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An Auxiliary Index for Reducing Brent Crude Investment Risk—Evaluating the Price Relationships between Brent Crude and Commodities

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Abstract: Examining the price relationships of Brent Crude with 78 global commodities, our study shows that the spot price of a certain commodity, New York Harbor No. 2 Heating Oil Spot Price FOB, can serve as an auxiliary forecasting index of the rise and fall of the monthly Brent Crude oil price. With an innovative view for evaluating the price relationship and prediction based on simple, practical measurement, our findings provide a helpful auxiliary index tool for investors and analysts by offering a high success rate (82.98%) and predicting the rise and fall of the monthly Brent Crude oil price three weeks in advance.

Keywords: commodity price; forecasting; Brent Crude; West Texas Intermediate (WTI); two paths index

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1. Introduction

Crude oil is the most actively traded commodity in the world by the Futures Industry Association (FIA) does. Its influence spreads all over the world [1]. While oil industries cover upstream, midstream and downstream markets, predicting crude oil prices remains a major challenge for oil companies, oil-related industries, and countries that demand or supply petroleum. The rise and fall of crude oil prices are related to the sustainable development plans and investment policies of many companies. However, the market trend and fluctuation of international oil prices often lead to unpredictable market conditions. An accurate crude oil price forecast is crucial so that institutions may take significant precautions and plan necessary policies in advance.

Since the first oil crisis erupted in October 1973, crude oil prices have been volatile. The price is determined not only by the rules of supply and demand, but also by more factors. Some of them are as follows: the quality of crude, oil futures contracts, geopolitical events and related catastrophes, financial crises, GDP growth, economic recession, wars, oil embargos, inflation, seasonal variations, changes in stocks, the fluctuations of US dollar exchange rates, speculations, psychological expectations, OPEC's oil strategies, disasters (e.g., the COVID-19 pandemic), etc. Many of these factors are hard to parameterize effectively in the prediction process.

The volatility of crude oil prices often shows the characteristics of complexity, uncertainty and randomness. The financial crisis in 2008 was a major event which resulted in a structural breakdown in oil prices and changed the long-run equilibrium price relationships and the short-run price dynamics, according to Armando et al. (2015) [2].

The movements of crude oil price are often influenced by particular industries or sectors. For example, the responses of crude oil prices to OPEC's policies are often greater

than responses to seasonal variations. Today, oil prices still fluctuate greatly due to uncertain conditions and events. Athanasia et al. (2019) provided evidence that points to the rejection of even a weak form of efficiency in the crude oil market, implying that a ceiling exists in predicting oil prices [3]. Therefore, it is necessary to surmount this ceiling in order to reduce crude oil investment risks by finding other auxiliary methods.

Many methods have been developed to predict crude oil price, and some of them have reached accuracies of around 61% [4]. A lot of room for improvement still exists for effective forecasting of crude oil prices. Finding out how closely a commodity's price is related to the Brent Crude oil price by using mathematical methods to serve as a predicting index may be a feasible and simple way to help reduce Brent Crude investment risks.

On top of this, focusing on the price relationships between commodities and Brent Crude may reduce the cost and time of complex factor selection, processing load, decision weighting, etc. By this simple method, the selected commodities can be used as auxiliary indices for the Brent Crude investment and, when appropriate, can also be used as the direct indices for forecasting the rise and fall of Brent Crude oil price.

To predict the Brent Crude oil price using commodity prices requires not only a close relationship but also an accurate directional variation. It is important to find a useful leading index of the rise and fall of the price. Achieving these goals at the same time is not easy. Before making a forecast, using a commodity's price as an auxiliary predicting tool is a good way to improve the success rate and to help better guide investing decisions.

The price trends of Brent Crude, Dubai Crude and WTI Crude show a close price relationship, as shown in Figure 1. However, it is less meaningful to speculate on the target crude oil price using other crude oil prices, unless other crude oil prices can accurately be predicted first. One of the famous investors' fallacies for crude oil is that past historical price changes can always reflect future price changes. Making predictions using other commodity prices may allow investors to avoid the results of this fallacy.

Figure 1. Monthly Price Trends of Brent, Dubai and WTI. Note: 1. The figures in the plot are Brent Crude oil prices; 2. Data range is from August 1988 to August 2020. Unit: US\$/bbl Source: World Bank.

The aim of this study is to find out what commodities demonstrate high price relationships with Brent Crude according to past data and design an auxiliary leading index for predicting the rise and fall of the monthly Brent Crude oil price.

2. Literature Review

Market research related to forecasting crude oil prices has a long history. Structural models and time series models are the two main approaches to predicting oil price. In structural models, oil price is predicted using many explanatory variables, such as supply and demand, consumption, production, available stock of oil, net oil imports, etc. In time series models, oil price could be predicted solely using the past price trend, either linear (ARMA) or non-linear trends (GARCH or ANN) (Bostanchi, 2017) [5]. The time series method is often referred to as the Box-Jenkins approach [6].

Many factors strongly influence crude oil prices and show the complex characteristics of nonlinearity, dynamic variation, and high irregularity, as pointed out by Plourde and Watkins (1994) [7]. Bopp and Sitzler (1987) found that futures prices of heating are useful predictors of spot prices [8]. Asche et al. (2003) showed that in a multivariate framework, crude oil price is weakly exogenous and that the spread is constant in some but not all relationships [9].

Since the 1980s, many typical statistical and econometric techniques have been widely applied to oil price forecasting, such as: Linear regression (LinR), Co-integration analysis, Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, Naive Random Walk, Vector Auto-Regression (VAR), error correction models (ECM), Autoregressive Integrated Moving Average (ARIMA), autoregressive distributed lag (ARDL) test, etc. Many studies have employed conventional empirical methods.

Morana (2001) showed how the GARCH properties of oil price changes can be employed to forecast the oil price distribution over short-term horizons [10]. Lanza et al. (2005) used ECM specifications to predict crude oil prices [11]. Akpanta and Okorie (2014) applied the ARIMA method in modeling and forecasting Nigerian crude oil prices [12]. Oil prices are time related with a great deal of nonlinearity and irregularity. Numerous experiments have demonstrated that typical statistical and econometric models have not been promising for forecasting, such as complex series, according to Moshiri and Foroutan (2006) [13].

Chang and Lee (2015) found evidence of a long-run co-integration price relationship between oil spot prices and the prices of futures [14]. Arfaoui (2018) drew the conclusion that the short-run and long-run elasticities exist between spot and futures prices and between crude and refined oil prices, with the exception of gasoline, by using an ARDL bounds testing approach [15]. Yin et al. (2018) made an oil price forecast by using many predictor variables with a new time-varying weight combination approach to improve the accuracy. Good prediction results can be provided by using the above model based on the near linear relationships [4]. Zhao et al. (2020) proposed a crude oil price forecasting method based on time-varying trend decomposition to describe the changes in trends over time and forecast crude oil prices [16].

Over two decades ago, machine learning attracted wide attention in various fields due to development of AI. Researchers have used the machine learning (or deep learning) model to fit and predict time series data to improve the accuracy of oil price forecasting.

Mirmirani and Li (2003) suggested that the BPN-GA model noticeably outperforms the VAR model [17]. Alvarez-Ramirez et al. (2008) analyzed the auto-correlations of international crude oil prices [18]. Yu et al. (2008) made a forecast on crude oil prices with an EMD-based neural network [19]. Slim (2015) improved the forecasting of crude oil prices with statistical learning methods (SLM) based on a combined fuzzy system (FS), artificial neural network (ANN), and support vector regression (SVR) [20]. He et al. (2016) showed a significant performance improvement in forecasting WTI and Brent Crude oil prices using a multivariate EMD-based model [21].

The Long Short-Term Memory (LSTM) method based on recurrent neural networks (RNN) techniques were initially developed in 1980 and have since been widely used to deal with sequence-based problems, such as machine translation, text classification, power demand forecasting, stock market prediction, and behavior recognition. Gupta and Pandey (2018) predicted crude oil prices by using LSTM networks [22]. Cen and Wang (2019)

applied LSTM with a deep learning algorithm to predict the volatility behaviors of crude oil prices [23]. Liab et al. (2019) proposed a hybrid model for forecasting monthly crude oil spot prices by using various mode decompositions and AI, such as the GASVM and GABP methods [24]. Yin and Wang (2019) discovered the best prediction model of WTI Crude oil prices by constructing a new HWP-CHAOS model which outperforms prediction models such as ANN + Chaos-type, Chaos-type and ANN-type [25].

Kevin et al. (2003) used the crude oil and heating oil markets for diesel fuel purchasing decisions [26]. Some studies have discussed the price relationship between oil prices and various commodities, such as agro, food or metal commodities. Kousik et al. (2020) stated it is still uncertain whether oil prices have relationships with agro-commodity prices [27]. Roman et al. (2020) indicated that there are long-term relationships between crude oil and meat prices. The linkage of crude oil prices occurred with food, cereal, and oil prices has been observed in the short term [28]. Vo et al. (2019) indicated that not every oil shock has the same effect on agricultural price fluctuations, and similarly the effects of aggregate demand shocks on the agricultural market vary [29]. Farhad et al. (2019) found that the impact of biofuel prices on food prices is statistically significant but explains less than 2% of the food price variance [1].

3. Methodology

The procedure in this study can be roughly divided into two phases. The purpose in the first phase is selecting the commodities having close price relationships with Brent Crude through evaluation and verification. The purpose in the second phase is calculating the success rate of each selected commodity, verifying the accuracy of its predictions in the COVID-19 period, and picking out the best commodity according to the success rate and verification results.

1. First phase

Pearson's correlation coefficient, simple linear regression and scatter plotting play major roles in the first phase for the following reasons.

- (1) Covariance is likely to be misleading if the same data are compared in different units. This weakness was compensated for by the correlation coefficient (r_{xy}).
- (2) Any method using too many parameters is complex and difficult to change, which causes more errors. Therefore, multiple or nonlinear modeling can also be very case dependent and time-consuming, and cause collinear problems.
- (3) A pure mathematical relationship in which the R^2 (Sample coefficient of determination) has exactly the square of r_{xy} allows us to easily eliminate low R^2 commodities by the r_{xy} method.
- (4) A scatter plot allows us to avoid many pitfalls such as outliers, misleading results, etc., thus enhancing study quality.
- (5) These methods are simple and commonly used, and they conform to the simplicity notion of the razor principle.

Owing to the good results in the first phase, multiple regressions or non-linear regressions, which are highly complex and time-consuming, are not adopted. Descriptions of the study methods are provided as follows.

2. Second phase

Two paths were designed to measure the success rate of predicting the rise and fall of Brent Crude oil price.

3.1. Correlation Coefficient and Simple Linear Regression

Pearson's correlation coefficient is denoted by r_{xy} and is defined by (1).

$$r_{xy} = \frac{\text{Cov}(x, y)}{\sqrt{\text{Var}(x)\text{Var}(y)}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{n \sum xy - \sum x \sum y}{\sqrt{\{n \sum x^2 - (\sum x)^2\} \{n \sum y^2 - (\sum y)^2\}}} \quad (1)$$

where x and y are the variables.

$$\frac{|\text{cov}(x, y)|^2}{S_{xx}S_{yy}} = |r_{xy}|^2 \leq 1 \quad (2)$$

which implies that r_{xy} is always in the range of -1.0 to 1.0 .

The r_{xy} , an indicator of the goodness of fit between x and y variables, measures the strength of association and the direction of the relationships between both variables. It provides evidence of association, not causation, which also tells us how strong a linear relationship between two variables is under a situation with no outliers. Rodgers and Nicewander (1988) published a detailed description [30].

The relevant sizes of two samples are not necessarily proportional to r_{xy} . A high r_{xy} does not mean a causal relationship exists in both. Krehbiel (2004) [31] and Asuero et al. (2006) [32] suggested modified rules of thumb to describe the r_{xy} strength, which can be divided into seven levels ranging from no correlation to perfect correlation, as shown in Table 1.

Table 1. The Rule of Thumb for Interpreting the Size of the Correlation Coefficient.

Size of $ r $		Interpretation
(Timothy C. Krehbiel)	(A. G. Asuero, et al.)	
$0.0 = r $	$0.0 = r $	no correlation
$0.0 < r < 0.2$	$0.0 < r < 0.3$	very weak correlation
$0.2 \leq r < 0.4$	$0.3 \leq r < 0.5$	weak correlation
$0.4 \leq r < 0.6$	$0.5 \leq r < 0.7$	moderately strong correlation
$0.6 \leq r \leq 0.8$	$0.7 \leq r < 0.9$	strong correlation
$0.8 \leq r < 1.0$	$0.9 \leq r < 1.0$	very strong correlation
$1.0 = r $	$1.0 = r $	perfect correlation

Source: The Rule of Thumb suggested by T. C. Krehbiel (2004) [31] and by Asuero et al. (2006) [32].

The requirements of $|r|$ still vary in different fields. For natural/science/social science/economics studies, a $|r|$ equal to 0.6 may be enough. For instrument calibration, a $|r|$ equal to 0.9 may be unacceptable. The greater the “ n ” (data range) is, the higher the acceptability (significance) of $|r|$. A satisfactory $|r|$ is dependent on the purpose of usage or on the quality of raw data. Due to the fact that the r_{xy} may be strongly affected by outliers, the raw data and results in this study must be evaluated by the scatter plot.

Regression is a generic term that is often used in finance, investment, economic forecasting and other disciplines such as commodities, interest rates, machine learning, etc. Stanton et al. (2001) [33] also described regression in detail. A simple linear regression equation is given in (3).

$$\text{Linear regression : } Y = a + bX + \varepsilon \quad (3)$$

where,

Y = the dependent variable to be predicted;

X = the independent variable used to predict Y ;

a = the intercept of the regression line (the value of Y when $X = 0$);

b = the slope of the regression line;

ε = the regression residuals which are known as the error term.

Correlation coefficient (r_{xy}) and linear regression are different statistical techniques and differ in meaning. In general, the study of interdependence leads to the study of correlations, while the study of dependence leads to the theory of regression. However, both have the same valid expressions that are not mutually exclusive. Asuero et al. (2006) provided a full description [32].

3.2. Sample Coefficient of Determination (R^2)

The R^2 of simple linear regression is a measure of how well observed outcomes are replicated by the model, based on the proportion of total variation of outcomes that can be explained by the model, which is a useful tool to check the fitness of a model. The R^2 is given in (4).

$$R^2 = 1 - \frac{SS_{\text{res}}}{S_{YY}} = \frac{SS_{\text{Reg}}}{S_{YY}} \quad (4)$$

where SS_{res} is the sum of squares of residuals (i.e., SSE); SS_{Reg} is the sum of squares due to regression; S_{YY} is the total sum of squares (proportional to the variance of the data); and R^2 ranges from 0 to 1.

It is a mathematical relationship where the R^2 is exactly the square of the r_{xy} in simple linear regression. That is, when there is no association between both variables, linear regression probably will not provide a useful fitting model. Following this concept, we can eliminate the low R^2 commodities by the r_{xy} method roughly.

3.3. Scatter Plot

A scatter plot can show more information than the r_{xy} does, which merely gives a value to show how closely the relationship between both variables is. It can show what categories different types of data belong to, such as linearity, nonlinearity, separated points, etc. Several scatter plots can be combined on one page, making it easy to understand the whole dataset's structure. The regression line on the scatter plot is difficult to detect when the matching relationship is weak (e.g., $|r_{xy}| \leq 0.3$). The shape becomes clearer when the relationship is strong (e.g., $|r_{xy}| \geq 0.90$).

By scatter plotting, researchers can avoid many pitfalls such as outliers, misleading results, etc., and have more confidence in the study quality. Therefore, the raw data and the study results can be checked by using a scatter plot and line chart to ensure the quality. Furthermore, the fit of a model provided is more convincing when a scatter plot is combined with linear regression.

3.4. The Proposed Auxiliary Leading Index for Predicting the Rise and Fall of Brent Crude Oil Price

Two paths were designed to measure the success rate of predicting the rise and fall of Brent Crude oil price as follows.

1. First step

Path 1: The spread ($Sp_{\text{path } 1}$) is the first weekly price of each month minus the monthly price of previous month, which is expressed as follows:

$$Sp_{\text{path } 1 \ t} = P_{\text{first week } t} - P_{\text{month } t-1} \quad Sp_{\text{path } 1 \ t} = P_{\text{first week } t} - P_{\text{month } t-1} \quad (5)$$

where Sp is the spread; P is the commodity price; t is monthly unit.

Path 2: The spread ($Sp_{\text{path } 2}$) is the first weekly price of each month minus the first weekly price of previous month, which is expressed as follows:

$$Sp_{\text{path } 2 \ t} = P_{\text{first week } t} - P_{\text{first week } t-1} \quad Sp_{\text{path } 2 \ t} = P_{\text{first week } t} - P_{\text{first week } t-1} \quad (6)$$

2. Second step

The spread of each Path is compared with the direction of the rise and fall of the monthly Brent Crude oil price in the same time slot to calculate the success rate (S_r) of each Path. The S_r is expressed as follows:

$$\text{Success Rate}(S_r) = \frac{1}{n} \sum_{i=1}^n s_t \quad (7)$$

where $s_t = 1$ if $(Sp_{path\ 1} \text{ or } Sp_{path\ 2}) * (P_{Brent\ t} - P_{Brent\ t-1}) > 0$ and $s_t = 0$ otherwise; n is the number of spreads (i.e., $n = \text{data range} - 1$). The S_r is in the range from 0 to 1. The higher the S_r is, the more accurate the model.

The auxiliary index using commodity price in this study focuses on finding commodity prices with a high success rate to predict the rise and fall of Brent Crude monthly oil price in that month three weeks in advance. The proposed index does not predict crude oil price directly. This index shows the rise and fall of Brent Crude monthly oil price in the short-term, not in the long-term price or according to target price. It could be used conjunction with other predicting methods such as ARIMA, LSTM, etc., rather than being used separately.

3.5. Remarks

In the first phase, we are more interested in determining the strength of r_{xy} for the following reasons:

1. High r_{xy} is the basis of accurate crude price estimates using commodity price. If the r_{xy} is weak or moderate, this method will be less meaningful.
2. In a mathematical relationship, the fact that the r_{xy}^2 is equal to the R^2 can reduce the complexity of the process, and less collinear problems will occur, unlike other methods such as multiple regression.

In the second phase, we focus on the success rate of empirical analysis, which is the basis of this auxiliary leading index.

Thus, high r_{xy} is the critical point in the first phase (i.e., correlation analysis or linear regression). Evaluating based on use of a scatter plot is another critical point for ensuring the study quality. The high success rate of the empirical results is of course a major point of concern.

4. Analysis and Empirical Results

A close price relationship is indispensable when using commodity prices to predict the rise or fall of Brent Crude oil price. Such a relationship can alleviate the effect of factors which are difficult to effectively parameterize in other kinds of studies, such as those related to geopolitics. If scientifically proven, the proposed index may be more convenient and convincing than many previous theories and models due to its simplicity and practicality.

4.1. Data Description

We selected past monthly Brent Crude oil prices as one variable. The other variable is the monthly price of each commodity. Brent Crude is extracted from the North Sea and is a major trading classification of sweet light crude oil that serves as a benchmark price for purchases of crude oil worldwide. Prices for other crude oils are generally priced as a differential (discount or premium) to Brent Crude. The prices of the same commodity vary from place to place, but we can estimate this price more accurately and easily (but not all) if a benchmark price of the commodity exists. Brent Crude oil price is more globally representative than that of Dubai Crude and WTI Crude, which serve as the comparative references in this study.

The study's dataset structure, shown in Appendix A and taken from the Mundi company website, includes 81 types of monthly commodity prices that are divided into 11 categories. Each commodity price herein is treated as one dataset with the attributes of "training data" or "testing data". The training data are mostly dated from December 1988 to October 2019. That is, the training data length of each commodity normally has 371 consecutive price data points (pts), owing to these data are not affected by COVID-19. The recorded data dated after October 2019 is treated as testing data, which can be used to verify the training data study results, even if the data range is insufficient.

To reduce the uncertainty of processing results, each commodity's training data must have at least 156 consecutive data points, including the price data dated October 2019. That

is, each commodity analyzed has at least 13 years of monthly-recorded data. None of the data has been adjusted for inflation, weighting, time lag, etc.

Each defective dataset is wholly replaced or deleted for the sake of quality, thus avoiding data incompleteness, massive data losses, data discontinuity, and poor accuracy. Replaced datasets were extracted from the World Bank, IMF or EIA (U.S. Energy Information Administration) based on Mundi's data source indexes. A big context gap may exist for the same commodity with different data sources. The replaced datasets were selected based on past experience when dual data sources existed.

After replacing and deleting data, there were still 79 qualified commodities in the datasets as shown in Appendix B, included Brent Crude oil price. Moreover, they include seven incomplete commodities: Ultra Sulfur No. 2 Diesel (No. 7; 171 pts), Kerosene-Type Jet Fuel (No. 11; 345 pts), TX Propane (No. 13; 339 pts), RBOB Regular Gasoline (No. 14; 207 pts), Palm Kernel Oil (No. 41; 296 pts), Rapeseed oil (No. 45; 223 pts) and Sunflower oil (No. 49; 212 pts). Each commodity has at least 171 consecutive monthly prices, include "testing data", and only one data source.

The datasets also include silver and zinc commodities with dual data sources from the World Bank and IMF. Full descriptions of each commodity can be reviewed in Mundi [34], World Bank [35], IMF [36] or EIA [37] reports. For the sake of consistency of price correlation and price change rate (or rate of price change) correlation, a maximum of 370 data points were used in the first phase.

4.2. Stability Analysis

Stability is an important basic trait when predicting crude oil prices using commodity prices. Without it, we would face great uncertainties. If a stable and close price relationship shows between the commodity and Brent Crude with different time lengths, it could be persuasive for investors or analysts.

Since commodity price belongs to a time series type, the r_{xy} has two types: price and price change rate, in this analysis. For the example of price r_{xy} , according to our design, the monthly price of each commodity in October 2019 is taken as the point of origin. The time length of the first r_{xy} is 18 months for the sake of comparison. The time length of the second r_{xy} is 24 months. Thereafter, the time length of each r_{xy} is extended forward until the last data point by an interval unit of 24 months to calculate each r_{xy} in different time lengths, i.e., 48 months, 72 months, 96 months . . . and so on. (See Appendices D and C).

For further analysis, each selected commodity price must be closely related to the Brent Crude oil price, whether in the short-term or long-term. That is, the average of r_{xy} is high and the SD of r_{xy} is small. Moreover, short-term prediction is often more valuable than that of long-term in the real world. To improve the prediction accuracy of Brent Crude, the time slot between predicted data and real data should be shortened. That is, the higher the short-term r_{xy} is, the better the accuracy is. We take these conditions as the basis to select commodities.

4.2.1. Correlation Analysis of Price

Price r_{xy} with different time lengths between 78 commodities and Brent Crude are shown in Appendix C. A big gap in price r_{xy} appears for different commodities or categories. We also find that the r_{xy} of many commodities are over 0.90, reaching as high as 0.98, thus surpassing our expectations.

For further analysis, we select the commodities with average r_{xy} over 0.90 and with SD of r_{xy} under 0.05. In total, six commodities met these conditions, including commodity No. 5, 6, 7, 8, 9 and 11 (see Table 2).

Table 2. The Commodities Selected through Correlation Analyses.

No.	Commodity	Source	Price r_{xy}				Price Change Rate r_{xy}			
			pts	Full	Average	SD	pts	Full	Average	SD
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	WB	370	0.9987	0.9972	0.0031	370	0.9600	0.9746	0.0138
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	WB	370	0.9910	0.9809	0.0103	370	0.9366	0.9283	0.0106
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	EIA	161	0.9857	0.9817	0.0136	160	0.9345	0.9432	0.0054
8	New York Harbor Conventional Gasoline Regular Spot Price FOB	EIA	370	0.9921	0.9710	0.0362	370	0.7948	0.8286	0.0212
9	New York Harbor No. 2 Heating Oil Spot Price FOB	EIA	370	0.9956	0.9869	0.0161	370	0.7998	0.8816	0.0591
11	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	EIA	355	0.9941	0.9874	0.0104	354	0.8677	0.8934	0.0335

Note: 1. Data range is from Jan. 1989 to October 2019 (Training Data); 2. SD, Standard Deviation.

4.2.2. Correlation Analysis of Price Change Rate

By this analysis, we can understand the sensitivity of the price change rate between each commodity and Brent Crude. The selection of the commodity is verified scientifically if the price change rate r_{xy} shows a close relationship.

The price change rate is the change rate of two neighboring prices, as shown in (8).

$$\text{Price Change Rate} = \frac{(\text{Price}_t - \text{Price}_{t-1})}{\text{Price}_{t-1}} \quad (8)$$

where t is the monthly unit.

The price change rates r_{xy} with different time lengths are shown in Appendix D. For the same commodity, the price change rate r_{xy} is mostly less than the price r_{xy} because the former is not as easy to match as the latter. By sorting the average of price change rate r_{xy} , we see a sharp gap between commodity no 8 ($r_{xy} = 0.8286$) which ranked 6th and no 14 ($r_{xy} = 0.6916$), which ranked 7th.

We selected the commodities with average r_{xy} of over 0.80 and SD of r_{xy} under 0.06. In total, six commodities met these conditions, including commodity No. 5, 6, 7, 8, 9 and 11, which are the same as the commodities selected by price r_{xy} (see Table 2). The price change rates r_{xy} of the four selected commodities, with the exception of crude oil, were above 0.82, reaching as high as 0.94.

4.2.3. Discussion of Commodities Selection

Lots of commodities were excluded by the r_{xy} method, such as Coal Export Price (South African, no. 2), Indonesian Liquefied Natural Gas Price (no. 10), TX Propane Spot Price FOB (no. 13), Russian Natural Gas Price (no. 15), Wheat Price (no. 26), Soybeans Price (no. 48), Copper Price (no. 62), Iron Ore Price (no. 64), Platinum Price (no. 68), etc. Fortunately, some commodities were not excluded. Moreover, we also found that a weak or moderate price relationship exists between Brent Crude oil and agro-commodities.

For further analysis, the commodities selected are New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price (no. 7; No 2 Diesel Spot Price), New York Harbor Conventional Gasoline Regular Spot Price FOB (no. 8; Gasoline Regular Spot Price FOB), New York Harbor No. 2 Heating Oil Spot Price FOB (no. 9; Heating Oil Spot Price FOB), and U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB (no. 11; Kerosene-Type Jet Fuel Spot Price FOB). All of them belong to the energy category and are the downstream products of crude oil. There seems to be causalities between these commodities and Brent Crude.

4.3. Scatter Plot and Linear Regression

Scatter plots combined with regression lines are more convincing, offering a useful tool to help make choices. After excluding commodities through correlation analysis, scatter plots and line charts of six commodities plotted by price and price change rate are shown in Figures 2 and 3. The regression formulae and R^2 of each commodity are shown also.

Figure 2. (a) Crude Oil, Dubai

Figure 2. (b) Crude Oil, WTI

Figure 2. (c) Ultra-Low Sulfur No 2 Diesel Spot Price

Figure 2. (d) Conventional Gasoline Regular Spot Price FOB

Figure 2. (e) No. 2 Heating Oil Spot Price FOB

Figure 2. (f) Kerosene-Type Jet Fuel Spot Price FOB

Figure 2. Scatter Plot (left) and Line Chart (right) for the Price of Each Selected Commodity and Brent Crude. Note: Data range is from December 1988 to October 2019 except for Figure 2c,f. (See Appendix B)

Figure 3. (a) Crude Oil, Dubai

Figure 3. (b) Crude Oil, WTI

Figure 3. (c) Ultra-Low Sulfur No 2 Diesel Spot Price

Figure 3. (d) Conventional Gasoline Regular Spot Price FOB

Figure 3. (e) No. 2 Heating Oil Spot Price FOB

Figure 3. (f) Kerosene-Type Jet Fuel Spot Price FOB

Figure 3. Scatter Plot (left) and Line Chart (right) for the Price Change Rate of Each Selected Commodity and Brent Crude. Note: Data range is from December 1988 to October 2019 except for Figure 3c,f. (See Appendix B)

As seen in the pictures, three selected commodities clearly show a nice in-phase shape. The Gasoline Regular Spot Price FOB ($R^2 = 0.6317$) is removed because neither R^2 nor the

short-term r_{xy} of the price change rate are good enough. High R^2 commodities are one option (but not the only option) in this study.

4.3.1. Commodities' Selection in the Datasets

New datasets are made from five commodities, including the reference commodities, WTI Crude and Dubai Crude. The others are the commodities selected based on the price relationship. We reprocess the price r_{xy} and the price change rate r_{xy} by reducing the data range to 161 points for the sake of consistency, because the minimum training data range among the selected commodities is 161 points. Table 3 shows some characteristics as follows.

Table 3. Correlation between Selected Commodities and Brent Crude over Different Time Lengths.

Price r_{xy}		2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD	R ²
		2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2006.6			
No.	Data Item (Monthly)	18	24	48	72	96	120	144	161			
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	0.9945	0.9861	0.9964	0.9984	0.9989	0.9984	0.9982	0.9979	0.9961	0.0043	0.9957
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	0.9696	0.9562	0.9825	0.9917	0.9863	0.9773	0.9701	0.9687	0.9753	0.0114	0.9384
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	0.9626	0.9579	0.987	0.9889	0.9937	0.9919	0.9857	0.9857	0.9817	0.0136	0.9716
9	New York Harbor No. 2 Heating Oil Spot Price FOB	0.9551	0.9364	0.9858	0.9898	0.9941	0.9929	0.9885	0.9886	0.9789	0.0212	0.9773
11	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	0.9722	0.9539	0.983	0.9917	0.9951	0.995	0.9863	0.9857	0.9829	0.0139	0.9716
Price Change Rate r_{xy}		2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD	R ²
		2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2006.7			
No.	Data Item (Monthly)	18	24	48	72	96	120	144	160			
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	0.9905	0.9911	0.9895	0.9909	0.9872	0.986	0.9884	0.9857	0.9887	0.0022	0.9717
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	0.9417	0.9446	0.9393	0.927	0.9092	0.9024	0.9228	0.9242	0.9264	0.0152	0.8542
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	0.9478	0.9498	0.9494	0.9416	0.9415	0.9421	0.9388	0.9345	0.9432	0.0054	0.8733
9	New York Harbor No. 2 Heating Oil Spot Price FOB	0.9444	0.9422	0.9316	0.928	0.9287	0.9317	0.9359	0.9329	0.9344	0.0060	0.8704
11	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	0.9447	0.9504	0.8989	0.924	0.922	0.9256	0.9212	0.9207	0.9259	0.0158	0.8476

Note: 1. SD, Standard Deviation; 2. Data range is from June 2006 to October 2019.

1. In price r_{xy} , the average and R^2 of all selected commodities are over that of WTI Crude.
2. In price change rate r_{xy} , the average and R^2 of the No 2 Diesel Spot Price show better performance than those of WTI Crude. The Heating Oil Spot Price FOB also shows

good performance. The performance of Kerosene-Type Jet Fuel Spot Price FOB is also close to that of WTI Crude.

These characteristics have never been clearly reported before. Some selected commodities may serve as good auxiliary indexes for investment in Brent Crude.

However, three other selected commodities have close price relationships with Brent Crude, and are downstream co-products of crude oil. They have their own demand markets, traders, specific consumers (e.g., Kerosene-Type Jet Fuel Spot), usage etc. Kevin et al. (2003) suggested that diesel fuel prices forecasted using the crude oil or heating oil futures market are reasonably accurate [26].

4.3.2. Pre-Examination of the Time Series

The most important feature of prediction by using time series, we believe, is that it is related to the before and after, or is related between both. Thus, the characteristics of each commodity should have a basic understanding before research. We operate the tests as follows.

1. All selected and related commodities are non-stationary according to the results of ADF (Augmented Dickey-Fuller) test. (See Table 4)
2. All selected commodities have passed the RESET test (Ramsey, 1969). That is, there exists a less significant non-linear relationship when you have built a linear regression model according to the results of RESET tests. (See Tables 5 and 6)

Table 4. The results of ADF test.

Monthly Data	Crude Oil (For Reference)			Selected Commodity		
Commodity	Brent Crude	Dubai Crude	WTI Crude	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
Dickey-Fuller	−2.5835	−2.52	−3.0486	−2.6591	−2.7006	−2.8188
Lag order	5	5	5	5	5	5
<i>p</i> -value	0.3326	0.3591	0.1384	0.3011	0.2837	0.2344
Test Results	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

Note: 1. Data range is from June 2006 to August 2020; 2. FALSE means “non-stationary state”; 3. Using the ADF test in *r*.

4.3.3. Discussion of Price Relationships of Some Unselected Commodities

Some unselected commodities in the energy category are the derivatives of crude oil and show different price relationships especially in price change rate, such as “Mont Belvieu, TX Propane Spot Price FOB” (no 13, 0.6561), Los Angeles Reformulated RBOB Regular Gasoline Spot Price (no 14, 0.6916), which are products of crude oil. Moreover, “Indonesian Liquefied Natural Gas Price, Japan” (no 10, −0.1525), which the contract price is mostly linked to crude oil price, has a negative value. Apparently, crude oil price is not the only component of the prices of these unselected commodities. Surely, the increase or decrease in demand for the derivatives is not equal to that of Brent Crude oil in the short term.

Table 5. The Results of RESET Test.

Monthly Data	Crude Oil (For Reference)			Selected Commodity	
Commodity	Dubai Crude	WTI Crude	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
Data points	171	171	171	171	171
RESET	2.6351	2.0249	1.0351	0.68152	0.081086
<i>p</i> -value	0.1064	0.1566	0.3104	0.4102	0.7762

Note: 1. Data range is from June 2006 to August; 2. The other time series is Brent Crude; 3. Using Ramsey’s RESET test (1969) in *r*.

Table 6. The Formulae and R^2 of Linear Regression between Selected Commodities and Brent Crude.

No.	Commodity	pts	Price	
			Formulae	R^2
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	171	$Y = 1.0361X - 0.3291$	0.996
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	171	$Y = 1.08915X - 1.2924$	0.9442
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	171	$Y = 37.865X - 7.3022$	0.9733
9	New York Harbor No. 2 Heating Oil Spot Price FOB U.S. Gulf Coast	171	$Y = 38.506X - 5.5545$	0.9783
11	Kerosene-Type Jet Fuel Spot Price FOB	171	$Y = 36.694X - 2.1202$	0.9728

Note: Data range is from June 2006 to August 2020.

We did not know before that the three selected commodities have such close price relationships with Brent Crude. The 161 monthly-price data points indicate that the commodities have been through various events, such as the financial crisis, U.S. shale oil and gas boom, economic sanctions on Iran, global oil price war, Iran nuclear crisis and deal, OPEC's oil production cuts, Trump Trade war etc. Each event had a significant effect on global crude oil prices and its derivatives. It is not easy for a commodity to maintain such a good match with the Brent Crude oil price throughout this period.

4.4. Estimated Brent Crude Oil Prices Using a Linear Regression Formulae

Estimated Brent Crude oil prices, derived from each commodity price by its formulae of linear regression, are shown in Appendix E. Analysis of these data, given below, can help us understand the differences between estimated and real Brent Crude oil prices. The estimated price line charts with residuals are shown in Figure 4.

In Figure 4, the degree of price matching between Dubai crude and Brent Crude is not disputed. It is less necessary to discuss this in the later phases. The residuals of WTI Crude were once over \$20/bbl in the period of "gas effect". All selected commodities show high residuals in the period of the financial crisis.

4.5. Performance Metrics

Mean squared prediction error (MSPE) and directional accuracy ratio (DAR) are popular metrics for forecasting in economics and finance. Zhao et al. (2020), Gao and Lei (2017) also applied both metrics to evaluate the performance of crude oil price forecasting [16,38]. They are also useful metrics for picking out a commodity as the best auxiliary investing tool for predicting Brent Crude oil price. MSPE and DAR are described as follows.

1. MSPE

MSPE summarizes the predictive ability of a model by measuring the average of the squares of the price errors. The price error is the difference between real crude price and estimated price. Let $y_1, y_2, \dots, y_n$ be the real crude oil prices and $\hat{y}_1, \hat{y}_2, \dots, \hat{y}_n$ be the estimated prices from each commodity model. MSPE is expressed as follows:

$$MSPE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (9)$$

where n is data range. The lower the MSPE is, the more accurate the model.

Figure 4. (a) Crude Oil, Dubai

Figure 4. (b) Crude Oil, WTI

Figure 4. (c) Ultra-Low Sulfur No 2 Diesel Spot Price

Figure 4. (d) No. 2 Heating Oil Spot Price FOB

Figure 4. (e) Kerosene-Type Jet Fuel Spot Price FOB

Figure 4. Scatter Plots (left) and Line Charts (right) of the Estimated Price and Real Price of Brent Crude. Note: Data range is from June 2006 to August 2020.

2. DAR

DAR is a measure for predicting the accuracy of directional variation between real crude price and estimated price (i.e., The commodity price has the same rise or fall of crude price in the same time slot). DAR is expressed as follows:

$$DAR = \frac{1}{n} \sum_{i=1}^n d_t \quad (10)$$

where $d_t = 1$ if $(y_t - y_{t-1}) * (\hat{y}_t - \hat{y}_{t-1}) > 0$ and $d_t = 0$ otherwise. We herein modified $(\hat{y}_t - y_{t-1})$ as $(\hat{y}_t - \hat{y}_{t-1})$ reasonably in the field of commodity price. DARs are all in the range from 0 to 1. The higher the DAR is, the more accurate the model.

If the DAR of a model is greater than 0.5, it is better than “Random Walk”. Highly accurate directional variation of price is a key basis for predicting Brent Crude oil price using a commodity price. It herein is more important than MSPE. We assume that commodity prices can assist in price predicting under the following conditions (but not all).

1. The total ratio of the same price directional variation in one time slot for the commodity and Brent Crude is over 80%. That is, DAR is over 80%.
2. The average of squared residuals between the estimated price and real price of Brent Crude is close to or lower than that of WTI Crude or Dubai Crude.

The results of DAR and MSPE in the three different time ranges shown in Table 7 have the following characteristics.

1. Dubai Crude with high DAR and low MSPE performs very well and has significant improvement in DAR after June 2006.
2. The MSPE of WTI Crude is more than that of all the selected commodities in the three time ranges. The significance of MSPE gets worse after June 2006 and gets well during the COVID-19 period.
3. The DARs of Kerosene-Type Jet Fuel Spot Price FOB approach or exceed 0.90 in the three time ranges. Of all selected commodities, this is the only commodity that does so.

Table 7. The DAR and MSPE in Different Time Ranges.

Time Range	Items	Crude oil, Dubai (Petroleum); Dubai Fateh Price	Crude oil, WTI (Petroleum); West Texas Intermediate Price	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	New York Harbor No. 2 Heating Oil Spot Price FOB	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB
Full Range (2020.08~1988.12)	Data range	381	381	171	381	365
	Number of “+”	348	344	148	329	325
	DAR (%)	91.58%	90.53%	87.06%	86.58%	89.29%
	MSPE	2.81	19.27	17.90	9.58	12.91
Partial Range I (2020.08~2006.06)	Data range	171	171	171	171	171
	Number of “+”	167	156	148	148	155
	DAR (%)	98.24%	91.76%	87.06%	87.06%	91.18%
	MSPE	3.15	38.67	17.90	15.50	18.61
Partial Range II (2019.10~2006.06)	Data range	161	161	161	161	161
	Number of “+”	157	146	140	141	146
	DAR (%)	98.13%	91.25%	87.50%	88.13%	91.25%
	MSPE	3.05	40.16	18.20	15.37	18.65

Note: 1. “+” means the same directional variation with Brent Crude oil price; 2. DAR’s data range = Data range − 1; 3. Partial Range II: the minimum training data range among all selected commodities (161 pts); 4. Partial Range I: Partial Range II plus testing data range (10 pts); 5. EIA added the data of ultra-low sulfur No. 2 diesel spot price on 14 June 2006.

4.6. Discussion of the above Analytical Results

A summary of the analyses results of this study is shown in Table 8, including the performance metrics, where the data range is from June 2006 to October 2019. We identified the following characteristics.

Table 8. A Brief Summary of Performance Metrics.

Items	Crude Oil, Dubai (Petroleum); Dubai Fateh Price	Crude Oil, WTI (Petroleum); West Texas Intermediate Price	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	New York Harbor No. 2 Heating Oil Spot Price FOB	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB
Price r_{xy}	0.9979	0.9687	0.9857	0.9886	0.9857
Average & SD of Price r_{xy} with different time lengths	0.9961 ± 0.0043	0.9753 ± 0.0114	0.9817 ± 0.0136	0.9789 ± 0.0212	0.9829 ± 0.0139
Price Change rate r_{xy}	0.9857	0.9242	0.9345	0.9329	0.9207
Average & SD of Price Change Rate r_{xy} with different time lengths	0.9887 ± 0.0022	0.9264 ± 0.0152	0.9432 ± 0.0054	0.9344 ± 0.0060	0.9259 ± 0.0158
MSPE	3.06	40.16	18.20	15.37	18.65
DAR (%)	98.13%	91.25%	87.50%	88.13%	91.25%
DAR (price change rate > ±2%)	100%	97.54%	97.54%	96.72%	96.72%

Note: 1. SD, Standard Deviation; Explanatory ability (R^2) = r_{xy}^2 ; 2. Data range is from June 2006 to October 2019, and the data range is 161 data points; 3. Data number ignoring the price change rate of Brent Crude within ±2% per month has 122 data points.

1. As for DARs, all selected commodities show a high match ($\geq 87.50\%$), and Kerosene-Type Jet Fuel Spot Price FOB has the same DAR (91.25%) as that of WTI Crude. Besides, by ignoring the commodities prices for which monthly price change rates of Brent Crude are within ±2% in the same time slot, we can recalculate DARs. It is interesting that the DARs of three commodities are over 96.72%, and the No 2 Diesel Spot Price has the same DAR (97.54%) as that of WTI Crude. That is, these selected commodities show a high degree of consistency in directional variation when Brent Crude oil prices fluctuate significantly. All of them should be eligible for assisting in prediction.
2. For MSPE, all selected commodities are at the same level. Each commodity's MSPE is less than that of WTI Crude (40.16), and No. 2 Heating Oil Spot Price FOB (15.37) shows excellent performance.

Briefly, the No 2 Diesel Spot Price shows a good match with the average and SD of price change rates r_{xy} . Heating Oil Spot Price shows a low MSPE. Kerosene-Type Jet Fuel Spot Price has a DAR which is as good as that of WTI Crude. Each selected commodity has its advantages. It is not easy to select a suitable commodity as the auxiliary index for prediction.

4.7. Verification of Testing Data

During the COVID-19 pandemic is a good time to verify the degree to which prices of selected commodities match with Brent Crude. The results' summary in Table 9 shows a similar trend with the previous analytical results of this study. The performance of the Dubai Crude model is outstanding and surpasses other models. The performance of the WTI Crude model is not good enough when compared with the models of the selected commodities, and the following differences appear.

Table 9. A Brief Summary of All Selected Commodity Models after Verification.

Performance Metrics	Crude Oil, Dubai; Dubai Fateh Price	Crude Oil, WTI; West Texas Intermediate Price	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	New York Harbor No. 2 Heating Oil Spot Price FOB	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB
DARs	100%	100%	80%	70%	90%
MSPEs	4.81	14.67	13.11	17.5	17.94

Note: Testing data range is from Nov. 2019 to August 2020.

1. In the DAR, the Kerosene-Type Jet Fuel Spot Price FOB model shows good performance (90%). It has the same DARs as the Dubai Crude model and WTI Crude model since the outbreak of COVID-19 in Jan. 2020. This model is better than the other commodity models as both “training data” and “testing data”. Moreover, the DAR (70%) of the No. 2 Heating Oil Spot Price FOB is less than 80%. (See Appendix F)
2. In the MSPE, all selected commodity models reach the same level. Among of them, the No 2 Diesel Spot Price model shows a good performance because its MSPE is lower than that of the WTI Crude model.

There was a turning point at which price soared in May 2020 after the crude oil price slump resulting from the COVID-19 outbreak. The Kerosene-Type Jet Fuel Spot Price FOB model shows a better price response (turning point) than do the other selected commodity models. After recalculating the r_{xy} of price and price change rate from June 2006 to August 2020, we find the following characteristics (see Table 10).

Table 10. Correlation between Selected Commodities and Brent Crude Included “Testing Data”.

Price r_{xy}		2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	Average	SD	R ²
		2019.3	2018.9	2016.9	2014.9	2012.9	2010.9	2008.9	2006.9	2006.6			
No.	Data Item (Month)	18	24	48	72	96	120	144	168	171			
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	0.9990	0.9990	0.9970	0.9965	0.9988	0.9988	0.9985	0.9981	0.9980	0.9982	0.0009	0.9960
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	0.9904	0.9905	0.9819	0.9836	0.9866	0.9843	0.9770	0.9722	0.9717	0.9820	0.0071	0.9442
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	0.9783	0.9793	0.9806	0.9802	0.9925	0.9935	0.9888	0.9868	0.9866	0.9852	0.0058	0.9734
9	New York Harbor No. 2 Heating Oil Spot Price FOB	0.9755	0.9766	0.9789	0.9819	0.9928	0.9939	0.9899	0.9892	0.9891	0.9853	0.0071	0.9783
11	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	0.9842	0.9820	0.9789	0.9847	0.9929	0.9946	0.9885	0.9865	0.9863	0.9865	0.0050	0.9728
Price Change Rater $_{xy}$		2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	2020.8	Average	SD	R ²
		2019.3	2018.9	2016.9	2014.9	2012.9	2010.9	2008.9	2006.9	2006.7			
No.	Data Item (Monthly)	18	24	48	72	96	120	144	168	170			
5	Crude oil, Dubai (petroleum); Dubai Fateh Price	0.9961	0.9962	0.9944	0.9940	0.9925	0.9912	0.9912	0.9891	0.9891	0.9926	0.0027	0.9783
6	Crude oil, WTI (petroleum); West Texas Intermediate Price	0.9486	0.9436	0.9355	0.9233	0.9186	0.9100	0.9169	0.9184	0.9182	0.9259	0.0134	0.8430
7	New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	0.8680	0.8804	0.8836	0.9096	0.9034	0.9084	0.9158	0.9121	0.9122	0.8993	0.0173	0.8321
9	New York Harbor No. 2 Heating Oil Spot Price FOB	0.8519	0.8654	0.8682	0.8914	0.8886	0.8944	0.9058	0.9086	0.9070	0.8868	0.0205	0.8227
11	U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	0.9218	0.9241	0.9097	0.9249	0.9219	0.9235	0.9195	0.9215	0.9213	0.9209	0.0045	0.8488

Note: 1. SD, Standard Deviation; 2. Data range is from June 2006 to August 2020.

1. In price r_{xy} , the average and the R^2 of all selected commodities are higher than those of WTI Crude. The high averages and low SDs of selected commodities show a stable and reliable relationship with Brent Crude, both in the short-term and long-term.
2. Some selected commodities show big variations compared with training data in price change rate r_{xy} due to the soaring price in May 2020, even if their price r_{xy} is high still. The commodities, No. 2 Diesel Spot Price and No. 2 Heating Oil Spot Price FOB, show less capability to capture the price turning point in time.
3. In price change rate r_{xy} , the high average (0.9209) and low SDs (0.0045) of Kerosene-Type Jet Fuel Spot Price FOB demonstrate that it has short-term and long-term stability and reliability with Brent Crude. Moreover, its R^2 is better than the others, except for Dubai Crude.

In this analysis, the Kerosene-Type Jet Fuel Spot Price FOB shows good stability, reliability and turning point capturing ability compared with the others. It may be a good auxiliary tool for Brent Crude investments.

4.8. The Empirical Results of the Auxiliary Leading Index for Predicting the Rise and Fall of Brent Crude

The selected commodity price is not only contemporaneous to the price of Brent Crude. It assists in reducing the investment risks of Brent Crude if it can serve as a leading index for predicting the rise and fall of Brent Crude oil price. That is, the index is helpful for investors and analysts.

In the second phase, we quote the first weekly price data of each month of WTI Crude and three selected commodities and go on to further analyze the prediction. All of the price data are from EIA. There are 176 spreads (or 177 pts) in the data ranging from June 2006 to February 2021 (see Appendix G). The empirical results of this auxiliary leading index are shown in Table 11.

Table 11. Success Rate Comparison of WTI Crude and the Selected Commodities.

Selected Commodity	WTI Crude (For Reference)		Ultra-Low Sulfur No 2 Diesel Spot Price		No. 2 Heating Oil Spot Price FOB		Kerosene-Type Jet Fuel Spot Price FOB	
	pts	Success rate	pts	Success rate	pts	Success rate	pts	Success rate
Number of spreads in the same directional variation								
Path 1 & Brent	137	80.59%	128	75.29%	133	78.24%	131	77.06%
Path 2 & Brent	121	71.18%	132	77.65%	130	76.47%	123	72.35%
Path 1 & Path 2 & Brent	115	80.99%	116	81.69%	117	82.98%	112	80.00%
Path 1 & Path 2	142		142		141		140	

Note: Data range is from Jun 2006 to August 2020. (170 spreads).

Taking the example of Kerosene-Type Jet Fuel Spot Price FOB, there are 131 spreads with the same direction as the rise or fall of the monthly Brent Crude oil price, in Path 1. That is, the success rate is 77.06%. In Path 2, there are 123 spreads with the same direction as the monthly Brent Crude oil price. The success rate is 72.35%.

In Table 11, the success rate of WTI Crude is the highest (80.59%) in Path 1, followed by No. 2 Heating Oil Spot Price FOB and Kerosene-Type Jet Fuel Spot Price FOB (77.06%). The success rate of Ultra-Low Sulfur No 2 Diesel Spot Price is the highest (77.65%) in Path 2, followed by No. 2 Heating Oil Spot Price FOB (76.47%).

Moreover, there are 140 spreads following the same direction between Path 1 and Path 2. Among them, there are 112 spreads following the same directions with the monthly Brent Crude oil spreads. The success rate rises to 80.00%. That is, we have an 80.00% success rate for predicting the rise and fall of Brent Crude oil monthly price of that month three weeks in advance, when the spreads of both paths (Path 1 & Path 2) have the same direction.

Table 11 also shows the best commodity for predicting the rise and fall of Brent Crude oil price when combining both paths, which is No. 2 Heating Oil Spot Price FOB with an 82.98% success rate. The second ranked path is the No. 2 Diesel Spot Price with an 81.69% success rate.

Actually, all three selected commodities are at the same high level of success rates for predicting the rise and fall of Brent Crude oil monthly price three weeks in advance. Investors and analysts can evaluate whether precaution measurements need to be taken or plan necessary policies in advance to reduce investment risks or losses, particularly when the auxiliary index shows a different direction from the current investment on the first day after the first week in that month. Thus, this auxiliary index should be a useful tool for investors and analysts.

4.8.1. The Empirical Results of Consecutive Prediction Mistakes in the Auxiliary Leading Index

No price prediction index is always right. Investors or analysts could lose confidence in this predicting index if many cases of consecutive price prediction mistakes occur. The setting for maximum allowable number of consecutive prediction mistakes by us is three. Table 12 shows that the commodities with no three consecutive prediction mistakes are WTI Crude, Ultra-Low Sulfur No 2 Diesel Spot Price and No. 2 Heating Oil Spot Price FOB over a period of 170 months. Consecutive prediction mistakes were made for the Kerosene-Type Jet Fuel Spot Price FOB. Furthermore, using the same change direction in both paths as the discrimination criterion is an effective method to improve the prediction success rate obviously.

Table 12. Number of Consecutive Prediction Mistakes for WTI Crude and the Selected Commodities.

Item	Selected Commodity	WTI Crude (For Reference)	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
1	Number of two consecutive prediction mistakes	2	4	5	6
2	Max. months per consecutive prediction mistake	2	2	2	5
3	Number of three consecutive prediction mistakes	0	0	0	3
4	Number of three consecutive prediction mistakes in “Path 1 & Path 2 & Brent” only	3	9	7	15

Note: 1. Data range is from June 2006 to August 2020 (170 spreads); 2. The items for this empirical analysis are “Path 1 & Path 2” and “Path 1 & Path 2 & Brent”; 3. This row data is no count when the cell in the “Path 1 & Path 2” item is “-”, except for item 4.

4.8.2. The Empirical Verifying of the Auxiliary Leading Index during the COVID-19 Period

During the COVID-19 period, there are 13 spreads from February 2020 to February 2021. We extend data range from August 2020 to February 2021 for operating the detail empirical analysis. Table 13 shows the price directional variations between Brent Crude and three selected commodities. Three selected commodities were able to capture the trend of price slump and surge, except for the loss in May 2020 due to the rebound of crude oil price. They could be a nice rechecking tool for Brent Crude investment.

Table 13. The monthly price variation between Brent Crude and the Selected Commodities during the COVID-19 period.

Date	Brent Crude	Ultra-Low Sulfur No 2 Diesel Spot Price				New No. 2 Heating Oil Spot Price FOB				Kerosene-Type Jet Fuel Spot Price FOB			
		Path 1 & Brent	Path 2 & Brent	Path 1 & Path 2	Path 1 & 2 & Brent	Path 1 & Brent	Path 2 & Brent	Path 1 & Path 2	Path 1 & 2 & Brent	Path 1 & Brent	Path 2 & Brent	Path 1 & Path 2	Path 1 & 2 & Brent
Feb-20	55.000	+	+	+	+	+	+	+	+	+	+	+	+
Mar-20	32.980	+	+	+	+	+	+	+	+	+	+	+	+
Apr-20	23.340	+	+	+	+	+	+	+	+	+	+	+	+
May-20	31.020	-	-	+	-	-	-	+	-	-	-	+	-
Jun-20	39.930	+	+	+	+	+	+	+	+	+	+	+	+
Jul-20	42.810	+	+	+	+	+	+	+	+	+	+	+	+
Aug-20	44.260	+	+	+	+	+	+	+	+	+	+	+	+
Sep-20	41.090	+	+	+	+	+	+	+	+	+	+	+	+
Oct-20	40.470	-	+	-	nc	-	+	-	nc	+	+	+	+
Nov-20	43.230	-	+	-	nc	-	+	-	nc	+	+	+	+
Dec-20	49.870	+	+	+	+	+	+	+	+	+	+	+	+
Jan-21	54.550	+	+	+	+	+	+	+	+	+	+	+	+
Feb-21	61.960	+	+	+	+	+	+	+	+	+	+	+	+
Number of “+”		10	12	11	10	10	12	11	10	12	12	13	12
Success rate		76.92%	92.31%		90.91%	76.92%	92.31%		90.91%	92.31%	92.31%		92.31%

Note: 1. “+” means the item has the same directional variation; 2. Unit: US\$/bbl (Brent Crude oil price); 3. “nc” means not count because the cell in “Path 1 & Path 2” item is “-”; 4. Data range is from February 2020 to February 2021. (13 pts).

Among them, Kerosene-Type Jet Fuel Spot Price FOB shows a full match between Path 1 and Path 2 over a period of 13 months. Two other selected commodities show two mismatched points between Path 1 and Path 2, and thus do not count. All selected commodities show a prediction mistake in May 2020. It is also interesting that the prediction results of the “Path 2 and Brent” items are better than those of the “Path 1 & Brent” item, unlike in Table 11.

COVID-19 pandemic is an unusual and unexpected event, having a heavy impact on Brent Crude oil price. This index was able to capture the trend of the rise and fall of Brent Crude oil price during the COVID-19 period from February 2020 to February 2021. Thus, the findings surpass our expectations.

4.8.3. The Empirical Results of the Auxiliary Leading Index after Seasonal Adjustment

Considering seasonal effect on crude oil, we operate the seasonal adjustment of all selected commodities, even if seasonality tests (Wo, 2019) could “not identify seasonality”. The empirical results under STL decomposition concepts show that the success rates of mostly selected commodities are slightly better than that of no-seasonal adjusting. (See Tables 11, 14 and 15)

Table 14. The Seasonality Test’s Results.

Monthly Data	Crude Oil (For Reference)			Selected Commodity	
Commodity	Brent Crude	WTI Crude	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
<i>p</i> -value	0.8415456	0.8415199	0.8157994	0.9519999	0.9246915
Test Results	FALSE	FALSE	FALSE	FALSE	FALSE
First Weekly data					
<i>p</i> -value	0.5679684	0.5878043	0.9296676	0.9555963	0.7810095
Test Results	FALSE	FALSE	FALSE	FALSE	FALSE

Note: 1. Data range is from June 2006 to August 2020; 2. FALSE means “not identify seasonality”; 3. Using seasonality test in r.

Table 15. Success Rate Comparison after Seasonal Adjustment.

Selected Commodity	WTI Crude (For Reference)		Ultra-Low Sulfur No 2 Diesel Spot Price		No. 2 Heating Oil Spot Price FOB		Kerosene-Type Jet Fuel Spot Price FOB	
Number of spreads in the same directional variation	pts	Success rate	pts	Success rate	pts	Success rate	pts	Success rate
Path 1 & Brent	134	78.82%	128	75.29%	138	81.18%	132	77.65%
Path 2 & Brent	132	77.65%	130	76.47%	131	77.06%	128	75.29%
Path 1 & Path 2 & Brent	123	82.00%	115	80.99%	121	84.62%	117	81.25%
Path 1 & Path 2	150		142		143		144	

Note: Data range is from June 2006 to August 2020. (170 spreads).

This seasonal adjusted auxiliary index is a nice tool but a bit time consuming when investors want to predict with a nice success rate. The non-seasonally adjusted index is also a nice choice when investors want to get the response quickly, owing to its simplicity, ease of use, and its success rates is nice also.

5. Discussion and Conclusions

The main purpose of this paper is to serve as a practical investment reference for investors or analysts by offering a simple, less time-consuming (<3 min.), and high success rate index that requires less data. Investors could evaluate whether the precaution measurements need to be taken or plan necessary policies in advance to reduce investment risks or losses, particularly when the auxiliary index shows a different direction from the

Brent Crude current investment on the first day after the first week of that month. This index can serve as one rechecking tool during Brent Crude investing, rather than to replace other predicting price methods. Brent Crude investing risks can be further reduced when using this index in conjunction with other prediction methods. Making predictions based on commodities is the initial stage of our research. Many applied fields and associated causalities await further exploration. Nonetheless, the empirical results show the proposed auxiliary index works.

The concept, direction, basis and usage purpose of the proposed index vary from most of price prediction methods, such as LSTM, ARIMA. For example, this auxiliary index is based on empirical results and is designed specifically to predict the short-term rise and fall of Brent Crude oil price (about one month). The advantages of our method are that it is less time-consuming (<3 min.), requires less data for calculation (3 price pts), has a high success rate (>80%), requires no-preprocessing, and is simple and easy to calculate. The autoregressive methods such as ARIMA, which are the famous time-series predicting methods, operate under the assumption of a stationary state. These methods are not applied specifically for predicting the Brent Crude oil price. Using ARIMA is time-consuming (several days), requires preprocessing (seasonal adjustment, trend adjustment and unit root test, etc.), needs software and large amounts of data, and is complex. The same is true in AI methods such as LSTM. The concept, direction, basis and usage purpose of the proposed index vary from most previous price prediction methods, such as LSTM, ARIMA, etc. It is difficult to compare these methods with the proposed index.

After examining the price relationships of Brent Crude with 78 global commodities, some commodities show good price relationships with Brent Crude, all of which belong to the energy sector, but the degrees of closeness vary. The commodities can be ranked in the following order: crude oil, refined oil, natural gas, coal and carbon, etc. In the non-energy sector, the prices of metal commodities such as copper and iron ore are closely related to the Brent Crude oil price, but they cannot be used as auxiliary investing tools due to the low r_{xy} in price change rate. Moreover, we conclude that a weak or moderate price relationship exists between Brent Crude oil and agro-commodities, which supports the finding of several papers [28].

The price relationships between Brent Crude, Dubai Crude ($Y = 1.0361X - 0.3291$) and WTI Crude ($Y = 1.0892X - 1.2927$) are the closest, theoretically. However, the study results show that the price trends of some refined oil products outperform even WTI Crude, including No 2 Diesel Spot Price, Heating Oil Spot Price FOB and Kerosene-Type Jet Fuel Spot Price FOB. These commodities are the downstream co-products of crude oil and seem to be causally linked with Brent Crude.

All the selected commodities show close price relationships, reaching a high success rate level for forecasting the rise and fall of Brent Crude oil price. However, we recommend using the Heating Oil Spot Price FOB model to serve as an auxiliary investing index of Brent Crude, since it has a highest success rate (82.98%), which is better than that of WTI Crude and other selected commodities. Moreover, no three consecutive prediction mistakes occurred over a period of 170 months. The second index is Ultra-Low Sulfur No 2 Diesel Spot Price. The findings can be helpful for investors or analysts in the real world. In particular, investment risks can be reduced when measures are taken at least three weeks in advance.

Moreover, this seasonal adjusted auxiliary index is nice tool but time consuming a bit, when investors want to predict with a nice success rate. The no-seasonal adjusted index is also a nice choice when Investors want to get the response quickly, owing to its simple, and its success rates is at the high level also. Investors can adopt either case by case.

After all, the Brent Crude investment is huge. It is necessary to take all reasonable measures to minimize investment risks. This index likes a reference indicator of investors before taking precaution measurements or remedial measurements, simple but important.

This study shows that a close price relationship is one (but not the only) prerequisite for predicting the rise and fall of Brent Crude oil price based on a commodity's price, and

the combination of different commodities or methods could improve the success rate of prediction. Our next step will be to create a more accurate forecasting model for crude oil price using "+" or "-" signs, rather than just forecasting a wide price range, or to set the investment threshold using commodities to obtain good returns on investment.

The indexes of each selected commodity do not always achieve a close price relationship with Brent Crude. Therefore, the price relationships should be re-evaluated every one or two years, especially when major events occur, or the market's key factors change. AI technology is improving in the field of crude oil price prediction. Making predictions based on commodity prices, which has been ignored in the past, could be helpful in the AI field.

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Appendix A

Figure A1. Commodity Categories and Structures of the Datasets in this Study. Note: Commodity categories in dotted frames are applied in this study. Source: Mundi Website in Jan. 2020.

Appendix B

Table A1. Data Items and Sources of Commodities in This Study.

No.	Category	Commodity Item	Unit	Source	Initial Record	Training Data
1	Energy	Coal, Australian Thermal coal Price	\$/mt	WB	1988.12	371
2		Coal, South African export price	\$/mt	WB	1988.12	371
3		Crude oil, average (petroleum) Price	\$/bbl	WB	1988.12	371
4		Crude oil, Brent (petroleum); Dated Brent Price	\$/bbl	WB	1988.12	371
5		Crude oil, Dubai (petroleum); Dubai Fateh Price	\$/bbl	WB	1988.12	371
6		Crude oil, WTI (petroleum); West Texas Intermediate Price	\$/bbl	WB	1988.12	371
7		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	\$/gallon	EIA	2006.06	161
8		New York Harbor Conventional Gasoline Regular Spot Price FOB	\$/gallon	EIA	1988.12	371
9		New York Harbor No. 2 Heating Oil Spot Price FOB	\$/gallon	EIA	1988.12	371
10		Indonesian Liquefied Natural Gas Price, Japan	\$/mmbtu	WB	1988.12	371
11		U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	\$/gallon	EIA	1990.04	355
12		Natural gas Price, US	\$/mmbtu	WB	1988.12	371
13		Mont Belvieu, TX Propane Spot Price FOB	\$/gallon	EIA	1992.06	329
14		Los Angeles Reformulated RBOB Regular Gasoline Spot Price	\$/gallon	EIA	2003.06	197
15		Russian Natural Gas Price	\$/mmbtu	WB	1988.12	371
16		Natural Gas, Russian Natural Gas border price in Germany	\$/mmbtu	IMF	1988.12	371
17	Beverages	CoCea beans Price	\$/kg	WB	1988.12	371
18		Coffee, Other Mild Arabicas Price	\$/kg	WB	1988.12	371
19		Coffee, Robusta Price	\$/kg	WB	1988.12	371
20		Tea Price, Mombasa	\$/kg	WB	1988.12	371
21	Cereals	Barley Price	\$/mt	WB	1988.12	371
22		Maize (corn) Price	\$/mt	WB	1988.12	371
23		Rice Price	\$/mt	WB	1988.12	371
24		Soft Red Winter Wheat Price, US SRW	\$/mt	WB	1988.12	371
25		Sorghum Price	\$/mt	WB	1988.12	371
26		Wheat Price, US HRW	\$/mt	WB	1988.12	371
27	Fruits	Bananas, Central American and Ecuador, FOB U.S. Ports	\$/mt	IMF	1988.12	371
28		Oranges Price	\$/kg	WB	1988.12	371
29	Meat	Beef Price	\$/kg	WB	1988.12	371
30		Poultry (Chicken) Price	\$/kg	WB	1988.12	371
31		Lamb, frozen carcass Smithfield London	\$ cents/pd	IMF	1988.12	371
32		Swine (pork), 51-52% lean Hogs, U.S. price	\$ cents/pd	IMF	1988.12	371
33	Seafood	Fish (salmon), Farm Bred Norwegian	\$/kg	IMF	1988.12	371
34		Salmon, export price Shrimps Price, Mexican	\$/kg	WB	1988.12	371
35	Sugar	Sugar Price, world	\$/kg	WB	1988.12	371
36		Sugar, European import price	\$/kg	WB	1988.12	371
37		Sugar, U.S. import price	\$/kg	WB	1988.12	371

Table A1. Cont.

No.	Category	Commodity Item	Unit	Source	Initial Record	Training Data
38	Vegetable Oils and Protein Meal	Coconut oil	\$/mt	WB	1988.12	371
39		Fish meal Price	\$/mt	WB	1988.12	371
40		Olive Oil, extra virgin less than 1% free fatty acid, ex-tanker price U.K.	\$/mt	IMF	1988.12	371
41		Palm Kernel Oil Price	\$/mt	WB	1996.01	286
42		Palm oil Price	\$/mt	WB	1988.12	371
43		Groundnut oil Price	\$/mt	WB	1988.12	371
44		Groundnuts (peanuts) Price	\$/mt	WB	1988.12	371
45		Rapeseed oil	\$/mt	WB	2002.02	213
46		Soybean meal	\$/mt	WB	1988.12	371
47		Soybean Oil Price	\$/mt	WB	1988.12	371
48		Soybeans Price	\$/mt	WB	1988.12	371
49		Sunflower oil	\$/mt	WB	2003.01	202
50	Agricultural Raw Materials	Wool, coarse, 23 micron, Australian Wool Exchange spot quote	\$ cents/kg	IMF	1988.12	371
51		Cotton, A Index Cotton Price	\$/kg	WB	1988.12	371
52		Wool, fine, 19 micron, Australian Wool Exchange spot quote	\$ cents/kg	IMF	1988.12	371
53		Logs, Malaysian Hard Logs Price	\$/cm	WB	1988.12	371
54		Sawnwood, Malaysian Hard Sawnwood Price	\$/cm	WB	1988.12	371
55		Hides, Heavy native steers, over 53 pounds, wholesale dealer's price, US, Chicago, fob Shipping Point	\$ cents/Pd.	IMF	1988.12	371
56		Plywood Price	\$ cents/sheet	WB	1988.12	371
57		Rubber Price, SGP/MYS	\$/kg	WB	1988.12	371
58		Soft Logs, Average Export price from the U.S. for Douglas Fir	\$/cm	IMF	1988.12	371
59		Soft Sawnwood, average export price of Douglas Fir, U.S. Price	\$/cm	IMF	1988.12	371
60		Wood Pulp Price	\$/mt	WB	1988.12	371
61	Metals	Aluminum Price	\$/mt	WB	1988.12	371
62		Copper, grade A cathode Price	\$/mt	WB	1988.12	371
63		Gold Price	\$/troy oz	WB	1988.12	371
64		Iron Ore Price, cfr spot price	\$/dmt	WB	1988.12	371
65		Lead Price	\$/mt	WB	1988.12	371
66		Nickel Price	\$/mt	WB	1988.12	371
67		Palladium, LME spot price	\$/troy oz	IMF	1988.12	371
68		Platinum	\$/troy oz	WB	1988.12	371
69		Silver Price	\$/troy oz	IMF	1988.12	371
70		Silver	\$/troy oz	WB	1988.12	371
71		Tin Price	\$/mt	WB	1988.12	371
72		Uranium, NUEXCO, Restricted Price, Nuexco exchange spot	\$/pd	IMF	1988.12	371
73		Zinc Price	\$/mt	IMF	1988.12	371
74		Zinc	\$/mt	WB	1988.12	371
75	Fertilizers	DAP fertilizer Price	\$/mt	WB	1988.12	371
76		Potassium chloride (Muriate of Potash) Standard Grade: FOB Vancouver Spot Price,	\$/mt	WB	1988.12	371
77		Phosphate Rock Price	\$/mt	WB	1988.12	371
78		Triple Superphosphate Price	\$/mt	WB	1988.12	371
79		Urea Price	\$/mt	WB	1988.12	371

Note: 1. WB: World Bank; 2. IMF: International Monetary Fund; 3. EIA: U.S. Energy Information Administration; 4. MT: Metric Ton; 5. MMBTU: Million Metric British Thermal Unit; 6. CM: Cubic Meter; 7. OZ: Ounce; 8. PD: Pound; 9. \$/mt: US Dollars per Metric Ton; 10. \$/bbl: US Dollars per Barrel; 11. \$/gallon: US Dollars per Gallon; 12. \$/mbtu: US Dollars per Million Metric British Thermal Unit; 13. \$/dmt: US Dollars per Dry Metric Ton; 14. \$ cents/sheet: US Cents per Sheet; 15. \$/pd: US Dollars per Pound; 16. \$/troy oz: US Dollars per Troy Ounce; 17. \$/cm: US\$ per Cubic Meter. Source: Mundi, World Bank, IMF, EIA. Data type: Monthly Average Price.

Appendix C

Table A2. Price r_{xy} Comparisons of Commodities and Brent Crude in Different time Ranges.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	2001.11	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
1	371	0.6581	0.6009	0.7606	0.2289	0.3957	0.4980	0.5765	0.5920	0.6601	0.7568	0.7979	0.8264	0.8363	0.8448	0.8515	0.8544	0.8558	0.6820	0.1825	
2	371	0.6058	0.5447	0.7223	0.3397	0.4859	0.5719	0.6215	0.6301	0.6846	0.7737	0.8119	0.8423	0.8552	0.8655	0.8748	0.8797	0.8818	0.7054	0.1624	
3	371	0.9968	0.9965	0.9987	0.9992	0.9986	0.9974	0.9968	0.9968	0.9974	0.9981	0.9984	0.9987	0.9989	0.9990	0.9990	0.9991	0.9991	0.9981	0.0010	
4	371																	1.0000	1.0000	0.0000	
5	371	0.9945	0.9861	0.9964	0.9984	0.9989	0.9984	0.9982	0.9976	0.9969	0.9977	0.9980	0.9983	0.9985	0.9987	0.9987	0.9987	0.9987	0.9972	0.0031	
6	371	0.9696	0.9562	0.9825	0.9917	0.9863	0.9773	0.9701	0.9683	0.9730	0.9803	0.9836	0.9868	0.9883	0.9895	0.9903	0.9907	0.9910	0.9809	0.0103	
7	161	0.9626	0.9579	0.9870	0.9889	0.9937	0.9919	0.9857	0.9857										0.9817	0.0136	
8	371	0.8861	0.8776	0.9343	0.9769	0.9850	0.9848	0.9833	0.9791	0.9813	0.9852	0.9872	0.9894	0.9904	0.9913	0.9918	0.9920	0.9921	0.9710	0.0362	
9	371	0.9551	0.9364	0.9858	0.9898	0.9941	0.9929	0.9885	0.9886	0.9900	0.9919	0.9928	0.9941	0.9947	0.9952	0.9955	0.9956	0.9956	0.9869	0.0161	
10	371	(0.1367)	0.0265	0.6789	0.8108	0.8866	0.8506	0.7411	0.7280	0.7831	0.8349	0.8599	0.8852	0.8970	0.9069	0.9132	0.9164	0.9184	0.7353	0.3074	
11	355	0.9722	0.9539	0.9830	0.9917	0.9951	0.9950	0.9863	0.9856	0.9844	0.9881	0.9899	0.9919	0.9928	0.9935	0.9939	0.9941	0.9941	0.9874	0.0104	
12	371	0.0590	0.0708	0.4259	0.7994	0.5215	0.4838	0.4288	0.2128	0.0461	0.0819	0.1016	0.2250	0.2821	0.3491	0.3925	0.4267	0.4404	0.3145	0.2067	
13	329	0.8091	0.6482	0.7310	0.8506	0.8279	0.7919	0.8008	0.7528	0.7438	0.7946	0.8117	0.8482	0.8623	0.8768	0.8835			0.8022	0.0632	
14	197	0.7127	0.7108	0.8698	0.8984	0.9435	0.9457	0.9521	0.9396	0.9478	0.9511								0.8872	0.0962	
15	371	0.5264	0.4934	0.6479	0.8166	0.9053	0.8998	0.7129	0.7033	0.7616	0.8210	0.8456	0.8748	0.8888	0.8997	0.9069	0.9102	0.9124	0.7957	0.1355	
16	371	0.5706	0.4950	0.5711	0.7686	0.8810	0.8728	0.6525	0.6498	0.7220	0.7918	0.8205	0.8530	0.8681	0.8801	0.8884	0.8922	0.8947	0.7689	0.1334	
17	371	0.0811	0.1609	(0.5265)	0.1428	(0.0989)	0.0343	0.0571	0.1750	0.3659	0.4753	0.5922	0.6318	0.6583	0.6850	0.7109	0.7262	0.7317	0.3296	0.3630	
18	371	0.1207	0.0124	(0.6547)	0.2885	0.3497	0.4794	0.4266	0.4596	0.5738	0.6929	0.7313	0.6738	0.5819	0.5155	0.5583	0.5790	0.5752	0.4096	0.3354	
19	371	0.6030	0.3538	(0.1984)	0.2070	0.4124	0.4630	0.5404	0.5464	0.6506	0.7584	0.7951	0.6932	0.6133	0.3602	0.4069	0.4347	0.4226	0.4743	0.2335	
20	371	0.6935	0.1986	(0.1730)	(0.5034)	(0.1821)	(0.1193)	(0.0792)	0.0599	0.2895	0.4620	0.5025	0.5067	0.5440	0.5897	0.6105	0.6314	0.6388	0.2747	0.3714	
21	371	0.0876	0.1488	0.5371	0.7329	0.8317	0.8490	0.8625	0.8449	0.8566	0.8407	0.8608	0.8702	0.8494	0.8570	0.8673	0.8733	0.8761	0.7439	0.2495	
22	371	(0.2609)	(0.1084)	0.2180	0.7005	0.7918	0.8086	0.8058	0.7846	0.8115	0.8448	0.8686	0.8727	0.8323	0.8336	0.8420	0.8454	0.8463	0.6669	0.3552	
23	371	(0.0135)	0.0958	0.4276	0.4676	0.7480	0.7209	0.5490	0.5583	0.6589	0.7545	0.7972	0.7878	0.7676	0.7639	0.7762	0.7791	0.7769	0.6127	0.2458	
24	371	(0.2729)	(0.0180)	0.4738	0.7362	0.8213	0.8041	0.7406	0.7210	0.7604	0.8080	0.8428	0.8633	0.8260	0.8200	0.8220	0.8244	0.8202	0.6702	0.3225	
25	371	0.2130	0.1023	0.2717	0.5262	0.7798	0.7748	0.7762	0.7477	0.7822	0.8245	0.8527	0.8655	0.8375	0.8412	0.8497	0.8541	0.8557	0.6915	0.2511	
26	371	0.5006	0.5043	0.5881	0.8460	0.8902	0.8789	0.8244	0.8195	0.8407	0.8631	0.8859	0.8963	0.8544	0.8487	0.8532	0.8559	0.8523	0.8001	0.1313	
27	371	(0.0355)	0.0156	0.7213	(0.2095)	(0.3798)	(0.3252)	(0.2595)	(0.0311)	0.2613	0.4741	0.5586	0.6290	0.6590	0.6889	0.7063	0.7066	0.7077	0.2875	0.4248	
28	371	0.6604	0.5341	(0.1919)	0.0045	0.3059	0.2876	0.3941	0.3716	0.3418	0.4832	0.5689	0.6526	0.6899	0.7157	0.7330	0.7431	0.7494	0.4732	0.2694	
29	371	(0.4891)	(0.4626)	0.3174	0.3038	(0.0010)	(0.0490)	0.0448	0.1504	0.3162	0.5045	0.5960	0.6740	0.7128	0.7284	0.7276	0.7254	0.7261	0.3250	0.4079	
30	371	0.5357	0.5600	0.5185	0.5371	0.3746	0.2390	0.1757	0.2593	0.3918	0.5655	0.6505	0.7077	0.7401	0.7701	0.7854	0.7919	0.7971	0.5529	0.2051	
31	371	0.0055	(0.0011)	0.7529	0.8351	0.3607	0.4187	0.4060	0.2821	0.0402	(0.0656)	0.0113	0.0955	0.0408	0.0838	0.1221	0.1604	0.1824	0.2194	0.2614	
32	371	0.1758	0.1455	0.0411	0.6726	0.6535	0.6644	0.5940	0.6056	0.5731	0.6587	0.6528	0.7032	0.5145	0.5210	0.5254	0.3551	0.3622	0.4952	0.2058	

Table A2. Cont.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	2001.11	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
33	371	0.5231	0.5010	0.2667	0.1015	(0.3094)	(0.2738)	(0.1747)	(0.0655)	0.1984	0.4246	0.5102	0.5593	0.5803	0.5492	0.4910	0.4534	0.4386	0.2808	0.3103	
34	371	(0.3094)	(0.2682)	0.2698	0.6942	0.2878	0.2088	0.1944	0.2528	0.3253	0.3336	0.1636	0.0520	(0.0221)	(0.0621)	(0.0239)	0.0006	0.0073	0.1238	0.2426	
35	371	(0.2971)	(0.3138)	(0.5412)	0.1037	0.5282	0.5421	0.4126	0.4307	0.5644	0.6679	0.7085	0.7326	0.7156	0.6869	0.6993	0.6906	0.6832	0.4126	0.4142	
36	371	0.6952	0.3097	0.5342	0.8880	0.9105	0.8014	0.4470	0.2256	(0.0352)	(0.1152)	(0.1488)	(0.2256)	(0.3125)	(0.3682)	(0.3993)	(0.3996)	(0.3846)	0.1425	0.4885	
37	371	(0.2118)	(0.2598)	(0.3999)	(0.5982)	(0.1502)	0.1459	0.1162	0.1741	0.2884	0.3698	0.4389	0.4527	0.4632	0.4667	0.4835	0.4859	0.4854	0.1618	0.3560	
38	371	0.6134	0.0983	(0.5422)	(0.1428)	(0.2470)	0.0071	0.1465	0.2201	0.3518	0.5118	0.5886	0.5913	0.5953	0.6121	0.6343	0.6544	0.6600	0.3149	0.3727	
39	371	0.4653	0.3300	0.2032	0.5198	0.5135	0.2638	0.1942	0.2771	0.4688	0.6142	0.6929	0.7356	0.7613	0.7881	0.8047	0.8145	0.8192	0.5451	0.2284	
40	371	0.4445	0.0594	(0.3124)	(0.3745)	(0.5218)	(0.5502)	(0.2981)	(0.3420)	(0.4287)	(0.1682)	0.0203	0.0824	(0.0725)	(0.0352)	0.0386	0.0812	0.1159	−0.1330	0.2700	
41	286	0.6379	0.1508	(0.3758)	0.0100	(0.0007)	0.2122	0.3220	0.3708	0.4621	0.5927	0.6541	0.6395	0.6359					0.3316	0.3206	
42	371	0.5968	0.2398	(0.3971)	0.5004	0.7341	0.7300	0.7496	0.7397	0.7747	0.8231	0.8540	0.8046	0.7989	0.7831	0.7996	0.8080	0.8125	0.6560	0.3108	
43	371	0.2058	0.0661	0.3641	0.2849	0.6878	0.6651	0.6677	0.6665	0.6746	0.7434	0.7910	0.8120	0.8224	0.8256	0.8371	0.8413	0.8450	0.6353	0.2472	
44	371	0.5797	0.5284	(0.1533)	(0.3467)	0.5100	0.5062	0.5380	0.5568	0.6184	0.6873	0.7205	0.7240	0.7323	0.7331	0.7372	0.6779	0.6853	0.5315	0.3079	
45	213	(0.4283)	(0.4108)	0.1460	0.6255	0.8089	0.7904	0.7973	0.8027	0.8319	0.8697								0.4833	0.5203	
46	371	0.6250	0.4984	0.3524	0.8186	0.7826	0.6929	0.6969	0.6479	0.6985	0.7754	0.8134	0.8440	0.8353	0.8504	0.8595	0.8657	0.8636	0.7365	0.1432	
47	371	0.1837	(0.0362)	(0.1180)	0.6293	0.8162	0.8054	0.8187	0.7966	0.8189	0.8610	0.8833	0.8692	0.8705	0.8587	0.8688	0.8750	0.8780	0.6870	0.3331	
48	371	0.1236	0.0798	(0.1490)	0.7237	0.8289	0.8365	0.8568	0.7903	0.8029	0.8560	0.8816	0.8912	0.8789	0.8836	0.8898	0.8940	0.8920	0.7036	0.3342	
49	202	0.2380	0.1041	(0.7170)	0.2789	0.7177	0.7372	0.7511	0.7530	0.7774	0.7934								0.4434	0.4850	
50	371	0.5827	0.5303	0.8639	0.1765	0.1352	0.1452	0.1628	0.2397	0.3842	0.4863	0.5917	0.6639	0.6921	0.7125	0.7348	0.7452	0.7452	0.5054	0.2489	
51	371	0.7157	0.6826	0.7290	0.5388	0.6551	0.4837	0.4514	0.4834	0.5206	0.5912	0.6264	0.6159	0.5498	0.4707	0.4941	0.4634	0.4565	0.5605	0.0947	
52	371	0.6157	0.5903	0.8658	0.0488	0.0571	0.2098	0.2434	0.3066	0.4386	0.5554	0.6098	0.6669	0.6860	0.6899	0.7132	0.6970	0.6679	0.5096	0.2463	
53	371	(0.5076)	(0.3147)	0.1283	0.5712	0.7537	0.7232	0.6007	0.6144	0.6980	0.7845	0.8198	0.8429	0.7945	0.6722	0.5258	0.5520	0.5614	0.5188	0.3887	
54	371	0.6920	0.3748	(0.3250)	0.5752	0.7039	0.7365	0.7115	0.7152	0.7724	0.8386	0.8569	0.8774	0.8191	0.7171	0.7036	0.7181	0.7221	0.6594	0.2785	
55	371	0.7784	0.4717	(0.5621)	0.4737	0.4923	0.4802	0.5207	0.4482	0.3596	0.2956	0.2456	0.2639	0.2035	0.1491	0.1616	0.1282	0.1117	0.2954	0.2845	
56	371	(0.5076)	(0.3147)	0.1283	0.5712	0.8055	0.8013	0.6738	0.5289	0.6046	0.7196	0.7605	0.7995	0.7417	0.5665	0.4864	0.5228	0.5385	0.4957	0.3807	
57	371	(0.0444)	(0.0315)	0.0399	0.4822	0.7786	0.6740	0.6847	0.6896	0.7309	0.7864	0.8223	0.8423	0.8328	0.8221	0.8324	0.8395	0.8409	0.6249	0.3179	
58	371	0.4648	0.2150	0.6291	0.0652	(0.4420)	(0.3753)	(0.2983)	(0.3237)	(0.2849)	0.0193	0.0159	0.0775	(0.1096)	(0.2625)	(0.2627)	(0.1284)	(0.0694)	−0.0629	0.2941	
59	371	0.3608	0.2964	0.7064	0.4091	0.0210	(0.1696)	(0.1952)	(0.2230)	(0.1383)	0.2054	0.3155	0.4001	0.4018	0.4201	0.4708	0.4873	0.4978	0.2510	0.2847	
60	371	—	—	—	0.0265	(0.6605)	(0.3168)	0.0799	0.1712	0.3714	0.5895	0.6341	0.7040	0.7033	0.6611	0.6859	0.6401	0.6097	0.3500	0.4319	
61	371	0.7246	0.5561	0.7647	0.3366	0.4338	0.5154	0.6118	0.3889	0.4588	0.5969	0.6460	0.7008	0.7115	0.7040	0.7290	0.7326	0.7036	0.6068	0.1353	
62	371	0.5799	0.3211	0.7886	0.7969	0.8953	0.8371	0.8351	0.7890	0.8153	0.8656	0.8892	0.9092	0.9156	0.9177	0.9231	0.9253	0.9244	0.8193	0.1544	
63	371	(0.6041)	(0.5486)	0.3152	0.2218	0.6637	0.6613	0.4009	0.4165	0.5624	0.6863	0.7452	0.7862	0.8057	0.8187	0.8299	0.8368	0.8396	0.4963	0.4481	
64	371	(0.4425)	(0.3653)	0.5146	0.7578	0.8765	0.7945	0.7975	0.7939	0.8305	0.8669	0.8868	0.8948	0.9030	0.9094	0.9132	0.9154	0.9170	0.6920	0.4240	
65	371	0.3006	0.0855	0.4798	0.3328	0.2853	0.3929	0.4269	0.3910	0.5497	0.6927	0.7527	0.7944	0.8119	0.8293	0.8422	0.8471	0.8496	0.5685	0.2488	

Table A2. Cont.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	2001.11	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
66	371	(0.0817)	(0.0209)	0.7407	0.7807	0.8326	0.6914	0.6812	0.2969	0.3299	0.4648	0.5362	0.6095	0.6385	0.6623	0.6826	0.6891	0.6744	0.5417	0.2654	
67	371	(0.7284)	(0.5790)	0.6358	0.1497	(0.1629)	(0.1567)	(0.0723)	0.0234	0.2096	0.3275	0.2628	0.3391	0.4170	0.4750	0.5178	0.5452	0.5553	0.1623	0.3961	
68	371	0.0184	(0.0427)	(0.4549)	0.6923	0.8619	0.8024	0.8096	0.8093	0.8377	0.8820	0.9025	0.9221	0.9307	0.9380	0.9421	0.9435	0.9441	0.6905	0.4212	
69	371	(0.3416)	(0.3262)	(0.1991)	0.5833	0.7700	0.7744	0.6827	0.6886	0.7501	0.8115	0.8416	0.8550	0.8677	0.8759	0.8843	0.8888	0.8902	0.6057	0.4362	
70	371	(0.3518)	(0.3346)	(0.2107)	0.5777	0.7691	0.7727	0.6809	0.6870	0.7488	0.8104	0.8407	0.8542	0.8669	0.8751	0.8836	0.8882	0.8895	0.6028	0.4397	
71	371	0.3753	0.3234	0.5534	0.7052	0.7129	0.6647	0.6527	0.6028	0.6858	0.7797	0.8194	0.8445	0.8590	0.8699	0.8781	0.8827	0.8802	0.7112	0.1720	
72	371	(0.3521)	(0.2234)	(0.3381)	0.1874	0.6365	0.6428	0.5176	0.2655	0.3867	0.5258	0.6012	0.6491	0.6727	0.6969	0.7157	0.7285	0.7333	0.4145	0.3781	
73	371	0.4711	0.1816	0.6721	0.0382	(0.2961)	(0.2717)	(0.0858)	(0.1625)	0.1282	0.3785	0.4723	0.5411	0.5731	0.6079	0.6271	0.6297	0.6209	0.3015	0.3452	
74	371	0.4711	0.1816	0.6721	0.0382	(0.2961)	(0.2720)	(0.0861)	(0.1628)	0.1272	0.3775	0.4713	0.5400	0.5720	0.6069	0.6262	0.6290	0.6202	0.3010	0.3450	
75	371	0.5766	0.5692	0.5528	0.5005	0.6907	0.6540	0.5433	0.5650	0.6407	0.7165	0.7597	0.7738	0.7834	0.7955	0.8103	0.8183	0.8213	0.6807	0.1124	
76	371	(0.4059)	(0.3265)	(0.4405)	0.3293	0.7267	0.7148	0.3434	0.3876	0.5110	0.6155	0.6706	0.7076	0.7322	0.7502	0.7638	0.7743	0.7789	0.4490	0.4281	
77	371	(0.0329)	0.0370	(0.7077)	(0.1545)	0.5451	0.5829	0.0475	0.1224	0.2459	0.3480	0.4084	0.4628	0.5016	0.5318	0.5545	0.5708	0.5773	0.2730	0.3516	
78	371	0.4377	0.4555	0.4025	0.4564	0.6935	0.7043	0.5511	0.5759	0.6381	0.7039	0.7424	0.7518	0.7619	0.7773	0.7917	0.8002	0.8026	0.6498	0.1411	
79	371	(0.1785)	(0.1576)	0.5133	0.6344	0.7828	0.7370	0.6763	0.6825	0.7273	0.7939	0.8297	0.8579	0.8599	0.8665	0.8740	0.8768	0.8789	0.6621	0.3287	

Note: SD, Standard Deviation.

Appendix D

Table A3. Price Change Rate r_{xy} between Commodities and Brent Crude in Different Time Ranges.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	200.111	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
1	370	0.2820	0.2767	0.1146	0.1785	0.2021	0.2206	0.3546	0.3741	0.3583	0.3418	0.2996	0.2862	0.2820	0.2698	0.2640	0.2386	0.2364	0.2694	0.0681	
2	370	0.1755	0.2527	0.1894	0.3066	0.3090	0.3047	0.4352	0.4330	0.3745	0.3440	0.2518	0.2192	0.2163	0.2122	0.2096	0.1872	0.1854	0.2710	0.0854	
3	370	0.9918	0.9922	0.9917	0.9915	0.9885	0.9881	0.9910	0.9902	0.9887	0.9882	0.9870	0.9863	0.9864	0.9859	0.9857	0.9874	0.9873	0.9887	0.0023	
4	370																	1.0000	1.0000	0.0000	
5	370	0.9905	0.9911	0.9895	0.9909	0.9872	0.9860	0.9884	0.9838	0.9690	0.9679	0.9647	0.9626	0.9605	0.9587	0.9575	0.9602	0.9600	0.9746	0.0138	
6	370	0.9417	0.9446	0.9393	0.9270	0.9092	0.9024	0.9228	0.9252	0.9279	0.9302	0.9265	0.9269	0.9285	0.9272	0.9282	0.9370	0.9366	0.9283	0.0106	
7	160	0.9478	0.9498	0.9494	0.9416	0.9415	0.9421	0.9388	0.9345										0.9432	0.0054	
8	370	0.8239	0.8320	0.7837	0.8359	0.8302	0.8363	0.8720	0.8561	0.8458	0.8362	0.8306	0.8399	0.8237	0.8224	0.8200	0.8032	0.7948	0.8286	0.0212	
9	370	0.9444	0.9422	0.9316	0.9280	0.9287	0.9317	0.9359	0.9285	0.9143	0.9001	0.8202	0.8303	0.8245	0.8117	0.8137	0.8012	0.7998	0.8816	0.0591	
10	370	(0.3328)	(0.1892)	(0.1190)	(0.2662)	(0.2360)	(0.1758)	(0.1630)	(0.1532)	(0.1382)	(0.1236)	(0.1200)	(0.0972)	(0.0994)	(0.0971)	(0.0947)	(0.0930)	(0.0944)	−0.1525	0.0693	
11	354	0.9447	0.9504	0.8989	0.9240	0.9220	0.9256	0.9212	0.8968	0.8718	0.8739	0.8586	0.8651	0.8642	0.8558	0.8541	0.8677		0.8934	0.0335	
12	370	(0.2751)	0.0414	0.1000	0.1569	0.0883	0.0407	0.1450	0.1288	0.1585	0.2082	0.1325	0.1521	0.1414	0.1348	0.1327	0.1401	0.1328	0.1035	0.1059	
13	328	0.7206	0.7012	0.7553	0.6693	0.6518	0.6196	0.7001	0.6985	0.6739	0.6829	0.5920	0.6049	0.5943	0.5899	0.5869			0.6561	0.0548	
14	196	0.8112	0.7638	0.5865	0.6481	0.6574	0.6611	0.6987	0.7064	0.6957	0.6866								0.6916	0.0622	
15	370	0.1339	0.1624	0.2904	0.1423	0.1186	0.1252	0.0729	0.0832	0.0948	0.0679	0.0632	0.0567	0.0627	0.0633	0.0603	0.0358	0.0284	0.0978	0.0627	
16	370	0.1600	0.0752	0.1212	0.0858	0.0635	0.1066	0.0222	0.0447	0.0700	0.0431	0.0445	0.0330	0.0353	0.0350	0.0315	(0.0084)	(0.0133)	0.0559	0.0445	
17	370	0.0868	0.0670	0.1232	0.1656	0.1778	0.1763	0.2498	0.2889	0.2567	0.2630	0.1752	0.1405	0.1177	0.1202	0.0953	0.0781	0.0663	0.1558	0.0724	
18	370	0.1399	0.1415	0.0817	0.0502	0.0626	0.0999	0.1881	0.2031	0.1461	0.1175	0.1499	0.0930	0.0508	0.0658	0.0518	0.0574	0.0590	0.1034	0.0501	
19	370	0.1696	0.0847	0.1986	0.1607	0.1652	0.1901	0.2436	0.2343	0.1648	0.1612	0.1799	0.1303	0.0971	0.1087	0.0889	0.1044	0.1005	0.1519	0.0495	
20	370	0.2907	0.1843	0.0479	(0.0637)	(0.0302)	(0.0015)	0.1209	0.1104	0.0990	0.0933	0.0841	0.0910	0.0833	0.0858	0.0842	0.1154	0.1267	0.0895	0.0794	
21	370	0.0407	0.1364	0.0495	0.0006	0.0123	0.1032	0.3396	0.3244	0.2778	0.2399	0.1952	0.1805	0.1720	0.1455	0.1505	0.1638	0.1730	0.1591	0.0998	
22	370	(0.2223)	(0.1596)	0.0338	0.0384	0.0829	0.1369	0.3139	0.2422	0.2024	0.1796	0.1377	0.1133	0.0946	0.0652	0.0648	0.0206	0.0244	0.0805	0.1308	
23	370	(0.0000)	0.2727	0.0851	0.0598	(0.0057)	(0.0130)	0.1441	0.1439	0.1355	0.1226	0.1102	0.0786	0.0718	0.0358	0.0349	0.0223	0.0213	0.0776	0.0728	
24	370	(0.3409)	(0.3239)	(0.0937)	(0.0777)	0.0092	0.0336	0.0854	0.0804	0.0735	0.0754	0.0650	0.0740	0.0882	0.0627	0.0572	0.0354	0.0391	−0.0034	0.1345	
25	370	0.0406	0.0018	0.1512	0.1129	0.1120	0.1657	0.3056	0.2363	0.1983	0.1767	0.1536	0.1223	0.1073	0.0821	0.0742	0.0364	0.0378	0.1244	0.0787	
26	370	(0.0882)	0.1013	(0.1244)	(0.0190)	0.0289	0.0623	0.1565	0.1636	0.1353	0.1318	0.1363	0.1217	0.1120	0.0794	0.0691	0.0466	0.0512	0.0685	0.0820	
27	370	0.1497	(0.0700)	(0.2491)	(0.1512)	(0.0274)	(0.0154)	(0.0642)	(0.0731)	(0.0716)	0.0037	0.0196	0.0214	(0.0041)	(0.0309)	(0.0356)	(0.0592)	(0.0502)	−0.0416	0.0820	
28	370	(0.0201)	0.0199	0.0462	(0.0355)	(0.0745)	(0.0475)	0.1066	0.0730	0.1182	0.1321	0.1188	0.1287	0.1202	0.1303	0.1189	0.1198	0.1193	0.0691	0.0724	

Table A3. Cont.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	200.111	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
29	370	(0.2365)	(0.2352)	(0.0081)	(0.0657)	(0.0360)	0.0604	0.2746	0.2418	0.2204	0.2243	0.2139	0.1973	0.1787	0.1626	0.1610	0.1543	0.1471	0.0973	0.1594	
30	370	0.0067	0.1592	(0.0612)	0.0428	0.0191	0.0154	0.0312	0.0308	0.0306	0.0356	0.0436	0.0287	0.0131	0.0134	0.0087	0.0014	0.0009	0.0247	0.0422	
31	370	(0.1421)	(0.0427)	0.1099	0.2209	0.2185	0.2306	0.3640	0.3668	0.3107	0.2968	0.2421	0.2701	0.2615	0.2542	0.2292	0.2009	0.1994	0.2112	0.1309	
32	370	0.3120	0.2583	0.1844	0.1830	0.1391	0.1385	0.1599	0.1300	0.1260	0.1118	0.0831	0.1383	0.1246	0.1123	0.1128	0.0822	0.0792	0.1456	0.0614	
33	370	0.2726	0.2650	0.0786	0.0029	0.0451	0.0970	0.1834	0.2054	0.1947	0.1800	0.1514	0.1450	0.1448	0.1449	0.1328	0.1383	0.1387	0.1483	0.0691	
34	370	(0.2241)	(0.1841)	(0.2831)	0.0010	0.0118	(0.0229)	0.0190	0.0473	0.0252	(0.0028)	0.0119	0.0205	0.0026	0.0123	0.0198	(0.0421)	(0.0353)	−0.0367	0.0967	
35	370	0.1868	0.1393	0.2759	0.2479	0.2451	0.1707	0.2117	0.2260	0.2253	0.2220	0.2035	0.1582	0.1468	0.1317	0.1408	0.1054	0.1012	0.1846	0.0526	
36	370	(0.1929)	(0.0239)	0.0249	0.1985	0.2237	0.2599	0.3744	0.3713	0.3173	0.3127	0.2267	0.2030	0.1921	0.1919	0.1701	0.1967	0.1968	0.1908	0.1427	
37	370	0.0046	0.0197	0.1243	0.0486	0.0895	0.0469	0.1764	0.1615	0.1498	0.1270	0.0824	0.0684	0.0681	0.0630	0.0651	0.0651	0.0616	0.0836	0.0485	
38	370	(0.2011)	(0.0388)	0.0772	0.1193	0.1428	0.1848	0.3129	0.3087	0.2721	0.2606	0.2078	0.1890	0.1721	0.1331	0.1247	0.1126	0.1090	0.1463	0.1261	
39	370	(0.0001)	0.1637	0.0727	0.1028	0.1040	0.1183	0.0721	0.0463	0.0277	0.0408	0.0084	0.0045	0.0151	(0.0007)	0.0038	0.0372	0.0304	0.0498	0.0485	
40	370	(0.5112)	(0.3454)	(0.1644)	(0.1973)	(0.0670)	(0.0291)	0.0996	0.1218	0.1309	0.1372	0.0765	0.1250	0.1546	0.1541	0.1480	0.1580	0.1578	0.0088	0.2019	
41	285	(0.1025)	(0.0217)	0.2152	0.1959	0.2088	0.1998	0.3410	0.3383	0.3113	0.3000	0.2531	0.2407	0.2302					0.2085	0.1313	
42	370	0.2852	0.2336	0.3038	0.3345	0.3469	0.3281	0.4392	0.4337	0.3850	0.3305	0.2074	0.1583	0.1577	0.1259	0.1219	0.1179	0.1123	0.2601	0.1136	
43	370	(0.2821)	(0.2611)	(0.1049)	(0.0410)	(0.0786)	(0.0201)	0.1476	0.1553	0.1322	0.1137	0.0692	0.0308	0.0189	0.0059	0.0200	0.0346	0.0471	−0.0007	0.1258	
44	370	0.2741	0.2296	(0.0005)	(0.0796)	(0.0670)	(0.0405)	0.1053	0.0730	0.0338	0.0168	0.0603	0.0521	0.0556	0.0406	0.0313	0.2453	0.2439	0.0750	0.1102	
45	212	(0.0862)	(0.0642)	0.1153	0.2205	0.2371	0.2854	0.4766	0.4793	0.4202	0.3657								0.2450	0.2050	
46	370	(0.0874)	(0.0081)	0.2358	0.1737	0.1929	0.1758	0.3278	0.2856	0.2541	0.2280	0.1907	0.1958	0.1897	0.1794	0.1669	0.1744	0.1780	0.1796	0.0974	
47	370	(0.0468)	0.0200	0.1562	0.2126	0.2670	0.2818	0.5220	0.5040	0.4575	0.3453	0.1652	0.1465	0.1374	0.1047	0.1049	0.1158	0.1160	0.2124	0.1635	
48	370	(0.3396)	(0.2825)	0.0945	0.0827	0.1416	0.1597	0.3995	0.3745	0.3112	0.2751	0.2301	0.2081	0.1868	0.1664	0.1565	0.1355	0.1379	0.1434	0.1936	
49	201	(0.0011)	(0.0107)	0.1407	0.1336	0.1616	0.1896	0.3921	0.3826	0.3569									0.1939	0.1537	
50	370	0.2620	0.2806	0.0668	0.1835	0.1648	0.2430	0.3692	0.3215	0.2624	0.2290	0.2082	0.1904	0.1533	0.1460	0.1530	0.1404	0.1394	0.2067	0.0764	
51	370	0.1007	0.2590	0.1014	0.1953	0.2380	0.2672	0.3285	0.3254	0.2866	0.2584	0.2543	0.2271	0.2134	0.2015	0.2087	0.1377	0.1407	0.2202	0.0692	
52	370	0.3035	0.3816	0.2091	0.2961	0.2616	0.3066	0.3855	0.3352	0.3054	0.2613	0.2503	0.2704	0.2283	0.2250	0.2163	0.2192	0.2183	0.2749	0.0560	
53	370	(0.3680)	(0.3145)	(0.1773)	0.0020	(0.0521)	(0.0512)	(0.1886)	(0.1409)	(0.1408)	(0.0844)	(0.0393)	0.0183	0.0109	0.0552	0.0535	0.0450	0.0361	−0.0786	0.1274	
54	370	0.2705	0.3262	0.3047	0.3510	0.3021	0.3028	0.3019	0.3064	0.3133	0.3071	0.2629	0.2333	0.2044	0.2046	0.1786	0.1612	0.1266	0.2622	0.0653	
55	370	0.5140	0.5186	0.2350	0.2550	0.2282	0.2399	0.2702	0.2343	0.2255	0.2159	0.1941	0.1869	0.1856	0.1938	0.2007	0.1603	0.1663	0.2485	0.1052	
56	370	(0.3680)	(0.3145)	(0.1773)	0.0020	0.0087	0.0255	(0.0191)	0.0280	(0.0320)	(0.0209)	0.0224	0.0678	0.0572	0.0811	0.0604	0.0564	0.0444	−0.0281	0.1321	
57	370	0.0848	0.0899	0.3078	0.3435	0.3236	0.3578	0.4964	0.4429	0.4132	0.3989	0.3851	0.3411	0.3184	0.3179	0.3165	0.2980	0.2983	0.3255	0.1051	
58	370	(0.0984)	(0.0107)	(0.1158)	(0.0952)	(0.1296)	(0.1059)	(0.1439)	(0.1130)	(0.1057)	(0.0898)	(0.0505)	(0.0640)	(0.0806)	(0.0700)	(0.0652)	(0.0595)	(0.0474)	−0.0850	0.0337	
59	370	0.2321	0.2986	0.0664	(0.0275)	(0.0202)	(0.0446)	0.0199	0.0444	0.1359	0.0953	0.1735	0.1681	0.1779	0.1652	0.1580	0.1194	0.1010	0.1096	0.0945	

Table A3. Cont.

Date	From	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	2019.10	Average	SD
	TO	2018.05	2017.11	2015.11	2013.11	2011.11	2009.11	2007.11	2005.11	2003.11	200.111	1999.11	1997.11	1995.11	1993.11	199.111	1989.11	1989.01			
No.	pts	18	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	370			
60	370				0.0066	(0.0355)	0.0415	0.3285	0.3054	0.2289	0.1442	0.0875	0.1185	0.1049	0.1031	0.1086	0.1097	0.1105	0.1259	0.1019	
61	370	0.4264	0.4465	0.2976	0.3396	0.3229	0.3695	0.5348	0.4849	0.4498	0.4305	0.4145	0.4296	0.4069	0.3830	0.3684	0.3929	0.3759	0.4043	0.0588	
62	370	0.3035	0.3509	0.2945	0.3941	0.4251	0.4416	0.5762	0.5496	0.5155	0.4857	0.4617	0.4641	0.4258	0.4056	0.3920	0.3743	0.3710	0.4254	0.0785	
63	370	(0.2879)	(0.1771)	(0.0327)	(0.0206)	0.0363	0.0562	0.1216	0.1893	0.1587	0.1760	0.1604	0.1385	0.1337	0.1278	0.1259	0.1687	0.1672	0.0731	0.1341	
64	370	0.4247	0.3359	0.2926	0.2475	0.2183	0.2523	0.3642	0.3488	0.3287	0.3114	0.2875	0.2575	0.2502	0.2451	0.2451	0.2192	0.2226	0.2854	0.0588	
65	370	(0.1165)	(0.0955)	0.0196	0.1793	0.2214	0.2881	0.3536	0.3328	0.2883	0.2727	0.2477	0.2254	0.2194	0.2025	0.1968	0.1685	0.1592	0.1861	0.1336	
66	370	0.2924	0.3156	0.2436	0.3032	0.2857	0.3282	0.4467	0.4125	0.3537	0.3457	0.3497	0.3611	0.3427	0.3130	0.3135	0.2932	0.2839	0.3285	0.0491	
67	370	(0.0330)	0.0101	0.1518	0.2243	0.2373	0.2788	0.3677	0.3536	0.3120	0.2865	0.2882	0.2676	0.2362	0.2381	0.2282	0.1984	0.2033	0.2264	0.1051	
68	370	0.3867	0.3617	0.2586	0.2165	0.2678	0.2965	0.4278	0.4244	0.3934	0.3974	0.3888	0.3510	0.3260	0.3232	0.3191	0.2959	0.2920	0.3369	0.0613	
69	370	(0.0822)	0.0061	0.0543	0.0830	0.1434	0.2449	0.3150	0.3363	0.3053	0.2875	0.2631	0.2082	0.1954	0.1925	0.1885	0.1774	0.1773	0.1821	0.1140	
70	370	(0.1043)	(0.0106)	0.0393	0.0592	0.1291	0.2348	0.3086	0.3304	0.2984	0.2808	0.2574	0.2028	0.1900	0.1872	0.1835	0.1730	0.1729	0.1725	0.1189	
71	370	0.1883	0.1592	0.2927	0.1908	0.2322	0.2972	0.4832	0.4563	0.4116	0.3820	0.3622	0.3315	0.3250	0.3053	0.3081	0.2749	0.2773	0.3105	0.0904	
72	370	(0.1248)	(0.0128)	0.0223	0.0003	(0.0056)	0.0258	0.0844	0.0800	0.0712	0.0752	0.0567	0.0925	0.0932	0.0882	0.0866	0.0442	0.0400	0.0422	0.0560	
73	370	0.2408	0.2248	0.2983	0.3504	0.3554	0.3698	0.4256	0.3831	0.3628	0.3609	0.3379	0.3255	0.2897	0.2832	0.2725	0.2171	0.2102	0.3122	0.0640	
74	370	0.2408	0.2248	0.2983	0.3504	0.3562	0.3689	0.4281	0.3861	0.3639	0.3635	0.3417	0.3296	0.2941	0.2876	0.2768	0.2212	0.2141	0.3145	0.0637	
75	370	0.0710	0.2300	0.3466	0.0904	0.0527	0.0188	0.1535	0.1576	0.1518	0.1295	0.1137	0.1010	0.1042	0.0957	0.0994	0.0920	0.0931	0.1236	0.0742	
76	370	0.2552	0.1814	0.0993	0.0808	0.0916	0.0899	0.1319	0.1215	0.1154	0.1074	0.0995	0.0921	0.0895	0.0885	0.0882	0.0782	0.0749	0.1109	0.0451	
77	370	(0.1960)	(0.1350)	(0.2074)	(0.2033)	(0.0874)	(0.0529)	(0.1005)	(0.0721)	(0.0703)	(0.0674)	(0.0617)	(0.0597)	(0.0589)	(0.0589)	(0.0570)	(0.0526)	(0.0525)	−0.0937	0.0558	
78	370	0.1043	0.2446	0.1198	0.0870	0.0145	0.1014	0.2614	0.2640	0.2477	0.2280	0.2007	0.1761	0.1751	0.1682	0.1676	0.1549	0.1530	0.1687	0.0690	
79	370	(0.3012)	(0.1808)	(0.0774)	(0.0473)	(0.0236)	(0.0317)	0.1619	0.1530	0.1605	0.1597	0.1526	0.1547	0.1523	0.1581	0.1595	0.1404	0.1382	0.0605	0.1439	

Note: SD, Standard Deviation.

Appendix E

Table A4. Estimated Brent Crude Oil Price, Change Rate and Residuals of Each Selected Commodity.

Brent Crude Oil Price		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price					New York Harbor No. 2 Heating Oil Spot Price FOB					U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB				
Formulae		Y = 38.023X – 7.7037					Y = 37.149X – 2.4578					Y = 36.358X – 1.8652				
Items	(1)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Training Data																
Jun-06	68.8577	2.091	71.802	2.945	4.28%		1.925	69.054	0.196	0.29%		2.081	73.796	4.938	7.17%	
Jul-06	73.8971	2.217	76.593	2.696	3.65%	+	1.935	69.426	−4.472	−6.05%	+	2.154	76.450	2.553	3.45%	+
Aug-06	73.6122	2.247	77.734	4.122	5.60%	−	1.984	71.246	−2.366	−3.21%	−	2.133	75.686	2.074	2.82%	+
Sep-06	62.7719	1.81	61.118	−1.654	−2.63%	+	1.699	60.658	−2.114	−3.37%	+	1.81	63.943	1.171	1.87%	+
Oct-06	58.3800	1.794	60.510	2.130	3.65%	+	1.648	58.764	0.384	0.66%	+	1.739	61.361	2.981	5.11%	+
Nov-06	58.4832	1.811	61.156	2.673	4.57%	+	1.648	58.764	0.281	0.48%	−	1.733	61.143	2.660	4.55%	−
Dec-06	62.3147	1.833	61.992	−0.322	−0.52%	+	1.684	60.101	−2.214	−3.55%	+	1.81	63.943	1.628	2.61%	+
Jan-07	54.2991	1.63	54.274	−0.025	−0.05%	+	1.528	54.306	0.007	0.01%	+	1.654	58.271	3.972	7.31%	+
Feb-07	57.7570	1.759	59.179	1.422	2.46%	+	1.693	60.435	2.678	4.64%	+	1.74	61.398	3.641	6.30%	+
Mar-07	62.1436	1.891	64.198	2.054	3.31%	+	1.742	62.256	0.112	0.18%	+	1.846	65.252	3.108	5.00%	+
Apr-07	67.3984	2.053	70.358	2.959	4.39%	+	1.864	66.788	−0.610	−0.91%	+	2.036	72.160	4.761	7.06%	+
May-07	67.4761	2.044	70.015	2.539	3.76%	−	1.884	67.531	0.055	0.08%	+	2.044	72.451	4.974	7.37%	+
Jun-07	71.3162	2.12	72.905	1.589	2.23%	+	1.991	71.506	0.190	0.27%	+	2.099	74.450	3.134	4.39%	+
Jul-07	77.2041	2.146	73.894	−3.310	−4.29%	+	2.072	74.515	−2.689	−3.48%	+	2.137	75.832	−1.372	−1.78%	+
Aug-07	70.7965	2.106	72.373	1.576	2.23%	+	1.984	71.246	0.449	0.63%	+	2.092	74.196	3.399	4.80%	+
Sep-07	77.1270	2.32	80.510	3.383	4.39%	+	2.179	78.490	1.363	1.77%	+	2.265	80.486	3.359	4.35%	+
Oct-07	82.8578	2.417	84.198	1.340	1.62%	+	2.282	82.316	−0.542	−0.65%	+	2.372	84.376	1.518	1.83%	+
Nov-07	92.5282	2.7	94.958	2.430	2.63%	+	2.586	93.610	1.081	1.17%	+	2.673	95.320	2.792	3.02%	+
Dec-07	91.4500	2.631	92.335	0.885	0.97%	+	2.574	93.164	1.714	1.87%	+	2.601	92.702	1.252	1.37%	+
Jan-08	91.9205	2.59	90.776	−1.145	−1.25%	−	2.558	92.569	0.649	0.71%	−	2.605	92.847	0.927	1.01%	+
Feb-08	94.8167	2.774	97.772	2.955	3.12%	+	2.644	95.764	0.947	1.00%	+	2.728	97.319	2.503	2.64%	+
Mar-08	103.2775	3.253	115.985	12.708	12.30%	+	3.066	111.441	8.164	7.90%	+	3.124	111.717	8.440	8.17%	+
Apr-08	110.1877	3.443	123.209	13.022	11.82%	+	3.226	117.385	7.197	6.53%	+	3.365	120.479	10.292	9.34%	+
May-08	123.9362	3.821	137.582	13.646	11.01%	+	3.615	131.836	7.900	6.37%	+	3.738	134.041	10.105	8.15%	+
Jun-08	133.0486	3.887	140.092	7.043	5.29%	+	3.801	138.746	5.697	4.28%	+	3.878	139.131	6.083	4.57%	+
Jul-08	133.8730	3.845	138.495	4.622	3.45%	−	3.759	137.185	3.312	2.47%	−	3.886	139.422	5.549	4.14%	+
Aug-08	113.8490	3.273	116.746	2.897	2.54%	+	3.169	115.267	1.418	1.25%	+	3.271	117.062	3.213	2.82%	+
Sep-08	99.0641	3.041	107.924	8.860	8.94%	+	2.911	105.683	6.619	6.68%	+	3.375	120.843	21.779	21.98%	−
Oct-08	72.8426	2.397	83.437	10.595	14.54%	+	2.239	80.719	7.876	10.81%	+	2.315	82.304	9.461	12.99%	+
Nov-08	53.2410	1.923	65.415	12.174	22.86%	+	1.843	66.008	12.767	23.98%	+	1.88	66.488	13.247	24.88%	+
Dec-08	41.5809	1.436	46.897	5.316	12.79%	+	1.402	49.625	8.044	19.35%	+	1.375	48.127	6.546	15.74%	+

Table A4. Cont.

Brent Crude Oil Price		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price					New York Harbor No. 2 Heating Oil Spot Price FOB					U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB				
Formulae		Y = 38.023X – 7.7037					Y = 37.149X – 2.4578					Y = 36.358X – 1.8652				
Items	(1)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Jan-09	44.8600	1.496	49.179	4.319	9.63%	+	1.465	51.965	7.105	15.84%	+	1.469	51.545	6.685	14.90%	+
Feb-09	43.2425	1.306	41.954	−1.288	−2.98%	+	1.279	45.056	1.813	4.19%	+	1.259	43.910	0.667	1.54%	+
Mar-09	46.8391	1.338	43.171	−3.668	−7.83%	+	1.277	44.981	−1.858	−3.97%	−	1.268	44.237	−2.602	−5.56%	+
Apr-09	50.8452	1.449	47.392	−3.454	−6.79%	+	1.357	47.953	−2.892	−5.69%	+	1.369	47.909	−2.936	−5.78%	+
May-09	57.9410	1.52	50.091	−7.850	−13.55%	+	1.473	52.263	−5.678	−9.80%	+	1.488	52.236	−5.705	−9.85%	+
Jun-09	68.6168	1.82	61.498	−7.119	−10.37%	+	1.747	62.442	−6.175	−9.00%	+	1.805	63.761	−4.856	−7.08%	+
Jul-09	64.9100	1.722	57.772	−7.138	−11.00%	+	1.628	58.021	−6.889	−10.61%	+	1.712	60.380	−4.530	−6.98%	+
Aug-09	72.5048	1.927	65.567	−6.938	−9.57%	+	1.865	66.825	−5.680	−7.83%	+	1.885	66.670	−5.835	−8.05%	+
Sep-09	67.6868	1.795	60.548	−7.139	−10.55%	+	1.728	61.736	−5.951	−8.79%	+	1.749	61.725	−5.962	−8.81%	+
Oct-09	73.1941	1.982	67.658	−5.536	−7.56%	+	1.929	69.203	−3.991	−5.45%	+	1.942	68.742	−4.452	−6.08%	+
Nov-09	77.0367	1.998	68.266	−8.770	−11.38%	+	1.981	71.134	−5.902	−7.66%	+	1.986	70.342	−6.695	−8.69%	+
Dec-09	74.6695	1.98	67.582	−7.088	−9.49%	+	1.968	70.651	−4.018	−5.38%	+	1.979	70.087	−4.582	−6.14%	+
Jan-10	76.3730	2.066	70.852	−5.521	−7.23%	+	2.046	73.549	−2.824	−3.70%	+	2.052	72.741	−3.632	−4.76%	+
Feb-10	74.3120	2.035	69.673	−4.639	−6.24%	+	1.978	71.023	−3.289	−4.43%	+	1.989	70.451	−3.861	−5.20%	+
Mar-10	79.2748	2.159	74.388	−4.887	−6.16%	+	2.083	74.924	−4.351	−5.49%	+	2.108	74.777	−4.497	−5.67%	+
Apr-10	84.9786	2.284	79.141	−5.838	−6.87%	+	2.204	79.419	−5.560	−6.54%	+	2.243	79.686	−5.293	−6.23%	+
May-10	76.2510	2.136	73.513	−2.738	−3.59%	+	2.04	73.326	−2.925	−3.84%	+	2.063	73.141	−3.110	−4.08%	+
Jun-10	74.8382	2.108	72.449	−2.389	−3.19%	+	2.032	73.029	−1.809	−2.42%	+	2.058	72.960	−1.879	−2.51%	+
Jul-10	74.7355	2.066	70.852	−3.884	−5.20%	+	1.979	71.060	−3.675	−4.92%	+	2.019	71.542	−3.194	−4.27%	+
Aug-10	76.6932	2.122	72.981	−3.712	−4.84%	+	2.016	72.435	−4.259	−5.55%	+	2.083	73.869	−2.825	−3.68%	+
Sep-10	77.7868	2.175	74.996	−2.790	−3.59%	+	2.09	75.184	−2.603	−3.35%	+	2.114	74.996	−2.791	−3.59%	+
Oct-10	82.9181	2.301	79.787	−3.131	−3.78%	+	2.242	80.830	−2.088	−2.52%	+	2.248	79.868	−3.051	−3.68%	+
Nov-10	85.6700	2.377	82.677	−2.993	−3.49%	+	2.32	83.728	−1.942	−2.27%	+	2.323	82.594	−3.076	−3.59%	+
Dec-10	91.7965	2.5	87.354	−4.443	−4.84%	+	2.468	89.226	−2.571	−2.80%	+	2.453	87.321	−4.476	−4.88%	+
Jan-11	96.2943	2.642	92.753	−3.541	−3.68%	+	2.604	94.278	−2.016	−2.09%	+	2.619	93.356	−2.938	−3.05%	+
Feb-11	103.9555	2.84	100.282	−3.674	−3.53%	+	2.77	100.445	−3.511	−3.38%	+	2.839	101.355	−2.600	−2.50%	+
Mar-11	114.4413	3.131	111.346	−3.095	−2.70%	+	3.034	110.252	−4.189	−3.66%	+	3.125	111.754	−2.688	−2.35%	+
Apr-11	123.0700	3.271	116.670	−6.400	−5.20%	+	3.196	116.270	−6.800	−5.52%	+	3.267	116.916	−6.154	−5.00%	+
May-11	114.4582	3.035	107.696	−6.762	−5.91%	+	2.952	107.206	−7.252	−6.34%	+	3.085	110.299	−4.159	−3.63%	+
Jun-11	113.7577	3.054	108.419	−5.339	−4.69%	−	2.967	107.763	−5.994	−5.27%	−	3.046	108.881	−4.876	−4.29%	+
Jul-11	116.4600	3.17	112.829	−3.631	−3.12%	+	3.068	111.515	−4.945	−4.25%	+	3.131	111.972	−4.488	−3.85%	+
Aug-11	110.0813	3.01	106.746	−3.336	−3.03%	+	2.946	106.983	−3.098	−2.81%	+	3.008	107.500	−2.582	−2.35%	+
Sep-11	110.8791	2.98	105.605	−5.274	−4.76%	−	2.921	106.054	−4.825	−4.35%	−	2.948	105.318	−5.561	−5.02%	−
Oct-11	109.4686	3.008	106.669	−2.799	−2.56%	−	2.953	107.243	−2.225	−2.03%	−	2.966	105.973	−3.496	−3.19%	−
Nov-11	110.5041	3.095	109.977	−0.527	−0.48%	+	3.054	110.995	0.491	0.44%	+	3.046	108.881	−1.623	−1.47%	+
Dec-11	107.9090	2.937	103.970	−3.939	−3.65%	+	2.891	104.940	−2.969	−2.75%	+	2.873	102.591	−5.318	−4.93%	+

Table A4. Cont.

Brent Crude Oil Price		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price					New York Harbor No. 2 Heating Oil Spot Price FOB					U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB				
Formulae		Y = 38.023X – 7.7037					Y = 37.149X – 2.4578					Y = 36.358X – 1.8652				
Items	(1)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Jan-12	111.1562	3.08	109.407	−1.749	−1.57%	+	3.054	110.995	−0.161	−0.14%	+	3.087	110.372	−0.784	−0.71%	+
Feb-12	119.7024	3.223	114.844	−4.858	−4.06%	+	3.196	116.270	−3.432	−2.87%	+	3.207	114.735	−4.967	−4.15%	+
Mar-12	124.9286	3.302	117.848	−7.080	−5.67%	+	3.217	117.051	−7.878	−6.31%	+	3.256	116.516	−8.412	−6.73%	+
Apr-12	120.4635	3.242	115.567	−4.897	−4.06%	+	3.15	114.562	−5.902	−4.90%	+	3.226	115.426	−5.038	−4.18%	+
May-12	110.5217	2.987	105.871	−4.651	−4.21%	+	2.913	105.757	−4.765	−4.31%	+	2.974	106.263	−4.258	−3.85%	+
Jun-12	95.5890	2.71	95.339	−0.250	−0.26%	+	2.619	94.835	−0.754	−0.79%	+	2.678	95.502	−0.088	−0.09%	+
Jul-12	103.1409	2.921	103.361	0.221	0.21%	+	2.813	102.042	−1.099	−1.07%	+	2.892	103.282	0.141	0.14%	+
Aug-12	113.3400	3.179	113.171	−0.169	−0.15%	+	3.045	110.661	−2.679	−2.36%	+	3.156	112.881	−0.459	−0.41%	+
Sep-12	113.3825	3.229	115.073	1.690	1.49%	+	3.134	113.967	0.585	0.52%	+	3.191	114.153	0.771	0.68%	+
Oct-12	111.9735	3.236	115.339	3.365	3.01%	−	3.14	114.190	2.217	1.98%	−	3.111	111.245	−0.729	−0.65%	+
Nov-12	109.7118	3.18	113.209	3.498	3.19%	+	3.009	109.324	−0.388	−0.35%	+	2.96	105.754	−3.957	−3.61%	+
Dec-12	109.6765	3.07	109.027	−0.650	−0.59%	+	2.995	108.803	−0.873	−0.80%	+	2.94	105.027	−4.649	−4.24%	+
Jan-13	112.9736	3.103	110.282	−2.692	−2.38%	+	3.068	111.515	−1.458	−1.29%	+	3.091	110.517	−2.456	−2.17%	+
Feb-13	116.5195	3.237	115.377	−1.143	−0.98%	+	3.168	115.230	−1.289	−1.11%	+	3.218	115.135	−1.385	−1.19%	+
Mar-13	109.2400	3.037	107.772	−1.468	−1.34%	+	2.943	106.872	−2.368	−2.17%	+	2.969	106.082	−3.158	−2.89%	+
Apr-13	102.8755	2.9	102.563	−0.312	−0.30%	+	2.742	99.405	−3.471	−3.37%	+	2.808	100.228	−2.647	−2.57%	+
May-13	103.0270	2.888	102.107	−0.920	−0.89%	−	2.739	99.293	−3.734	−3.62%	−	2.725	97.210	−5.817	−5.65%	−
Jun-13	103.1100	2.891	102.221	−0.889	−0.86%	+	2.752	99.776	−3.334	−3.23%	+	2.769	98.810	−4.300	−4.17%	+
Jul-13	107.7161	3.018	107.050	−0.666	−0.62%	+	2.885	104.717	−2.999	−2.78%	+	2.894	103.355	−4.361	−4.05%	+
Aug-13	110.9645	3.068	108.951	−2.014	−1.81%	+	2.958	107.429	−3.536	−3.19%	+	3.003	107.318	−3.647	−3.29%	+
Sep-13	111.6214	3.048	108.190	−3.431	−3.07%	−	2.961	107.540	−4.081	−3.66%	+	2.934	104.809	−6.812	−6.10%	−
Oct-13	109.4787	2.995	106.175	−3.304	−3.02%	+	2.94	106.760	−2.718	−2.48%	+	2.885	103.028	−6.451	−5.89%	+
Nov-13	108.0762	2.938	104.008	−4.068	−3.76%	+	2.923	106.129	−1.947	−1.80%	+	2.83	101.028	−7.048	−6.52%	+
Dec-13	110.6740	3.045	108.076	−2.598	−2.35%	+	3.032	110.178	−0.496	−0.45%	+	2.955	105.573	−5.101	−4.61%	+
Jan-14	107.4200	3.117	110.814	3.394	3.16%	−	3.063	111.330	3.910	3.64%	−	2.921	104.337	−3.083	−2.87%	+
Feb-14	108.8100	3.227	114.997	6.187	5.69%	+	3.061	111.255	2.445	2.25%	−	2.965	105.936	−2.874	−2.64%	+
Mar-14	107.4000	2.993	106.099	−1.301	−1.21%	+	2.912	105.720	−1.680	−1.56%	+	2.89	103.209	−4.191	−3.90%	+
Apr-14	107.7900	2.958	104.768	−3.022	−2.80%	−	2.888	104.829	−2.961	−2.75%	−	2.888	103.137	−4.653	−4.32%	−
May-14	109.6800	2.936	103.932	−5.748	−5.24%	−	2.86	103.788	−5.892	−5.37%	−	2.868	102.410	−7.270	−6.63%	−
Jun-14	111.8700	2.962	104.920	−6.950	−6.21%	+	2.882	104.606	−7.264	−6.49%	+	2.883	102.955	−8.915	−7.97%	+
Jul-14	106.9800	2.879	101.765	−5.215	−4.88%	+	2.776	100.668	−6.312	−5.90%	+	2.817	100.555	−6.425	−6.01%	+
Aug-14	101.9200	2.85	100.662	−1.258	−1.23%	+	2.753	99.813	−2.107	−2.07%	+	2.839	101.355	−0.565	−0.55%	−
Sep-14	97.3400	2.74	96.479	−0.861	−0.88%	+	2.633	95.356	−1.984	−2.04%	+	2.729	97.356	0.016	0.02%	+
Oct-14	87.2700	2.532	88.571	1.301	1.49%	+	2.424	87.591	0.321	0.37%	+	2.46	87.575	0.305	0.35%	+
Nov-14	78.4400	2.43	84.692	6.252	7.97%	+	2.249	81.090	2.650	3.38%	+	2.297	81.649	3.209	4.09%	+
Dec-14	62.3300	2.034	69.635	7.305	11.72%	+	1.856	66.491	4.161	6.68%	+	1.801	63.616	1.286	2.06%	+

Table A4. Cont.

Brent Crude Oil Price		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price					New York Harbor No. 2 Heating Oil Spot Price FOB					U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB				
Formulae		Y = 38.023X – 7.7037					Y = 37.149X – 2.4578					Y = 36.358X – 1.8652				
Items	(1)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Jan-15	48.0700	1.678	56.099	8.029	16.70%	+	1.616	57.575	9.505	19.77%	+	1.496	52.526	4.456	9.27%	+
Feb-15	57.9300	1.991	68.000	10.070	17.38%	+	1.873	67.122	9.192	15.87%	+	1.766	62.343	4.413	7.62%	+
Mar-15	55.7900	1.856	62.867	7.077	12.69%	+	1.632	58.169	2.379	4.26%	+	1.629	57.362	1.572	2.82%	+
Apr-15	59.3900	1.843	62.373	2.983	5.02%	-	1.722	61.513	2.123	3.57%	+	1.702	60.016	0.626	1.05%	+
May-15	64.5600	1.965	67.011	2.451	3.80%	+	1.832	65.599	1.039	1.61%	+	1.849	65.361	0.801	1.24%	+
Jun-15	62.3400	1.888	64.084	1.744	2.80%	+	1.762	62.999	0.659	1.06%	+	1.732	61.107	-1.233	-1.98%	+
Jul-15	55.8700	1.684	56.327	0.457	0.82%	+	1.557	55.383	-0.487	-0.87%	+	1.549	54.453	-1.417	-2.54%	+
Aug-15	46.9900	1.516	49.939	2.949	6.28%	+	1.387	49.068	2.078	4.42%	+	1.389	48.636	1.646	3.50%	+
Sep-15	47.2400	1.516	49.939	2.699	5.71%	-	1.423	50.405	3.165	6.70%	+	1.395	48.854	1.614	3.42%	+
Oct-15	48.1200	1.472	48.266	0.146	0.30%	-	1.404	49.699	1.579	3.28%	-	1.391	48.709	0.589	1.22%	-
Nov-15	44.4200	1.413	46.023	1.603	3.61%	+	1.316	46.430	2.010	4.53%	+	1.326	46.346	1.926	4.33%	+
Dec-15	37.7200	1.138	35.566	-2.154	-5.71%	+	1.037	36.066	-1.654	-4.39%	+	1.082	37.474	-0.246	-0.65%	+
Jan-16	30.8000	0.979	29.521	-1.279	-4.15%	+	0.939	32.425	1.625	5.28%	+	0.93	31.948	1.148	3.73%	+
Feb-16	33.2000	1.036	31.688	-1.512	-4.55%	+	0.972	33.651	0.451	1.36%	+	0.973	33.511	0.311	0.94%	+
Mar-16	39.0700	1.19	37.544	-1.526	-3.91%	+	1.132	39.595	0.525	1.34%	+	1.069	37.002	-2.068	-5.29%	+
Apr-16	42.2500	1.247	39.711	-2.539	-6.01%	+	1.188	41.675	-0.575	-1.36%	+	1.147	39.837	-2.413	-5.71%	+
May-16	47.1300	1.423	46.403	-0.727	-1.54%	+	1.356	47.916	0.786	1.67%	+	1.299	45.364	-1.766	-3.75%	+
Jun-16	48.4800	1.491	48.989	0.509	1.05%	+	1.416	50.145	1.665	3.43%	+	1.382	48.382	-0.098	-0.20%	+
Jul-16	45.0700	1.37	44.388	-0.682	-1.51%	+	1.292	45.539	0.469	1.04%	+	1.272	44.382	-0.688	-1.53%	+
Aug-16	46.1400	1.399	45.490	-0.650	-1.41%	+	1.325	46.765	0.625	1.35%	+	1.295	45.218	-0.922	-2.00%	+
Sep-16	46.1900	1.418	46.213	0.023	0.05%	+	1.351	47.730	1.540	3.34%	+	1.319	46.091	-0.099	-0.21%	+
Oct-16	49.7300	1.559	51.574	1.844	3.71%	+	1.488	52.820	3.090	6.21%	+	1.457	51.108	1.378	2.77%	+
Nov-16	46.4400	1.463	47.924	1.484	3.20%	+	1.39	49.179	2.739	5.90%	+	1.356	47.436	0.996	2.15%	+
Dec-16	54.0700	1.642	54.730	0.660	1.22%	+	1.553	55.235	1.165	2.15%	+	1.491	52.345	-1.725	-3.19%	+
Jan-17	54.8900	1.623	54.008	-0.882	-1.61%	-	1.551	55.160	0.270	0.49%	-	1.514	53.181	-1.709	-3.11%	+
Feb-17	55.4900	1.624	54.046	-1.444	-2.60%	+	1.562	55.569	0.079	0.14%	+	1.547	54.381	-1.109	-2.00%	+
Mar-17	51.9700	1.528	50.395	-1.575	-3.03%	+	1.492	52.969	0.999	1.92%	+	1.445	50.672	-1.298	-2.50%	+
Apr-17	52.9800	1.586	52.601	-0.379	-0.72%	+	1.523	54.120	1.140	2.15%	+	1.51	53.035	0.055	0.10%	+
May-17	50.8700	1.514	49.863	-1.007	-1.98%	+	1.454	51.557	0.687	1.35%	+	1.412	49.472	-1.398	-2.75%	+
Jun-17	46.8900	1.417	46.175	-0.715	-1.53%	+	1.332	47.025	0.135	0.29%	+	1.295	45.218	-1.672	-3.56%	+
Jul-17	48.6900	1.521	50.129	1.439	2.96%	+	1.423	50.405	1.715	3.52%	+	1.417	49.654	0.964	1.98%	+
Aug-17	51.3700	1.631	54.312	2.942	5.73%	+	1.519	53.972	2.602	5.06%	+	1.561	54.890	3.520	6.85%	+
Sep-17	55.1600	1.791	60.395	5.235	9.49%	+	1.708	60.993	5.833	10.57%	+	1.8	63.579	8.419	15.26%	+
Oct-17	57.6200	1.799	60.700	3.080	5.34%	+	1.707	60.956	3.336	5.79%	-	1.659	58.453	0.833	1.45%	-
Nov-17	62.5700	1.916	65.148	2.578	4.12%	+	1.823	65.265	2.695	4.31%	+	1.76	62.125	-0.445	-0.71%	+
Dec-17	64.2100	1.944	66.213	2.003	3.12%	+	1.863	66.751	2.541	3.96%	+	1.817	64.197	-0.013	-0.02%	+

Table A4. Cont.

Brent Crude Oil Price		New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price					New York Harbor No. 2 Heating Oil Spot Price FOB					U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB				
Formulae		Y = 38.023X – 7.7037					Y = 37.149X – 2.4578					Y = 36.358X – 1.8652				
Items	(1)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Jan-18	68.9900	2.074	71.156	2.166	3.14%	+	2.017	72.472	3.482	5.05%	+	1.952	69.106	0.116	0.17%	+
Feb-18	65.4200	1.937	65.947	0.527	0.81%	+	1.853	66.379	0.959	1.47%	+	1.849	65.361	−0.059	−0.09%	+
Mar-18	66.4500	1.93	65.681	−0.769	−1.16%	−	1.874	67.159	0.709	1.07%	+	1.858	65.688	−0.762	−1.15%	+
Apr-18	71.6300	2.079	71.346	−0.284	−0.40%	+	2.037	73.215	1.585	2.21%	+	2.015	71.396	−0.234	−0.33%	+
May-18	76.6500	2.23	77.088	0.438	0.57%	+	2.187	78.787	2.137	2.79%	+	2.155	76.486	−0.164	−0.21%	+
Jun-18	75.1900	2.146	73.894	−1.296	−1.72%	+	2.112	76.001	0.811	1.08%	+	2.09	74.123	−1.067	−1.42%	+
Jul-18	74.4400	2.14	73.666	−0.774	−1.04%	+	2.108	75.852	1.412	1.90%	+	2.098	74.414	−0.026	−0.04%	−
Aug-18	73.1300	2.153	74.160	1.030	1.41%	−	2.125	76.484	3.354	4.59%	−	2.118	75.141	2.011	2.75%	−
Sep-18	78.8600	2.252	77.924	−0.936	−1.19%	+	2.225	80.199	1.339	1.70%	+	2.189	77.722	−1.138	−1.44%	+
Oct-18	80.4700	2.332	80.966	0.496	0.62%	+	2.313	83.468	2.998	3.73%	+	2.249	79.904	−0.566	−0.70%	+
Nov-18	65.1700	2.048	70.167	4.997	7.67%	+	2.033	73.066	7.896	12.12%	+	1.945	68.851	3.681	5.65%	+
Dec-18	56.4600	1.801	60.776	4.316	7.64%	+	1.784	63.816	7.356	13.03%	+	1.696	59.798	3.338	5.91%	+
Jan-19	59.2700	1.846	62.487	3.217	5.43%	+	1.819	65.116	5.846	9.86%	+	1.784	62.997	3.727	6.29%	+
Feb-19	64.1300	1.962	66.897	2.767	4.32%	+	1.931	69.277	5.147	8.03%	+	1.914	67.724	3.594	5.60%	+
Mar-19	66.4100	1.988	67.886	1.476	2.22%	+	1.965	70.540	4.130	6.22%	+	1.902	67.288	0.878	1.32%	−
Apr-19	71.2000	2.062	70.700	−0.500	−0.70%	+	2.036	73.178	1.978	2.78%	+	1.977	70.015	−1.185	−1.66%	+
May-19	70.5300	2.031	69.521	−1.009	−1.43%	+	2.006	72.063	1.533	2.17%	+	1.972	69.833	−0.697	−0.99%	+
Jun-19	63.3000	1.846	62.487	−0.813	−1.28%	+	1.82	65.153	1.853	2.93%	+	1.817	64.197	0.897	1.42%	+
Jul-19	64.0000	1.92	65.300	1.300	2.03%	+	1.888	67.680	3.680	5.75%	+	1.913	67.688	3.688	5.76%	+
Aug-19	59.2500	1.824	61.650	2.400	4.05%	+	1.795	64.225	4.975	8.40%	+	1.8	63.579	4.329	7.31%	+
Sep-19	62.3300	1.935	65.871	3.541	5.68%	+	1.919	68.831	6.501	10.43%	+	1.874	66.270	3.940	6.32%	+
Oct-19	59.3700	1.937	65.947	6.577	11.08%	−	1.92	68.868	9.498	16.00%	−	1.861	65.797	6.427	10.83%	+
Testing Data																
Nov-19	62.7400	1.933	65.795	3.055	4.87%	−	1.915	68.683	5.943	9.47%	−	1.824	64.452	1.712	2.73%	−
Dec-19	65.8500	1.988	67.886	2.036	3.09%	+	1.966	70.577	4.727	7.18%	+	1.893	66.960	1.110	1.69%	+
Jan-20	63.6000	1.848	62.563	−1.037	−1.63%	+	1.829	65.488	1.888	2.97%	+	1.77	62.488	−1.112	−1.75%	+
Feb-20	55.0000	1.622	53.970	−1.030	−1.87%	+	1.589	56.572	1.572	2.86%	+	1.509	52.999	−2.001	−3.64%	+
Mar-20	32.9800	1.19	37.5437	4.5637	13.84%	+	1.156	40.486	7.506	22.76%	+	0.953	32.7840	−0.1960	−0.59%	+
Apr-20	23.3400	0.88	25.7565	2.4165	10.35%	+	0.852	29.1931	5.8531	25.08%	+	0.606	20.1677	−3.1723	−13.59%	+
May-20	31.0200	0.887	26.0227	−4.9973	−16.11%	+	0.843	28.8588	−2.1612	−6.97%	−	0.686	23.0764	−7.9436	−25.61%	+
Jun-20	39.9300	1.124	35.0342	−4.8958	−12.26%	+	1.073	37.4031	−2.5269	−6.33%	+	0.983	33.8747	−6.0553	−15.16%	+
Jul-20	42.8100	1.24	39.4448	−3.3652	−7.86%	+	1.187	41.6381	−1.1719	−2.74%	+	1.084	37.5469	−5.2631	−12.29%	+
Aug-20	44.2600	1.226	38.9125	−5.3475	−12.08%	−	1.18	41.3780	−2.8820	−6.51%	−	1.112	38.5649	−5.6951	−12.87%	+

Note: 1. Data range is from June 2006 to August 2020; 2. Dubai Crude and WTI Crude are not listed; 3. Item: (1) Real price; (2) Estimated Brent Crude oil price; (3) Residuals; (4) Change (%); (5) Directional variation; 4. “+” means the commodity and Brent Crude have the same directional variation of price.

Appendix F

Table A5. Summary of All Selected Models after Verification.

Commodity	Items	19-Nov	19-Dec	20-Jan	20-Feb	20-Mar	20-Apr	20-May	20-June	20-July	20-Aug	DARs	MSPEs
Brent Crude Oil Price	(1)	62.74 5.68%	65.85 4.96%	63.6 −3.42%	55 −13.52%	32.98 −40.04%	23.34 −29.23%	31.02 32.90%	39.93 28.72%	42.81 7.21%	44.26 3.39%	-	-
Crude oil, Dubai; Dubai Fateh Price	(1)	61.41 5.03%	64.41 4.89%	63.76 −1.01%	54.51 −14.51%	33.75 −38.08%	23.27 −31.05%	31.56 35.63%	40.14 27.19%	42.64 6.23%	43.71 2.51%	100%	4.81
	(2)	64.036	67.088	66.427	57.016	35.895	25.232	33.667	42.396	44.939	46.028		
	(3)	1.296 2.07%	1.238 1.88%	2.827 4.44%	2.016 3.67%	2.915 8.84%	1.892 8.11%	2.647 8.53%	2.466 6.18%	2.129 4.97%	1.768 3.99%		
	(4)	+	+	+	+	+	+	+	+	+	+		
Crude oil, WTI; West Texas Intermediate Price	(1)	57.06 5.71%	59.8 4.80%	57.52 −3.81%	50.53 −12.15%	29.88 −40.87%	16.52 −44.71%	28.56 72.88%	38.3 34.10%	40.75 6.40%	42.36 3.95%	100%	14.67
	(2)	59.615	62.670	60.128	52.335	29.312	14.417	27.840	38.699	41.431	43.226		
	(3)	−3.125 −4.98%	−3.180 −4.83%	−3.472 −5.46%	−2.665 −4.85%	−3.668 −11.12%	−8.923 −38.23%	−3.180 −10.25%	−1.231 −3.08%	−1.379 −3.22%	−1.034 −2.34%		
	(4)	+	+	+	+	+	+	+	+	+	+		
New York Harbor Ultra-Low Sulfur No 2 Diesel Spot Price	(1)	1.933 −0.21%	1.988 2.85%	1.848 −7.04%	1.622 −12.23%	1.19 −26.63%	0.88 −26.05%	0.887 0.80%	1.124 26.72%	1.24 10.32%	1.226 −1.13%	80%	13.11
	(2)	65.795	67.886	62.563	53.970	37.544	25.757	26.023	35.034	39.445	38.913		
	(3)	3.055 4.87%	2.036 3.09%	−1.037 −1.63%	−1.03 −1.87%	4.564 13.84%	2.417 10.35%	−4.997 −16.11%	−4.896 −12.26%	−3.365 −7.86%	−5.348 −12.08%		
	(4)	−	+	+	+	+	+	+	+	+	−		

Table A5. Cont.

Commodity	Items	19-Nov	19-Dec	20-Jan	20-Feb	20-Mar	20-Apr	20-May	20-June	20-July	20-Aug	DARs	MSPEs
New York Harbor No. 2 Heating Oil Spot Price FOB	(1)	1.915 −0.26%	1.966 2.66%	1.829 −6.97%	1.589 −13.12%	1.156 −27.25%	0.852 −26.30%	0.843 −1.06%	1.073 27.28%	1.187 10.62%	1.18 −0.59%	70%	17.50
	(2)	68.683	70.577	65.488	56.572	40.486	29.193	28.859	37.403	41.638	41.378		
	(3)	5.943 9.47%	4.727 7.18%	1.888 2.97%	1.572 2.86%	7.506 22.76%	5.853 25.08%	−2.161 −6.97%	−2.527 −6.33%	−1.172 −2.74%	−2.882 −6.51%		
	(4)	−	+	+	+	+	+	−	+	+	−		
U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB	(1)	1.824 −1.99%	1.893 3.78%	1.77 −6.50%	1.509 −14.75%	0.953 −36.85%	0.606 −36.41%	0.686 13.20%	0.983 43.29%	1.084 10.27%	1.112 2.58%	90%	17.94
	(2)	64.452	66.960	62.488	52.999	32.784	20.168	23.076	33.875	37.547	38.567		
	(3)	1.712 2.73%	1.110 1.69%	−1.112 −1.75%	−2.001 −3.64%	−0.196 −0.59%	−3.172 −13.59%	−7.944 −25.61%	−6.055 −15.16%	−5.263 −12.29%	−5.695 −12.87%		
	(4)	−	+	+	+	+	+	+	+	+	+		

Note: 1. Item: (1) Real Price (\$) & Monthly Change Rate (%); (2) Estimated Brent Price (\$/bbl); (3) Residuals (\$/bbl) & Change Rate (%); (4) Monthly Directional Variation (+/−); 2. Residuals = Estimated Brent Crude oil price—Brent Crude oil price; Change Rate = Residuals/Brent Crude oil price; 3. Directional Variation: (1) Estimated prices derived from commodities have the same direction of price change as that of Brent Crude, excluding unchanged prices; (0) otherwise.

Appendix G

Table A6. The First Weekly Price of Each Month of the Selected Commodities and WTI Crude.

Items	Crude oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB	Items	Crude Oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
Jun-06	71.53	2.099	1.975	2.09	Nov-13	96.94	2.95	2.923	2.875
Jul-06	74.65	2.169	1.947	2.124	Dec-13	96.21	3.052	3.044	2.952
Aug-06	75.2	2.37	2.047	2.226	Jan-14	96.47	3.04	3.035	2.96
Sep-06	70.01	2.108	1.928	2.038	Feb-14	97.78	3.389	3.059	2.881
Oct-06	59.77	1.761	1.618	1.739	Mar-14	103.07	3.114	3.028	2.992
Nov-06	58.55	1.721	1.599	1.657	Apr-14	100.46	2.905	2.854	2.833
Dec-06	62.02	1.917	1.735	1.848	May-14	100.51	2.945	2.865	2.869
Jan-07	57.76	1.683	1.551	1.669	Jun-14	103.23	2.87	2.795	2.799
Feb-07	57.11	1.713	1.643	1.721	Jul-14	105.52	2.944	2.845	2.845
Mar-07	61.64	1.857	1.765	1.827	Aug-14	102.19	2.879	2.777	2.842
Apr-07	64.82	2.046	1.854	2.013	Sep-14	94.06	2.825	2.714	2.841
May-07	63.82	2.018	1.866	2.024	Oct-14	91.44	2.654	2.566	2.626
Jun-07	63.94	2.056	1.893	2.045	Nov-14	78.24	2.473	2.31	2.395
Jul-07	71.78	2.179	2.074	2.138	Dec-14	67.18	2.177	1.981	2.03
Aug-07	76.75	2.149	2.055	2.151	Jan-15	53.44	1.836	1.703	1.582
Sep-07	75.96	2.25	2.103	2.181	Feb-15	50.58	1.818	1.746	1.679
Oct-07	80.59	2.305	2.171	2.287	Mar-15	50.38	2.109	1.788	1.751
Nov-07	93.46	2.605	2.487	2.575	Apr-15	48.91	1.719	1.582	1.575
Dec-07	88.71	2.603	2.509	2.56	May-15	57.98	1.949	1.823	1.806
Jan-08	98.17	2.719	2.697	2.721	Jun-15	59.66	1.897	1.769	1.765
Feb-08	91.14	2.546	2.516	2.562	Jul-15	57.92	1.842	1.72	1.696
Mar-08	103.44	3.136	2.935	3.026	Aug-15	44.94	1.532	1.405	1.419
Apr-08	103.46	3.197	3.016	3.081	Sep-15	46.73	1.591	1.476	1.45
May-08	115.42	3.408	3.213	3.347	Oct-15	45	1.486	1.416	1.398
Jun-08	128.16	3.812	3.701	3.782	Nov-15	45.98	1.508	1.43	1.426
Jul-08	142.52	4.045	3.992	4.109	Dec-15	40.4	1.314	1.199	1.226
Aug-08	124.57	3.582	3.472	3.599	Jan-16	36.99	1.058	1.004	1.027
Sep-08	108.37	3.126	3.02	3.224	Feb-16	31.26	1.048	0.979	1.004
Oct-08	96.59	2.923	2.742	2.865	Mar-16	34.43	1.102	1.046	1.007

Table A6. Cont.

Items	Crude oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB	Items	Crude Oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
Nov-08	64.31	2.101	2.004	2.048	Apr-16	36.82	1.163	1.101	1.032
Dec-08	45.6	1.574	1.525	1.536	May-16	44.22	1.338	1.279	1.204
Jan-09	42.4	1.376	1.334	1.361	Jun-16	49	1.49	1.422	1.369
Feb-09	40.78	1.374	1.355	1.329	Jul-16	48.17	1.467	1.401	1.389
Mar-09	43.18	1.218	1.18	1.161	Aug-16	40.82	1.269	1.18	1.158
Apr-09	50.34	1.463	1.367	1.38	Sep-16	45.11	1.422	1.355	1.333
May-09	50.2	1.382	1.312	1.314	Oct-16	49.48	1.567	1.496	1.463
Jun-09	68.11	1.786	1.732	1.756	Nov-16	45.51	1.469	1.4	1.368
Jul-09	69.32	1.782	1.698	1.778	Dec-16	48.63	1.561	1.477	1.433
Aug-09	71.58	1.965	1.889	1.916	Jan-17	53.34	1.676	1.601	1.539
Sep-09	68.39	1.788	1.721	1.733	Feb-17	53.33	1.621	1.547	1.518
Oct-09	68.84	1.81	1.742	1.754	Mar-17	53.56	1.606	1.573	1.527
Nov-09	79	2.024	2.007	2.002	Apr-17	51.26	1.59	1.513	1.512
Dec-09	76.81	2.013	1.994	2.006	May-17	47.21	1.456	1.391	1.364
Jan-10	79.07	2.108	2.106	2.074	Jun-17	48.48	1.518	1.446	1.404
Feb-10	74.57	2.002	1.965	1.956	Jul-17	44.96	1.465	1.38	1.352
Mar-10	80.19	2.139	2.06	2.091	Aug-17	49.52	1.646	1.542	1.551
Apr-10	83.01	2.225	2.15	2.182	Sep-17	46.68	1.718	1.582	1.759
May-10	80.24	2.277	2.182	2.198	Oct-17	50.23	1.764	1.689	1.645
Jun-10	72.91	2.071	1.983	2.016	Nov-17	54.59	1.875	1.781	1.706
Jul-10	74.96	2.032	1.976	1.988	Dec-17	57.81	1.927	1.839	1.782
Aug-10	81.79	2.229	2.13	2.178	Jan-18	61.36	2.07	2.006	1.923
Sep-10	74.02	2.113	2.012	2.05	Feb-18	65.32	2.074	2.003	1.973
Oct-10	78.41	2.258	2.178	2.194	Mar-18	62.07	1.917	1.843	1.861
Nov-10	84.93	2.366	2.316	2.312	Apr-18	63.07	1.98	1.935	1.919
Dec-10	86.75	2.448	2.382	2.382	May-18	68.38	2.147	2.104	2.076
Jan-11	89.54	2.54	2.509	2.51	Jun-18	66.96	2.211	2.169	2.138
Feb-11	89.52	2.782	2.739	2.773	Jul-18	73.73	2.17	2.137	2.112
Mar-11	101.05	3.112	3.024	3.126	Aug-18	69.46	2.131	2.104	2.104
Apr-11	105.08	3.149	3.062	3.146	Sep-18	68.51	2.231	2.202	2.183
May-11	105.84	3.132	3.05	3.175	Oct-18	75.13	2.406	2.384	2.32
Jun-11	100.92	3.108	3.035	3.122	Nov-18	65.06	2.231	2.215	2.144
Jul-11	93.7	2.959	2.863	2.935	Dec-18	52.63	1.885	1.87	1.785

Table A6. Cont.

Items	Crude oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB	Items	Crude Oil, WTI	Ultra-Low Sulfur No 2 Diesel Spot Price	No. 2 Heating Oil Spot Price FOB	Kerosene-Type Jet Fuel Spot Price FOB
Aug-11	90.85	3.077	3.001	3.063	Jan-19	47	1.709	1.688	1.621
Sep-11	88.07	3.097	3.035	3.082	Feb-19	53.63	1.879	1.851	1.842
Oct-11	79.43	2.87	2.794	2.857	Mar-19	56.12	2.003	1.97	1.934
Nov-11	93.24	3.073	3.029	3.027	Apr-19	62.36	2.014	1.99	1.936
Dec-11	99.91	3.029	2.987	2.974	May-19	62.9	2.072	2.046	1.984
Jan-12	102.39	3.105	3.067	3.075	Jun-19	52.97	1.803	1.777	1.772
Feb-12	97.8	3.083	3.071	3.089	Jul-19	57.32	1.91	1.873	1.883
Mar-12	107.52	3.281	3.238	3.245	Aug-19	56.55	1.909	1.878	1.905
Apr-12	103.52	3.303	3.198	3.269	Sep-19	55.73	1.857	1.839	1.815
May-12	103.47	3.199	3.114	3.188	Oct-19	53.12	1.894	1.873	1.839
Jun-12	87.06	2.793	2.723	2.772	Nov-19	55.17	1.933	1.925	1.847
Jul-12	85.74	2.85	2.725	2.798	Dec-19	57.64	1.919	1.896	1.821
Aug-12	89.1	2.967	2.86	2.955	Jan-20	61.74	2.03	2.013	1.951
Sep-12	95.68	3.262	3.13	3.227	Feb-20	50.36	1.622	1.594	1.499
Oct-12	90.81	3.245	3.141	3.144	Mar-20	45.57	1.512	1.481	1.392
Nov-12	85.87	3.154	3.049	2.951	Apr-20	21.69	1.025	0.989	0.717
Dec-12	87	3.045	2.979	2.928	May-20	15.71	0.692	0.654	0.461
Jan-13	92.77	3.07	3.037	2.99	Jun-20	37.32	1.026	0.995	0.901
Feb-13	97.33	3.168	3.129	3.179	Jul-20	39.85	1.194	1.141	1.023
Mar-13	92.19	3.119	3.017	3.087	Aug-20	41.57	1.242	1.193	1.111
Apr-13	95.07	3.039	2.929	2.962	Sep-20	41.84	1.166	1.124	1.074
May-13	93.4	2.859	2.668	2.686	Oct-20	38.99	1.124	1.071	0.995
Jun-13	94.25	2.869	2.731	2.755	Nov-20	37.71	1.144	1.073	1.05
Jul-13	100.65	2.934	2.809	2.812	Dec-20	45.37	1.368	1.302	1.26
Aug-13	105.54	3.05	2.898	2.944	Jan-21	47.98	1.478	1.397	1.348
Sep-13	108.77	3.14	3.04	3.036	Feb-21	55.39	1.69	1.567	1.498
Oct-13	103.14	2.987	2.911	2.857					

Note: Data time range is from June 2006 to February 2021. Source: EIA; 2. the third weekly price of Ultra-Low Sulfur No 2 Diesel Spot Price on June 2006 treat as the first weekly price due to no data.

References

1. Taghizadeh-Hesary, F.; Rasoulinezhad, E.; Yoshino, N. Energy and Food Security: Linkages through Price Volatility. *Energy Policy* **2019**, *128*, 796–806. [\[CrossRef\]](#)
2. Zavaleta, A.; Walls, W.; Rusco, F.W. Refining for export and the convergence of petroleum product prices. *Energy Econ.* **2015**, *47*, 206–214. [\[CrossRef\]](#)
3. Dimitriadou, A.; Gogas, P.; Papadimitriou, T.; Plakandaras, V. Oil Market Efficiency under a Machine Learning Perspective. *Forecasting* **2018**, *1*, 11. [\[CrossRef\]](#)
4. Yin, X.; Peng, J.; Tang, T. Improving the Forecasting Accuracy of Crude Oil Prices. *Sustainability* **2018**, *10*, 454. [\[CrossRef\]](#)
5. Bostanchi, H. *WTI Oil Price Prediction Modeling and Forecasting*; Project Report; Faculty of California State Polytechnic University: Pomona, CA, USA, 2017.
6. Box, G.E.P.; Jenkins, G.M. *Time Series Analysis: Forecasting and Control*; Holden Day: San Francisco, CA, USA, 1976.
7. Plourde, A.; Watkins, G. How volatile are crude oil prices? *OPEC Rev.* **1994**, *18*, 431–444. [\[CrossRef\]](#)
8. Bopp, A.E.; Sitzler, S. Are petroleum futures prices good predictors of cash value? *J. Futures Mark.* **1987**, *7*, 705–719. [\[CrossRef\]](#)
9. Asche, F.; Gjølberg, O.; Völker, T. Price relationships in the petroleum market: An analysis of crude oil and refined product prices. *Energy Econ.* **2003**, *25*, 289–301. [\[CrossRef\]](#)
10. Morana, C. A semiparametric approach to short-term oil price forecasting. *Energy Econ.* **2001**, *23*, 325–338. [\[CrossRef\]](#)
11. Lanza, A.; Manera, M.; Giovannini, M. Modeling and forecasting cointegrated relationships among heavy oil and product prices. *Energy Econ.* **2005**, *27*, 831–848. [\[CrossRef\]](#)
12. Akpanta, A.C.; Okorie, I.E. Application of box-Jenkins techniques in modelling and forecasting Nigeria crude oil prices. *Int. J. Stat. Appl.* **2014**, *4*, 283–291. [\[CrossRef\]](#)
13. Moshiri, S.; Foroutan, F. Forecasting nonlinear crude oil future prices. *Energy J.* **2006**, *27*, 81–96. [\[CrossRef\]](#)
14. Chang, C.-P.; Lee, C.-C. Do oil spot and futures prices move together? *Energy Econ.* **2015**, *50*, 379–390. [\[CrossRef\]](#)
15. Arfaoui, M. On the spot-futures relationship in crude-refined petroleum prices: New evidence from an ARDL bounds testing approach. *J. Commod. Mark.* **2018**, *11*, 48–58. [\[CrossRef\]](#)
16. Zhao, L.-T.; Wang, S.-G.; Zhang, Z.-G. Oil Price Forecasting Using a Time-Varying Approach. *Energies* **2020**, *13*, 1403. [\[CrossRef\]](#)
17. Mirmirani, S.; Li, H.C. A Comparison of Var And Neural Networks With Genetic Algorithm In Forecasting Price Of Oil. *Econom. Netw.* **2004**, *19*, 203–223. [\[CrossRef\]](#)
18. Alvarez-Ramirez, J.; Alvarez, J.; Rodriguez, E. Short-term predictability of crude oil markets: A detrended fluctuation analysis approach. *Energy Econ.* **2008**, *30*, 2645–2656. [\[CrossRef\]](#)
19. Yu, L.; Wang, S.; Lai, K.K. Forecasting crude oil price with an EMD-based neural network ensemble learning paradigm. *Energy Econ.* **2008**, *30*, 2623–2635. [\[CrossRef\]](#)
20. Slim, C. Improved Crude Oil Price Forecasting With Statistical Learning Methods. *J. Mod. Account. Audit.* **2015**, *11*, 51–62. [\[CrossRef\]](#)
21. He, K.; Zha, R.; Wu, J.; Lai, K.K. Multivariate EMD-Based Modeling and Forecasting of Crude Oil Price. *Sustainability* **2016**, *8*, 387. [\[CrossRef\]](#)
22. Gupta, V.; Pandey, A. Crude oil price prediction using LSTM networks. *Int. J. Comput. Inf. Eng.* **2018**, *12*, 226–230. [\[CrossRef\]](#)
23. Cen, Z.; Wang, J. Crude oil price prediction model with long short term memory deep learning based on prior knowledge data transfer. *Energy* **2019**, *169*, 160–171. [\[CrossRef\]](#)
24. Li, J.; Zhu, S.; Wu, Q. Monthly crude oil spot price forecasting using variational mode decomposition. *Energy Econ.* **2019**, *83*, 240–253. [\[CrossRef\]](#)
25. Yin, T.; Wang, Y. Predicting the Price of WTI Crude Oil Using ANN and Chaos. *Sustainability* **2019**, *11*, 5980. [\[CrossRef\]](#)
26. Kevin, C.D.; Erik, D.; Joseph, L.P. Using the crude oil and heating oil markets for diesel fuel purchasing decisions. *Am. J. Agric. Econ.* **2003**, *21*, 213–229. Available online: <https://www.researchgate.net/publication/23942728> (accessed on 26 March 2021).
27. Guhathakurta, K.; Dash, S.R.; Maitra, D. Period specific volatility spillover based connectedness between oil and other commodity prices and their portfolio implications. *Energy Econ.* **2020**, *85*, 104566. [\[CrossRef\]](#)
28. Roman, M.; Górecka, A.; Domagała, J. The Linkages between Crude Oil and Food Prices. *Energies* **2020**, *13*, 6545. [\[CrossRef\]](#)
29. Vo, D.H.; Vu, T.N.; Vo, A.T.; McAleer, M. Modeling the Relationship between Crude Oil and Agricultural Commodity Prices. *Energies* **2019**, *12*, 1344. [\[CrossRef\]](#)
30. Rodgers, J.L.; Nicewander, W.A. Thirteen Ways to Look at the Correlation Coefficient. *Am. Stat.* **1988**, *42*, 59–66. [\[CrossRef\]](#)
31. Krehbiel, T.C. Correlation Coefficient Rule of Thumb. *Decis. Sci. J. Innov. Educ.* **2004**, *2*, 97–100. [\[CrossRef\]](#)
32. Asuero, A.G.; Sayago, A.; González, A.G. The Correlation Coefficient: An Overview. *Crit. Rev. Anal. Chem.* **2006**, *36*, 41–59. [\[CrossRef\]](#)
33. Stanton, J.M. Galton, Pearson, and the Peas: A Brief History of Linear Regression for Statistics Instructors. *J. Stat. Educ.* **2001**, *9*. [\[CrossRef\]](#)
34. Mundi. Commodity Prices. Available online: <https://www.indexmundi.com/commodities/> (accessed on 1 January 2020).
35. World Bank. World Bank Monthly Commodity Feed Prices. 2018. Available online: <http://www.worldbank.org/en/research/commodity-markets> (accessed on 9 April 2021).

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36. IMF. Primary Commodity Prices Web. Available online: <https://www.imf.org/en/Research/commodity-prices> (accessed on 9 April 2021).
 37. EIA. Price Web in U.S. Available online: <https://www.eia.gov/petroleum/data.php#prices> (accessed on 9 April 2021).
 38. Gao, S.; Lei, Y. A new approach for crude oil price prediction based on stream learning. *Geosci. Front.* **2017**, *8*, 183–187. [[CrossRef](#)]