

## Article

# Application of Data Validation and Reconciliation to Improve Measurement Results in the Determination Process of Emission Characteristics in Co-Combustion of Sewage Sludge with Coal

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**Abstract:** One of the actions popularized worldwide to reduce the consumption of fossil fuels is the combustion of renewable fuels and the co-combustion of both of these fuels. To properly implement combustion and co-combustion processes in power-generation installations, operational characteristics, including emission characteristics are required. To determine these characteristics, tests must be conducted, within the scope of which, for individual operating stages of the installation's work, the readings collected from a relatively large number of control and measurement instruments should be taken into account. All these instruments have different levels of accuracy, which, among other factors, bring about lower adequacy of the characteristics determined on the basis of these measurements. The objective of this study is to present possible adaptations of data validation and reconciliation methods to increase the adequacy of emission characteristics for the process of co-combustion of fuels. The methodology is discussed based on the example of studies on the co-combustion process of sewage sludge with coal in a grate furnace. The aforementioned characteristics were determined based on measurement tests of gaseous emissions of flue gas components. The tests were carried out for various preset operational conditions of the process, such as the thickness of fuel layer on the grate, the share of sludge in the fuel, the humidity of the sludge, the theoretical ratio of excess air to combustion, and the distribution of air stream during the process. The research object is described and detailed research results concerning two exemplary measurement tests are given, as well as the most important results referring to the whole research. The performed calculations indicate the necessity to take into account often significant corrections, which can amount to about 10% of the measured value.

**Keywords:** data validation and reconciliation; emission characteristics; co-combustion; sewage sludge; coal



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## 1. Introduction

Many types of wastes have combustible properties with low or medium calorific value (heating value). One of the management methods of such wastes is to subject them to a co-combustion process with substances of higher calorific value, including non-renewable or renewable fuels.

To forecast the effects of such a co-combustion process, for example, the emission of CO<sub>2</sub>, CO, NO<sub>x</sub>, SO<sub>2</sub>, as well as the content of combustible parts in solid products (in slag and ash), the temperature of the obtained flue gases, and the thermal efficiency of the process, we apply process characteristics [1–5]. They are used as analytical functional relationships between the parameters defining each of the mentioned effects and the quantities controlling the process, for example, the amount of fuels (share of waste), their composition, excess air for combustion, and the method of their supply to the process [1].

The dependencies concerning the emission of harmful factors to the environment are referred to as emission characteristics. The characteristics are determined using an analytical and experimental method of study. Two stages can be distinguished in the method. In the first stage, the mathematical type of the function describing the characteristics is determined. It often takes the form of first- or second-degree polynomials, depending on the theoretically predicted or experimentally determined impact of the independent (control) variables on the dependent variables being determined. It is recommended [6,7] to use second-degree polynomials.

In the adopted general form of the function, apart from independent and dependent variables, there are also constant coefficients. In the second stage, on the basis of the performed measurements, the values of such coefficients are determined using statistical methods [8], and the adequacy of the determined characteristic in terms of measurement results is assessed [9,10].

The adequacy of the emission characteristic depends on the accuracy of the measurements performed. Higher certainty can be obtained by using more accurate measuring instruments for the measurements. However, it should be noted that the cost of more accurate instruments often increases exponentially with higher accuracy. Another, cheaper and at the same time effective method of improving the results is to reconcile them using the technology of data validation and reconciliation [11–14]. Another cheaper and, at the same time, effective method of improving the results involves their reconciliation with the use of data validation and reconciliation. This procedure is used to correct measurement data and to determine unknown quantities, so that the mathematical equations describing the process under consideration are met (data reconciliation), while minimizing, at the same time, the deviations of the obtained measurement data from the corrected values [13]. One of the architects of data validation and reconciliation technology is K. Weigl [15]. Data validation and reconciliation is commonly used in geodesy and cartography [16].

From 1950 to 2020, in Poland, at the Silesian University of Technology in Gliwice and at the Academy of Mining and Metallurgy (AGH) in Cracow, many studies were conducted, presenting research that significantly extending the application scope of data validation and reconciliation. They substantiated the potential and desirability to apply this technology for the analysis of thermal processes in power plants and in combined heat and power plants [13,17–21], in ferrous and non-ferrous metallurgy [22,23], and in the coking industry [24]. An interesting proposition, it seems, is the use of data validation and reconciliation to authenticate the weights of criteria in the multi-criteria optimization [25].

During the same period, numerous interesting research studies were published in world literature concerning both general principles involving the application of data validation and reconciliation [11,12,14,26–30] as well as its application for solving more specific problems. For example, they were presented in the publications covering the problems of nuclear power plants [31–33], chemical industry [34–40], power plants and combined heat and power plants [41–46], refrigeration [47], and biotechnology [48].

The review of the literature presented earlier indicates universal potential of using the concept of data validation and reconciliation in various fields of experimental research.

The novelty of data validation and reconciliation presented in this paper is that it is applied to improve measurement results carried out as part of determining emission characteristics of the co-combustion process of sewage sludge with hard coal. It further extends the scope of applications of the discussed method to studies on waste management, and thus to the research in the field of environmental protection. We are not aware of any publications to date, presenting the application of data validation and reconciliation in this area. Another innovative element of this study involves the fact that the analyses were carried out based on the measurements of a dynamic test object (the distribution of the supplied primary air stream was changed over time, as well as the realization time of the entire process).

It should also be noted that the determination of the emission characteristics on the basis of the measurement results corrected by subjecting them to data validation and reconciliation procedures, increased their adequacy.

## 2. Materials and Methods

### 2.1. Validation Principles of Measurement Results

Mathematical notation of the laws used to validate measurement results is referred to as the equations of conditions. The conditions can be defined based on the following laws:

- Chemical, for example, mass conservation (balances of chemical elements and total shares of components in individual substances);
- Physical, for example, conservation of energy (energy balance) by Newton, Kirchhoff, and Ohm;
- Mathematical, for example, Pythagorean equation and sums of angles in polygons.

In the equations of conditions, apart from the quantities measured or determined on the basis of measurement results, there may be quantities whose values are unknown. In general, the above equations can be written in the following form [18]:

$$F_k(Z_1, \dots, Z_j, \dots, Z_n, Y_1, \dots, Y_l, \dots, Y_u) = 0 \quad (1)$$

where  $k = 1, \dots, r$ ;  $Z_j$  is the  $j$ -th measured quantity or determined based on measurement results,  $j = 1, \dots, n$ ;  $Y_l$  is the  $l$ -th unknown quantity,  $l = 1, \dots, u$ .

Obviously, the preliminary determination of the value of unknown quantities is possible only when  $r \geq n$ .

Equation (1) is satisfied only for the error-free values  $\hat{z}_j$  and values  $\hat{y}$  of the determined unknowns:

$$F_k(\hat{z}_1, \dots, \hat{z}_j, \dots, \hat{z}_n, \hat{y}_1, \dots, \hat{y}_l, \dots, \hat{y}_u) = 0 \quad (2)$$

The values of measured quantities  $z_j$  are burdened with error. Thus, the values of the determined unknowns are also uncertain. Hence:

$$F_k(z_1, \dots, z_j, \dots, z_n, y_1, \dots, y_l, \dots, y_u) = w_k \neq 0 \quad (3)$$

where  $z_j$  is measured values,  $y_l$  is initially determined values of unknown quantities, and  $w_k$  is inconsistency of the  $k$ -th condition of the equation.

To ensure that the equation of conditions is satisfied, each value obtained from the measurement should be corrected:

$$\hat{z}_j = z_j + \vartheta_j \quad (4)$$

where  $\vartheta_j$  is the correction of the value  $z_j$  obtained from the measurement.

The same should be done with the values of initially determined unknowns:

$$\hat{y}_l = y_l + \mu_l \quad (5)$$

where  $\mu_l$  is the correction of the initially determined value  $y_l$ .

Equations (4) and (5) are the basis for validating measurement results and the initially estimated values of the unknowns.

The basic criterion for determining the corrections  $\vartheta_j$  is the minimum of the weighted sum of squared corrections:

$$\phi = \sum_{j=1}^n \left( \frac{\vartheta_j}{m_j} \right)^2 \rightarrow \min \quad (6)$$

where  $m_j$  is the average determination inaccuracies of the value  $z_j$ .

The determination of the value of correction  $\vartheta_j$  and  $\mu_l$  is narrowed down to solving the optimization problem consisting of obtaining the minimum of the objective function, Equation (6), taking into account the constraints, Equations (3) and (4). For this purpose,

the Lagrange method of undetermined coefficients can be used. The methods of solving the above optimization problem are provided in detail in the literature [18,49].

## 2.2. Object of Research

The studies discussed in this paper were carried out on a laboratory stand consisting of a combustion chamber with a 100 kW grate furnace, a flue gas analyzer with a logger of measurement results, an air fan, and a set of rotameters. The combustion chamber was used for the co-combustion of sewage sludge and hard coal. The elemental composition of both substances was known. The said compositions were not subject to reconciliation.

### 2.2.1. Independent Variables

The studies involving the determination of emission characteristics were carried out with the following independent variables (the range of their changes accounted for in the research is given in brackets):

- Share of dry matter of sludge in the mixture with coal  $x_1$  (from 0 to 30%);
- Moisture content in sludge  $x_2$  (from 20 to 60%);
- Initial thickness of the fuel layer on the grate  $x_3$  (from 50 to 150 mm);
- Theoretical ratio of excess primary air supplied to the combustion process  $x_4$  (from 1.2 to 1.6);
- Execution time of the process  $x_5$  (from 30 to 50 min);
- Change of the position of the maximum of primary air stream supplied under the grate  $x_6$  (from 1/6 to 2/3 of the execution time).

Each combination of the values of dependent variables makes up one test. Assuming six independent variables and three values of each variable, the required number of tests in compliance with the principle of “each value with one another” (the so-called complete 3-value plan) is  $3^6 = 729$ . In order to limit the number of tests and to select appropriate and representative values of individual independent variables, the methods of planned experiment were used [1,8–10]. For the detailed analyses, the so-called static determined poly-selective plan of type B was applied which requires that 51 tests are carried out for 6 independent variables. For each test, the density of the coal-sludge mixture was measured.

Since in the combustion process of solid fuels there are different phases (drying, degassing, gasification, and combustion), the demand for combustion air varies over time.

We attempted to change the air stream in accordance with the following equation [1]:

$$f(\tau) = a \cdot \left(1 - \frac{\tau}{x_4}\right) \cdot \tau \cdot \exp(c \cdot \tau) \quad (7)$$

where  $a$  is the coefficient,  $\text{m}^3/\text{min}^2$ ;  $c$  is the coefficients,  $\text{min}^{-1}$ ;  $\tau$  is the stay time of fuel on the grate, min;  $x_4$  is the total duration of the process, min.

The coefficients “ $a$ ” and “ $c$ ” in the dependence (7) are calculated from the following conditions:

- The maximum of air stream occurs for  $\tau = x_6$  as follows:

$$f'(\tau)|_{x_6} = 0 \quad (8)$$

- The area under the curve  $f(\tau)$  should correspond with the amount of air supplied for combustion in particular tests:

$$V_a = \int_0^{x_4} f(\tau) \cdot d\tau \quad (9)$$

During each test, five equal subperiods of air supply to the furnace were determined. The duration of the subperiods was  $1/5x_5$ . In the  $e$ -th zone ( $e = 1, \dots, 5$ ), we were trying to maintain a constant air stream which was:

$$f_e = \frac{1}{2} \left\{ f \left( \tau = \frac{x_5}{5} \cdot e \right) + f \left[ \tau = \frac{x_5}{5} \cdot (e - 1) \right] \right\} \quad (10)$$

where  $e$  is the number of the subperiod zone of air supply to the furnace ( $e = 1, \dots, 5$ );

## 2.2.2. Dependent Variables

During the individual tests, the dependent variables were measured. They comprised average, minute values of the concentrations of the flue gas components:

- Oxygen  $O_2$ ,  $r_{O_2}$ , mg/m<sup>3</sup><sub>n</sub>;
- Carbon dioxide,  $CO_2$ ,  $r_{CO_2}$ , mg/m<sup>3</sup><sub>n</sub>;
- Carbon oxide,  $CO$ ,  $r_{CO}$ , mg/m<sup>3</sup><sub>n</sub>;
- Sulfur dioxide,  $SO_2$ ,  $r_{SO_2}$ , mg/m<sup>3</sup><sub>n</sub>;
- Nitrogen oxides,  $NO_x$ ,  $r_{NO_x}$ , mg/m<sup>3</sup><sub>n</sub>;
- Share of combustible parts in slag,  $r_s$ , %.

The average, minute values of the concentrations of flue gas components were not directly used to determine the characteristics [1]. When determining the characteristics, the total emissions of flue gas components in individual tests were used. These values were determined from the average minute values, in line with the Formula (17).

The following instruments were used in the research:

- The measurements of the concentration of flue gas components were made with an MGA 5 analyzer manufactured by MRU GmbH;
- Air stream measurements were made with a set of rotameters, type RDN 65, as well as RIN 402 and 405;
- Laboratory scales were used to measure the mass of fuel;
- Shares of combustible parts in slag were determined using the weight method from the samples of post-process residues from individual tests.

## 2.3. Quantities Included in the Equations of Conditions

To validate the measurement results needed to determine the emission characteristics, they were subjected to reconciliation (see Section 2.1) based on the balance equations of conditions (see Section 2.3). Some of the measurements were used directly in the condition equations. Others required appropriate modification. Ultimately, the following quantities related to the independent variables were used in the equations of conditions:

- Mass of coal,  $Z_1$ , kg;
- Mass of dry sludge,  $Z_2$ , kg;
- Mass of moisture in the sludge,  $Z_3$ , kg;
- Amount of air for combustion,  $Z_4$ , m<sup>3</sup><sub>n</sub>.

The dependencies between the independent factors presented in the paper and the quantities directly included in the equations of conditions were as follows:

$$Z_1 = M_f \cdot \frac{(1 - x_1) \cdot (1 - x_2)}{1 - x_2 \cdot (1 - x_1)} \quad (11)$$

$$Z_2 = M_f \cdot \frac{x_1 \cdot (1 - x_2)}{1 - x_2 \cdot (1 - x_1)}, \text{ kg}, \quad (12)$$

$$Z_3 = M_f \cdot \frac{x_1 \cdot x_2}{1 - x_2 \cdot (1 - x_1)} \quad (13)$$

with:

$$M_f = x_3 \cdot \rho \cdot S \quad (14)$$

where  $M_f$  is the fuel mass in a particular test, kg;  $\rho$  is the fuel density, kg / m<sup>3</sup>; and  $S$  is the area of the grate in the combustion chamber, m<sup>2</sup>.

The amount of air was determined from the relationship:

$$Z_4 = x_4(V_{ac} \cdot Z_1 + V_{as} \cdot Z_2) \quad (15)$$

where  $V_{ac}$  and  $V_{as}$  are the theoretical air demand for the combustion of coal and sludge.

With reference to Equations (9) and (10), it should be noted that there is a relation:

$$Z_4 = V_a = \frac{x_5}{5} \cdot \sum_{e=1}^5 f_e \quad (16)$$

The analyses also accounted for the following quantities, in which the values were determined on the basis of measurements related to the determination of dependent variables:

- O<sub>2</sub> emissions in flue gas,  $Z_5$ , kmol;
- CO<sub>2</sub> emissions in flue gas,  $Z_6$ , kmol;
- CO emissions in flue gas,  $Z_7$ , kmol;
- SO<sub>2</sub> emissions in flue gas,  $Z_8$ , kmol;
- NO<sub>x</sub> emissions in flue gas,  $Z_9$ , kmol;
- share of combustible parts in slag,  $Z_{10}$ , %.

The emission of flue gas components  $Z_j$  ( $j = 5, \dots, 9$ ) was determined based on the measurements of their average, minute concentrations  $r_d$ , where  $d \in (\text{O}_2, \text{CO}_2, \text{CO}, \text{SO}_2, \text{NO}_x)$ . The values  $d$  are closely assigned to the values  $j$ , for example,  $j_5 \rightarrow d = \text{O}_2$ ,  $j_9 \rightarrow d = \text{NO}_x$ . Due to difficulties involving the measurement of flue gas flow  $\dot{V}$  during the test, it was initially assumed that the flue gas flow  $\dot{V}_{wg} = \dot{V}_a$ . Then, the following dependence is obtained:

$$Z_j = \tau \sum_{e=1}^5 \dot{V}_{a,e} \left( \sum_{p=R_1}^{R_p} r_{j,p} \right) \quad (17)$$

with:

$$R_1 = \frac{x_5}{5} (e - 1) + 1 \quad (18)$$

$$R_2 = \frac{x_5}{5} e \quad (19)$$

where  $Z_j$  is the emission of the  $j$ -th flue gas component, kmol;  $\dot{V}_{a,e}$  is the air flow in the  $e$ -th subperiod of test execution ( $e = 1, \dots, x_5$ ), m<sup>3</sup><sub>n</sub>/min;  $\Delta\tau$  is the time step ( $\Delta\tau = 1$  min).

The share of combustible parts in the slag was:

$$Z_{10} = r_s, \% \quad (20)$$

In the conducted analyses it was assumed that the combustible parts in the slag contain only the carbon C.

Moreover, the calculations included the following unknown quantities:

- Total amount of nitrogen in flue gas,  $Y_1$ , kmol;
- Mass of slag,  $Y_2$ , kg;
- Total moisture in flue gas,  $Y_3$ , kmol;
- Amount of dry flue gas generated in the combustion process,  $Y_4$ , m<sup>3</sup><sub>n</sub>.

All quantities and their values were referenced to each of the analyzed tests.

Apart from the above parameters, the equations of conditions include the composition of hard coal and that of dry sewage sludge. The compositions are given in Table 1. They were not subject to reconciliation.

**Table 1.** Composition of hard coal and sewage sludge (dry substance).

Parameter	Carbon	Hydrogen	Nitrogen	Sulphur	Oxygen	Moisture
Hard coal	0.7301	0.0457	0.0153	0.0037	0.0966	0.0479
Sewage sludge (dry mass)	0.3013	0.0435	0.0367	0.0141	0.1930	–

#### 2.4. Applied Equations of Conditions

Below 7 balance equations are presented, used as equations of conditions for the reconciliation of measured quantities:

I. Balance of carbon:

$$\frac{1}{M_C} \cdot (\hat{z}_1 \cdot g_{Cw} + \hat{z}_2 \cdot g_{Cos} - \hat{y}_2 \cdot g_C) - A \cdot (\hat{z}_6 + \hat{z}_7) = 0 \quad (21)$$

II. Balance of nitrogen:

$$\frac{1}{M_{N_2}} \cdot (\hat{z}_1 \cdot g_{Nw} + \hat{z}_2 \cdot g_{Nos} - \hat{y}_2 \cdot g_{Nzuz}) + D \cdot \hat{z}_{10} - \hat{y}_1 - 0.5 \cdot A \cdot \hat{z}_9 = 0 \quad (22)$$

III. Balance of sulphur:

$$\frac{1}{M_S} \cdot (\hat{z}_1 \cdot g_{Sw} + \hat{z}_2 \cdot g_{Sos} - \hat{y}_2 \cdot g_{Szuz}) - A \cdot \hat{z}_8 = 0 \quad (23)$$

IV. Balance of oxygen:

$$\frac{1}{M_{O_2}} \cdot (\hat{z}_1 \cdot g_{Ow} + \hat{z}_2 \cdot g_{Oos}) + \frac{0.5}{M_{H_2O}} \cdot \hat{z}_3 + \hat{z}_{10} \cdot (B + 0.5 \cdot G) - A \cdot (\hat{z}_5 + \hat{z}_6 + \hat{z}_8) - 0.5 \cdot (y_3 + A \cdot (\hat{z}_7 + \hat{z}_9)) = 0 \quad (24)$$

V. Balance of hydrogen:

$$\frac{1}{M_{H_2}} \cdot (\hat{z}_1 \cdot g_{H_2w} + \hat{z}_2 \cdot g_{H_2os}) + \frac{1}{M_{H_2O}} \cdot (\hat{z}_1 \cdot g_{H_2Ow} + \hat{z}_3) + G \cdot \hat{z}_4 - \hat{y}_3 = 0 \quad (25)$$

VI. Balance of slag and ash:

$$\hat{y}_2 - \hat{y}_2 \cdot \hat{z}_{11} - \hat{z}_1 \cdot g_{Pw} - \hat{z}_2 \cdot g_{Pos} = 0 \quad (26)$$

VII. Balance of flue gases:

$$C \cdot \hat{y}_4 - A \cdot (\hat{z}_5 + \hat{z}_6 + \hat{z}_7 + \hat{z}_8 + \hat{z}_9) - \hat{y}_1 = 0 \quad (27)$$

where:

$g_{Nw}, g_{O_2w}, g_{Cw}, g_{Sw}, g_{H_2w}, g_{Pw}$  are the gram fraction in coal, respectively, of nitrogen, oxygen, carbon element, sulfur, hydrogen, and ash;

$g_{Nos}, g_{O_2os}, g_{Cos}, g_{Sos}, g_{H_2os}, g_{Pos}$  are the gram fraction in dry mass of sludge, respectively, of nitrogen, oxygen, carbon, sulfur, hydrogen, and ash;

$M_{N_2}, M_{O_2}, M_C, M_{H_2}, M_S, M_{H_2O}$  are the Molar mass, respectively, of nitrogen, oxygen, carbon, hydrogen, sulfur, and water (kg/kmol);

$m_w, m_{os}, m_{sl}, m_{H_2Ocal}$ , respectively, are the mass of coal, dry mass of sludge, mass of slag, and total mass of moisture in the combustion mixture (kg);

$n''_{N_2}, n''_{O_2}, n''_{CO_2}, n''_{CO}, n''_{NO}, n''_{SO_2}, n''_{H_2O}$  are the amount contained in flue gases, respectively, of  $N_2, O_2, CO_2, CO, NO_x, SO_2$ , and  $H_2O$  (kmol);

$g_{cz.sl}, g_C, g_{Nsl}, g_{Ssl}$  are the gram share in slag, respectively, of combustible parts, carbon, nitrogen, and sulfur;

$A = \frac{\hat{y}_4}{\hat{z}_4}$  is the correction factor for the calculation of the emissions of individual pollutants resulting from the initial adoption of the assumption  $\hat{y}_4 = \hat{z}_4$ ;

B, D, G are the coefficients accounting for the degree of air humidity ( $\text{kmol}/\text{m}^3_{\text{n}}$ );  
C is the coefficient ( $\text{kmol}/\text{m}^3_{\text{n}}$ ).

The particular coefficients used in the balance equations are expressed by the following formulas:

$$D = \frac{1}{22.42} \cdot \frac{0.79}{1 + X'_{zpow}} \text{ kmol}/\text{m}^3_{\text{n}} \quad (28)$$

where  $X'_{zpow}$  is the molar degree of air humidity (average conditions were adopted for the place of air intake for the combustion process  $\phi = 60\%$ ,  $t = 18^\circ\text{C}$ );

$$B = \frac{1}{22.42} \cdot \frac{0.21}{1 + X'_{zpow}} \text{ kmol}/\text{m}^3_{\text{n}} \quad (29)$$

$$G = \frac{1}{22.42} \cdot \frac{X'_{zpow}}{1 + X'_{zpow}} \text{ kmol}/\text{m}^3_{\text{n}} \quad (30)$$

$$C = \frac{1}{22.42} \text{ kmol}/\text{m}^3_{\text{n}} \quad (31)$$

### 3. Results

To illustrate the upgrading method of measurement results presented in this paper, an exemplary and detailed calculation for two tests is presented below:

- Test No. 1 concerning the lowest values of the independent variables;
- Test No. 32 concerning the highest values of the independent variables.

The mentioned limit tests provide the variability of measurement values and the level of improvement of these results effected by the application of the procedure proposed in the study.

When determining emission characteristics, the measurement results for all the performed tests were subjected to reconciliation. We present a more detailed presentation of only two tests in this study because they sufficiently illustrate the presented method and at the same time reduce the volume of the publication.

#### 3.1. Measurements Results of Independent and Dependent Variables

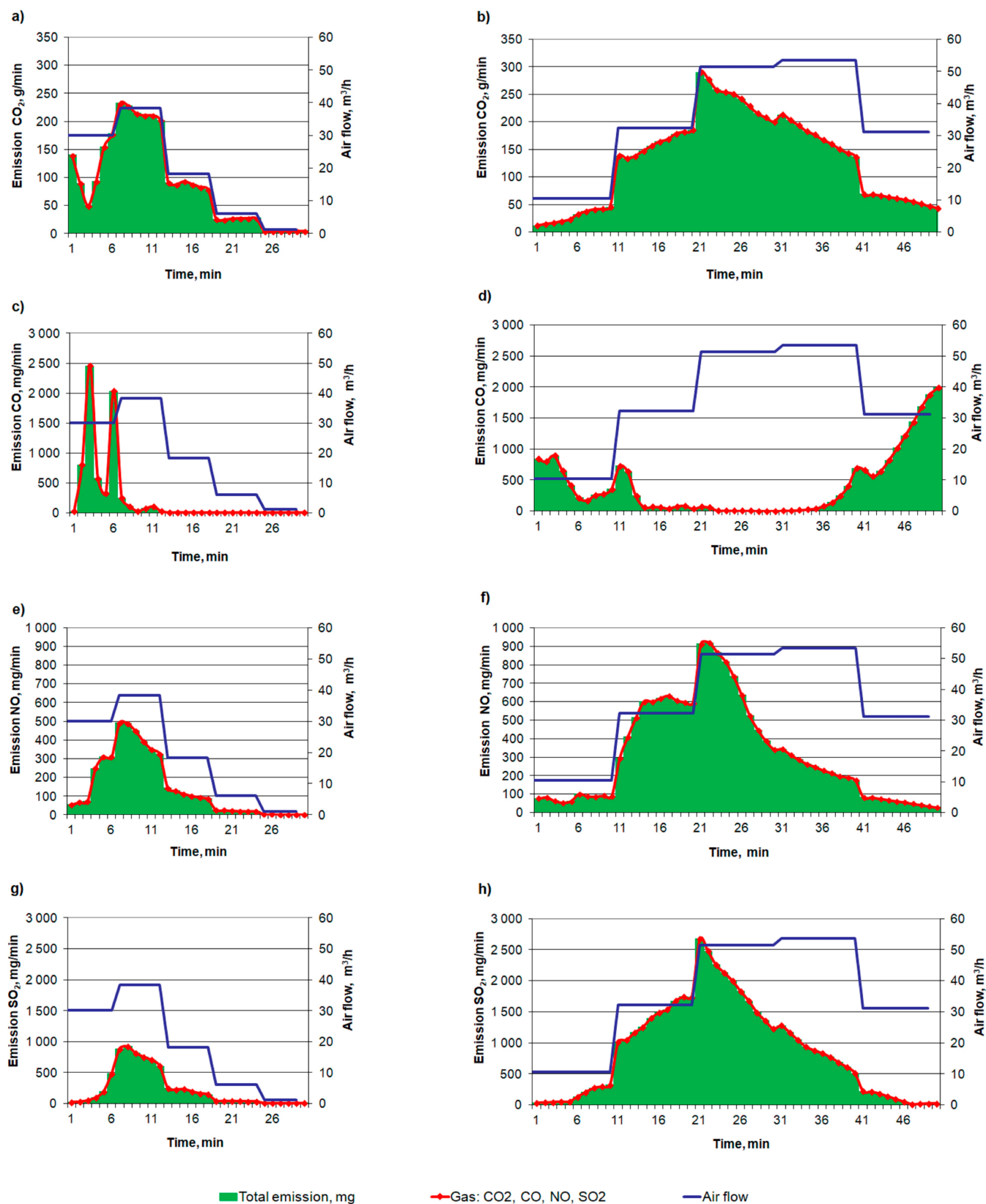
The measurement results of the quantities determining the independent variables in the emission characteristics are given in Table 2.

Figure 1 provide a graphic interpretation of the results of the performed measurements for the exemplary tests No. 1 and No. 32. The graphs show the average minute emission of individual gaseous pollutants, and the field illustrating the total emission of a given pollutant was indicated (in line with Formula (17)). Additionally, the curve illustrating the variability of air stream fed under the grate was plotted.

In order to make it easier to compare the quantities and changes of emissions over time, for individual gases the same ranges of values on the vertical axes were used (axes of minute average emissions). Moreover, the scale on the horizontal axes (time axes) is identical.

**Table 2.** Values of measured quantities of dependent and independent variables, before and after the reconciliation ( $z_j, \hat{z}_j$ ) and the average values of measurement uncertainty ( $m_j$ ), as well as corrections of the value ( $v_j$ ), for tests No. 1 and No. 32.

Quantity		Coal Mass	Mass of Dry Sludge	Mass of Moisture in Sludge	Volume of Air for Combustion	Emission of O <sub>2</sub> in Flue Gas	Emission of CO <sub>2</sub> in Flue Gas	Emission of CO in Flue Gas	Emission of SO <sub>2</sub> in Flue Gas	Emission of NO <sub>x</sub> in Flue Gas	Share of Combustible Parts in Slag
Symbol		Z <sub>1</sub>	Z <sub>2</sub>	Z <sub>3</sub>	Z <sub>4</sub>	Z <sub>5</sub>	Z <sub>6</sub>	Z <sub>7</sub>	Z <sub>8</sub>	Z <sub>9</sub>	Z <sub>10</sub>
Unit		kg	kg	kg	m <sup>3</sup> <sub>n</sub>	kmol	kmol	kmol	kmol	kmol	–
Test No. 1	$z_j$	1.1330	0.0000	0.0000	10.1000	0.0184	0.0675	0.0002	0.0001	0.0001	0.1521
	$m_j$	0.0200	0.0000	0.0000	0.5000	0.0012	0.0009	0.0000	0.0000	0.0000	0.0500
	$v_j$	−0.0081	0.0000	0.0000	0.1063	0.0029	0.0015	0.0000	0.0000	0.0000	−0.0061
	$\hat{z}_j$	1.1249	0.0000	0.0000	10.2063	0.0213	0.0690	0.0002	0.0001	0.0001	0.0401
Test No. 32	$z_j$	2.2280	1.1140	1.6710	32.0500	0.1098	0.1498	0.0007	0.0007	0.0005	0.1764
	$m_j$	0.0200	0.0050	0.0061	0.5000	0.0012	0.0009	0.0000	0.0000	0.0000	0.0500
	$v_j$	0.0036	0.0003	0.0000	−0.0034	0.0022	0.0014	0.0000	0.0000	0.0000	0.0109
	$\hat{z}_j$	2.2316	1.1143	1.6710	32.0466	0.1120	0.1512	0.0007	0.0007	0.0005	0.1873



**Figure 1.** Changes in emissions during the realization of tests, exemplary tests No. 1 (a, c, e, g) and No. 32 (b, d, f, h): (a,b) changes in CO<sub>2</sub> emission; (c,d) changes in CO emission; (e,f) changes in NO<sub>x</sub> emission; (g,h) changes in SO<sub>2</sub> emission.

As it can be observed in the figure above, the independent variables accounted for in the research significantly affect the observed values of instantaneous emissions as well as the total emissions. Thus, in the case of CO<sub>2</sub>, the total emission for the test No. 32 is over two times higher than that for the test No. 1. For CO, the ratio is 3.5 and for NO<sub>x</sub> it is 5. As to the highest minute average emissions observed, the largest differences in emission values are observed for SO<sub>2</sub>, i.e., more than 2.5 times and for NO<sub>x</sub>, i.e., 1.8 times. In the case of test No. 1, we observe distinct changes in the average minute emissions of CO<sub>2</sub> and NO<sub>x</sub> caused by successive changes in the primary air stream. In the case of test No. 32, there is a clear increase in the average minute emissions of CO<sub>2</sub>, NO<sub>x</sub>, and SO<sub>2</sub> associated with the first two changes (increases) in the primary air stream. In the case of test No. 1 for the average CO emissions in the first range of primary air, there are two distinct peaks. This is probably affected by non-uniform ignition of fuel on the grate. This can also be confirmed by the minimum of average minute CO<sub>2</sub> emissions occurring in the same range as primary air. In the case of test No. 32, a significant increase in the average minute CO emissions is visible in the final phase of the combustion process. This effect is often observed in the processes of unsteady fuel combustion on grates, when in the final phase of the combustion process, due to the lack of hydrocarbon radicals and too low temperatures, the combustion reactions of CO to CO<sub>2</sub> are slowed down.

The changes in emissions presented in Figure 1 and discussed above confirm the advisability of determining the operating characteristics (including emission characteristics) for combustion and co-combustion processes.

Table 2 presents the total emissions of O<sub>2</sub>, CO<sub>2</sub>, CO, NO<sub>x</sub>, and SO<sub>2</sub> (calculated in compliance with the Formula (17)) for the analyzed exemplary tests.

To determine the share of combustible parts in slag, as part of each test, three 10 g samples of slag were collected from the combustion chamber of the laboratory installation. For the samples, in compliance with PN 93/Z-15008/03 [50], three determinations of combustible parts were performed. The average results of the share of combustible parts in the slag for tests No. 1 and No. 32 are also presented in Table 3.

**Table 3.** Predetermined values of unknowns  $y_l$ , corrections of value  $\mu_l$ , and values of unknowns after the reconciliation for tests No. 1 and No. 32.

Quantity		Total Amount of Nitrogen in Flue Gas	Mass of Slag	Total Amount of Moisture in Flue Gas	Volume of Dry Flue Gases Generated in Combustion Process
Symbol		$Y_1$	$Y_2$	$Y_3$	$Y_4$
Unit		kmol	kg	kmol	m <sup>3</sup> <sub>n</sub>
Test No. 1	$y_l$	0.35595	0.0848	0.0294	9.7721
	$\mu_l$	0.0037	−0.0028	0.0000	0.2181
	$\hat{y}_l$	0.3597	0.0820	0.0294	9.9902
Test No. 32	$y_l$	1.0755	0.6410	0.1892	29.8540
	$\mu_l$	0.0001	0.0089	0.0025	0.0957
	$\hat{y}_l$	1.0756	0.6499	0.1917	29.9497

### 3.2. Average Measurement Uncertainty

The procedures pertaining to data validation and reconciliation (Equation (6)) require that we determine the average uncertainty  $m_j$ , involving the determination of the values of the reconciled quantities  $z_j$ .

The recommended measure of uncertainty is the average standard deviation of measurement results [15,35]. It can be determined based on the results of separate measurements or based on the average measurement uncertainty arising from the class of measuring instruments.

The average uncertainty  $m_j$  is strongly influenced by the determination method of the particular reconciled quantities  $z_j$ . If their determination requires an additional measurement of at least two quantities  $x_a$ :

$z_j = z_j(x_1, \dots, x_b)$ ,  $b \geq 2$ , then the error propagation law should be used:

$$m_j = \sqrt{\sum_{a=1}^b \left( \frac{\partial z_j}{\partial x_a} \right)_0^2 m_a^2} \quad (32)$$

where  $\left( \frac{\partial z_j}{\partial x_a} \right)_0$  is the derivative of complex quantities, in line with the  $a$ -th quantity measured at the point "0".

The point "0" determines the mentioned values of the measured quantities  $x_a$  ( $a = 1, \dots, b$ ).

The uncertainties of the individual quantities present in the data validation and reconciliation are as follows:

1. For the measurement of coal mass, the inaccuracy was determined at 20 g (this quantity resulted from the coal assortment, the method of fuel dosing into the combustion chamber, etc.). It accounts for approximately 1% of the average mass of coal combusted in all tests performed during the tests. The said quantity was determined experimentally.
2. For the determination of sludge mass, it was assumed that the inaccuracy was 5 g (this quantity results from sludge grain size, the method of fuel dosing into the combustion chamber, etc.). It accounts for approximately 1% of the average mass of sludge combusted during the tests with its use. This quantity was determined experimentally.
3. For the determination of moisture mass added to the mixture in order to ensure its proper composition, the inaccuracy was assumed at the level of 2 g (it results from the accuracy of the applied volumetric flask).
4. For the volume of supplied air, the inaccuracy was assumed to be 0.5 m<sup>3</sup>. This quantity accounts for about 2.5% of the average volume of air supplied to the combustion process in the course of all tests. The adopted value takes into account measurement inaccuracy of the flowing air, disturbances occurring when the flow changes during individual tests, and air sucking through furnace leaks (especially when a small stream of air is fed).
5. For the share of combustible parts in the slag, the inaccuracy of 5% was assumed. This quantity was determined based on the experiments carried out in previous studies.
6. For the emissions of CO<sub>2</sub>, O<sub>2</sub>, CO, NO<sub>x</sub>, and SO<sub>2</sub> (Formula (17)), in compliance with error propagation law, the measurement error were determined from the formula:

$$m_j = \Delta\tau \cdot \sqrt{\left( \sum_{p=1}^{x_5} r_{j,p} \right)^2 \cdot m^2 + \frac{x_5^2}{25} \cdot \dot{V}_a^2 \cdot m^2(r_j)} \quad (33)$$

where  $m(\Delta\tau)$  and  $m(r_j)$  are the average uncertainty involving the determination of time step and the emission of the  $j$ -th flue gas component, determined from the class of measuring instruments.

The inaccuracy class of the analyzer used in the tests was 1%, and hence the errors in the case of single measurements of the measured quantities adopt the values given in Table 3.

In the course of the analysis of the results involving the conducted research, it was assumed that no gross error was made if the following condition was satisfied:

$$|\vartheta_j| \leq 3 \cdot |m_j| \quad (34)$$

In the course of the analyses, no case was found for which the above relationship would not occur.

### 3.3. Calculation Results

The optimization task defined by the objective function, Equation (6), as well as the constraints, Equations (3) and (4), were solved with the use of a library program (Jełowicki A, balance reconciliation library computer program).

The calculations yielded the following values of the reconciled quantities:

- Average determination uncertainties  $m_j$  for  $j = 1, \dots, 10$ ;
- Corrections of values  $v_j$  and  $\mu_l$  for  $l = 1, \dots, 4$ ;
- Values after the reconciliation of  $\hat{z}_j = z_j + \vartheta_j$  and  $\hat{y}_l = y_l + \mu_l$ ;
- Inaccuracies of the equations of conditions  $w_k$  for  $k = 1, \dots, 7$ .

For the tests No. 1 and 32, the values of the measured quantities of dependent and independent variables, before and after the reconciliation ( $z_j, \hat{z}_j$ ), and the average values of measurement uncertainty ( $m_j$ ), as well as the corrections of the values ( $v_j$ ) are given in Table 2.

The predetermined values of the unknowns  $y_1$ , corrections of the value  $\mu_1$ , and the values of unknowns after the reconciliation for the tests No. 1 and No. 32 are given in Table 3.

It can be observed that for the data presented in Table 2, for almost all dependent and independent variables, the relation  $m_j \geq |v_j|$  occurs. For both of the discussed tests, a different dependence occurs for the variables  $Z_5$  and  $Z_6$ . Moreover, most frequently as an effect of reconciliation, the values of the variables increased.

The main objective of the validation of measurement results was to determine the values of independent variables  $x_1$ – $x_4$  (the values  $x_5$  and  $x_6$  were not reconciled) and the values of dependent variables  $Z_5$ – $Z_9$  used to determine the emission characteristics of the co-combustion process of sewage sludge with coal. The values of the independent variables  $x$ , before and after the reconciliation for all performed tests as part of the research, are given in Table 4.

As can be observed from the data presented in the table, the corrections were on average 0.3% of the value before the reconciliation for  $x_1$ , 1.1% for  $x_2$ , 2.5% for  $x_3$ , and 1.7% for  $x_4$ .

Table 5 presents the values of the dependent variables  $Z_5$ – $Z_9$ , before and after the reconciliation, for all 51 tests included in the research. In the case of tests No. 1 and No. 32, these values are the same as those presented in Table 2. Tables 4 and 5 highlight (gray background) the results of the tests analyzed in detail in the article.

Based on the results of all tests carried out in the determining process of emission characteristics, we can observe that the mean correction value  $v_j$  was:

- For total emission of  $O_2$ , approximately 3.5% of the measured value;
- For total emission of  $CO_2$ , approximately 1.6% of the measured value;
- For the emission of  $SO_2$ , approximately 0.5% of the measured value;
- For the share of combustible parts in slag, over 10% of the measured value.

**Table 4.** Values of independent variables  $x$ , before and after reconciliation for all tests performed within the research.

Test No	Before Reconciliation				After Reconciliation			
	Share of Dry Mass of Sludge in the Mixture	Share of Moisture in Sludge	Thickness of Fuel Layer on the Grate	Theoretical Ratio of Excess Primary Air	Share of Dry Mass of Sludge in the Mixture	Share of Moisture in Sludge	Thickness of Fuel Layer on the Grate	Theoretical Ratio of Excess Primary Air
	$x_1$	$x_2$	$x_3$	$x_4$	$x_1$	$x_2$	$x_3$	$x_4$
	%	%	mm	–	%	%	mm	–
1	0	–	50	1.2	0.0	–	48.8	1.23
2	30	20	50	1.2	30.1	20.2	48.4	1.15
3	0	–	50	1.2	0.0	–	49.1	1.17
4	30	60	50	1.2	30.0	60.2	51.1	1.21
5	0	–	150	1.2	0.0	–	147.5	1.21
6	30	20	150	1.2	30.0	20.3	145.5	1.16
7	0	–	150	1.2	0.0	–	147.4	1.22
8	30	60	150	1.2	30.1	60.1	153.3	1.19
9	0	–	50	1.6	0.0	–	49.1	1.59
10	30	20	50	1.6	30.0	20.3	48.3	1.64
11	0	–	50	1.6	0.0	–	49.0	1.59
12	30	60	50	1.6	30.3	60.1	50.9	1.59
13	0	–	150	1.6	0.0	–	147.8	1.60
14	30	20	150	1.6	30.0	20.3	145.2	1.56
15	0	–	150	1.6	0.0	–	147.9	1.59
16	30	60	150	1.6	30.0	60.1	153.4	1.56
17	0	–	50	1.2	0.0	–	49.2	1.19
18	30	20	50	1.2	30.2	20.2	48.2	1.18
19	0	–	50	1.2	0.0	–	48.9	1.24
20	30	60	50	1.2	30.1	60.2	51.1	1.15
21	0	–	150	1.2	0.0	–	147.1	1.24
22	30	20	150	1.2	30.2	20.3	144.6	1.24
23	0	–	150	1.2	0.0	–	147.3	1.22
24	30	60	150	1.2	30.1	60.1	153.3	1.18
25	0	–	50	1.6	0.0	–	48.8	1.64
26	30	20	50	1.6	30.0	20.3	48.3	1.63
27	0	–	50	1.6	0.0	–	49.0	1.61
28	30	60	50	1.6	30.2	60.2	50.9	1.61
29	0	–	150	1.6	0.0	–	147.8	1.60
30	30	20	150	1.6	30.1	20.3	145.1	1.57
31	0	–	150	1.6	0.0	–	148.0	1.58
32	30	60	150	1.6	30.0	60.1	153.6	1.55
33	0	–	100	1.4	0.0	–	98.2	1.44
34	30	40	100	1.4	30.1	40.0	96.1	1.40
35	15	20	100	1.4	15.0	20.8	97.4	1.41
36	15	60	100	1.4	15.1	60.2	96.9	1.43
37	15	40	50	1.4	15.1	40.5	48.4	1.44
38	15	40	150	1.4	15.1	40.5	145.6	1.43
39	15	40	100	1.2	15.0	40.5	97.3	1.20
40	15	40	100	1.6	15.0	40.5	97.5	1.59
41	15	40	100	1.4	15.0	40.5	97.2	1.41
42	15	40	100	1.4	14.9	40.5	97.7	1.36
43	15	40	100	1.4	15.1	40.4	97.1	1.41
44	15	40	100	1.4	15.1	40.5	97.0	1.43
45	15	40	100	1.4	15.0	40.5	97.2	1.42
46	15	40	100	1.4	15.0	40.5	97.4	1.40
47	15	40	100	1.4	15.0	40.5	97.1	1.43
48	15	40	100	1.4	15.1	40.4	97.3	1.40
49	15	40	100	1.4	15.0	40.6	97.1	1.43
50	15	40	100	1.4	15.0	40.6	97.1	1.43
51	15	40	100	1.4	15.0	40.5	97.3	1.41

(–) In the absence of sludge in the mixture, moisture cannot occur in it.

**Table 5.** Values of dependent variables from Z<sub>5</sub> to Z<sub>9</sub>, for all 51 tests carried out in the research, before and after the reconciliation.

Test No	Before Reconciliation					After Reconciliation				
	Emission of O <sub>2</sub> in Flue Gas	Emission of CO <sub>2</sub> in Flue Gas	Emission of CO in Flue Gas	Emission of SO <sub>2</sub> in Flue Gas	Emission of NO <sub>x</sub> in Flue Gas	Emission of O <sub>2</sub> in Flue Gas	Emission of CO <sub>2</sub> in Flue Gas	Emission of CO in Flue Gas	Emission of SO <sub>2</sub> in Flue Gas	Emission of NO <sub>x</sub> in Flue Gas
	Z <sub>5</sub>	Z <sub>6</sub>	Z <sub>7</sub>	Z <sub>8</sub>	Z <sub>9</sub>	Z <sub>5</sub>	Z <sub>6</sub>	Z <sub>7</sub>	Z <sub>8</sub>	Z <sub>9</sub>
kmol										
1	0.0184	0.0675	0.00024	0.000093	0.000145	0.0213	0.0690	0.00024	0.000111	0.000145
2	0.0341	0.0271	0.00027	0.000116	0.000089	0.0330	0.0265	0.00027	0.000116	0.000089
3	0.0301	0.0608	0.00037	0.000143	0.000123	0.0272	0.0595	0.00037	0.000142	0.000123
4	0.0317	0.0343	0.00025	0.000120	0.000098	0.0315	0.0340	0.00025	0.000120	0.000098
5	0.0721	0.2011	0.00225	0.000395	0.000314	0.0718	0.2009	0.00225	0.000395	0.000314
6	0.0671	0.1108	0.00209	0.000476	0.000296	0.0665	0.1103	0.00209	0.000476	0.000296
7	0.1036	0.1768	0.00091	0.000383	0.000402	0.1007	0.1751	0.00091	0.000383	0.000402
8	0.0769	0.1204	0.00129	0.000517	0.000394	0.0753	0.1196	0.00129	0.000517	0.000394
9	0.0543	0.0600	0.00047	0.000134	0.000155	0.0558	0.0613	0.00047	0.000133	0.000155
10	0.0427	0.0408	0.00037	0.000136	0.000120	0.0419	0.0401	0.00037	0.000137	0.000120
11	0.0583	0.0642	0.00058	0.000110	0.000108	0.0556	0.0620	0.00058	0.000111	0.000108
12	0.0499	0.0419	0.00025	0.000190	0.000094	0.0477	0.0408	0.00025	0.000190	0.000094
13	0.1793	0.1880	0.00086	0.000360	0.000375	0.1817	0.1894	0.00086	0.000360	0.000375
14	0.1250	0.1238	0.00195	0.000427	0.000409	0.1220	0.1220	0.00195	0.000427	0.000409
15	0.1919	0.1745	0.00024	0.000379	0.000496	0.1948	0.1762	0.00024	0.000379	0.000496
16	0.1481	0.1231	0.00113	0.000417	0.000399	0.1455	0.1214	0.00113	0.000417	0.000399
17	0.0274	0.0582	0.00053	0.000118	0.000079	0.0279	0.0583	0.00053	0.000119	0.000079
18	0.0284	0.0280	0.00024	0.000124	0.000084	0.0301	0.0293	0.00024	0.000124	0.000084
19	0.0286	0.0629	0.00035	0.000117	0.000096	0.0258	0.0610	0.00035	0.000118	0.000096
20	0.0215	0.0376	0.00059	0.000144	0.000049	0.0245	0.0396	0.00059	0.000144	0.000049
21	0.0626	0.2030	0.00218	0.000392	0.000312	0.0655	0.2052	0.00218	0.000393	0.000312
22	0.0702	0.1092	0.00163	0.000422	0.000262	0.0695	0.1092	0.00163	0.000422	0.000262
23	0.0629	0.2160	0.00180	0.000388	0.000301	0.0605	0.2145	0.00180	0.000388	0.000301
24	0.0619	0.1356	0.00215	0.000489	0.000314	0.0598	0.1344	0.00215	0.000489	0.000314
25	0.0507	0.0625	0.00063	0.000133	0.000086	0.0529	0.0644	0.00063	0.000132	0.000086
26	0.0435	0.0383	0.00049	0.000147	0.000115	0.0436	0.0383	0.00049	0.000148	0.000115
27	0.0486	0.0679	0.00039	0.000121	0.000074	0.0494	0.0679	0.00039	0.000113	0.000074
28	0.0382	0.0458	0.00043	0.000187	0.000108	0.0408	0.0477	0.00043	0.000188	0.000108
29	0.1540	0.2232	0.00163	0.000397	0.000408	0.1524	0.2218	0.00163	0.000397	0.000408
30	0.1029	0.1323	0.00119	0.000514	0.000312	0.1061	0.1346	0.00119	0.000514	0.000312
31	0.1470	0.2182	0.00165	0.000414	0.000428	0.1500	0.2197	0.00165	0.000414	0.000428
32	0.1098	0.1498	0.00074	0.000689	0.000529	0.1120	0.1512	0.00074	0.000689	0.000529
33	0.0772	0.1245	0.00073	0.000254	0.000190	0.0783	0.1256	0.00073	0.000254	0.000190
34	0.0469	0.0852	0.00083	0.000387	0.000276	0.0474	0.0859	0.00083	0.000387	0.000276
35	0.0622	0.1086	0.00089	0.000328	0.000215	0.0599	0.1072	0.00089	0.000328	0.000215
36	0.0605	0.0820	0.00035	0.000246	0.000211	0.0605	0.0822	0.00035	0.000246	0.000211
37	0.0496	0.0341	0.00020	0.000098	0.000106	0.0464	0.0322	0.00020	0.000098	0.000106
38	0.1229	0.1581	0.00185	0.000470	0.000402	0.1231	0.1586	0.00185	0.000470	0.000402
39	0.0622	0.0756	0.00101	0.000190	0.000223	0.0591	0.0736	0.00100	0.000190	0.000223
40	0.1060	0.1109	0.00015	0.000311	0.000305	0.1060	0.1109	0.00015	0.000311	0.000305
41	0.0752	0.0871	0.00103	0.000239	0.000341	0.0719	0.0850	0.00103	0.000239	0.000341
42	0.0520	0.1089	0.00070	0.000298	0.000286	0.0494	0.1070	0.00070	0.000298	0.000286
43	0.0968	0.0650	0.00029	0.000233	0.000279	0.0938	0.0635	0.00029	0.000232	0.000279
44	0.0559	0.0971	0.00100	0.000308	0.000257	0.0575	0.0986	0.00100	0.000308	0.000257
45	0.0734	0.0883	0.00085	0.000278	0.000289	0.0704	0.0870	0.00085	0.000278	0.000289
46	0.0729	0.0864	0.00078	0.000249	0.000242	0.0714	0.0858	0.00078	0.000249	0.000242
47	0.0637	0.0977	0.00086	0.000289	0.000272	0.0609	0.0960	0.00086	0.000289	0.000272
48	0.0787	0.0774	0.00065	0.000212	0.000252	0.0791	0.0784	0.00065	0.000213	0.000252
49	0.0629	0.0990	0.00072	0.000256	0.000242	0.0601	0.0970	0.00072	0.000257	0.000242
50	0.0639	0.0940	0.00068	0.000235	0.000266	0.0633	0.0936	0.00068	0.000236	0.000266
51	0.0747	0.0878	0.00065	0.000271	0.000285	0.0713	0.0861	0.00065	0.000271	0.000285

#### 4. Discussion

The measured values are almost always burdened with inaccuracy. Thus, in view of the above [18,49]:

- The calculation of unknown quantities from different systems of balance equations result in different values;

- The substitution of the calculated values of the unknowns from the adopted system of equations to the remaining equations results in the fact that they are not satisfied.

At the same time, during the research, the number of equations that could be written, and from which unknown quantities could be determined, exceeded the number of the unknowns. Thus, we observed a phenomenon referred to as excess of measurement information.

We can eliminate the above-mentioned inconveniences by the reconciliation of measurement results by means of data validation and reconciliation technology. Such processing of measurement data, in addition to eliminating the above disadvantages, ensures the realization of several other goals, such as [11–13,18]:

- Unambiguous calculation of the most probable values of unknown;
- Control to maintain the assumed accuracy of measurements (it is particularly important when the individual tests are carried out once only and there is a possibility of making, e.g., gross errors);
- Reduction in the inaccuracy of measurement result;
- Assessment of the accuracy of the corrected measurement results and the calculated values of unknowns.

Thus, the data validation and reconciliation allow for a more unambiguous determination of the values of quantities which are difficult to measure. By specifying corrections of the measured values, we can identify measurements burdened with the so-called “gross errors”.

In view of the above, one of the advantages of using data validation and reconciliation is that it reduces the costs of performed measurements, as it reduces the need to use more accurate (and thus more expensive) measuring instruments.

In this study, the calculations with data validation and reconciliation were carried out for seven model equations involving:

- Balance of carbon;
- Balance of nitrogen;
- Balance of oxygen;
- Balance of hydrogen;
- Balance of sulfur;
- Balance of the mineral part (ash and slag);
- Balance of flue gas.

The searched unknowns were as follows:

- Total amount of nitrogen in flue gas,  $Y_1$ ;
- Mass of slag,  $Y_2$ ;
- Total amount of moisture in flue gas,  $Y_3$ ;
- Volume of dry flue gas from the combustion process,  $Y_4$ .

In this paper, we concisely discuss the calculation procedures involving data validation and reconciliation and presents literature which discusses the said issue in detail.

The example discussed in this paper can be regarded as another argument confirming the reasonability, or even the necessity to reconcile test results which comprise measurements. We can observe that even in the case of tests carried out in laboratory conditions (as presented in the paper), on average, the corrections of the values of some variables amounted (for 51 tests) to over 10% of the measured value.

When determining emission characteristics of the process, the use of reconciled measurement results for individual variables provides credibility of such characteristics.

In the study, we propose and present the application of data validation and reconciliation to improve the results of measurements carried out as part of the determination process of emission characteristics of co-combustion of sewage sludge with hard coal. Applications of the applied method could be extended to studies on waste management, co-combustion processes, and the environmental impact of these processes. Another innovative element of this studies involves the fact that the analyses are based on the measurements of a dynamic

research object, i.e., during the individual tests, the distribution of the supplied stream of primary air changed over time. Additionally, during the tests, the total test execution times (co-combustion processes) were variable. As a result, the emissions of flue gas components also changed over time during the individual tests.

In connection with the above, the proposed procedure presented in this study should constitute one of the elements of the development and evaluation of measurement results carried out for the following:

- As part of research aimed at developing operational characteristics (including emission characteristics) for the processes of co-combustion and combustion of fuels in furnaces;
- For all types working in steady conditions or similar to steady conditions,
- For all types powered periodically;
- As part of research aimed at determining the efficiency of processes for combustion and co-combustion of fuels are;
- During the operation of combustion installation, in order to identify malfunctioning measuring devices;
- During the operation of combustion installation, in order to determine the unmeasured values.

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