sustainability

Article

Research on Joint Decision-Making of Timely Delivery, Product Quality and Marketing in Supply Chain Based on Differential Game

Hongfang Qiao [1], Xiaowei Lin [1,*], Xideng Zhou [2,*] and Minglin Jiang [1]

1 Business School, Minnan Normal University, Zhangzhou 363000, China
2 School of Economics and Management, Yuzhang Normal University, Nanchang 330099, China
* Correspondence: lxw949@163.com (X.L.); chowxd@foxmail.com (X.Z.)

Abstract: Due to the sudden surge of orders, it is difficult for suppliers with a limited capacity to ensure that all orders are delivered in time and all the products are qualified. Suppliers are likely to put more limited capacity into completing orders, thus ignoring the quality of products. This will easily lead to the occurrence of product quality events, and then affect the goodwill of enterprise products. The innovations of this paper are as follows: first, based on the above facts, a negative and dynamic correlation between the delivery level and the quality level is established, which has been involved in previous studies. Second, the joint decision model of timely delivery, product quality, and marketing is constructed. Thirdly, centralized decision-making is the best way of supply chain cooperation, and cost sharing contracts can coordinate the supply chain. This paper provides guidance for enterprise managers when making decisions on quality, marketing and delivery. It also provides the basis for enterprise managers to formulate effective cooperation models. We can draw some research implications: when consumers are less sensitive to timely delivery, enterprises should give some coupons and small gifts to consumers in exchange for the extension of delivery time and put their limited capacity into improving the product quality. When consumers are highly sensitive to timely delivery, they can outsource some orders to cost-effective and professional third-party enterprises, which not only improves the delivery rate but also improves the product quality.

Keywords: timely delivery; product quality; marketing; differential game; coordination

check for updates

Citation: Qiao, H.; Lin, X.; Zhou, X.; Jiang, M. Research on Joint Decision-Making of Timely Delivery, Product Quality and Marketing in Supply Chain Based on Differential Game. *Sustainability* **2022**, *14*, 10774. https://doi.org/10.3390/su141710774

Academic Editors: Yanzhi Li and Ke Fu

Received: 15 July 2022
Accepted: 26 August 2022
Published: 29 August 2022

1. Introduction

Improving product quality is not only of great significance to the development of enterprises, but also will have a far-reaching impact on society. The quality of products or services is the main factor that determines the quality, development, economic strength, and competitive advantage of enterprises. Quality is also the most critical factor to compete for the market. Whoever can provide users with satisfactory products or services in a flexible and fast way will win the competitive advantage in the market. Product quality safety is an important issue in supply chain management.

Empirical evidence shows that improving product quality and ensuring product quality safety can improve consumers' willingness to pay, expand consumers' market demand, and improve the business performance of supply chain enterprises [1–3]. This shows that supply chain enterprises have great enthusiasm to improve product quality. However, it can be found that quality and safety incidents are still common in recent years. In 2010, Toyota spent nearly USD 1.3 billion recalling more than 8 million vehicles due to defects in the accelerator pedal of its supplier. In 2020, General Motors announced the recall of about 7 million vehicles worldwide due to the quality problems of airbags that were produced by supplier Takata, and assumed the total recall cost of nearly USD 1.2 billion. In 2022, according to Reuters, the German Federal transport administration said that due to the failure of the automatic emergency call system, Tesla's Model 3 and Model y electric vehicles would be recalled, and 59,129 vehicles worldwide would be affected. This recall undoubtedly made Tesla's situation worse, which undoubtedly damaged Tesla's market reputation. From these quality safety events, we can see that the product quality defects of upstream enterprises in the supply chain are the main reasons for the occurrence of quality safety events.

With the continuous development of social productivity, customers have multi-level and diversified requirements for products under the condition of market economy, which is both an opportunity and a challenge for enterprises. Enterprises can adapt to the changes of market economy only by continuously improving their rapid response ability. Among them, delayed delivery is the biggest obstacle for enterprises to achieve rapid response, and it is also a difficult problem that enterprises must solve. Delayed delivery has a negative impact on manufacturing enterprises and customers. At the same time, the level of timely delivery is a symbol to measure the comprehensive strength of enterprises. On the one hand, due to the delayed delivery, the manufacturing enterprises have confused the internal production system, and face the delay penalty from customers, which reduces the credibility of the whole enterprise which gradually loses its market share. On the other hand, it causes serious losses to customers. Due to the rapid upgrading of contemporary products, delayed delivery makes downstream demanders lose sales opportunities and suffer heavy losses.

Generally speaking, the phenomenon of delayed delivery can be divided into internal factors and external factors. The external factors are unavoidable factors, such as the global financial crisis, earthquakes, and other natural disasters. Internal factors are the expanding influence of internal problems of enterprises, such as the organizational management ability of the supplier. This paper mainly considers the internal factors of the enterprise, that is, the loss that can be reduced through the continuous improvement of the enterprise itself.

When merchants promote sales, the demand for certain goods (orders) suddenly increases. According to the order requirements, the supplier needs to deliver qualified products to downstream customers in time. However, it is difficult for suppliers with capacity constraints to ensure the timely delivery of all products and qualified product quality. Suppliers may tend to invest more resources (human, material, and financial resources) to complete customer orders on time, while ignoring product quality. This can easily lead to the occurrence of product quality events, and then affect the goodwill of enterprise products.

Therefore, the research objective of this paper is to build a negative correlation between timely delivery and product quality based on the realistic background, which has rarely been involved in previous studies. By comparing strategy and profit, it provides effective guidance for enterprises to make decisions. In addition, this paper analyzes how consumer sensitivity factors affect decisions and profit changes. The conclusions of the analysis can provide suggestions for enterprise managers to take effective measures.

The innovations of this paper are as follows: first, a negative and dynamic correlation between the delivery level and the quality level is established. Secondly, it studies the joint decision-making of timely delivery, product quality, and marketing under different decision-making modes. Thirdly, it compares the timely delivery, product quality, marketing strategy, and supply chain profit under different decision-making modes, and designs the cost sharing contract mechanism.

This paper proceeds as follows: a literature review is presented in Section 2. Section 3 describes the notations and proposes the basic hypothesis. The optimal decisions and profits under the two scenarios are obtained and compared in Sections 4 and 5. A coordinating mechanism is designed in Section 6. Section 7 provides a numerical analysis. Finally, the concluding remarks are obtained in Section 8.

2. Literature Review

The research issues that are involved in this paper include product quality, timely delivery, and marketing.

The quality level of the product and the delivery date affect each other. The improvement of the delivery level will affect the quality level [4], and the two are mostly dynamic. For this, scholars have carried out extensive research by using differential games and other methods. Hong et al. [5] used the method of differential game to study the optimal quality management strategy and optimal profit of a two-level supply chain under four quality management games. Huang et al.'s [6] research showed that under the uncertainty of market demand, there is an optimal combination of product price, delivery time, and quality, which enables enterprises achieve the objectives of optimizing pricing strategy, improving delivery punctuality, and reducing operating costs. Wang et al. [7] used the differential game model to draw the conclusion that the higher the duopoly manufacturer's investment in the unit product warranty service, the longer the warranty period, and the higher the equilibrium price. Voros [8] established a dynamic model in which demand depends on both the price and quality. The research showed that the quality can be improved by investing in the development process, and the value of accumulated quality knowledge can be incorporated into the model. Chen et al. [9] used the method of game theory to theoretically analyze the impact of the input capacity constraints on food quality and quality regulation. Their research results show that both capacity input constraints lead to the reduction of the output level, quality level, and consumer surplus level, but the price level increases. They also found that rewards are more effective than punishment or fines in food quality assurance programs. Cellini et al. [10] put forward a dynamic model about price and quality competition by using a differential game to analyze the impact of competition on quality. Their research results show that if the suppliers adopt closed-loop decision-making rules, the more intense the competition is, the higher the quality will be, but the quality is still lower than that under open-loop rules. Heydari et al. [11] studied a two-level reverse supply chain that was composed of a single remanufacture and a single recycler, which has uncertainties in the quality of recycled products and remanufacturing capacity and designed a customized revenue sharing contract and shared risks together. Supply chain members' reciprocal altruism and consumers' quality and service reference effects are important behavioral factors that affect the decision-making of supply chain members [12,13]. Zhou et al. [13] established a dynamic model including product quality reference effect and service quality reference effect. Using differential game theory, the optimal decisions of product quality and service quality under different decision scenarios are obtained. Ruidas al. [14] explored a production inventory model considering two high-tech products of the same kind and believed that the demand of the updated product is also dependent on the quality of the primary product. Qiu et al. [15] considered the dynamic reference quality effect under the O2O environment.

As the level of timely delivery (i.e., service level) that is invested by the supplier has an impact on profits, high service level needs to bear the loss of profits and taking the joint decision of ordering can reduce the loss of profits that are caused by the pursuit of service level [16]. Therefore, how to optimize the service level of products is also the focus of academic research in recent years. For example, Wang et al. [17] analyzed the impact of product service system value, cost, and service value ratio on consumer strategic behavior by building a dynamic game model of two sales stages. Tian and Ge [18] proposed a service-oriented dynamic multi-level prediction maintenance grouping strategy, constructed the penalty cost and grouping service cost, and designed an improved k-means method to dynamically group the predicted optimal services. The research on delivery level is more about how to determine the delivery date. The hot spots mainly focus on inventory management, such as optimal price and optimal order quantity. Many scholars at home and abroad have conducted in-depth research in this field. Modak and Kelle [19] examined such a dual-channel supply chain under price- and delivery-time-dependent stochastic customer demand. Glock et al. [20] assumed that the delivery lead time is not fixed, but that both the retailer and the manufacturer have the option to shorten it. Shorter lead times enable the retailer to place orders closer to the start of the selling season where additional information on customer preferences has become available, reducing demand uncertainty. Qiu et al. [21] studied the decision-making problem of dual channel supply chain under the sensitive demand of price and delivery date. Roy and Sana [22] addressed an inter-dependent reduction strategy of lead time and ordering cost in a two-stage single vendor and single buyer supply chain model. The marketing efforts of enterprises such as advertising, channel expansion, product display, and salesperson's explanation also affect the market demand of products.

In the previous literature on supply chain decision-making considering product sales efforts, Yue et al. [23] studied the pricing and advertising investment decisions of the supply chain when both manufacturers and the retailer offered price discounts and found that the situation that was dominated by the manufacturer was more beneficial to the manufacturer than the situation of equal power between the two sides. Song et al. [24] first considered the impact of different channel power structures on the production decisions of manufacturers and the retailer when product quality and sales efforts affect demand at the same time. Pu et al. [25] studied the impact of the retailer' fair preference on promotion efforts and supply chain operation under the manufacturer-led framework. Gao et al. [26] studied the decision-making model of retailer-led closed-loop supply chain when product greenness and sales effort affect demand at the same time, and compared and analyzed the impact of the greenness effect and sales effort utility on enterprise decision-making. Ma et al. [27] studied a supply chain decision-making model considering marketing efforts to improve corporate social responsibility under symmetric and asymmetric information The above literature studies the influence of marketing efforts on the decision-making of supply chain members. This paper comprehensively analyzes the interactive mechanism of marketing efforts and innovation ability on the decision-making of supply chain members. Ranjan and Jha [28] investigated the pricing strategies and coordination mechanism between the members in a dual-channel supply chain. Ezimadu [29] used Stackelberg differential game theory to model the direct involvement of both the distributor and the retailer in advertising.

See Table 1 for the summary of the main research issues that were raised in the current relevant research literature. The existing research on product quality, timely delivery, and marketing has the following shortcomings, which is proven by literature review. First, although the topic of product quality, timely delivery, and marketing has attracted the interest of many scholars, there is not much literature on the joint decision-making of product quality, timely delivery, and marketing. Secondly, although some scholars consider the supply chain equilibrium strategy of timely delivery and quality, or marketing and quality, most of these studies are based on the static framework, and few studies consider the dynamic characteristics of the supply chain. From the dynamic point of view, it is obviously not enough to build models and methods from the static architecture, and it is difficult to provide appropriate decision-making suggestions for supply chain management. Thirdly, there are few studies about the relationship between timely delivery and product quality.

As mentioned above, the previous literature rarely studies the negative correlation between timely delivery and product quality, and rarely involves the joint decision-making of product quality, timely delivery, and marketing. Therefore, the theoretical contributions of this paper include the following aspects: first, the negative correlation between timely delivery and product quality is considered in the supply chain decision model; Secondly, the joint decision model of timely delivery, product quality, and marketing is constructed. It enriches the theory of supply chain quality, marketing, and delivery decision. The practical contributions include the following aspects: first, this paper provides guidance for enterprise managers when making decisions on quality, marketing, and delivery. Secondly, it provides the basis for enterprise managers to formulate effective cooperation models.

Table 1. Comparison of recent relevant literature research points.

Literature	Publish Time	Timely Delivery as a Decision Variable	Product Quality as a Decision Variable	Marketing as a Decision Variable	Using Differential Gaming Methods
Huang et al.	2016	yes	yes	no	no
Modak and Kelle	2018	yes	no	no	no
Ranjan and Jha	2019	no	no	yes	no
Chakraborty et al.	2019	no	yes	no	no
Zhou et al.	2020	no	yes	no	yes
Glock et al.	2020	yes	no	no	no
Roy and Sana	2021	yes	yes	no	no
Qiu et al.	2021	yes	no	no	no
Qiu et al.	2022	no	yes	no	yes
Zhan et al.	2022	no	yes	yes	yes
Ezimadu	2022	no	no	yes	yes
This paper	2022	yes	yes	yes	yes

3. Symbols and Assumptions

We study a supply chain system that was composed of a retailer and a supplier. The products that are produced by the supplier are sold through the retailer, considering the dynamic change of demand. The supplier determines the quality level and timely delivery level of the products; the retailer determines the marketing level of the products. Based on this, the relevant symbols and descriptions are shown in Table 2.

Table 2. Symbols and descriptions.

Symbol	Description	Symbol	Description
E_m	The supplier's effort in timely delivery, which is a decision variable.	$\prod_m$	The marginal profit of the supplier.
q	The product quality level of the supplier, which is a decision variable.	$\prod_r$	The marginal profit of the retailer.
E_r	The marketing level of the retailer, which is a decision variable.	β	The impact factors of product quality level on timely delivery level, $\beta > 0$.
x	Timely delivery level of products.	δ	Decay rate of timely delivery level, $\delta > 0$.
η_{m1}	The supplier's quality cost coefficient, $\eta_{m1} > 0$.	γ_0	Sensitivity coefficient of consumers to the level of timely delivery, $\gamma_0 > 0$.
η_{m2}	The supplier's timely delivery cost coefficient, $\eta_{m2} > 0$.	γ_m	Sensitivity coefficient of consumers to the product quality level, $\gamma_m > 0$.
η_{m3}	The retailer's marketing cost coefficient, $\eta_{m3} > 0$.	γ_r	Sensitivity coefficient of consumers to marketing level, $\gamma_r > 0$.
α	The impact factors of timely delivery investment effort on timely delivery level, $\alpha > 0$.	ρ	The discount rate, $\rho > 0$.

Hypothesis 1. *Since the supplier's quality level, timely delivery effort, and the retailer's marketing level are convex functions of quality input, delivery input, and marketing input cost, respectively [30], then the costs of quality input, delivery input, and marketing input at time t are, respectively:*

$$C_q = \frac{1}{2}\eta_{m1}q^2(t) \tag{1}$$

$$C_e = \frac{1}{2}\eta_{m2}E_m^2(t) \tag{2}$$

$$C_r = \frac{1}{2}\eta_r E_r^2(t) \tag{3}$$

where C_q, C_e, and C_r represent the costs of quality input, delivery input, and marketing input at time t, respectively.

Hypothesis 2. *During the busy period of production, the supplier devotes more energy to timely delivery, and it is easy to ignore product quality* [4]. *The level of timely delivery of products (i.e., on-time delivery rate) has a certain correlation with the quality level. It is assumed that the timely delivery level of the supplier is not only related to the delivery input efforts of the supplier (used to increase workers, update equipment, etc.), but also has a certain negative correlation with the quality of products. Then, the state equation of the change of timely delivery level with time can be expressed as:*

$$\dot{x}(t) = \alpha E_m(t) - \beta q(t) - \delta x(t) \tag{4}$$

where $x(t)$ represents the timely delivery level at time t; the initial delivery level is $x(0) = x_0$, $(x_0 > 0)$; $E_m(t)$ is the supplier's timely delivery effort at time t; and $\alpha(\alpha > 0)$ is the supplier's delivery effort impact coefficient, which indicates the impact of timely delivery efforts on the timely delivery level. $q(t)$ is the product quality level at time t. $\beta(\beta > 0)$ is the impact factor of the product quality level on timely delivery level. δ is the decay rate of the timely delivery level, which is caused by the accumulation of orders during the hot sales period, season change period, or large-scale promotion period.

Hypothesis 3. *According to the assumption in reference* [31], *the market demand can be expressed as:*

$$D(t) = \gamma_0 x(t) + \gamma_m q(t) + \gamma_r E_r(t) \tag{5}$$

where $\gamma_0 > 0$, $\gamma_m > 0$, $\gamma_r > 0$ are the sensitivity coefficients of consumers to the level of timely delivery, product quality, and marketing, respectively.

Hypothesis 4. *The discount rate of both the supplier and the retailer is $\rho(\rho > 0)$, and the members of the supply chain make decisions based on their optimal profits in the infinite time zone.*

4. Decentralized Decision-Making

Under decentralized decision-making, the supplier and the retailer, as independent individuals, make decisions based on the principle of their own best interests. Superscript d indicates decentralized decision-making in the supply chain. The instantaneous profit function of the supplier and retailer can be expressed as:

$$\pi_m^d(t) = \prod\nolimits_m[\gamma_0 x(t) + \gamma_m q(t) + \gamma_r E_r(t)] - \frac{1}{2}\eta_{m1} q^2(t) - \frac{1}{2}\eta_{m2} {E_m}^2 \tag{6}$$

$$\pi_r^d = \prod\nolimits_r[\gamma_0 x(t) + \gamma_m q(t) + \gamma_r E_r(t)] - \frac{1}{2}\eta_r {E_r}^2 \tag{7}$$

In this case, the goal of the supplier and the retailer is to find out the optimal quality level, timely delivery effort, and marketing level in continuous time, so as to maximize the discount value of their supplier's and retailer's profits. Therefore, the objective function of supplier is:

$$J_m = \int_0^\infty e^{-\rho t}[\prod\nolimits_m[\gamma_0 x(t) + \gamma_m q(t) + \gamma_r E_r(t)] - \frac{1}{2}\eta_{m1} q^2 - \frac{1}{2}\eta_{m2} {E_m}^2]dt \tag{8}$$

$s.t.\ \dot{x}(t) = \alpha E_m(t) - \beta q(t) - \delta x(t)$.

Next, we solve the optimal quality level and timely delivery effort to maximize the objective function, and the corresponding present value Hamilton Function is:

$$H_m = \prod\nolimits_m[D_0 + \gamma_0 x + \gamma_m q + \gamma_r E_r] - \frac{1}{2}\eta_{m1} q^2 - \frac{1}{2}\eta_{m2} {E_m}^2 + X_{m1}(\alpha E_m - \beta q - \delta x) \tag{9}$$

The supplier's optimal decision-making needs to meet the following conditions:

$$\frac{dH_m}{dE_m} = -\eta_{m2}E_m + \alpha X_{m1} = 0 \tag{10}$$

$$\frac{dH_m}{dq} = \prod\nolimits_m \gamma_m - \eta_{m1}q - \beta X_{m1} = 0 \tag{11}$$

$$\dot{X}_{m1}(t) = \rho X_{m1} - \frac{dH_m}{dx} = X_{m1}(\rho+\delta) - \prod\nolimits_m \gamma_0 \tag{12}$$

Hessian matrix of H_d with respect to E_m and q is:

$$H_d = \begin{bmatrix} \frac{d^2H_{mr}}{dE_m{}^2} & \frac{d^2H_{mr}}{dE_m dq} \\ \frac{d^2H_{mr}}{dE_m dq} & \frac{d^2H_{mr}}{dq^2} \end{bmatrix} = \begin{bmatrix} -\eta_{m2} & 0 \\ 0 & -\eta_{m1} \end{bmatrix}$$

Now $|H_d| > 0, \frac{d^2H_{mr}}{dE_m{}^2} < 0$. This shows that the Hessian matrix H_d is negative definite. Hence, we have unique critical point which maximizes the supplier profit.

From Equations (10) and (11), we obtain:

$$E_m = \frac{\alpha X_{m1}}{\eta_{m2}} \tag{13}$$

$$q = \frac{\prod_m \gamma_m - \beta X_{m1}}{\eta_{m1}} \tag{14}$$

By solving the differential equation of (12), we obtain:

$$X_{m1}(t) = c_1 e^{(\rho+\delta)t} + \frac{\prod_m \gamma_0}{\rho+\delta} \tag{15}$$

Substituting Equation (15) into (13), we obtain:

$$E_m = \frac{\alpha}{\eta_{m2}}[c_1 e^{(\rho+\delta)t} + \frac{\prod_m \gamma_0}{\rho+\delta}] \tag{16}$$

Substituting Equation (15) into (14), we obtain

$$q = \frac{\prod_m \gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}}(c_1 e^{(\rho+\delta)t} + \frac{\prod_m \gamma_0}{\rho+\delta}) \tag{17}$$

When $t \to \infty$, the supplier's quality level and timely delivery effort are limited, so $\lim_{t\to\infty} E_m(t) < \infty$, $\lim_{t\to\infty} q(t) < \infty$. Therefore, it can be judged that c_1 in Equations (16) and (17) is equal to 0. Simplify the supplier's quality level q and delivery effort E_m, that is:

$$E_m* = \frac{\prod_m \alpha}{\eta_{m2}(\rho+\delta)}\gamma_0 \tag{18}$$

$$q* = \frac{\prod_m \gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}}\frac{\prod_m \gamma_0}{\rho+\delta} \tag{19}$$

Since the quality level q is greater than 0, the optimal quality level also needs to meet the condition $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$.

Then, to solve the optimal marketing level of the retailer, and its optimal decision-making meets the following conditions:

$$\frac{d\pi_r}{dE_r} = \prod\nolimits_r \gamma_r - \eta_r E_r = 0 \tag{20}$$

And it is easy to get $\frac{d^2\pi_r}{dE_r{}^2} < 0$.

From Equation (20), the optimal marketing level of the retailer can be obtained as:

$$E_r = \frac{\prod_r}{\eta_r}\gamma_r \tag{21}$$

Substituting Equations (18) and (19) into (4), we can get the change track of the supplier's timely delivery level with time:

$$x*(t) = \frac{\alpha E_m * - \beta q*}{\delta} + e^{-\delta t}(x_0 - \frac{\alpha E_m * - \beta q*}{\delta}) \tag{22}$$

Based on the above analysis, Theorem 1 is obtained.

Theorem 1. *Under decentralized decision-making, when $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0 > 0$, the supplier's optimal product quality level, the timely delivery effort, and the retailer's optimal marketing level are, respectively:*

$$E_m{}^d* = \frac{\prod_m \alpha}{\eta_{m2}(\rho+\delta)}\gamma_0$$

$$q^d* = \frac{\prod_m \gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}}\frac{\prod_m \gamma_0}{\rho+\delta}$$

$$E_r{}^d = \frac{\prod_r}{\eta_r}\gamma_r$$

The timely delivery level is:

$$\begin{aligned} x^d*(t) = {} & \tfrac{\prod_m}{\delta}[\alpha\tfrac{\alpha\gamma_0}{\eta_{m2}(\rho+\delta)} - \tfrac{\beta\gamma_m}{\eta_{m1}} + \tfrac{\beta\gamma_0}{\eta_{m1}(\rho+\delta)}] + e^{-\delta t}\{x_0 - \tfrac{\prod_m}{\delta}[\alpha\tfrac{\alpha\gamma_0}{\eta_{m2}(\rho+\delta)} \\ & - \tfrac{\beta\gamma_m}{\eta_{m1}} + \tfrac{\beta\gamma_0}{\eta_{m1}(\rho+\delta)}]\} \end{aligned}$$

Substituting the above Theorem 1 into Equations (6) and (7) and simplifying the present value profit function of the retailer and the supplier, we obtain the following equations.

$$\begin{aligned} J_r{}^d = {} & \tfrac{1}{\rho}\{\prod_r[\gamma_0(\tfrac{\alpha E_m{}^{d*} - \beta q^{d*}}{\delta}) + \tfrac{\gamma_m \prod_m}{\eta_{m1}}(\gamma_m - \tfrac{\beta\gamma_0}{\rho+\delta}) + \tfrac{\prod_r \gamma_r{}^2}{\eta_r}] - \tfrac{1}{2}\eta_r E_r{}^{d*2}\} \\ & + \tfrac{\prod_r \gamma_0}{\rho+\delta}(x_0 - \tfrac{\alpha E_m{}^{d*} - \beta q^{d*}}{\delta}) \end{aligned} \tag{23}$$

$$\begin{aligned} J_m{}^d = {} & \tfrac{1}{\rho}\{\prod_m[\gamma_0(\tfrac{\alpha E_m{}^{d*} - \beta q^{d*}}{\delta}) + \tfrac{\gamma_m \prod_m}{\eta_{m1}}(\gamma_m - \tfrac{\beta\gamma_0}{\rho+\delta}) + \tfrac{\prod_r \gamma_r{}^2}{\eta_r}] - \tfrac{\prod_m{}^2}{2\eta_{m1}}(\gamma_m - \tfrac{\beta\gamma_0}{\rho+\delta})^2 \\ & - \tfrac{1}{2}\eta_{m2}E_m{}^{d*2}\} + \tfrac{\prod_m \gamma_0}{\rho+\delta}(x_0 - \tfrac{\alpha E_m{}^{d*} - \beta q^{d*}}{\delta}) \end{aligned} \tag{24}$$

5. Centralized Decision-Making

Under centralized decision-making, the supplier and the retailer are regarded as two departments in an enterprise, which maximize the profits of the supply chain by designing the optimal quality level, timely delivery efforts, and marketing level. Superscript c indicates centralized decision-making.

The goal of the supply chain system is to find the optimal quality level, timely delivery efforts, and marketing level in a continuous time $t \in [0, \infty)$ to maximize the discount value of its profits. The objective function of the supply chain system is:

$$J_{mr}^c = \int_0^\infty e^{-\rho t}[(\prod_m + \prod_r)[\gamma_0 x(t) + \gamma_m q(t) + \gamma_r E_r(t)] - \frac{1}{2}\eta_{m1}q^2 - \frac{1}{2}\eta_{m2}E_m{}^2 - \frac{1}{2}\eta_{m3}E_r{}^2]dt \tag{25}$$

$s.t.\ \dot{x}(t) = \alpha E_m(t) - \beta q(t) - \delta x(t)$. The optimal decision-making problem of the supply chain is described as the optimal control problem $\max\limits_{\substack{q>0,E_r>0 \\ E_m>0}} J^c_{mr}$. Using the maximum principle, the Hamilton function is constructed.

$$\begin{aligned} H_{mr}{}^c = {} & (\textstyle\prod_m + \prod_r)[\gamma_0 x + \gamma_m q + \gamma_r E_r] - \frac{1}{2}\eta_{m1} q^2 - \frac{1}{2}\eta_{m2} E_m{}^2 - \frac{1}{2}\eta_{m3} E_r{}^2 \\ & + X_{r2}(\alpha E_m - \beta q - \delta x) \end{aligned} \tag{26}$$

The optimal decision-making of supply chain meets the following conditions:

$$\frac{dH_{mr}}{dE_m} = -\eta_{m2} E_m + X^c_{r2}\alpha \tag{27}$$

$$\frac{dH_{mr}}{dq} = (\prod\nolimits_r + \prod\nolimits_m)\gamma_m - \eta_{m1} q - X_{r2}\beta \tag{28}$$

$$\frac{dH_{mr}}{dE_r} = (\prod\nolimits_r + \prod\nolimits_m)\gamma_r - \eta_r E_r \tag{29}$$

$$\dot{X}_{r2}(t) = \rho X_{r2} - \frac{dH_{mr}}{dx} = (\rho + \delta) X_{r2} - \gamma_0(\prod\nolimits_r + \prod\nolimits_m) \tag{30}$$

Hessian matrix of H_c with respect to E_m, q and E_r is

$$H_c = \begin{bmatrix} \frac{d^2 H_{mr}}{dE_m{}^2} & \frac{d^2 H_{mr}}{dE_m dq} & \frac{d^2 H_{mr}}{dE_m dE_r} \\ \frac{d^2 H_{mr}}{dE_m dq} & \frac{d^2 H_{mr}}{dq^2} & \frac{d^2 H_{mr}}{dq dE_r} \\ \frac{d^2 H_{mr}}{dE_m dE_r} & \frac{d^2 H_{mr}}{dq dE_r} & \frac{d^2 H_{mr}}{dE_r{}^2} \end{bmatrix} = H_c = \begin{bmatrix} -\eta_{m2} & 0 & 0 \\ 0 & -\eta_{m1} & 0 \\ 0 & 0 & -\eta_r \end{bmatrix}$$

Now $|H_c| < 0$, $\frac{d^2 H_{mr}}{dE_m{}^2} < 0$, $\begin{vmatrix} -\eta_{m2} & 0 \\ 0 & -\eta_{m1} \end{vmatrix} > 0$. This shows that the Hessian matrix H_c is negative definite. Hence, we have unique critical point which maximizes the supply chain profit.

From Equations (27) and (28), we get:

$$E_m = \frac{\alpha X_{r2}}{\eta_{m2}} \tag{31}$$

$$q = \frac{(\prod_r + \prod_m)\gamma_m - \beta X_{r2}}{\eta_{m1}} \tag{32}$$

By solving Equation (30), we get:

$$X_{r2}(t) = c_1 e^{(\rho+\delta)t} + \frac{(\prod_r + \prod_m)\gamma_0}{\rho + \delta} \tag{33}$$

Substituting Equation (33) into (31) and (32), we get:

$$E_m = \frac{\alpha}{\eta_{m2}}[c_2 e^{(\rho+\delta)t} + \frac{(\prod_r + \prod_m)\gamma_0}{\rho + \delta}] \tag{34}$$

$$q = \frac{(\prod_r + \prod_m)\gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}}(c_2 e^{(\rho+\delta)t} + \frac{(\prod_r + \prod_m)\gamma_0}{\rho + \delta}) \tag{35}$$

When $t \to \infty$, the supplier's quality level and delivery effort are limited, that is $\lim\limits_{t\to\infty} E_m(t) < \infty$, $\lim\limits_{t\to\infty} q(t) < \infty$. Therefore, it can be judged that c_2 in Equations (34) and (35) is equal to 0. The optimal quality level and delivery effort of the supply chain system are obtained:

$$E_m = \frac{\alpha(\prod_r + \prod_m)}{\eta_{m2}(\rho+\delta)}\gamma_0 \tag{36}$$

$$q = \frac{(\prod_r + \prod_m)}{\eta_{m1}}\gamma_m - \frac{\beta(\prod_r + \prod_m)}{\eta_{m1}(\rho+\delta)}\gamma_0 \tag{37}$$

From Equation (29), we get:

$$E_r = \frac{(\prod_r + \prod_m)}{\eta_r}\gamma_r \tag{38}$$

Substituting Equations (36) and (37) into (4), the change track of delivery level with time in the case of centralized decision-making is obtained:

$$x*(t) = \frac{\alpha E_m * - \beta q*}{\delta} + e^{-\delta t}(x_0 - \frac{\alpha E_m * - \beta q*}{\delta}) \tag{39}$$

Based on the above analysis, Theorem 2 is obtained.

Theorem 2. *In the case of centralized decision-making, when $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0 > 0$, the optimal product quality level, timely delivery effort, and the optimal marketing level of the supply chain system are, respectively:*

$$E_m{}^c* = \frac{(\prod_m + \prod_r)\alpha}{\eta_{m2}(\rho+\delta)}\gamma_0$$

$$q^c* = \frac{(\prod_m + \prod_r)\gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}}\frac{(\prod_m + \prod_r)\gamma_0}{\rho+\delta}$$

$$E_r{}^c = \frac{(\prod_m + \prod_r)}{\eta_r}\gamma_r$$

The level of timely delivery is:

$$\begin{aligned} x^c*(t) &= \tfrac{(\prod_m + \prod_r)}{\delta}[\alpha\tfrac{\alpha\gamma_0}{\eta_{m2}(\rho+\delta)} - \tfrac{\beta\gamma_m}{\eta_{m1}} + \tfrac{\beta\gamma_0}{\eta_{m1}(\rho+\delta)}] \\ &+ e^{-\delta t}\{x_0 - \tfrac{(\prod_m + \prod_r)}{\delta}[\alpha\tfrac{\alpha\gamma_0}{\eta_{m2}(\rho+\delta)} - \tfrac{\beta\gamma_m}{\eta_{m1}} + \tfrac{\beta\gamma_0}{\eta_{m1}(\rho+\delta)}]\} \end{aligned}$$

Substituting the above Theorem 2 into Equation (25), we obtain the profit present value function of the supply chain system, as shown below:

$$\begin{aligned} J_{sc}{}^c &= \tfrac{1}{\rho}\{(\prod_r + \prod_m)[D_0 + \gamma_0(\tfrac{\alpha E_m{}^c* - \beta q^c*}{\delta}) + \tfrac{\gamma_m(\prod_r + \prod_m)}{\eta_{m1}}(\gamma_m - \tfrac{\beta\gamma_0}{\rho+\delta}) + \tfrac{(\prod_r + \prod_m)\gamma_r{}^2}{\eta_r}] \\ &- \tfrac{(\prod_r + \prod_m)^2}{2\eta_{m1}}(\gamma_m - \tfrac{\beta\gamma_0}{\rho+\delta})^2 - \tfrac{1}{2}\eta_{m2}E_m{}^{c2} - \tfrac{1}{2}\eta_r E_r{}^{c2}\} + \tfrac{(\prod_r{}^c + \prod_m{}^c)\gamma_0}{\rho+\delta}(x_0 - \tfrac{\alpha E_m{}^c* - \beta q^c*}{\delta}) \end{aligned} \tag{40}$$

Then, the optimal decisions and profits are compared under the above two different decision modes.

Comparing the optimal delivery effort, we get:

$$\begin{aligned} E_m^c - E_m^d &= \tfrac{\alpha(\prod_r + \prod_m)}{\eta_{m2}(\rho+\delta)}\gamma_0 - \tfrac{\prod_m \alpha}{\eta_{m2}(\rho+\delta)}\gamma_0 \\ &= \tfrac{\alpha\prod_m}{\eta_{m2}(\rho+\delta)}\gamma_0 > 0 \end{aligned}$$

Comparing the optimal product quality level, we get:

$$\begin{aligned} q_m^c - q_m^d &= \tfrac{(\prod_r + \prod_m)}{\eta_{m1}}\gamma_m - \tfrac{\beta(\prod_r + \prod_m)}{\eta_{m1}(\rho+\delta)}\gamma_0 - \tfrac{\prod_m \gamma_m}{\eta_{m1}} + \tfrac{\beta}{\eta_{m1}}(\tfrac{\prod_m \gamma_0}{\rho+\delta}) \\ &= \tfrac{\prod_r}{\eta_{m1}}\gamma_m - \tfrac{\beta\prod_r}{\eta_{m1}(\rho+\delta)}\gamma_0 > 0 \end{aligned}$$

Comparing the optimal marketing level, we get:

$$E_r^c - E_r^d = \frac{(\prod_r + \prod_m)}{\eta_r}\gamma_r - \frac{\prod_r}{\eta_r}\gamma_r = \frac{\prod_m}{\eta_r}\gamma_r > 0$$

Comparing the overall profits of the supply chain under different decision-making modes, we get:

$$\begin{aligned} & \int_0^\infty e^{-\rho t}(\pi_{sc}^c - \pi_{sc}^d)d(t) \\ = & \int_0^\infty e^{-\rho t}\left\{ \begin{array}{l} (E_m^c - E_m)[(\prod_m + \prod_r)M_1 - \frac{1}{2}\eta_{m2}(E_m^c + E_m)] + (q^c - q)[(\prod_m + \prod_r)M_2 \\ -\frac{1}{2}\eta_{m1}(q + q^c)] + (E_r^c - E_r)[(\prod_m + \prod_r)\gamma_r - \frac{1}{2}\eta_r(E_r^c + E_r)] \end{array} \right\} d(t) \end{aligned}$$

where $M_1 = \frac{\alpha}{\eta_{m2}(\rho+\delta)}\gamma_0$, $M_2 = \frac{\gamma_m}{\eta_{m1}} - \frac{\beta}{\eta_{m1}(\rho+\delta)}\gamma_0$, $M_3 = \gamma_r$. As $E_m^c > {E_m}^d$, $q^c > q^d$, $E_r^c > {E_r}^d$, it is easy to get $(\prod_m + \prod_r)M_1 > \frac{1}{2}\eta_{m2}(E_m^c + E_m)$, $(\prod_m + \prod_r)M_2 = \frac{1}{2}\eta_{m1}(q + q^c)$, $(\prod_m + \prod_r)M_3 > \frac{1}{2}\eta_r(E_r^c + E_r)$. Therefore, we obtain $\int_0^\infty e^{-\rho t}(\pi_{mr}^c - \pi_{mr}^d)d(t) > 0$.

Based on the above analysis, the following inference is obtained.

Inference 1. *In the case of centralized and decentralized decision-making, when the condition $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$ is satisfied, there is ${E_m}^c* > {E_m}^d*$, $q^c* > q^d*$, ${E_r}^c* > {E_r}^d*$, $\int_0^\infty e^{-\rho t}(\pi_{mr}^c)d(t) > \int_0^\infty e^{-\rho t}(\pi_{mr}^d)d(t)$.*

6. Cost Sharing Mechanism for Timely Delivery

From the above analysis, it can be concluded that the present value profit of the supply chain system under the centralized decision-making mode is greater than the present value profit of the supply chain system under the decentralized decision-making mode. Therefore, it is necessary to design an effective coordination mechanism to realize that the present value profit under the decentralized decision-making mode is equal to the present value profit under the centralized decision-making mode. This paper designs a timely delivery effort cost sharing contract to coordinate the supply chain. In this contract coordination mechanism, both the supplier and retailer adopt the optimal product quality decision, delivery effort decision, and marketing decision of the centralized decision-making model. The retailer makes a reasonable delivery cost sharing proportion k to distribute the total profits of the supply chain of the centralized decision-making. Then, the present value of the profits of the supplier and retailer can be expressed as:

$$\begin{aligned} {J_r}^c = & \frac{1}{\rho}\{ \prod_r[\gamma_0(\frac{\alpha E_m* - \beta q*}{\delta}) + \frac{\gamma_m(\prod_r + \prod_m)}{\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta}) + \frac{(\prod_r + \prod_m){\gamma_r}^2}{\eta_r}] \\ & - \frac{(\prod_r + \prod_m)^2}{2\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta})^2 - \frac{(\prod_r + \prod_m)^2}{2\eta_r}{\gamma_r}^2 - \frac{1}{2}k\eta_{m2}{E_m}^2\} + \frac{\gamma_0\prod_r}{\rho+\delta}(x_0 - \frac{\alpha E_m* - \beta q*}{\delta}) \end{aligned} \tag{41}$$

$$\begin{aligned} {J_m}^c = & \frac{1}{\rho}\{ \prod_m[\gamma_0(\frac{\alpha E_m* - \beta q*}{\delta}) + \frac{\gamma_m(\prod_r + \prod_m)}{\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta}) + \frac{(\prod_r + \prod_m){\gamma_r}^2}{\eta_r}] \\ & - \frac{(\prod_r + \prod_m)^2}{2\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta})^2 - \frac{1}{2}(1-k)\eta_{m2}{E_m}^2 - \frac{1}{2}\eta_r{E_r}^2\} + \frac{\prod_m\gamma_0}{\rho+\delta}(x_0 - \frac{\alpha E_m* - \beta q*}{\delta}) \end{aligned} \tag{42}$$

Add Equations (41) and (42) left and right to get Equation (43), which is equal to the total profit of the supply chain of centralized decision-making. How to set the proportion of delivery cost sharing (in the case of cooperation mechanism) will affect whether the profits of the supplier and the retailer are reasonably distributed.

$$\begin{aligned} {J_{mr}}^c = & \frac{1}{\rho}\{(\prod_r + \prod_m)[\gamma_0(\frac{\alpha E_m* - \beta q*}{\delta}) + \frac{\gamma_m(\prod_r + \prod_m)}{\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta}) + \frac{(\prod_r + \prod_m){\gamma_r}^2}{\eta_r}] \\ & - \frac{(\prod_r + \prod_m)^2}{2\eta_{m1}}(\gamma_m - \frac{\beta\gamma_0}{\rho+\delta})^2 - \frac{1}{2}\eta_{m2}{E_m}^2 - \frac{1}{2}\eta_r{E_r}^2\} + \frac{(\prod_r + \prod_m)\gamma_0}{\rho+\delta}(x_0 - \frac{\alpha E_m* - \beta q*}{\delta}) \end{aligned} \tag{43}$$

One of the necessary conditions to realize supply chain coordination is that the retailer's profit meets the following constraints. That is, in the case of coordination mechanism, when the cost sharing proportion of timely delivery efforts meets $k_{\max} \in [0,1]$, the retailer's profit is not less than the retailer's optimal profit under decentralized decision-making.

$$J_r{}^{c*} \geq J_r{}^{d*}, 0 \leq k_{\max} \leq 1 \tag{44}$$

According to Equation (44), the maximum proportion of retailers that are willing to share the cost of timely delivery of the supplier is:

$$k_{\max} = \frac{2\rho\left\{\begin{array}{c} \frac{1}{\rho}\left\{\prod_r[\gamma_0(\frac{\alpha E_m*-\beta q*}{\delta}) + \gamma_m q + \gamma_r E_r] - \frac{1}{2}\eta_{m1}q^2 - \frac{1}{2}\eta_r E_r{}^2\right\} \\ +\frac{\prod_r \gamma_0}{\rho+\delta}(x_0 - \frac{\alpha E_m*-\beta q*}{\delta}) \end{array}\right\}}{\eta_{m2}E_m{}^{c*2}}$$

Similarly, only when the supplier's profit meets the following constraints can the supplier accept the coordination mechanism, that is, under the coordination mechanism, when the cost sharing proportion of timely delivery efforts meets $k_{\min} \in [0,1]$, the supplier's profit is not less than the supplier's optimal profit under decentralized decision-making.

$$J_m{}^{c*} \geq J_m{}^{d*},\ 0 \leq k^c_{\min} \leq 1 \tag{45}$$

From Equation (45), it can be seen that the minimum proportion of the supplier requiring the retailer to share the cost of timely delivery efforts cannot be less than:

$$k_{\min} = 1 - \frac{2\rho\left\{\begin{array}{c} \frac{1}{\rho}\left\{\prod_m[\gamma_0(\frac{\alpha E_m*-\beta q*}{\delta}) + \gamma_m q + \gamma_r E_r] - \frac{1}{2}\eta_{m1}q^2 - \frac{1}{2}\eta_r E_r{}^2\right\} \\ +\frac{\prod_m \gamma_0}{\rho+\delta}(x_0 - \frac{\alpha E_m*-\beta q*}{\delta}) \end{array}\right\}}{\eta_{m2}E_m{}^2}$$

Moreover, it is easy to get $k_{\max} - k_{\min} > 0$, so the following theorem is obtained.

Theorem 3. *When the cost sharing proportion of timely delivery meets $k \in [k_{\max}, k_{\min}]$, $k_{\min} \in [0,1]$, and $k_{\max} \in [0,1]$, the supply chain system can reach a coordinated state, and the profit Pareto of the supplier and the retailer can be improved.*

Theorem 3 shows that when the cost sharing proportion of timely delivery meets $k \in [k_{\max}, k_{\min}]$, $k_{\min} \in [0,1]$, and $k_{\max} \in [0,1]$, both the supplier and retailer adopt the optimal strategy of centralized decision-making, so that the profits of both supply and demand sides are not lower than those under decentralized decision-making. The value of k depends on the dominant position of the supplier and retailer in the supply chain.

7. Numerical Analysis

Through numerical experiments, this section analyzes the differences of supply chain profits and strategies in different situations, in order to verify the previous theoretical results.

Suppose that the values of the parameters in the model are $\rho = 0.1$, $\prod_m = 10$, $\prod_r = 4$, $n_{m1} = 1$, $n_{m2} = 1$, $n_r = 1$, $\alpha = 2$, $\beta = 1.5$, $\delta = 1.4$, $x_0 = 2$, $\gamma_0 = 1$, $\gamma_m = 2$, $\gamma_r = 1$, respectively.

First, according to the benchmark parameters, the profits of the supplier and the retailer under the timely delivery cost sharing coordination mechanism change with the proportion of cost sharing, and is compared with the profit of corresponding enterprises without the coordination mechanism, as shown in Figure 1, where the superscript c represents the timely delivery cost sharing contract coordination mechanism, the superscript d represents the non-coordination mechanism, the subscript r represents the retailer, and the subscript m represents the supplier.

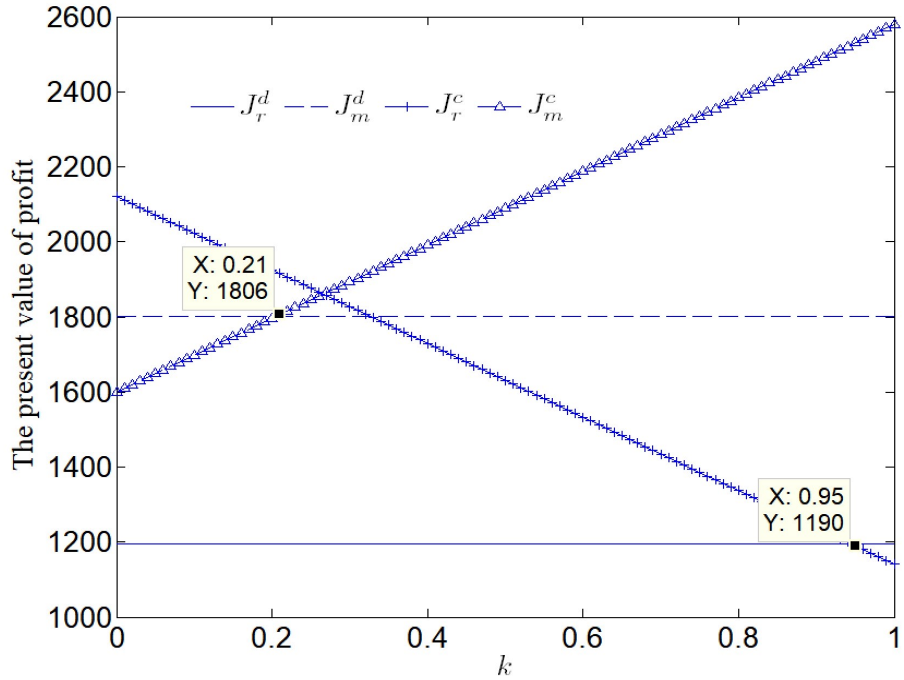
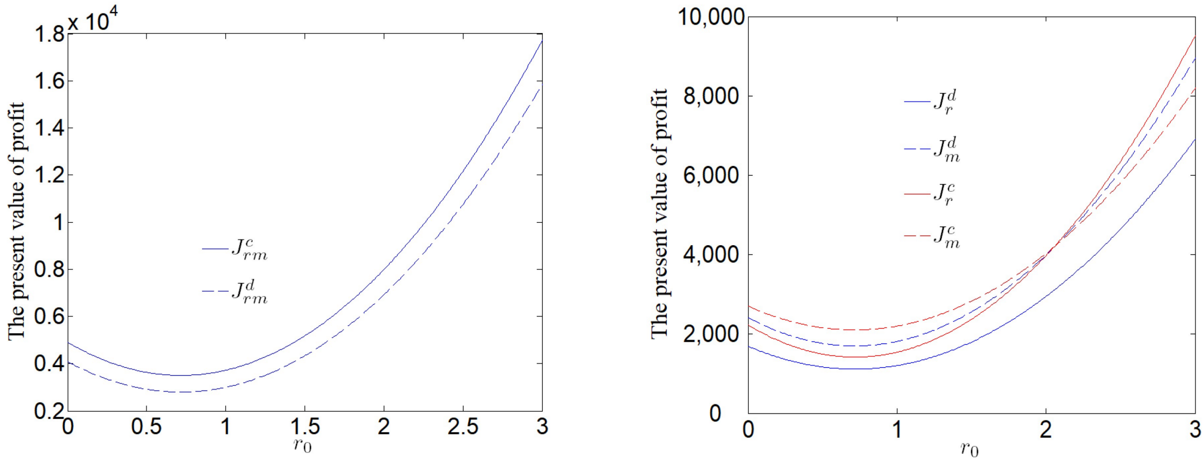


Figure 1. Changes of supply chain profits under different decision-making situations.

As shown in Figure 1, the profit of supplier under the timely delivery cost sharing mechanism increases with the increase of the proportion of cost sharing. Within the range of [0.21, 1], the supplier's profit is not less than the supplier's profit without contract mechanism. Within the range of [0, 0.21], the supplier's profit is less than that under the non-contract mechanism. When within the range of [0, 0.95], the retailer's profit is not less than that under the non-contract mechanism. Within the range of [0.91, 1], the retailer's profit is less than that under the non-contract mechanism. Therefore, it can be seen that when the cost sharing proportion is within the range of [0.21, 0.95], the profits of the supplier and the retailer are not less than that under the non-contract mechanism.

Next, the changes of the sensitive factors of consumers' timely delivery level to profits under the cost sharing coordination mechanism and without coordination mechanism are plotted, as shown in Figure 2.

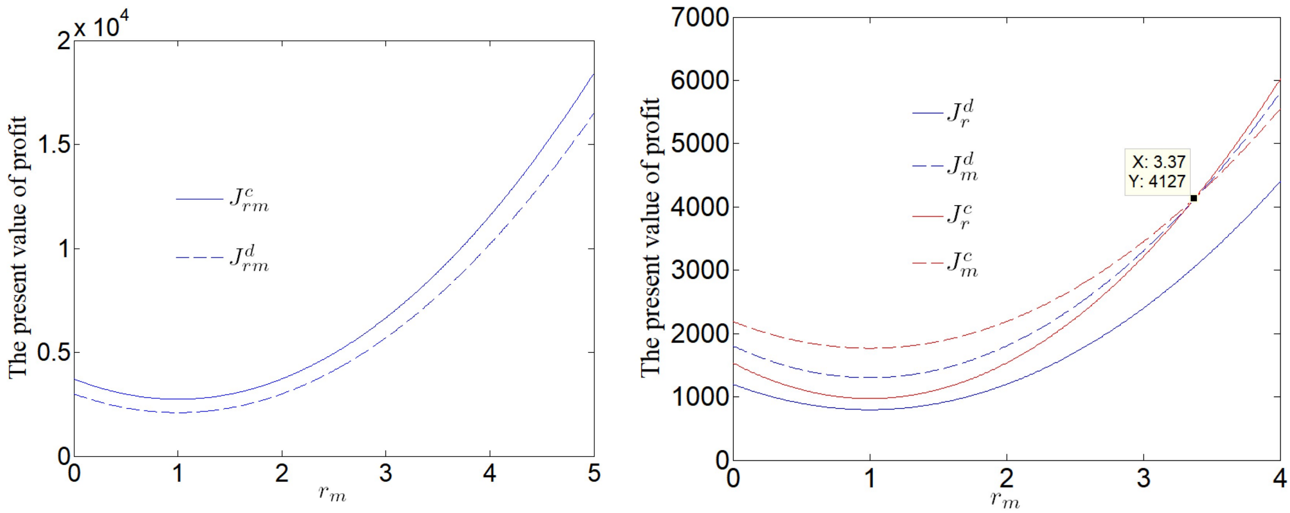
From the above, $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$, we can get the value range of γ_0 as $(0, 2)$. Figure 2a shows that within the range $\gamma_0 \in (0, 2)$, the overall profit of the supply chain decreases first and then increases with the increase of the sensitive factor of the timely delivery level of consumers under centralized decision-making and decentralized situations. When the sensitivity factor of timely delivery is small, the increase of the timely delivery level sensitive factor will increase the overall sales revenue of the supply chain, but at the same time, it will lead to the increase of the input cost of the timely delivery efforts of the supply chain, so that the increase of the cost is greater than the increase of the sales revenue, which will reduce the profits of the supply chain. When the sensitivity factor of timely delivery level is high, the increase of timely delivery cost is less than that of sales revenue, resulting in the increase of supply chain profits. From Figure 2a, it can also be concluded that the supply chain profit under centralized decision-making is greater than that under decentralized decision-making. Figure 2b shows that when the cost sharing proportion is $k = 0.6$, the profits of enterprises under centralized decision-making and decentralized decision-making will decrease first and then increase with the increase of the sensitivity factor of timely delivery level. When the sensitivity factor of timely delivery is small, the increase of the sensitivity factor of timely delivery level will increase the income of supplier and retailer, but at the same time, it will lead to the increase of the input cost of enterprise delivery efforts, so that the increase of cost is greater than the increase of sales revenue, which will reduce the profits of supplier and retailer. When the sensitive factor of timely delivery level is high, the increase of delivery cost is less than that of sales revenue, resulting in the increase of profits of supplier and retailer. It can also be seen that the profits of supplier and retailer under centralized decision-making are greater than those of enterprises under decentralized decision-making.



(**a**) Impact of γ_0 on supply chain profit. (**b**) Impact of γ_0 on enterprises profits.

Figure 2. Impact of sensitive factor γ_0 on profits under decentralized and centralized decisions.

Next, the changes of the sensitive factor of product quality to profits under the cost sharing coordination mechanism and without the coordination mechanism are plotted, as shown in Figure 3.



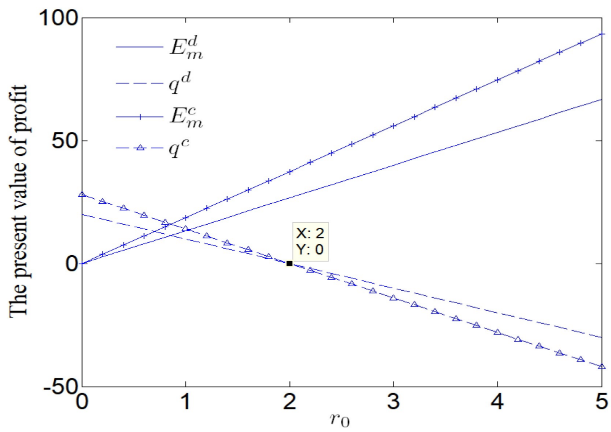
(**a**) Impact of γ_m on supply chain profit. (**b**) Impact of γ_m on enterprises profits.

Figure 3. Impact of sensitive factor γ_m on profits under decentralized and centralized decisions.

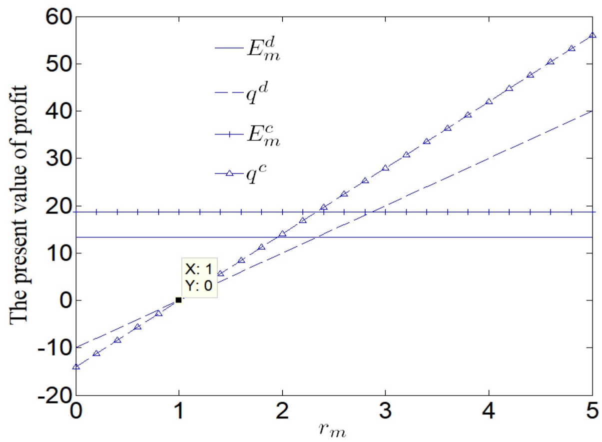
From the above, we can know that $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$, and the value range of γ_m is $(1, \infty)$. Figure 3a shows that in the range of $\gamma_m \in (1, \infty)$, the overall profit of the supply chain increases with the increase of consumer quality-sensitive factor under centralized decision-making and decentralized decision-making. As the increase of quality-sensitive factor will lead to the improvement of product quality, and then the increase the overall sales revenue of the supply chain. Although it will also lead to the increase of supply chain quality input cost, the increase of the cost is less than that of the sales revenue, resulting in the increase of supply chain profit. It can also be seen that the supply chain profit under centralized decision-making is greater than that under decentralized decision-making. Figure 3b shows that when the delivery cost sharing proportion $k = 0.6$, in the range $\gamma_m \in (1, \infty)$, the profits of enterprises in the supply chain increase with the increase of consumer quality-sensitive factor under centralized decision-making and decentralized decision-making. For the supplier, the increase of quality-sensitive factors will lead to the improvement of the product quality level, and then increases the income of enterprises. Although it will also lead to the increase of supplier quality input cost, the increase of the cost is less than that of the sales revenue, resulting in the increase of supplier profit. For the retailer, the increase of quality-sensitive factors will increase the sales revenue of the retailer without increasing the quality input cost.

It is also noted that when $\gamma_m > 3.37$, the supplier's profit under the cost sharing mechanism is less than that under the decentralized decision-making, which shows that when the cost sharing proportion $k = 0.6$, the supply chain cannot coordinate effectively. Therefore, when the cost sharing proportion $k = 0.6$, the value range of γ_m is $(1, 3.37)$. When $\gamma_m \in (1, 3.37)$, the retailer's profit under the cost sharing mechanism is greater than that under the decentralized decision-making.

Next, we analyze how the sensitive factor γ_0 or γ_m affects the changes of timely delivery or product quality under the cost sharing coordination mechanism and no coordination mechanism, as shown in Figure 4.



(**a**) Impact of γ_0 on decision variables.



(**b**) Impact of γ_m on decision variables.

Figure 4. Impact of sensitive factors γ_0 and γ_m on decision variables in decentralized and centralized decision-making situations.

From the above, we can know that $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$, and the value range of γ_0 is $(1, 2)$. Figure 4a shows that in the range of $\gamma_0 \in (1, 2)$, the product quality level decreases with the increase of consumer delivery sensitivity under centralized decision-making and decentralized decision-making. In practice, timely delivery is very important to consumers, so the retailer will put forward higher requirements for the supplier to deliver on time. Then, the supplier will put limited capacity and resources into timely delivery, but this will also affect the product quality. In addition, we can also find that the timely delivery level of suppliers is positively correlated with the delivery sensitivity coefficient of consumers.

When $\gamma_0 = 1$, from $\gamma_m > \frac{\beta}{\rho+\delta}\gamma_0$, the value range of γ_m is $(1, \infty)$. Figure 4b shows that when $\gamma_m \in (1, \infty)$, the more sensitive consumers are to quality, it will help to improve the quality of products, but will not affect the level of timely delivery. From Figure 4a,b, it can be found that the product quality level and timely delivery level under centralized decision-making are greater than those under decentralized decision-making.

Finally, the change of timely delivery level with time is plotted, as shown in Figure 5.

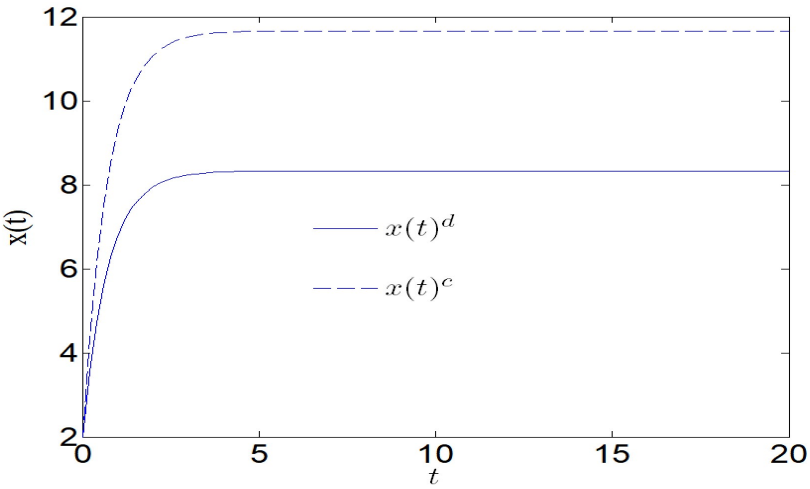


Figure 5. Changes of supplier's timely delivery level with time.

It can be seen from Figure 5 that in the case of decentralized decision-making and centralized decision-making, the timely delivery level of the supplier increases with time and tends to a stable value. In addition, the timely delivery level of the supplier under centralized decision-making is higher than that under decentralized decision-making.

8. Results and Discussion

Generally speaking, when an order is placed, consumers expect to receive the product in time. In particular, during the shopping festival, Christmas day, and the promotion period of merchants, the large-scale purchases of consumers have put more production pressure on the upstream manufacturers. However, in order to complete the order, the manufacturers put limited human, material, and other resources into timely delivery, and do not pay more attention to product quality. Thus, it is easy to cause product quality disputes and lead to the decline of enterprise reputation. In view of this, the supply chain that is composed of a single supplier and a single retailer is taken as the research object. The differential equation is constructed to reflect the negative correlation and dynamic relationship between product quality and timely delivery. The timely delivery, product quality, marketing strategies, and supply chain profits under different decision-making modes are analyzed and compared.

By comparing the optimal value and profit of different decision-making modes, the following findings are obtained:

(1) Under certain conditions, the product quality, marketing, and delivery strategies under centralized decision-making are greater than those under decentralized decision-making. Under centralized decision-making and decentralized decision-making, the product quality level decreases with the increase of consumers' timely delivery sensitivity. The level of timely delivery of the supplier increases with the increase of consumer-sensitive factor of timely delivery. The increase of consumer quality-sensitive factors will also improve the quality level of products but will not affect the level of timely delivery.

(2) For the supply chain profits, the supply chain profit, the supplier profit, and the retailer under centralized decision-making are greater than those under decentralized decision-making. First of all, there is a threshold for the timely delivery-sensitive factor of consumers. When the timely delivery-sensitive factor of consumers is lower than the threshold, the higher the timely delivery-sensitive factor is, the smaller the profit of the supply chain system is. When the timely delivery-sensitive factor of consumers is higher than the threshold, the higher the timely delivery-sensitive factor, the greater the profit of the supply chain system. Further, it can be concluded that under the centralized decision-making and decentralized decision-making situations,

the profits of enterprises in the supply chain decrease first and then increase with the increase of timely delivery-sensitive factor. Finally, supply chain profit, supplier profit, and retailer profit increase with the increase of the consumer quality-sensitive factor under centralized decision-making and decentralized decision-making.

(3) Timely delivery cost sharing contract can coordinate the supply chain. That is, there is a range of timely delivery cost sharing proportions in which the profits of the supplier and the retailer are greater than their respective profit without a cost sharing contract. Under the condition that both the supplier and retailer accept the cost sharing contract of timely delivery, the cost sharing proportion that is provided by the retailer is negatively related to the profit of the retailer. On the contrary, the cost sharing ratio that is provided by the retailer is positively related to the profit of the supplier.

(4) Under the two decision-making situations, the level of timely delivery increases with time and tends to a stable value. The level of timely delivery under centralized decision-making is greater than that under decentralized decision-making.

With the rapid development of social economy, the effective strategies of timely delivery, product quality, and marketing are more and more important to enterprises, and have been studied by many scholars. This paper analyzes and compares the supply chain strategies and profits under different decision-making modes and believes that the profit of supply chain under centralized decision-making is greater than that under decentralized decision-making (Ghosh et al. [32]), and the cost sharing contract can achieve the Pareto optimization of supply chain members (Zhou et al. [13], Chakraborty et al. [33]). Consumers' timely delivery, product quality, and marketing sensitivity coefficient have a positive impact on their respective strategies (Roy and Sana [22], Modak and Kelle [19], Glock et al. [20], Ranjan and Jha [28]), and, with time, each decision value tends to be stable (Zhou et al. [13], Qiu et al. [15]).

However, the biggest difference between this paper and the previous literature is that we consider the negative correlation between product quality and timely delivery. The following findings were obtained: (i) the higher the sensitivity of consumers to timely delivery is not conducive to improving the product quality of enterprises. As the supplier has put limited resources into timely delivery in order to complete the order, and has not paid more attention to product quality, this results in the decline of product quality. (ii) When the consumer's sensitivity to timely delivery is low, the sensitivity to timely delivery is negatively related to the enterprise's profit. When the consumer's sensitivity to timely delivery exceeds a certain threshold, the sensitivity to timely delivery is positively related to the enterprise's profit. The main reasons are as follows: first, the lower sensitivity to timely delivery causes the enterprise to pay more attention to product quality, resulting in an increase in quality cost. Second, the lower sensitivity has little impact on demand, resulting in lower sales revenue for enterprises. According to the above, when the sensitivity is less than a certain threshold, the increase in sales revenue due to the increase in sensitivity is less than the increase in cost, resulting in a decrease in enterprise profits. Although this paper is similar to some of the conclusions of the research literature Huang et al. [6], the essential difference between this paper and the previous literature is that this paper considers the negative correlation between timely delivery and product quality. This is also the innovation of this paper.

We only consider the Nash non-cooperative game in the decentralized decision model. If repeated game, evolutionary game, non-zero -um game, and Stackelberg game are studied (Abdalzaher et.al. [34,35], Abdalzaher and Muta [36], He et al. [37]), then compared with this paper, what are the differences between optimal decisions and how sensitive factors affect optimal decisions, etc., these topics will become more interesting.

9. Conclusions

When some social public events occur or merchants promote sales, the demand (orders) for some commodities suddenly increases. According to the order, the supplier needs to deliver the qualified products to the downstream customers in time. However, due to limited resources, it is difficult for suppliers to ensure that all the products are delivered in time and the quality of products is qualified. Suppliers are likely to put more resources into completing customer orders on time, while ignoring the product quality. This will easily lead to the occurrence of product quality events, and then that affects the goodwill of enterprise products. The previous literature rarely studies this phenomenon, that is, the relationship between the delivery level and product quality. The innovations of this paper are as follows: first, based on the above facts, a negative and dynamic correlation between the delivery level and the quality level is established, which has rarely been involved in previous studies. Secondly, it studies the joint decision-making of timely delivery, product quality, and marketing under different decision-making modes. Thirdly, it compares the timely delivery, product quality, marketing strategies, and supply chain profits under different decision-making modes and designs the cost sharing contract mechanism.

The theoretical contributions of this paper include the following aspects: first, the negative correlation between timely delivery and product quality is considered in the supply chain decision model. Secondly, the joint decision models of timely delivery, product quality, and marketing are constructed. It enriches the theory of supply chain quality, marketing, and delivery decision-making. The practical contributions include the following aspects: first, this paper provides guidance for enterprise managers when making decisions on quality, marketing, and delivery. Secondly, it provides the basis for enterprise managers to formulate effective cooperation models.

By comparing the strategies and profits under different decision-making modes, the following conclusions are obtained: (i) Under certain conditions, the product quality, marketing, and delivery strategies under centralized decision-making are greater than those under decentralized decision-making. The more sensitive the consumers are to timely delivery, the more unfavorable it is to improve product quality. (ii) Supply chain profit, supplier profit, and retailer profit under centralized decision-making are all greater than those under decentralized decision-making. There is a nonlinear relationship between the sensitivity coefficient of on-time delivery and the profit of the supply chain. (iii) Timely delivery and cost sharing contracts can coordinate the supply chain. That is to say, there is a range of cost sharing proportions of timely delivery, in which the profits of the supplier and the retailer are greater than those of the respective profits without cost sharing contracts.

From the above conclusions, we can draw some research implications. (i) Enterprises should realize that the way to increase profits is not only limited to the way to reduce costs, but also a more effective way to improve timely delivery, product quality, and marketing level. For example, optimize the process, shorten the delivery time of orders, improve the reliability and performance of products, increase the investment in product quality testing, cultivate the knowledge level of enterprise marketing staff, and create an experiential shopping environment. (ii) When making decisions, enterprises should not only collect their own data, but also obtain consumer data through market survey questionnaires. Especially, when consumers are less sensitive to timely delivery, enterprises should give some coupons and small gifts to consumers in exchange for the extension of delivery time, and invest limited resources to improve product quality. When consumers are highly sensitive to timely delivery, they can outsource some orders to cost-effective and professional third-party enterprises, which not only improves the delivery rate but also improves the product quality. (iii) Alliance is a way for enterprises to realize resource complementarity and gain greater competitive advantages. Suppliers can establish strategic alliances with retailers, form cooperative partnerships, and share the marketing costs of retailers, so as to encourage retailers to conduct marketing more actively and improve market demand.

However, the proposed model has some drawbacks. These can be overcome by extending it in many ways in the future. (i) The marginal profits of the supplier and the retailer is fixed. The model will be more realistic if we consider price as a variable. (ii) The proposed model considers a supply chain that is composed of a single supplier and a single retailer. Supplier competition or retailer competition can be considered in the future.

(iii) This paper assumes that consumers are homogeneous. In fact, consumers have different preferences for the level of timely delivery, and the demand function will also be different.

Author Contributions: Conceptualization, H.Q. and X.Z.; methodology, M.J. and X.L.; software, X.L.; formal analysis, M.J.; investigation, X.L.; writing—original draft, M.J.; writing—review and editing, H.Q.; project administration H.Q. and X.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Social Science Foundation of China Western Project Research on the Measurement and Countermeasures of Factor Market Distortion in China from the Perspective of Spatial Heterogeneity"(No. 20XJL004), the National Social Science Foundation of China Western Project "Research on the Dilemma, Causes and Strategies of High-quality Development of Rural Logistics Driven by Digital Economy"(No.20XJY011), the Fujian Innovation Strategy Research Project" Research on the Difference of Factor Market Distortion Affecting Enterprise Innovation"(No.2020R0072), Jiangxi Provincial Situation Investigation Project(22SQ12), and Science and Technology Research Project of Jiangxi Provincial Education Department (No.GJJ213106).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Yang, H.; Chen, W. Retailer-driven carbon emission abatement with consumer environmental awareness and carbon tax: Revenue-sharing versus cost-sharing. *Omega* **2018**, *78*, 179–191. [CrossRef]
2. Guars, Z.; Ye, T.; Yin, R. Channel coordination under Nash bargaining fairness concerns in differential games of goodwill accumulation. *Eur. J. Oper. Res.* **2020**, *285*, 916–930.
3. Xu, X.; He, P.; Xu, H.; Zhang, Q. Supply chain coordination with green technology under cap-and-trade regulation. *Int. J. Prod. Econ.* **2017**, *183*, 433–442. [CrossRef]
4. Tian, H.P.; Liu, C.X.; Liu, S.K. Research on joint decision of quality and delivery time in manufacturing outsourcing. *J. Manag. Eng.* **2010**, *24*, 161–166.
5. Hong, J.T.; Huang, P. Research on quality coordination of supply chain based on differential game. *Chin. J. Manag. Sci.* **2016**, *24*, 100–107.
6. Huang, J.; Yang, W.; Li, L. Correlation pricing decisions of lead time and quality under stochastic demand. *Comput. Integr. Manuf. Syst.* **2016**, *22*, 1747–1758.
7. Wang, Y.; Tam, D.Q. Research on product warranty period and quality assurance service investment strategy of inferior durable goods manufacturers. *Chin. J. Manag. Sci.* **2018**, *26*, 142–151.
8. Voros, J. An analysis of the dynamic price-quality relationship. *Eur. J. Oper. Res.* **2019**, *277*, 1037–1045. [CrossRef]
9. Chen, Y.; Huang, S.L.; Mishra, A.K.; Wang, X.H. Effects of input capacity constraints on food quality and regulation mechanism design for food safety management. *Ecol. Model.* **2018**, *385*, 89–95. [CrossRef]
10. Cellini, R.; Siciliani, L.; Straume, O.R. A dynamic model of quality competition with endogenous prices. *J. Econ. Dyn. Control* **2018**, *94*, 190–206. [CrossRef]
11. Heydari, J.; Ghasemi, M. A revenue sharing contract for reverse supply chain coordination under stochastic quality of returned products and uncertain re-manufacturing capacity. *J. Clean. Prod.* **2018**, *197*, 607–615. [CrossRef]
12. Ma, D.; Hu, J.; Wang, W. Differential game of product–service supply chain considering consumers' reference effect and supply chain members' reciprocity altruism in the online-to-offline mode. *Ann. Oper. Res.* **2021**, *304*, 263–297. [CrossRef]
13. Zhou, X.; Xu, B.; Xie, F.; Li, Y. Research on Quality Decisions and Coordination with Reference Effect in Dual-Channel Supply Chain. *Sustainability* **2020**, *12*, 2296. [CrossRef]
14. Ruidas, S.; Seikh, M.R.; Nayak, P.K. A production inventory model for high-tech products involving two production runs and a product variation. *J. Ind. Manag. Optim.* **2021**, *in press*. [CrossRef]
15. Qiu, R.; Yu, Y.; Sun, M. Supply chain coordination by contracts considering dynamic reference quality effect under the O2O environment. *Comput. Industr. Eng.* **2022**, *163*, 107802. [CrossRef]
16. Duan, Y.R.; Xu, C.; Huo, J.Z. Dynamic pricing and management under service level Research and Management Science constraints. *Oper. Res. Manag. Sci.* **2019**, *28*, 1–7.
17. Wang, D.E.; Zhang, X.M.; Zhou, M.S.; Gao, H.L.; Dan, B. Dynamic pricing and coordination of product and service supply chain considering consumer strategic behavior. *Sys. Eng. Theory Pract.* **2017**, *37*, 3052–3065.
18. Tian, W.; Ge, B. Differential dame analysis of two-channel supply chain under competition and cooperation of manufacturers' service efforts. *Lnd. Eng. Manag.* **2019**, *24*, 136–143.

19. Modak, N.M.; Kelle, P. Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand. *Eur. J. Oper. Res.* **2018**, *272*, 147–161. [CrossRef]
20. Glock, C.H.; Rekik, Y.; Ries, J.M. A coordination mechanism for supply chains with capacity expansions and order-dependent lead times. *Eur. J. Oper. Res.* **2020**, *285*, 247–262. [CrossRef]
21. Qiu, R.; Chu, X.; Sun, Y. Dual-channel supply chain decision model based on robust optimization under price and delivery-time dependent demand. *Chin. J. Manag. Sci.* **2021**, *29*, 1–12.
22. Roy, M.D.; Sana, S.S. Inter-dependent lead-time and ordering cost reduction strategy: A supply chain model with quality control, lead-time dependent backorder and price-sensitive stochastic demand. *Opsearch* **2021**, *58*, 690–710.
23. Yue, J.; Austin, J.; Huang, Z.; Chen, B. Pricing and advertisement in a manufacturer–retailer supply chain. *Eur. J. Oper. Res.* **2013**, *231*, 492–502. [CrossRef]
24. Song, J.; Li, F.; Wu, D.; Liang, L.; Dolgui, A. Supply chain coordination through integration of innovation effort and advertising. *Appl. Math. Model.* **2017**, *49*, 108–123. [CrossRef]
25. Pu, X.; Gong, L.; Zhang, X. The incentive mechanism design for promotion effort considering the retailer's fairness preference. *Sys. Eng. Theory Pract.* **2015**, *35*, 2271–2279.
26. Gao, J.H.; Han, H.S.; Hou, L.T.; Wang, H.Y. Decision-making in closed-loop supply chain with retailer considering products' green degree and sales effort. *Manag. Rev.* **2015**, *27*, 187–196.
27. Ma, P.; Shang, J.; Wang, H. Enhancing corporate social responsibility: Contract design under information asymmetry. *Omega* **2017**, *67*, 19–30. [CrossRef]
28. Ranjan, A.; Jha, J.K. Pricing and coordination strategies of a dual-channel supply chain considering green quality and sales effort. *J. Clean. Prod.* **2019**, *218*, 409–424. [CrossRef]
29. Ezimadu, P. A mathematical model of cooperative advertising support to the followers in a manufacturer-distributor-retailer supply chain. *Int. J. Oper. Res.* **2022**, *44*, 141–170. [CrossRef]
30. Lee, S.; Park, S.J. Who should lead carbon emissions reductions upstream vs. downstream firms. *Int. J. Prod. Econ.* **2020**, *230*, 107790. [CrossRef]
31. Zhao, L.; Sun, J.; Zhang, H. Coordination mechanism of marketing cooperation in low-carbon product supply chain based on differential game. *J. Ind. Eng. Manag.* **2018**, *32*, 105–110.
32. Ghosh, S.K.; Seikh, M.R.; Chakrabortty, M. Pricing Strategy and Channel Co-ordination in a Two-Echelon Supply Chain Under Stochastic Demand. *Int. J. Appl. Comput. Math* **2020**, *6*, 28. [CrossRef]
33. Chakraborty, T.; Satyaveer, S.C.; Mustapha, O. Cost-sharing mechanism for product quality improvement in a supply chain under competition. *Int. J. Prod. Econ.* **2019**, *208*, 566–587. [CrossRef]
34. Abdalzaher, M.S.; Samy, L.; Muta, O. Non-zero-sum game-based trust model toenhance wireless sensor networks securityfor IoT applications. *IET Wirel. Sens. Syst.* **2019**, *9*, 218–226. [CrossRef]
35. Abdalzaher, M.S.; Seddik, K.; Elsabrouty, M.; Muta, O.; Furukawa, H.; Abdel-Rahman, A. Game Theory Meets Wireless Sensor Networks Security Requirements and Threats Mitigation: A Survey. *Sensors* **2016**, *16*, 1003. [CrossRef]
36. Abdalzaher, M.S.; Muta, O. A Game-Theoretic Approach for Enhancing Security and Data Trustworthiness in IoT Applications. *IEEE Internet Things* **2020**, *7*, 11250–11261. [CrossRef]
37. He, X.; Prasad, A.; Sethi, S.P.; Gutierrez, G.J. A survey of Stackelberg differential game models in supply and marketing channels. *J. Syst. Sci. Syst. Eng.* **2007**, *16*, 385–413. [CrossRef]