Supplementary Materials

All Tables S1–S4

Table S1. Outcomes in scenario *R and A*

Optimal value \ Power structure	*R*			*A*		
	ps	*vn*	*ss*	*ps*	*vn*	*ss*
$w_1^{x,y*}$	$a/4$	$a/3$	$a/2$	-	-	-
$p_1^{x,y*}$	$3a/4$	$2a/3$	$3a/4$	$a/2$	$a/2$	$a/2$
$q_1^{x,y*}$	$a/2$	$a/3$	$a/4$	$a/2$	$a/2$	$a/2$
$\pi_{s,1}^{x,y*}$	$a^2/16$	$a^2/9$	$a^2/8$	$(1-\alpha)a^2/4-F$	$(1-\alpha)a^2/4-F$	$(1-\alpha)a^2/4-F$
$\pi_{p,1}^{x,y*}$	$a^2/8-F$	$a^2/9-F$	$a^2/16-F$	$\alpha a^2/4$	$\alpha a^2/4$	$\alpha a^2/4$

Table S2. Outcomes in scenario *RR*

Optimal value \ Power structure	*RR*		
	ps	*vn*	*ss*
$w_j^{x,RR*}$	$\frac{a(1-d)^2(2+2d-5d^2-5d^3+2d^4+2d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$	$\frac{a(1-d)^2(1+d)}{3-2d-3d^2}$	$\frac{a(1-d)^2(1+d)(2+d-2d^2)}{4-d-10d^2+d^3+4d^4}$
$p_j^{x,RR*}$	$\frac{2a(1-d)^2(3+3d-7d^2-7d^3+3d^4+3d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$	$\frac{2a(1-d)^2(1+d)}{3-2d-3d^2}$	$\frac{2a(1-d)^2(3+3d-7d^2-7d^3+3d^4+3d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$
$q_j^{x,RR*}$	$\frac{a(1-d)^2(2+2d-5d^2-5d^3+2d^4+2d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$	$\frac{a(1-d)^2(1+d)}{3-2d-3d^2}$	$\frac{a(1-d)^2(2+2d-5d^2-5d^3+2d^4+2d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$
$\pi_{s,j}^{x,RR*}$	$\frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2}$	$\frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2}$	$\frac{a^2(1-d)^4(1+d)^2(2+d-2d^2)(2-5d^2+2d^4)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)^2}$
$\pi_{p,j}^{x,RR*}$	$\frac{a^2(1-d)^4(1+d)^2(4+2d-14d^2-5d^3+14d^4+2d^5-4d^6)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)^2}-F$	$\frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2}-F$	$\frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2}-F$

Table S3. Outcomes in scenario *RA*

Optimal value \ Power structure	*RA*		
	ps	*vn*	*ss*
$w_j^{x,RA*}$	$\frac{a(1-d)^2(1+d)}{2(2-d-2d^2)}$	$\frac{a(1-d)^2(1+d)(2+d-2d^2)}{6-14d^2+6d^4}$	$\frac{a(1-d)(4+2d-9d^2-2d^3+4d^4)}{8-21d^2+8d^4}$

$p_1^{x,RA*}$	$\frac{a(1-d)^2(3+3d-7d^2-7d^3+3d^4+3d^5)}{(2-d-2d^2)(2-5d^2+2d^4)}$	$\frac{2a(1-d)^2(1+d)(2+d-2d^2)}{6-14d^2+6d^4}$	$\frac{3a(1-d)(4+2d-9d^2-2d^3+4d^4)}{2(8-21d^2+8d^4)}$
$p_2^{x,RA*}$	$\frac{a(1-d)^2(1+d)(4+d-10d^2-d^3+4d^4)}{2(2-d-2d^2)(2-5d^2+2d^4)}$	$\frac{a(1-d)^2(3+5d-d^2-3d^3)}{6-14d^2+6d^4}$	$\frac{a(1-d)^2(1+d)(4+3d-4d^2)}{8-21d^2+8d^4}$
$q_1^{x,RA*}$	$\frac{a(1-d)^2(1+d)}{2(2-d-2d^2)}$	$\frac{a(1-d)^2(2+3d-d^2-2d^3)}{6-14d^2+6d^4}$	$\frac{a(4-2d-11d^2+7d^3+6d^4-4d^5)}{2(8-21d^2+8d^4)}$
$q_2^{x,RA*}$	$\frac{a(1-d)^2(4+5d-9d^2-11d^3+3d^4+4d^5)}{2(2-d-2d^2)(2-5d^2+2d^4)}$	$\frac{a(1-d)^2(3+5d-d^2-3d^3)}{6-14d^2+6d^4}$	$\frac{a(8+6d-28d^2-15d^3+28d^4+6d^5-8d^6)}{2(1+d)(8-21d^2+8d^4)}$
$\pi_{s,1}^{x,RA*}$	$\frac{a^2(1-d)^4(1+d)^2}{4(2-d-2d^2)^2}$	$\frac{a^2(1-d)^4(1+d)^2(2+d-2d^2)^2}{4(3-7d^2+3d^4)^2}$	$\frac{a^2(1-d)^2(4+2d-9d^2-2d^3+4d^4)^2}{2(8-21d^2+8d^4)^2}$
$\pi_{s,2}^{x,RA*}$	$\frac{a^2(1-d)^4(4+5d-9d^2-11d^3+3d^4+4d^5)^2(1-\alpha)}{4(2-d-2d^2)^2(2-5d^2+2d^4)^2}-F$	$\frac{a^2(1-d)^4(3+5d-d^2-3d^3)^2(1-\alpha)}{4(3-7d^2+3d^4)^2}-F$	$\frac{a^2(4-d-7d^2+4d^3)^2(2-5d^2+2d^4)(1-\alpha)}{2(8-21d^2+8d^4)^2}-F$
$\pi_{p,1}^{x,RA*}$	$\frac{a^2(1-d)^4(1+d)^2(2+d-2d^2)}{4(2-d-2d^2)(2-5d^2+2d^4)}-F$	$\frac{a^2(1-d)^4(2+3d-d^2-2d^3)^2}{4(3-7d^2+3d^4)^2}-F$	$\frac{a^2(4-2d-11d^2+7d^3+6d^4-4d^5)^2}{4(8-21d^2+8d^4)^2}-F$
$\pi_{p,2}^{x,RA*}$	$\frac{a^2(1-d)^4(1+d)^2(4+d-10d^2-d^3+4d^4)^2\alpha}{4(2-d-2d^2)^2(2-5d^2+2d^4)^2}$	$\frac{a^2(1-d)^4(3+5d-d^2-3d^3)^2\alpha}{4(3-7d^2+3d^4)^2}$	$\frac{a^2(1-d)^2(4+3d-4d^2)^2(2-5d^2+2d^4)\alpha}{2(8-21d^2+8d^4)^2}$

Table S4. Outcomes in scenario *AA*

Optimal value \ Power Structure	**AA (*ps* /*vn* /*ss*)**
$p_j^{x,AA*}$	$\frac{a(1-d)^2(1+d)}{2-d-2d^2}$
$q_j^{x,AA*}$	$\frac{a(1-d)^2(1+d)}{2-d-2d^2}$
$\pi_{s,j}^{x,AA*}$	$\frac{a^2(1-d)^4(1+d)^2(1-\alpha)}{(2-d-2d^2)^2}-F$
$\pi_{p,j}^{x,AA*}$	$\frac{a^2(1-d)^4(1+d)^2\alpha}{(2-d-2d^2)^2}$

All Listings and Proofs

Listing S1. Thresholds in Proposition 3

$$\bar{F}_{p2}^{ps}=\frac{a^2(1-d)^4(1+d)^2\left(\frac{4(8-38d^2+61d^4-38d^6+8d^8)}{(4-d-10d^2+d^3+4d^4)^2}-\frac{(4+d-10d^2-d^3+4d^4)^2\alpha}{(2-5d^2+2d^4)^2}\right)}{4(2-d-2d^2)^2},$$

$$\bar{F}_{p2}^{vn}=\frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2}-\frac{a^2(1-d)^4(3+5d-d^2-3d^3)^2\alpha}{4(3-7d^2+3d^4)^2},$$

$$\bar{F}_{p2}^{ss}=\frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2}-\frac{a^2(1-d)^2(4+3d-4d^2)^2(2-5d^2+2d^4)\alpha}{2(8-21d^2+8d^4)^2}.$$

$$\bar{F}_{p3}^{ps}=\frac{a^2(1-d)^4(1+d)^2(4-8\alpha-d^2(9-20\alpha)+d^4(4-8\alpha))}{4(2-d-2d^2)^2(2-5d^2+2d^4)},\quad \bar{F}_{p3}^{vn}=\frac{a^2(1-d)^4(2+3d-d^2-2d^3)^2}{4(3-7d^2+3d^4)^2}-\frac{a^2(1-d)^4(1+d)^2\alpha}{(2-d-2d^2)^2},$$

$$\bar{F}_{p3}^{ss} = \frac{a^2\left(4-2d-11d^2+7d^3+6d^4-4d^5\right)^2}{4\left(8-21d^2+8d^4\right)^2} - \frac{a^2(1-d)^4(1+d)^2\alpha}{\left(2-d-2d^2\right)^2} .$$

$$\overline{\alpha}_p^{ps} = \frac{\left(2+d-2d^2\right)\left(2-5d^2+2d^4\right)\left(16+6d-57d^2-16d^3+57d^4+6d^5-16d^6\right)}{\left(4-d-10d^2+d^3+4d^4\right)^2\left(8+d-20d^2-d^3+8d^4\right)} , \overline{\alpha}_p^{vn} = \frac{\left(2+d-2d^2\right)^2\left(12-d-28d^2+d^3+12d^4\right)}{\left(3-2d-3d^2\right)^2\left(12+d-28d^2-d^3+12d^4\right)} ,$$

$$\overline{\alpha}_p^{ss} = \frac{\begin{pmatrix}512-320d-5632d^2+3152d^3+26352d^4-12880d^5-68512d^6+28355d^7+108620d^8-36618d^9\\ -108620d^{10}+28355d^{11}+68512d^{12}-12880d^{13}-26352d^{14}+3152d^{15}+5632d^{16}-320d^{17}-512d^{18}\end{pmatrix}}{\left(2\left(4-d-10d^2+d^3+4d^4\right)^2\left(64+8d-376d^2-20d^3+820d^4+15d^5-820d^6-20d^7+376d^8+8d^9-64d^{10}\right)\right)} .$$

Listing S2. Thresholds in Lemma 1

$$\bar{F}_{s2}^{ps} = \frac{a^2(1-d)^4\left(\frac{\left(4+5d-9d^2-11d^3+3d^4+4d^5\right)^2(1-\alpha)}{\left(2-5d^2+2d^4\right)^2} - \frac{4\left(2+2d-5d^2-5d^3+2d^4+2d^5\right)^2}{\left(4-d-10d^2+d^3+4d^4\right)^2}\right)}{4\left(2-d-2d^2\right)^2} ,$$

$$\bar{F}_{s2}^{vn} = \frac{1}{4}a^2(1-d)^4\left(\frac{\left(3+5d-d-3d^3\right)^2(1-\alpha)}{\left(3-7d^2+3d^4\right)^2} - \frac{4(1+d)^2}{\left(3-2d-3d^2\right)^2}\right) ,$$

$$\bar{F}_{p2}^{ss} = \frac{a^2(1-d)^4\left(2+2d-5d^2-5d^3+2d^4+2d^5\right)^2}{\left(2-d-2d^2\right)^2\left(4-d-10d^2+d^3+4d^4\right)^2} - \frac{a^2(1-d)^2\left(4+3d-4d^2\right)^2\left(2-5d^2+2d^4\right)\alpha}{2\left(8-21d^2+8d^4\right)^2} .$$

$$\bar{F}_{s3}^{ps} = \frac{a^2(1-d)^4(1+d)^2(3-4\alpha)}{4\left(2-d-2d^2\right)^2} , \qquad \bar{F}_{s3}^{vn} = \frac{a^2(1-d)^4(1+d)^2(1-\alpha)}{\left(2-d-2d^2\right)^2} - \frac{a^2(1-d)^4(1+d)^2\left(2+d-2d^2\right)^2}{4\left(3-7d^2+3d^4\right)^2} ,$$

$$\bar{F}_{s3}^{ss} = \frac{a^2(1-d)^4(1+d)^2(1-\alpha)}{\left(2-d-2d^2\right)^2} - \frac{a^2(1-d)^2\left(4+2d-9d^2-2d^3+4d^4\right)^2}{2\left(8-21d^2+8d^4\right)^2} .$$

$$\overline{\alpha}_s^{ps} = \frac{96-44d-720d^2+265d^3+2088d^4-586d^5-2940d^6+586d^7+2088d^8-265d^9-720d^{10}+44d^{11}+96d^{12}}{\left(4-d-10d^2+d^3+4d^4\right)^2\left(8+d-20d^2-d^3+8d^4\right)} ,$$

$$\overline{\alpha}_s^{vn} = \frac{60-83d-240d^2+286d^3+348d^4-286d^5-240d^6+83d^7+60d^8}{\left(3-2d-3d^2\right)^2\left(12+d-28d^2-d^3+12d^4\right)} ,$$

$$\overline{\alpha}_s^{ss} = \frac{2\begin{pmatrix}256-160d-2752d^2+1596d^3+12588d^4-6628d^5-32114d^6+14807d^7+50354d^8-19246d^9-50354d\\ +14807d^{11}+32114d^{12}-6628d^{13}-12588d^{14}+1596d^{15}+2752d^{16}-160d^{17}-256d^{18}\end{pmatrix}}{\left(4-d-10d^2+d^3+4d^4\right)^2\left(64+8d-376d^2-20d^3+820d^4+15d^5-820d^6-20d^7+376d^8+8d^9-64d^{10}\right)} .$$

Listing S3. Thresholds in Proposition 8

$$\bar{F}_{AA}^{x} = \frac{a^2\left(4-12d-d^2+28d^3-12d^4-16d^5+8d^6\right)}{4\left(2-d-2d^2\right)^2} .$$

$$\bar{F}_{RA}^{ps} = \frac{a^2\left(12-32d-73d^2+254d^3+133d^4-792d^5+11d^6+1232d^7-300d^8-1004d^9+353d^{10}+406d^{11}-165d^{12}-64d^{13}+28d^{14}\right)}{4\left(-4+2d+14d^2-5d^3-14d^4+2d^5+4d^6\right)^2} ,$$

$$\bar{F}_{RA}^{vn} = \frac{a^2\left(8-14d-43d^2+84d^3+81d^4-180d^5-50d^6+164d^7-14d^8-54d^9+17d^{10}\right)}{4\left(3-7d^2+3d^4\right)^2} ,$$

$$\bar{F}_{RA}^{ss} = \frac{a^2\left(48-80d-296d^2+564d^3+632d^4-1346d^5-431d^6+1260d^7-120d^8-368d^9+112d^{10}\right)}{4\left(8-21d^2+8d^4\right)^2} .$$

$$\bar{F}_{RR}^{ps} = \frac{a^2\left(32-96d-164d^2+764d^3+127d^4-2376d^5+662d^6+3688d^7-1649d^8-3004d^9+1516d^{10}+1216d^{11}-624d^{12}-192d^{13}+96d^{14}\right)}{4\left(2-d-2d^2\right)^2\left(4-d-10d^2+d^3+4d^4\right)^2} ,$$

$$\bar{F}_{RR}^{vn} = \frac{a^2\left(7-20d-2d^2+52d^3-25d^4-32d^5+16d^6\right)}{4\left(3-2d-3d^2\right)^2} ,$$

$$\bar{F}_{RR}^{ss}=\frac{a^2\left(32-96d-164d^2+764d^3+127d^4-2376d^5+662d^6+3688d^7-1649d^8-3004d^9+1516d^{10}+1216d^{11}-624d^{12}-192d^{13}+96d^{14}\right)}{4\left(2-d-2d^2\right)^2\left(4-d-10d^2+d^3+4d^4\right)^2}.$$

Proof of Table S1.

Scenario R:

Platform-led structure.

We solve the optimal profit functions by backward induction. In the second stage, the supplier decides her wholesale price on the basis of platform's marginal profit $m_1^{ps,R}$. For a given marginal profit, we have $\partial^2\pi_{s,1}^{ps,R}/(\partial w_1^{ps,R})^2<0$, leading to supplier's profit funcation is concave in her wholesale price. Let $\partial\pi_{s,1}^{ps,R}/\partial w_1^{ps,R}=0$, we obtain supplier's wholesale price responses to the marginal profit. In the first stage, the platform determines his marginal profit, taking into account the supplier's wholesale price. We have $\partial^2\pi_{p,1}^{ps,R}/(\partial m_1^{ps,R})^2<0$, hence platform's profit is concave in his marginal profit. Let $\partial\pi_{p,1}^{ps,R}/\partial m_1^{ps,R}=0$, we obtain that the platform's optimal marginal profit is $m_1^{ps,R*}=a/2$. By substituting $m_1^{ps,R*}$ into $w_1^{ps,R}$, we derive that the supplier's optimal wholesale price is $w_1^{ps,R*}=a/4$. Finally, we take $m_1^{ps,R*}$ and $w_1^{ps,R*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,1}^{ps,R*}=a^2/16$ and $\pi_{p,1}^{ps,R*}=a^2/8-F$.

Vertical-Nash structure.

We solve the optimal profit functions by backward induction. We have $\partial^2\pi_{s,1}^{vn,R}/(\partial w_1^{vn,R})^2<0$ and $\partial^2\pi_{p,1}^{vn,R}/(\partial m_1^{vn,R})^2<0$, leading to both supplier's and platform's profits are concove in the wholesale price and marginal profit, respectively. By solving the combination of first-order condition functions $\partial\pi_{s,1}^{vn,R}/\partial w_1^{vn,R}=0$ and $\partial\pi_{p,1}^{vn,R}/\partial m_1^{vn,R}=0$, we derive the optimal wholesale price $w_1^{vn,R*}=a/3$ and optimal marginal profit $m_1^{vn,R*}=a/3$. Finally, we take $w_1^{vn,R*}$ and $m_1^{vn,R*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,1}^{vn,R*}=a^2/9$ and $\pi_{p,1}^{vn,R*}=a^2/9-F$.

Supplier-led structure.

We solve the optimal profit functions by backward induction. In the second stage, the platform decides his sales price $p_1^{ss,R}$ on the basis of platform's wholesale price $w_1^{ss,R}$. For a given wholesale price, we have $\partial^2\pi_{s,1}^{ss,R}/(\partial p_{s,1}^{ss,R})^2<0$, leading to supplier's profit function is concave in his sales price. Let $\partial\pi_{s,1}^{ss,R}/\partial p_1^{ss,R}=0$, we obtain platform's sales price responses to the wholesale price. In the first stage, the supplier determines her wholesale price, taking into account the platform's sales price. We have $\partial^2\pi_{p,1}^{ss,R}/(\partial w_1^{ss,R})^2<0$, hencesupplier's profit is concave in her wholesale price. Let $\partial\pi_{p,1}^{ss,R}/\partial w_1^{ss,R}=0$, we obtain that the optimal wholesale price for the platform is $w_1^{s,R*}=a/2$. By substituting $w_1^{ss,R*}$ into $p_1^{ss,R}$, it is easy to derive the platform's optimal sales price is $p_1^{ss,R*} = 3a/4$. Finally, we take $w_1^{ss,R*}$ and $p_1^{ss,R*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,1}^{ss,R*} = a^2/8$ and $\pi_{p,1}^{ss,R*} = a^2/16 - F$, respectively.

Scenario A:

For any power structure.

We solve the optimal profit functions by backward induction. We have $\partial^2 \pi_{s,1}^{x,A} / (\partial p_1^{x,A})^2 < 0$, leading to both supplier's and platform's profits are concove in the sales price, respectively. By solving the combination of first-order condition functions $\partial \pi_{s,1}^{x,A} / \partial p_1^{x,A} = 0$, we derive the optimal sales price $p_1^{x,A*} = a/2$. Finally, we take $p_1^{x,A*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,1}^{x,A*} = (1-\alpha)a^2/4 - F$ and $\pi_{p,1}^{x,A*} = \alpha a^2/4$.

Proof of Proposition 1.

For each power structure, compare $\pi_{p,1}^{x,R*}$ and $\pi_{p,1}^{x,A*}$: we can hold that $\pi_{p,1}^{x,R*} \geq \pi_{p,1}^{x,A*}$ when $F \in (0, \overline{F}_{p1}^x]$, $\pi_{p,1}^{x,R*} < \pi_{p,1}^{x,A*}$ otherwise, where $x \in \{ps, vn, ss\}$, $\overline{F}_{p1}^{ps} = (1-2\alpha)a^2/8$, $\overline{F}_{s1}^{ps} = (3-4\alpha)a^2/16$, $\overline{F}_{p1}^{vn} = (4-9\alpha)a^2/36$.

For each power structure, compare $\pi_{s,1}^{x,R*}$ and $\pi_{s,1}^{x,A*}$: we can hold that $\pi_{s,1}^{x,R*} \leq \pi_{s,1}^{x,A*}$ when $F \in (0, \overline{F}_{s1}^x]$, $\pi_{s,1}^{x,R*} < \pi_{s,1}^{x,A*}$ otherwise, where $x \in \{ps, vn, ss\}$, $\overline{F}_{s1}^{vn} = (5-9\alpha)a^2/36$, $\overline{F}_{p1}^{ss} = (1-4\alpha)a^2/16$ *and* $\overline{F}_{s1}^{ss} = (1-2\alpha)a^2/8$.

For each power structure, compare $\overline{F}_{p1}^x$ and $\overline{F}_{s1}^x$: we can hold that $\overline{F}_{s1}^x > \overline{F}_{p1}^x$.

Proof of Proposition 2.

For each power structure, compare $\pi_{p,1}^{x,R*} + \pi_{s,1}^{x,R*}$ and $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*}$: we can hold that $\pi_{p,1}^{x,R*} + \pi_{s,1}^{x,R*} < \pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*}$.

Proof of Table S2.

Platform-led structure.

We solve the optimal profit functions by backward induction. In the second stage, the supplier decides her wholesale price on the basis of platform *j*'s marginal profit $m_j^{ps,RR}$. For a given marginal profit, we have $\partial^2 \pi_{s,j}^{ps,RR} / (\partial w_j^{ps,R})^2 < 0$, leading to supplier *j*'s profit funcation is concave in her wholesale price. Let $\partial \pi_{s,j}^{ps,RR} / \partial w_j^{ps,RR} = 0$, we obtain supplier *j*'s wholesale price responses to the marginal profit. In the first stage, each platform determines his marginal profit, taking into account the supplier's wholesale price. We have $\partial^2 \pi_{p,j}^{ps,RR} / (\partial m_j^{ps,RR})^2 < 0$, hence platform *j*'s profit is concave in his marginal profit. Let $\partial \pi_{p,j}^{ps,RR} / \partial m_j^{ps,RR} = 0$, we obtain that the platform *j*'s optimal marginal profit is

$m_j^{ps,RR*}=\frac{a(1-d)^2(2+2d-5d^2-5d^3+2d^4+2d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$. By substituting $m_j^{ps,RR*}$ into $w_j^{ps,RR}$, we derive that the supplier *j*'s optimal wholesale price is $w_j^{ps,RR*}=\frac{a(1-d)^2(2+2d-5d^2-5d^3+2d^4+2d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$. Finally, we take $m_j^{ps,RR*}$ and $w_j^{ps,RR*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,j}^{ps,RR*}=\frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2}$ and $\pi_{p,j}^{ps,RR*}=\frac{a^2(1-d)^4(1+d)^2(4+2d-14d^2-5d^3+14d^4+2d^5-4d^6)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)^2}-F$.

Vertical-Nash structure.

We solve the optimal profit functions by backward induction. We have $\partial^2\pi_{s,j}^{vn,RR}/(\partial w_j^{vn,RR})^2<0$ and $\partial^2\pi_{p,j}^{vn,RR}/(\partial m_j^{vn,RR})^2<0$, leading to both supplier *j*'s and platform *j*'s profits are concove in the wholesale price and marginal profit, respectively. By solving the combination of first-order condition functions $\partial\pi_{s,j}^{vn,RR}/\partial w_j^{vn,RR}=0$ and $\partial\pi_{p,j}^{vn,RR}/\partial m_j^{vn,RR}=0$, we derive the optimal wholesale price $w_j^{vn,RR*}=\frac{a(1-d)^2(1+d)}{3-2d-3d^2}$ and optimal marginal profit $m_j^{vn,RR*}=\frac{a(1-d)^2(1+d)}{3-2d-3d^2}$. Finally, we take $w_j^{vn,RR*}$ and $m_j^{vn,RR*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,j}^{vn,RR*}=\frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2}$ and $\pi_{p,j}^{vn,RR*}=\frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2}-F$.

Supplier-led structure.

We solve the optimal profit functions by backward induction. In the second stage, Platform *j* decides her sales price $p_j^{ss,RR}$ on the basis of platforms' wholesale price $w_j^{ss,RR}$. For a given wholesale price, we have $\partial^2\pi_{s,j}^{ss,R}/(\partial p_{s,j}^{ss,R})^2<0$, leading to supplier *j*'s profit function is concave in his sales price. Let $\partial\pi_{s,j}^{ss,RR}/\partial p_j^{ss,RR}=0$, we obtain platforms' sales price responses to the wholesale price. In the first stage, each supplier determines her wholesale price, taking into account the platform *j*'s sales price. We have $\partial^2\pi_{p,j}^{ss,RR}/(\partial w_j^{ss,RR})^2<0$, hence supplier *j*'s profit is concave in her wholesale price. Let $\partial\pi_{p,j}^{ss,RR}/\partial w_j^{ss,RR}=0$, we obtain that the optimal wholesale price for the platform *j* is $w_j^{s,RR*}=\frac{a(1-d)^2(1+d)(2+d-2d^2)}{4-d-10d^2+d^3+4d^4}$. By substituting $w_j^{ss,RR*}$ into $p_j^{ss,RR}$, it is easy to derive the platform *j*'s optimal sales price is $p_j^{ss,RR*}=\frac{2a(1-d)^2(3+3d-7d^2-7d^3+3d^4+3d^5)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)}$. Finally, we take $w_1^{ss,RR*}$ and $p_1^{ss,RR*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,j}^{ss,RR*}=\frac{a^2(1-d)^4(1+d)^2(2+d-2d^2)(2-5d^2+2d^4)}{(2-d-2d^2)(4-d-10d^2+d^3+4d^4)^2}$

and $\pi_{p,j}^{ss,RR*} = \dfrac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2} - F$, respectively.

Proof of Table S3.

The proof is similar to that of Scenario RR with three different power structure which has shown in the proof of Table 4 above and hence is omitted.

Proof of Table S4.

For any power structure.

We solve the optimal profit functions by backward induction. We have $\partial^2 \pi_{s,j}^{x,AA} / (\partial p_j^{x,AA})^2 < 0$, leading to both supplier's and platform *j*'s profits are concove in the sales price, respectively. By solving the combination of first-order condition functions $\partial \pi_{s,j}^{x,AA} / \partial p_j^{x,AA} = 0$, we derive the optimal sales price $p_j^{x,AA*} = \dfrac{a(1-d)^2(1+d)}{2-d-2d^2}$. Finally, we take $p_j^{x,AA*}$ into each firm's profit function, and obtain the corresponding optimal outcomes $\pi_{s,j}^{x,AA*} = \dfrac{a^2(1-d)^4(1+d)^2(1-\alpha)}{(2-d-2d^2)^2} - F$ and $\pi_{p,j}^{x,AA*} = \dfrac{a^2(1-d)^4(1+d)^2\alpha}{(2-d-2d^2)^2}$.

Proof of proposition 3.

For each power structure:

RR will be an equilibrium strategy if and only if $\begin{cases} \pi_{p,1}^{x,RR*} \geq \pi_{p,1}^{x,AR*} \\ \pi_{p,2}^{x,RR*} \geq \pi_{p,2}^{x,RA*} \end{cases}$. Thus, we derive that both platforms will choose reselling mode when $F \in (0, \bar{F}_{p2}^{x}]$, where $x \in \{ps, vn, ss\}$,

$$\bar{F}_{p2}^{ps} = \frac{a^2(1-d)^4(1+d)^2\left(\dfrac{4(8-38d^2+61d^4-38d^6+8d^8)}{(4-d-10d^2+d^3+4d^4)^2} - \dfrac{(4+d-10d^2-d^3+4d^4)^2\alpha}{(2-5d^2+2d^4)^2}\right)}{4(2-d-2d^2)^2},\quad \bar{F}_{p2}^{vn} = \frac{a^2(1-d)^4(1+d)^2}{(3-2d-3d^2)^2} - \frac{a^2(1-d)^4(3+5d-d^2-3d^3)^2\alpha}{4(3-7d^2+3d^4)^2},$$

$$\bar{F}_{p2}^{ss} = \frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2} - \frac{a^2(1-d)^2(4+3d-4d^2)^2(2-5d^2+2d^4)\alpha}{2(8-21d^2+8d^4)^2}.$$

AA will be an equilibrium strategy if and only if $\begin{cases} \pi_{p,1}^{x,AA*} \geq \pi_{p,1}^{x,RA*} \\ \pi_{p,2}^{x,AA*} \geq \pi_{p,2}^{x,AR*} \end{cases}$. Thus, we derive that both platforms will choose agency selling mode when $F \in (\bar{F}_{p3}^{x}, +\infty)$, where $x \in \{ps, vn, ss\}$,

$$\bar{F}_{p3}^{ps} = \frac{a^2(1-d)^4(1+d)^2(4-8\alpha-d^2(9-20\alpha)+d^4(4-8\alpha))}{4(2-d-2d^2)^2(2-5d^2+2d^4)},\quad \bar{F}_{p3}^{vn} = \frac{a^2(1-d)^4(2+3d-d^2-2d^3)^2}{4(3-7d^2+3d^4)^2} - \frac{a^2(1-d)^4(1+d)^2\alpha}{(2-d-2d^2)^2},$$

$$\bar{F}_{p3}^{ss} = \frac{a^2(4-2d-11d^2+7d^3+6d^4-4d^5)^2}{4(8-21d^2+8d^4)^2} - \frac{a^2(1-d)^4(1+d)^2\alpha}{(2-d-2d^2)^2}.$$

RA will be an equilibrium strategy if and only if $\begin{cases} \pi_{p,1}^{x,RA*} \geq \pi_{p,1}^{x,AA*} \\ \pi_{p,2}^{x,RA*} \geq \pi_{p,2}^{x,RR*} \end{cases}$. Thus, we derive that both platforms will choose opposite mode when $F \in (\bar{F}_{p2}^{x}, \bar{F}_{p3}^{x}]$.

AR will be an equilibrium strategy if and only if $\begin{cases} \pi_{p,1}^{x,AR*} \geq \pi_{p,1}^{x,RR*} \\ \pi_{p,2}^{x,AR*} \geq \pi_{p,2}^{x,AA*} \end{cases}$. Thus, we derive that both platforms will choose opposite g mode when $F \in (\bar{F}_{p2}^{x}, \bar{F}_{p3}^{x}]$.

Comparing $\bar{F}_{p2}^{x}$ and $\bar{F}_{p3}^{x}$, we can derive:

When $\alpha \in (0, \bar{\alpha}_{p}^{x}]$, $\bar{F}_{p3}^{x} \leq \bar{F}_{p2}^{x}$, we can hold that RR when $F \in (0, \bar{F}_{p3}^{x}]$, AA/RR when $F \in (\bar{F}_{p3}^{x}, \bar{F}_{p2}^{x}]$, AA when $F \in (\bar{F}_{p2}^{x}, +\infty)$. We continue to compare the platforms' profit between RR and AA when $F \in (\bar{F}_{p3}^{x}, \bar{F}_{p2}^{x}]$. And we can derive that $\begin{cases} \pi_{p,1}^{x,RR*} > \pi_{p,1}^{x,AA*} \\ \pi_{p,2}^{x,RR*} > \pi_{p,2}^{x,AA*} \end{cases}$, thereby RR is the optimal result when $F \in (\bar{F}_{p3}^{x}, \bar{F}_{p2}^{x}]$.

When $\alpha \in (\bar{\alpha}_{p}^{x}, +\infty)$, $\bar{F}_{p3}^{x} > \bar{F}_{p2}^{x}$, we can hold that RR when $F \in (0, \bar{F}_{p2}^{x}]$, RA/AR when $F \in (\bar{F}_{p2}^{x}, \bar{F}_{p3}^{x}]$, AA when $F \in (\bar{F}_{p3}^{x}, +\infty)$.

Where

$$\bar{\alpha}_{p}^{ps} = \frac{(2+d-2d^2)(2-5d^2+2d^4)(16+6d-57d^2-16d^3+57d^4+6d^5-16d^6)}{(4-d-10d^2+d^3+4d^4)^2(8+d-20d^2-d^3+8d^4)}, \quad \bar{\alpha}_{p}^{vn} = \frac{(2+d-2d^2)^2(12-d-28d^2+d^3+12d^4)}{(3-2d-3d^2)^2(12+d-28d^2-d^3+12d^4)},$$

$$\bar{\alpha}_{p}^{ss} = \frac{\begin{pmatrix} 512-320d-5632d^2+3152d^3+26352d^4-12880d^5-68512d^6+28355d^7+108620d^8-36618d^9 \\ -108620d^{10}+28355d^{11}+68512d^{12}-12880d^{13}-26352d^{14}+3152d^{15}+5632d^{16}-320d^{17}-512d^{18} \end{pmatrix}}{\left(2(4-d-10d^2+d^3+4d^4)^2(64+8d-376d^2-20d^3+820d^4+15d^5-820d^6-20d^7+376d^8+8d^9-64d^{10})\right)}.$$

Therefore, we can obtain the proposition 3.

Proof of lemma 1.

For each power structure:

RR will be an equilibrium strategy if and only if $\begin{cases} \pi_{s,1}^{x,RR*} \geq \pi_{s,1}^{x,AR*} \\ \pi_{s,2}^{x,RR*} \geq \pi_{s,2}^{x,RA*} \end{cases}$. Thus, we derive that both suppliers will choose reselling mode when $F \in (\bar{F}_{s2}^{x}, +\infty)$, where $x \in \{ps, vn, ss\}$,

$$\bar{F}_{s2}^{ps} = \frac{a^2(1-d)^4\left(\frac{(4+5d-9d^2-11d^3+3d^4+4d^5)^2(1-\alpha)}{(2-5d^2+2d^4)^2} - \frac{4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(4-d-10d^2+d^3+4d^4)^2}\right)}{4(2-d-2d^2)^2},$$

$$\bar{F}_{s2}^{vn} = \frac{1}{4}a^2(1-d)^4\left(\frac{(3+5d-d-3d^3)^2(1-\alpha)}{(3-7d^2+3d^4)^2} - \frac{4(1+d)^2}{(3-2d-3d^2)^2}\right),$$

$$\bar{F}_{p2}^{ss} = \frac{a^2(1-d)^4(2+2d-5d^2-5d^3+2d^4+2d^5)^2}{(2-d-2d^2)^2(4-d-10d^2+d^3+4d^4)^2} - \frac{a^2(1-d)^2(4+3d-4d^2)^2(2-5d^2+2d^4)\alpha}{2(8-21d^2+8d^4)^2}.$$

AA will be an equilibrium strategy if and only if $\begin{cases} \pi_{s,1}^{x,AA*} \geq \pi_{s,1}^{x,RA*} \\ \pi_{s,2}^{x,AA*} \geq \pi_{s,2}^{x,AR*} \end{cases}$. Thus, we derive that both suppliers will choose agency selling mode when $F \in (0, \bar{F}_{s3}^{x}]$, where $x \in \{ps, vn, ss\}$,

$$\bar{F}_{s3}^{ps} = \frac{a^2(1-d)^4(1+d)^2(3-4\alpha)}{4(2-d-2d^2)^2}, \quad \bar{F}_{s3}^{vn} = \frac{a^2(1-d)^4(1+d)^2(1-\alpha)}{(2-d-2d^2)^2} - \frac{a^2(1-d)^4(1+d)^2(2+d-2d^2)^2}{4(3-7d^2+3d^4)^2},$$

$$\bar{F}_{s3}^{ss}=\frac{a^2(1-d)^4(1+d)^2(1-\alpha)}{\left(2-d-2d^2\right)^2}-\frac{a^2(1-d)^2\left(4+2d-9d^2-2d^3+4d^4\right)^2}{2\left(8-21d^2+8d^4\right)^2}.$$

RA will be an equilibrium strategy if and only if $\begin{cases}\pi_{s,1}^{x,RA*}\geq\pi_{s,1}^{x,AA*}\\ \pi_{s,2}^{x,RA*}\geq\pi_{s,2}^{x,RR*}\end{cases}$. Thus, we derive that both suppliers will choose opposite mode when $F\in(\bar{F}_{s3}^{x},\bar{F}_{s2}^{x}]$.

AR will be an equilibrium strategy if and only if $\begin{cases}\pi_{s,1}^{x,AR*}\geq\pi_{s,1}^{x,RR*}\\ \pi_{s,2}^{x,AR*}\geq\pi_{s,2}^{x,AA*}\end{cases}$. Thus, we derive that both suppliers will choose opposite g mode when $F\in(\bar{F}_{s3}^{x},\bar{F}_{s2}^{x}]$.

Comparing $\bar{F}_{s2}^{x}$ and $\bar{F}_{s3}^{x}$, we can derive:

When $\alpha\in(0,\bar{\alpha}_{s}^{x}]$, $\bar{F}_{s3}^{x}\leq\bar{F}_{s2}^{x}$, we can hold that AA when $F\in(0,\bar{F}_{s3}^{x}]$, RA/AR when $F\in(\bar{F}_{s3}^{x},\bar{F}_{s2}^{x}]$, RR when $F\in(\bar{F}_{s2}^{x},+\infty)$.

When $\alpha\in(\bar{\alpha}_{s}^{x},+\infty)$, $\bar{F}_{s3}^{x}>\bar{F}_{s2}^{x}$, we can hold that RR when $F\in(0,\bar{F}_{s2}^{x}]$, RR/AA when $F\in(\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}]$, AA when $F\in(\bar{F}_{s3}^{x},+\infty)$. We continue to compare the platforms' profit between RR and AA when $F\in(\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}]$. And we can derive that $\begin{cases}\pi_{s,1}^{x,RR*}>\pi_{s,1}^{x,AA*}\\ \pi_{s,2}^{x,RR*}>\pi_{s,2}^{x,AA*}\end{cases}$, thereby RR is the optimal result when $F\in(\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}]$.

Where $\bar{\alpha}_{s}^{ps}=\dfrac{96-44d-720d^2+265d^3+2088d^4-586d^5-2940d^6+586d^7+2088d^8-265d^9-720d^{10}+44d^{11}+96d^{12}}{\left(4-d-10d^2+d^3+4d^4\right)^2\left(8+d-20d^2-d^3+8d^4\right)}$,

$$\bar{\alpha}_{s}^{vn}=\frac{60-83d-240d^2+286d^3+348d^4-286d^5-240d^6+83d^7+60d^8}{\left(3-2d-3d^2\right)^2\left(12+d-28d^2-d^3+12d^4\right)},$$

$$\bar{\alpha}_{s}^{ss}=\frac{2\begin{pmatrix}256-160d-2752d^2+1596d^3+12588d^4-6628d^5-32114d^6+14807d^7+50354d^8-19246d^9-50354d\\ +14807d^{11}+32114d^{12}-6628d^{13}-12588d^{14}+1596d^{15}+2752d^{16}-160d^{17}-256d^{18}\end{pmatrix}}{\left(4-d-10d^2+d^3+4d^4\right)^2\left(64+8d-376d^2-20d^3+820d^4+15d^5-820d^6-20d^7+376d^8+8d^9-64d^{10}\right)}.$$

Therefore, we can obtain the lemma 1.

Proof of Proposition 4. According to lemma 1, Combining all players' selling mode sub-equilibrium, we can hold that:

RR will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,RR*}\geq\pi_{p,1}^{x,AR*}\\ \pi_{p,2}^{x,RR*}\geq\pi_{p,2}^{x,RA*}\end{cases}$ and $\begin{cases}\pi_{s,1}^{x,RR*}\geq\pi_{s,1}^{x,AR*}\\ \pi_{s,2}^{x,RR*}\geq\pi_{s,2}^{x,RA*}\end{cases}$. Thus, we derive that all players will choose opposite mode when $F\in(\bar{F}_{s2}^{x},\bar{F}_{p2}^{x}]$. Comparing $\bar{F}_{p2}^{x}$ and $\bar{F}_{s2}^{x}$: we can derive that $\bar{F}_{p2}^{x}>\bar{F}_{s2}^{x}$ if and only if $d\in(0.543,0.664)$ / $d\in(0.498,0.664)$ / $d\in(0.554,0.664)$ for ps /vn /ss structure.

AA will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,AA*}\geq\pi_{p,1}^{x,RA*}\\ \pi_{p,2}^{x,AA*}\geq\pi_{p,2}^{x,AR*}\end{cases}$ and $\begin{cases}\pi_{s,1}^{x,AA*}\geq\pi_{s,1}^{x,RA*}\\ \pi_{s,2}^{x,AA*}\geq\pi_{s,2}^{x,AR*}\end{cases}$. Thus, we derive that all players will choose agency selling mode when $F\in(\max\{\bar{F}_{p2}^{x},\bar{F}_{p3}^{x}\},\min\{\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}\}]$. Compare $\max\{\bar{F}_{p2}^{x},\bar{F}_{p3}^{x}\}$ and $\min\{\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}\}$, we can derive that $\max\{\bar{F}_{p2}^{x},\bar{F}_{p3}^{x}\}\leq\min\{\bar{F}_{s2}^{x},\bar{F}_{s3}^{x}\}$ if and only if $d\in(0,0.543]$ / $d\in(0,0.498]$ / $d\in(0,0.554]$ for ps /vn /ss structure.

RA/ AR will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,RA*} \ge \pi_{p,1}^{x,AA*} \\ \pi_{p,2}^{x,RA*} \ge \pi_{p,2}^{x,RR*}\end{cases}$ / $\begin{cases}\pi_{p,1}^{x,AR*} \ge \pi_{p,1}^{x,RR*} \\ \pi_{p,2}^{x,AR*} \ge \pi_{p,2}^{x,AA*}\end{cases}$ and $\begin{cases}\pi_{s,1}^{x,RA*} \ge \pi_{s,1}^{x,AA*} \\ \pi_{s,2}^{x,RA*} \ge \pi_{s,2}^{x,RR*}\end{cases}$ / $\begin{cases}\pi_{s,1}^{x,AR*} \ge \pi_{s,1}^{x,RR*} \\ \pi_{s,2}^{x,AR*} \ge \pi_{s,2}^{x,AA*}\end{cases}$. Thus, we derive that all players will choose opposite mode when $F \in (\max\{\overline{F}_{s3}^{ps}, \overline{F}_{p2}^{ps}\}, \min\{\overline{F}_{s2}^{ps}, \overline{F}_{p3}^{ps}\}]$. Compare $\max\{\overline{F}_{p2}^{x}, \overline{F}_{p3}^{x}\}$ and $\min\{\overline{F}_{s2}^{x}, \overline{F}_{s3}^{x}\}$, we can derive that $\max\{\overline{F}_{s3}^{ps}, \overline{F}_{p2}^{ps}\} \le \min\{\overline{F}_{s2}^{ps}, \overline{F}_{p3}^{ps}\}$ if and only if $\{d \in (0, 0.466], \alpha \in (\overline{\alpha}_p^{ps}, \overline{\alpha}_s^{ps}]\}$ / $\{d \in (0, 0.375], \alpha \in (\overline{\alpha}_p^{vn}, \overline{\alpha}_s^{vn}]\}$ / $\{d \in (0, 0.517], \alpha \in (\overline{\alpha}_p^{ss}, \overline{\alpha}_s^{ss}]\}$ for ps /vn /ss structure.

Therefore, we can obtain the Proposition 4.

Proof of Proposition 5.

RR will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,RR*} + \pi_{s,1}^{x,RR*} \ge \pi_{p,1}^{x,AR*} + \pi_{s,1}^{x,AR*} \\ \pi_{p,2}^{x,RR*} + \pi_{s,2}^{x,RR*} \ge \pi_{p,2}^{x,RA*} + \pi_{s,2}^{x,RA*}\end{cases}$. Thus, we derive that any supply chain will choose reselling mode when $d \in (0.543, 0.664)$ / $d \in (0.498, 0.664)$ / $d \in (0.554, 0.664)$ for ps /vn /ss structure.

AA will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,AA*} + \pi_{s,1}^{x,AA*} \ge \pi_{p,1}^{x,RA*} + \pi_{s,1}^{x,RA*} \\ \pi_{p,2}^{x,AA*} + \pi_{s,2}^{x,AA*} \ge \pi_{p,2}^{x,AR*} + \pi_{s,2}^{x,AR*}\end{cases}$. Thus, we derive that any supply chain will choose agency selling mode when $d \in (0, 0.543]$ / $d \in (0, 0.498]$ / $d \in (0, 0.517]$ for ps /vn /ss structure.

RA/ AR will be an equilibrium strategy if and only if $\begin{cases}\pi_{p,1}^{x,RA*} + \pi_{s,1}^{x,RA*} \ge \pi_{p,1}^{x,AA*} + \pi_{s,1}^{x,AA*} \\ \pi_{p,2}^{x,RA*} + \pi_{s,2}^{x,RA*} \ge \pi_{p,2}^{x,RR*} + \pi_{s,2}^{x,RR*}\end{cases}$ / $\begin{cases}\pi_{p,1}^{x,AR*} + \pi_{s,1}^{x,AR*} \ge \pi_{p,1}^{x,RR*} + \pi_{s,1}^{x,RR*} \\ \pi_{p,2}^{x,AR*} + \pi_{s,2}^{x,AR*} \ge \pi_{p,2}^{x,AA*} + \pi_{s,2}^{x,AA*}\end{cases}$. Thus, we derive that any supply chain will choose opposite mode. And this scenario only appears when $d \in (0.517, 0.554]$ for ss structure.

Therefore, we can obtain the Proposition 5.

Proof of Corollary 1.

Comparing the platforms' profit between RR and AA. And we can derive that $\begin{cases}\pi_{p,1}^{x,RR*} > \pi_{p,1}^{x,AA*} \\ \pi_{p,2}^{x,RR*} > \pi_{p,2}^{x,AA*}\end{cases}$ when $F \in (0, \tilde{F}_p^x]$, and we can hold that $\max\{\overline{F}_{p1}^{x}, \overline{F}_{p2}^{x}\} < \tilde{F}_p^x$. Combining Lemma 1 and we can find that there is a "Prisoner's dilemma" for both platforms, i.e., each platform choosing agency selling mode results in this "Prisoner's Dilemma", whereas both platforms can obtain more payoffs with reselling mode.

Comparing the suppliers' profit between RR and AA. And we can derive that $\begin{cases}\pi_{s,1}^{x,RR*} > \pi_{s,1}^{x,AA*} \\ \pi_{s,2}^{x,RR*} > \pi_{s,2}^{x,AA*}\end{cases}$ when $F \in (\tilde{F}_s^x, +\infty)$, and we can hold that $\tilde{F}_s^x < \min\{\overline{F}_{s1}^{x}, \overline{F}_{s2}^{x}\}$. Combining Lemma 2 and we can find that there is a "Prisoner's dilemma" for both suppliers, i.e., each supplier choosing agency selling mode results in this "Prisoner's Dilemma", whereas both suppliers can obtain more payoffs with reselling mode.

Comparing the supply chains' profit between RR and AA. And we can derive that

$\begin{cases}\pi_{p,1}^{x,RR*}+\pi_{s,1}^{x,RR*}>\pi_{p,1}^{x,AA*}+\pi_{s,1}^{x,AA*}\\ \pi_{p,2}^{x,RR*}+\pi_{s,2}^{x,RR*}>\pi_{p,2}^{x,AA*}+\pi_{s,2}^{x,AA*}\end{cases}$ when $d\in(0.372,0.664)$ / $d\in(0.271,0.664)$ / $d\in(0.372,0.664)$, and we can hold that $d\in(0.372,0.543]$ / $d\in(0.271,0.498]$ / $d\in(0.372,0.517]$,. Combining Proposition 4 and we can find that there is a "Prisoner's dilemma" for both supply chains, i.e., any supply chain choosing agency selling mode results in this "Prisoner's Dilemma", whereas both supply chains can obtain more payoffs with reselling mode.

Proof of Proposition 6.

Comparing $\sum_{j=1}^{2}(\pi_{p,j}^{x,RR*}+\pi_{s,j}^{x,RR*})$, $\sum_{j=1}^{2}(\pi_{p,j}^{x,RA*}+\pi_{s,j}^{x,RA*})$ and $\sum_{j=1}^{2}(\pi_{p,j}^{x,AA*}+\pi_{s,j}^{x,AA*})$:

RR will be an equilibrium strategy if and only if $d\in(0.390,0.664)$ / $d\in(0.279,0.664)$ / $d\in(0.417,0.664)$. Thus, we derive that competitive supply chain system will choose reselling mode.

RA/AR will be an equilibrium strategy if and only if $d\in(0.349,0.390]$ / $d\in(0.263,0.279]$ / $d\in(0.332,0.417]$. Thus, we derive that competitive supply chain system will choose opposite mode.

AA will be an equilibrium strategy if and only if $d\in(0,0.349]$ / $d\in(0,0.263]$ / $d\in(0,0.332]$. Thus, we derive that competitive supply chain system will choose agency selling mode.

Therefore, we can obtain the Proposition 6.

Proof of Proposition 7.

According to Proposition 2 and 5, by comparing a single supply chain's optimal profit under competitive supply chains circumstance and the single supply chain's optimal profit under monopolistic supply chain circumstance.

When $d\in(0,0.349]$ / $d\in(0,0.263]$ / $d\in(0,0.332]$, comparing $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}$ and $\pi_{p,1}^{x,AA*}+\pi_{s,1}^{x,AA*}$, we can derive that $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}>\pi_{p,1}^{x,AA*}+\pi_{s,1}^{x,AA*}$.

When $d\in(0.349,0.390]$ / $d\in(0.263,0.279]$ / $d\in(0.332,0.417]$, comparing $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}$ and $\pi_{p,1}^{x,RA*}+\pi_{s,1}^{x,RA*}$, we can derive that $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}>\pi_{p,1}^{x,RA*}+\pi_{s,1}^{x,RA*}$.

When $d\in(0.390,0.664)$ / $d\in(0.279,0.664)$ / $d\in(0.417,0.664)$, comparing $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}$ and $\pi_{p,1}^{x,RR*}+\pi_{s,1}^{x,RR*}$, we can derive that $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}<\pi_{p,1}^{x,RR*}+\pi_{s,1}^{x,RR*}$ if and only if $d\in(0.559,0.664)$ / $d\in(0.598,0.664)$ / $d\in(0.559,0.664)$ for ps / vn / ss structure, $\pi_{p,1}^{x,A*}+\pi_{s,1}^{x,A*}\geq\pi_{p,1}^{x,RR*}+\pi_{s,1}^{x,RR*}$ otherwise.

Therefore, we can hold that for any single supply chain, competitive supply chains circumstance is better off than monopolistic supply chain circumstance if and only if $d\in(0.559,0.664)$ / $d\in(0.598,0.664)$ / $d\in(0.559,0.664)$ for ps / vn / ss structure.

Proof of Proposition 8.

According to Proposition 2 and 5, by comparing a single supply chain's optimal profit under competitive supply chains circumstance and the single supply chain's optimal profit under monopolistic supply chain circumstance.

When $d \in (0, 0.349]$ / $d \in (0, 0.263]$ / $d \in (0, 0.332]$, comparing $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*}$ and $\sum_{j=1}^{2}(\pi_{p,j}^{x,AA*} + \pi_{s,j}^{x,AA*})$, we can derive that $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} \le \sum_{j=1}^{2}(\pi_{p,j}^{x,AA*} + \pi_{s,j}^{x,AA*})$ if and only if $F \in (0, \bar{F}_{AA}^{ps}]$ / $F \in (0, \bar{F}_{AA}^{vn}]$ / $F \in (0, \bar{F}_{AA}^{ss}]$ for ps / vn / ss structure, $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} > \sum_{j=1}^{2}(\pi_{p,j}^{x,AA*} + \pi_{s,j}^{x,AA*})$ otherwise. Where $\bar{F}_{AA}^{x} = \frac{a^2\left(4 - 12d - d^2 + 28d^3 - 12d^4 - 16d^5 + 8d^6\right)}{4\left(2 - d - 2d^2\right)^2}$.

When $d \in (0.349, 0.390]$ / $d \in (0.263, 0.279]$ / $d \in (0.332, 0.417]$, comparing $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*}$ and $\sum_{j=1}^{2}(\pi_{p,j}^{x,RA*} + \pi_{s,j}^{x,RA*})$, we can derive that $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} \le \sum_{j=1}^{2}(\pi_{p,j}^{x,RA*} + \pi_{s,j}^{x,RA*})$ if and only if $F \in (0, \bar{F}_{RA}^{ps}]$ / $F \in (0, \bar{F}_{RA}^{vn}]$ / $F \in (0, \bar{F}_{RA}^{ss}]$ for ps / vn / ss structure, $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} > \sum_{j=1}^{2}(\pi_{p,j}^{x,RA*} + \pi_{s,j}^{x,RA*})$ otherwise. Where

$$\bar{F}_{RA}^{ps} = \frac{a^2\left(12 - 32d - 73d^2 + 254d^3 + 133d^4 - 792d^5 + 11d^6 + 1232d^7 - 300d^8 - 1004d^9 + 353d^{10} + 406d^{11} - 165d^{12} - 64d^{13} + 28d^{14}\right)}{4\left(-4 + 2d + 14d^2 - 5d^3 - 14d^4 + 2d^5 + 4d^6\right)^2},$$

$$\bar{F}_{RA}^{vn} = \frac{a^2\left(8 - 14d - 43d^2 + 84d^3 + 81d^4 - 180d^5 - 50d^6 + 164d^7 - 14d^8 - 54d^9 + 17d^{10}\right)}{4\left(3 - 7d^2 + 3d^4\right)^2},$$

$$\bar{F}_{RA}^{ss} = \frac{a^2\left(48 - 80d - 296d^2 + 564d^3 + 632d^4 - 1346d^5 - 431d^6 + 1260d^7 - 120d^8 - 368d^9 + 112d^{10}\right)}{4\left(8 - 21d^2 + 8d^4\right)^2}.$$

When $d \in (0.390, 0.664)$ / $d \in (0.279, 0.664)$ / $d \in (0.417, 0.664)$, comparing $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*}$ and $\sum_{j=1}^{2}(\pi_{p,j}^{x,RR*} + \pi_{s,j}^{x,RR*})$, we can derive that $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} \le \sum_{j=1}^{2}(\pi_{p,j}^{x,RR*} + \pi_{s,j}^{x,RR*})$ if and only if $F \in (0, \bar{F}_{RR}^{ps}]$ / $F \in (0, \bar{F}_{RR}^{vn}]$ / $F \in (0, \bar{F}_{RR}^{ss}]$ for ps / vn / ss structure, $\pi_{p,1}^{x,A*} + \pi_{s,1}^{x,A*} > \sum_{j=1}^{2}(\pi_{p,j}^{x,RR*} + \pi_{s,j}^{x,RR*})$ otherwise. Where

$$\bar{F}_{RR}^{ps} = \frac{a^2\left(32 - 96d - 164d^2 + 764d^3 + 127d^4 - 2376d^5 + 662d^6 + 3688d^7 - 1649d^8 - 3004d^9 + 1516d^{10} + 1216d^{11} - 624d^{12} - 192d^{13} + 96d^{14}\right)}{4\left(2 - d - 2d^2\right)^2\left(4 - d - 10d^2 + d^3 + 4d^4\right)^2},$$

$$\bar{F}_{RR}^{vn} = \frac{a^2\left(7 - 20d - 2d^2 + 52d^3 - 25d^4 - 32d^5 + 16d^6\right)}{4\left(3 - 2d - 3d^2\right)^2},$$

$$\bar{F}_{RR}^{ss} = \frac{a^2\left(32 - 96d - 164d^2 + 764d^3 + 127d^4 - 2376d^5 + 662d^6 + 3688d^7 - 1649d^8 - 3004d^9 + 1516d^{10} + 1216d^{11} - 624d^{12} - 192d^{13} + 96d^{14}\right)}{4\left(2 - d - 2d^2\right)^2\left(4 - d - 10d^2 + d^3 + 4d^4\right)^2}.$$

Therefore, we can derive Proposition 8.

Proof of Corollary 2.

Comparing $\bar{F}_{RR}^{x}$, $\bar{F}_{RA}^{x}$ and $\bar{F}_{AA}^{x}$: We can hold that $\bar{F}_{RR}^{x} > \bar{F}_{RA}^{x} > \bar{F}_{AA}^{x}$ if and only if $d \in (0.390, 0.664)$ / $d \in (0.279, 0.664)$ / $d \in (0.417, 0.664)$ in the ps / vn / ss structure.

Proof of Proposition 9.

Under monopolistic supply chain circumstance, comparing the order fulfillment cost threshold through the comparison of players profits with reselling mode and players profits with agency selling profits in Proposition 1, we can derive that $\overline{F}_{p1}^{ss} < \overline{F}_{p1}^{vn} < \overline{F}_{p1}^{ps}$ and $\overline{F}_{s1}^{ss} < \overline{F}_{s1}^{vn} < \overline{F}_{s1}^{ps}$.

Under competitive supply chains circumstance, comparing the order fulfillment cost threshold in lemma 1 and lemma 2, we can derive that $\overline{F}_{p2}^{ss} < \overline{F}_{p2}^{vn} < \overline{F}_{p2}^{ps}$, $\overline{F}_{p3}^{ss} < \overline{F}_{p3}^{vn} < \overline{F}_{p3}^{ps}$, $\overline{F}_{s2}^{ss} < \overline{F}_{s2}^{vn} < \overline{F}_{s2}^{ps}$ and $\overline{F}_{s3}^{ss} < \overline{F}_{s3}^{vn} < \overline{F}_{s3}^{ps}$. Comparing the platform fee rate threshold in proposition 3 and lemma 1, we can derive that $\overline{\alpha}_{p}^{ss} < \overline{\alpha}_{p}^{vn} < \overline{\alpha}_{p}^{ps}$ and $\overline{\alpha}_{s}^{ss} < \overline{\alpha}_{s}^{vn} < \overline{\alpha}_{s}^{ps}$.

Proof of Proposition 10.

According to Proposition 8, we can derive the order fulfillment cost threshold through the comparison of the monopolistic supply chain system and the competitive supply chain system. Next, we compare these thresholds among three power structure.

Comparing $\overline{F}_{AA}^{ss}, \overline{F}_{AA}^{ps}, \overline{F}_{AA}^{vn}$, we can derive that $\overline{F}_{AA}^{ss} = \overline{F}_{AA}^{ps} = \overline{F}_{AA}^{vn}$.

Comparing $\overline{F}_{RA}^{ss}, \overline{F}_{RA}^{ps}, \overline{F}_{RA}^{vn}$, we can derive that $\overline{F}_{RA}^{ps} < \overline{F}_{RA}^{ss} < \overline{F}_{RA}^{vn}$ when $d \in (0, 0.392]$, $\overline{F}_{RA}^{ps} > \overline{F}_{RA}^{ss} > \overline{F}_{RA}^{vn}$ otherwise.

Comparing $\overline{F}_{RR}^{ss}, \overline{F}_{RR}^{ps}, \overline{F}_{RR}^{vn}$, we can derive that $\overline{F}_{RR}^{ps} = \overline{F}_{RR}^{ss} < \overline{F}_{RR}^{vn}$ when $d \in (0, 0.471]$, $\overline{F}_{RR}^{ps} = \overline{F}_{RR}^{ss} > \overline{F}_{RR}^{vn}$ otherwise.