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# A Statistical Simulation Model for the Analysis of the Traffic Flow Reliability and the Probabilistic Assessment of the Circulation Quality on a Freeway Segment

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**Abstract:** Measuring the traffic quality and congestion level is fundamental in highway engineering, and several decades of studies and research have pursued this specific objective, especially for freeways. Nowadays, smart technologies on personal devices and information shared by users have made available various online information platforms that provide dynamic representations of the use of the road network. If, on the one hand, these tools provide a simple and direct representation of the quality of circulation, on the other hand, their aggregated information is only partial for those dealing with traffic and highway engineering. This branch of engineering relies on multidimensional knowledge of traffic flow phenomena, and only through their in-depth knowledge, we can assess traffic quality and congestion risk. After identifying the different approaches for analyzing in quantitative terms the traffic quality on the freeway, the paper deepens the reliability approach. From this point of view, the paper aims to unite the two perspectives in the literature, namely, the probabilistic analysis of traffic instability with the characterization of speed random processes and the analysis of breakdowns with the survival analysis. For this purpose, the work outlines a procedure based on the estimation and simulation of ARIMA models for speed random processes in a freeway section, particularly on the leftmost lane, to assess the traffic reliability function. Applying the Product Limit Method to the Monte Carlo simulation results makes it possible to obtain probabilistic assessments of congestion, considering the Level of Service density limits defined in the Highway Capacity Manual. Its application to a case study makes it possible to illustrate the application of the method, which can be easily applied to historical and near-real-time data using a continuous flow of information.

**Keywords:** traffic flow reliability; freeway traffic reliability; traffic congestion; Level of Service; breakdown analysis; speed random process

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## 1. Introduction

The availability of an accurate and effective method for measuring the traffic quality and congestion level on a segment or part of the network represents a fundamental aspect of the planning, designing, and controlling of a transport system. In the field of road transport, especially for highways, several decades of studies and research have pursued this specific objective. Various discussions have been proposed about the meaning of the quality of circulation and congestion to define models and operating procedures that can represent and observe it.

Over the years, the growing availability of information about mobility and traffic has led to further considerations related to using these data to monitor traffic quality and control/manage infrastructure systems and mobility demand. These systems are known as Intelligent Transportation Systems (ITSs) [1]. As in [2], the ITS research topic in the highway sector, although central in the last twenty years, has only occasionally been transformed into medium- and long-term projects. However, today, the proliferation of intelligent technology, which we often refer to as smart technology, is the primary driver of innovation in the highway sector [3]. The smart road represents the frontier of innovation in the road and highway sector, where the most innovative automation and communication technologies, big data, and artificial intelligence integrate each other to offer a transport service suitable for the highly dynamic needs of modern mobility [3].

From this point of view, it is evident that the availability of data on the movements of road users has experienced an incredible boost in recent years; today, data perform the lion's share of the work of smart technology, especially in the highway field. The ubiquitous deployment of technology for tracking movement using personal and compact GPS or cell phone devices has made available, for instantaneous and massive use, various online information systems and platforms capable of providing representations of the state of use of the road network. The analysis of this information, commonly referred to as big data and complemented by social applications with information shared by users, has made it possible to offer the public some immediate tools to inquire about the traffic situation in a specific territorial context. Online platforms—among which we can mention the widely used services provided by Google Traffic, TomTom Move, Bing Traffic, Inrix, and Waze—provide some information on travel times and travel speeds on roads or sections of roads on simple and easily navigable maps, both in real time (live traffic) and as average values over selectable daily and hourly intervals (typical seasonal daily situations). On the one hand, we can say that these tools provide a simple and direct representation of the quality of circulation. However, on the other hand, the information transferred in terms of average speeds or travel times, or even more aggregated qualitative indices, is only partial for those dealing with the planning, designing, and controlling of transport systems and networks.

Traffic engineering relies on multidimensional knowledge of traffic flow phenomena, and average information about speed or travel time on a sample of users, if not accompanied by additional information, is inadequate to define the infrastructure operations. Only through the in-depth knowledge of traffic phenomena and the macroscopic and microscopic study of the involved variables, we can represent traffic conditions and circulation quality and evaluate the distance from the congestion.

Starting with Greenshields's seminal works in the first half of the last century [4], several perspectives have been proposed, searching for a method to study traffic behaviors and measure circulation quality. From this point of view, first, we can differentiate two approaches [5]:

- A first one includes those methods that define the quality of circulation through a functional relationship with some qualitative and quantitative parameters of driving operations in the traffic flow (index approach);
- A second one defines the quality of circulation through the relationships between the macroscopic variables of flow, speed, and density, using steady-state models to estimate the Fundamental Diagram [1] and, starting from the latter, identifying typical situations of the state of circulation, or Level of Service (LOS), from free to congested (LOS approach).

Among the first traffic flow scholars of the last century, Greenshields, Platt, and Drew proposed studies and methods related to the first approach, adopting time-to-time specific quality indices [6–10]. As highlighted in [5], this approach has not had significant applications, probably due to what appears to be an abstract definition of the quality of circulation, to a certain arbitrariness in the selection of variables, and often to the difficulty of measuring them in actual situations on the road.

The second approach includes the methods and procedures in the different editions, up to the latest, current one, of the Highway Capacity Manual (HCM) [11], which is widespread internationally and is adopted in various national contexts. The generalized speed–flow curves for standard cases periodically updated in the various HCM editions and their easy applicability for technicians and practitioners have made the fortune of this approach, which is now extremely widespread all over the world for the analysis of the quality of circulation.

In addition to these two approaches, we can add a third one that links the traffic circulation quality to some reliability measures, i.e., the reliability approach. As in [12], we can consider a system reliable if it can fully perform the tasks for which it was designed. From this point of view, system reliability is the probability that it adequately performs its tasks within a given time interval and under specified environmental conditions. Going beyond descriptive studies and applications based on the analysis of the travel time reliability (e.g., [13–19]), the approach that we define here in terms of reliability collects the theoretical contributions of the analysis of the speed random processes [1,12,20–25] and the probabilistic analysis of breakdown phenomena and capacity [26–30]. In general, we can observe that the contributions of the probabilistic analysis of breakdowns have found various applications in recent years, up to also being included in the HCM (in a simplified way) for the estimation of the probabilistic distribution of the lane capacity [11]. Despite its numerous applications in the estimation of statistical models for short-term forecasts, of which we find an interesting review in [31], the analysis of the speed processes has been scarcely used in the assessment of the circulation quality, perhaps due to the greater theoretical complexity and burden of calculation in practical applications.

*Aim of Research*

In the context briefly outlined here, the work presented in this paper addresses the two lines of the investigation cited above for the analysis of reliability—i.e., stochastic speed processes and the probabilistic analysis of breakdown phenomena and capacity—to propose a single procedural framework able to integrate both and to connect them with the critical elements of LOS analyses according to the HCM.

Thus, the paper wants to examine the probability of a traffic breakdown, the probabilistic distribution of lane capacity, and the probability of a lane operating at any LOS, based on the analysis of the stochastic processes of vehicle speeds in a freeway section. The objective is to propose a more general approach, going beyond the relationships that can we find in the literature, obtained under certain initial and boundary conditions.

The proposed procedure can be useful for evaluating the circulation quality and the Level of Service on a freeway segment from the reliability point of view by observing how the speed processes develop in the situation that is under observation from time to time, for historical data and applications in near real-time.

The paper is organized as follows: Section 2 addresses the primary literature references and the theoretical and applicative assumptions for measuring the quality of freeway traffic. Section 3 proposes some fundamental aspects related to the analysis of speed processes, with some details in the Appendix A. Section 4 describes the model characterization and the quantitative procedure that the paper proposes. Finally, Section 5 proposes an explicative application to a real case on the Italian Brenner A22 freeway.

## 2. Approaches to the Analysis of the Quality of Circulation

### *2.1. Quality of Service*

As anticipated in the introduction, we can identify three approaches to measuring traffic circulation quality: indices, LOS, and reliability analyses.

An example of the first approach was proposed by Greenshields (Greenshields, 1961) with the QI (Quality Index), based on the assumption that drivers evaluate the driving experience on the infrastructure based on the speed they reach and the degree of uniformity of this speed. Platt (Platt, 1963) also suggests something similar with the LTSI (Level of Traffic Service Index), which considers some additional factors experienced by the driver, such as the rate of change in speed, the counter-steering rate (which describes the frequency in the modification of the direction of movement of the steering wheel from counterclockwise to clockwise or vice versa), the rate of reversal of accelerations (which describes the rate of the shift from acceleration to deceleration, or vice versa), and the braking rate. Some studies by Drew [9,10,32] highlighted how the quality indices proposed by Greenshields and Platt link to a description of average performance over the entire trip or, in any case, over a relatively long stretch of infrastructure. The same author—remarking how some variables involved in the indices calculations link to the infrastructure characteristics, which usually vary along the travel route—highlighted the need to link any quantitative parameter to short and homogeneous infrastructure sections. Drew (Drew, 1965) introduced a new quantity in the parameters that describe the quality of the circulation, namely, the acceleration noise, considering the three fundamental aspects for assessing traffic circulation quality: the driver, the road, and traffic conditions.

In general, the LOS approach defines the quality of circulation based on the relationships between the macroscopic traffic variables of flow, speed, and density, using steady-state models to estimate the Fundamental Diagram [1] and, starting from the latter, identifying typical situations of the state of circulation, namely, LOS, from free to congested. This approach includes the analysis and evaluation methods in the HCM, with its distinctive feature of associating the traffic circulation quality to some areas in the speed–flow plane.

The attribution to a given situation of the six Levels of Service in HCM is based on a measure that represents the traffic flow condition, the so-called Measure of Effectiveness (MOE). In the case of interest concerning the basic highway sections, there have been various hypotheses regarding the MOE over time considering speed, flow-to-capacity ratio, and density. Since 1975 [33], the assumption of density as an MOE has been used in all subsequent editions of the manual until the last edition in 2022 [11].

### *2.2. Traffic Flow Reliability and Stochastic Capacity*

Regarding the last of the three approaches, i.e., the reliability one, we can say that a system is reliable if it can perform the tasks for which it was designed [12]. Reliability is, thus, the probability that engineering works can adequately perform their functions for a given time and under certain environmental conditions. Initially developed for aviation purposes, especially in the military field, the reliability analysis [34] has been increasingly employed and is used in various fields of industrial engineering, such as mechanical, chemical, and nuclear engineering, as well as in software communication. In the last decade, significant developments related to the concept of reliability have influenced civil engineering, especially the building sector [35], as well as transport and highway engineering [1].

In highway engineering, the probabilistic analysis of the reliability of road infrastructure can be assessed by considering the reliability of the flow parameters along the lanes of its section, using a stochastic approach that examines these quantities and their link with congestion and service quality in probabilistic terms. As already mentioned in the introduction, we can assess the reliability of a freeway section as the traffic flow reliability, detailing it in the following [12]:

- The probabilistic analysis of traffic flow breakdowns;
- The probabilistic analysis of the traffic flow instability.

As regards the probabilistic and forecast analysis of breakdowns, this is essentially based on models of analysis of traffic block phenomena (breakdown models) [26,36–38] and capacity estimation starting from the observation of flow and velocity values [27–29,39,40] according to the theoretical–operational approach of the survival analysis or lifetime analysis and of the Product Limit [41].

Probabilistic breakdown analysis models are based on representing the lane capacity with a random variable characterized by its probability distribution. Among the first to introduce and document the probabilistic behavior of congestion and capacity with empirical observations and interpretative models on freeway ramps and sections, we can find [36]. According to the probabilistic point of view, the probability of congestion is usually represented as a function of the traffic flow; low traffic volumes correspond to a low probability of congestion; at very high volumes, congestion likely occurs. From an initial free traffic flow, the onset of congestion characterizes what we usually identify as a traffic breakdown.

Several studies have calculated the breakdown probability function from empirical data, such as [26] or [36], by grouping the flows, for example, by multiples of 100 vehicles per hour, and finding the frequency of the breakdowns in each group. In terms of capacity, which we intend here as the value of the flow that occurs before a breakdown phenomenon, the empirical distribution function of the capacity $F_c(q) = prob(q_c \leq q)$ can be identified based on the probability of having a capacity value $c = q_c$ less than or equal to $q$. The first method for estimating the capacity distribution was proposed by [42] based on the analogy with the statistical analysis of the life duration and on the observation of traffic flows downstream of a traffic bottleneck. A more in-depth discussion was reported by Brilon (e.g., [27,28]), who identified the criteria for the correct identification of the survey section and for the classification of traffic observations (flow and speed) and defined the methodologies for the non-parametric and parametric estimation of the $F_c(q)$ distribution.

### 2.3. Traffic Flow Instability

Regarding traffic flow instability, the starting point is the observations of the stochastic process of vehicle speeds [1,12,20–25,43]. When the traffic flow on a freeway carriageway exceeds a certain threshold, there can be sudden drops in speed, causing hiccups (stop and go) and often leading to a complete traffic blockage. This typical phenomenon is what we can indicate as instability of the traffic flow, caused above all by the instability of the distancing implemented among the vehicles on a freeway segment. In general, it can be observed that a vehicle driving on a stretch of highway varies its driving speed due to random phenomena related to the driving behavior of individual drivers or due to the need to adapt its speed to the speed changes of the vehicle in front [1,44]. On a multilane carriageway, lane change maneuvers are among the most frequent causes of speed fluctuations. A vehicle moving from the right to the left lane reduces the gap between two consecutive vehicles in transit on the latter, causing a reduction in speed in the following vehicle. At the same time, in the right lane, the increase in the gap between vehicles induces a speed increase in the follower. Speed change propagation in the flow depends on the drivers' reaction delay to changes in the speeds of the vehicle in front of them. This phenomenon has random components, which concern the driver's reaction to the accelerations and decelerations of the previous vehicle and which vary randomly from driver to driver and according to different situations. Traffic flow instability, therefore, arises when a reduction in the speed of one vehicle causes an even more significant reduction in the speeds of the following vehicles [20,21].

Thus, speed reductions in a freeway section without on- and off-ramps produce a density increase, with increased maneuvering difficulties and interactions among vehicles and instability. Therefore, studying speed variations allows us to address the problem of traffic instability on a highway lane and traffic flow reliability. Experimental observations show that, under many circumstances, traffic instability on a multilane carriageway first occurs in the leftmost lane [20]. Thus, the study of traffic flow instability on the highway carriageway coincides with the seemingly critical lane among all in the same driving direction [45]; ultimately, the investigation of the probability of traffic flow instabilities on a freeway carriageway can be approached by deepening the analysis of the speed variations in the leftmost lane [12,20].

## 3. Speed Random Process

### 3.1. Speed Random Process Definition

To analyze the reliability of freeway traffic, it is necessary to investigate the behaviors that characterize a vehicular flow made by vehicles that follow each other. Considering the sequence of vehicles passing a section $S$ of the leftmost lane of a freeway carriageway, we consider $t = 1, 2, \ldots, n$ the instants in which each component of a vehicle succession passes through section $S$ and $v_t$ the speed of the vehicle passing at instant $t$. Therefore, the sequence $\ldots v_{t-1}, v_t, v_{t+1} \ldots$ is the realization of the random process that we indicate as the speed process.

If we denote by $\overline{v}_t$ the dynamic mean value, or level, at instant $t$ conditioned by the previous realization of the speed process up to $t-1$, it can be verified that $v_t = \overline{v}_t + a_t$ [12,20,25]. It means that the vehicle passing at time $t$ has a speed deviation $a_t$ with respect to dynamic speed level $\overline{v}_t$ due to the above realization, where $a_t$ is a random variable with mean zero and variance $\sigma^2$ [12,20,25]. If we consider the whole vehicle succession in $t = 1, 2, \ldots, n$, all $a_t$ are independent and identically distributed random variables with zero mean and variance $\sigma^2$. Some experiments have proved [20] that if at instant $t$, there is a deviation $a_t$ from level $\overline{v}_t$, this deviation influences the level at $t+1$, i.e., the conditional mean of the previous realization up to $t$, for a quantity $\lambda a_t$, with $\lambda$ being a coefficient between 0 and 1. Thus, we can write $\overline{v}_{t+1} = \overline{v}_t + \lambda a_t$, and setting $(1-\lambda) = \vartheta$, it results that $v_{t+1} = v_t - a_t\vartheta + a_{t+1}$ (see Appendix A), which corresponds to an ARIMA (0,1,1) model [12,20,25].

It can be shown (see Appendix A) that the study of traffic reliability involves the analysis of the speed process defined by $\overline{v}_t = \overline{v}_{t-k} + \lambda\sum_{j=1}^{k} a_{t-j}$. With $a_t \sim N(0,\sigma^2)$ and the distribution of $\lambda a_t$ completely defined by $\lambda\sigma$, the same quantity $\lambda\sigma$ represents a measure of the reliability of the speed process, i.e., of the greater or lesser probability that the flow is stable over time. If, on the one hand, $\lambda$ is small (at the limit equal to zero, with low flow rate and large spacings) or if $\sigma$ is small even in the presence of a non-low $\lambda$, the probability that the process deviates enough from a constant value over time is low, and the process can be considered to be stable, with vehicles that condition themselves little; speeds appear normally distributed with constant mean equal to $\overline{v}$ and variance $\sigma^2$ coinciding with that of residuals $a_t$ and whose sequence is the realization of a renewal process. If, on the other hand, $\lambda$ and $\sigma$ are not too small, the probability of a consistent deviation from the constant value over time is high, and the speed process is constantly unstable. In any case, parameters $\lambda$ and $\sigma^2$ are estimable considering the process defined by $w_t = v_{t+1} - v_t = a_t(\lambda - 1) + a_{t+1}$, which is a stationary process with $a_t$ and $a_{t+1}$ identically distributed, that is, a first-order moving average (MA) (1) process [12,20,25].

*3.2. Speed Process and Flow Rate Analysis*

Going back to section $S$ on the leftmost lane of a freeway carriageway, we can consider a succession $\Delta t^{(1)}, \Delta t^{(2)}, \ldots, \Delta t^{(i-1)}, \Delta t^{(i)}, \Delta t^{(i+1)}, \ldots$ of time intervals of duration $\Delta t$ covering an interval of total duration $T$ with an almost constant flow rate $q_{\Delta t^{(i)}}$. Let $n^{(1)}$, $n^{(2)}, \ldots, n^{(i-1)}, n^{(i)}, n^{(i+1)}, \ldots$ be the sequence of vehicles transited in each interval, each one as the realization of a random process. Considering succession $n^{(i)}$ of the vehicles passing during $\Delta t^{(i)}$ as the realization of a first-order stationary process, if $q_{\Delta t^{(i)}}$ is the flow rate in generic interval $\Delta t^{(i)}$, i.e., $q_{\Delta t^{(i)}} = n^{(i)}/\Delta t^{(i)}$, and $\overline{v}_t^{(i)}$ is the dynamic mean speed at the instant of the passage of the $t$-th vehicle ($1 \leq t \leq n^{(i)}$), density $k_t^{(i)}$ can be estimated as the ratio between $q_{\Delta t^{(i)}}$ and $\overline{v}_t^{(i)}$, i.e., $k_t^{(i)} = q_{\Delta t^{(i)}}/\overline{v}_t^{(i)}$. If flow rate $q_{\Delta t^{(i)}}$ is constant during $\Delta t^{(i)}$, random variations in speed level $\overline{v}_t^{(i)}$ generate random variations in density $k_t^{(i)}$. If these random variations produce a $k_t^{(i)}$ exceeding a limit value

, we have traffic instability at the instant of the passage of the $t$-th vehicle. Thus, the probability of having a certain instant $t$ during $\Delta t^{(i)}$ with $k_t^{(i)} > k^*$ can identify probability $P^*(x|k^*)$ that a traffic block phenomenon $x$ may occur in $S$ during $\Delta t^{(i)}$, assigned a density threshold $k^*$.

Instability, as a random event, depends on the random process of the speed level. Thus, the probability that the instability event does not occur in a certain instant defines reliability $\varnothing$ of the flow. In these terms, the reliability of a constant traffic flow $q_{\Delta t}$ for a time interval $\Delta t$ can be defined as the conditional probability that dynamic speed level $\overline{v}_t$ does not decrease during $\Delta t$ in such a way that density $k_t = q_{\Delta t}/\overline{v}_t$ can reach limit density $k^*$ (provided that in $t_0 = 1$, the level of speed $\overline{v}_0$ corresponds to a stable flow). If this occurs, the control mechanism implemented by the drivers goes into crisis, and the flow becomes unstable.

Thus, if we consider a sequence of speed values measured over a reasonably long period of time $T$, during which flow $Q$ varies in a sufficiently wide range for the same environmental conditions, traffic composition, and driver population, the whole sequence of the detected speeds can be subdivided into sequences that are the realization of homogeneous processes. The generic $i$-th process is characterized by specific values of $\lambda^{(i)}$ and $\sigma^{2(i)}$, lasting $\Delta t^{(i)}$, with a certain flow $q_{\Delta t^{(i)}}$, and by a density $k_t{}^{(i)} = q_{\Delta t^{(i)}} / \overline{v}_t{}^{(i)}$ at the instant of the passage of the $t$-th vehicle of the process realization. The values of $\lambda^{(i)}$ and $\sigma^{2(i)}$ can be estimated considering that for each identified sequence, $w_t{}^{(i)}$ is the realization of a first-order moving average process (MA) (1).

### 3.3. Speed Process and Traffic Stream Reliability

Parameter $\lambda^{(i)}$, which we can estimate for each sequence $i$, appears to be an increasing linear function of the logarithm of average density $k$. Some previous studies [12,20–22] demonstrated that regardless of the different circumstances in which the observations are made, $\lambda$ can be well represented by the following linear equation: $\lambda = -0.126 + 0.274\ln(k)$. The same studies also confirmed that the trend of $\sigma^{2(i)}$ is also linear with respect to $\ln(k)$ and passes through point [$k$=30 ($\ln(k)$ = 3.4; $\sigma^2$ = 1.5 m$^2$s$^{-2}$]. Linear equation $\sigma^2 = f(\ln(k))$, and in particular angular coefficient $M$, summarizes the current characteristics of the traffic flow with respect to its vehicular composition, the population of drivers, and the environment. Its intersection with the axis of density $k$ identifies average limit density $k_{lim}$, at which the traffic flow is only possible if the speed fluctuations do not exist and beyond which there is a high probability of a crisis of the traffic flow. In these terms, the limit value represents a measure of critical density $k^*$.

In consideration of the above, we can observe that reliability $\varnothing$ depends on the following:

- A time interval $\Delta t$, in which we evaluate the probability that an instability event does not occur;
- A flow rate $q_{\Delta t}$, which we assume is constant throughout $\Delta t$;
- An angular coefficient $M$ of straight line $\sigma^2 = f(\ln(k))$, which summarizes the characteristics of the traffic flow.

Through simulations of the random process of the speed levels between zero speed and maximum speed, it is possible to obtain [12,20–23] the expressions of $\varnothing$ for a traffic flow in the leftmost lane of a freeway carriageway depending on $\Delta t$ (in minutes), $q_{\Delta t}$ (in vehicles/hour/lane), and $M$ (in m$^2$ km s$^{-2}$). Considering the level of speed $\overline{v}_0$ at $t_0$ (corresponding to a stable flow) expressed by equation $\overline{v}_0 = 125 - 0.020q$, according to [20], we can consider the regression of $\varnothing$ with respect to $T$, $q$, and $M$, given by

$$\varnothing = 1 - 19.80\left(\frac{q_{\Delta t}}{10000}\right)^{8.82} \Delta t^{\,1.933}\, M^2 \quad (1)$$

Furthermore, starting from this expression, it is possible to obtain the value of flow rate $q$, which transits on the leftmost lane of a freeway carriageway with a reliability value $\varnothing$:

$$\widetilde{q} = 10000\left(\frac{1-\varnothing}{19.80\,\Delta t^{1.933}\, M^2}\right)^{\frac{1}{8.82}} \quad (2)$$

If we set a reliability value $\widetilde{\varnothing}$ close to one (generally, $\widetilde{\varnothing}$ = 0.8–0.9) and a conventional period (e.g., $\Delta t$ = 15 min), the $\widetilde{q}$ value obtained with Equation (2) is capacity $C_{pass}$ of the fastest lane (i.e., the leftmost lane). The instability, which is reached with a probability $1-\widetilde{\varnothing}$ when the $q_{pass}$ flow rate on the leftmost lane reaches its capacity $C_{pass}$ ($q_{pass}/C_{pass}$ = 1), determines the flow instability over the entire carriageway. In this regard, in a two-lane carriageway, we can determine carriageway capacity $C_r$ by considering the experimental relationships between flow rates $q_{right}$ and $q_{pass}$ on the two lanes, as the total flow rate, $q_r$, on the entire carriageway varies [12,46].

The knowledge of reliability $\varnothing$ in a section and of its variation over time can be used to prepare functional criteria for activating traffic control strategies [1]. These can prevent the onset of instability phenomena by acting on the values of $q$ and $M$. Based on Equation (1), in fact, if $q$ and $M$ are high, it is possible to increase $\varnothing$ by reducing $M$ (hence, variance $\sigma^2$) even without acting on traffic flow rate $q$. If, on the other hand, the value of $\varnothing$ below the reliability threshold occurs with a value of $M$ that is already low, then the increase in $\varnothing$ cannot be achieved, except with an adjustment of flow rate $q$ [20].

As we have seen, the analysis of the speed process allows us to obtain a formulation of reliability $\varnothing$ expressed by Equation (1) and to derive the probabilistic distribution of capacity based on Equation (2), according to the method introduced by [12,20]. It should be emphasized, however, that Equations (1) and (2) are regressions obtained by the simulating of random processes under specific boundary conditions. Therefore, they are not directly generalizable but require a simulation and regression process for each application to contextualize them time to time. On the other hand, the current simulation techniques and the improvement of the computational performances make it possible to study the stability through the analysis of the speed processes in near-real-time mode, with evaluations that we can carry out under the current flow conditions. This type of analysis is the novelty proposed in this paper. Details about the proposed model are presented in Section 4, and an exemplificative application is presented in Section 5. Thus, the main goal of the proposed method is to go beyond the use of standardized functions in the literature. As we said, these functions result from regression using simulations made once and for all with well-defined assumptions and boundary conditions [12,20–23], leading to Equations (1) and (2).

## 4. Simulation and Analysis Procedure

### *4.1. Speed Process Simulation Model for Reliability Analysis*

As we mention in the previous sections, the study of the reliability of traffic in the leftmost lane of a freeway carriageway, with hourly flow rate $q_{\Delta t}$ passing through a section $S$ in a time interval $\Delta t$ with a dynamic average speed level $\overline{v}_t$, involves the analysis of the speed random process defined by $\overline{v}_t = \overline{v}_{t-k} + \lambda \sum_{j=1}^{k} a_{t-j}$, with $a_t \sim N(0, \sigma^2)$ and the distribution of $\lambda a_t$ fully defined by $\lambda\sigma$, according to an ARIMA (0,1,1) model.

To try to estimate the probability that during a certain time interval $\Delta t$, hourly flow rate $q_{\Delta t}$ can determine a congestion phenomenon, i.e., when average density $k_t = q_{\Delta t}/\overline{v}_t$ exceeds a threshold value $k^*$ with $\overline{v}_t$ below a limit value $v^*$, it is possible to simulate the realization of a large number $m$ of speed sequences on the basis of parameters $\lambda$ and $\sigma^2$. For crisis probability estimation, we can verify how many simulated sequences produce $k_t > k^*$ or $\overline{v}_t < v^*$. Parameters $\lambda$ and $\sigma^2$, which particularize the ARIMA model (0,1,1), can be estimated on the basis of the recorded speed sequence, considering the random process defined by $v_{t+1} - v_t = -a_t(1-\lambda) + a_{t+1}$, i.e., $w_t = a_t(\lambda - 1) + a_{t+1}$, which is a stationary process with $a_t$ and $a_{t+1}$ being identically distributed according to a first-order moving average (MA) (1) process.

In a real-life situation, with a succession of vehicle transits $n = q_{\Delta t}\,\Delta t$, we can assume that the related speed sequence is the realization of the MA process with parameters $\lambda$ and $\sigma^2$. Considering the recorded $n$ speed values $v_t$, the estimates of $\lambda$ and $\sigma^2$ can be obtained using the models of econometric statistics, taking care to test the primary hypothesis of adequacy of the auto-regressive model.

Let us consider a speed sequence over a time period $T$, during which hourly flow rate $Q$ varies within a sufficiently wide range of values for the same environmental conditions, traffic composition, and driver population. The sequence of the detected speeds can be divided into some sub-sequences, which are the realization of homogeneous speed processes. The generic $i$-th random process consisting in $n^{(i)}$ speed values can be characterized by specific values of $\lambda^{(i)}$ and $\sigma^{2(i)}$, by a certain hourly flow rate $q_{\Delta t^{(i)}}$, and by a density that we can approximate with ${k_t}^{(i)} = q_{\Delta t^{(i)}}/{\overline{v}_t}^{(i)}$, regarding instants $t$ of sub-period $\Delta t^{(i)}$ that sees the $t$-th vehicle.

Following [12,20], in practical applications, it is possible to identify sub-sequences with a fixed number of transits $n$ (e.g., 50 vehicles each) and with a variable duration $\Delta t^{(i)}$. For each sub-sequence of $n$ vehicles, it is possible to determine the corresponding hourly flow rate ($q_{\Delta t^{(i)}}$) and the average speed level ($\overline{v}_t{}^{(i)}$) and to approximate the average density ($k_t{}^{(i)} = q_{\Delta t^{(i)}}/\overline{v}_t{}^{(i)}$). As indicated above, for each sub-sequence, the values of $\sigma^{2(i)}$ and $\lambda^{(i)}$ can be estimated, considering the first-order moving average process (MA) (1) of the deviations of the vehicle velocities $w_t{}^{(i)} = v_{t+1}{}^{(i)} - v_t{}^{(i)} = a_t{}^{(i)}\left(\lambda^{(i)} - 1\right) + a_{t+1}{}^{(i)}$. After verifying the adequacy of the MA (1) model according to specific hypothesis tests, for each interval $\Delta t^{(i)}$ that subdivides $T$, the estimated models can be used to simulate a large number $m$ of speed sub-sequences that are homogeneous with respect to the real one. Based on the estimates obtained for the parameters of each process, we can simulate the speeds of the vehicles in transit in a particular test interval $\tau$, e.g., the 5 min interval following $\Delta t^{(i)}$. Thus, if $q_{\Delta t^{(i)}}$ is constant even in test interval $\tau$ and speeds continue to be generated according to the same random process, a large number $m$ of sequences of deviations $\widetilde{w}^{(i)}$ in $\tau$ can be obtained through a Monte Carlo simulation. From the $j$-th sequence of deviations $\widetilde{w}^{(i,j)}$, we can generate the $j$-th speed sequence in $\tau$, assuming, for example, a starting value equal to the overall average ($\overline{v}_{\Delta t}{}^{(i)}$) of speeds in $\Delta t^{(i)}$. In this way, assuming that $v_1{}^{(i,j)} = \overline{v}_{\Delta t}{}^{(i,j)}$ is the speed of the first vehicle of the $j$-th simulated sequences, the speed values of the other vehicles in the sequence can be generated accordingly as $v_2{}^{(i,j)} = v_1{}^{(i,j)} + \widetilde{w_1}{}^{(i,j)}$ for the second vehicle, $v_3{}^{(i,j)} = v_2{}^{(i,j)} + \widetilde{w_2}{}^{(i,j)}$ for the third vehicle, etc., until the end of period $\tau$.

Assuming that density is the MOE to identify the onset of a crisis phenomenon, following the view proposed by the latest editions of the HCM and supposing $q_{\Delta t^{(i)}}$ to be constant in test interval $\tau$ after $\Delta t^{(i)}$, we can calculate for each 50 veh sub-sequence $i$ and for each Monte Carlo iteration $j$ mean speed $\widetilde{v}_{\tau|\Delta t}{}^{(i,j)}$ at the end of $\tau$ and density $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} = q_{\Delta t^{(i)}}/\widetilde{v}_{\tau|\Delta t}{}^{(i,j)}$. Having identified a density threshold value $k^*$ for the traffic crisis, we can calculate the number ($m^{*(i)}$) of sequences of simulated speed for which $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} < k^*$. Thus, ratio $m^{*(i)}/m$ can be used to evaluate reliability $\varnothing(k^*)^{(i)} = 1 - P^*(x|k^*)^{(i)}$ for flow rate $q_{\Delta t^{(i)}}$ recorded in $\Delta t^{(i)}$ during the next $\tau$-long interval.

### *4.2. Application of the Product Limit Method for the Probabilistic Analysis of Traffic Performance*

The results obtained through the simulations of the speed processes according to the methodology proposed in Section 4.1 can be used to produce further assessments regarding the distribution of the flow values with respect to various density thresholds on the leftmost lane of the carriageway. A first analysis can be obtained by considering $k^*$ as the limit density for reaching the capacity. For this purpose, we can use the Product Limit Method (PLM) [41], considering the sequence of flow rates ($q_{\Delta t^{(i)}}$) during $T$ and, for each of them in correspondence with a certain $\Delta t^{(i)}$ and a fixed following time interval $\tau$, the $m$ values of $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)}$. Using average speed level $\overline{v}_{\Delta t}{}^{(i)}$, we evaluate density $k_{\Delta t}{}^{(i)} = q_{\Delta t^{(i)}}/\overline{v}_{\Delta t}{}^{(i)}$ for each term $q_{\Delta t^{(i)}}$ of the flow rate sequence. If $k_{\Delta t}{}^{(i)} \geq k^*$, then $q_{\Delta t^{(i)}}$ is beyond the capacity; therefore, it is excluded from the subsequent analysis, since it does not contain information on the value of the same capacity. If $k_{\Delta t}{}^{(i)} < k^*$, then the $m$ values of $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)}$ take on importance in evaluations. Thus, the entire set of $q_{\Delta t^{(i)}}$ values that we find during $T$ with $k_{\Delta t}{}^{(i)} < k^*$ is divided into two sub-sets:

- {A} is the set of $q_{\Delta t^{(i)}}\ _{|k_{\Delta t}{}^{(i)} < k^*}$ values for which $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} < k^*$, indicating the density threshold not exceeded both in $\Delta t^{(i)}$ and in test time $\tau$ in Monte Carlo iteration $j$;
- {B} is the set of $q_{\Delta t^{(i)}}\ _{|k_{\Delta t}{}^{(i)} < k^*}$ values for which $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} \geq k^*$, indicating the density threshold not exceeded in $\Delta t^{(i)}$ and exceeded in test time $\tau$ in Monte Carlo iteration $j$.

In this way, in the case of $k_{\Delta t}{}^{(i)} < k^*$, based on $m$ Monte Carlo simulations, the value of each $q_{\Delta t^{(i)}}\ _{|k_{\Delta t}{}^{(i)} < k^*}$ appears $m^*$ times in dataset {A} and $m - m^*$ in dataset {B}.

The Product Limit Method that we propose in these analyses presents similarities with that indicated by Brilon (e.g., [27,28]), based on van Toorenburg's approach [42]. The difference is that in the application we discuss here, the breakdown is not evaluated in the different $\Delta t^{(i)}$ , but based on the probabilistic results in the $m$ simulations for test interval $\tau$

that would hypothetically follow each $\Delta t^{(i)}$ , under the conditions of flow constancy and speed process homogeneity.

The method is based on the estimate of the survival function of the hourly flow rate in consideration of the crisis limit imposed by threshold $k^*$. The capacity distribution (i.e., for $k$ exceeding $k^*$) can be written as $F_c(q) = \text{prob}(c \leq q)$ and can be estimated with

$$\hat{F}_c(q) = 1 - \prod\nolimits_{i:q_i \leq q} \frac{l_i - d_i}{l_i} \text{ with } i \in \{\text{B}\} \tag{3}$$

where $q$ is a certain value for the hourly flow rate between a minimum and a maximum considered for evaluation; $q_{\Delta t^{(i)}}$ is the hourly flow rate of generic interval $\Delta t^{(i)}$; $l_i$ is the number of times when $q \geq q_{\Delta t^{(i)}}$; $d_i$ is the number of times when $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} \geq k^*$ for $q_{\Delta t^{(i)}}$; and finally, {B} is the set of $q_{\Delta t^{(i)}}\ _{|k_{\Delta t}{}^{(i)} < k^*}$ values for which $\widetilde{k}_{\tau|\Delta t}{}^{(i,j)} \geq k^*$.

As known, the PLM does not require the assumption of a specific type of distribution function. However, it should be noted that the maximum value of the estimated distribution function reaches the unit value only if the maximum flow observed in the section belongs to set {B}. On the other hand, if the maximum observed flow does not belong to {B}, then the estimated distribution function $\hat{F}_c(q)$ stops at a value of less than 1;therefore, the complete trend cannot be estimated. To overcome this problem, it is necessary to hypothesize the mathematical form assumed by distribution function $F_c(q)$. As in [27], we can consider Weibull functions as follows:

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \qquad \text{with } x \geq 0 \tag{4}$$

where $\alpha$ and $\beta$ are distribution parameters that can be estimated with the maximum likelihood approach [27].

For $k^*$, the threshold values can be assumed based on the analysis of the dispersion of the experimental points in the two-dimensional diagrams of the macroscopic variables of the traffic or through the calibration of the Fundamental Diagram according to a preselected mathematical formulation that allows the value of the critical density at capacity to be identified [45]. However, a conventional value can also be assumed, for example, using the density threshold that identifies the limit between LOS E and LOS F according to the latest editions of the HCM [11], i.e., considering ${k^*}_{E\to F} = 28$ vehicles/kilometer/lane.

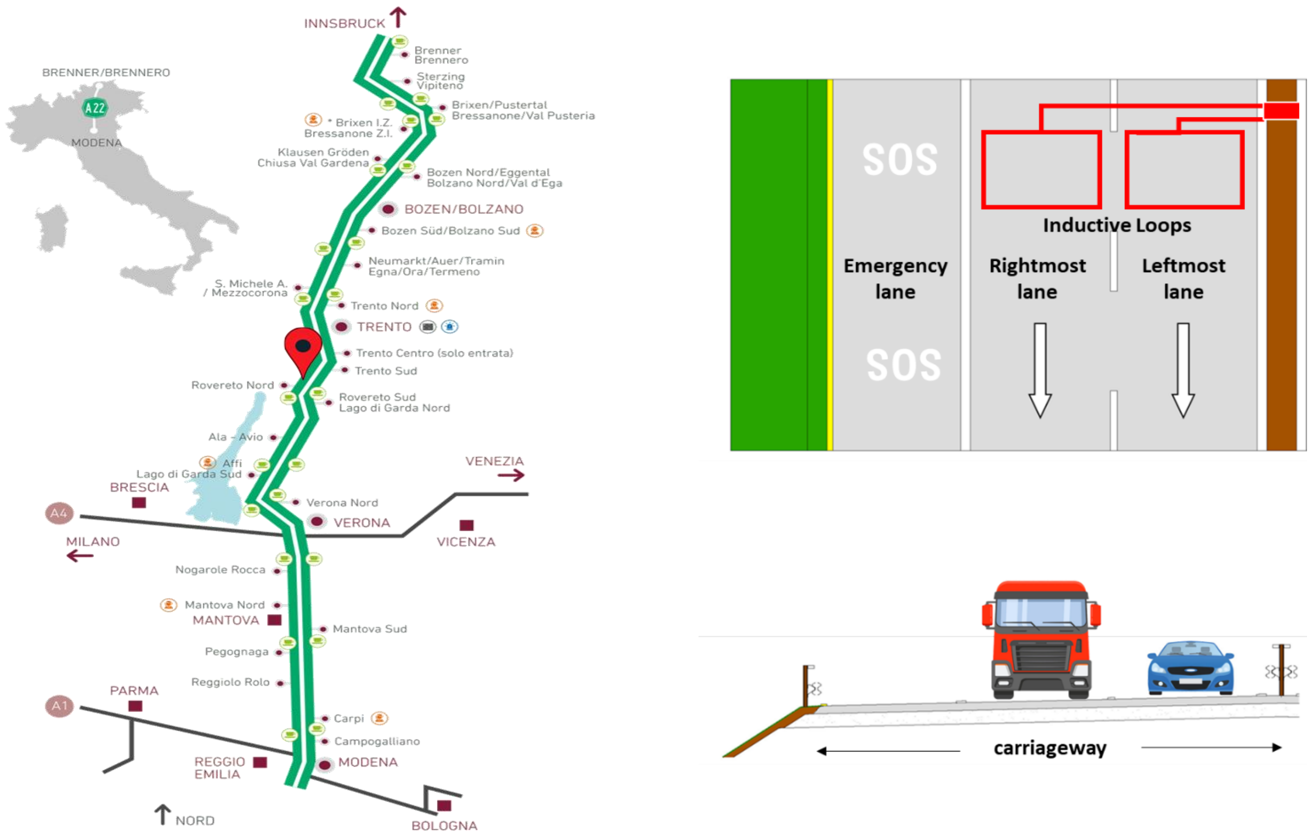
The PLM analysis can be extended towards the probability of exceeding the limit density for Levels of Service that come before LOS F in the HCM density ranges. To this end, we can proceed by reiterating the PLM after choosing a new value for $k^*$. To explore the probabilities of exceeding the flow rates for LOS A, B, C, and D (thus of having LOS B, LOS C, D, and E), we can use the limit density values provided by the HCM, i.e., ${k^*}_{A\to B} = 7$, ${k^*}_{B\to C} = 11$, ${k^*}_{C\to D} = 16$, and ${k^*}_{D\to E} = 22$ vehicles/kilometer/lane. In this way, probabilistic charts for the LOS analysis based on the hourly flow can be created, as we see in the case study in Section 5 of this paper.

It should be noted that no considerations are made regarding the type of vehicles in transit. We only consider vehicles, which we can assume here belong to a single homogeneous class, i.e., passenger cars. If the flow consists of different vehicle classes, for example, passenger cars and freight vehicles, this classification should be taken into consideration. Even the reference densities of the HCM consider equivalent vehicle units, based on adequate homogenization coefficients [11]. These aspects, excluded in this study stage, will be the subject of future investigations and may produce an even more robust generalization of the procedure.

## 5. Application of the Simulation Model to a Real Case

In this section, for illustrative purposes, we propose applying the simulation model described in Section 4 to a case study on the Italian freeway network, a two-lane section of the A22 Brenner freeway between the Trento Sud and Rovereto Nord toll stations. The detection devices were placed at Km 156 on the southbound carriageway on a straight and flat segment, at a sufficient distance from the exit ramp at Rovereto Nord (about 2.5 km).

Figure 1 shows the location along the route of A22 and the positioning of the counting devices. Data received from the freeway concessionaire were analyzed relating to the transits between 1 June 2014 and 30 June 2014, previously validated by the same concessionaire with the relative quality controls and consistency of measurements.



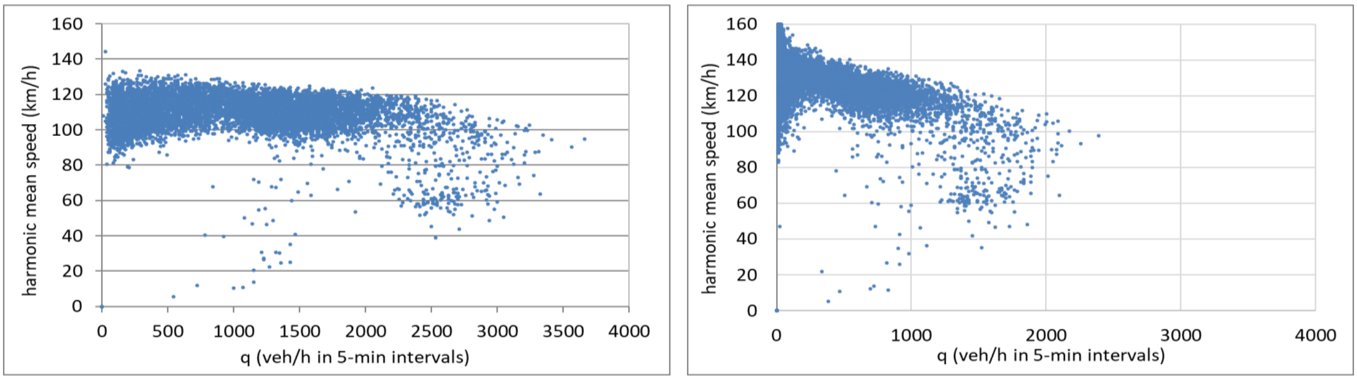

(**a**) (**b**)

**Figure 1.** A22 del Brennero freeway: (**a**) location of the monitoring section—Km 156 Trento Sud/Rovereto Nord; (**b**) inductive loop position on the southbound carriageway.

The data collected using the cross-sectional monitoring systems (inductive loop detectors) allowed us to qualify each transit concerning the following information: transit instant (Unix date format in seconds from 1 January 1970); identification code of the detection apparatus; lane identification; space headway; time headway; compliant/wrong direction flag; speed in km/h; vehicle length in cm.

The database included 732,700 vehicle passages on the two lanes, 416,057 on the rightmost lane and 316,643 on the leftmost lane, with an average daily traffic of about 24,400 total daily vehicles. Individual transit data were aggregated by 5 min intervals carrying out the flow rate in veh/h/lane and the harmonic mean of the speed as an approximation of the space average speed. Figure 2 shows the scatter diagrams of the experimental points in the speed–flow plane in 5 min intervals, both as regards the carriageway as a whole (a) and with detail of the leftmost lane (b).

The analysis of the reliability using the method proposed in Section 4 can be based on observing the vehicle sequences on the leftmost lane of the freeway carriageway grouped into sub-sequences consisting of 50 vehicles each. In the case study, we proceeded by dividing the whole vehicle succession on the leftmost lane per monitoring interval (i.e., 316,643 between 1 June 2014 and 30 June 2014) into sequences of 50 vehicles and calculating for each of them the corresponding hourly flow rate ($q$), the harmonic mean speed ($v$), and the mean density ($k$). However, the application, made here on historical data, can also be produced in near-real-time mode, based on the sequences of 50 vehicles found In the current state as time progresses.




(**a**) (**b**)

**Figure 2.** Scatter diagram of the experimental points ($q$-$v$) in 5 min intervals: (**a**) two-lane carriageway; (**b**) leftmost lane.

These time series considered a term for each sub-sequence of 50 vehicles in instant $t$ in which the passage of the last vehicle took place, measured in seconds. The time origin was the instant of the first transit of the first sub-sequence (start monitoring) on the leftmost lane.

For each sub-sequence of 50 vehicles, we can assume that a first-order moving average process (MA) (1) is the generating process for speed deviations $w_{t+1} = v_{t+1} - v_t = a_t(\lambda - 1) + a_{t+1}$. Based on this assumption, discussed in the previous sections and as detailed in Appendix A, we can estimate the $\lambda$ and $\sigma^2$ parameters of the 6332 MA (1) models (i.e., one model for each sequence of 50 vehicles). It should be specified that before estimating the MA (1) models, we verified the time series stationarity with the ADF (augmented Dickey–Fuller) unit root test. Based on the ADF test, all 6332 series of $w_t$ confirmed their stationarity with 95% confidence.

The MA (1) models were estimated using the regARIMA function in Matlab 2020a with specification (0,0,1). For each of the 6332 series of $w_t$, the adequacy of the MA (1) model was evaluated using the portmanteau test by Ljung and Box, verifying the null hypothesis that the residuals did not show autocorrelation. In 92% of cases (5845 series out of 6332), there was insufficient evidence to reject the null hypothesis of no residual autocorrelation (20 lags). Thus, also in this case study, it was possible to confirm, as in the literature, the adequacy of the MA (1) model to represent the succession of speed deviations $w_t$.

Using each of the 6332 MA (1) models estimated considering the sequences of 50 vehicles passing the leftmost lane, we simulated 200 alternative realizations for each sequence. The simulations were obtained with the Monte Carlo method using the simulate function in Matlab 2020a, which allows one to simulate a sample path from an ARIMA model (MA (1) in this case). For each sub-sequence $i$ of 50 vehicles, which took place in an interval of variable duration $\Delta t^{(i)}$ , $j$=1, 2, . . . , 200, the vehicle speed successions were simulated for a test interval $\tau$ represented by the following 5 min. As mentioned, the hypothesis was to consider hourly flow rate $q_{\Delta t^{(i)}}$ , corresponding to 50 passages in each $\Delta t^{(i)}$ , to also be constant in test interval $\tau$ and their speeds to be generated according to the same random process of the original sub-sequence.

Thus, for each sub-sequence $i$, we simulated m = 200 sequences of deviations $\widetilde{w}^{(i,j)}$ in $\tau$, and starting from them, we generated the relative speed sequences, assuming a starting value equal to mean speed ${\overline{v}_{\Delta t}}^{(i)}$ in each $\Delta t^{(i)}$ . In this way, for each sequence $i$ and each simulation $j$, we found ${v_1}^{(i,j)} = {\overline{v}_{\Delta t}}^{(i,j)}$, ${v_2}^{(i,j)} = {v_1}^{(i,j)} + {\widetilde{w_1}}^{(i,j)}$, ${v_3}^{(i,j)} = {v_2}^{(i,j)} + {\widetilde{w_2}}^{(i,j)}$, up to the last simulated vehicle. Based on the simulated speeds, for each of the 6332 sub-sequences and each of the 200 alternative realizations, we calculated average speed ${\widetilde{v}_{\tau|\Delta t}}^{(i,j)}$ at the end of $\tau$ and density ${\widetilde{k}_{\tau|\Delta t}}^{(i,j)} = q_{\Delta t^{(i)}} / {\widetilde{v}_{\tau|\Delta t}}^{(i,j)}$. Figures 3 and 4 show two heatmap graphs of the average speed and density values obtained in the simulation with 200 iterations ($x$-axis) for the 6332 sequences of 50 vehicles ($y$-axis).

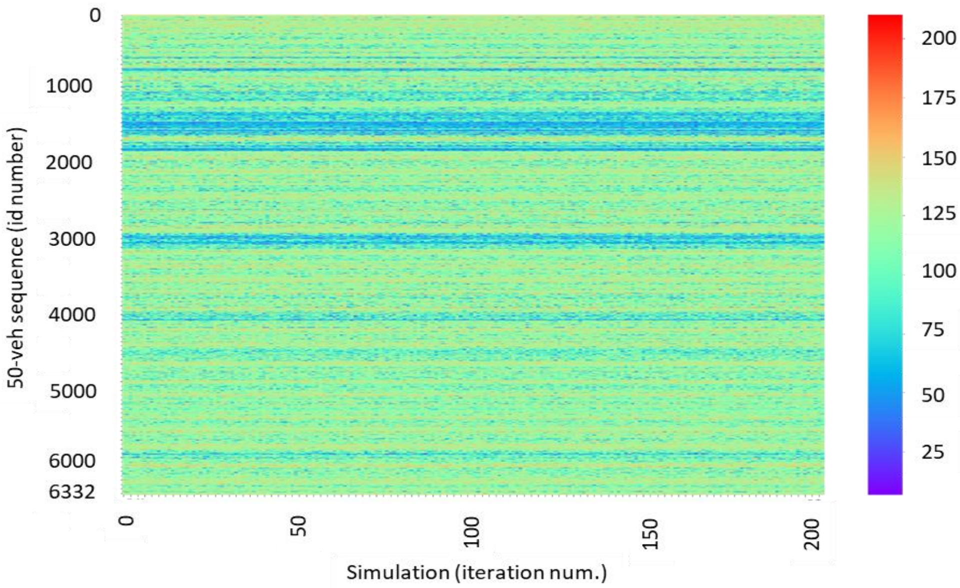



**Figure 3.** Heatmap graph of the average speeds obtained in the simulation with 200 iterations ($x$-axis) for the 6332 sequences of 50 vehicles ($y$-axis).

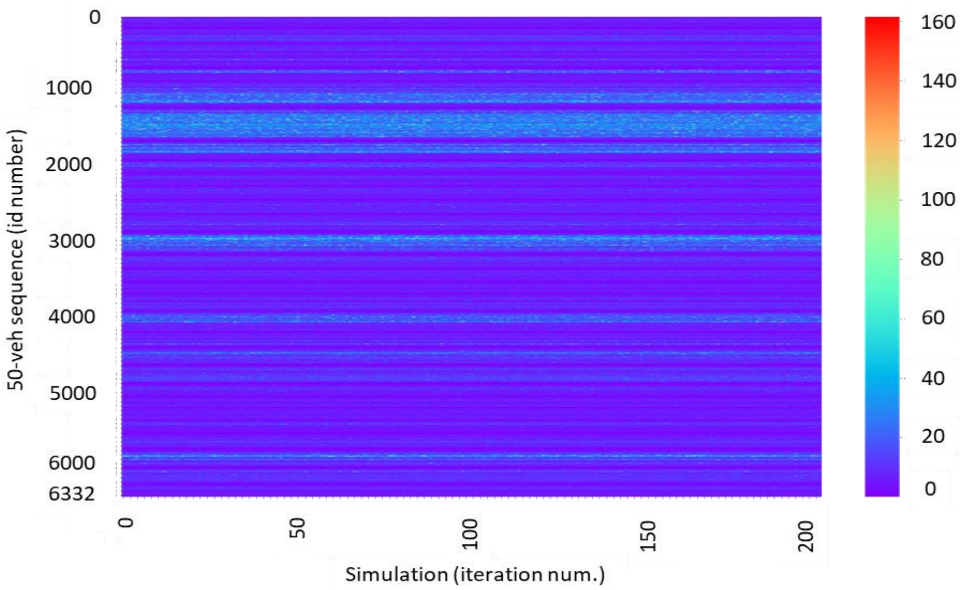



**Figure 4.** Heatmap graph of the average densities obtained in the simulation with 200 iterations ($x$-axis) for the 6332 sequences of 50 vehicles ($y$-axis).

Using ${\widetilde{k}_{\tau|\Delta t}}^{(i,j)}$, it is possible to identify the number of sequences $m^{*(i)}$ out of the total $m$ = 200 simulated for which ${\widetilde{k}_{\tau|\Delta t}}^{(i,j)} < k^*$. This value can be used to estimate reliability $\varnothing(k^*)^{(i)} = m^*/m$ of the flow rate in $\Delta t^{(i)}$, i.e., $q_{\Delta t^{(i)}}$, during the subsequent test interval $\tau$. For threshold value $k^*$, we assumed the conventional density threshold that identifies the entrance to the LOS F according to the latest editions of the [11], i.e., ${k^*}_{E \to F}$ = 28 vehicles/kilometer/lane. Figure 5 shows the trend of the reliability over the entire duration of the time series, $\varnothing({k^*}_{E\to F} = 28)^{(i)}$, while Figure 6 shows a zoom of a time window in which the reliability shows substantial reductions. Figure 7 shows the trend of $q_{\Delta t^{(i)}}$ for the whole monitoring period, with the superimposition (red dots) of values for which

$\varnothing(k^*_{E\to F} = 28)^{(i)}$ resulted to be less than 80%, i.e., with a probability of exceeding the conventional density at capacity per lane identified by the HCM greater than 20%. Figure 8 shows the dispersion diagram of the $q_{\Delta t^{(i)}}$ values and of the respective reliability values, $\varnothing(k^*_{E\to F} = 28)^{(i)}$, during the monitoring period.

As mentioned in Section 4.2, the results obtained through the simulations of the speed processes can be used to produce further evaluations concerning the distribution of the flow rates for various density thresholds. A first analysis can be obtained by considering $k^*_{E\to F} = 28$ as the limit density for reaching lane capacity and using the PLM [41]. The PLM can also be applied by proceeding with an extension of the analysis to investigate the probability that the limit ranges of LOS C, D, and E are exceeded using the limit density values provided by the HCM, i.e., $k^*_{B\to C} = 11$, $k^*_{C\to D} = 16$, and $k^*_{D\to E} = 22$. The dotted curves in Figure 9 show the trend of the reliability functions with different values of $k^*$, which correspond to the probabilities of exceeding $k^*$ as a function of $q$, which were obtained considering the m = 200 simulations of the 6632 sub-sequences of 50 vehicles. The continuous curves in Figure 9 show the reliability trend using Weibull functions. The interpolating functions showed an excellent fit, as shown by the graphical comparison with the discontinuous data, with high values of $R^2$.

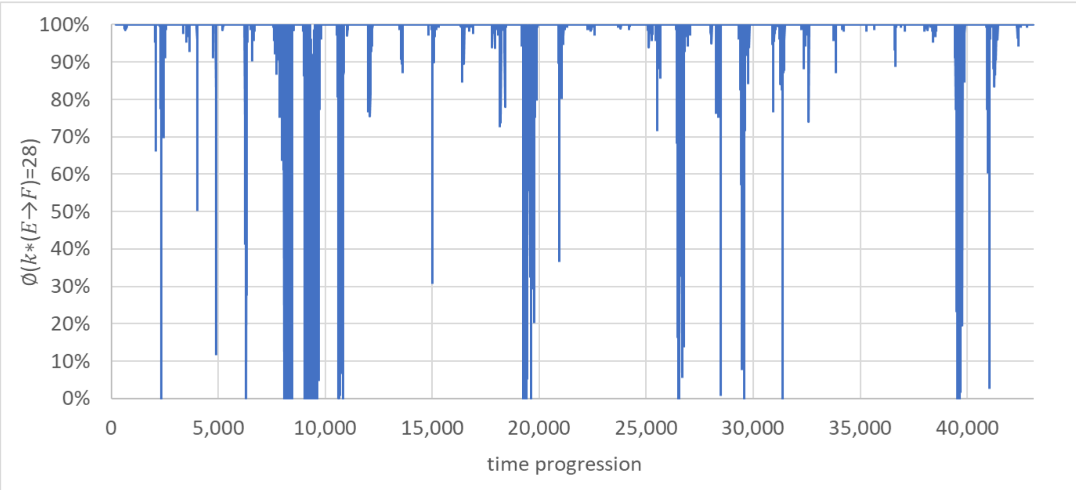



**Figure 5.** Evolution of $\varnothing(k^*_{E\to F} = 28)$ over the entire monitoring period.

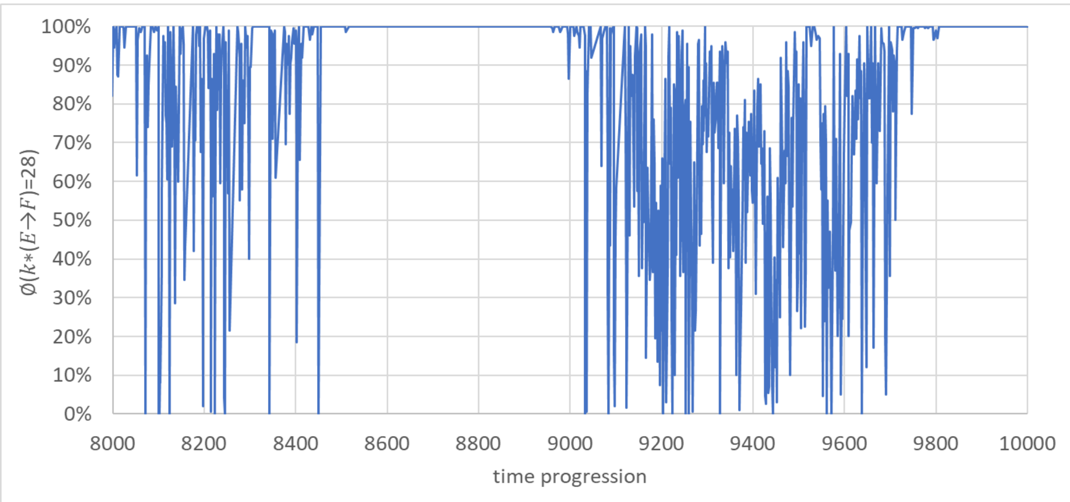



**Figure 6.** Evolution of $\varnothing(k^*_{E\to F} = 28)^{(i)}$ over the entire monitoring period—selected time window showing 8000–10,000 s from the first transit.

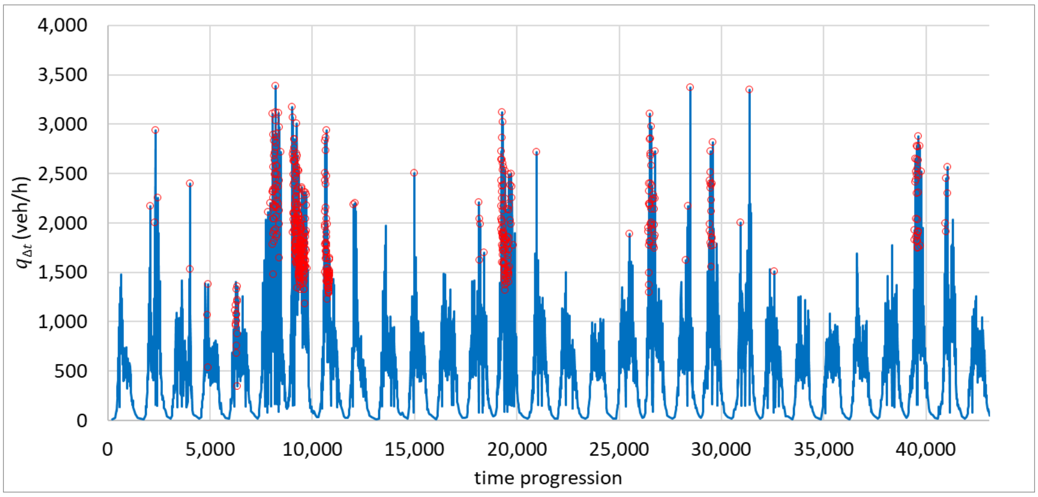



**Figure 7.** Trend of $q_{\Delta t^{(i)}}$ for the whole monitoring period, with the superimposition (red dots) of $q_{\Delta t^{(i)}}$ values with $\varnothing(k^*{}_{E \to F} = 28)^{(i)}$ <80%.

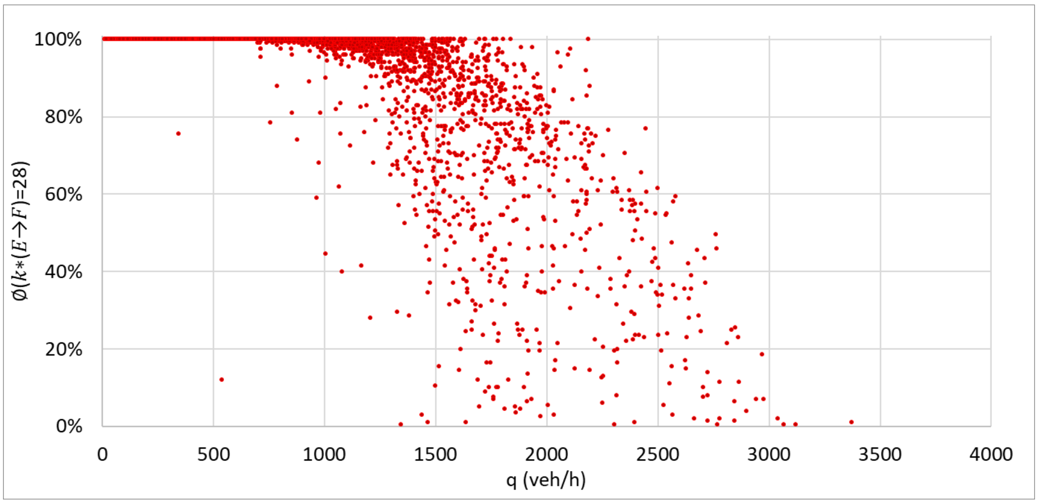



**Figure 8.** Scatter plot of points ($q_{\Delta t^{(i)}}$ ; $\varnothing(k^*{}_{E \to F} = 28)^{(i)}$ ).

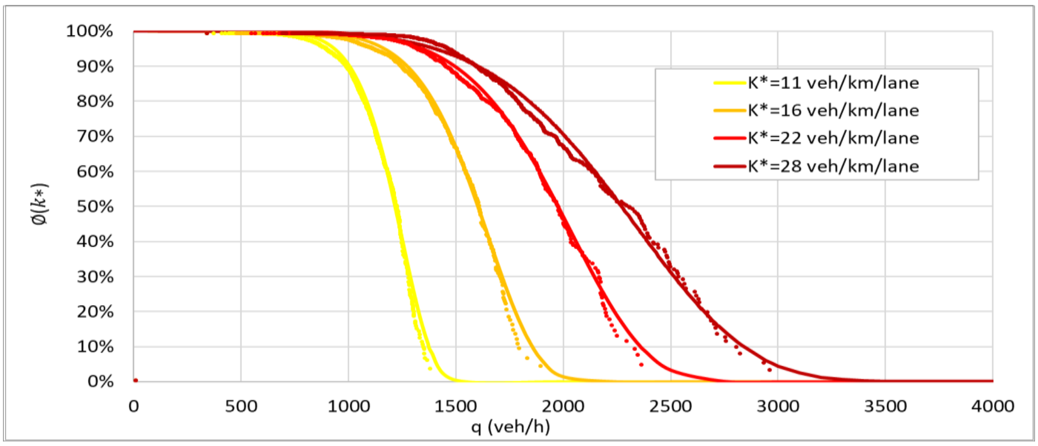



**Figure 9.** Reliability values and estimated Weibull curves with $k^*{}_{B \to C} = 11, k^*{}_{C \to D} = 16, k^*{}_{D \to E} = 22$ and $k^*{}_{E \to F} = 28$ as functions of $q_{\Delta t}$.

Figure 10 shows the trend of the reliability function of the hourly flow rates for exceeding $k^*_{E \to F} = 28$ together with the Weibull function estimated according to [27]. The latter is represented by points (green dots), obtained using the same dataset and considering, as the breakdown threshold, a speed value of 80 km/h and 5 min fixed intervals for data aggregation. Table 1 shows the two estimated functions and the relative parameters, which appear essentially superimposable as in Figure 10.

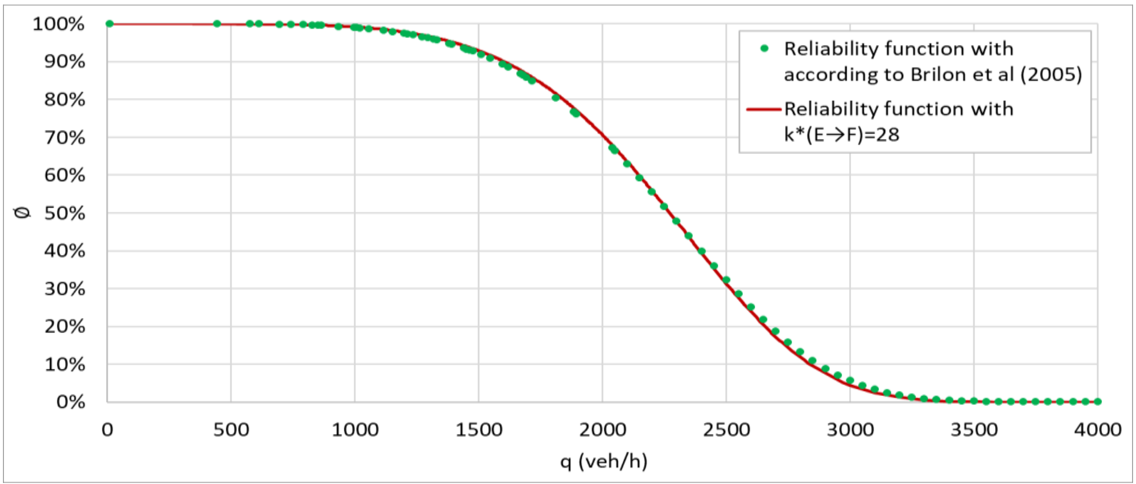



**Figure 10.** Comparison between the Weibull curve estimated for the reliability function with $k^*_{E \to F} = 28$ and the Weibull curve obtained with the method by [27], with breakdown speed being equal to 80 km/h and aggregation intervals of 5 min.

**Table 1.** Weibull curves and related calibration parameters with $k^*_{E \to F} = 28$ and with the method by Brilon et al. (2005) [27].

| Reliability Function with $k^*_{E \to F}$=28 | Reliability Function According to [27] |
|---|---|
| General model:<br>f($x$) = exp(−($x$/$\beta$)^$\alpha$)<br>Coefficients (with 95% confidence bounds):<br>$\alpha$ = 5.395 (5.341; 5.448)<br>$\beta$ = 2431 (2426; 2437)<br>Goodness-of-fit: R-square: 0.9909 | General model:<br>f($x$) = exp(−($x$/$\beta$)^$\alpha$)<br>Coefficients (with 95% confidence bounds):<br>$\alpha$ = 5.127 (4.945; 5.31)<br>$\beta$ = 2440 (2410; 2469)<br>Goodness-of-fit: R-square: 0.9937 |

Based on the reliability functions with the different values of $k^*$ as the limits between LOS B and C, LOS C and D, LOS D and E, and LOS E and F, Figure 11 shows the probability curves $1 - \varnothing(k^*)$ of exceeding the limits of density for each LOS.

These curves can represent a helpful tool for the probabilistic analysis of the performance of the leftmost lane and the entire freeway section. Figure 11 shows as an example the case of a flow rate equal to 1500 vehicles/hour/lane. From the intersection with the different probabilistic curves, it is possible to identify the probability of exceeding each density limit. In the case study, based on the data collected for the entire monitoring period and the simulations of the speed processes, we can state that a flow rate of 1500 vehicles/hour in the leftmost lane is such as to exceed the LOS B limit in this lane with a probability of 99.5%, 33.2% for LOS C, 10.3% for LOS D, and 7.1% LOS E. Using the same curves, we can say that a flow rate of 1500 vehicles/hour/lane is such as to generate LOS A or B with 0.5% probability, LOS C with 66.3% probability, LOS D with 22.9% probability, LOS E with 3.2% probability, and LOS F with 7.1% probability. For this analysis, we can use the probabilistic distributions of the LOSs for each value of the hourly flow shown in Figure 12.

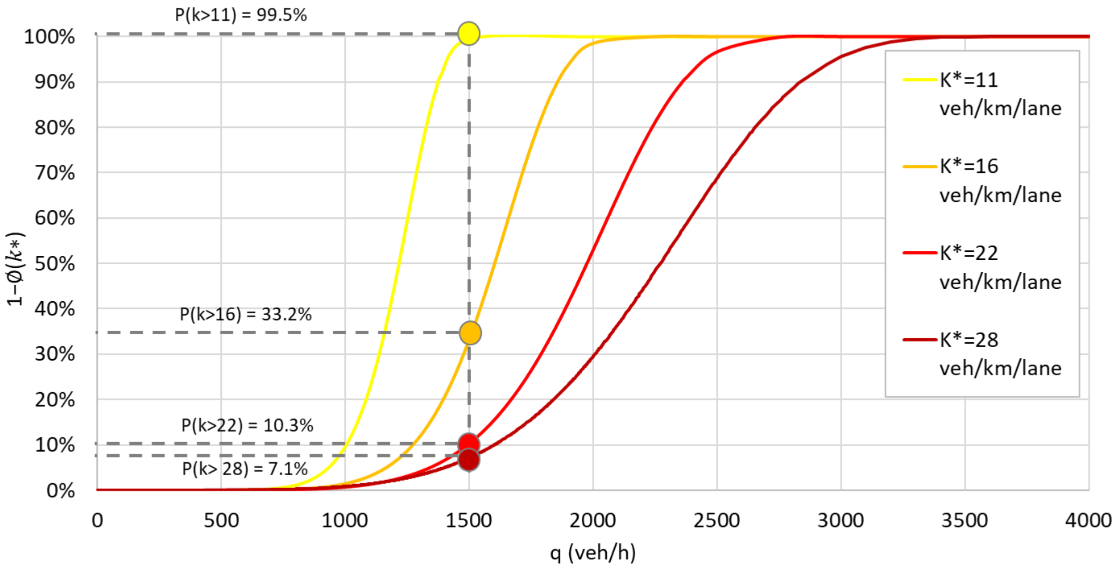



**Figure 11.** LOS-limit-exceeding curves and example case with flow rate equal to 1500 veh/h.

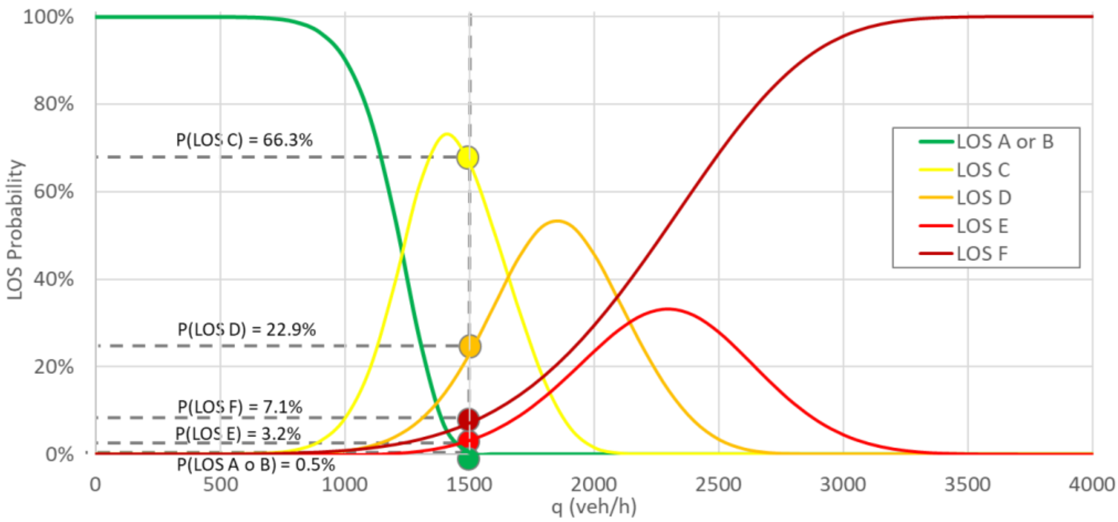



**Figure 12.** Probability distribution curves of LOSs and example case with flow rate equal to 1500 veh/h.

## 6. Conclusions

The reliability approach to circulation quality involves the most recent research, resorting to a probabilistic description of traffic phenomena and the effects of interactions between vehicles on the quality of traffic. In this context, the paper retraces the probabilistic point of view of the traffic circulation quality, which concerns the analysis of breakdown phenomena and capacity on the one hand and the analysis of the reliability based on the random processes of vehicle speeds on the other hand. In particular, the paper explores the main aspects of this latter approach, addressing and deepening the literature regarding the description of the speed random processes in the leftmost lane of a motorway carriageway, in which congestion phenomena occur first.

According to the reviewed literature, the study of the reliability of the traffic in the leftmost lane of a freeway carriageway involves the analysis of the speed random process according to an ARIMA (0,1,1) model. The literature provides regression equations for the reliability function and the capacity distribution. However, these regressions were obtained by simulating random processes under specific boundary conditions. For these reasons, the paper highlights that the regression equations for traffic reliability and lane capacity are not directly generalizable, requesting the reiteration of the simulation and regression process more correctly to particularize them each time to the specific case.

For this purpose, the work outlines a general procedure based on the estimation and simulation of ARIMA models for speed random processes in a freeway section to assess the traffic reliability function using historical data or near-real-time information flow obtained using speed monitoring devices. The paper shows a further novelty in the reliability analysis through the estimation and simulation of ARIMA models for speed random processes, proposing a method for the analysis of the distribution of the flow rate concerning various density thresholds on the leftmost lane of the carriageway. For these analyses, the study shows the use of the PLM, widely used in the breakdown and capacity analysis approach. This operation is performed precisely in consideration of the purpose of this research, which was to enclose in a single operative framework the probabilistic approaches to the quality of circulation—i.e., the random speed processes and the probabilistic analysis of breakdown phenomena and capacity in the critical lane—and to connect them with the critical elements of LOS analyses according to the HCM.

The procedure outlined in the paper starts from analyzing and simulating the speed processes of vehicle sequences to evaluate the traffic reliability, i.e., the probability of not exceeding a certain criticality density threshold in the leftmost lane in a 5 min interval with a constant flow rate. Thus, it obtains the probabilistic distribution of the corresponding flow rate values through the application of the PLM. This distribution represents the probabilistic distribution of the capacity if we consider the critical density at capacity as the threshold, e.g., by setting the value indicated by the HCM as the limit for LOS E. Further distributions can be characterized by varying the threshold value considering the density values that separate the Levels of Service, from A/B to E/F, according to the HCM.

At the end of these conclusions, we would like to underline the current limitation of this procedure. This limit consists in considering only one vehicle class. In illustrating the procedure, we consider vehicles that we can assume here belong to a single homogeneous class, i.e., passenger cars. On the other hand, this is true in the case study concerning A22 del Brennero, where heavy vehicles cannot transit on the leftmost lane due to the overtaking ban on the infrastructure. In the most general case, this classification should be considered if the flow consists of different vehicle classes—for example, passenger cars and freight vehicles. Even the reference densities of the HCM consider equivalent vehicle units based on adequate homogenization coefficients. These aspects, which the study does not deal with in this stage, will be the subject of future investigations and may produce an even more robust generalization of the procedure.

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## Appendix A

The speed levels conditioned by the previous realizations between two successive vehicular passages at instants $t$ and $t+1$, respectively $\overline{v}_t$ and $\overline{v}_{t+1}$, are linked by a quantity

$\lambda a_t$, with $\lambda$ being a coefficient between 0 and 1 and $a_t$ being a random variable with zero mean and variance $\sigma^2$.

We can write $\overline{v}_{t+1} = \overline{v}_t + \lambda a_t$, that is, $\overline{v}_{t+1} - \overline{v}_t = \lambda a_t$. Since $\overline{v}_t = v_t - a_t$ and $\overline{v}_{t+1} = v_{t+1} - a_{t+1}$, we have that $v_{t+1} - a_{t+1} = \overline{v}_{t+1} = \overline{v}_t + \lambda a_t = v_t - a_t + \lambda a_t$; therefore, $v_{t+1} - a_{t+1} = v_t - a_t + \lambda a_t$.

Ultimately, we can write $v_{t+1} = v_t - a_t(1-\lambda) + a_{t+1}$; therefore, by setting $(1-\lambda) = \vartheta$, it results that $v_{t+1} = v_t - a_t\vartheta + a_{t+1}$.

Considering the generic form of a self-regressive and integrated moving average ARIMA(p,q,d) model expressed as $x_t - \varphi_1 x_{t-1} - \ldots - \varphi_p x_{t-p} - \ldots \varphi_{p+d} x_{t-p-d} = a_t - \vartheta_1 x_{t-1} - \ldots - \vartheta_q x_{t-q}$, the expression $v_t = v_{t-1} - a_{t-1}\vartheta + a_t$ corresponds to an ARIMA (0,1,1) model, where $x_t = v_t$ and $\varphi_1 = 1$.

To proceed more easily, it is useful to consider the following operators:

- Difference operator, $\nabla x_t = x_t - x_{t-1}$;
- Sum operator, $\mathrm{S}x_t = \nabla^{-1} x_t = \sum_{-\infty}^{t} x_j$;
- Back operator, $\mathrm{B}x_t = x_{t-1}$;
- Forward operator, $\mathrm{F}x_t = \mathrm{B}^{-1} x_t = x_{t+1}$.

The associative property, the commutative property of the sum and of the product, and the distributive property of the product are valid for these operators.

Since $v_t = v_{t-1} - a_{t-1}\vartheta + a_t$, we can write $v_t - v_{t-1} = \nabla v_t$ and $a_t - a_{t-1}\vartheta = a_t - \vartheta B a_t = (1 - \vartheta B) a_t$.

Thus, $(1-\vartheta B) = (1-\vartheta)B + (1-B)$ and $(1-\vartheta B) = \lambda B + (1-B)$.

Since $(1-\mathrm{B})x_t = x_t - Bx_t = x_t - x_{t-1} = \nabla x_t$, it holds in general that $((1-B) = \nabla$, and we obtain $(1-\vartheta B) = \lambda B + \nabla$.

With this result, we can write $\nabla v_t = \lambda B a_t + \nabla a_t$.

Now, we apply sum operator $S$ to both sides, and we have $S(\nabla v_t) = S(\lambda B a_t + \nabla a_t) = S(\lambda B a_t) + S(\nabla a_t) = S(\lambda a_{t-1}) + S(\nabla a_t)$, with $S(\nabla v_t) = \nabla^{-1}(\nabla v_t) = v_t$, $S(\lambda a_{t-1}) = \lambda S(a_{t-1}) = \lambda \sum_{j=1}^{\infty} a_{t-j}$, and $S(\nabla a_t) = \nabla^{-1}(\nabla a_t) = a_t$.

Thus, we have that $v_t = \lambda \sum_{j=1}^{\infty} a_{t-j} + a_t$, i.e., the speed of the vehicle passing in instant $t$, can be defined as a function of all the deviations occurring from the beginning of the process up to time $t$. Furthermore, since $\overline{v}_t = v_t - a_t = \lambda \sum_{j=1}^{\infty} a_{t-j} + a_t - a_t$, the conditional mean of the speeds results to be $\overline{v}_t = \lambda \sum_{j=1}^{\infty} a_{t-j}$.

Considering an instant $t - f$, we have that $\overline{v}_{t-f} = \lambda \sum_{j=f+1}^{\infty} a_{t-j}$, and being $\overline{v}_t = \lambda \sum_{j=1}^{f} a_{t-j} + \lambda \sum_{j=f+1}^{\infty} a_{t-j}$, we have $\overline{v}_t = \overline{v}_{t-f} + \lambda \sum_{j=1}^{f} a_{t-j}$.

This last equation shows that the trend of the levels of $v_t$ starting from instant $t - f$ is the result of a random walk generated by variable $\lambda a_t$. It is a non-stationary process with constant mean and variance that grows indefinitely over time.

If $a_t \sim WN(0, \sigma^2)$ and $\sigma^2$ are constant over time, $\lambda a_t$ has standard deviation $\lambda\sigma$.

Therefore, if $a_t \sim N(0, \sigma^2)$, $\lambda\sigma$ fully defines the distribution of $\lambda a_t$ and thus represents the measure of the stability of the speed process generated on the lane by a traffic flow that is affected by the interactions among vehicles. It can be observed that the random walk that produces the speed level trend is limited by the two barriers, consisting of the zero speed value and the maximum speed value that the vehicles can maintain in the lane section.

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