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Impact of Government Subsidy Strategies on Supply Chains Considering Carbon Emission Reduction and Marketing Efforts

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Abstract: This study analyzes the impact of different government subsidies on supply chain members under the low-carbon policy. Using the theory and the Stackelberg game method, we derive the equilibrium decision of diverse government subsidy models on the carbon emission reduction efforts and marketing efforts of supply chain members when manufacturers are dominant. We found that government subsidies positively influenced the carbon emission reduction efforts and marketing efforts of supply chain members and could increase the overall profit of the supply chain and the overall welfare of society. Meanwhile, social welfare increased first and then decreased with the subsidy, and there was a maximum value. Within a certain threshold, when the market demand was sensitive to carbon emission reduction efforts, it was more beneficial to subsidize manufacturers, and when it was sensitive to marketing efforts, subsidizing retailers was more beneficial. Regardless of the subsidy situation, an optimal subsidy rate exists among supply chain members. Meanwhile, adjusting government subsidy measures can decrease the profit gap between supply chain members, and it provides potential possibilities for cooperation among supply chain members.

Keywords: low-carbon economy; government subsidies; sustainability; carbon emission reduction efforts; marketing efforts; Stackelberg game

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1. Introduction

With the growth of the world's industrial economy, carbon dioxide emissions have snowballed, and a series of ensuing issues have endangered the sustainable development of humans and posed a threat to the health of human beings [1]. A growing number of countries believe that developing a low-carbon economy can alleviate global warming, improve the ecological environment, and promote coordinated social development. Sustainable development of the supply chain has received growing attention in a low-carbon economy. Charles, head of the UNEP, highlighted that in the next decade, sustainable supply chains will become the only supply chain model in the world (China–EU high-level forum on sustainable supply chains, 2016). Accordingly, countries have started formulating relevant low-carbon policies and measures. In 2017, the Chinese government formulated guidelines on promoting innovation and application of supply chains, as well as encouraged enterprises to engage in low-carbon manufacturing and green and sustainable management. In 2021, the South Korean government emphasized expanding cooperation with public companies in low-carbon projects, and it is expected to invest 94 trillion won in the sector by 2025. The EU introduced a plan called "Carbon Border Adjustment Mechanism" in the same year; this plan requires that all products imported into the EU must bear the cost of carbon emissions. It aims to promote the management of carbon emission reduction in the supply chain and avoid carbon leakage. In building a low-carbon city and advocating green development, Chile has fully respected the law of sustainable development and established an emission reduction office as a public institution to provide management functions, quantify carbon emission monitoring, and formulate plans to decrease carbon emissions.

The triple bottom line (TBL), first proposed by Elkington [2], believes that it is more conducive to the sustainable development of enterprises only by always adhering to the unity of corporate profitability, social responsibility, and environmental responsibility. Thus, the sustainable development of supply chains is based on the TBL, which requires strategic integration and system optimization of economic, social, and environmental objectives, involves multiple stakeholders, and requires the joint participation of supply chain members, governments, and consumers [3,4]. Facing the new trend of modern enterprise development and the new characteristics of market demand, problems related to low-carbon sustainable supply chain management (SSCM) have gradually developed around government regulation, supply chain member operation, and consumer purchase behavior. Meanwhile, scholars have started exploring from the standpoints of low carbon, sustainable development, and green development [5–7], and making suggestions for the promotion of a low-carbon economy and sustainable development of supply chains. In the context of low carbon, since the production and sales process of products is realized through the supply chain, to reduce the carbon emissions of products, different members of the supply chain play different roles; not only manufacturers need to invest in low-carbon R&D, but also retailers need to guide consumers' low-carbon purchasing behavior through marketing means (electronic advertising, shelf display, and promotional activities). In addition, in a global Accenture survey, >80% of respondents showed a high degree of green preference when making purchase choices, suggesting that more enterprises will expand the green and low-carbon commodity market and reinforce the low-carbon and sustainable management of the supply chain. However, in the early stage of policy implementation and manufacturing and promotion of low-carbon products, enterprises often face adverse conditions, such as constraints on R&D and technological innovation funds and lack of innovation resources. Therefore, the regulatory role of the government must be brought into play, as government subsidies are indispensable [8]. Various forms of government input subsidies can accelerate the transformation and upgrading of enterprises [9], promote enterprises to conduct R&D and innovation activities, enhance the efficiency of resource use, and improve the greenness of products [10]. Exploring the investment of sustainable low-carbon products under centralized and decentralized supply chains, Dong et al. [11] reported that the government's sustainable innovation fund stimulated the increase of orders while decreasing carbon emissions.

Our research primarily focuses on the new energy vehicle industry. Under the background of global low-carbon policies and the active promotion of the automobile industry to the transformation of new energy vehicles by governments worldwide, global well-known automobile enterprises have increased R&D investment and collaboration in supporting energy of new energy vehicles, as well as actively promoted product upgrading and carbon reduction. For example, Ford Motor announced a new sustainable Development Goal in March 2021, aiming to decrease greenhouse gas emissions from its global operations by 76% by 2035 compared with 2017. Toyota, BMW, and other companies are doing the same. Domestic car brands, such as Geely, have launched high-powered and long-endurance electric cars, and BYD has pioneered plug-in hybrid technology, which has significantly enhanced the energy efficiency of vehicles. Data reveal that China's existing new energy passenger cars decrease carbon emissions by about 15 million tons every year. To expand the advantages of the new energy vehicle industry in carbon emission reduction, several countries started vigorously subsidizing the new energy industry to support its development. The Chinese government has provided about $5 billion in subsidies to new energy vehicle companies between 2016 and 2020, pushing companies to continue making efforts to decrease carbon emissions. The American Clean Energy Act of 2021 provides a total of about $259.5 billion in clean energy tax credits. It also offers manufacturers a 30% tax credit to help them restructure or build new plants. Moreover, the EU approved a European battery industry innovation project, which will provide a total of 2.9 billion euros of financial assistance to 42 companies such as BMW in January 2021. Besides, owing to the role of retail enterprises in the sales of new energy vehicles, the government encourages retailers to actively publicize and play their role to make the market more accepting of environment-friendly new energy vehicle products. Some provinces and cities, such as Xi'an city in 2021, issued several measures to promote key consumption 100-day action, and highlighted that the top 10 automobile sales enterprises that made significant contributions to the 100-day action would be given certain subsidies. For example, home appliance energy-saving products usually need to turn to a business channel of large retailers, such as Suning, HBS, and GOME, to sell them. HBS got 11.7853 million yuan of government subsidies in the first half of 2021.

Based on the above enterprise cases, the following research questions are proposed:

Question 1: Different types of enterprises play different roles in the supply chain system (manufacturing enterprises focus on carbon emission reduction; retail enterprises focus on marketing); how will these roles affect the overall market demand?

Question 2: How do different government subsidy strategies, such as subsidizing manufacturers or retailers, affect the pricing and profits of supply chain members?

Question 3: How do the government's subsidy measures affect the sustainable development of the supply chain? Are they conducive to promoting the development of a low-carbon economy and society?

Hence, it is of great practical significance to explore the low-carbon supply chain. Our research problem is related to the literature about government subsidies under a low carbon supply chain, and carbon emission reduction efforts or marketing efforts of supply chain members. Our paper is closely related to Li et al. [12], Wang et al. [13], Wang et al. [7], Wang et al. [14], and Gao et al. [15]. Li et al. [12] explored government subsidies and enterprise cooperation to decrease emissions. Accordingly, we explored the government subsidy and marketing efforts to decrease emissions under the supply chain members joint effect of market demand. Wang et al. [13] proposed building the government subsidies to the manufacturers and retailers under the differential game model and the introduction of the cost-sharing contract. It is proved that carbon emission reduction and promotion efforts of supply chain members closely correlate with subsidy rate and game status. The difference is that we further explore the impact of government subsidies on supply chain members and deepen relevant research conclusions. Wang et al. [7] reported that carbon trading policy exerts a positive impact on carbon emission reduction and low-carbon publicity levels in the supply chain. However, we considered different subsidy strategies of the government to enrich relevant conclusions. However, some studies are dominated by retailers [14,15]; in practice, most government subsidies support manufacturing enterprises to decrease carbon emissions and upgrade products, and manufacturing enterprises shoulder more crucial responsibilities for resource conservation, environmental protection, and social development. Thus, research with manufacturers in the leading position is more in line with the actual situation and has more practical significance.

To answer the abovementioned research questions and fill in these research gaps, we consider investigating different government subsidy strategies and their impact on the decision-making of supply chain members in a manufacturer's dominant position. First, the market demand function is constructed by abstracting the real example and combining it with the research problem. Then, we considered the manufacturer dominant hypothesis and model, established the government subsidies, not the government subsidies to manufacturers and government subsidies retailers game model, and attained the corresponding pricing under different government subsidy policy, efforts to decrease emissions, marketing efforts, and supply chain profit, such as the social welfare equilibrium solution. Finally, the equilibrium price, supply chain profit, and total social welfare under different government subsidy strategies are compared and analyzed.

Overall, the main contributions of this study are as follows:

1. Among the current research, this is the first study to consider the combined impact of carbon emission reduction input and marketing effort level of supply chain members on market demand under the different roles played by government subsidy mechanism and supply chain members.

2. This study considers three subsidy strategies for supply chain members: no subsidy, manufacturer subsidy, and retailer subsidy. These strategies have rarely been explored in relevant studies. In government subsidy implementation, the government can maximize the overall profit of the supply chain and the total social welfare based on the different subsidy objects and the size of the subsidy proportion.

3. This study comparatively analyzes the decisions under equilibrium conditions, such as carbon emission reduction input, marketing effort level, supply chain profit, and social welfare under each strategy, and acquires more beneficial management enlightenment, which provides a reference for the government to implement subsidy measures and supply chain members to implement sustainable development strategy.

The remainder of this paper is structured as follows. First, we give the literature review in Section 2. Section 3 describes the problem and model assumptions; Section 4 presents the three decisions of government subsidies for supply chain members and the corresponding equilibrium solutions; Section 5 compares the optimal decisions of supply chain members and the government under different government subsidy strategies; Section 6 validates the study conclusion by a numerical analysis method; Section 7 summarizes the conclusions; Section 8 explains the practical significance and management enlightenment brought by this paper; Section 9 elaborates the limitations of this study and proposes future research directions.

2. Literature Review

Our paper is closely related to two streams of literature: government subsidies under a low carbon supply chain, and carbon emission reduction efforts or marketing efforts of supply chain members. At the end of each section, we highlight the key differences between the existing research and our work.

2.1. Government Subsidies under Low Carbon Supply Chain

In recent years, many studies have investigated government subsidies and corporate carbon emission reduction and promotion. The government can play a role in macro-control. Some studies focused on the performance of government subsidies and carbon emissions. Resnier et al. [16], Toshimitsu et al. [17], and Hong et al. [18] explored the impact of government subsidies on the development of low-carbon industries. The research shows that government subsidies can not only promote the development and development of clean energy, but also promote the production and marketing of low-carbon products. Furthermore, Sheu et al. [19] found through research that government subsidies can improve the income of low-carbon enterprises. Jiang et al. [20] reported that government carbon subsidies could promote the profits of supply chain members and reduce carbon emissions. Xia and Xu [21] demonstrated that government subsidies are conducive to decreasing the impact of supply chains on the environment and improving the overall benefits of low-carbon supply chains. Li et al. [12] considered supply chain emission reduction cooperation and explored the optimal manufacturer under different government subsidies, emission reduction investment, and the government's optimal subsidy rate. While some studies are based on different government subsidy strategies, Hafezalkotob et al. [22] compared the role of government subsidy policies and carbon tax policies, revealing that appropriate government policies can inspire manufacturers to implement low-cost policies. Carbon production reduces the carbon emissions of enterprises and promotes the low-carbon sustainable development of enterprises. Sinayi and Rasti-Barzoki [23] considered the manufacturer's own carbon emission reduction efforts and examined the impact on the profits of the supply chain and social welfare without government subsidies and subsidies based on emission reduction levels, emphasizing that subsidies based on carbon emission reduction levels exert a positive effect on social welfare and social development. These studies have laid the foundation for establishing the model under the subsidy policy in this paper. Unlike our study, all the abovementioned studies did not consider the net utility of government subsidies for supply chain members.

2.2. Carbon Emission Reduction Efforts or Marketing Efforts of Supply Chain Members

Meanwhile, in the study of carbon emission reduction efforts and promotion efforts of supply chain members, research has demonstrated that enterprises' carbon emission reduction and advertising marketing of supply chain members can increase supply chain profits [24]. Green marketing of supply chain members can increase market demand [25] and enhance supply chain performance [15]. Many studies focused on the coordinated design of a cooperative emission reduction supply chain [26–28], and some studies examined the cooperation of manufacturers to guide retailers' low-carbon publicity [14,29,30]. Wang et al. [26] explored the problem of carbon emission reduction when retailers are dominant within the power balance, and reported that cost sharing and wholesale price premium contracts can attain the objective of reducing total carbon emissions. Huang et al. [27] proposed cooperative emission reduction, cooperative promotion, and that there are three cooperative modes of mutual cost-sharing, as well as that mutual cost-sharing is beneficial to manufacturers' emission reduction and retailers' efforts to promote sales. Zu et al. [28] established a differential game model between manufacturers and their upstream suppliers, and studied the best outcomes of the two parties with different game positions. They optimized emission reduction strategies and profit levels, and further pointed out that the cooperation between the two parties can effectively reduce the carbon emissions of products and increase the profits of the supply chain. Xie et al. [29] studied the supply chain system composed of upstream manufacturers and downstream two competing retailers, indicating that all subjects of the supply chain vigorously publicize low-carbon products, to expand sales to make up for the capital investment in emission reduction. Wang et al. [14] considered that the retailer bears part of the cost of emission reduction efforts of the manufacturer, and used the differential game theory to give the optimal emission reduction effort level and publicity effort level of the manufacturer and the retailer. Zhou et al. [30] studied how to optimize the management decision-making of the low-carbon supply chain, and analyzed the impact of a cooperative publicity contract, with cooperative publicity and emission reduction cost sharing a contract on the optimal decision-making and coordination of low-carbon supply chain. Unlike our research, most of the abovementioned papers do not consider the joint impact of carbon emission reduction efforts and marketing efforts on market demand.

3. Model Formulation

Under the mechanism of government subsidies, consider a low-carbon supply chain based on a single manufacturer and a single retailer and consumers. Of these, manufacturers play a leading role. Manufacturers adopt new low-carbon technologies and production modes to produce green and low-carbon products and reduce carbon emissions, such as hybrid vehicles produced by Toyota, BYD, and Geely's ultra-long-endurance electric vehicles. Retailers can have close contact with consumers in product sales to give play to their marketing advantages and overcome the manufacturers' limitations. To support emerging industries and alleviate the capital and resource problems of enterprises, the government provides subsidies to manufacturers for carbon emission reduction. To make the market accept low-carbon products more quickly, the government encourages retailers to execute marketing propaganda and provides certain subsidies to retailers for their marketing efforts. Figure 1 shows the model structure of the supply chain studied in this paper.

Table 1 shows the model parameters and descriptions to express the problem clearly.

In addition, we made the following assumptions based on the study problem. We assume that supply chain member manufacturers and retailers seek to maximize their interests, and members' decision-making risks are neutral. To simplify the calculation and not affect the results, it is assumed that the product manufacturing cost under the carbon trading and carbon allowance policy is zero.

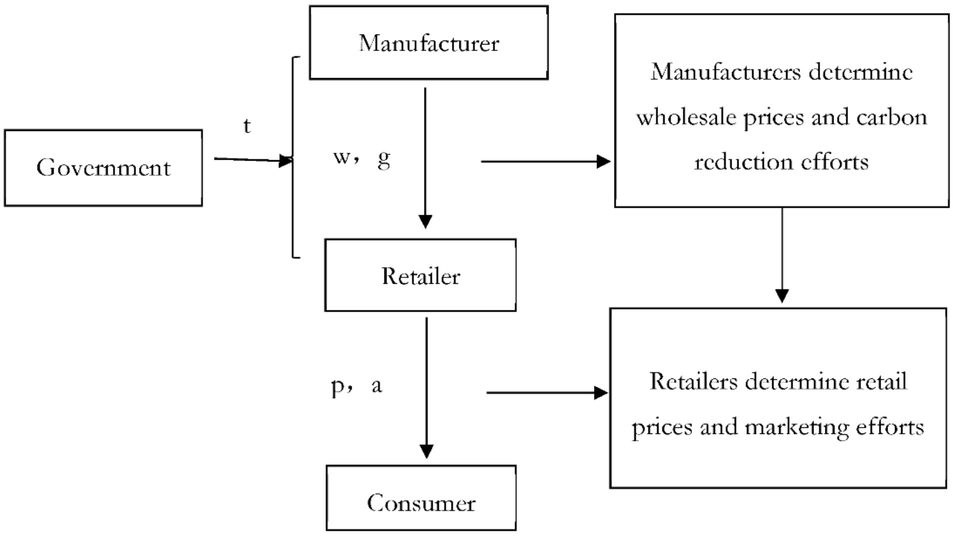

Figure 1. Low-carbon supply chain operation mode under government subsidies.

Table 1. Model notations.

Notation	Definition
p^i	The retail price of the product in Strategy i (decision variable, $i = e, s, q$)
w^i	The unit wholesale price of the product in Strategy i (decision variable, $i = e, s, q$)
g^i	Manufacturer's carbon reduction efforts in Strategy i (decision variable, $i = e, s, q$)
a^i	Retailer marketing efforts in Strategy i (decision variable, $i = e, s, q$)
t^i	The proportion of government subsidies ($0 < t < 1$) in Strategy i (decision variable, $i = e, s, q$)
T	Basic market demand scale
b	Consumers' sensitivity coefficient to retail prices indicates the impact of product price changes on the supply chain market demand
v^i	Profit margin per unit product ($i = e, s, q$)
f	The sensitivity coefficient of carbon emission reduction efforts to market demand reflects consumer support for carbon emission reduction efforts
γ	The sensitivity coefficient of the retailer's marketing efforts to demand, that is, the impact of changes in marketing efforts on the supply chain market demand
m	The cost factor for manufacturers' carbon reduction efforts
h	Cost coefficient of retailer's marketing efforts
U_c^i	Consumer surplus in Strategy i ($i = e, s, q$)
${\pi_j}^i$	$j = m, r, sc$ where m denotes the manufacturer; r denotes the retailer; sc denotes the entire supply chain in Strategy i ($i = e, s, q$)
D_{sc}^i	Market demand in Strategy i ($i = e, s, q$)
U_{sc}^i	Social welfare in Strategy i ($i = e, s, q$)

Market demand. To characterize the demand function, assume that the product market demand in the product supply chain is primarily decided by the manufacturer's new carbon emission reduction efforts and the retailer's marketing efforts. Based on the hypothesis of Gurnani and Erkoc [31], the market demand function is as follows:

$$D_{sc} = T - bp + fg + \gamma a \tag{1}$$

As the supply chain members are profit-oriented, there is usually $p > w$. For the convenience of calculation, let $p = w + v$, where $v > 0$ denotes the marginal profit of the retailer's unit product.

Social welfare. The total social welfare conforms to the principle of linear addition, that is, the sum of the total profit of the supply chain and consumer surplus; consumer surplus can be expressed as follows:

$$U_c = \int_{p_{min}}^{p_{max}} D_{sc}(p)dp = \frac{(D_{sc}(p))^2}{2b} \tag{2}$$

Carbon emission reduction efforts. Based on the study of Huang et al. [27], the fixed cost of the manufacturer's carbon emission reduction efforts is $\frac{1}{2}mg^2$, where m denotes the cost coefficient of the manufacturer's carbon emission reduction efforts.

Marketing effort. Based on Gao et al. [15], the retailer's marketing effort cost is $\frac{1}{2}ha^2$, where h denotes the cost coefficient of the retailer's marketing effort.

Decision sequence. We model pricing and government subsidies decisions as a Stackelberg game, in which the manufacturer and retailer are the leader and follower, respectively. The decision sequence is depicted as follows. First, the government decides the subsidy rate t, then the manufacturer decides the market wholesale price w, the carbon emission reduction effort g, and then the retailer decides the retail price of the product p and marketing effort a.

Subsidy strategy. Given the results of our model, we considered the three situations where the government does not subsidize all supply chain members (Strategy e), only subsidizes manufacturers' carbon emission reduction efforts (Strategy s), and only subsidizes retailers' marketing efforts (Strategy q), and multiple parties in the game make their decisions to maximize profits.

4. Equilibrium Analysis

In this section, we derive the equilibrium decision of diverse government subsidy models on the carbon emission reduction efforts and marketing efforts of supply chain members when manufacturers are dominant, in which the manufacturer and retailer make independent decisions to maximize their own expected revenue under the three strategies. We used backward induction to solve the problem. In the following, superscript "e", "s", and "q" are used to represent the optimal decision-making solutions in these three situations, respectively.

4.1. Strategy e

In this strategy, the manufacturer decides the market wholesale price w^e and the carbon emission reduction effort g^e; then, the retailer decides the commodity retail price p^e and the marketing effort a^e. At this time, the profit and social welfare functions of the manufacturer, retailer, and supply chain are as follows:

$$\pi_m^e = w^e D_{sc} - \frac{1}{2} m (g^e)^2 \tag{3}$$

$$\pi_r^e = v^e D_{sc} - \frac{1}{2} h (a^e)^2 \tag{4}$$

$$\pi_{sc}^e = \pi_m^e + \pi_r^e \tag{5}$$

$$U_{sc}^e = \pi_{sc}^e + U_c \tag{6}$$

In a manufacturer-led supply chain, the retailer's profit function is the joint concave function of p^e and a^e, while the manufacturer's profit function is the joint concave function of w^e and g^e, and there is a unique optimal solution. The reverse induction method can be used to attain the equilibrium results of the decision-making when the government does not subsidize the supply chain members when the conditions $2bh - \gamma^2 > 0$ and $-f^2h + 4bhm - 2m\gamma^2 > 0$ are met.Solving the supply chain's equilibrium results, we can derive the following Lemma 1. as shown in Table 2. The proofs of all Lemmas are in Appendix A.1.

Table 2. Equilibrium results under three different strategies.

Equilibrium Result	*Lemma 1*	*Lemma 2*	*Lemma 3*
w^i	$\frac{mT(2bh-\gamma^2)}{b(-f^2h+4bhm-2m\gamma^2)}$	$\frac{(-m+mt)T(2bh-\gamma^2)}{b(f^2h-4bhm+4bhmt+2m\gamma^2-2mt\gamma^2)}$	$\frac{mT(-2bh+2bht+\gamma^2)}{b(f^2h-4bhm-f^2ht+4bhmt+2m\gamma^2)}$
g^i	$-\frac{fhT}{f^2h-4bhm+2m\gamma^2}$	$-\frac{fhT}{f^2h-4bhm+4bhmt+2m\gamma^2-2mt\gamma^2}$	$-\frac{f(-h+ht)T}{-f^2h+4bhm+f^2ht-4bhmt-2m\gamma^2}$
p^i	$\frac{mT(3bh-\gamma^2)}{b(-f^2h+4bhm-2m\gamma^2)}$	$\frac{m(-1+t)T(3bh-\gamma^2)}{b(f^2h+2m(-1+t)(2bh-\gamma^2))}$	$\frac{mT(3bh(-1+t)+\gamma^2)}{b(f^2(h-ht)+2m(2bh(-1+t)+\gamma^2))}$
a^i	$\frac{mT\gamma}{-f^2h+4bhm-2m\gamma^2}$	$\frac{m(-1+t)T\gamma}{f^2h+2m(-1+t)(2bh-\gamma^2)}$	$-\frac{mT\gamma}{f^2(h-ht)+2m(2bh(-1+t)+\gamma^2)}$
v^i	$-\frac{hmT}{f^2h-4bhm+2m\gamma^2}$	$\frac{hm(-1+t)T}{f^2h+2m(-1+t)(2bh-\gamma^2)}$	$\frac{hm(-1+t)T}{f^2(h-ht)+2m(2bh(-1+t)+\gamma^2)}$
D^i_{sc}	$\frac{bhmT}{-f^2h+4bhm-2m\gamma^2}$	$\frac{bhm(-1+t)T}{f^2h+2m(-1+t)(2bh-\gamma^2)}$	$\frac{bhm(-1+t)T}{f^2(h-ht)+2m(2bh(-1+t)+\gamma^2)}$
U^i_c	$\frac{bh^2m^2T^2}{2(f^2h+2m(-2bh+\gamma^2))^2}$	$\frac{bh^2m^2(-1+t)^2T^2}{2(f^2h+2m(-1+t)(2bh-\gamma^2))^2}$	$\frac{bh^2m^2(-1+t)^2T^2}{2(f^2h(-1+t)-2m(2bh(-1+t)+\gamma^2))^2}$
π^i_m	$\frac{hmT^2}{-2f^2h+8bhm-4m\gamma^2}$	$\frac{hm(-1+t)T^2}{2f^2h+4m(-1+t)(2bh)-\gamma^2}$	$-\frac{hm(-1+t)T^2}{2f^2h(-1+t)-4m(2bh(-1+t)+\gamma^2)}$
π^i_r	$\frac{hm^2T^2(2bh-\gamma^2)}{2(f^2h+2m(-2bh+\gamma^2))^2}$	$\frac{hm^2(-1+t)^2T^2(2bh-\gamma^2)}{2(f^2h+2m(-1+t)(2bh-\gamma^2))^2}$	$\frac{hm^2(-1+t)T^2(2bh(-1+t)+\gamma^2)}{2(f^2h(-1+t)-2m(2bh(-1+t)+\gamma^2))^2}$
π^i_{sc}	$\frac{hmT^2(-f^2h+6bhm-3m\gamma^2)}{2(f^2h+2m(-2bh+\gamma^2))^2}$	$\frac{hm(-1+t)T^2(f^2h+3m(-1+t)(2bh-\gamma^2))}{2(f^2h+2m(-1+t)(2bh-\gamma^2))^2}$	$-\frac{hm(-1+t)T^2(f^2h(-1+t)-3m(2bh(-1+t)+\gamma^2))}{2(f^2h(-1+t)-2m(2bh(-1+t)+\gamma^2))^2}$
U^i_{sc}	$\frac{hmT^2(-f^2h+7bhm-3m\gamma^2)}{2(f^2h+2m(-2bh+\gamma^2))^2}$	$\frac{hmT^2(-f^2h+m(-1+t)^2(7bh-3\gamma^2))}{2(f^2h+2m(-1+t)(2bh-\gamma^2))^2}$	$\frac{hmT^2(-f^2h(-1+t)^2+7bhm(-1+t)^2+m(-3+2t)\gamma^2)}{2(f^2h(-1+t)-2m(2bh(-1+t)+\gamma^2))^2}$

4.2. Strategy s

In this strategy, the government subsidizes manufacturers' carbon emission reduction efforts. At this time, the decision sequence of the supply chain is as follows: first, the government decides the subsidy rate t^s, then the manufacturer decides the market wholesale price w^s, the carbon emission reduction effort g^s, and then the retailer decides the retail price of the product p^s and marketing effort a^s. At this time, the profit and social welfare functions of the manufacturer, retailer, and supply chain are as follows:

$$\pi^s_m = w^s D_{sc} - \frac{1}{2}m(g^s)^2(1-t) \tag{7}$$

$$\pi^s_r = v^s D_{sc} - \frac{1}{2}h(a^s)^2 \tag{8}$$

$$\pi^s_{sc} = \pi^s_m + \pi^s_r \tag{9}$$

$$U^s_{sc} = \pi^s_{sc} + U_c - \frac{1}{2}tm(g^s)^2 \tag{10}$$

The equilibrium results of the decision-making when the government only subsidizes manufacturers can be attained when the conditions $2bh - \gamma^2 > 0$ and $f^2h + 2m(-1+t)$ $(2bh - \gamma^2) < 0$ are met.Solving the supply chain's equilibrium results, we can derive the following Lemma 2. as shown in Table 2.

4.3. Strategy q

In this strategy, the government subsidizes the retailer's marketing efforts. At this time, the decision sequence of the supply chain is as follows: first, the government decides the subsidy rate t^q, then the manufacturer decides the market wholesale price w^q and the carbon emission reduction effort g^q, finally the retailer decides the retail price p^q and marketing efforts a^q. At this time, the profit and social welfare functions of the manufacturer, retailer, and supply chain are as follows:

$$\pi^q_m = w^q D_{sc} - \frac{1}{2}m(g^q)^2 \tag{11}$$

$$\pi^q_r = v^q D_{sc} - \frac{1}{2}h(a^q)^2(1-t) \tag{12}$$

$$\pi^q_{sc} = \pi^q_m + \pi^q_r \tag{13}$$

$$U^q_{sc} = \pi^q_{sc} + U_c - \frac{1}{2}th(a^q)^2 \tag{14}$$

The equilibrium results of the supply chain decision-making when the government only subsidizes retailers can be attained under conditions $2bh(1-t)-\gamma^2 > 0$ and $f^2(h-ht)+2m(2bh(-1+t)+\gamma^2) < 0$. Solving the supply chain's equilibrium results, we can derive the following Lemma 3. as shown in Table 2.

Corollary 1. *(i)* $\frac{\partial w^e}{\partial f} > 0$; *(ii)* $\frac{\partial g^e}{\partial f} > 0$; *(iii)* $\frac{\partial v^e}{\partial f} > 0$; *(iv)* $\frac{\partial p^e}{\partial f} > 0$; *(v)* $\frac{\partial D^e_{sc}}{\partial f} > 0$; *(vi)* $\frac{\partial U^e_c}{\partial f} > 0$; *(vii)* $\frac{\partial \pi^e_m}{\partial f} > 0$; *(viii)* $\frac{\partial \pi^e_r}{\partial f} > 0$; *(ix)* $\frac{\partial \pi^e_{sc}}{\partial f} > 0$ *and* $\frac{\partial U^e_{sc}}{\partial f} > 0$.

The proofs of all Corollaries, Propositions are in Appendix A.2.

Corollary 1 presents that the wholesale price of products, the level of carbon emission reduction efforts, retailer's marginal profit, product's retail price, product's market demand, manufacturer's profit, retailer's profit, and social welfare, all follow the consumer's reduction of corporate carbon emissions. In addition, the degree of recognition of effort increases because of consumers; as the demand side of market products increases, the introduction and implementation of carbon emission reduction policies promotes the development of a green and low-carbon society, aligning with people's pursuit of a green and low-carbon life, and people's efforts to decrease corporate carbon emissions. Thus, the demand for corresponding products will also rise, increasing the market demand space. Furthermore, manufacturers and retailers increase prices to respond to the expanded market demand and promote the sustainable development of the supply chain; thus, wholesale prices and retail prices increase, and manufacturers' and retailers' business profits and social welfare also increase because of rising demand. The increase in social welfare naturally promotes the good development of society.

Corollary 2. *(i)* $\frac{\partial w^e}{\partial \gamma} > 0$; *(ii)* $\frac{\partial v^e}{\partial \gamma} > 0$; *(iii)* $\frac{\partial a^e}{\partial \gamma} > 0$; *(iv)* $\frac{\partial p^e}{\partial \gamma} > 0$; *(v)* $\frac{\partial D^e_{sc}}{\partial \gamma} > 0$; *(vi)* $\frac{\partial {U_c}^e}{\partial \gamma} > 0$; *(vii)* $\frac{\partial \pi^e_m}{\partial \gamma} > 0$; *(viii)* $\frac{\partial \pi^e_r}{\partial \gamma} > 0$; *(ix)* $\frac{\partial \pi^e_{sc}}{\partial \gamma} > 0$ *and* $\frac{\partial U^e_{sc}}{\partial \gamma} > 0$.

The logic behind Corollary 2 is identical to that of Corollary 1. Therefore, we omit the details for brevity.

Corollary 3. *(i)* $\frac{\partial w^s}{\partial t} > 0$; *(ii)* $\frac{\partial g^s}{\partial t} > 0$; *(iii)* $\frac{\partial v^s}{\partial t} > 0$; *(iv)* $\frac{\partial a^s}{\partial t} > 0$; *(v)* $\frac{\partial p^s}{\partial t} > 0$; *(vi)* $\frac{\partial D^s_{sc}}{\partial t} > 0$; *(vii)* $\frac{\partial U^s_c}{\partial t} > 0$; *(viii)* $\frac{\partial \pi^s_m}{\partial t} > 0$; *(ix)* $\frac{\partial \pi^s_r}{\partial t} > 0$ *and* $\frac{\partial \pi^s_{sc}}{\partial t} > 0$.

Corollary 3 demonstrates that the products' wholesale price, the level of carbon emission reduction efforts, retailer's marginal profit, marketing efforts, products' retail price, product demand, consumer surplus, manufacturer's profit, retailer's profit, and the total supply chain profit increase with the increase in the proportion of government subsidies. With the increase in government subsidies, the cost of carbon emission reduction efforts by manufacturers has been decreased, and manufacturers can obtain higher profits. They are more willing to use new processes and equipment to improve the energy conversion efficiency of products and improve the green level of products to decrease enterprise carbon emissions, promote low-carbon sustainable development of enterprises, and increase the market demand for their products. In this case, both retailers and manufacturers have the incentive to increase prices; thus, the profit of the entire supply chain increases. Meanwhile, the increase in product market demand causes the consumer surplus to continue to increase, thereby improving the total social welfare level.

Corollary 4. *(i)* $\frac{\partial w^q}{\partial t} > 0$; *(ii)* $\frac{\partial g^q}{\partial t} > 0$; *(iii)* $\frac{\partial v^q}{\partial t} > 0$; *(iv)* $\frac{\partial a^q}{\partial t} > 0$; *(v)* $\frac{\partial p^q}{\partial t} > 0$; *(vi)* $\frac{\partial D^q_{sc}}{\partial t} > 0$; *(vii)* $\frac{\partial U^q_c}{\partial t} > 0$; *(viii)* $\frac{\partial \pi^q_m}{\partial t} > 0$; *(ix)* $\frac{\partial \pi^q_r}{\partial t} > 0$ *and* $\frac{\partial \pi^q_{sc}}{\partial t} > 0$.

Similarly, the logic behind Corollary 4 is identical to that of Corollary 3. Thus, we omit the details for brevity.

5. Comparison

This section compares the equilibrium results derived in Section 3.

Proposition 1. *(i)* $w^s > w^e$, *(ii)* $g^s > g^e$, *(iii)* $v^s > v^e$, *(iv)* $a^s > a^e$, *(v)* $p^s > p^e$, *(vi)* $D^s_{sc} > D^e_{sc}$, *(vii)* $U^s_c > U^e_c$, *(viii)* $\pi^s_m > \pi^e_m$, *(ix)* $\pi^s_r > \pi^e_r$, *(x)* $\pi^s_{sc} > \pi^e_{sc}$ *and* $U^s_{sc} > U^e_{sc}$.

Proposition 1 demonstrates that when the government subsidizes manufacturers' carbon emission reduction efforts, the products' wholesale price, the level of carbon emission reduction efforts, retailer's marginal profit, marketing efforts, products' retail price, product demand, consumer surplus, manufacturing business profits, retailer profits, total supply chain profits, and social welfare are increased because of government subsidies. It demonstrates that government subsidies to manufacturers are conducive to the increase of economic profits, the improvement of carbon emission reduction level is conducive to the improvement of carbon emission reduction performance, and can achieve a win–win situation of social benefits and profitability, because when the government only subsidizes manufacturers, and the cost of carbon emission reduction efforts by manufacturers is decreased, it increases the manufacturers' profitability. Manufacturers gain more profits, which, in turn, encourages them to promote product conversion and upgrade; this is conducive to the output of low-carbon products and also increases customer recognition of the product and increases market demand. Owing to the increase in market demand, retailers wholesale more goods and increase their retail prices to increase their profit margins. Meanwhile, manufacturers also increase the wholesale prices of goods to increase their profit margins, and the retailers' profitability increases further. When the retailer's profit increases, it is more active in marketing the product; thus, the market demand increases again, which is conducive to forming a virtuous circle and promote the sustainable development of the supply chain.

Proposition 2. *(i)* $w^q > w^e$, *(ii)* $g^q > g^e$, *(iii)* $v^q - v^e$, *(iv)* $a^q > a^e$, *(v)* $p^q > p^e$, *(vi)* $D^q_{sc} > D^e_{sc}$, *(vii)* $U^q_c > U^e_c$, *(viii)* $\pi^q_m > \pi^e_m$, *(ix)* $\pi^q_r > \pi^e_r$, *(x)* $\pi^q_{sc} > \pi^e_{sc}$ *and* $U^q_{sc} > U^e_{sc}$.

The logic behind Proposition 2 is identical to that of Proposition 1. Thus, we omit the details for brevity.

Proposition 3. *(i) if* $\pi_1 = \pi^s_{sc} - \pi^e_{sc}$, *then* $\frac{\partial \pi_1}{\partial t} > 0$. *(ii) if* $\pi_2 = \pi^q_{sc} - \pi^e_{sc}$, *then* $\frac{\partial \pi_2}{\partial t} > 0$.

Proposition 3 demonstrates that compared with the situation where the government does not subsidize, the profit margins of government-subsidized manufacturers and retailers increase with the increase in the subsidy rate; this is because as the government subsidies increase, it is equivalent to decreasing the cost of supply chain. Supply chain members are more willing to use new processes and equipment to enhance the green quality of products or to actively market products, which is conducive to increasing product market demand. Thus, when the commodity market demand increases, the total profit margin of the entire supply chain also become larger, which is conducive to forming a virtuous circle and promotes the sustainable development of the supply chain.

Proposition 4. *Under different government subsidies, the correlation between supply chain profits is as follows:* $\pi^q_{sc} \geq \pi^s_{sc} \geq \pi^e_{sc}$.

Proposition 4 demonstrates that compared with the situation where the government subsidizes manufacturers or does not subsidize, government subsidies to retailers are the best state, which can maximize the overall supply chain profit, also showing that retailers' marketing efforts in the supply chain play a vital role. Retailers can use their marketing efforts to deliver information about products and services to consumers, increase consumers' confidence in products and services, and expand the scale of demand.

Proposition 5. *(i) if* $f^2h + 2m\gamma^2 < 4bhm < 2\left(f^2h + m\gamma^2\right)$*, then* $\pi_r^e > \pi_m^e$*; (ii) let* $A = m\left(2bh - \gamma^2\right)$*, if* $\frac{A-f^2h}{A} < t < \frac{2A-f^2h}{2A}\left(A > f^2h\right)$*, then* $\pi_r^s > \pi_m^s$*,* $t = \frac{-f^2h+2bhm-m\gamma^2}{m(2bh-\gamma^2)}$*; (iii) if* $\frac{f^2h-2bhm+m\gamma^2}{h(f^2-2bm)} < t < \frac{f^2h-4bhm+2m\gamma^2}{h(f^2-4bm)}\left(f^2 < 2bm\right)$*, then* $\pi_r^q > \pi_m^q$*,* $t = \frac{f^2h-2bhm+m\gamma^2}{h(f^2-2bm)}$*.*

Proposition 5 demonstrates that if the government does not subsidize, the government subsidizes manufacturers, and the government subsidizes retailers, retailers within the supply chain can earn higher profits, showing that under the mechanism of government subsidies, a certain balance can be attained among the supply chain members. Even if the manufacturer occupies a dominant position and has the advantage of first decision-making, this does not imply that the retailer's benefit is lower than that of the manufacturer. Conversely, owing to the uncertainty of the benefits between supply chain members, the adjustment of government subsidies can decrease the profit gap between the two, which also lays the foundation for the study of the likelihood of collaboration between supply chain members. Simultaneously, it shows the necessity and importance of the government to adopt macro-incentive measures, such as subsidies, in the context of carbon trading and carbon quota policies.

Proposition 6. *(i) when the government subsidizes manufacturers, then* $t^s = \frac{3bh-\gamma^2}{7bh-3\gamma^2}$*,* $\frac{\partial t^s}{\partial \gamma} > 0$*; (ii) when the government subsidizes retailers, then* $t^q = -\frac{2\left(3bhm-m\gamma^2\right)}{h\left(f^2-10bm\right)}$*,* $\frac{\partial t^q}{\partial f} > 0$*.*

Proposition 6 demonstrates that under the government subsidy mechanism, whether the government only subsidizes manufacturers' carbon emission reduction efforts or the government only subsidizes retailers' marketing efforts, the profits of the supply chain and the total social welfare increase. Nevertheless, the government will not always subsidize manufacturers or retailers, but will control the subsidy rate at a certain level; this can optimize the overall welfare of society. It is conducive to the healthy development of society. Simultaneously, the more sensitive the market demand regarding carbon emission reduction efforts and marketing efforts, the higher the optimal carbon emission reduction efforts and marketing efforts. In addition, the higher the consumer's recognition of carbon emission reduction efforts, the more actively manufacturers will respond to national policies and use new production models and new processes to manufacture products. Similarly, consumers will be more positively affected by marketing efforts and products. The higher the reputation, the higher the consumer demand for products, and retailers will increase their marketing efforts to influence consumer decision-making.

6. Numerical Analysis

To validate the correctness of the model conclusions, we used MATLAB to conduct numerical simulation experiments to further analyze the impact of key parameters on supply chain decision-making. Meanwhile, based on the idea of parameter assignment in related studies [15] and the conditions that the equilibrium solution meets in different situations, the values of each parameter are as follows: $T = 100, b = 8, f = 3, \gamma = 3$, $h = 3, m = 3$. At this time, we can make $t \in (0, 0.75)$.

We explored the respective effects of the two on the profit and social welfare of the supply chain under different strategic models; Figures 2 and 3 show the results. The profit of the supply chain and total social welfare increases with the increase of the sensitivity coefficient of carbon emission reduction efforts or the sensitivity coefficient of marketing efforts under three strategies; this aligns with the conclusions of Corollary 1 and 2. Furthermore, Figure 2 shows that when the subsidy rate exceeds a certain threshold, market demand is more sensitive to carbon emission reduction efforts, supply chain profits and social welfare under Strategy s are higher than Strategy q. Similarly, Figure 3 shows the opposite result.

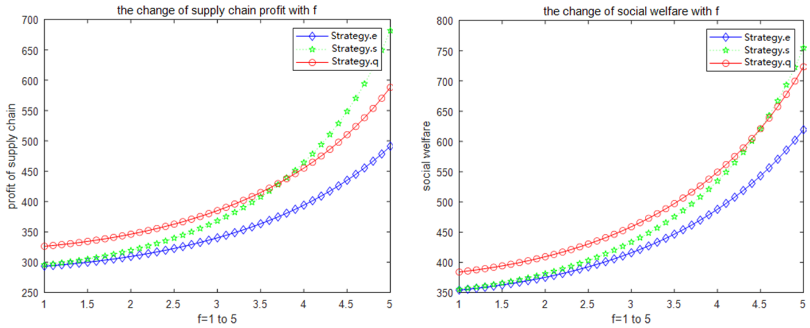

Figure 2. The impact of carbon emission reduction efforts sensitivity coefficient f on supply chain profits and social welfare in Strategies e, s, and q ($T = 100$ and $b = 8$).

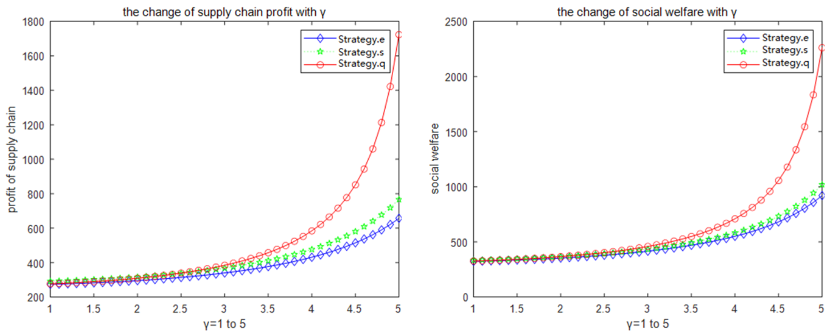

Figure 3. The impact of marketing effort sensitivity coefficient γ on supply chain profits and social welfare in Strategies e, s, and q ($T = 100$ and $b = 8$).

To further demonstrate the role of government subsidies on the sustainable development of supply chain, Figure 4 shows that compared with Strategy e, government subsidies increase consumer surplus, manufacturer profits, retailer profits and supply chain profit; this is consistent with the conclusions of Propositions 1 and 2. Meanwhile, it can also be seen that with the increase of subsidy rate, consumer surplus, profits of manufacturers and retailers, and supply chain profit also increase; this is consistent with the conclusions of Corollary 3 and 4. In addition, it demonstrates that it expands the profit space of the supply chain under the condition of government subsidies; this is consistent with the conclusions of Proposition 3.

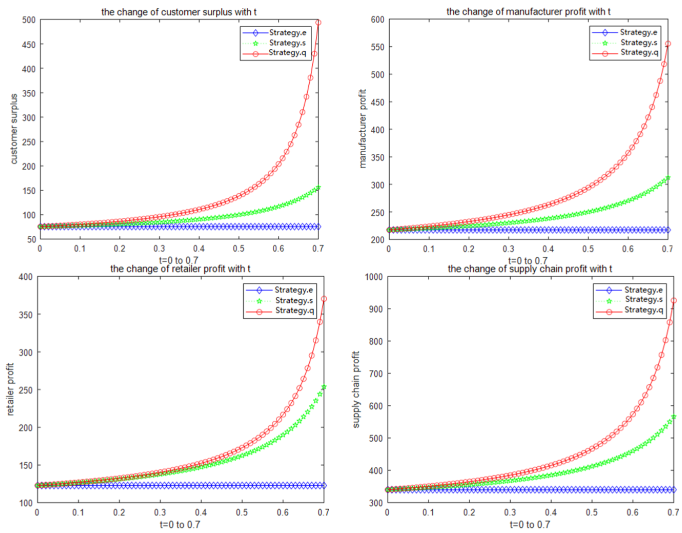

Figure 4. The impact of the government subsidy rate on consumer surplus, manufacturer's profit, retailer's profit, and supply chain profit in Strategies e, s, and q ($T = 100$ and $b = 8$).

Figure 5 shows that in the case of government subsidies, social welfare increases first and then decreases with the subsidy rate, and there is a maximum value; this is consistent with the conclusions of Proposition 6. Besides, within a certain threshold, social welfare is the maximum when the government subsidizes retailers' marketing efforts.

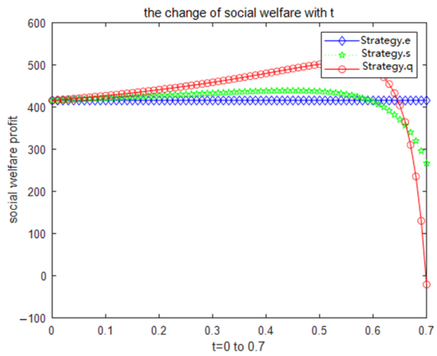

Figure 5. The impact of government subsidy rate on social welfare in Strategies e, s, and q ($T = 100$ and $b = 8$).

In addition, we conducted further research. Figures 6 and 7 show that within the range where the subsidy rate can be discussed, in Strategy s, when the variation of the marketing effort cost coefficient decreases, the sensitivity coefficient increases. As the magnitude of change increases, the optimal subsidy rate increases. In Strategy q, the subsidy rate shows a convex change with the change of the marketing effort sensitivity coefficient and the cost coefficient. The greater the change in the marketing effort cost, the higher the optimal subsidy rate. Second, the subsidy rate varies concavely with the change of the carbon emission reduction effort sensitivity coefficient and cost coefficient. The more sensitive the market demand is to carbon emission reduction efforts, the higher the optimal subsidy rate will be, and vice versa.

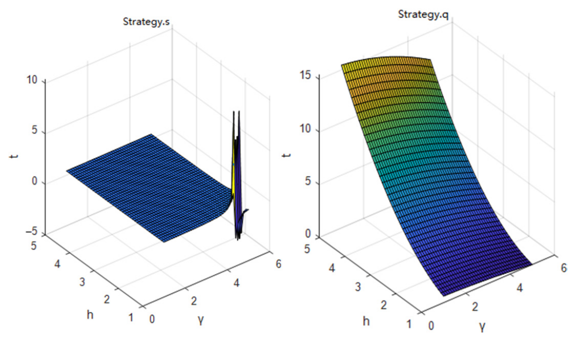

Figure 6. Changes in the optimal subsidy rate with the sensitivity coefficient and cost coefficient of marketing effort ($T = 100$ and $b = 8$).

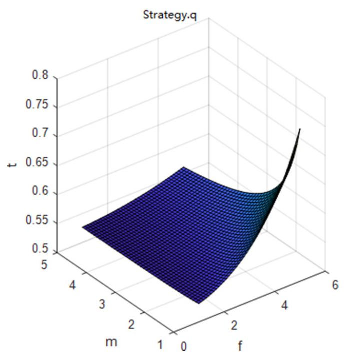

Figure 7. Changes in the optimal subsidy rate with the sensitivity coefficient and cost coefficient of carbon emission reduction efforts ($T = 100$ and $b = 8$).

7. Conclusions

This study explored the influence of different government subsidy strategies on the decision-making of supply chain members and the government's role in promoting sustainable development of the supply chain. In addition, the impact of the carbon emission reduction efforts of retailers and the marketing efforts of retailers is compared and analyzed, as well as the optimal pricing, optimal carbon emission reduction effort level, optimal marketing effort level, and optimal subsidy rate under different government subsidy situations when manufacturers are dominant. Based on our research, we derive the following theoretical results.

First, government subsidies for manufacturers' carbon emission reduction input and retailers' marketing efforts have a positive impact, which is conducive to encouraging manufacturers to improve their carbon emission reduction performance and retailers to enhance their marketing efforts. Our findings extend the results from Wang et al. [13], which focused more on cost-sharing among members of the supply chain and did not consider the separate effects of government subsidies on manufacturers and retailers.

Second, in the case of government subsidy, the social welfare level increases first and then decreases with the subsidy, and there is an optimal value.Wang et al. [13], Jiang et al. [20], and others highlighted the positive effect of government subsidies on social welfare, but did not point out the changing trend of social welfare, lacking extensive research.

Third, as the market demand is primarily the combined effect of carbon emission reduction investment and marketing efforts, when the market demand is sensitive to carbon emission reduction efforts, it is more advantageous for the government to subsidize manufacturers. When market demand is sensitive to the level of marketing effort, it is better for the government to subsidize retailers. Although Wang et al. [7] found that different carbon trading policies are beneficial to retailers and manufacturers, our study also found that different government subsidy strategies are beneficial to manufacturers and retailers.

Fourth, whether the government subsidizes or not, a certain balance can be reached among the members of the supply chain, and the optimal subsidy rate exists. Besides, within a certain threshold, government subsidies are beneficial to shrink the profit margin among supply chain members. These findings extend the work of Li et al. [1], Wang et al. [13], Xia et al. [21], and Jiang et al. [20].

The conclusion of this study shows that the reasonable government subsidies to supply chain members are conducive to the increase of economic profits, the improvement of social welfare, and the improvement of carbon emission reduction performance of enterprises. It can realize the win–win of enterprise economic and social benefits. The research results of this study fulfill the three dimensions of sustainability (TBL) emphasized by Elkington [2]. Overall, while the research conclusions deepen and expand the theme and scope of the existing research, they also coincide with the existing policies. For example, to further improve environmental and economic benefits, Shenzhen plans to increase support for green and low-carbon industries in 2022, with a maximum subsidy of 10 million.

8. Practical Implications

This study has crucial practical significance. First, the results have key implications for the application of macro-control measures such as government subsidies. From the management perspective, policy formulation often needs to rely on certain innovative resource conditions, taking into account the current situation of resources faced by enterprises of different scales. From a practical perspective, in the process of supply chain members responding to low-carbon policies, enterprises inevitably encountering difficulties in innovative resources, such as funds, would hinder the transformation and upgrading of enterprises and affect the sustainable development of enterprises, while the government can play its own role. To help enterprises solve the problems they face, it will not only increase the overall profits of the entire supply chain but also improve the overall welfare of the society. In the entire supply chain system, manufacturers and retailers have different roles in the entire supply chain system. Manufacturers and retailers should actively apply for relevant government subsidies. Manufacturers should improve carbon emission reduction performance, and retailers should improve their marketing level to jointly promote the sound development of the supply chain.

Second, the low-carbon performance and social benefits of the supply chain involve the government, enterprises, consumers, and other stakeholders. The government, supply chain members and consumers, as important members of society, look at these elements from a systematic viewpoint and are closely interconnected. Enterprises need to realize the integrated development of economy, environment, and society for SSCM. Supply chain members create a supply system of products and services, and consumers are the needs of the system, driving the production and service of the supply chain. The government can help solve the problems encountered in the supply chain, maximize the overall welfare of society, and better promote the sustainable development of society.

9. Limitations and Future Research Directions

This study can provide a crucial reference value for the practice of governments at all levels and related enterprises to some extent, and can be applied to the subsidy policies formulated by the government to help the production process of some industries. However, there are also some limitations. First, this study showed the possibility of cooperation among supply chain members. Whether it is manufacturers or retailers, they hope to better serve consumers with products and services, which aligns with certain requirements. From a system perspective, the cooperation of supply chain members is conducive to attaining win–win results or economies of scale. Future research can investigate the cooperation mechanism of supply chain members in the context of government subsidies.

Second, this study assumed that supply chain members are risk-neutral, which is not consistent with the real-life situation. The decision-makers of enterprises will have risk preferences in decision-making, and it is hard to attain risk-neutrality. Thus, future research can consider the decision-making issuer's risk appetite. Third, this study emphasized the problem of government subsidies in the case of manufacturers occupying a dominant position. However, in some industries, the dominant power of retailers is not lower than that of manufacturers. Future research can be based on the characteristics of different industries and explore the impact of different dominance of supply chain members.

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Appendix A

Appendix A.1. Part A (Solution of Equilibrium Results under Different Strategies)

Strategy e.

Proof of Lemma 1. Find the first derivative of profit margin and marketing effort from Equation (4):

$$\frac{\partial \pi_r^e}{\partial v} = fg + T - 2bv - bw + a\gamma \tag{A1}$$

$$\frac{\partial \pi_r^e}{\partial a} = -ah + v\gamma \tag{A2}$$

We obtain the Hessian matrix of profit function about marginal profit and marketing effort: $H = \begin{bmatrix} -2b & \gamma \\ \gamma & -h \end{bmatrix} = 2bh - \gamma^2$. As $-2b < 0$, when $2bh - \gamma^2 > 0$, the Hessian matrix is definitely negative. At this time, the expected profit function is the joint concave function of v and a. Therefore,v and a has optimal values. Let Equations (A1) and (A2) be equal to 0; the simultaneous equations can be obtained as follows:

$$v^e = \frac{h(fg + T - bw)}{2bh - \gamma^2} \tag{A3}$$

$$a^e = \frac{(fg + T - bw)\gamma}{2bh - \gamma^2} \tag{A4}$$

Incorporating Equations (A3) and (A4) into Equation (3), the simplified version is as follows:

$$\pi_m^e = \frac{-2b^2hw^2 + 2bh\left(-g^2m + fgw + Tw\right) + g^2m\gamma^2}{4bh - 2\gamma^2} \tag{A5}$$

From Equation (A5), find the first derivative of the wholesale price and carbon emission reduction efforts:

$$\frac{\partial \pi_m^e}{\partial w} = \frac{bh(fg + T - 2bw)}{2bh - \gamma^2} \tag{A6}$$

$$\frac{\partial \pi_m^e}{\partial g} = \frac{-2bghm + bfhw + gm\gamma^2}{2bh - \gamma^2} \tag{A7}$$

Equations (A6) and (A7) show a second derivative, and the Hessian matrix of the profit function on the wholesale price and carbon emission reduction efforts can be obtained as follows: $H = \begin{bmatrix} -\frac{2b^2h}{2bh-\gamma^2} & \frac{bfh}{2bh-\gamma^2} \\ \frac{bfh}{2bh-\gamma^2} & \frac{-2bhm+m\gamma^2}{2bh-\gamma^2} \end{bmatrix} = \frac{b^2h\left(-f^2h+4bhm-2m\gamma^2\right)}{\left(-2bh+\gamma^2\right)^2}$. As $-\frac{2b^2h}{2bh-\gamma^2} < 0$, when $-f^2h + 4bhm - 2m\gamma^2 > 0$, the Hessian matrix is definitely negative. At this time, the expected profit function is the joint concave function of w and g, which have optimal values. Let Equations (A6) and (A7) be equal to 0, the simultaneous equations can be obtained as follows:

$$w^e = \frac{mT\left(2bh - \gamma^2\right)}{b(-f^2h + 4bhm - 2m\gamma^2)} \tag{A8}$$

$$g^e = -\frac{fhT}{f^2h - 4bhm + 2m\gamma^2} \tag{A9}$$

Add w^e and g^e to Equations (A3) and (A4) respectively, to obtain the optimal value of marginal profit and marketing effort:

$$v^e = -\frac{hmT}{f^2h - 4bhm + 2m\gamma^2} \tag{A10}$$

$$a^e = \frac{mT\gamma}{-f^2h + 4bhm - 2m\gamma^2} \tag{A11}$$

Add w^e and v^e to get the best retail price:

$$p^e = \frac{mT(3bh - \gamma^2)}{b(-f^2h + 4bhm - 2m\gamma^2)} \tag{A12}$$

Add w^e, g^e, v^e, and a^e to Equations (1)–(6), respectively, to obtain market demand, consumer utility, manufacturer profit, retailer profit, supply chain profit, and social welfare. □

Strategy s.

Proof of Lemma 2. The proof process is similar to Lemma 1. □

Strategy q.

Proof of Lemma 3. The proof process is similar to Lemma 1. □

Appendix A.2. Part B (Proof)

Proof of Corollary 1.

Proof. Equation (A8) takes the first-order derivative of f: $\frac{\partial w^e}{\partial f} = \frac{2fhmT(2bh-\gamma^2)}{b(-f^2h+4bhm-2m\gamma^2)^2} > 0$
Other parameters proved similar results. □

Proof of Corollary 2.

Proof. Equation (A8) takes the first-order derivative of γ: $\frac{\partial w^e}{\partial \gamma} = \frac{2f^2hmT\gamma}{b(f^2h+2m(-2bh+\gamma^2))^2} > 0$
Other parameters proved similar results. □

Proof of Corollary 3.

Proof. Take the first derivative of t on w^s: $\frac{\partial w^s}{\partial t} = \frac{f^2hmT(2bh-\gamma^2)}{b(f^2h+2m(-1+t)(2bh-\gamma^2))^2} > 0$
Other parameters proved similar results. □

Proof of Corollary 4.

Proof. Take the first derivative of t on w^q: $\frac{\partial w^q}{\partial t} = \frac{f^2hmT\gamma^2}{b(f^2h(-1+t)-2m(2bh(-1+t)+\gamma^2))^2} > 0$
Other parameters proved similar results. □

Proof of Proposition 1.

Proof. $w^s - w^e = -\frac{f^2hmtT(2bh-\gamma^2)}{b(-f^2h+4bhm-2m\gamma^2)(f^2h-4bhm+4bhmt+2m\gamma^2-2mt\gamma^2)} > 0$

According to the conditions $2bh - \gamma^2 > 0$, $-f^2h + 4bhm - 2m\gamma^2 > 0$, and $f^2h - 4bhm + 4bhmt + 2m\gamma^2 - 2mt\gamma^2 < 0$ are easily judged to be positive numbers. Other parameters proved similar in this study. □

Proof of Proposition 2.

Proof. $w^q - w^e = -\frac{f^2hmtT\gamma^2}{b(-f^2h+4bhm-2m\gamma^2)(f^2h-4bhm-f^2ht+4bhmt+2m\gamma^2)} > 0$

According to the conditions, $-f^2h + 4bhm - 2m\gamma^2 > 0$ and $f^2h - 4bhm - f^2ht + 4bhmt + 2m\gamma^2 < 0$ are easily judged to be positive numbers. Other parameters proved similar. □

Proof of Proposition 3.

Proof. $\frac{\partial\pi_1}{\partial t} = \frac{f^2h^2mT^2(f^2h+4m(-1+t)(2bh-\gamma^2))}{2(f^2h+2m(-1+t)(2bh-\gamma^2))^3} > 0$

According to the conditions, $2bh - \gamma^2 > 0$, $-f^2h + 4bhm - 2m\gamma^2 > 0$, and $f^2h + 2m(-1+t)(2bh - \gamma^2) < 0$ are easily judged to be positive numbers. Other parameters proved similar. □

Proof of Proposition 4.

Proof. According to the condition $\pi_r^s > \pi_r^e$, $\pi_m^s > \pi_m^e$, $\pi_r^q > \pi_r^e$, $\pi_m^q > \pi_m^e$, it is easy to know $\pi_{sc}^s \geq \pi_{sc}^e$, $\pi_{sc}^q \geq \pi_{sc}^e$. Moreover, we can get $\pi_{sc}^q \geq \pi_{sc}^s$. From the above, we can get $\pi_{sc}^q \geq \pi_{sc}^s \geq \pi_{sc}^e$. □

Proof of Proposition 5.

Proof. $\pi_m^e - \pi_r^e = -\frac{hmT^2(f^2h+m(-2bh+\gamma^2))}{2(f^2h+2m(-2bh+\gamma^2))^2}$

When no government subsidy is present, according to the condition $2bh - \gamma^2 > 0$, $-f^2h + 4bhm - 2m\gamma^2 > 0$, it is easy to know $4bhm > 2m\gamma^2, f^2h + 2m\gamma^2 < 4bhm$; when $\pi_m^e < \pi_r^e$, there is $f^2h + m(-2bh + \gamma^2) > 0$. Obtain $4bhm < 2(f^2h + m\gamma^2)$. From the above, we can get $f^2h + 2m\gamma^2 < 4bhm < 2(f^2h + m\gamma^2)$. Other situations proved similar. □

Proof of Proposition 6.

Proof. When the government subsidizes manufacturers, calculate the first and second derivatives of the subsidy ratio t $(0 < t < 1)$ for social welfare U_{sc}. Moreover, because $\frac{\partial^2 U_{sc}}{\partial t^2} < 0$. Of note, social welfare U_{sc} has a unique optimal solution t. Assuming $\frac{\partial U_{sc}}{\partial t} = 0$, we can find $t^s = \frac{3bh-\gamma^2}{7bh-3\gamma^2}$ and $\frac{\partial t^s}{\partial \gamma} = \frac{4bh\gamma}{(7bh-3\gamma^2)^2} > 0$. The optimal subsidy rate when the government subsidizes retailers proves to be similar. □

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