Article

Dynamic Model of a Transcritical CO₂ Heat Pump for Residential Water Heating

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Abstract: This paper presents a distributed mathematical model for a carbon dioxide direct expansion solar-assisted heat pump used to heat bath water. The main components are a gas cooler, a needle valve, an evaporator/collector, and a compressor. To develop the heat exchange models, mass, energy, and momentum balances were used. The model was validated for transient as well as steady state conditions using experimental data. A reasonably good agreement was observed between the predicted temperatures and experimental data. The simulations showed that the time step required to demonstrate the behavior of the heat pump in the transient regime is greater than the time step required for the steady state. The results obtained with the mathematical model revealed that a reduction in the water mass flow rate results in an increase in the water outlet temperature. In addition, when the carbon dioxide mass flow rate is reduced, the compressor inlet and outlet temperatures increase as well as the water outlet temperature.

Keywords: dynamic model; carbon dioxide; heat pump; water heating; transcritical cycle

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1. Introduction

The increase in the electrical energy consumption reflects the improvement in the quality of life as well as economic growth. Between 1974 and 2017, world gross production of electricity increased annually at an average rate of 3.3%, and it has grown continuously since 1974. In addition, the total electricity produced from fossil fuels accounted for 64.5% in 2017 [1]. The emission of greenhouse gases into the atmosphere caused an increase in the Earth's average temperature with a consequent effect on climate change [2]. According to the World Health Organization, more than seven million people die every year due to poor air quality, with three million of these deaths being premature [3]. The concern with global warming and the reduction of CO₂ emissions observed in recent years have become research topics worldwide [4]. Currently, different researchers have worked on solutions to increase the energy efficiency of the equipment used in industrial, commercial, and domestic applications. In many cases, the aim of these studies was to contribute to the sustainable development, and in this context, the heat pump appears as an alternative for heating water for residential use. In developed countries, there is an increasing demand for heat pumps [5]. However, in Brazil, the electric shower [6] is still extensively used to heat water. The use of a heat pump to produce hot water could result in a significant electrical energy saving, since it absorbs the environment thermal energy and consumes only a fraction of the electricity consumed by the electric shower [7–9].

The vapor compression cycle was developed in 1834; since then, different refrigerant fluids have been used, in both refrigerating and heat pump systems. In the beginning,

the refrigerants were the natural fluids. However, in the second half of the 20th century, these were gradually replaced, first by chlorofluorocarbons (CFCs) that were considered safer and more efficient at that time. The Montreal Protocol in 1987 phased out the CFCs, due to their contribution to the ozone layer depletion [10]. The alternatives were hydrochlorofluorocarbons (HCFCs) and hydrofluorocarbons (HFCs), which were, in turn, restricted by the Kyoto protocol in 1997 [11] and later on by the Kigali amendment in 2016 [12]. Due to these restrictions, the scientific community has been looking for alternative refrigerant fluids. In that context, a return to natural fluids represents a viable option.

Carbon dioxide (CO_2) is an alternative among the natural refrigerants [13]. It offers zero ozone depletion ($\text{ODP} = 0$) and is the reference in global warming potential ($\text{GWP} = 1$), meeting not only the requirements of Montreal and Kyoto protocols but also the Kigali amendment. It has no flammability and no toxicity. It is also cheap, plentiful, compatible with normal lubricants and construction materials, and there is no need for recovery. Furthermore, CO_2 has a high volumetric capacity, which allows the use of smaller and more compact compressors and heat exchangers [14].

There are heat pumps that are integrated with a solar thermal collector (SAHP) [8]. One special type of these is the direct-expansion solar assisted heat pump (DX-SAHP) [15]. The DX-SAHP receives, in addition to the energy from natural and forced convection as well as from the condensation of water vapor in the external atmospheric air, the energy from solar radiation. The increase of the coefficient of performance (COP) in a DX-SAHP has been presented in the literature [16,17]. Furthermore, the DX-SAHP operates with better COPs than a non-solar assisted heat pump [18]. A study about refrigerants selection for a DX-SAHP for domestic hot water was presented in [19] by the authors evaluated alternatives to replace R134a, and the results showed that propane followed by CO_2 are a good option.

The use of the CO_2 in a DX-SAHP was recently reported in the literature by Oliveira et al. [20], who developed a distributed mathematical model to study the transient condition in the gas cooler of a CO_2 DX-SAHP. That mathematical model was validated using experimental data. The difference noted between the experimental and theoretical data of the inlet and outlet CO_2 temperatures at the gas cooler was 1°C . Faria et al. [21] also developed a distributed mathematical model to study the transient condition in the evaporator and expansion device of a CO_2 DX-SAHP. Their mathematical model was validated using experimental data with good agreement. Paulino et al. [22] studied the effects of an abrupt change in the solar radiation on a CO_2 DX-SAHP. The distributed model was, once again, validated using experimental data. The results showed that after a variation in the solar radiation, the expansion device needs to act almost immediately to maintain the superheat in the correct value range. In addition, Rabelo et al. [23,24] presented two experimental studies with a CO_2 DX-SAHP. In the first one, the authors discussed the influence of water mass flow rate on the performance of the equipment. The results showed that the increase in the mass flow rate causes a reduction in the outlet water temperature, in the CO_2 outlet temperature, and in the evaporating temperature. In the second study, the authors demonstrated that there is an expansion valve opening at which the heat pump operates at an optimum high pressure and, consequently, at a maximum COP, regardless of the environmental conditions.

The development of mathematical models to simulate the transient and steady state regimes in CO_2 DX-SAHPs is a topic that still demands continued scientific effort. It should be highlighted that due to the fact that the CO_2 heat pump operates in a transcritical cycle, its thermodynamic and transport properties are different from other refrigerants [21]. Additionally, a DX-SAHP is more affected by a solar radiation variation than a non-solar assisted heat pump.

Based on a literature review, it was observed that some theoretical and experimental studies have already been made about CO_2 DX-SAHP. However, none of them have presented a distributed model to study, in a same simulation configuration, for the behavior of all components of a CO_2 DX-SAHP. Thus, the objective of this paper is to study the

dynamic behavior of the CO₂ DX-SAHP with a more inclusive approach. For this purpose, a distributed model is used to find, as a typical application, the steady state of the system as well as the transient response after a variation in the water mass flow rate and in the internal area of the expansion device, followed by an experimental validation. This paper presented here also differs from the others in that it uses the water outlet temperature as the model input data. This type of treatment is most suitable for heating bath water and allows for a better analysis of COP variation under different environmental conditions. Furthermore, the computational effort of the model is assessed.

2. Materials and Methods

2.1. Prototype Description

The prototype used in this study was previously presented by Faria et al. [21] and by Oliveira et al. [20]. In this study, the equipment needs to produce hot water for a family of four people. For this purpose, it is necessary to heat 200 L of water up to 40 °C [25,26]. A schematic of the prototype heat pump is given in Figure 1, where “T” and “P” are the thermocouples and pressure sensors, respectively. The temperature is measured with T-type thermocouples, with an accuracy of ± 0.5 °C. Three FuelTech pressure sensors with an accuracy of $\pm 0.5\%$ (from 0 to 102 bar) were used to measure the pressure at the compressor suction, the compressor discharge, and the gas cooler outlet. A mechanical pressure gauge ZURICH, with an accuracy of $\pm 0.25\%$ (from 0.1 to 200 bar), was installed at the compressor inlet. The water volumetric flow rate is measured with a beaker and a chronometer with an accuracy of $\pm 2.5\%$.

Figure 1. Design layout of the CO₂ heat pump prototype.

The CO₂ compressor model chosen is the EK 6210 CD manufactured by Embraco. The compressor is the hermetic and reciprocating type, has constant speed, operates with a voltage of 110–127 V, and has a displacement of $1.75 \text{ cm}^3 \cdot \text{rev}^{-1}$ and a maximum operating pressure of 12 MPa. This equipment was chosen because of its capability of working

at high pressures and its high cooling capacity, which consequently generates a high heating capacity.

The gas cooler was designed according to the methodology proposed by Cavallini et al. [27], which suggests temperature differences between the fluids at the inlet and outlet of 20 and 5 °C, respectively. The use of energy conservation equations resulted in a gas cooler tube length of 21 m. The tube was arranged in a spiral shape around the water storage tank. This configuration made the heat pump assembly more compact.

The expansion device is a needle valve model SS-31RS4, manufactured by Swagelok with an orifice area of 1.6 mm².

The evaporator/collector is a flat solar plate panel made of copper with an area of 1.57 m². It receives energy from solar radiation, from natural convection, as well as from the condensation of water vapor present in the atmospheric air. The total length of the tube (16.3 m) was obtained according to the methodology presented by Cheng et al. [28].

The data acquisition system receives the signals generated by the thermocouples and by the pressure sensors. A LabVIEW code was used to save and to show in real time the temperatures and pressures. Figure 2 shows a photograph of the test facility.

Figure 2. Prototype of a CO₂ heat pump for domestic water heating.

2.2. Model Descriptions

To simulate the transient and the steady-state behavior of the CO₂ heat pump, a distributed dynamic model was written in Fortran. This section presents the mathematical model of the evaporator, compressor, gas cooler, and expansion device. Furthermore, it explains how these models are coupled to obtain the overall CO₂ heat pump dynamic model.

2.2.1. Compressor Model

The compressor model has the function of providing the outlet mass flow rate for the evaporator model, and the mass flow rate and enthalpy at the gas cooler inlet. In order to formulate the mathematical model for the compressor, the following premises were considered: (i) an irreversible adiabatic compression, (ii) a negligible pressure loss at the suction and discharge, (iii) a constant mass flow rate, and (iv) a refrigerant without any contamination of lubricating oil.

The isentropic efficiency (η_{is}), of the compressor presented in Equation (1) was obtained using a multiple regression as described by Sarkar et al. [29]. Then, using Equation

(2), the compressor outlet enthalpy (h_{comp2}) was obtained, which is also the value of the gas cooler inlet enthalpy (h_{gc1}).

$$\eta_{is} = a_1 + a_2 \left(\frac{P_{comp2}}{P_{comp1}} \right) + a_3 \left(\frac{P_{comp2}}{P_{comp1}} \right)^2 + a_4 \left(\frac{P_{comp2}}{P_{comp1}} \right)^3 + a_5 \left(\frac{P_{comp2}}{P_{comp1}} \right)^4 \quad (1)$$

$$h_{comp2} = \left(\frac{h_{comp2s} - h_{comp1}}{\eta_{is}} \right) + h_{comp1} \quad (2)$$

where a_1 to a_5 are the regression coefficients, P refers to pressure, and the subscripts $comp$, 1, 2, and s , refer to compressor, inlet, outlet, and isentropic process, respectively.

Similarly, using a multiple regression, the compressor volumetric efficiency (η_{vol}) was obtained as presented Equation (3). Then, it was possible to find the compressor mass flow rate ($\dot{m}_{comp}$) as shown in Equation (4).

$$\eta_{vol} = b_1 + b_2 \left(\frac{P_{comp2}}{P_{comp1}} \right) + b_3 \left(\frac{P_{comp2}}{P_{comp1}} \right)^2 \quad (3)$$

$$\dot{m}_{comp} = \frac{NV\eta_{vol}}{v_{comp1}} \quad (4)$$

where b_1 to b_3 are the regression coefficients, N is the rotation speed, V is the volumetric displacement, and v is the specific volume. It is important to highlight that the mass flow rate presented in Equation (4) is the same in evaporator outlet and in the gas cooler inlet, as previously discussed.

2.2.2. Expansion Device Model

The expansion valve model provides the mass flow rate at the gas cooler outlet and at the evaporator inlet. To obtain a simplified numerical simulation, the following assumptions were made: (i) the process is adiabatic, (ii) the flow is one-dimensional, (iii) the relation between orifice of the expansion device (d_{eq}) and number of turns of the valve (nv) is linear, and (iv) the refrigerant is without any contamination of lubricating oil.

The Equation (5) provides the function to calculate the mass flow rate through the expansion device.

$$\dot{m}_{ed} = f(d_{eq}, P_{ed1}, T_{ed1}, \rho_{ed1}, P_c, T_c) \quad (5)$$

where T is temperature, ρ is the density, and the subscripts ed and c refer to expansion device and critical point, respectively.

Using the Buckingham-Pi theorem, it was possible to obtain Equation (6), for the expansion device mass flow rate [30]. The parameters k_1 to k_3 were obtained experimentally, which makes the mathematical model semi-empirical. Martin et al. [31] obtained these parameters for a CO₂ system operating under similar conditions for pressure and mass flow rate.

$$\dot{m}_{ed} = d_{eq}^2 k_1 (\rho_{ed1} P_{ed1})^{0.5} \left(\frac{P_{ed1}}{P_c} \right)^{k_2} \left(\frac{T_{ed1}}{T_c} \right)^{k_3} \quad (6)$$

2.2.3. Heat Exchangers Models

The heat exchangers models have the function of providing the spatial profile of pressure, temperature, enthalpy, specific volume, and mass flow rate for each time step. The following simplifying assumptions were considered: (i) the flows of refrigerant and water were one dimensional, (ii) in the two-phase region, the liquid and vapor were in thermodynamic equilibrium, (iii) the refrigerant was without any contamination by lubricating oil, (iv) there was no axial heat conduction, (v) the radial temperature gradient along the tube wall was negligible, and (vi) the external heat transfer coefficient was uniform.

In case of the gas cooler, there was no heat loss to the ambient, and the internal tube is exactly in center of the external tube. For the evaporator, the fin was considered well fixed to the tube. Therefore, there was no contact resistance.

The heat exchangers models can be separated in external and internal balance. For both of them, the solution is similar. The heat exchanger was divided into “n” control volumes and the balances of mass, energy, and momentum applied for the refrigerant in each coil section generate the Equations (7)–(9).

$$\frac{\partial G_f}{\partial z} = \frac{\rho_f - \rho_f^0}{\Delta t} \quad (7)$$

$$\frac{\partial h_f}{\partial z} = \frac{1}{G_f} \left[\frac{P_f - P_f^0}{\Delta t} - \rho_f \frac{h_f - h_f^0}{\Delta t} + \frac{H_i p_i}{A_i} (T_p - T_f) \right] \quad (8)$$

$$\frac{\partial \bar{P}}{\partial z} = \left(\frac{G_f - G_f^0}{\Delta t} \right) - \left(\frac{dP}{dz} \right)_f \quad (9)$$

where G and $\bar{P}$ are the mass flux and the refrigerant modified pressure, respectively. The terms A_i and p_i are the cross-section area and the internal perimeter of the tube. H_i is the heat transfer coefficient between the wall and the refrigerant. This coefficient was calculated for the gas cooler using the methodology proposed by Gnielinski [32], and for the evaporator, the methodology proposed by Cheng et al. [28,33]. $(dP/dz)_f$ is the pressure loss due to the friction. In the boiling region, the pressure loss was calculated by the correlation proposed by Friedel and recommended by Cheng et al. [28]. In the superheat region, the pressure loss was given by the Fanning equation, and the friction factor was determined by Blasius equation as described by Ozisik [34]. The subscripts f and p refer to the refrigerant and wall. The superscript “0” refers to the previous instant of time. Finally, Δt is the time step.

The input data for the first control volume are the mass flow rate and the enthalpy provided by the compressor model (for the gas cooler) or by the expansion device model (for the evaporator). Furthermore, the assumptions of the inlet pressure and the temperature of the tube wall profile are important as initial conditions. After the first control volume, the equations are solved by the finite difference method in which the output conditions for the current control volume are the input conditions for the next one. In the last control volume, the outlet mass flow rate is compared to that provided by the compressor (for the gas cooler) or by the expansion device (for the evaporator). If the maximum error is greater than 0.1%, the process is repeated until the convergence occurs. After the second iteration, the Newton–Raphson algorithm is used [6] to accelerate the convergence. Then, having achieved the first convergence, the model carries out the external energy balance, from which a tube wall temperature profile is obtained and compared with that was assumed initially. If this temperature profile presents a maximum error greater than 0.01 or 0.1 °C [6] for the phase change regions and for the region close to the critical point, the internal energy balance is restarted. The current temperature of the tube wall profile is used as the input data until convergence occurs. The thermodynamic and transport properties of carbon dioxide are obtained following the methodology proposed by Span and Wagner [35]. Figure 3 shows the flowchart for the heat exchanger models.

Figure 3. Heat exchanger model flowchart.

2.2.4. Gas Cooler Model

The models of the heat exchangers are similar to each other, the main difference occurs in the external energy balance. In the gas cooler, the CO₂ and water flow take place in counter-current in concentric tubes, as shown in Figure 4.

Figure 4. Gas cooler configuration in millimeters.

In the gas cooler, the external energy balance applied between water and the copper tube is represented by Equations (10) and (11). The first one represents the energy gain over time for the water and the second one for the copper tube.

$$\rho_w A_o c_{p_w} \frac{T_w(i) - T_w^0(i)}{\Delta t} = -G_w A_o c_{p_w} \frac{T_w(j-1) - T_w(j)}{\Delta z} - H_o p_o (T_w - T_p) \quad (10)$$

$$\rho_w A_o c_{p_w} \frac{T_p(i) - T_p^0(i)}{\Delta t} = -G_w A_o c_{p_w} \frac{\partial T_w}{\partial z} - H_o p_o (T_w - T_p) \quad (11)$$

where c_{p_w} is the specific heat at constant pressure and the subscripts w and o are the water and the internal tube. The external convective coefficient H_o is calculated using the Dittus–Boelter equations [36].

2.2.5. Evaporator Model

The evaporator model is different from gas cooler model with respect to the external energy balance, as previously discussed. The evaporator/collector is a flat solar plate panel. It receives energy from solar radiation, from natural convection, as well as from the condensation of the water vapor present in the atmospheric air. Equation (12) presents the external energy balance for the evaporator.

$$\rho_w A_w c_{p_w} \frac{\partial T_p}{\partial t} = [(w - d_e)F + d_e][S - U_L(T_w - T_{sky})] - H_f A_f (T_w - T_f) \quad (12)$$

where S and U_L are solar radiation and overall heat transfer coefficient, respectively. F and w are fin efficiency and distance between tubes. The subscript e indicates external. For more details on external heat transfer in the evaporator, see Faria et al. [21]. Moreover, the evaporator is shown in Figure 5.

Figure 5. Gas cooler configuration.

2.2.6. Heat Pump Model

The heat pump model was written in Fortran language, and each component of the model was developed in different subroutines. The integration of these subroutines is carried out in the main program. First, the outlet pressure and the evaporator superheat are estimated. After that, the gas cooler model generates the spatial profile using the data provided by the expansion device and compressor models. Using the output data of the gas cooler, the evaporator model subroutine starts. Similarly, the evaporator model generates the spatial profile of temperature and pressure using the data provided by the expansion

device and compressor models. The evaporator model provides the new values of pressure and superheat, and they are compared with the previously estimated values. If there is no convergence, the process starts again, using the new values for the outlet pressure and superheat. Once the convergence between them has been reached, the model moves on to the next time step.

The model runs until it reaches a steady state configuration. At this moment, the water outlet temperature in the gas cooler is compared to that suitable for bath (40 °C). If they do not converge, the model is restarted, and a correction in the water mass flow rate is made to obtain a new value of water outlet temperature. This methodology makes the water outlet temperature an input data instead of an output data. Furthermore, this convergence process also follows the Newton–Raphson methodology and represents a great differential of this model in relation to those that have been presented in the literature up to now. Figure 6 shows the flowchart for the heat pump distributed model.

Figure 6. Flowchart of the heat pump model.

3. Results

3.1. Time Step Selection

Figure 7 presents the heat pump startup for three different time steps, 5, 10, and 25 s. It shows the values of compressor suction and discharge pressures in the transient and steady state conditions. In both of them, the transient condition occurs within 100 s, having compressor suction and discharge pressures around 4 and 8.1 MPa, respectively. The required time to heat water is around 3 h; consequently, the transient state occurs in a small range of the DX-SAHP operating time. If the object of the analysis is the operation in a steady state regime, a large time step, such as 25 s, produces good results with reduced computational effort. On the other hand, if the object of the analysis is a transient operational state, a small-time step, such as 5 s, produces better results, despite demanding a higher computational effort. The simulations below, in order to present the CO₂ DX-SAHP behavior in steady state and transient condition, consider a time step of 5 s.

Figure 7. Effect of time step in compressor suction and discharge pressure.

3.2. Volumetric Flow Rate Step—Model Validation

First of all, in order to validate the mathematical model, in transient and steady states, step change in the water volumetric flow rate was adopted. The model and experiment were running for 1500 s, while the water volumetric flow rate changed from 1.0 to 1.5 L/min (Step 1), as presented Figure 8. The results of the model and experiments are closer in the steady state as well as in the transient condition. Comparing the data, average and maximum deviations of 5% and 10% can be verified, respectively. Figure 9 shows the effects of the step in the following parameters: CO₂ inlet (T_{gc1}) and outlet (T_{gc2}) temperatures at the gas cooler, CO₂ inlet (T_{ev1}) and outlet (T_{ev2}) temperatures at the evaporator, and water outlet temperature (T_{w2}).

In addition of the first step in the water volumetric flow rate at 1500 s, a second one at 2200 s (Step 2) occurred as presented Figure 9. In the first, the water volumetric flow rate increased, and in the second, it decreased. It should be noted that the variation of the water volumetric flow rate produced, as expected, an inverse effect on water outlet temperature. The deviations between model and experimental temperatures in Figure 9 are shown in Table 1.

Figure 8. Variation of CO₂ volumetric flow rate regarding the expansion device opening area step.

Figure 9. Variation of temperatures regarding the water volumetric flow rate steps.

Table 1. Deviation between model and experiment temperatures presented in Figure 9.

Variable	Average Deviation (°C)	Maximum Deviation (°C)
T_{gc1}	0.9	2.1
T_{gc2}	1.7	1.9
T_{ev1}	1.4	1.5
T_{ev2}	1.9	2.7
T_{w2}	−0.3	1.3

Figure 10 presents the processing time for the model to perform the analysis presented in Figure 9. In the moments close to the steps (1 and 2) in the water volumetric flow rate and until the system reaches the steady state regime, the processing times were longer. In the steady state, the processing time is around 0.5 s. However, under the transient condition, the process time varied between 0.5 and 7.5 s. This is due to the greater computational effort required to represent the heat pump behavior in the transient regime. Finally, the

processing time returns to the previous values when the system returns to the steady state condition.

Figure 10. Processing time due to the volumetric flow rate step.

3.3. Expansion Valve Opening Area Step

Figure 11 shows the variation of temperatures regarding the expansion valve opening area step. The step occurs at 1500 s, when the internal area of the expansion device is reduced. The reduction of the internal area of the expansion device produces a reduction in the CO₂ mass flow rate. This leads to an increase in the superheat as previously discussed by Paulino et al. [22]. In other words, the compressor inlet temperature (evaporator outlet temperature, T_{ev2}) increases. Consequently, the compressor outlet temperature (gas cooler inlet temperature, T_{gc1}) increases as well. The gas cooler is a counter-current type. Therefore, if the CO₂ gas cooler inlet temperature rises, the water outlet temperature must, of course, increase (T_{w2}).

Figure 11. Variation of temperatures regarding the expansion valve opening area step.

If, however, the water mass flow rate and water inlet temperature are kept constant, then the gas cooler outlet temperature (T_{gc2}) maintained almost the same value. The energy received in the evaporator is the same during the simulation procedure, and it is possible to notice that the evaporator inlet temperature (T_{ev1}) was almost unchanged due to the reduction of the internal area of expansion device.

The COP of the CO₂ DX-SAHP varies because of the adjustment of the expansion valve opening area, as shown in Figure 12. When the internal area of the expansion device changes, the COP also changes. This behavior is discussed by Rabelo et al. [24]. However, to analyze the COP trend (if it will increase or decrease for a specific operational point), it is important to know about the optimum high pressure.

Figure 12. Variation of COP regarding the expansion valve opening area step.

A higher step in the expansion valve opening area was executed. The effects of the greater reduction of the internal area are shown in Figure 13 for temperatures and in Figure 14 for the COP. The trends of all variables are the same as those presented in Figures 11 and 12. However, as the step was higher, the effects on the evaporator outlet temperature, gas cooler inlet temperature and water outlet temperature were more pronounced.

Figure 13. Variation of temperatures regarding the higher expansion valve opening area step.

Figure 14. Variation of COP regarding the higher expansion device opening area step.

4. Conclusions

In this paper, a distributed mathematical model for a CO₂ DX-SAHP is presented and validated experimentally. Based on the analysis of the dynamic behavior of the system, the following information was observed:

- the comparison between the experimental results and those obtained with the model show small deviations for the analyzed temperatures and volumetric flow rate;
- the computational processing time is longer for the transient condition. To show the variables with good representativeness, time steps of 25 and 5 s are appropriate for steady state and transient conditions, respectively;
- an increase in the volumetric water flow rate produces a decrease in the water outlet temperature. On the other hand, a reduction in the volumetric water flow rate produces a rise in the water outlet temperature, as one would expect;
- the reduction of the internal area of the expansion device shows an increase of inlet and outlet compressor temperatures as well as in the water outlet temperature;
- the reduction of the internal area of the expansion device shows a variation in the CO₂ DX-SAHP coefficient of performance, as one would expect;
- in the model presented in this paper, differently from what is usual in the literature for the CO₂ DX-SAHP, the water outlet temperature is an input data. This makes it possible to use the model under real operating conditions for which this variable is controlled.

At least, the presented model is a useful tool to analyze the CO₂ DX-SAHP in the transient and steady state regimes, demonstrating its behavior under different operating conditions.

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Abbreviations

Abbreviations

COP	Coefficient of performance
DX	Direct expansion
GWP	Global Warming Potential
ODP	Ozone depletion potential
SAHP	Solar assisted heat pump;

Latin Symbols

A	Area [m^2]
a, b	Regression coefficients
c_p	Specific heat at constant pressure [$\text{Jkg}^{-1}\text{K}^{-1}$]
d	Diameter [m]
F	Fin efficiency
G	Mass flux [$\text{kgs}^{-1}\text{m}^{-2}$]
h	Enthalpy [Jkg^{-1}]
H_i	Heat transfer coefficient between the wall and the refrigerant [$\text{Wm}^{-2}\text{K}^{-1}$]
N	Rotation speed [rps]
nv	Number of turns of the valve
P	Pressure [Pa]
p	Perimeter [m]
S	Solar radiation [Wm^{-2}]
T	Temperature [$^{\circ}\text{C}$]
U_L	Overall heat transfer coefficient [$\text{Wm}^{-2}\text{K}^{-1}$]
V	Volumetric displacement [m^3]
w	Distance between the tubes [m]

Greek Symbols

Δt	Time step
η	Efficiency
v	Specific volume [m^3kg^{-1}]
ρ	Density

Special symbols

$(dP/dz)_f$	Pressure loss due to the friction
$\dot{m}$	Mass flow rate [kgs^{-1}]
$\bar{P}$	Refrigerant modified pressure

Common subscripts and superscript

0	Previously instant of time
1	Inlet
2	Outlet
c	Critical point
$comp$	Compressor
e	External
ed	Expansion device
eq	Equivalent orifice of the expansion device
ev	Evaporator
f	Refrigerant
gc	Gas cooler
i	Internal
is	Compressor isentropic

<i>o</i>	Internal tube
<i>p</i>	Wall
<i>s</i>	Isentropic
<i>vol</i>	Volumetric
<i>w</i>	Water

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