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A Decision Model for Free-Floating Car-Sharing Providers for Sustainable and Resilient Supply Chains

Wei Zhou [1], Haixia Wang [2], Victor Shi [3] and Xiding Chen [4,*]

[1] School of International Business, Southwestern University of Finance and Economics, Chengdu 611130, China; zw117120291001@smail.swufe.edu.cn
[2] School of Business Administration, Guangdong University of Finance, Guangzhou 510320, China; 47-156@gduf.edu.cn
[3] Lazaridis School of Business & Economics, Wilfrid Laurier University, Waterloo, ON N2L3C5, Canada; cshi@wlu.ca
[4] School of Finance and Trade, Wenzhou Business College, Wenzhou 325035, China
* Correspondence: 00201142@wzbc.edu.cn

Abstract: For green and sustainable supply chains, transportation resilience is a critical issue. Car Sharing is an effective way to improve transportation resilience. The emerging car-sharing industry continues to attract a lot of investment, but few companies in the industry are profitable. Indeed, numerical experiments based on dynamic models in this paper showed that it was challenging for a car-sharing company to be profitable. As the numerical experiments followed the fractional factorial designs, from the factor analysis, it is suggested that a new car-sharing business first study the external business environment. Even if the external environment is sound, the company still needs to pay attention to internal operations management. Moreover, when the company decides the number of cars it owns and the fleet size, it should consider factors including variable daily expenses, maintenance costs, salvage value, and commission.

Keywords: car-sharing; green supply chain; sustainability; supply chain resilience

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1. Introduction

The green and resilient supply chain has been highly valued these years. However, due to its objectives such as reducing waste, there are relatively fewer resources as safety stock. On the other hand, disruption risk becomes a vital issue considered in the green supply chain management [1]. Pandemics such as COVID-19 can result in severe supply chain disruptions worldwide. Extensive research has been done on sustainable resilient supply chains ([2–4]). In particular, car-sharing possibly improves the robustness of supply chain networks, to succeed, many key elements need to be identified and in-depth research is required [5]. This paper considers optimal decisions for car-sharing providers to support their development.

The sharing economy has become a popular concept in recent years. The car-sharing industry has been expanding rapidly since its first appearance in the late 1980s in Europe. In October 2010, over 1100 cities in the world had car-sharing companies [6] . Experts' forecasts for the industry are optimistic [7] , but this does not mean that car-sharing companies are very profitable. For example, Zipcar, founded in 2000, took 12 years to become profitable [8]. This paper identifies critical factors for a car-sharing company to be profitable where it can operate in one of three ways: do not own the cars, own a proportion of the cars, or own all the cars.

For improving the sustainable supply chain, people consider electric car sharing. However, until now, the government subsidy is still crucial for the service provider of electric cars [9]. This paper studies several factors excluding the government subsidy. First, this research can be useful for the government to make subsidy policies on the sustainable car-sharing industry. Second, when people consider whether to start a car-sharing business, they can apply the models in this paper to estimate the expected profits, and business risk. Third, the results derived based on these models can help them design business plans. Fourth, based on the identified critical factors, managers can quickly focus on key management problems.

The remainder of this paper is organized as follows: Section 2 provides the literature review. Section 3 describes a model. Section 4 provides the analysis of it. Section 5 discusses some numerical experiments. Section 6 focuses the question of whether a car-sharing company should own cars. Section 7 explores the scenario in which a car-sharing company considers its profit in infinite time. Section 8 provides conclusions and ideas for future research.

2. Literature Review

The car-sharing industry has been studied from different views. Research related to this work mainly includes the following aspects: significant factors, classification of car-sharing systems, fleet size, and bicycle-sharing business.

Factors such as the perceived risk of scarcity, monetary motivations, environmental and social concerns, flexibility, and political motives drive customers to participate in the sharing economy ([10–14]). However, [15] found that environmental concern was of minor importance. One possible reason for the different conclusions about environmental concern is the divided opinions on environmental issues. In addition, the conclusion depends on the research sample. [12] also pointed out that the behavior of consumers differed across cultures, though materialism was a global phenomenon.

Published works on the car-sharing industry also discussed some factors (besides those factors mentioned in the previous paragraph) in different application backgrounds, but the discussed factors were not classified well. [16] observed that since not many people were likely to take a trip together in San Francisco, small and fuel-efficient cars became popular in the car-sharing business. [17] identified key factors such as utility, trust, and cost savings that would induce people to use a sharing option. [18] found that an auto manufacturer could improve fuel efficiency and profit by introducing car-sharing into its business, but this might not always benefit the environment. [19,20] concluded that potential customers are concerned about car availability when deciding whether to join a car-sharing organization. [21] argued that social networking is also an essential factor in the decision. [22] claimed self-efficacy as the most important factor according to data from Beijing. Through a survey conducted in London, Madrid, Paris, and Tokyo [23] identified some sociodemographic reasons why people choose car clubs or private vehicles. [24] showed that, when a potential customer considers whether to use a shared car, he/she may need to consider departure time, route, activity sequence, location, duration, and parking location. They demonstrated that fleet size, fleet distribution, and parking fees could significantly impact the decision. [25] considered vehicle rental fee, fixed vehicle cost, variable vehicle cost, vehicle relocation cost, station operating cost, etc. Similarly, [26] considered vehicle rental fees, vehicle maintenance costs, vehicle relocation costs, depot maintenance costs, and vehicle depreciation costs when they studied a company's profitability. [27] took manpower cost and maintenance costs into consideration. Overall, similar costs were considered, although some costs were described by different ways. As there are many related factors, for management convenience, this paper not only identifies key factors, but also classifies key factors into external environment factors and internal operation factors.

It is easy to be confused by terms used in the literature on the general sharing economy. [10] compared access with ownership and sharing by considering six dimensions: temporality, anonymity, market mediation, consumer involvement, the type of accessed object, and political consumerism, where political consumerism means that customers show their ideological interests in society, business, and government through their choice of mode of consumption. [28] stated that collaborative consumption is different from other forms of consumption because collaborative consumption involves three actors (a platform provider, a peer service provider, and a customer).

Car-sharing systems can be classified into round-trip and one-way. In terms of stations, the systems can be classified into free-floating car-sharing systems, peer-to-peer (also known as person-to-person), and station-based. In a free-floating system, customers can return a car anywhere in a certain area; in peer-to-peer, a car is available only when its owner is not using it and would be returned to a pre-arranged point; in the station-based system, customers pick up and return cars at fixed stations. Thus, a free-floating system is more flexible for customers than a station-based system.

In the one-way car-sharing mode, the relationship between the parking capacity of each station and fleet size is important. [29] provided a mixed queueing network model for a car-sharing operator to determine the fleet size and station capacities simultaneously. In a one-way, station-based car-sharing system, there is often an imbalance between demand and availability. [30] presented an approach with low computation time to relocate cars to match demand and thus improve the usage rate of vehicles and profit. [31] developed a mixed integer nonlinear programming model to make a one-way, station-based system achieve relative balance on vehicle stock by determining dynamic trip prices and relocating vehicles. On average, with the model, prices will be higher, and less demand will be served, but the profit will be higher since expenses on the fleet of vehicles and parking spaces are reduced. The models introduced in this paper can provide comments for people to start the business, and also provide help on daily operations.

Research on fleet size decisions, the number of parking spots, and maintenance costs is still limited. This paper will demonstrate the profit change when a company shifts from providing all cars to not providing any car, thus becoming a pure platform provider. It will be shown that a car-sharing company operating in the access-based and free-floating form may have a high risk of losing money if it cannot control the scrap rate of its cars. A high scrap rate means that the company has to spend a lot of money to purchase new cars or maintain its current cars. It is a challenging task to control the scrap rate. Regulations have been developed to protect consumers [32]. [33] conducted an experiment with 355 participants to study whether consumers support a governance system. In the experiment, 81.7% of participants supported governance because they believed that human beings were egoistic. The other participants did not support governance because they worried about its negative consequences, such as losing self-determination. However, it may be necessary to regulate car quality so that both service providers and customers can benefit from maintenance cost reduction.

The car-sharing business is close to the bicycle-sharing business, and many reports have discussed the attrition rate in the bicycle-sharing industry. Attrition here means that a bicycle is damaged or lost. For example, a high attrition rate was claimed as a key factor for bicycle-sharing providers going bankrupt, and a high attrition rate is partly due to consumers' vandalism. In China, the first bankrupt bicycle-sharing provider lost 90% of its bicycles [34], while the second one lost over 90% of its bicycles [35]. [36] stated that bicycle-sharing providers had to significantly increase their costs to reduce the attrition rate.

A lot of consumer research has been done on the car-sharing business. [37] studied a company that simultaneously provided a station-based car-sharing scheme and a free-floating scheme. Customers had to register for each scheme separately if they wanted to join both schemes. The study found that customers of each scheme were different in age and consuming behavior. [38] confirmed that most customers selecting the free-floating scheme used it for discretionary trips when there was insufficient public transport in an area. [39] investigated by multi-agent simulation the impacts of different parking prices on free-floating car-sharing. They found that demand for free-floating car-sharing would increase if parking prices increased because fewer people would drive private cars. [40] argued that it is not enough to estimate the demand for car-sharing from the data of the current transportation system. They suggested that travel behavior in an area should be simulated before estimating the demand in that area. [41] simulated the operation of two station-based and one free-floating service provider for one week. They found that customers might have very different behaviors on a weekday and a day on a weekend. On the other hand, the car-sharing business has changed to the transportation system. [42] observed that some one-car households gave up their cars, so the average number of vehicles per household dropped significantly. [43] suggested using simulation to find the effects of car-sharing on transportation. [44] used a multi-stage models to study relocation strategies.

3. Model

A company is considering investing in a new car-sharing business. Without loss of generality, assume that the company starts at time 0 without any cars, and plans to purchase Q cars at time 0. The company wants to profit from providing car-sharing in T periods under the free-floating form. Let t be the index of the period, which may be a subscript, but which may be omitted if there is no possibility of confusion. Suppose c is the price of each new car. Assume that each car can be in one of five states—new, available, in use, undergoing maintenance, and scrapped-indexed as 1, 2, 3, 4, or 5, respectively. (See Table 1.) The maintenance state means that the car needs cleaning, quality check, repair, etc. The scrapped state means that the car will never be able to provide service. If the company purchases a new car in a period, the new car will become available in service in the next period. Assuming that an available car in a period will still be available in the next period with probability p_{22}, it will be available for customers' use for the next period with probability p_{23}, and it needs maintenance in the next period to match the service requirements with probability p_{24}. If a car is being used, it will still be being used if the customer uses it for more than one period; it will be in maintenance if the customer uses it just for one period, and some maintenance works are needed. If a car is in maintenance, it will be available in the next period if the maintenance work can fix all the problems in one period; it will be still in maintenance if the maintenance work needs more than one period; it will be scrapped if the maintenance staff finds out its quality is too bad to perform maintenance works. Suppose scrapped cars are sold with the salvage value of r_5 per car. Moreover, suppose the company always owns a total of Q cars in states 1 to 4.

Table 1. Possible states and state changes of a car, where "$\surd$" indicates a possible change.

State Index	Meaning	From State	To State 1	2	3	4	5
1	A car is new.	1		$\surd$			
2	A car is available.	2		$\surd$	$\surd$	$\surd$	
3	A car is being used.	3			$\surd$	$\surd$	
4	A car is in maintenance.	4		$\surd$		$\surd$	$\surd$
5	A car is scrapped.	5					

Suppose each of the arriving customers needs/demands one car. Suppose each of the customers pays r_3 for using a car for a period, where r_3 is a positive constant. Customers are sensitive to the convenience of picking up a car. If Q is large, customers will have a high probability for obtaining an available car nearby. Thus, suppose that the arrival of customers follows a Poisson distribution with arriving rate $\lambda(Q)$ per period, where $\lambda(Q)$ is an increasing function of Q.

Assume that, for $t = 1, \ldots, T-1$, in period t, the following events happen at the beginning of each period in the sequence:

(1) The company counts the number of available cars, the number of cars being used, and the number of cars in the maintenance. They are denoted by m_2, m_3, and m_4, respectively.
(2) The company posts the information on the number of available cars.
(3) Customers arrive.

(4) The company spends daily expenses $C_0(c, Q)$ for the daily operations.
(5) The company spends $C_2(c)$, which includes the finance costs and basic operation costs, on each of the remaining available cars.
(6) Suppose the company spends $C_3(c)$, which includes the finance costs and basic operation costs, on each car being used per period, and earns an income of r_3 from each car being used per period.
(7) Some cars in the maintenance state will be scrapped because of their conditions.
(8) The company purchases $Q - m_2 - m_3 - (m_4 - n_{45})$ new cars to keep total Q cars owned by the company. The new cars will be available in the next period.

Since the company spends $C_2(c)$ on each of the remaining available cars, the expense on the remaining available cars will be $(m_2 - d)C_2(c)$ if $d < m_2$; otherwise, it is zero. Suppose that $C_2(c)$ is an increasing function of c. If $d < m_2$, each of the $m_2 - d$ available cars will be in the maintenance state with the probability $p_{24}(C_0)$. If daily expenses are high, a car will have a high probability of being available. Thus, assume that $p_{24}(C_0)$ is a decreasing function of C_0. Let n_{24} be the number of available cars with the change from state 2 to 4 in a period. Hence, $0 \leq n_{24} \leq m_2 - d$.

It follows from (6) above that the difference between incomes and expenses in a period is $(m_3 + d)(r_3 - C_3(c))$. After a car has been used, it moves to the maintenance state, after which it may return to service. A customer will use a car for only one period with a probability p_{34} and will use it for more than one period with a probability $1 - p_{34}$. Let n_{34} be the number of cars that change from state 3 to 4 in one period. We have $0 \leq n_{34} \leq m_3 + d$.

Suppose the probability of scrapping a car in maintenance is $p_{45}(C_0)$. If the company spends more money, then it has a better chance of maintaining the quality of its cars. Therefore, assume that $p_{45}(C_0)$ is a decreasing function of C_0. Let n_{45} be the number of cars in maintenance being scrapped in a period. Thus, $0 \leq n_{45} \leq m_4$. Assume that the company will repair the remaining $m_4 - n_{45}$ cars at the cost of $C_{42}(c)$ per car. Moreover, assume that these $m_4 - n_{45}$ cars will be available in the next period.

Suppose that each car moves from state 3 to 4 or from state 4 to 5 independently of the other cars. A system state is denoted by a vector (m_2, m_3, m_4). Thus, a system state (m_2, m_3, m_4) in a period will change to the system state $(Q - m_3 - d - n_{24},\ m_3 + d - n_{34},\ n_{24} + n_{34})$ in the next period.

At the beginning of the last period, assume that the company will not purchase any new car or repair any car. Assume that the company still spends $C_3(c)$ on each car being used and spends $C_2(c)$ on each of the available cars and cars in maintenance. At the end of the Tth period, assume that all cars will be sold at the average price r_5. Given the system state (m_2, m_3, m_4) at the beginning of the Tth period, the system state at the end of the Tth period will become $(0, 0, d)$ if $d < m_2$. It will become $(0, 0, m_2)$ if $d \geq m_2$. By selling cars, the company adds $(1 - \alpha)Qr_5$ to the revenue of the last period, where α is the discount rate of a period, and the factor $(1 - \alpha)$ is because cars are sold at the end of the last period.

Suppose the company is without any car at time 0. The company does not have any income in the first period, but it has to purchase Q new cars and pay daily expenses. The profit each period later will be discounted to time 0. The company is going to maximize the total discounted expected profit at time 0, and the decision variable is the number of cars, i.e., Q. We will use the superscript "$*$" to denote "optimal".

4. Analysis

If d is the number of customers who arrive in a period, then d cars will be picked up. Hence, the probability of customers picking up d cars in a period is $P(d) = \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)}$, if $d < m_2$. The probability of customers picking up m_2 cars is $P(m_2) = \sum_{d=m_2}^{\infty} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)}$, if $d \geq m_2$.

It is natural to use a three-dimensional vector (m_2, m_3, m_4) to denote a system state. Here, we can convert the three-dimension vector index to a one dimensional index. Given m_2, the value m_3 falls between 0 and $Q - m_2$. Given m_2 and m_3, m_4 falls between 0 and

$Q - m_2 - m_3$. Given Q, since $\sum_{m_2=0}^{Q}\sum_{m_3=0}^{Q-m_2}(Q - m_2 - m_3 + 1) = \frac{(Q+1)(Q+2)(Q+3)}{6}$, there are a total of $\frac{(Q+1)(Q+2)(Q+3)}{6}$ system states. For simplicity, all the system states are arranged in ascending order such as system state $(0, 0, Q)$ is before $(0, 1, Q-1)$, and the arranged system states are indexed from 1 to $\frac{(Q+1)(Q+2)(Q+3)}{6}$. For the system state (m_2, m_3, m_4), its index is given by $\sum_{i=0}^{m_2-1}\sum_{j=0}^{Q-i}(Q - i - j + 1) + \sum_{j=0}^{m_3-1}(Q - m_2 - j + 1) + m_4 + 1 = \frac{m_2(Q+1)(Q+2)}{2} - \frac{(m_2-1)m_2(3Q-m_2+5)}{6} + m_3(Q - m_2 + 1) - \frac{(m_3-1)m_3}{2} + m_4 + 1$. For simplicity, either the vector (m_2, m_3, m_4) or an integral index to present a system state will be used.

Let $\mathbf{P}_{t,(m_2,m_3,m_4),(Q-m_3-d-n_{24},\ m_3+d-n_{34},\ n_{24}+n_{34})}$ be the transition probability matrix, and $\mathbf{R}_{t,(m_2,m_3,m_4),(Q-m_3-d-n_{24},\ m_3+d-n_{34},\ n_{24}+n_{34})}$ be the profit matrix in period t when the system state in period t is (m_2, m_3, m_4), and the system state in the next period is $(Q - m_3 - d - n_{24},\ m_3 + d - n_{34},\ n_{24} + n_{34})$. Each of the matrices has $(Q+1)(Q+2)(Q+3)/6$ rows and columns. An element of these matrices will have three indexes in its subscript: the first index is the period number; the second is the system state before transition; the third is the system state after the transition. When a system state (m_2, m_3, m_4) changes to the system state $(Q - m_3 - d - n_{24},\ m_3 + d - n_{34},\ n_{24} + n_{34})$ in period t, the corresponding transition probability and profit are given by the following expressions:

(1) If $d < m_2$, $0 \le n_{24} \le m_2 - d$, $0 \le n_{34} \le m_3 + d$, $0 \le n_{45} \le m_4$, and $t = 1, \ldots, T-1$, then

$$
\begin{aligned}
&P_{t,(m_2,m_3,m_4),(Q-m_3-d-n_{24},\ m_3+d-n_{34},\ n_{24}+n_{34})} \\
&= \frac{(\lambda(Q))^d}{d!}e^{-\lambda(Q)}\frac{(m_2-d)!}{n_{24}!(m_2-d-n_{24})!}p_{24}^{n_{24}}(C_0)(1-p_{24}(C_0))^{m_2-d-n_{24}} \\
&\quad \frac{(m_3+d)!}{n_{34}!(m_3+d-n_{34})!}p_{34}^{n_{34}}(1-p_{34})^{m_3+d-n_{34}} \\
&\quad \frac{m_4!}{n_{45}!(m_4-n_{45})!}p_{45}^{n_{45}}(C_0)(1-p_{45}(C_0))^{m_4-n_{45}}
\end{aligned}
$$

and

$$
\begin{aligned}
&R_{t,(m_2,m_3,m_4),(Q-m_3-d-n_{24},\ m_3+d-n_{34},\ n_{24}+n_{34})} = -C_0 - C_{42}(c)(m_4 - n_{45}) \\
&- c(Q - m_2 - m_3 - m_4 + n_{45}) + (m_3 + d)(r_3 - C_3(c)) + n_{45}r_5 - (m_2 - d)C_2(c).
\end{aligned} \tag{1}
$$

(2) If $d \ge m_2$, $0 \le n_{34} \le m_3 + m_2$, $0 \le n_{45} \le m_4$, and $t = 1, \ldots, T-1$, then

$$
\begin{aligned}
&P_{t,(m_2,m_3,m_4),(Q-m_3-m_2,\ m_3+m_2-n_{34},\ n_{34})} \\
&= \left[\sum_{d=m_2}^{\infty}\frac{(\lambda(Q))^d}{d!}e^{-\lambda(Q)}\right]\frac{(m_3+m_2)!}{n_{34}!(m_3+m_2-n_{34})!}p_{34}^{n_{34}}(1-p_{34})^{m_3+m_2-n_{34}} \\
&\quad \frac{m_4!}{n_{45}!(m_4-n_{45})!}p_{45}^{n_{45}}(C_0)(1-p_{45}(C_0))^{m_4-n_{45}}
\end{aligned} \tag{2}
$$

and

$$
\begin{aligned}
&R_{t,(m_2,m_3,m_4),(Q-m_3-m_2,\ m_3+m_2-n_{34},\ n_{34})} \\
&= -C_0 - C_{42}(c)(m_4 - n_{45}) - c(Q - m_2 - m_3 - m_4 + n_{45}) + (m_3 + m_2)(r_3 - C_3(c)) + n_{45}r_5.
\end{aligned} \tag{3}
$$

(3) If $d < m_2$ and $t = T$, then

$$P_{T,(m_2,m_3,m_4),(0,\ 0,\ d)} = \frac{(\lambda(Q))^d}{d!}e^{-\lambda(Q)} \tag{4}$$

and

$$R_{T,(m_2,m_3,m_4),(0,\ 0,\ d)} = -C_0 + (m_3 + d)(r_3 - C_3(c)) - (Q - m_3 - d)C_2(c) + (1-\alpha)Qr_5. \tag{5}$$

(4) If $d \geq m_2$ and $t = T$, then

$$P_{T,(m_2,m_3,m_4),(0,\,0,\,m_2)} = \sum_{d=m_2}^{\infty} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)} \tag{6}$$

and

$$R_{T,(m_2,m_3,m_4),(0,\,0,\,m_2)} = -C_0 + (m_3 + m_2)(r_3 - C_3(c)) - (Q - m_2 - m_3)C_2(c) + (1-\alpha)Qr_5. \tag{7}$$

(5) Otherwise, the system state change is impossible. Thus, set $P_{\cdot,(\cdot),(\cdot)} = 0$ and $R_{\cdot,(\cdot),(\cdot)} = 0$ in the matrices P and R.

Let the row vector $\mathbf{e} = (1, 0, \ldots, 0)$ and the column vector $\mathbf{u} = (1, 1, \ldots, 1)$, where each of them has $\frac{(Q+1)(Q+2)(Q+3)}{6}$ elements. Let $\mathbf{S}_t$ be a matrix with $(Q+1)(Q+2)(Q+3)/6$ rows and columns, and its element $S_{t,i,j}$ is given by $S_{t,i,j} = P_{t,i,j}R_{t,i,j}$. Let the row vector $\mathbf{e}_t = \mathbf{e}\left(\prod_{k=1}^{t-1} \mathbf{P}_k\right)$. Given Q, the total discounted expected profit at time 0 when the company starts at the system state $(0,0,0)$ is given by $\sum_{t=1}^{T}(1-\alpha)^{t-1}\mathbf{e}_t\mathbf{S}_t\mathbf{u}$. If this expected profit is negative, the company would not like to operate. Thus, let $W(Q) = \max\left\{0, \sum_{t=1}^{T}(1-\alpha)^{t-1}\mathbf{e}_t\mathbf{S}_t\mathbf{u}\right\}$ be the total discounted expected profit at time 0 when the company starts at the system state $(0,0,0)$. The optimization problem that the company is going to solve is

$$\max_Q\{W(Q)\} = \max_Q\left\{\max\left\{0, \sum_{t=1}^{T}(1-\alpha)^{t-1}\mathbf{e}_t\mathbf{S}_t\mathbf{u}\right\}\right\}. \tag{8}$$

If this expected profit $\max\limits_Q\{W(Q)\} = 0$, the best decision for the company is not to start the business, i.e., let $Q^* = 0$.

5. Numerical Experiments

Thus far, we have built the model and obtained the objective function that includes a set of parameters. It is difficult to directly conduct theoretical analysis. Hence, this section is to study the effects of costs, charge, salvage value, etc. on the company's expected profit through factor analysis based on numerical experiment results.

Most of the parameters were set at two levels in numerical experiments. Here, one day was one period, and the total time was two years. In other words, let the number of periods $T = 730$. The functions are shown in Table 2. The cost of new cars and the usage charges were set with reference to data from the companies car2go and Zipcar in a city in North America in July 2018. The price of one type of new car was between \$22,000 and \$33,000. Here, let the price per new car $c =$ \$27,000. The price for using car2go was \$15 to \$19 per hour or \$89 to \$129 per day. Users have to become members, but the membership fee of car2go was negligible. The price for using Zipcar was \$8 to \$10 per hour or \$60 to \$72 per day, and the membership fee was \$7 per month or \$70 per year. Thus, let the low level of charge be \$60, and the high level be \$120.

Since car-sharing companies have different business environments, experiments of identifying critical factors were performed at different operating periods and arriving sensitivity functions to improve the observation robustness.

Table 2. Function settings.

$C_0(c, Q) = a_0 c Q + b_0$	$a_0 > 0, b_0 > 0$
$C_2(c) = a_2 c + b_2$	$a_2 > 0, b_2 \geq 0$
$C_3(c) = a_3 c + b_3$	$a_3 > 0, b_3 \geq 0$
$C_{42}(c) = a_{42} c + b_{42}$	$a_{42} > 0, b_{42} \geq 0$
$p_{24}(C_0) = -a_{24} C_0 + b_{24}$	$a_{24} > 0, b_{24} \geq 0$
$p_{45}(C_0) = -a_{45} C_0 + b_{45}$	$a_{45} > 0, b_{45} \geq 0$
$\lambda(Q) = aQ$	$a > 0$

Experiment 1. *The aim of the experiment is to identify the factors that have significant effects on the expected profit.*

To reduce the number of parameter combinations, let $b_2 = b_3 = b_{42} = 0$ *and* $b_{24} = b_{45} = 0.0002$. *Each of the other parameters has the two levels shown in Table 3. For example, if a car had the probability* 0.0002 *to be scrapped each period, the probability that the car would still be good after two years is* $0.9998^{730} = 0.8641$. *If* $\alpha = 0.0002$, *the annual interest rate was about* 7.57%, *since* $1/(1 - 0.0002)^{365} = 1.0757$.

There were 2^{12} *(= 4096) combinations of the parameters. We selected a resolution IV factional factorial design in which the main effects were un-confounded by two-factor interactions, with 128 combinations generated through Minitab 17. To find the optimal Q that maximizes the expected profit for each combination, let Q increase from* 1 *to* 20. *(Note that* 20 *is set as the upper bound due to memory requirement.) For each Q, the expected profit was evaluated. Then, the maximum expected profit among these* 20 *Qs as the expected profit of the combination, and the corresponding optimal Q was recorded.*

Table 3. Parameter setting.

Parameters	a	a_0	a_2	a_3	a_{24}	a_{42}
Low	0.6	0.0001	10^{-6}	10^{-6}	10^{-7}	0.001
High	1	0.0005	5×10^{-6}	5×10^{-6}	3×10^{-7}	0.02
Parameters	a_{45}	b_0	p_{34}	r_3	r_5	α
Low	4×10^{-7}	20	0.80	60	5000	0.0002
High	6×10^{-7}	50	0.99	120	20,000	0.0004

Among the 128 combinations, only 34 combinations (or 26.56%) produced a positive expected profit, and all the optimal Q for these 34 combinations were at the upper bound 20. For the other combinations, the company would lose some money. This matched the observation that it was difficult for a car-sharing company to be profitable. Suppose when the company would lost money in a combination, the company would not open the business. Then, the expected profit in the combination was set zero in our numerical experiments. Factor analysis showed that a, a_0, a_{42}, p_{34}, r_3, and r_5 were the only factors that had significant effects on the expected profit. Table 4 shows the means and other statistics of the expected profit on the two levels of each of the six important factors. The company had a good chance to be profitable under the following conditions:

1. The demand was high (a was at the high level).
2. The variable daily expense was (a_0 at the low level).
3. The maintenance fees were (a_{42} at the low level).
4. Customers that used the cars for a long time (p_{34} were at the low level).
5. Rental income was high (r_3 at the high level).
6. A scrapped car could be sold for a good price (the salvage value r_5 was at the high level).

In the car-sharing business, if an available car is idle for a period, it means that the recourses are wasted. Conditions 1 and 4 implied that the idle time of available cars was relatively small, and high utilization resulted in positive profits.

Table 4. Statistics on two levels of each of six significant factors where L is the number of combinations that resulted in a loss

	a		a_0	
	Low	**High**	**Low**	**High**
Mean of expected profit	54,736.80	71,217.20	78,440.50	47,513.50
Maximum expected profit	489,408.58	563,814.59	563,814.59	437,209.31
Minimum expected profit	0	0	0	0
Number of combinations	64	64	64	64
L	48	46	43	51
	a_{42}		p_{34}	
	Low	High	Low	High
Mean of expected profit	125,954.00	0	79,378.60	46,575.40
Maximum expected profit	563,814.59	0	563,814.59	447,678.55
Minimum expected profit	0	0	0	0
Number of combinations	64	64	64	64
L	30	64	44	50
	r_3		r_5	
	Low	High	Low	High
Mean expected profit	3143.74	122,810.00	25,146.80	100,807.00
Maximum expected profit	77,579.81	563,814.59	247,909.02	563,814.59
Minimum expected profit	0	0	0	0
Number of combinations	64	64	64	64
L	59	35	51	43

Remark 1. *It took about* 11.3 *s to obtain the maximum expected profit of one combination when the experiments were run by using MATLAB on a notebook with a 3.1 gigahertz processor and* 8 *gigabytes RAM.*

To test how sensitive the computation time is to Q and T, experiments were performed with different T or Q. When $T = 1825$ and when Q ran from 1 *to* 20*, the computation time per combination was about* 28 *s. When $T = 730$ and when Q ran from* 1 *to* 30*, it was about* 150 *s.*

In Experiment 1, we expected that the scrap rate was a significant factor, but it was not significant according to data analysis. To further understand the effects of the scrap factor and other factors, Experiment 2 was conducted.

Experiment 2. *Let $b_2 = b_3 = b_{42} = 0$ and $b_{24} = b_{45} = 0.0002$. Let $a = 0.8$, $a_0 = 0.0003$, $a_{42} = 0.001$, $p_{34} = 0.9$, $r_3 = 90$, and $r_5 = 12,500$. The remaining six parameters (b_0, a_2, a_3, a_{24}, a_{45}, and α) were the same as in Table 3. There were 2^6 or 64 combinations, and we selected the full factional factorial design.*

In Experiment 2, all the six factors a_2, a_3, a_{24}, a_{45}, b_0, and α had significant effects on the expected profit. The company could earn a profit for each of the 64 combinations. The minimum expected profit was $16,180.12, and the maximum was $46,963.70. Of course, it does not mean that a company can always earn a profit no matter how high the scrap rate is. For example, given $a_2 = 10^{-6}$, $a_3 = 10^{-6}$, $a_{24} = 10^{-7}$, $b_0 = 20$, and $\alpha = 0.0002$, the expected profit decreased from \$35,266.82 to $-$\$117.81 when the scrap probability p_{45} increased from 0.0001272 to 0.000253.

Let us consider the results of Experiments 1 and 2 together. If a company does not have a monopoly in the market, it usually has little impact on the six factors a (demand), a_0 (variable daily expense), a_{42} (maintenance fee), p_{34} (probability of a customer ending consumption), r_3 (rental rate), and r_5 (salvage value). These six factors are called external factors here. On the other hand, the six factors α, a_2, a_3, a_{24}, a_{45}, and b_0 are internal factors. From the results of Experiments 1 and 2, the following conclusions are obtained:

(1) It is not easy for a car-sharing company to be profitable. Before a company decides to join the car-sharing industry, it should first consider the external factors, which determine whether the company has a chance to be profitable. In an external sound environment, it may be profitable; in a poor external environment, the company will suffer losses no matter how good its management is.

(2) Given a favorable external environment, whether the company is profitable depends on internal factors of the company. If its internal management is good, the company will be able to be very profitable; otherwise, it will not be very profitable or may lose money. For example, a high scrap rate may bankrupt the company, as the examples in the introduction showed.

Remark 2. *Experiments were conducted to study the effects of various factors on the expected profit if C_2 and C_3 were independent of c, and p_{24} and p_{45} were independent of C_0. The same two conclusions as in Experiments 1 and 2 were reached.*

Experiment 3. *The time $T = 730$ (i.e., 2 years) in Experiment 1 was changed to $T = 1825$ (i.e., 5 years). Everything else remained the same.*

Here, 45 combinations produced positive expected profits. Recall that only 34 combinations produced positive expected profits in Experiment 1. More combinations produced positive expected profits here because some cars were of good quality at the end of 2 years and could still be used. Factor analysis showed that the external factors a, a_0, a_{42}, p_{34}, r_3, and r_5 had significant effects on the difference between the expected profit in two years and the expected profit in five years. The two conclusions of Experiments 1 and 2 were also true at $T = 1825$.

Finally, experiments were performed with a different demand function.

Experiment 4. *The experiment was the same as Experiment 1 except that the demand function was $\lambda(Q) = 10a(1 - e^{-\frac{Q}{2}})$.*

The results of the experiment were almost the same as those of Experiment 1, except:

(1) Besides all the external factors (i.e., a, a_0, a_{42}, p_{34}, r_3, and r_5), the internal factor α also had significant effects on the expected profit here.

(2) The optimal Q^* lay between 7 and 18 for combinations that led to a positive expected profit.

(3) Factor analysis showed that only the six external factors, a, a_0, a_{42}, p_{34}, r_3, and r_5, had significant effects on the optimal Q^* although the internal factor α had significant effects on the expected profit. When a, r_3, and r_5 were at the low level and a_0, a_{42}, and p_{34} were at the high level, the averages of the optimal Q^* were lower than that when a, r_3, and r_5 were at the high level, and a_0, a_{42}, and p_{34} were at the low level.

Remark 3. *If the constant 10 in $\lambda(Q) = 10a(1 - e^{-\frac{Q}{2}})$ was increased to 15, the optimal Q^* would be 20 in some combinations. (1) and (3) above were still true.*

Of course, if the constant 10 *in* $\lambda(Q) = 10a(1 - e^{-\frac{Q}{2}})$ *was replaced by a large enough constant, all the optimal* Q^* *would be* 20 *for combinations, which led to a positive expected profit.*

Experiment 5. *The experiment was the same as Experiment 2 except for the demand function, which was now* $\lambda(Q) = 10a(1 - e^{-\frac{Q}{2}})$.

The results were different from those of Experiment 2 in two ways: The internal factors α and a_2 did not have significant effects on the expected profit here. From Experiments 4 and 5, the same two conclusions as those from Experiments 1 and 2 were obtained.

To apply the model in general situation, users may have to first collect data to determine the arriving rate function, cost functions, probability functions, etc. Then, they can go along the same way in this section to conduct experiments, and identify key factors. As a result, they can obtain some management strategies to improve their car-sharing business.

6. Identifying the Optimal Strategy

The car-sharing business includes a platform service and the car-sharing service. In practice, the whole business can be operated by a single company or cooperators. As an example, Uber is a platform provider and Uber driver is a peer service provider. However, Zipcar is considered an access-based service company because it is not only a platform provider but also supplies cars.

Suppose that a car-sharing company is going to provide Q cars in an area. How many cars should be provided by the company itself and how many should be provided by other providers that pay commissions to the company? We will answer this question with the following assumptions: (1) Cars provided by the company or other providers are the same for customers. (2) The company operates as in Section 3, but it does not replace the cars of the other providers or maintain their cars. (3) The company obtains the commission θr_3 for each car provided by another provider. (4) The salvage value of a car belongs to its owner. (5) When the demand is less than the number of available cars, a binomial probability distribution is used to divide the demand among the company and the other providers.

The model here is very similar to the model in Section 3, but the number of cars Q is now the sum of two variables, Q' and Q'', where Q' is the number of cars provided by the car-sharing company, and Q'' is the number of cars provided by the other providers. Likewise, the same notations as in the previous sections will be used, with the additional superscript "$\prime$", which indicates a parameter related to the company, and the superscript "$\prime\prime$", which indicates a parameter related to the other providers. Then, along the same way, we can obtain expressions of those functions in Sections 3 and 4 such as a system state is presented by $(m_2', m_3', m_4', m_2'', m_3'', m_4'')$.

When a system state $(m_2', m_3', m_4', m_2'', m_3'', m_4'')$ changes to the system state $(Q' - m_3' - d' - n_{24}',\ m_3' + d' - n_{34}',\ n_{24}' + n_{34}', Q'' - m_3'' - d'' - n_{24}'',\ m_3' + d' - n_{34}',\ n_{24}'' + n_{34}'')$, the corresponding transition probability and the profit for the period are as follows:

(1) If $d < m_2' + m_2''$, $0 \leq n_{24}' \leq m_2' - d'$, $0 \leq n_{34}' \leq m_3' + d'$, $0 \leq n_{45}' \leq m_4'$, $0 \leq n_{24}'' \leq m_2'' - d''$, $0 \leq n_{34}'' \leq m_3'' + d''$, $0 \leq n_{45}'' \leq m_4''$, $t = 1, \ldots, T-1$, and $d'' = d - d'$, then

$$\begin{aligned}
&p_{t,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(Q'-m_3'-d'-n_{24}',\ m_3'+d'-n_{34}',\ n_{24}'+n_{34}',Q''-m_3''-d''-n_{24}'',\ m_3'+d'-n_{34}',\ n_{24}''+n_{34}'')} \\
&= \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)} \sum_{d'=\max\{0,\ d-m_2''\}}^{d'=\min\{d,\ m_2'\}} \frac{\binom{m_2'}{d'}\binom{m_2''}{d''}}{\binom{m_2'+m_2''}{d}} \\
&\quad \frac{(m_2'-d')!}{n_{24}'!(m_2'-d'-n_{24}')!} p_{24}^{n_{24}'}(C_0)(1-p_{24}(C_0))^{m_2'-d'-n_{24}'} \\
&\quad \frac{(m_3'+d')!}{n_{34}'!(m_3'+d'-n_{34}')!} p_{34}^{n_{34}'}(1-p_{34})^{m_3'+d'-n_{34}'} \\
&\quad \frac{m_4'!}{n_{45}'!(m_4'-n_{45}')!} p_{45}^{n_{45}'}(C_0)(1-p_{45}(C_0))^{m_4'-n_{45}'} \\
&\quad \frac{(m_2''-d'')!}{n_{24}''!(m_2''-d''-n_{24}'')!} p_{24}^{n_{24}''}(C_0)(1-p_{24}(C_0))^{m_2''-d''-n_{24}''} \\
&\quad \frac{(m_3''+d'')!}{n_{34}''!(m_3''+d''-n_{34}'')!} p_{34}^{n_{34}''}(1-p_{34})^{m_3''+d''-n_{34}''} \\
&\quad \frac{m_4''!}{n_{45}''!(m_4''-n_{45}'')!} p_{45}^{n_{45}''}(C_0)(1-p_{45}(C_0))^{m_4''-n_{45}''}
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
&R_{t,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(Q'-m_3'-d'-n_{24}',\ m_3'+d'-n_{34}',\ n_{24}'+n_{34}',Q''-m_3''-d''-n_{24}'',\ m_3'+d'-n_{34}',\ n_{24}''+n_{34}'')} \\
&= -C_0 - C_{42}(c)(m_4'-n_{45}') - c(Q'-m_2'-m_3'-m_4'+n_{45}') \\
&\quad + (m_3'+d')(r_3 - C_3(c)) + n_{45}'r_5 - (m_2'-d+m_2'')C_2(c) + (m_3''+d'')(\theta r_3 - C_3(c)).
\end{aligned} \tag{10}$$

(2) If $d \geq m_2' + m_2''$, $0 \leq n_{34}' \leq m_3' + m_2'$, $0 \leq n_{45}' \leq m_4'$, $0 \leq n_{34}'' \leq m_3'' + m_2''$, $0 \leq n_{45}'' \leq m_4''$, and $t = 1, \ldots, T-1$, then

$$\begin{aligned}
&p_{t,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(Q'-m_3'-m_2',\ m_3'+m_2'-n_{34}',\ n_{34}',Q''-m_3''-m_2'',\ m_3''+m_2''-n_{34}'',\ n_{34}'')} \\
&= \left[\sum_{d=m_2'+m_2''}^{\infty} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)}\right] \\
&\quad \frac{(m_3'+m_2')!}{n_{34}'!(m_3'+m_2'-n_{34}')!} p_{34}^{n_{34}'}(1-p_{34})^{m_3'+m_2'-n_{34}'} \\
&\quad \frac{m_4'!}{n_{45}'!(m_4'-n_{45}')!} p_{45}^{n_{45}'}(C_0)(1-p_{45}(C_0))^{m_4'-n_{45}'}. \\
&\quad \frac{(m_3''+m_2'')!}{n_{34}''!(m_3''+m_2''-n_{34}'')!} p_{34}^{n_{34}''}(1-p_{34})^{m_3''+m_2''-n_{34}''} \\
&\quad \frac{m_4''!}{n_{45}''!(m_4''-n_{45}'')!} p_{45}^{n_{45}''}(C_0)(1-p_{45}(C_0))^{m_4''-n_{45}''}
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
&R_{t,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(Q'-m_3'-m_2',\ m_3'+m_2'-n_{34}',\ n_{34}',Q''-m_3''-m_2'',\ m_3''+m_2''-n_{34}'',\ n_{34}'')} \\
&= -C_0 - C_{42}(c)(m_4'-n_{45}') - c(Q'-m_2'-m_3'-m_4'+n_{45}') + (m_3'+m_2')(r_3 - C_3(c)) + n_{45}'r_5 \\
&\quad + (m_3''+m_2'')(\theta r_3 - C_3(c)).
\end{aligned} \tag{12}$$

(3) If $d < m_2' + m_2''$, and $t = T$, then

$$p_{T,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(0,\ 0,\ d',0,\ 0,\ d'')} = \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)} \sum_{d'=\max\{0,\ d-m_2''\}}^{d'=\min\{d,\ m_2'\}} \frac{\binom{m_2'}{d'}\binom{m_2''}{d''}}{\binom{m_2'+m_2''}{d}} \quad (13)$$

and

$$\begin{aligned} &R_{T,((m_2',m_3',m_4',m_2'',m_3'',m_4''),(0,\ 0,\ d',0,\ 0,\ d'')} \\ &= -C_0 + (m_3' + d')(r_3 - C_3(c)) - (Q' - m_3' - d')C_2(c) + (1-\alpha)Q'r_5 \\ &\quad - (Q'' - m_3'' - d'')C_2(c) + (m_3'' + d'')(\theta r_3 - C_3(c)). \end{aligned} \quad (14)$$

(4) If $d \geq m_2' + m_2''$, and $t = T$, then

$$p_{T,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(0,\ 0,\ m_2',0,\ 0,\ m_2'')} = \sum_{d=m_2'+m_2''}^{\infty} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)} \quad (15)$$

and

$$\begin{aligned} &R_{T,(m_2',m_3',m_4',m_2'',m_3'',m_4''),(0,\ 0,\ m_2',0,\ 0,\ m_2'')} \\ &= -C_0 + (m_3' + m_2')(r_3 - C_3(c)) - (Q' - m_2' - m_3')C_2(c) + (1-\alpha)Q'r_5 \\ &\quad - (Q'' - m_2'' - m_3'')C_2(c) + (m_3'' + m_2'')(\theta r_3 - C_3(c)). \end{aligned} \quad (16)$$

(5) If none of (1)–(4) is true, then $P_{\cdot,(\cdot),(\cdot)} = 0$ and $R_{\cdot,(\cdot),(\cdot)} = 0$.

Experiment 6. *This aim of the experiment is to determine how many cars the company should provide.*

To reduce the number of parameter combinations, let $T = 730$, $c = 27{,}000$, $b_2 = b_3 = b_{42} = 0$, $b_{24} = b_{45} = 0.0002$, *and* $a_2 = a_3 = 3 \times 10^{-6}$. *Let the other parameters have the two levels shown in Table 5. There are now* 2^{11} *(or 2048) combinations. A resolution V factional factorial design with 128 combinations was selected.*

Since the number of possible system states was large, simulation was applied to evaluate the expected profit. For each combination of the 11 parameters, we let Q *increase from 1 to 20. For each* Q*, we let* Q' *increase from 0 to* Q*. Given* Q *and* Q'*, simulation for 30 random instances were performed, and the average of expected profits of the 30 instances was considered as the company's expected profit. Then, the combination of* Q *and* Q' *with the largest expected profit was selected as the optimal decision.*

Table 5. Parameter setting for determining the number of cars that the company should own.

Parameters	a	a_0	a_{24}	a_{42}	a_{45}	b_0
Low	0.6	0.0001	10^{-7}	0.001	4×10^{-7}	20
High	1	0.0005	3×10^{-7}	0.02	6×10^{-7}	50
Parameters	p_{34}	r_3	r_5	α	θ	
Low	0.80	60	5000	0.0002	0.2	
High	0.99	120	20,000	0.0004	0.5	

Factor analysis showed that factors a_0, a_{42}, r_5, and θ had significant effects on the number of cars furnished by other providers. It is interesting that the first three of them are external factors. If a_{42} was at the high level (high maintenance fee), r_5 was at the low level (low salvage value), θ was at the high level (the company can obtain a large commission from the car-sharing of other providers' cars), the company would not like to own a lot of cars. It is not intuitive why, when a_0 was at the low level, the number of cars from other providers was greater than that when a_0 was at the high level (Table 6). The reason is that, if the daily expenses are high in a difficult business environment, the company and the other car providers would not like to enter the car-sharing business, but if the daily expenses are low, the company could obtain a profit by providing a car-sharing platform for other car providers since the company only has to spend a little money daily and does not need to purchase cars. Among the 128 combinations, 12 were too challenging for the company even though it did not need to purchase new cars or maintain current cars, while 48 combinations were attractive for the company to do the whole business by itself. The company and other car providers cooperated for 68 combinations.

Table 6. Statistics on two levels of each of four significant factors.

	a_0		a_{42}		r_5		θ	
	Low	**High**	**Low**	**High**	**Low**	**High**	**Low**	**High**
Mean of Q''	13.00	8.92	4.42	17.50	13.08	8.43	8.75	13.17

The experiment results showed that not anyone of the business models dominates another one. In general, users can make decisions on whether to provide cars by applying the model in this section. Before application, each user needs to estimate parameters and determine functions. Then, along the same process as that in this section, each user can find a business model that suits himself/herself.

7. Discussion: Infinite Time Horizon

Let $V_{t,m_2,m_3,m_4}(Q)$ be the sum of expected discounted profit from period t to the infinite future. However, the time information of period t will be ignored in the subscript in this section since the expected discounted profit $V_{t,m_2,m_3,m_4}(Q)$ is independent of t but depends on the system state (m_2, m_3, m_4) and the number of cars Q. We have,

$$
\begin{aligned}
&V_{m_2,m_3,m_4}(Q) \\
&= -C_0 \\
&+ \sum_{d=0}^{m_2-1} \sum_{n_{24}=0}^{m_2-d} \sum_{n_{34}=0}^{m_3+d} \sum_{n_{45}=0}^{m_4} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)} \frac{(m_2-d)!}{n_{24}!(m_2-d-n_{24})!} p_{24}^{n_{24}}(C_0)(1-p_{24}(C_0))^{m_2-d-n_{24}} \\
&\quad \frac{(m_3+d)!}{n_{34}!(m_3+d-n_{34})!} p_{34}^{n_{34}}(1-p_{34})^{m_3+d-n_{34}} \\
&\quad \frac{m_4!}{n_{45}!(m_4-n_{45})!} p_{45}^{n_{45}}(C_0)(1-p_{45}(C_0))^{m_4-n_{45}} \\
&\quad [(m_3+d)(r_3-C_3(c)) + n_{45}r_5 - (m_2-d)C_2(c) - C_{42}(c)(m_4-n_{45}) \\
&\qquad - c(Q-m_2-m_3-m_4+n_{45}) + (1-\alpha)V_{Q-m_3-d-n_{24},\, m_3+d-n_{34},\, n_{24}+n_{34}}(Q)] \\
&+ \left[\sum_{d=m_2}^{\infty} \frac{(\lambda(Q))^d}{d!} e^{-\lambda(Q)}\right] \sum_{n_{34}=0}^{m_3+m_2} \sum_{n_{45}=0}^{m_4} \\
&\quad \frac{(m_3+m_2)!}{n_{34}!(m_3+m_2-n_{34})!} p_{34}^{n_{34}}(1-p_{34})^{m_3+m_2-n_{34}} \\
&\quad \frac{m_4!}{n_{45}!(m_4-n_{45})!} p_{45}^{n_{45}}(C_0)(1-p_{45}(C_0))^{m_4-n_{45}} \\
&\quad [-c(Q-m_2-m_3-m_4+n_{45}) + (m_3+m_2)(r_3-C_3(c)) + n_{45}r_5 - C_{42}(c)(m_4-n_{45}) \\
&\qquad + (1-\alpha)V_{Q-m_2-m_3,\, m_3+m_2-n_{34},\, n_{34}}(Q)].
\end{aligned} \tag{17}
$$

Let $\mathbf{V}(Q) = (V_{0,0,0}(Q), \ldots, V_{Q,0,0}(Q))$ be a column vector. Let $\mathbf{r}(Q) = (r_{0,0,0}(Q), \ldots, r_{Q,0,0}(Q))$ be a column vector whose elements are net profits at all the system states in one period. Let $\mathbf{P}(Q)$ be the transition probability matrix. Then, the company is looking for the maximum expected profit $V^*_{0,0,0}(Q^*)$ by solving the following problem:

$$\begin{aligned} &\max_Q \{V_{0,0,0}(Q)\} \\ &\text{subject to} \\ &\qquad \mathbf{V}(Q) = \mathbf{r}(Q) + (1-\alpha)\mathbf{P}(Q)\mathbf{V}(Q). \end{aligned} \tag{18}$$

Finally, in practice, the company may like to determine the number of new cars and/or the number of maintainable cars chosen to be repaired in each period. For this, the expected discount profit $V_{m_2,m_3,m_4}(Q)$ needs to include the number of new cars and/or the number of maintainable cars chosen to be repaired as decision variables, but the expression of the expected discount profit will be the same as the expression at the right side of Equation (18). No matter whether the time horizon is finite or infinite, the models with minor changes are possible to cover a lot of application situations.

When the business time horizon is long enough, one possible approximation is to consider the model with an infinite time horizon here. Then, a stable plan in the car-sharing business can be obtained. In application, users need to keep updating parameters and functions in their models.

8. Conclusions

In the literature, many manuscripts studied the car-sharing industry by surveys and simulations. In this paper, models are provided to compute the expected profit in different scenarios, and factors were classified into external factors and internal ones. The external factors represent an essential part of the external business environment, and managers have to study the external business environment carefully before deciding whether to enter the car-sharing business. Furthermore, managers have to control the internal operation well. Sometimes, only if a company stays in business for a long time, does it have a chance to be profitable. Note that a company can profit from opening its business platform to car owners to earn commission.

Car-sharing can reduce disease spread among public transportation crowds during infectious disease pandemics such as the COVID-19 pandemic. The COVID-19 pandemic has led to severe problems such as supply chain interruptions, insufficient transportation capacity, and a backlog of goods. Car-sharing is a possible way to better utilize idle resources and improve the transportation in supply chains.

There are several limitations in this research. For example, this paper omits the issue of how government subsidy policies affect sustainable supply chains. Such an issue can be addressed in future research. Another interesting research topic may be the dependability of a car based on its age. To this end, each car's age has to be recorded and the probability p_{24}, costs such as C_2 and C_{42}, should depend on each car's age. Another interesting topic is location optimization. A station-based system may not have sufficient flexibility to satisfy consumers if a station is only an individual parking lot. A possible solution is to consider a certain area as a station. Another interesting problem is to determine the optimal number of stations, their locations, and relocation management strategies. It is natural to try to solve the location issue with a two-stage model. [25] studied the model where customers could drop off cars at any station. A lot of practical issues were considered. [45] used integer programming on medium-sized instances but had to apply heuristics on large-scale instances. In addition, research on the bicycle-sharing industry could be used to study the management of the car-sharing industry. For example, for short trips in a city, shared bicycles could replace cars. [46] first determined bicycle-sharing stations from a set of potential sites such as metro stations and shopping centers, then determined the capacity of the selected bicycle stations. It was a two-stage model. The models and analysis in this paper can be applied in other sharing industries.

Moreover, car-sharing is just one intermediate part of the supply chain, and the research can be considered in the whole supply chain. For sustainable green supply chains, [47] employed cases and found that a vertically integrated supply chain can help improve car production. [48] focused on the model of vehicle and route selection. [49] considered the repair and maintenance part. As car-sharing alone might not be enough to significant reduce the number of private cars [50], further studies on collaboration of different supply chain partners are needed to further improve sustainability.

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