exhibition Center. In the horizontal and vertical coordinate system of the main image factors, and the weight of the building and the distance to the International Convention and Exhibition Center, the vertical displacement is the z coordinate. It can be seen that the vertical displacement was toward the International Convention and Exhibition. The center position is gradually increasing, and the direction is constantly changing.

3.3. Discussion and Verification

This paper uses the actual measured data of the CBD cluster building load in Zhengzhou to calculate the horizontal displacement, vertical displacement, geoid undulation, and geoid inclination of the International Convention and Exhibition Center. The resulting horizontal displacement was 0.1027 mm; the resulting vertical displacement was -0.33 mm; the caused geoid undulation was 17.22 mm; the inclination of the geoid was -1.87 s. Wang [32] and others used the data provided by the Shanghai Institute of Surveying and Mapping to calculate the impact of the Shanghai urban building complex on the surface of the earth and the geoid in Shanghai and its surrounding areas. The calculation results show that under the load of the Shanghai building complex, the maximum vertical displacement of the ground was -1.3269 mm, the maximum undulation of the geoid caused by the building complex was -0.25 mm, and the maximum elevation change was 1.1052 mm. Through comparative analysis, under the action of the load of the building group, the horizontal displacement, vertical displacement, undulation of the ground, and the change of the ground inclination of the ground gradually decreased with the increase of the distance to the target; at the same time, it also decreased with the building group itself. The weight increased and gradually strengthened.

In order to verify the results calculated in this paper, they should be compared with the measured data. However, ground deformation is related to loading buildings, groundwater exploitation, urban population development, and other factors. Therefore, this paper was compared with the measured ground deformation trend in this area. Through InSAR data inversion, the land subsidence rate of the study area in 2013 was shown in Figure 16, with an average value of 9 mm/a. From 2007 to 2018, the accumulated settlement in the study area reached 150 mm in Figure 17. The overall trend of settlement. The settlement center was located in the international convention and exhibition center, which is consistent with the calculation trend in this paper.



Figure 16. Subsidence rate of Zhengzhou CBD in 2013 (mm/a).

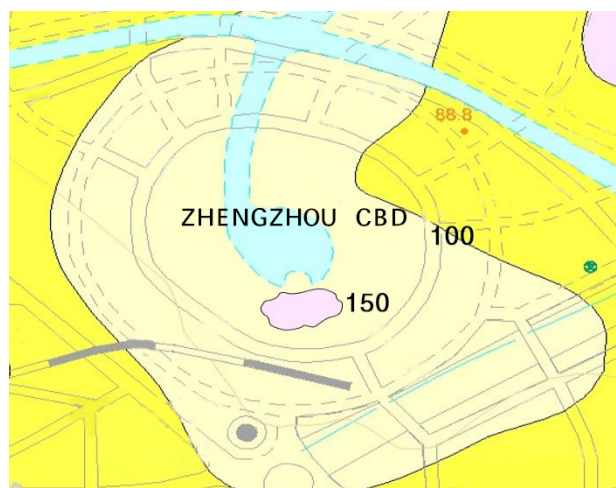


Figure 17. Cumulative settlement of Zhengzhou CBD in 2007–2018 (mm).

4. Conclusions and Recommendations

Based on the load tide theory and the measured building data in Zhengzhou, the influence of Zhengzhou Millennium Plaza Minsheng Bank as an independent building on the ground deformation was calculated. The impact of the cluster load buildings around the CBD on the ground horizontal and vertical displacements of the International Convention and Exhibition Center were analyzed. The calculation results show that the horizontal displacement, vertical displacement, geoid undulation, and geoid inclination of the ground caused by the independent single building, Millennium Plaza Minsheng Bank, were all centered on the building, and the distance continues to increase. With the increase of the non-linear attenuation and diffusion to the surrounding concentric circles, the attenuation speed gradually decreased. The horizontal displacement, vertical displacement, undulation, and tilt of the ground caused by the cluster building load were related to the distance to the target building and the cluster building load, but the relationship is non-linear; the greater the distance to the target building, the smaller the influencing deformation variable; the greater the weight of the building group itself, the smaller the influencing deformation variable.

In the research process, there is still some content to be further studied: (1) The model used in this article was to calculate the building as a mass point. The distance, r , was very small, and its influence cannot be calculated. How to calculate the close distance of the building and shape variable is yet to be further explored. (2) The influence of building group load is superimposed, and the superimposed influence relationship needs to be further studied.

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