


Article

Relations of Imperfect Repairs to Critical Infrastructure Maintenance Costs

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Abstract: This article presents an analysis of maintenance costs and maintenance policy of critical infrastructure systems, considering the case of imperfect repair. The maintenance costs of the systems, considering a multistate approach and imperfect repairs, are analyzed with reference to the renewal stream and renewal process theory. Availability characteristics of repairable critical infrastructure systems with non-negligible renovation time, necessary for the cost analysis, are determined based on classical renewal theory. We assume that the system is renewed after exceeding its critical reliability state. The multistate approach proposed to availability analysis, including imperfect repairs, allows considering various configurations of the system's renewals. The procedure for determining the total cost of maintenance up to certain point in time is proposed. Furthermore, the procedure comparing the costs of repair and operation of the system until a certain moment for different configurations of the number of repairs to various reliability states is put forward and its exemplary application is presented. Conducting perfect renewals of infrastructure to the state of entire availability seems to be the most profitable in the case study analyzed in the article. The exception is when imperfect repairs are much cheaper than perfect repairs (they are up to half of the perfect repair cost), or when costs of an infrastructure's renewals are considered in the short period of its exploitation (up to one year). The summary provides a conclusion on maintenance policy based on the proposed procedure analysis and a comparison of maintenance costs for various configurations of perfect and imperfect repairs.

Keywords: maintenance costs; optimal maintenance policy; critical infrastructure; availability; imperfect repair; multistate failure process



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1. Introduction

One of the key issues to ensure maintaining the appropriate exploitation and reliability parameters of critical infrastructure systems is conducting proper repairs and maintenance policy, to guarantee their uninterrupted functioning and adequate availability level with respective exploitation cost minimization. Critical Infrastructure (CI) systems are found as those with the special need to apply the above analysis. CI systems and systems existing within them, interconnected and interdependent objects, installations, appliances, and services, are essential for safety of state and its citizens and for efficient functionality of public institutions, administration, and business [1]. Thus, ensuring their uninterrupted functionality, associated with appropriate reliability and availability level, is one of the most important aspects that works on systems are based on [2–4]. Examples of systems belonging to CI are communication, food and water supply, IT networks, finances, transportation, and health protection [5].

To assure an appropriate level of CI system reliability and availability, there is a need to set up an adequate strategy for conducting the system's renewals and its maintenance [6,7]. The undertaken strategy strictly interacts with maintenance costs of the system [8]. Fang et al. [6], when planning maintenance and repair strategy for system

resilience and repair time optimization, also paid attention to prioritizing of restoration activities depending on components' importance. Stenstrom et al. [8] proposed finding the balance between preventive and corrective maintenance to minimize costs and presented results for a case study of rail infrastructure. Thus, optimization of costs, related to ensuring the functional continuity of systems, meaning adequate policy of planning of renewals, seems to be one of the most important challenges appearing nowadays in researches conducted on the reliability and availability of these systems [9,10].

The purpose of this paper is to present an analysis of the CI system exploitation costs, taking into account various types of repair—resulting with the system transitioning to a certain reliability state—and also to propose procedures of determining the optimal configuration of repairs by defining relations among numbers and types of repairs, and the cumulative cost of system maintenance within certain time frames.

2. Previous Researches and Approach Proposed

Cost analysis and optimization are vital issues in the reliability and availability analysis of technical systems [11–13]. However, not only should costs of setting up the system itself and its initial operation be considered, but analysis of the operational costs, expenses of repairing of the system over a longer usage period, and optimization of total life cycle costs are also of great importance [14]. For the purpose of analyzing the costs related to maintenance of repairable systems, it is necessary to think through a number of additional expenses associated with their usage and renewal, such as system suspension costs or failing to fulfil the order. The importance of the additional costs mentioned was highlighted by Lisnianski et al. in [15], who defined them as reliability associated costs and presented basic life-cycle cost concepts. According to the authors, basic cost types include design and development, production, operation, support, and disposal costs. By taking into account life-cycle cost as a function of the system reliability, an optimal level of system reliability can be ascertained. The authors in [15] also outlined a comparative analysis, basing on criteria of a cost type, such as financial losses and investment expenses.

Optimization, considering all costs appearing throughout the lifetime of the system, is also introduced in [11,16,17]. Creation of the optimal maintenance plan, reflecting minimization of total costs and maximization of system performance, makes accurate modeling of system reliability and availability one of the essential factors. Barone and Frangopol [18] underline that the model should also reflect loads impacting on the system throughout its life cycle. The loads, varying in time, were analyzed by Wen and Kang [19] for the purpose of minimizing the total life-cycle cost. Barone and Frangopol [18] exercise biobjective optimizations based on costs of total maintenance and annual indicators of performance (annual index of reliability, annual risk) or lifetime distribution (hazard and availability functions).

The topic of maintenance of complex systems and assessment of their management were widely discussed in [20,21]. Marquez [21] investigated, inter alia, the impact of failures and reliability of the system on life-cycle cost. De Almeida et al. [20] introduced multicriteria models for effective planning of maintenance. They also highlighted maintenance priorities importance during planning and, moreover, underlined the priorities of setting and choosing a strategy of maintenance as the key point prior to creating a maintenance plan. Optimization of production tasks and maintenance planning is the main goal of the study presented in [22]. Paprocka et al. [22] proposed a method allowing planning of technical inspections of machines and predictive maintenance schedules that optimize production and maximize stability and robustness.

Maintenance strategy, according to Stenstrom et al. [8], depends on the reliability characteristics of a system; its components; and other factors like direct and indirect costs, including cost of downtime and redundancy. Issues connected to the uncertainty of obtaining system reliability characteristics and estimating time to failure, which are used for planning maintenance policy, can be minimized by implementing condition-based maintenance. The condition-based maintenance advantages, especially when the cost of

inspection is relatively low, are emphasized by Kim et al. [7] as unexpected failures in the time-based maintenance approach, which can result in a significant increase in repair costs. Uncertainties of model parameters in optimization of maintenance policy were also pointed out by Wu et al. [23], who presented the concept of risk aggregation by using the conditional value-at-risk theory and integrating the uncertainty of model parameters for optimizing a maintenance policy.

Daneshkhah et al. [24] presented a sensitivity analysis on the maintenance optimization problem, taking into account fluctuations and uncertainties of the model's parameters and assuming continuous degradation of infrastructure and its assets. They propose Partial Expected Value of Perfect Information concept, to find the optimal maintenance strategy that combines age-based and condition-based maintenance policies. Maintenance cost and maintenance time reduction have been a subject of researches involving Big Data technologies proposed by Petrillo et al. [25]. Petrillo et al. proposed a cloud-based infrastructure, based on Big Data technologies, for prognostic analysis of automotive components. They estimated the End-of-Life and the Remaining Useful Life of the components by analysing these parameters' trends, with the use of Big Data technologies.

De Santo et al., 2020 [26] proposed a Long Short-Term Memory model for HDD health assessment, combined with a technique to address the unbalanced nature of data, showing more flexibility to the complex nature of the data compared to alternative models and approaches. As they indicate, hard drives often deteriorate gradually, which is why four health degree levels are defined, associated with the Time-to-failure feature determined for them. Similarly, this article pays attention to suitability of the multistate approach for precise estimation of system reliability, and the description of a system and its components' degradation, by defining the times when the system stays in certain reliability states subsets, corresponding to the Time-to-failure feature determined for different health degree levels in [26].

The approach proposed in this paper is based on the time-based approach, with use of reliability characteristics determined for a system. To reflect the infrastructure degradation process more precisely and more realistically, the multistate approach to its reliability analysis has been adopted, highlighting different reliability states of infrastructure and its components, linked with their aging process. The reliability parameters of particular assets are determined separately for each of the states, and coordinates of the reliability function of infrastructure and its components correspond to them. Moreover, to determine the infrastructure's reliability characteristics and its time-to-failure (in the multistate approach—more generally—time of departure from particular reliability states) more precisely, dependence of its components has been included. The dependence is expressed by the influence of degradation of one of the infrastructure components, and on degradation of its other components and whole infrastructure. In this paper, a mathematical model of the multistate approach to infrastructure reliability estimation is proposed, taking into account dependences of degradation of its components to obtain the optimal repair strategy, including possibilities of perfect and imperfect repairs; to the best of authors' knowledge, is the first reported attempt.

With use of the multistate approach to reliability and the availability analysis of renewable systems, the procedure allowing comparison of infrastructure exploitation and repair costs within a certain time period, taking into account possibilities of perfect and imperfect repairs initiated at different levels of infrastructure reliability states, has been proposed. The main goal of the paper and the procedure proposed is to try to determine, in general, which repair strategy of the infrastructure is the most optimal and beneficial with respect to its repair and exploitation costs, rather than determination of its precise repair cost within a certain time frame.

3. System Maintenance Policy Reflecting Imperfect Repairs

We analyze maintenance costs for a renewable system with non-negligible renewal durations. We consider a multistate renewable system [27,28] with availability states degrading from state z of system entire availability, where $z \geq 1$, through intermediate states of limited or reduced availability, to state 0 of absolute inability. We assume that reliability state $r, r \in \{1, 2, \dots, z\}$ is a state, after which exceeding the system is renewed. In a particular case, r can be state z , i.e., “as good as new”. In that case, a system is renewed after its transition to state $(z - 1)$ of limited availability (in case of a two-state system—after transition to state 0), and all its renewals are perfect to state z of entire availability. In the case when $r < z$, we also consider imperfect repairs to states of limited or reduced system availability, e.g., $z - 1$. In general, a system can be renewed to any availability state $z - x$, where $x = 0, 1, \dots, z - r$.

A random variable $T(r), r \in \{1, 2, \dots, z\}$ represents the time of system staying in availability states from the set $\{r, r + 1, \dots, z\}$, assuming the system was in state z of entire availability at the initial moment $t = 0$ (i.e., the system lifetime). Further, including the system's renewals, $T^{(1)}(r)$ is the time from the beginning of the system operation until the moment of first exceeding the availability state r that initiates the first renewal procedure for the system. Similarly, $T^{(N)}(r), N = 2, 3, \dots$, denotes the time from the moment of the $(N - 1)$ th renewal until the moment of the N th exceeding of availability state r , initiating the procedure of the N th system's renewal. We assume that random variables $T^{(1)}(r), T^{(2)}(r), \dots, r \in \{1, 2, \dots, z\}$ are independent.

The moments of successive renewal of a system define a sequence of random variables [29]:

$$S^{(N)}(r) = T^{(1)}(r) + T^{(2)}(r) + \dots + T^{(N)}(r), r \in \{1, 2, \dots, z\}, N = 1, 2, \dots, \quad (1)$$

representing time up to the N th exceeding availability state r by a system. From the classical renewal theory [29,30], it follows that $S^{(N)}(r)$ for sufficiently large N has an approximately normal distribution with the expected value given by

$$E[S^{(N)}(r)] \cong N \cdot \mu(r), r \in \{1, 2, \dots, z\}. \quad (2)$$

With these notations, a renewal stream is defined by a sequence of random variables $S^{(1)}(r), S^{(2)}(r), \dots, r \in \{1, 2, \dots, z\}$. A renewal process is defined as the random process $\{N(t, r), t \geq 0\}$, of the number $N(t, r)$ of renewals, after exceeding availability state r by a system, within the time interval $[0; t]$.

We assume that first $N_R(z)$ system's renewals include operations that make the system to return to state z of entire availability and times between successive renewals are independent and identically distributed. Subsequently, random variables $T^{(1)}(r), T^{(2)}(r), \dots, T^{(N_R(z)+1)}(r)$ have identical distribution with expected value $\mu(r)$ and standard deviation $\sigma(r)$, the same as for a new system. The successive times of system's renewals to state z of entire availability after exceeding availability state r are independent and identically distributed with expected value $\mu_R(z)$ and standard deviation $\sigma_R(z)$. The variable $S^{(N_R(z)+1)}(r)$, defined in (1), represents the time up to the moment of $(N_R(z) + 1)$ th exceeding availability state r by a system, and its expected value is estimated from the following formula [27]:

$$E[S^{(N_R(z)+1)}(r)] \cong (N_R(z) + 1) \cdot \mu(r), r \in \{1, 2, \dots, z\}. \quad (3)$$

Next, $N_R(z - 1)$ renewals designate that the system returns to an operational condition corresponding to availability state $(z - 1)$, and the expected value of the time of such recovery after exceeding availability state r is equal $\mu_R(z - 1)$. Further, we assume that $N_R(z - x)$ system's renewals mean repairs to availability state $(z - x)$, where $x = 0, 1, \dots, z - r$ and $r \in \{1, 2, \dots, z\}$. The expected value of time of system's recovery to state $(z - x)$ after exceeding availability state r is $\mu_R(z - x)$, $x = 0, 1, \dots, z - r$. Finally, $N_R(r)$ renewals

are performed as a system renewal to availability state r with an expected recovery time value $\mu_R(r)$.

The expected time $S^{(\omega)}(r)$ up to the moment of ω th exceeding availability state r is estimated from the following formula:

$$E[S^{(\omega)}(r)] \cong (N_R(z) + 1) \cdot \mu(r) + N_R(z) \cdot \mu_R(z) + \sum_{j=1}^{x-1} N_R(z-j) \cdot (\mu(r) - \mu(z-j+1) + \mu_R(z-j)) + \eta \cdot (\mu(r) - \mu(z-x+1) + \mu_R(z-x)), \quad (4)$$

where $\eta = \omega - (N_R(z) + \dots + N_R(z-x+1)) - 1$, i.e., $\eta = 1, \dots, N_R(z-x)$ for $\omega = N_R(z) + \dots + N_R(z-x+1) + 2, \dots, N_R(z) + \dots + N_R(z-x+1) + N_R(z-x) + 1$, and $x = 1, 2, \dots, z-r, r \in \{1, 2, \dots, z\}$.

In Formula (4), $\mu_R(u)$, $u = z, z-1, \dots, r$, is the expected recovery time needed to repair the system after exceeding state r , to an operational condition corresponding to availability state u , and $\mu(u)$ is the mean lifetime of the system in subset of states $\{u, u+1, \dots, z\}$, $u = z, z-1, \dots, r$.

4. Optimization of System Maintenance Costs Assessing Imperfect Repairs

The purpose of the analysis presented in this section is to estimate repair and maintenance costs within a fixed time interval and to propose the procedure that allows finding an optimal configuration of system's repairs, thus minimizing these costs. In the cost analysis, we take into account both direct costs of repair and associated costs [27,31].

The adopted notations related to cost and time of system's repairs, used in the further part of the article, are as follows:

- $C_R(z, r)$ —repair costs, including costs of materials and service, related to the repair restoring a system to state z of entire availability after departing an availability state r ;
- $T_R(z, r)$ —repair time related to operations restoring a system to state z of entire availability after departing an availability state r ;
- $C_R(u, r)$ —repair costs, including costs of materials and service, related to the repair restoring a system to its availability state u , $u = z-1, z-2, \dots, r$, after departing an availability state r ;
- $T_R(u, r)$ —repair time related to operations restoring a system to its availability state u , $u = z-1, z-2, \dots, r$, after departing an availability state r ;
- C_{Dt} —downtime costs per unit of time (hour, day, etc.);
- $C_{order_Ex}(n_{order}, T_R(u, r))$ —costs of disrupting the fulfillment of orders and their delays, depending on the average number of orders performed at a given moment and the system repair time to its availability state u , $u = z, z-1, z-2, \dots, r$.

Repair costs fulfill the following condition:

$$C_R(r, r) \leq \dots \leq C_R(z-2, r) \leq C_R(z-1, r) \leq C_R(z, r), \quad (5)$$

for each $r, r \in \{1, 2, \dots, z\}$.

Costs of restoring a system to particular availability states depend on these states. Condition (5) means that cost of restoring to state r is the lowest, cost of restoring to intermediate availability are higher, and repair costs to state z of entire availability is the highest.

A similar condition applies to repair times:

$$T_R(r, r) \leq \dots \leq T_R(z-2, r) \leq T_R(z-1, r) \leq T_R(z, r), r \in \{1, 2, \dots, z\}. \quad (6)$$

Costs of restoring a system to state z of entire availability are evaluated with the following formula:

$$C_T(z, r) = C_R(z, r) + C_{Dt} \cdot T_R(z, r) + C_{order_Ex}(n_{order}, T_R(z, r)). \quad (7)$$

Similarly, costs of restoring a system to any availability state u , where $u = z, z - 1, \dots, r$, are

$$C_T(u, r) = C_R(u, r) + C_{Dt} \cdot T_R(u, r) + C_{order_Ex}(n_{order}, T_R(u, r)), u = z, z - 1, \dots, r. \quad (8)$$

The total cost of the system's repair and maintenance [27], assuming conducting of perfect repairs, up to the time point t , $0 \leq t \leq S^{(N_R(z)+1)}(r)$, for fixed availability state r , $r \in \{1, 2, \dots, z\}$, is

$$C(t, r) \cong C_T(z, r) \cdot \left\lfloor \frac{t + \mu_R(z)}{\mu(r) + \mu_R(z)} \right\rfloor, \quad (9)$$

where $\lfloor t \rfloor$ denotes the integer part of t , costs of restoring a system to state z of entire availability $C_T(z, r)$ are given by (7), and the expected time $S^{(N_R(z)+1)}(r)$ until the moment of $(N_R(z) + 1)$ th departure from an availability state r is estimated from Formula (2).

Consequently, the complete cost of the system's repair and maintenance up to the point of time t ,

$$S^{(N_R(z)+\dots+N_R(z-x+1)+1)}(r) < t \leq S^{(N_R(z)+\dots+N_R(z-x+1)+N_R(z-x)+1)}(r), x = 1, 2, \dots, z - r, \quad (10)$$

for fixed availability state r , $r \in \{1, 2, \dots, z\}$, assuming that at the initial moment, system is in state z of entire availability, is given by the following formula:

$$C(t, r) \cong C_T(z, r) \cdot N_R(z) + \sum_{j=1}^{x-1} C_T(z-j, r) \cdot N_R(z-j) + C_T(z-x, r) \cdot \left\lfloor \frac{t - (N_R(z)+1) \cdot \mu(r) - N_R(z) \cdot \mu_R(z) - \sum_{j=1}^{x-1} N_R(z-j) \cdot (\mu(r) - \mu(z-j+1) + \mu_R(z-j))}{\mu(r) - \mu(z-x+1) + \mu_R(z-x)} \right\rfloor + 1, \quad (11)$$

where costs $C_T(u, r)$ for $u = z, z - 1, \dots, r$ are given by (8), and the expected value of time until the successive system's renewal is estimated from Formula (3).

Taking into account the above results, we propose a procedure comparing the costs of system's renewals and maintenance up to the fixed point of time t , $t \geq 0$, assuming that at the initial moment, a system is in a state z of entire availability. Its aim is to find the most beneficial configurations of the numbers of repairs $N_R(z)$, $N_R(z - 1)$, \dots , $N_R(r)$, to various availability states $z, z - 1, \dots, r$, thus minimizing these costs [27]. The procedure proposed below, compared to the one presented in [27], is extended by the possibility of taking into account the system's repairs after exceeding its any availability state. In addition, in this paper, we compare the costs of the system's renewals and maintenance in a given period, taking into account the possibility of repairs from any state of limited/reduced availability or absolute inability from $z - 1$ to 0, i.e., for $r = z, \dots, 1$. This way, we can compare whether immediate reaction and frequent restoration to the state of entire availability, in case of even slight reduction of the system's availability level, are more profitable. On the other hand, less frequent and more expensive repairs for some kinds of systems can be more effective. Conceivably, an intermediate solution can turn out to be the optimal one. To decide it, the following procedure can be used.

1. Assume $r = z$ and follow the procedure given in points 2 to 13.
2. Start with all system repairs only to the best state, i.e., state z . Determine the number of such repairs $N_R(z)$ and calculate the costs of such a system's renewal until time t . Record the cost of the system's renewal $C(t, r)$.
3. If $N_R(z) < 1$, then no system repair until time t was needed and costs of the system's renewal are equal to 0.
4. If $r < z$ and $N_R(z) > 0$, then set $x = 0$ and reduce the number of repairs $N_R(z - x)$ (i.e., for $x = 0$, $N_R(z)$ repairs to state z) by one and perform the last repair (repairs) to state $z - x - 1$ (i.e., for $x = 0$, to state $z - 1$). Determine the number of repairs $N_R(z - x - 1)$ (i.e., for $x = 0$, the number $N_R(z - 1)$ of repairs to state $z - 1$) and calculate the costs of all system's renewal until time t .
5. If the cost of repairs from point 4 is lower than the cost of system's renewal from point 2, record it and remember the number of repairs to particular states.
6. Increase the index number x by one ($x = x + 1$).
7. If $r < z - x$ and $N_R(z - x) > 0$, then repeat the procedure from step 4 for the new index x —that is, reduce the number of repairs $N_R(z - x)$ to state $z - x$ by one and perform the last repair (repairs) to state $z - x - 1$. Determine the number of repairs $N_R(z - x - 1)$ to state $z - x - 1$, and calculate the costs of all system's renewal until time t .
8. If the repair cost from point 7 is lower than the previously saved cost of system's renewal, update and record this cost. Record the number of repairs to particular states ($N_R(z), N_R(z - 1), \dots, N_R(r)$).
9. Increase the index number x by one ($x = x + 1$).
10. Repeat the procedure given in points 7–9 when the following conditions are met: $x < z - r$ and $N_R(z - x) > 0$. If $x > z - r$, it means that the last repair took place to state r .
11. If $x > z - r - 1$, then decrease index x one by one ($x = x - 1$) until one of the conditions, $x = 0$ or $N_R(z - x) > 0$, is met.
12. Repeat the procedure given in points 7–11 until $x = 0$ and $N_R(z) = 0$.
13. Provide the last recorded costs of the system's renewal and the configuration of repair numbers as the optimal result for a system and its availability state, related to the system's renewal, equal to r .
14. If $r > 1$, then decrease number of states related to the system's renewal r by one ($r = r - 1$) and repeat the procedure given in points 2 to 13.
15. If $r = 1$, then compare the results of renewal's costs obtained for $r = z, \dots, 1$, (recorded in point 13 of the procedure) and choose the smallest one as the minimal cost of system maintenance related to optimal configuration of the system's renewal.

A flowchart of the procedure allowing us to compare the costs of system's renewals for various configurations of renewals to different availability states, and to find the optimal configuration to minimize these costs, is presented in Figure 1.

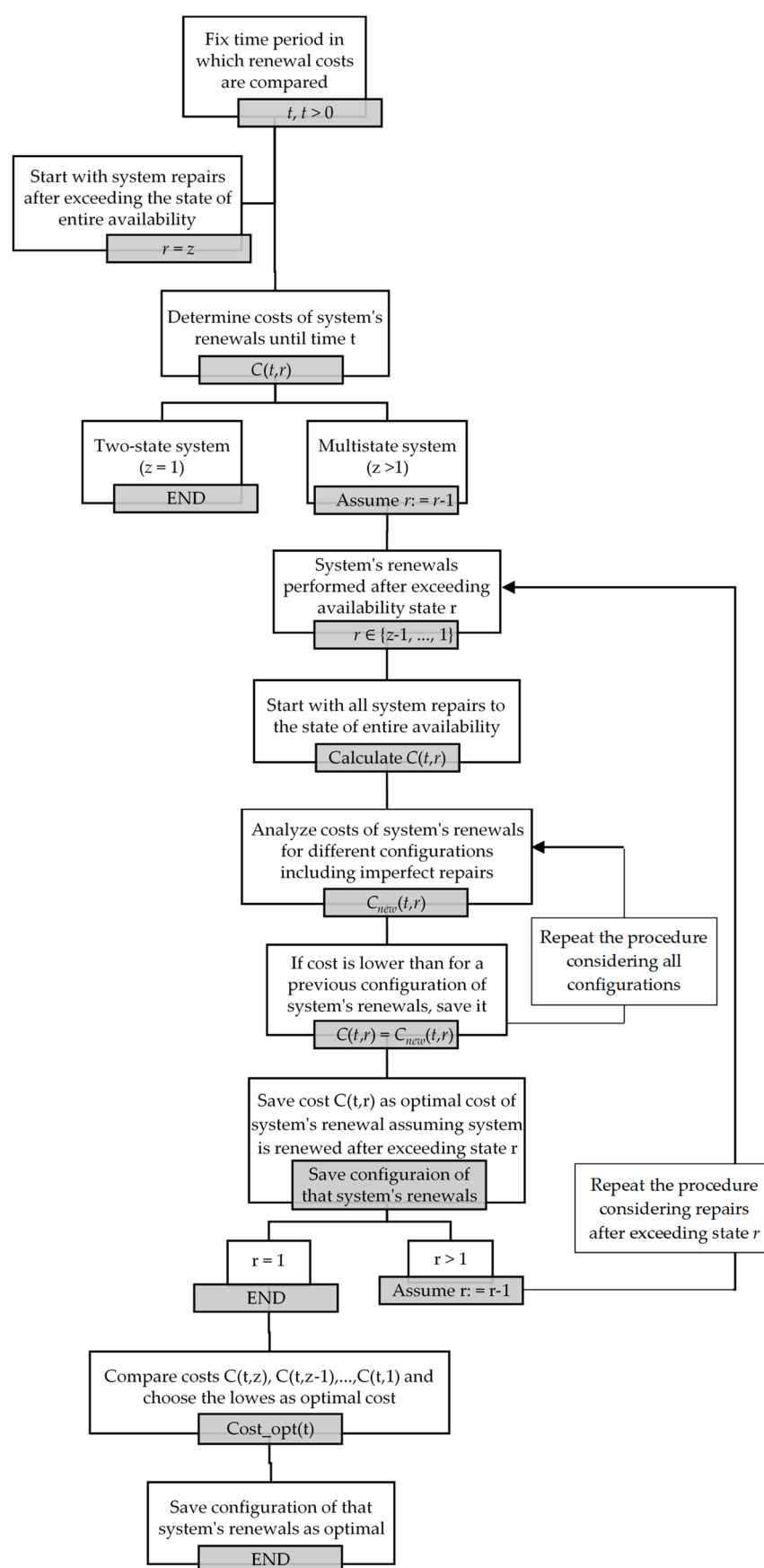


Figure 1. A scheme of the procedure for finding the optimal configuration of system's renewals to different availability states, thus minimizing costs of renewals in a fixed period.

The algorithm in pseudocode, allowing us to find the optimal cost of system's renewal and the most beneficial configuration of number of repairs, is proposed below (Algorithm 1).

Algorithm 1. Finding the optimal cost of system's renewal and the optimal configuration of number of repairs

```

1:  $r := z$ ;
2: do{
3:  $N_R(z) = \text{int}\left(\frac{t + \mu_R(z)}{\mu(r) + \mu_R(z)}\right)$ ; // integer part of it
4: for ( $i = 1$ ;  $i < z - r + 1$ ;  $i++$ )
5:    $N_R(z - i) = 0$ ;
6:  $C_T(z, r) = C_R(z, r) + C_{Dt} \cdot T_R(z, r) + C_{order\_Ex}(n_{order}, T_R(z, r))$ ;
7:  $C(t, r) \cong C_T(z, r) \cdot N_R(z)$ ;
8: for ( $i = 0$ ;  $i < z - r + 1$ ;  $i++$ )
9:    $N[i, r] := N_R(z - i)$ ;
10: if ( $r < z$ ) and ( $N_R(z) > 0$ ) then{
11:  $x := 0$ ;
12: do{
13: do{
14:    $N_R(z - x) := N_R(z - x) - 1$ ;
15:    $\omega = \frac{t - (N_R(z) + 1) \cdot \mu(r) - N_R(z) \cdot \mu_R(z) - \sum_{j=1}^x N_R(z - j) \cdot (\mu(r) - \mu(z - j + 1) + \mu_R(z - j))}{\mu(r) - \mu(z - x) + \mu_R(z - x - 1)}$ ;
16:    $N_R(z - x - 1) := 1 + \text{int}(\omega)$ ; // integer part of  $\omega$ 
17: for ( $j = 1$ ;  $j < x + 2$ ;  $j++$ )
18:    $C_T(z - j, r) = C_R(z - j, r) + C_{Dt} \cdot T_R(z - j, r) + C_{order\_Ex}(n_{order}, T_R(z - j, r))$ ;
19:    $C_{new}(t, r) \cong C_T(z, r) \cdot N_R(z) + \sum_{j=1}^{x+1} C_T(z - j, r) \cdot N_R(z - j)$ ;
20: if ( $C_{new}(t, r) < C(t, r)$ ) then{
21:    $C(t, r) = C_{new}(t, r)$ ;
22: for ( $i = x + 2$ ;  $i < z - r + 1$ ;  $i++$ )
23:    $N_R(z - i) = 0$ ;
24: for ( $i = 0$ ;  $i < z - r + 1$ ;  $i++$ )
25:    $N[i, r] := N_R(z - i)$ ;
26: }
27:  $x := x + 1$ ;
28: } while ( $x < z - r$ ) and ( $N_R(z - x) > 0$ );
29: if ( $x > z - r - 1$ ) then{
30: do{
31:  $x := x - 1$ ;
32: } while ( $N_R(z - x) = 0$ ) and ( $x > 0$ )
33: }
34: } while ( $N_R(z) > 0$ )
35: }
36: else if ( $N_R(z) < 1$ ) then
37:    $C(t, r) = 0$ ;
38: System.out.println("Optimal cost of system's renewal assuming that system is renewed after exceeding availability state  $r$ : " +  $C(t, r)$ );
39: for ( $i = 0$ ;  $i < z - r + 1$ ;  $i++$ )
40: System.out.println("Optimal number of renewals  $N_R(z - i)$  to availability state ( $z - i$ ): " +  $N[i, r]$ );
41:  $r := r - 1$ ;
42: } while ( $r > 0$ )
43: if ( $r = 0$ ) then{
44: //  $\text{Cost\_opt}(t) = \min\{C(t, z), C(t, z - 1), \dots, C(t, 1)\}$ 
45:    $\text{Cost\_opt}(t) = C(t, z)$ ;
46:    $r_{\text{opt}} = z$ ;
47: for ( $i = 1$ ;  $i < z$ ;  $i++$ )

```

Algorithm 1. *Cont.*

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48: {if ( $C(t, z - i) < \text{Cost\_opt}$ ) then{
49:      $\text{Cost\_opt} = C(t, z - i)$ ;
50:      $r\_opt = z - i$ ;
51: }
52: }
53:  $N\_total[r\_opt] = 0$ ;
54: for ( $i = 0; i < z - r\_opt + 1; i++$ )
55:  $N\_total[r\_opt] = N\_total[r\_opt] + N[i, r\_opt]$ ;
56: System.out.println("Optimal cost of system's renewal is: " +  $\text{Cost\_opt}$ );
57: for ( $i = 0; i < z - r\_opt + 1; i++$ )
58: System.out.println("Optimal number of renewals to state ( $z - i$ ): " +  $N[i, r\_opt]$ );
59: System.out.println("Total number of renewals in optimal solution is: " +  $N\_total[r\_opt]$ );
60: }

```

We compare the costs of system's renewals and maintenance and look for the optimal configuration of repairs that minimize them, by carrying out the procedure of determining the number of repairs to various infrastructure availability states for $r = z, z - 1, \dots, 1$. The procedure allows us to take into account the possibility of more frequent, but cheaper and faster repairs, e.g., from state $z - 1$ of limited availability to the state of entire availability and the possibility of less frequent, but more expensive repairs from state 1 of partial availability or state 0 of absolute inability. In the latter case, we additionally take into account the possibility of perfect repairs (i.e., to state z) and imperfect repairs to one of states of limited availability, e.g., $z - 1, z - 2$. The optimal and most beneficial repair configuration is specified by each system's availability characteristics and renewal costs. It depends on the system's lifetimes (times of system's staying in particular availability states), the operating time of the system (time within which we consider its maintenance and renewal costs), and, of course, the costs and duration of repairs to particular availability states. Nevertheless, based on the comparative analysis carried out in the next section for different cases (for different values of these characteristics), we drew some general conclusions regarding the determination of the optimal repair configuration to minimize the costs of a system's maintenance and renewal in a given period.

5. Critical Infrastructure Maintenance Costs Optimization—Exemplary Application

For the purpose of working on an application of procedures and the proposed results, we analyze the maintenance cost of Critical Infrastructure (CI) composed of eleven CI systems $S_i, i = 1, 2, \dots, 11$, [32]. CI systems, defined in Section 1, and the infrastructure they form, are analyzed as multistate systems. The following five reliability states of the CI and its systems are specified:

- State 4 of entire availability—meaning CI and its systems are fully functional, and all of its components work with no disruptions (so called "as good as new");
- State 3 of limited availability—referring to when functionality of the CI/CI systems is slightly disrupted, but entire functionality of the system is still ensured;
- State 2 of reduced availability—representing the situation when disruptions appearing in CI/CI systems reduce their functionality, but the key functions of the system are still at the appropriate level;
- State 1 of partial availability—appearing when disturbances in CI/CI systems cause their key functionality and exploitation parameters to fall below allowed limits;
- State 0 of absolute inability—indicates CI/CI system failure that stops its functionality.

5.1. Reliability and Availability Characteristics of CI and Its Systems

Taking into account the reliability states of CI systems described above, we define reliability functions of systems $S_i, i = 1, 2, \dots, 11$ as vectors:

$$R_i(t, \cdot) = [1, R_i(t, 1), R_i(t, 2), R_i(t, 3), R_i(t, 4)], \quad t \geq 0, \quad i = 1, 2, \dots, 11, \quad (12)$$

where their coordinates $R_i(t, u)$, $u = 1, 2, 3, 4$ express the probability of the system staying in one of the availability states from subset $\{u, u + 1, \dots, 4\}$ at moment t , under the assumption that it was at state 4 of entire availability at moment $t = 0$. We assume that CI systems S_i , $i = 1, 2, \dots, 11$ have exponential reliability functions and their coordinates are given by

$$R_i(t, u) = \exp[-\lambda_i(u)t], \quad u = 1, 2, 3, 4, \quad i = 1, 2, \dots, 11, \quad (13)$$

where $\lambda_i(u)$, $u = 1, 2, 3, 4$ denote the intensities of departure of systems S_i , $i = 1, 2, \dots, 11$ from the subset $\{u, u + 1, \dots, 4\}$.

CI systems are connected in series in a reliability sense, as all of them must be in the availability state and able to work to ensure correct functioning of the entire CI. In order to determine basic reliability and availability characteristics necessary for further analysis, we present the reliability function of the critical infrastructure. Taking into account dependencies among CI systems S_i , $i = 1, 2, \dots, 11$ forming the infrastructure, we assume the local load-sharing dependency model, introduced in [4,27]. Consequently, we assume that perturbations in one of the systems and changing/deterioration of its availability state affect functioning of other ones and the entire infrastructure. This influence on the deterioration of availability states and reliability characteristics of other CI systems, and the influence strength, are expressed by the coefficients $q(v, S_j, S_i)$, $i, j = 1, 2, \dots, 11$, $i \neq j$. The coefficient of influence $q(v, S_j, S_i)$ reflects the effect of deterioration of the availability state of S_j , $j = 1, 2, \dots, 11$ on lifetimes in the subset $\{v, v + 1, \dots, 4\}$, $v = 1, 2, 3$, of systems S_i , $i = 1, 2, \dots, 11$, $i \neq j$. Under the assumption of an exponential distribution of systems S_i , $i = 1, 2, \dots, 11$, (12)–(13), the reliability function of infrastructure, composed of dependent systems, is given by the formula [32]

$$R_{CI}(t, \cdot) = [1, R_{CI}(t, 1), R_{CI}(t, 2), R_{CI}(t, 3), R_{CI}(t, 4)], \quad t \geq 0, \quad (14)$$

where

$$R_{CI}(t, u) = \exp\left[-\sum_{i=1}^{11} \lambda_i(u+1)t\right] + \sum_{j=1}^{11} \frac{\lambda_j(u+1) - \lambda_j(u)}{\sum_{i=1}^{11} \lambda_i(u+1) - \sum_{i=1}^{11} \lambda_i(u)} \cdot [\exp\left[-\sum_{i=1}^{11} \frac{\lambda_i(u)}{1-q(u, S_j, S_i)}t\right] - \exp\left[-\left(\sum_{i=1}^{11} \lambda_i(u+1) - \sum_{i=1}^{11} \lambda_i(u) + \sum_{i=1}^{11} \frac{\lambda_i(u)}{1-q(u, S_j, S_i)}\right)t\right]], \quad t \geq 0, u = 1, 2, 3, \quad (15)$$

$$R_{CI}(t, 4) = \exp\left[-\sum_{i=1}^{11} \lambda_i(4)t\right], \quad t \geq 0. \quad (16)$$

Reliability parameters, such as intensities of departure and coefficients of influence between CI systems, are adopted from sample expert data to illustrate the results.

The mean lifetimes $\mu(u)$, $u = 1, 2, 3, 4$, of infrastructure in state subsets $\{u, u + 1, \dots, 4\}$ are determined from the following formula [27]:

$$\mu(u) = \int_0^{\infty} R_{CI}(t, u) dt, \quad u = 1, 2, 3, 4, \quad (17)$$

where coordinates of the CI reliability function $R_{CI}(t, u)$, $u = 1, 2, 3, 4$ are given by (15)–(16).

By using Formula (17) for data coming from experts in the field, for the purposes of cost analysis, the following mean lifetimes of the CI in state subsets $\{1, 2, 3, 4\}$, $\{2, 3, 4\}$, $\{3, 4\}$, $\{4\}$, expressed in years, have been assumed:

$$\mu(1) = 0.408, \quad \mu(2) = 0.297, \quad \mu(3) = 0.184, \quad \mu(4) = 0.133. \quad (18)$$

5.2. Optimal Exploitation Costs and Configuration of Renewals of CI

First, we assume that the CI availability state, after which exceeding the infrastructure is renewed, is state $r = 2$. That means the renewals are conducted after exceeding state 2 of reduced availability, and the transition of CI to state 1 of partial availability. In such case,

the possibilities of conducting perfect repairs to state 4 of entire availability, and imperfect repairs, resulting in the backward transition of CI to reliability state 3 or 2, are considered while looking for the optimal configuration of repairs. The type of repair is related to repair costs and repair time of the CI. The costs of repairs have been determined as percentage of the perfect repair in the article. It has thus been assumed that repair to state 4 of entire availability takes 14 days and its cost is 100%. The renewal to state 3 of limited availability, lasts 6 days, and the cost is 60% of the perfect repair. The repair to state 2 of reduced availability needs 2 days and the cost amounts to 40% of the cost of perfect repair. Additional downtime costs per day during CI system repair is 1% of perfect renewal. The costs of interruption of processing orders, or their delays, have not been taken into account, and they have been assumed to be zero. The analysis of optimal exploitation costs of CI has been conducted based on the assumptions mentioned for the time period of 1 year, 2 years, and successive periods up to 10 years. The results are shown in Figures 2 and 3. These results and others presented in this section are obtained by applying the procedure introduced in Section 4, from a computer program written based on pseudocode given in Section 4.

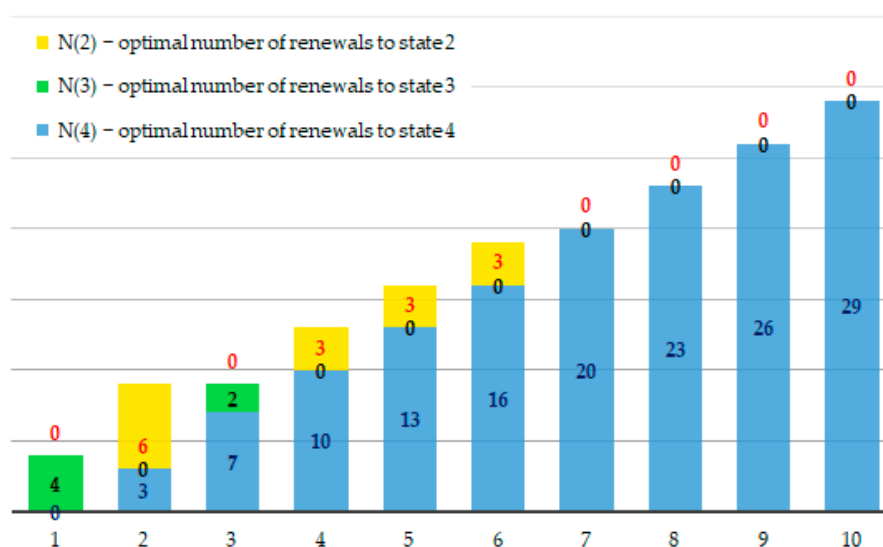


Figure 2. Optimal configuration of infrastructure renewals, minimizing maintenance costs within the period of 1 year, 2 years, and successive periods up to 10 years.

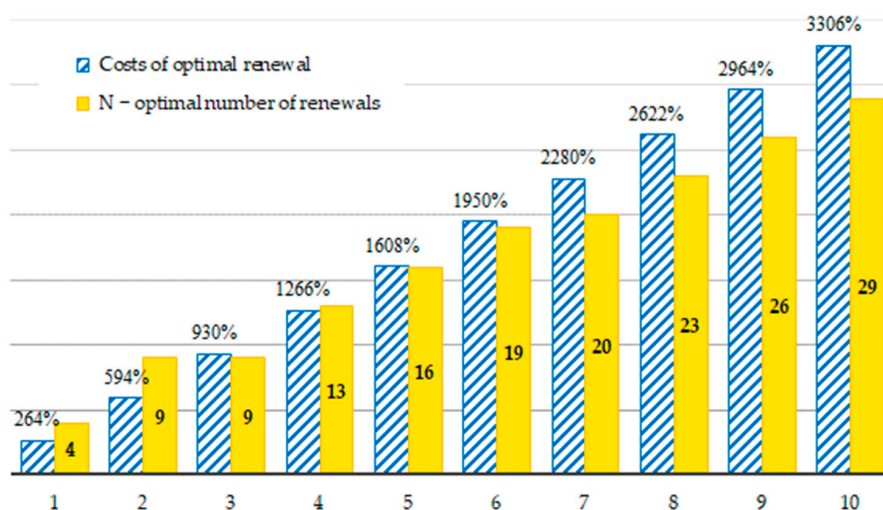


Figure 3. Cumulative number of repairs, and costs of infrastructure maintenance within the period of 1 year, 2 years, and successive periods up to 10 years.

Figure 2 shows numbers of repairs processed after the infrastructure to state 1, resulting with the CI transition to states 2, 3, and 4, respectively, within the assumed period of time of its exploitation. It has been assumed that at the initial moment, the infrastructure is at state 4 of entire availability. A high number of repairs to state 4, in analyzed circumstances, is caused by the reasonably long time of CI staying at this state compared with the time of the infrastructure staying at state 3, coming out of (12). Consequently—renewals to state 3 are unprofitable. The drawing shown in Figure 3 presents the cumulative numbers of repairs within analyzed time periods of the CI exploitation. The costs have been shown in percentages, while 100% reflects to the cost of single renewal to state 4. The results of the analysis performed would change if considering different costs of imperfect repairs. Respective comparison for the CI, for the time period up to 5 years, is shown in Table 1 and Figure 4.

Table 1. Optimal numbers of infrastructure renewals to particular availability states during the period of infrastructure exploitation ranging from 1 to 5 years, assuming various costs of imperfect renewal.

Maintenance Period t		40% and 50%	40% and 60%	40% and 70%	50% and 60%	50% and 70%	50% and 80%	60% and 80%
1 year	N (4) ¹	0	0	2	0	2	2	2
	N (3) ²	4	4	0	4	0	0	0
	N (2) ³	0	0	1	0	1	1	1
2 years	N (4) ¹	0	3	3	3	5	5	5
	N (3) ²	9	0	0	4	0	0	0
	N (2) ³	1	6	6	0	1	1	1
3 years	N (4) ¹	0	7	6	7	7	8	7
	N (3) ²	15	2	0	2	2	0	2
	N (2) ³	0	0	6	0	0	1	0
4 years	N (4) ¹	0	10	10	10	10	10	10
	N (3) ²	20	0	0	2	2	0	2
	N (2) ³	1	3	3	0	0	3	0
5 years	N (4) ¹	1	13	13	13	13	13	13
	N (3) ²	23	0	0	2	2	0	2
	N (2) ³	2	3	3	0	0	3	0

¹ Optimal number of renewals to availability state 4. ² Optimal number of renewals to availability state 3. ³ Optimal number of renewals to availability state 2.

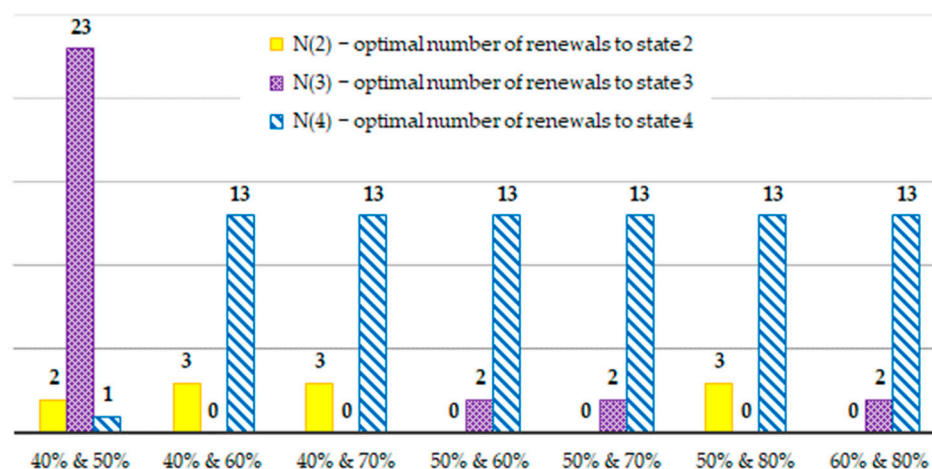


Figure 4. Optimal numbers of infrastructure renewals to particular availability states, during five years of infrastructure exploitation, assuming various costs of imperfect renewals.

Figure 4 shows the optimal numbers of repairs to particular availability states, in the case that the cost of renewal to state 2 is 40% of the perfect repair and the cost of repair to state 3 amounts to 50% of the single perfect repair. The cost of the single repair of infrastructure to state 4 of entire availability is 100%. Other cost level cases, for which a 5-year exploitation optimization procedure has been conducted, are indicated in the drawing.

From the conducted comparison, we conclude that only a significant reduction of cost of imperfect renewals can influence the optimal configuration of number of renewals. The most beneficial are renewals to state 3, in the case when the cost of renewal to state 3 of limited availability is 50% of the perfect repair and the cost of repair to state 2 of significantly reduced availability is 40% of the perfect repair. In other considered cases of various costs of imperfect renewals, the most beneficial are perfect renewals, allowing the infrastructure to return to an operational condition corresponding to the state of entire availability.

5.3. Finding Optimal Availability State Initiating Restoration Actions of CI

The aim of our analysis is to find optimal configuration of repairs to various availability levels of infrastructure, including imperfect and perfect repairs. Additionally, we analyze the possibility of an infrastructure's renewal immediately after leaving state 4 ($r = 4$) of entire availability, the possibility of an infrastructure's renewal after leaving state 3 ($r = 3$) of limited availability, and the possibility of an infrastructure's renewal after leaving state 2 ($r = 2$) of significantly reduced availability. As we include imperfect repairs in our analysis, infrastructure's renewal after leaving state 3 means the possibility to consider renewals to state 3 and to state 4. After the availability state of infrastructure exceeds state 2, we consider renewals to states 2, 3, and 4. Results of the total number of CI repairs and costs related to them within the period ranging from 1 year to 10 years are collected in Table 2.

Table 2. Total number of CI repairs and costs related to them depending on CI availability state initiating restoration actions, determined within the period ranging from 1 to 10 years.

Maintenance Period t	1	2	3	4	5	6	7	8	9	10
total number of renewals and their costs for $r = 2$	4	9	9	13	16	19	20	23	26	29
total number of renewals and their costs for $r = 3$	4	9	13	18	22	27	31	36	40	45
total number of renewals and their costs for $r = 4$	6	11	17	23	29	35	41	46	52	58
cost difference for $r = 3$ and $r = 2$	8%	46%	98%	142%	200%	250%	296%	346%	404%	454%
cost ratio for $r = 3$ and $r = 2$	97%	92%	89%	89%	88%	87%	87%	87%	86%	86%
cost difference for $r = 4$ and $r = 2$	−20%	73%	125%	177%	235%	293%	339%	445%	503%	561%
cost ratio for $r = 4$ and $r = 2$	108%	88%	87%	86%	85%	85%	85%	83%	83%	83%

The performed analyses allow us to outline that more frequent but cheaper restorations of infrastructure immediately after its departure from the state of entire availability are the most beneficial, except for the maintenance period of 1 year. The difference between costs related to such repairs compared to infrastructure's repairs conducted after leaving availability state 2 by the infrastructure increases with the time of infrastructure exploitation. The total cost related to infrastructure repairs over 10 years, in the case when corrective and repair actions are taken after infrastructure has left availability state 3, is approximately 14% lower compared to the total cost assuming infrastructure recovery after leaving availability state 2. The cost difference increases up to 17% when comparing the repair cost of infrastructure over 10 years when infrastructure is renewed after leaving state 4 with the cost assuming infrastructure recovery after leaving availability state 2. Costs of

the optimal repair configuration for $r = 2$, $r = 3$, and $r = 4$ are also compared graphically in Figure 5.

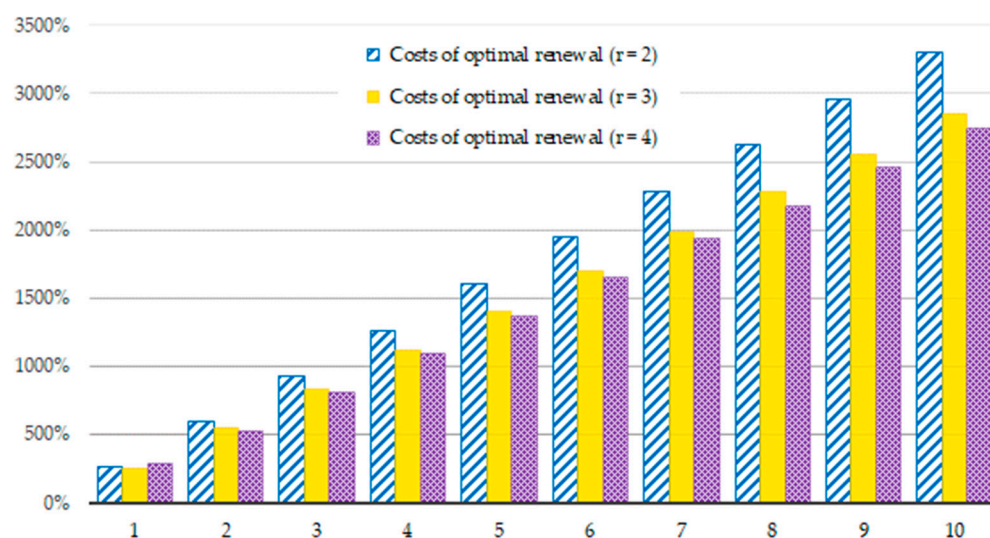


Figure 5. Costs of optimal renewal of a CI depending on its availability state initiating restoration actions.

We conclude that a quick response—even after slight reduction of infrastructure’s availability level—and carrying out repair and corrective actions, making infrastructure return to the state of entire availability, is especially profitable in a longer exploitation time. From the conducted analysis, it follows that this trend is invariable, regardless of what percentage of cost of perfect renovation are the costs of partial renovation of the infrastructure. Table 3 presents results for the case when the cost of renewal to state 2 is 40% of the perfect repair cost and the cost of repair to state 3 amounts to 50% of the single perfect repair cost. At imperfect repair cost percentages of 50% and 40%, repairs of infrastructure after exceeding availability state 2 (for $r = 2$) are the most profitable in the period of 1 year of infrastructure exploitation. Over a longer period of infrastructure exploitation, starting repair and corrective actions after leaving state 4 of entire availability is the best solution to minimize the cost of an infrastructure’s renewals and maintenance. In other cases, considered in Section 5.2, the results are almost identical with those presented in Table 2.

Table 3. Total number of CI repairs and costs related to them depending on CI availability state initiating restoration actions, determined within the period ranging from 1 to 10 years (cost of renewal to state 2 is 40% of the perfect repair cost, and cost of repair to state 3 is 50% of the perfect repair cost).

Maintenance Period t	1	2	3	4	5	6	7	8	9	10
total number of renewals and their costs for $r = 2$	4	10	15	21	26	32	37	43	49	54
total number of renewals and their costs for $r = 3$	4	9	13	18	22	27	31	36	40	45
total number of renewals and their costs for $r = 4$	6	11	17	23	29	35	41	46	52	58
cost difference for $r = 3$ and $r = 2$	−32%	3%	8%	43%	78%	83%	118%	137%	170%	177%
cost ratio for $r = 3$ and $r = 2$	114%	99%	99%	96%	95%	95%	94%	94%	94%	94%
cost difference for $r = 4$ and $r = 2$	−20%	73%	125%	177%	235%	293%	339%	445%	503%	561%
cost ratio for $r = 4$ and $r = 2$	127%	95%	96%	94%	92%	93%	92%	90%	90%	91%

5.4. Discussion on Optimal Maintenance Policy of CI

From the comparative analysis conducted in this section, we conclude that performing perfect repairs that allow the infrastructure to return to an operational condition corresponding to the state of entire availability are the most profitable for repair and maintenance cost optimization. Moreover, we conclude that values of reliability and availability characteristics of infrastructure, and of CI systems forming it, are also of great importance, besides repairs costs and further costs associated with them. We have analyzed the total costs of an infrastructure's renewals and maintenance in certain time periods, assuming the values of reliability characteristics of CI, determined in Section 5.1. By comparing results of the optimal repair configurations of infrastructure for different costs of imperfect repairs relative to costs of perfect repair, we noticed that the general tendency is convergent. The ratio of these costs has no significant influence on choosing a repair strategy and finding the optimal configuration of repairs to minimize their costs, especially over a long period of infrastructure exploitation. Moreover, perfect repairs are the most beneficial, even during several years of infrastructure exploitation. Apparent differences in results are found when reducing the cost of imperfect repair to availability state 3 down to 50% of the cost of perfect repair. In that case, carrying out renewals of infrastructure to availability state 3 is the most profitable.

The procedure proposed in the paper also allows to take into account the availability state of infrastructure, after exceeding of which, restoration actions are initiated. We concluded that within the period of 1 year, the most beneficial and optimal solution is carrying out all repairs when the infrastructure exceeds availability state 3 as perfect repairs to state 4 of entire availability of infrastructure. Within longer periods ranging from 2 to 10 years (the maximal period considered in our analysis), renewals just after the departure of infrastructure from state 4 and perfect repairs back to state 4 are the most beneficial.

The above conclusions result mainly from the mean lifetimes of the infrastructure in state subsets, determined in Section 5.1 and given by (18). The CI mean lifetime in state 4, which is 0.133 year, compared to the CI mean lifetime in availability state 3 ($\mu(4) - \mu(3) = 0.051$ year), is significantly longer. Assuming that mean lifetime of the CI in the state of entire availability is shorter, the optimal configuration of repairs of CI systems carried out, minimizing repair and maintenance cost in a certain period, can differ. Namely, if $\mu(4) \leq 0.11$ and values of other mean lifetimes are the same as in (18), the most profitable repair and maintenance policy is to start corrective and repair actions after the infrastructure's departure from state 3 instead of perfect repairs after its departure from state 4, as indicated in the analysis conducted in Section 5.3. Such tendency is independent of the period, within which the maintenance and repair costs of the infrastructure are determined.

The results can differ for particular infrastructures depending on their reliability structure and the reliability parameters of their components. Thus, precise determination of system reliability characteristics is a key issue for its further exploitation and repair costs analysis. For this purpose, the multistate approach to system reliability and availability analysis is justified, because it leads to analysing system and its components more precisely, taking into account their degradation process.

The method of optimization of infrastructure repair costs and maintenance policy introduced in the paper is based on determined reliability characteristics of the infrastructure that the optimal repair strategy then refers to. Uncertainty of the model's parameters, including uncertainty of the reliability characteristics of infrastructure, is an important issue that can influence repair optimization and maintenance planning. One of methods of accounting for uncertainty of the model can be the approach proposed by Wu et al. [23], which allows estimation of the optimal maintenance policy and minimizes the expected cost (of system maintenance and repairs) and its variance. In this context, the procedure of determining the optimal repair configuration can be conducted repeatedly for various values of infrastructure reliability characteristics. The values should match the determined distribution of the infrastructure reliability function, by taking into account, for example,

standard deviations of infrastructure lifetimes in reliability state subsets. For the case study analysed in the article, the strategy for conducting the repairs is consistent even for various values of infrastructure lifetimes in reliability state subsets. More precisely, the right reaction in the case of deterioration of infrastructure availability state and conducting of more frequent repairs seems to be more effective, than at applying repairs before the moment of infrastructure exceeding a certain critical state, or its complete failure.

Distinguishing aleatory and epistemic uncertainties in modeling and optimization is of great importance, while considering uncertainties in the analysis of the expected cost of repair and maintenance of infrastructure. In this context, by identifying sources of uncertainty of the model, appropriate activities to balance uncertainties at an acceptable risk level can be undertaken. One of the possible activities could be the integration of two methods of maintenance strategy, i.e., time-based and condition-based maintenance, and finding the optimal ratio of the two methods. Such an approach is proposed, inter alia, in [8,24].

With reference to the procedure of determining the optimal repair configuration that minimizes repair and exploitation cost within a certain time frame, as proposed in the article, conducting periodical system state inspections can be additionally taken into account. The inspections allow verifying the system's estimated state and its time to failure (more generally—time of system departure from particular states), and accuracy of the proposed repair-conducting procedure. Afterwards, by verifying values of model input parameters, the procedure determining the optimal repair configuration of infrastructure is conducted again. Such a proceeding proposal, taking into account conducting sensitivity analysis of infrastructure repair strategy and management with respect to uncertainties in model parameters, is briefly introduced by the scheme in Figure 6.

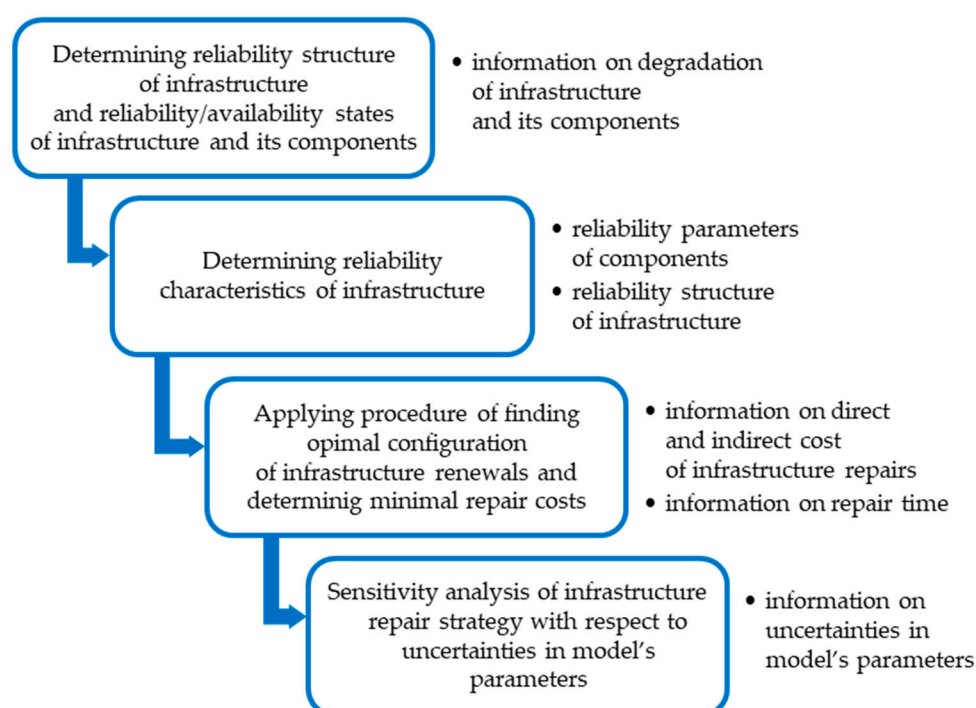


Figure 6. A scheme of the procedure for finding the optimal configuration of system's renewals, taking into account conducting sensitivity analysis of the repair strategy with respect to uncertainties in the model's parameters.

Continuous monitoring and real-time assessment of system reliability/availability states, corresponding to its functionality level, are used for a number of systems [25]. It usually involves the necessity to process large amount of data and using, for example, Big Data technologies [25], which is going to be the next challenge in further research works.

The researches are going to be enhanced with investigations on the optimal infrastructure repair strategy and its management, taking into account the uncertainties of the model.

6. Conclusions

The paper describes the analysis of the CI system exploitation costs, considering the multistate process of its degradation and possibility of its imperfect repairs. The procedure of determining cumulative costs of the system's exploitation and ensuring its availability within a certain time period has been also proposed. Moreover, for the purpose of optimization of the system's exploitation and minimization of renewal costs, a procedure allowing the comparison of exploitation and renewal costs considering different levels of imperfect and perfect repairs of a system within a certain time period has been proposed. The procedure allows us to determine the optimal configuration of the level and frequency of repairs, taking into account possibilities of imperfect repairs. It has been used to optimize CI exploitation and can also be used for planning renewals strategy of many other systems. Extension of the optimization procedure by taking into account the moment of initiating corrective and repair actions related to the availability state of a system, besides including the possibility of perfect and imperfect repairs, allows for a broad and holistic comparison of costs of various repair configurations. As indicated in the section above, values of reliability and availability characteristics of a system are no less important, and even more significant than repair costs and the costs associated with them for finding the optimal maintenance policy for a system, thus minimizing total repair costs.

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