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Determining the Optimal Density of *Phoebe bournei* Plantations Based on Dynamic Programming under Close-to-Nature Management Measures

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Abstract: Close-to-nature management (CTNM) is the most promising option for plantation silviculture and has received widespread attention in recent years. Stand density is a key variable in CTNM, as it directly influences growth and yield. Research for the optimal density that maximizes the total harvest has been ongoing. In this paper, a dynamic programming model was applied to the CTNM of *Phoebe bournei* plantations for the first time to solve the problem of stand density and target tree density control. This paper took *Phoebe bournei* plantations in Jindong Forest Farm of Hunan Province as the research object. Based on the data of seven consecutive years from 2015 to 2021, Richard's growth equation was used to fit the height growth equation and basal area growth equation of *Phoebe bournei*. Stand growth was divided into five development stages according to the forest growth process and characteristics. Stand density and basal area were selected as two-dimensional state variables, and the maximum total harvest in the entire stand growth process was used as the objective function to establish a dynamic programming model. The optimal stand density and target tree density at each growth stage of the stand under three different site conditions were determined. According to the results obtained, the objective forest shape was designed for the stand under three types of site conditions, which can provide a theoretical basis for the CTNM of *Phoebe bournei* plantations to make the stand achieve the maximum harvest.

Keywords: close-to-nature management; sustainable forest management; optimal stand density; target tree density; *Phoebe bournei* plantation; objective forest shape

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1. Introduction

A multitude of strategies to adapt forest management to climate change have been proposed [1,2]. Sustainable forest management targeting sustainable social, economic, and environmental development is a global trend under a climate change scenario [3]. Close-to-nature management (CTNM) is a promising forestry management approach to meet the criteria for sustainable forestry [4]. Compared with the other two management methods to mitigate climate change, that is, structure-based forest management (SBFM) [5,6] and secondary forest comprehensive silvicuture (SFCS) [7,8], the differences in CTNM are that it pays attention to the feasibility of ecology and environment [9], and simultaneously integrates wood production and ecological service functions of a forest at a relatively small spatial scale, such as the stand level, by developing a structure similar to that of an original forest [10]. A main objective of CTNM is to increase tree growth or improve tree quality on a sustainable basis.

A target tree management system (TMS) is not only an effective means to achieve CTNM [11], but also an important management model for cultivating large-diameter wood. TMS is an intermediate silvicultural treatment intended to provide increased growing space to selected trees (target trees) and improve the quality of individual trees through the removal of crown competition from adjacent trees (interference trees). Thus, it is often assumed to be synonymous with thinning, as well as cutting or timber stand improvement [12].

The determination of stand density and target tree density is the core issue of CTNM and TMS. In forest ecosystems, stand density directly affects the distribution of ecological factors such as light, water, and heat [13], and thus, affects forest productivity, stand growth, canopy structure, and soil fertility [14]. Overly dense areas will lead to excessive competition, and areas that are not dense enough will lead to wasted forest space; thus, it is necessary to seek optimal density. In a sense, forestry is the science of optimizing forest density [15]. Density is an important factor that can be artificially regulated and affect tree growth, and it is one of only a few things that we can control efficiently, while also often being profitable [15].

Stand density is mainly controlled by thinning. Any intermediate cutting may reduce the number of trees in a stand and thus concentrate the growth on fewer trees, which grow faster than under unthinned conditions [16]. The level of each thinning affects subsequent thinning, including the magnitude of the final harvest [17]. That is, each cutting decision has implications for the future growth and yield of the stand. Therefore, forest managers will face sequential or interdependent decision-making challenges when controlling density. An optimal sequence of such interrelated relationships can be derived by dynamic programming [18].

Dynamic programming is a branch of operations research that is used to solve the optimization problem of the multistage decision-making process [19]. It is a mathematical programming technique that searches through a defined network to find the optimal path that maximizes an objective [20]. Dynamic programming was first applied by Arimizu in forestry to seek the best thinning system with the goal of maximum harvest [21]. In the process of plantation management in the past, the maximum harvest was often used as the goal, and multiple thinning occurred until the final clear cutting was regarded as a multistage decision-making problem. Dynamic programming was used to determine the thinning interval, thinning volume, and times in the clear-cutting system of plantations, but it has rarely been applied to CTNM.

This paper uses a *Phoebe bournei* plantation in Jindong Forest Farm of Hunan Province as the research object. *Phoebe bournei*, a large evergreen tree of *Lauraceae*, is a unique and precious timber tree in China. The timber of *Phoebe bournei* is tough, dense, and glossy, and can be used for both architecture and furniture [22]. *Phoebe bournei* grows slowly and suffers significantly from human damage, and has been listed as a rare and endangered plant under *Class II* protection in China. However, in recent years, there have been some problems on *Phoebe bournei* plantations, such as monospecific structure, low stand productivity, and gradual deterioration of forest quality, which cannot be solved by relying solely on the traditional clear-cutting method. It is particularly important to use CTNM technology to gradually transform the existing even-aged and pure forest of *Phoebe bournei* into uneven-aged, multilayer, and mixed forests. CTNM can achieve the goal of accurately improving stand quality and cultivating large-diameter timber of precious tree species of *Phoebe bournei*.

The objectives of this paper are (1) to find a set of optimal stand densities and target tree densities at each stage under three different site conditions by applying dynamic programming and (2) to design the objective forest shape for the stand under three types of site conditions to provide the direction and goal of regulation and control for the CTNM of *Phoebe bournei* plantations.

2. Materials and Methods

2.1. Study Area

The Jindong forest farm is located in Yongzhou County of Hunan Province with geographical coordinates of 110°53′43″–112°13′37″ E and 26°2′10″–26°21′37″ N (Figure 1). It is approximately 33 km wide from east to west and 36 km long from north to south, with a total area of 635 km^2. The terrain of Jindong Forest Farm is high in the southwest and low in the northeast. There are many high and steep mountains in the territory, with an altitude of 108–1435 m. The study area experiences a middle subtropical southeast monsoon humid climate. The highest temperature in the summer can reach 40 °C, and the lowest temperature in the winter can reach −8 °C. The precipitation and sunlight are sufficient. The soil is mainly yellow-red soil and mountain yellow soil. The areas above 1000 m above sea level are mountain yellow-brown soil, and the hilly areas are mainly red soil. Jindong Forest Farm is rich in plant resources. According to the survey, the existing higher plants belong to 210 families and 1558 species, and woody plants belong to 98 families and 656 species. Among them, *Ginkgo biloba* L., *Taxus chinensis*, and *Bretschneidera sinensis* are listed as national *Class I* protected plants, and *Phoebe bournei*, *Cinnamum camphora*, *Pseudotsuga menziesii*, *Cephalotaxus oliveri*, *Ormosia henryi Prain*, *Itoa orientalis*, and *Eucommia ulmoides Oliver* are listed as national *Class II* protected plants.

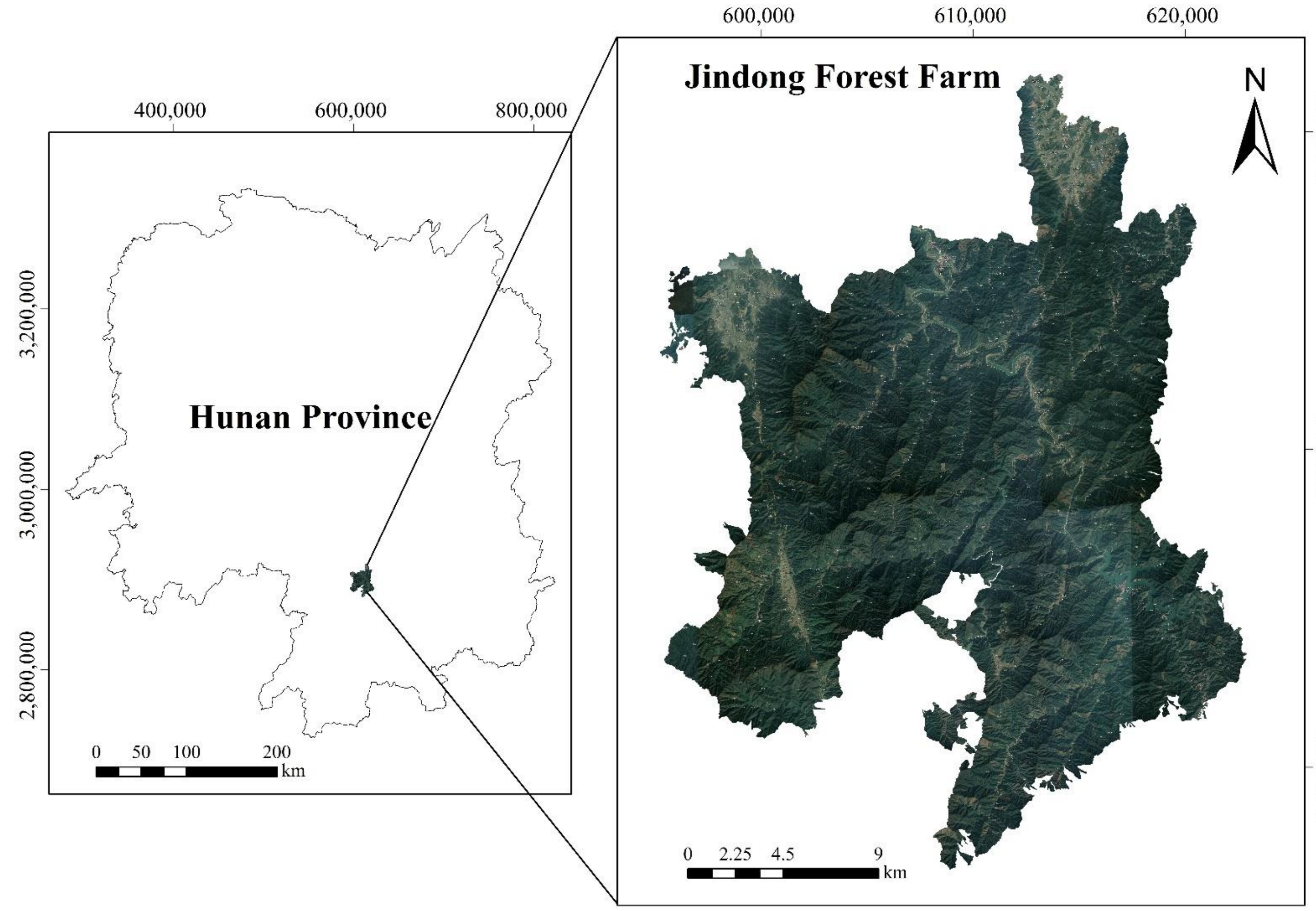

Figure 1. Location map of Jindong Forest Farm.

2.2. Sample Plot Setting and Investigation

Nine sample plots with an area of 20 m × 30 m were set in *Phoebe bournei* plantations using tools such as compass and tape measures, among which the number of sample plots with a site index of 24, 22, and 20 was three. The team investigated and retested these fixed sample plots every July from 2015 to 2021, and seven consecutive periods of data were obtained. The basic investigation factors of the sample plots include the DBH, tree height, number of trees, crown width, canopy density, age, and so on. The basic situation of the fixed sample plots of the *Phoebe bournei* plantation is shown in Table 1.

Table 1. Basic situation of nine fixed sample plots for *Phoebe bournei* plantations.

Site Index	Sample Number	Altitude/m	Slope/°	Stand Density/Trees ha^{-1}	Average DBH/cm	Average Tree Height/m	Afforestation Time
24	1	120	5	1350	15.54	14.46	2002
	2	120	8	1175	16.61	14.89	2002
	3	130	10	1117	14.23	13.39	2002
22	4	150	12	1500	11.35	8.47	2008
	5	175	14	1600	12.03	8.22	2008
	6	185	18	1650	13.51	12.51	2002
20	7	200	27	1875	11.63	10.74	2002
	8	260	25	1700	9.06	6.72	2012
	9	280	24	1850	8.42	7.56	2012

2.3. Methods

Dynamic programming is a mathematical method used to solve the optimal decision-making process [19]. The whole system can reach the best state through multistage decision-making. The global optimal solution of the dynamic programming model and the optimal density sequence of each stage can be obtained.

2.3.1. Stage Division

This classification divided the forest successional process under CTNM into five developmental stages [23] (Table 2), that is, the forest establishment stage (stage I), competition differentiation stage (stage II), quality selection stage (stage III), close-to-nature stage (stage IV), and continuous forest stage (stage V).

Table 2. Division of forest development stages.

Stages	Stand Development Stages	Age Range
Stage I	Forest establishment stage	<5
Stage II	Competitive differentiation stage	5–20
Stage III	Quality selection stage	20–40
Stage IV	Close-to-nature stage	40–50
Stage V	Continuous forest stage	>60

2.3.2. Determination of State Variables and Decision Variables

For the stand density control problem, the state variables used to describe stand growth should be satisfied if (1) they are closely related to stand density, (2) they can describe the change in stand growth, and (3) they are closely related to the target harvest and stand volume [15]. The basal area per hectare and the number of trees per hectare were selected as two-dimensional state variables to represent the change in density. The number of trees and basal area to be cut at each stage were selected as decision variables.

2.3.3. State Transition Equation

The state transition equation is written according to the meaning of state and decision variables and the related recursive relations of each stage. The state transition equation is as follows.

$$N_{i+1} = N_i - n_i \tag{1}$$

$$B_{i+1} = B_i - Y_i + \Delta G_i \tag{2}$$

In Formula (1), i represents the stage (in this paper, a stage is treated as a growth period), N_i represents the number of trees per hectare at the beginning of stage i, and n_i represents the number of trees per hectare removed in stage i. In Formula (2), B_i represents the basal area per hectare at the beginning of stage i, which is obtained by fitting Richard's growth function, and Y_i represents the basal area per hectare removed in stage i. The remaining basal area $B_i - Y_i$ is obtained at the end of stage i, which constitutes the growth basis of this stage [24]. ΔG_i represents the net periodic basal area growth increment per hectare from stage i to stage $i + 1$ (as shown in Figures 2 and 3).

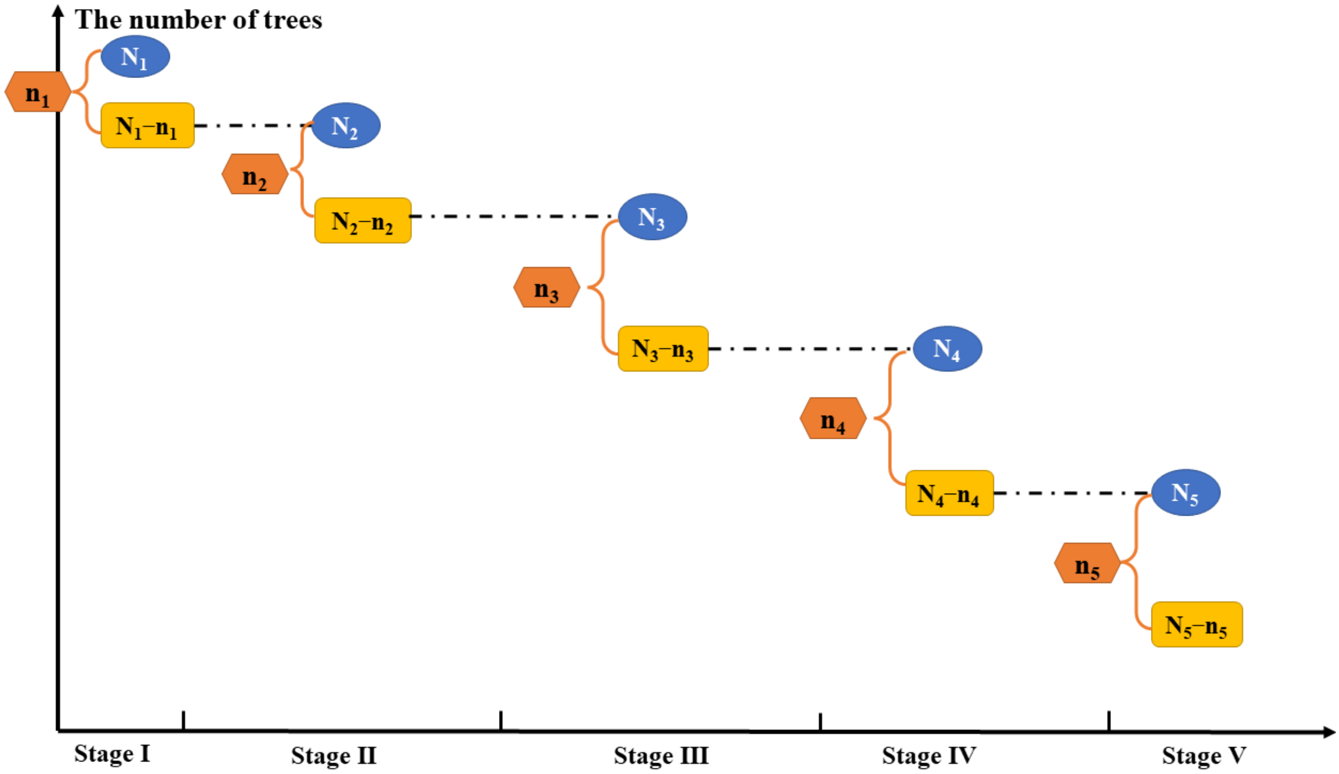

Figure 2. The change in number of trees over time.

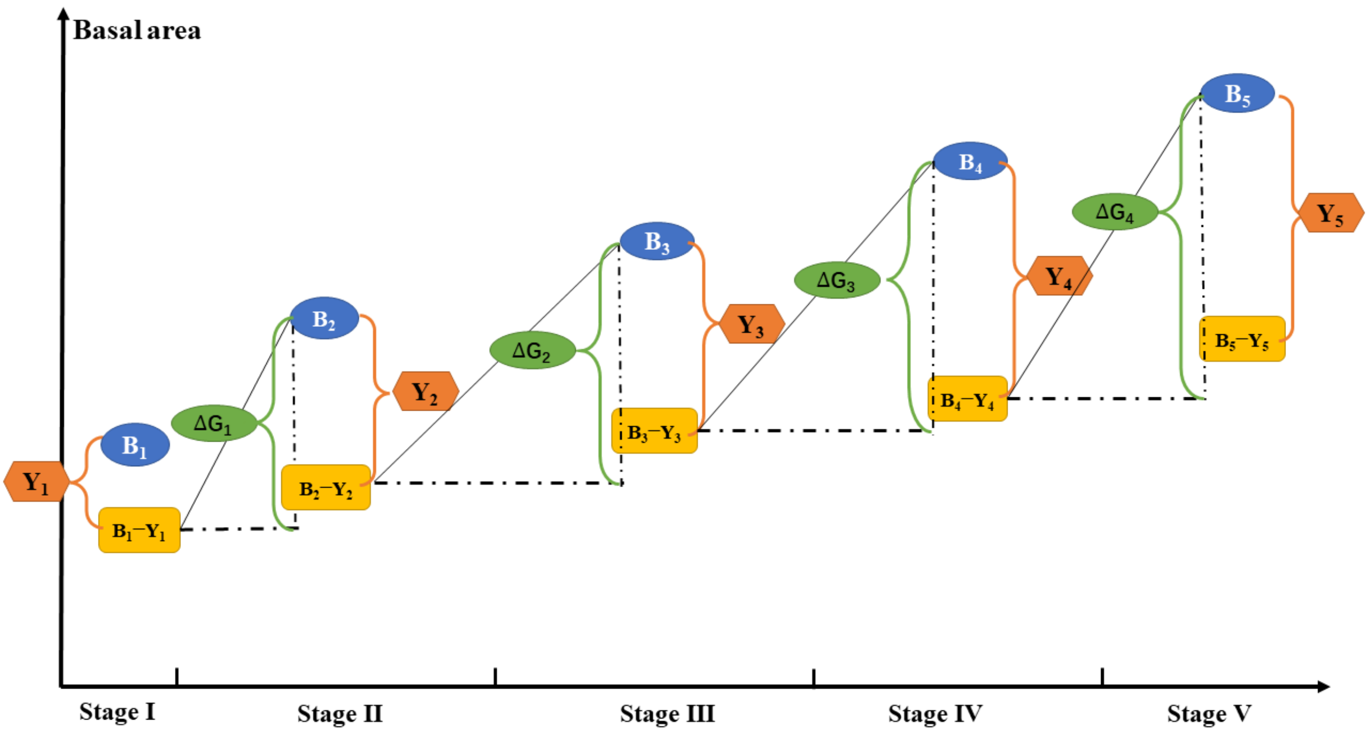

Figure 3. The change in basal area over time.

2.3.4. Objective Function

The goal of stand density control and CTNM is to improve stand productivity and obtain the maximum harvest during the management period [25]. The objective function is to determine the amount of thinning $Y_1, Y_2, \ldots, Y_i$ and $n_1, n_2, \ldots, n_i$ at stages $1, 2, \ldots, n$ such that the accumulated total harvest is maximized.

$$Max \quad M = \sum_{i=1}^{N} V_i(D_i) \times n_i \tag{3}$$

In Formula (3), M represents the sum of the stand volume harvested from each stage, N represents the number of the final stage, and V_i represents the individual volume of harvested trees with a diameter of D_i at stage i, according to the binary volume table of *Phoebe bournei* in Hunan Province:

$$V_i = 0.000041028005 * {D_i}^{1.80063} * {H_i}^{1.130599} \tag{4}$$

In Formula (4), H_i represents the average height of the stand at stage i, which is fitted by Richard's growth function.

D_i represents the average DBH of the removed trees at stage i.

$$D_i = \sqrt{\frac{40000 * Y_i}{n_i * \pi}} \tag{5}$$

In Formula (5), Y_i represents the basal area removed at stage i and n_i represents the number of trees removed at stage i.

2.3.5. Constraint Condition

According to the inspection method, the cutting amount of the basal area at each stage must be less than the growth amount [18]. The thinning intensity should not exceed 20% of the stand volume at stage I and stage II.

2.4. Data Processing and Analysis

The data were sorted in Excel 2019, and then stand factors such as age, tree height, DBH, and site conditions were introduced into SPSS 25.0 to fit the parameters of the tree height growth equation and basal area growth equation. The dynamic programming model was inputted into Lingo 11.0, and the global optimal solution and the optimal density at each stage were obtained.

3. Results

3.1. Height Growth Equation

Richard's growth equation has a wide range of applicability and can simulate all stages of tree growth well [26]. In this paper, Richard's growth equation was used to fit the average height growth of the stand at each stage under three types of site conditions.

$$H = a\left(1 - be^{-kt}\right)^{\frac{1}{1-m}} \tag{6}$$

In Formula (6), H represents the average height of the stand, t represents the stand age, and a, b, k and m represent the model parameters.

The stand age and average tree height data were analyzed by regression analysis in SPSS 25.0, and the tree height growth models under three types of site conditions were obtained as shown in Table 3.

Table 3. Tree height growth models under the three site conditions.

Site Index	Tree Height Growth Models	R^2
20	$H = 20.756 * (1 - 0.606 * e^{-0.065*t})^{3.745}$	0.901
22	$H = 22.764 * (1 - 0.673 * e^{-0.061*t})^{3.012}$	0.817
24	$H = 25.089 * (1 - 0.757 * e^{-0.056*t})^{2.119}$	0.802

According to the tree height growth model, the growth curves of the tree height under the three site conditions can be obtained. It can be seen from the tree height growth curves in Figure 4 that the site conditions affect the tree height growth. At the same age, the better the stand site condition is, the faster the tree height increases and the higher the average height of the stand is. The tree height increases rapidly in the young and middle-aged forest, and the growth rate decreases after entering the mature forest stage. When the stand age reaches 60 years, the tree height tends to the maximum and stops growing.

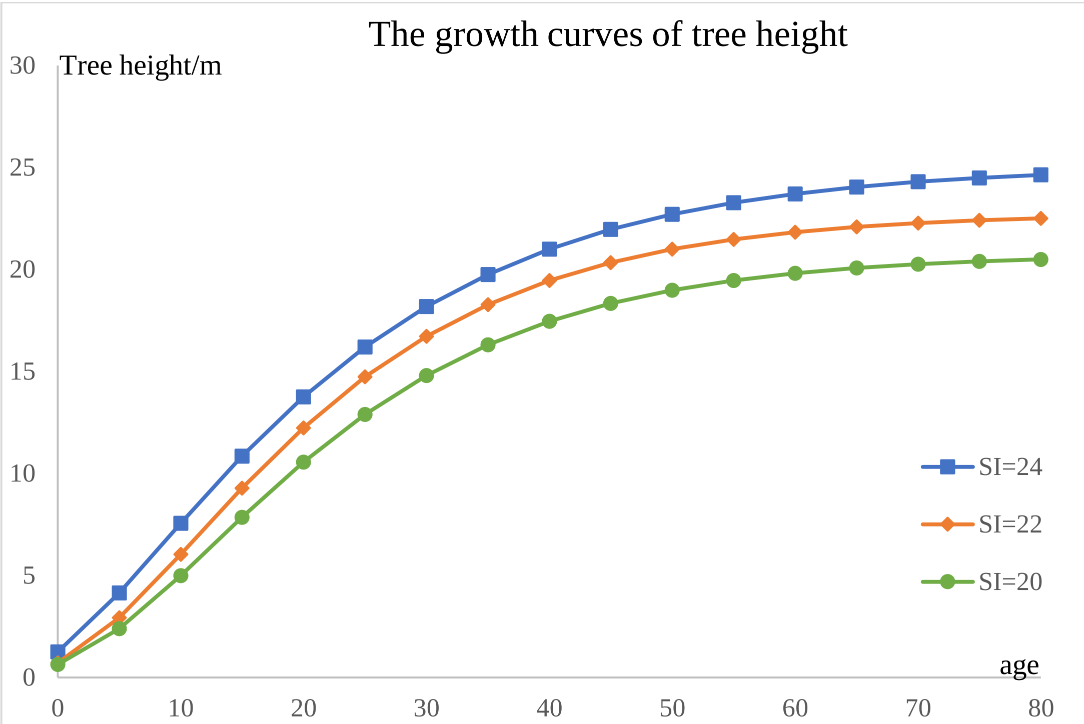

Figure 4. Tree height growth curves under the three site conditions.

3.2. Basal Area Growth Equation

In the process of natural growth and stand development, the growth and harvest of forests largely depend on the stand age, site quality, and stand density [27]. In fact, stand management measures indirectly affect stand growth and harvest by improving the site quality of the stand (such as fertilization) and adjusting the stand density (such as thinning). This paper used the site index to reflect site quality and the number of trees as the density index. Stand age, site index, and the number of trees were selected as independent variables to construct a stand basal area growth model. Richard's growth function was used to fit the growth equation of stand basal area at each stage:

$$B = a_1 SI^{a_2} \left\{ 1 - \exp\left[-a_3 \left(\frac{N}{1000} \right)^{a_4} \right] * t \right\}^{a_5} \quad (7)$$

In Formula (7), B represents the basal area, N represents the number of trees, SI represents the site index, t represents the stand age, and a_1, a_2, a_3, a_4, and a_5 represent the model parameters.

The model expression was obtained by regression analysis of stand age, site index, the number of trees, and basal area in SPSS 25.0.

$$B = 4.942 SI^{0.846}\left\{1 - \exp\left[-0.039 * \left(\frac{N}{1000}\right)^{0.292}\right] * t\right\}^{1.296} \tag{8}$$

$$R^2 = 0.869$$

Using the initial stand density of 2000 trees ha^{-1} as an example, the growth curves of basal area under the three different site conditions were obtained. It can be seen from the growth curves of basal area in Figure 5 that at the same age, the basal area growth was positively correlated with the site quality; that is, the larger the site index was, the larger the basal area was. The basal area grew faster in the young and middle age, then grew slowly after entering the mature stage, and gradually stopped growing until the age reached 60 years, where the basal area tended to reach its maximum.

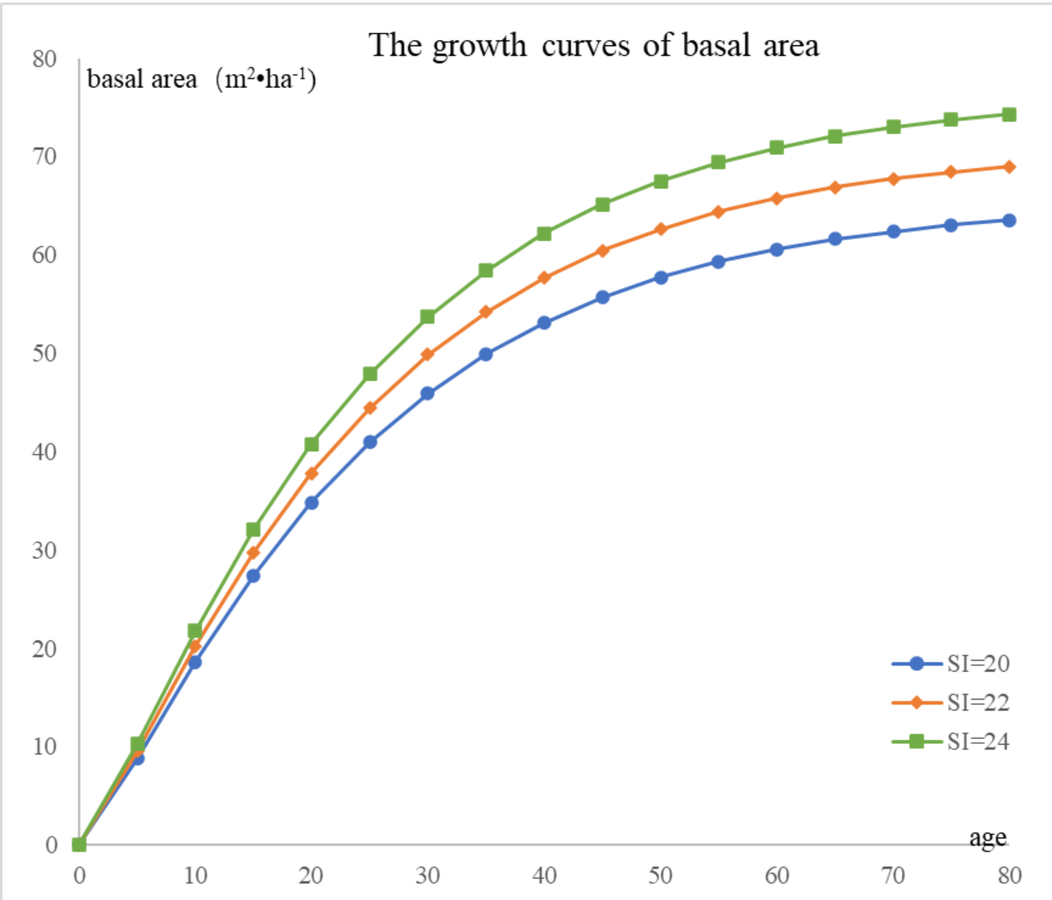

Figure 5. This is a figure. Schemes follow the same formatting.

3.3. The Optimal Stand Density and Target Tree Density

The optimal stand density varies according to the management, site conditions, stand age, tree species, and so on [28]. In the process of full-cycle forest management, there is an optimal density corresponding to each stage, but it is not always the best density for the entire growth process [25]. CTNM differs from other thinning types in that all trees in the stand are divided into target trees, interference trees, and other trees. In the stand, trees with well-developed crowns and straight trunks were selected as the target tree, and trees that affected the growth and development of the target tree were selected as the interference tree and needed to be cut in time. The selection of the target tree begins after the stand enters stage III because the differences in growth, development, and quality of the trees can be observed. From then on, all silvicultural activities and measures revolve around the protection and promotion of these target trees and their crowns and trunks to grow unimpeded.

This paper focuses on the maximum total harvest to determine the optimal density of different stages under three types of site conditions through the establishment of a dynamic programming model. The determination of the target tree density needs to fully consider the differences in local site conditions and tree species. Combined with the theory of CTNM, the trees felled in each stage are regarded as interference trees [29]. Then the ratio of interference trees to target trees is determined according to different site conditions, and the optimal target tree density at each stage under different site conditions can be obtained.

Using the initial density of 2000 trees ha^{-1} as an example, the optimal stand density and target tree density at the different stages under three types of site conditions were obtained by solving the dynamic programming model in Lingo 11.0, which is shown in Tables 4–6.

In the stand with better site conditions, the trees grew well and needed larger nutrition space, so there were fewer target trees to be retained. One to two interference trees were cut around each target tree with high thinning intensities. The ratio of target tree to interference tree was set to 1:2. According to the proportion, the optimal target tree density was 364 trees ha^{-1} in stage III, 153 trees ha^{-1} in stage IV, and 100 trees ha^{-1} in stage V, as shown in Table 4.

Table 4. The optimal stand density, target tree density, and harvest of stands with a site index of 24 at each stage.

Stages	Age	Height (m)	Optimal Stand Density (Trees ha^{-1})	Number of Interference Trees (Trees ha^{-1})	Target Trees (Trees ha^{-1})	Basal Area (m^2 ha^{-1})	Basal Area Cut (m^2 ha^{-1})	Stand Volume (m^3 ha^{-1})	Harvest (m^3 ha^{-1})
Stage I	5	4.15	2000	204	—	9.73	0.99	16.15	1.72
Stage II	20	13.76	1796	359	—	37.66	7.53	218.43	43.67
Stage III	40	21.00	1437	728	364	56.32	28.53	495.14	250.84
Stage IV	50	22.71	709	305	153	56.77	24.42	507.77	218.43
Stage V	60	23.71	404	199	100	57.26	28.20	508.08	250.27
								Total	764.93

Table 5. The optimal stand density, target tree density, and harvest of stands with a site index of 22 at each stage.

Stages	Age	Height (m)	Optimal Stand Density (Trees ha^{-1})	Number of Interference Trees (Trees ha^{-1})	Target Trees (Trees ha^{-1})	Basal Area (m^2 ha^{-1})	Basal Area Cut (m^2 ha^{-1})	Stand Volume (m^3 ha^{-1})	Harvest (m^3 ha^{-1})
Stage I	5	2.93	2000	135	—	9.04	0.61	10.65	0.72
Stage II	20	12.24	1865	373	—	35.29	7.06	181.26	36.26
Stage III	40	19.46	1492	702	702	52.60	24.75	428.83	201.77
Stage IV	50	21.00	790	327	327	53.52	22.15	445.54	184.42
Stage V	60	21.83	463	136	136	54.16	15.91	446.15	131.05
								Total	554.21

In the stand with the general site conditions, trees grew moderately in diameter, so the number of target trees to be retained per hectare was moderate, and one disturbance tree was cut around each target tree with moderate intensity. The ratio of target tree to interference tree was set to 1:1. According to the proportion, the optimal target tree density was 702 trees ha^{-1} in stage III, 327 trees ha^{-1} in stage IV, and 136 trees ha^{-1} in stage V, as shown in Table 5.

In the stand with poor site conditions, the trees grew slowly, and the average DBH of the stand was relatively small, so there were many target trees per hectare to be retained. One interference tree was cut for each target tree with high thinning intensity. The ratio of target tree to interference tree was set to 2:1. According to the proportion, the optimal target tree density was 1026 trees ha^{-1} in stage III, 702 trees ha^{-1} in stage IV, and 248 trees ha^{-1} in stage V, as shown in Table 6.

Table 6. The optimal stand density, target tree density, and harvest of stands with a site index of 20 at each stage.

Stages	Age	Height (m)	Optimal Stand Density (Trees ha^{-1})	Number of Interference Trees (Trees ha^{-1})	Target Trees (Trees ha^{-1})	Basal Area (m^2 ha^{-1})	Basal Area Cut (m^2 ha^{-1})	Stand Volume (m^3 ha^{-1})	Harvest (m^3 ha^{-1})
Stage I	5	2.40	2000	127	—	8.34	0.53	7.90	0.50
Stage II	20	10.56	1873	294	—	32.59	5.12	142.78	22.41
Stage III	40	17.47	1581	513	1026	48.90	19.39	357.46	141.72
Stage IV	50	18.99	1068	351	702	51.29	20.97	385.09	159.61
Stage V	60	19.82	717	124	248	52.63	11.36	386.98	86.00
								Total	410.23

3.4. Design Objective Forest Shape

The objective forest shape refers to the forest structure, which can continuously provide the maximum harvest and meet the maximum extent of the regulation purpose and describes the stand state when achieving the management goal qualitatively and quantitatively [30]. The formulation of the objective forest shape is an important link in CTNM that depends on the existing site conditions, management history, and vegetation types, and can reflect the future development goals of the forest [31].

The design of the objective forest shape is a long-term and meticulous task based on a complete understanding of the various factors of the stand [32]. Based on the current site conditions of the forest, this paper analyzes the current actual situation of the forest and predicts the ideal stand structure to provide economic, ecological, social, and other functional benefits.

First, it is necessary to analyze the current characteristics of the stand (Table 7), such as the basic stand factors and hierarchical structure, and then design objective forest shapes according to the site conditions, tree species characteristics, and forest vegetation types to determine the objectives and direction of forest development and succession.

Table 7. Stand status under three site conditions.

Site Index	Basic Factors		Hierarchical Structure		Site Conditions		
	Stand Density (Trees ha^{-1})	Average DBH (cm)	Main Forest Layer Tree Species	Secondary Forest Layer Tree Species	Altitude/m	Slope/°	Soil
24	1000–1400	14–17	*Phoebe bournei*	*Schima superba*, *Cinnamomum camphora*, *Altingia chinensis*	120–130	5–10	Yellow-red soil
22	1500–1650	11–14	*Phoebe bournei*	*Taxus wallichiana* *Koelreuteria paniculate*	150–185	12–18	Yellow-red soil
20	1700–1850	8–12	*Phoebe bournei*	*Cunninghamia lanceolata*, *Litsea pungens*, *Sassafras tzumu*	220–260	24–27	Mountain yellow soil

The characteristics of the objective forest shape are described by the indices of tree species composition, forest structure, stand density, target diameter, unit stand volume, and so on [30]. The stand state after entering stage V can be regarded as the stand terminal state, because the growth rate of the stand decreases and the growth tends to stop at 60 years. The number of target trees, stand volume, and basal area in stage V are obtained according to the established dynamic programming model (Table 8), which can provide a reference for the formulation of the objective forest shape.

Table 8. The stand state after entering stage V according to the established dynamic programming model.

Site Index	Stand Density (Trees ha^{-1})	Target Tree Density (Trees ha^{-1})	Basal Area (m^2 ha^{-1})	The Average DBH (cm)	Stand Volume (m^3 ha^{-1})
24	404	100	57.26	42.49	508.08
22	463	136	54.16	38.6	446.15
20	717	248	52.63	30.58	386.98

The general goal of the formulation of the objective forest shape is to develop even-aged forest into uneven-aged forest, single-layer forest into multilayer forest, and pure forest into mixed forest. The ultimate goal of CTNM is to explore stable, diverse, and high-value forests. In short, an unreasonable stand structure should be adjusted or transformed into the most stable and reasonable structure with the maximized harvest. The objective forest shape of the *Phoebe bournei* plantation was designed under three types of site conditions.

1. The objective forest shape of the stand with a site index of 24 was designed as follows:
   Species composition: The dominant tree species in the main forest layer is *Phoebe bournei*, accounting for 50–60%; *Schima superba*, *Cinnamomum camphora*, and *Altingia chinensis*, accounting for 30–40%; and other natural regeneration tree species, accounting for 10%. Interlayer trees and seedlings such as *Phoebe bournei* and *Schima superba* are evenly distributed under the forest;
   Hierarchical structure: *Phoebe bournei-Schima superba* multilayer forest;
   Target tree density: 100–130 trees ha^{-1};
   Stand volume per hectare: 450–500 m^3;
   60-year target tree diameter: 40–45 cm.
2. The objective forest shape of the stand with a site index of 22 was designed as follows:
   Species composition: The dominant tree species in the main forest layer is *Phoebe bournei*, accounting for 60–70%; *Taxus wallichiana* and *Koelreuteria paniculate*, accounting for 20–30%; and other natural regeneration tree species, accounting for 10%. Interlayer trees and seedlings such as *Phoebe bournei* and *Taxus wallichiana* are evenly distributed under the forest;
   Hierarchical structure: *Phoebe bournei-Taxus wallichiana* multilayer forest;
   Target tree density: 130–160 trees ha^{-1};
   Stand volume per hectare: 400–450 m^3;
   60-year target tree diameter: 35–40 cm.
3. The objective forest shape of the stand with a site index of 20 was designed as follows:
   Species composition: the dominant tree species in the main forest layer is *Phoebe bournei*, accounting for 70–80%; *Litsea pungens*, *Cunninghamia lanceolata* and *Sassafras tzumu*, accounting for 10–20%; and other natural regeneration tree species, accounting for 10%. Interlayer trees and seedlings such as *Phoebe bournei* and *Litsea pungens* are evenly distributed under the forest;
   Hierarchical structure: *Phoebe bournei-Litsea pungens* multilayer forest;
   Target tree density: 200–250 trees ha^{-1};
   Stand volume per hectare: 350–400 m^3;
   60-year target tree diameter: 30–35 cm.

4. Discussion

In this paper, a dynamic programming model was established to determine the optimal stand density and target tree density of a *Phoebe bournei* plantation at each stage under three different site conditions. It can be seen from the growth curves in Figures 4 and 5 that in stands with a site index of 24, the trees grew faster and well, and required larger nutrition space because of the better site conditions. The thinning intensity at each stage was high. There were fewer trees to be retained per hectare. The opposite was true in stands with a site index of 20. It can be seen from Tables 4–6 that the amount of harvesting was relatively small in stage I and stage II because in these two stages, the canopy of the trees had just closed, and the stand had initially formed a tree population structure, with sufficient space for tree growth and development. Only trees with poor stem shape and slow growth needed to be cut down. In stage III, the thinning intensity was relatively high, because the trees grew rapidly and the competition in the forest was fierce. It was necessary to solve the problem of excessive competition caused by excessive density. At this stage, trees in the main forest layer were differentiated, and the dominant trees and pressed trees could be distinguished from the vitality. From this stage, forest management can carry out interference tree cutting according to TMS. That is, the dominant trees with good growth and straight trunks were selected as target trees and the interference trees that affected the growth of target trees were removed. The thinning intensity decreased in stage IV and stage V because the stand structure was basically stable and the stand density decreased obviously due to the early thinning. In these two stages, we will continue to cultivate the target trees with large diameter, cut down interference trees, and harvest the target trees that reach the target diameter. A forest gap will form under each tree that has been cut down, which will give the understory vegetation a growth opportunity. Some native broad-leaved tree species should be replanted appropriately, which can lead to the renewal and succession of the forest and gradually transform the pure forest of *Phoebe bournei* into multilayer, mixed, and uneven-aged forests.

Stand density manipulation has the potential to have a major impact on individual tree size and stand yield [33], which is a key silvicultural decision to control stand growth in CTNM. Stand density is constantly changing at all stages of the forest and requires continuous adjustment in the process of stand growth based on the needs and objectives of forest management. When there are differences in site conditions, tree species, and stand age, the optimal density is also different. It is of great significance for forest managers to determine the optimal stand density at different ages under distinct site conditions according to specific objectives. Numerous studies have analyzed measures for stand density and target tree density quantification, among which thinning experiments or density experiments are the most typical. However, if there are enough samples for thinning experiments, it will take a long time, more manpower, and material resources. If there are not enough samples, the experimental results will inevitably be limited by conditions and affect its representativeness [34]. Li [35] determined the reasonable target tree density of *Phoebe bournei* from the perspective of the nutritional area and crown width of the target tree. However, in the actual conditions, when the crown width is simulated by a circle, the estimation error may be large. Wang [29] used the stand density control chart to determine the target tree density of *Phoebe bournei* plantations at each developmental stage, but it was difficult to consider both cultivation objectives and site conditions.

Dynamic programming is a process that uses states (e.g., potential treatments applied at each stand age) and stages (e.g., stand ages), where one must quantify how a stand changes from each state within a stage to other states at subsequent stages [36]. It contains time-varying factors and variables and divides the whole process into several interrelated stages; then, decisions are made at each stage. Decision-making in one stage not only affects the effect of the stage itself but also often affects the initial state of the next stage, thus affecting the subsequent process. Dynamic programming is a powerful approach to stand-level optimization problems because growth models are dynamic in nature [36]. The growth of trees, whether in individuals or in groups, is a dynamic process that changes over time (age). Age is divided into stages based on a determined interval. In each stage, the stand has some characteristic factors that reflect its growth status, such as DBH, tree height, basal area, number of trees, and volume, which vary at different site conditions and can be regarded as state variables of each stage. For each state, what kind of management measures (decisions) should be used, such as thinning intensity, to achieve the optimal goal forms a multistage decision-making optimization problem.

Once the dynamic programming network is "appropriately" constructed for the target problem, Bellman's principle of optimality works to seek an optimal solution [37]. After the introduction of dynamic programming by Bellman, the first application of dynamic programming in forestry was conducted by Arimizu for an optimal thinning regime in the USA. Later, other similar studies, including the works by Kao and Brodie [38,39], Riitter [40], Haight [41]. Chen [42] and Zhang [43], derived a discrete time continuous-state dynamic programming model to solve the optimal stand density, but they needed a sufficient number of corresponding basal area growth data under distinct ages, sites, and densities.

In this paper, the dynamic programming model related to stand growth. Stand age, site index, and number of trees were selected as independent variables, and Richard's growth function was used to fit the basal area growth equation. The accuracy of the growth function was reliable, and the stand growth process and the effects of stand density and site conditions on stand growth were dynamically simulated when there was a lack of growth data. The dynamic programming model constructed in this paper has strong applicability and flexibility. By adjusting the parameters of the growth model and considering the global optimum, the optimal stand density at each stage under different site conditions can be obtained. Combined with the theory of CTNM, the trees to be felled in each stage were regarded as interference trees. The ratio of interference trees to target trees was determined according to separate site conditions, and the optimal target tree density at each stage under different site conditions could be obtained.

However, this paper did not consider that thinning may promote stand growth when establishing the basal area growth model, so the growth model needs to be further improved. At the same time, this paper took the maximum total harvest in the whole growth process as the objective function to find the optimal density at each stage, and only considered the productivity function of the forest. Follow-up research can try to determine the optimal stand density to maximize the multiple functions of forests.

5. Conclusions

In this paper, a dynamic programming model was established based on the theory of CTNM. It was feasible in theory and practice to use dynamic programming to solve the density optimization problem. The optimal stand density and target tree density at each stage of *Phoebe bournei* plantations under three types of site conditions were determined. Using the initial stand density of 2000 trees ha^{-1} in stage I as an example, the results show that the optimal stand density under the three types of site condition in stage II was 1796 trees ha^{-1}, 1865 trees ha^{-1}, and 1873 trees ha^{-1}. After stage III, target trees were selected and interference trees were cut down according to CTNM and TMS. According to the ratio of target trees to interference trees under different site conditions, the optimal target tree density in stage III was 364 trees ha^{-1}, 702 trees ha^{-1}, and 1026 trees ha^{-1}; in stage IV was 153 trees ha^{-1}, 327 trees ha^{-1}, and 702 trees ha^{-1}; and in stage V was 100 trees ha^{-1}, 136 trees ha^{-1}, and 248 trees ha^{-1}.

Based on the results obtained, the objective forest shape was designed for the stand under three types of site conditions, which provided a theoretical basis for the CTNM of *Phoebe bournei* plantations and the cultivation of large-diameter *Phoebe bournei* wood, including technical support for improving the quality of *Phoebe bournei* plantations.

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