Supplementary Materials

This note aims to compare the predictive performance between the FSPSOSVR and the structural models of exchange rates, suggested by the referee. Section 1 discusses the famous structural models of exchange rates, and present the fundamentals that we consider. In section 2, we present the data, plots, descriptive statistics, and unit root and cointegration tests. We discuss the forecasting results in Section 3.

1. Structural forecasting models of exchange rates

1.1. Single forecasting equations

In this section, we introduce three well-known structural models for forecasting exchange rates. These models are standard in international economics and thus has been extensively examined in the literature [1,2], these models are the uncovered interest parity (UIRP) model [3], purchasing power parity (PPP) model [4], and simple monetary model (MM) [5,6]. The UIRP model describes the interest rate differential between home and foreign countries as being equal to the changes in the exchange rate over the same period [3]. The PPP model, which can be construed as an international version of the law of one price, describes a long-run relationship between the nominal exchange rate and the price differential between two countries when a common basket of goods and services, expressed in a common currency, costs the same between the two countries [4]. The simplest MM describes exchange rates as linear combinations of changes in money stocks and of outputs between home and foreign countries [5,6]. In particular, our forecasting model for exchange rate, which uses the fundamentals, is given by

$$s_t = \alpha + \beta f_t + e_t \tag{1}$$

where s_t is the logarithm of the exchange rate at time *t*, the f_t represents the fundamentals, e_t is the regression error, and α and β are parameters to be estimated. Specifically, the f_t term for the fundamentals is specified according to structural models and has the following forms:

$$\text{UIRP:}\ \ f_t = i_t - i_t^* \tag{2}$$

$$\text{PPP:}\ \ f_t = p_t - p_t^* \tag{3}$$

$$\text{MM:}\ \ f_t = (m_t - m_t^*) - (y_t - y_t^*) \tag{4}$$

where i_t, p_t, m_t, and y_t are the natural logarithms of a home country's nominal interest rate, price level, money stock, and output. The asterisks indicate variables for a foreign country (the United States in this study). We estimate Equation (19) using ordinary least square (OLS) regression and generate forecasts for each currency.

1.2. Multivariate forecasting equations

We further formulate forecasting models by fitting the exchange rate and fundamentals to either a vector autoregression (VAR) model or a vector error correction model (VECM). The VAR model is used when the two series are stationary, and the VECM model is used when the two series are nonstationary but cointegrated. Subsequently, forecasts are generated accordingly by the estimated VAR or VECM models. The basic *p*-lag VAR(*p*) model has the form

$$y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + u_t, \quad t = 1, \ldots, T. \tag{5}$$

where $y_t = (s_t, f_t)'$ is an 2×1 vector that contains, from left to right, the exchange rate and fundamentals; u_t are white noise processes that may be contemporaneously correlated; A_0 is a constant vector; and A_i are 2×2 coefficient matrices. Because each equation in Equation (5) has the same regressors, comprising lagged values of s_t and f_t, the VAR model can be estimated by OLS regression equation by equation.

If the exchange rate and fundamentals are nonstationary without cointegration, then a VAR model is fitted to differences in the data. Conversely, if the two series are nonstationary but cointegrated, we fit a VECM model, in which an error correction term is included in the VAR specification of differenced data and has the form

$$\Delta y_t = \alpha\beta' y_{t-1} + \mathrm{B}_1 \Delta y_{t-1} + \ldots + \mathrm{B}_{p-1} \Delta y_{t-p+1} + u_t, \quad t = 1, \ldots, T. \tag{6}$$

where α is a parameter that measures the speed of adjustment toward long-run equilibrium; β is the long-run coefficient matrix; and $\beta' y_{t-1}$ is the error correction term, reflecting the long-term equilibrium relationship between variables. Forecasts are generated from the estimated VAR or VECM models in a recursive manner.

2. Data and preliminary econometric analysis

2.1. Data sources

The paper uses monthly observations of the nominal exchange rate, money supply, industrial production index, consumer price index, and nominal interest rate for the seven countries, where the United States is designated as the foreign country. Due to the limited data, the sample period for China is changed from 1993M1 to 2018M9, and it remains unchanged for the other countries. The consumer and industrial production indices have 2015 as their base year. Due to limitations in data availability, different measures (all in US dollars) for money supply are used: M3 is used for Australia, Canada, the euro area, and the United Kingdom; M2 is used for China, Taiwan, and the United States; and M1 is used for Japan. Similarly, the nominal interest rates are also measured differently across countries: the money or interbank rate is used for China, the euro area, Japan, Taiwan, and the United Kingdom; the short-term interest rate is used for Australia and Canada; and the effective Federal funds rate is used for the United States. These data are mostly drawn from the Federal Reserve Economic Data (FRED) database, and parts of the data on interest rates are retrieved from OECD statistics. (Data for Taiwan are downloaded from Directorate-General of Budget, Accounting and Statistics). All series are expressed in logarithmic form except for those for interest rates. We construct the nominal interest rate differentials, price differentials, and monetary fundamentals according to Equations (2), (3), and (4).

2.2. Plots

Data on the exchange rate and fundamentals are drawn in pairs in Figure S1. For saving space, only the graphs of Australia and the Euro are presented, and others can be requested from the authors. As shown in the figure, the exchange rate and fundamentals of the two countries show a certain degree of variations over time, which becomes larger during the financial crisis in 2008, where price-diff denotes the price differentials, ir-diff denotes the nominal interest rate differentials, money_base devotes the monetary fundamentals. The second characteristic of the exchange rate and fundamentals in pairs is that they appear to move together, especially for the Euro, implying that the fundamentals may help to predict the behavior of the Euro. However, the relationship between the exchange rate and fundamentals seems not to be apparent for Australian dollar.

2.3. Descriptive statistics

Overall, each series exhibits varying degrees of variability, which hampers forecasts based on fundamentals. Table S1 shows the descriptive statistics of PPP, UIRP, and MM monthly data for seven countries. In PPP and UIRP, the starting year is 1993 for China, 1983 for Taiwan, and 1999 for the Euro, with descriptive statistics collected up to 2018M9 for each country. In MM, the starting year is 1974 for Australia and 1999 for China, respectively. The table displayed the following statistics of the PPP, UIRP, and MM: minimum (Min), maximum (Max), mean, median, first quartile (Q1), third quartile (Q3), interquartile range (IQR), standard deviation (SD), and coefficient of variance (CV%). For PPP descriptive statistics in Table S1, China exhibited the highest degree of variation in terms of its CV value (approximately 127.59 %). Japan exhibited the lowest CV value (41.14 %) of all countries, implying that Japan's PPP variability is low. In the UIRP descriptive statistics in Table S1, the extent of CV variability is greatest for the Euro (approximately 511.86%). The UK has a lower CV value of all countries (73.27%), suggesting a low UIRP variability level in the UK. In the MM descriptive statistics in Table S1, China has the highest degree of CV variability (approximately 218.52%). Canada had the lowest CV value for all countries (10.96%), indicating that Canada's MM variability is remarkably low. These results revealed that the seven countries exhibited different variations and trends in PPP, UIRP, and MM, respectively.

Table S1. Descriptive statistics of economic fundamentals monthly data for seven countries.

Country	Min	Max	Mean	Med	Q1	Q3	IQR	SD	CV (%)	Periods	OBS
PPP											
Australia	−0.582	−0.025	−0.295	−0.073	−0.386	−0.198	0.188	0.158	53.56	1971M1–2018M9	573
Canada	−0.012	0.163	0.074	0.049	0.024	0.121	0.097	0.055	74.32	1971M1–2018M9	573
China	−0.375	0.072	−0.058	−0.034	−0.084	−0.016	0.068	0.074	127.59	1993M1–2018M9	309
Euro	−0.032	0.059	0.017	0.014	0.004	0.038	0.034	0.021	123.53	1999M1–2018M9	237
Japan	0.532	0.941	0.734	0.479	0.654	0.850	0.196	0.302	41.14	1971M1–2018M9	573
Taiwan	0.096	0.350	0.215	0.152	0.193	0.232	0.039	0.101	46.98	1983M10–2018M9	420
UK	−0.644	0.009	−0.214	−0.026	−0.332	−0.050	0.282	0.168	78.50	1971M1–2018M9	573
UIRP											
Country	Min	Max	Mean	Med	Q1	Q3	IQR	SD	CV (%)	Periods	OBS
Australia	−6.470	12.190	3.360	2.570	−0.386	−0.198	4.840	2.789	83.01	1971M1–2018M9	573
Canada	−2.763	5.580	1.458	0.910	0.440	2.303	1.863	1.508	103.43	1971M1–2018M9	573
China	−3.490	6.960	1.477	2.030	−0.320	3.020	3.340	2.294	155.31	1993M1–2018M9	309
Euro	−2.786	2.850	−0.253	−0.071	−1.439	0.692	2.131	1.295	511.86	1999M1–2018M9	237
Japan	−12.850	3.460	−3.337	−2.310	−5.250	−0.800	4.450	2.750	82.41	1971M1–2018M9	573
Taiwan	−5.590	3.560	−0.631	−0.270	−1.530	0.280	1.810	1.567	248.34	1983M10–2018M9	420
UK	−4.870	10.550	2.959	2.331	1.590	4.080	2.490	2.168	73.27	1971M1–2018M9	573
MM											
Country	Min	Max	Mean	Med	Q1	Q3	IQR	SD	CV (%)	Periods	OBS
Australia	−3.847	−2.061	−2.837	−2.043	−3.221	−2.410	0.811	0.633	22.31	1974M1–2018M9	531
Canada	−2.867	−1.740	−2.537	−2.327	−2.681	−2.414	0.267	0.278	10.96	1971M1–2018M9	573
China	−1.517	0.697	−0.324	−0.288	−1.048	0.424	1.472	0.708	218.52	1999M1–2018M9	237
Euro	−0.404	0.151	−0.185	−0.214	−0.282	−0.129	0.153	0.133	71.89	1999M1–2018M9	237
Japan	−2.470	−1.373	−1.841	−1.062	−2.013	−1.686	0.328	0.593	32.21	1971M1–2018M9	573
Taiwan	−4.157	−1.173	−1.944	−1.955	−2.237	−1.552	0.686	0.417	21.45	1983M10–2018M9	420
UK	−4.656	−2.890	−3.709	−2.649	−4.077	−3.198	0.879	0.836	22.54	1971M1–2018M9	573

Abbreviations: PPP = Purchasing Power Parity, UIRP = Uncovered Interest Parity, MM = Monetary Model, Min = Minimum, Max = Maximum, Med = Median, Q1 = First Quartile, Q3 = Third Quartile, IQR = Interquartile range, SD = Standard deviation, CV = Coefficient of Variance, OBS = Observation.

Note: CV (%) is defined as the ratio of the standard deviation to the mean; a larger value indicates a larger variance.

2.4. Unit root and cointegration tests

The forecasting procedures adopted in this article consist of three steps. First, the Augmented Dickey-Fuller (ADF) (1979) [7] test is applied to test for the presence of a unit root in the series. Second, if the series is tested to have a unit root, then the co-integration test of Johansen and Juselius (1990) [8] is applied to examine the presence of cointegration relationship between the exchange rate and fundamentals for each country. Third, if the cointegration exists, use the VECM model to predict the exchange rate; otherwise, take the difference of the variables and a VAR model is used to predict. Results are shown in Table S2.

The third column of Table S2 indicates that the unit root null hypothesis cannot be rejected by the ADF test for the exchange rates and fundamentals across all seven currencies, since all test values are larger than −3.416 of the 5% critical value, implying that they are nonstationary. The lag length, varying from 1 to 18, is chosen by Akaike information criterion (AIC) with the maximum lag set to be 18.

Next, the cointegration test of Johansen and Juselius (1990) [8] is applied to the data, and the results are shown in the fourth column of Table S2. The evidence in Table S2 indicates that the long-run equilibrium relationship between the exchange rates and fundamentals exists for half of the 21 cases since the null hypothesis of no cointegration can be rejected by the two statistics of maximum eigenvalue test and trace test at 10% level. This implies that there is a long-run equilibrium relationship among the two variables. The chosen predicting model to forecast the exchange rate based on the results is displayed in the last column of Table S2.

3. Comparative evaluation of time series and structural forecasting models

In econometric models, structural models are often used to investigate the relationship between economic behavior, economic phenomena, and other variables. Therefore, a comparative evaluation of the time-series and structural models using machine learning can elucidate the benefits of our research approach. This section presents an experimental analysis that compares the predictive power of the FSPSOSVR with those of the structural models of exchange rates. The UIRP model, PPP model, and simple MM were fitted using the VAR/VECM method, OLS regression, support vector regression (SVR), random forest regression (RFR) and adaptive boosting (AdaBoost). MAPE and RMSE metrics were applied to each method to determine the predictive power of these methods, and the results are presented in Table S3. The results show that the FSPSOSVR outperforms all methods except the MAPE for Australia and the MAPE and RMSE for the euro. In overall performance, the FSPSOSVR outperformed all the other methods with an average RMSE of 2.296 and an average MAPE of 0.416. The experimental results demonstrate the robust predictive performance of FSPSOSVR for data of most countries relative to the other three structural models.

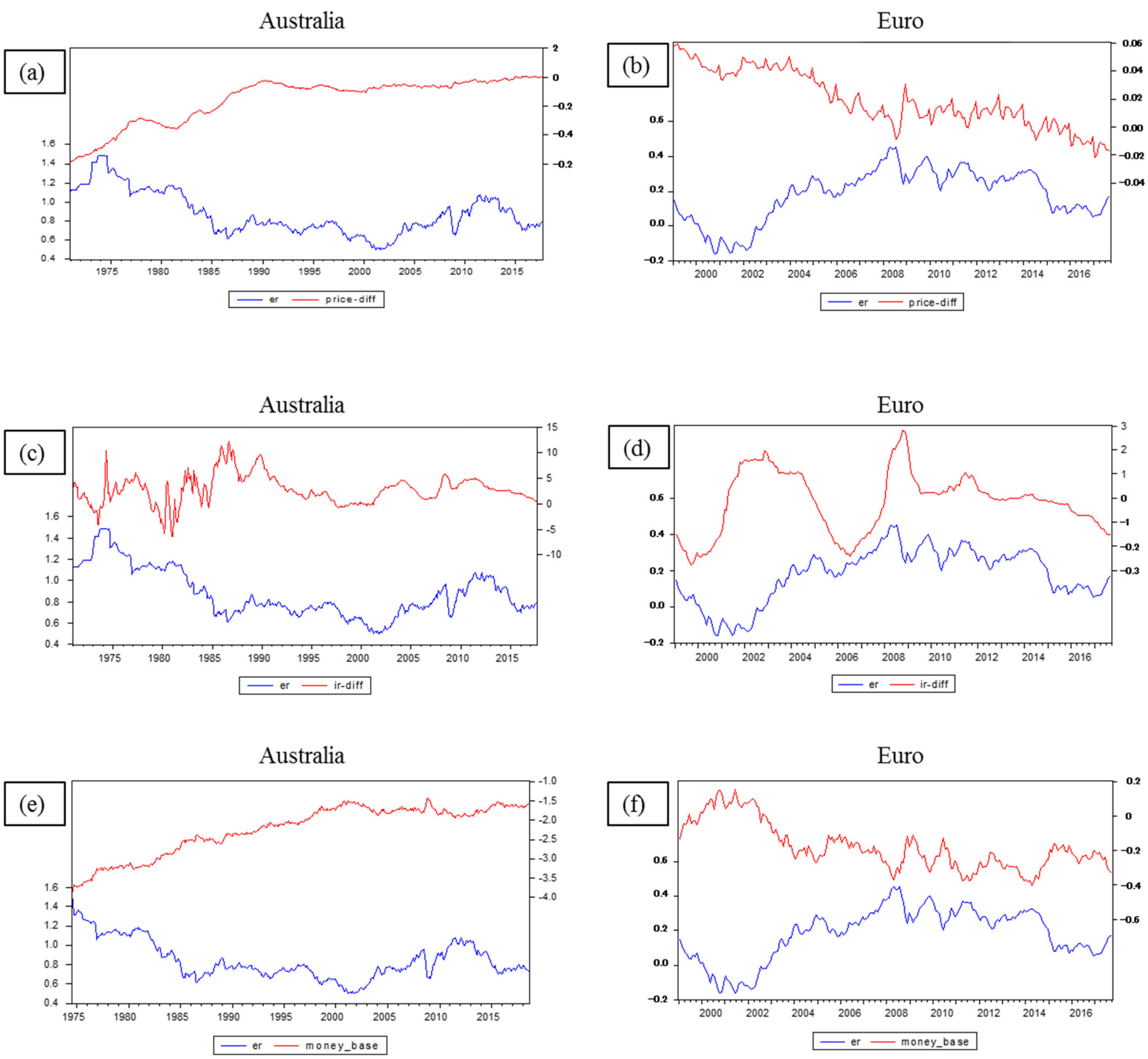

Figure S1. Time series plot of exchange rates and fundamentals.

Table S2. Results of unit root test and cointegration tests.

Country	Series	ADF unit root test		Johansen's Cointegration test					Prediction model
				Hypothesized No of CE(s)	Trace test		Maximum Eigenvalue test		
Australia	ER	−1.724	(2)						
	PPP	−2.533	(13)	At most 1	5.705	[0.214]	9.164	[0.214]	VECM
	UIRP	−3.055	(5)	At most 1	2.895	[0.088] *	2.895	[0.088] *	VECM
	MM	−1.485	(14)	At most 1	8.053	[0.081] *	8.053	[0.081] *	VECM
Canada	ER	−1.968	(4)						
	PPP	−1.918	(14)	None	4.950	[0.507]	4.950	[0.507]	VAR
	UIRP	−3.272*	(6)	At most 1	3.607	[0.507]	3.607	[0.057] *	VECM
	MM	−2.457	(13)	At most 1	2.647	[0.648]	2.647	[0.648]	VECM
China	ER	−2.241	(2)						
	PPP	−3.373*	(18)	At most 1	2.073	[0.149]	2.073	[0.149]	VECM
	UIRP	−2.332	(4)	None	18.852	[0.077] *	10.949	[0.255]	VAR
	MM	−0.666	(2)	At most 1	2.082	[0.761]	2.082	[0.761]	VECM
Euro	ER	−1.731	(1)						
	PPP	−2.098	(13)	None	5.416	[0.763]	2.919	[0.952]	VAR
	UIRP	−3.391*	(13)	None	10.117	[0.271]	7.302	[0.454]	VAR
	MM	−1.931	(14)	None	3.043	[0.964]	3.029	[0.944]	VAR
Japan	ER	−2.139	(15)						
	PPP	−0.262	(12)	At most 1	0.272	[0.601]	0.272	[0.601]	VECM
	UIRP	−3.380*	(4)	At most 1	2.771	[0.096] *	2.771	[0.096] *	VECM
	MM	−2.283	(13)	None	6.653	[0.618]	4.564	[0.795]	VAR
Taiwan	ER	−2.186	(1)						
	PPP	−2.008	(16)	None	10.014	[0.279]	9.772	[0.227]	VAR
	UIRP	−3.057	(2)	At most 1	7.230	[0.114]	7.230	[0.114]	VECM
	MM	−2.151	(13)	None	24.770	[0.068] *	16.046	[0.143]	VAR
UK	ER	−3.059	(3)						
	PPP	−3.372*	(3)	At most 1	9.385	[0.157]	9.385	[0.157]	VECM
	UIRP	−2.988	(15)	None	24.467	[0.074] *	15.511	[0.167]	VAR
	MM	−1.928	(14)	None	19.054	[0.277]	13.310	[0.303]	VAR

Note: 1. PPP = Purchasing Power Parity, UIRP = Uncovered Interest Parity, MM = Monetary Model. 2. Numbers in the parenthesis are the optimal lag length selected by the Akaike information criterion with the maximum lag set at 18. 3. Critical values of the ADF test are −3.416, and −3.130 for the 5%, 10% level, respectively. The intercept and time trend are included in the testing equation. 4. Number in the brackets denotes the *p*-value of the cointegration test. 5. Statistical significance is denoted by an asterisk (*) for the 10% level.

Table S3. Out-of-sample forecast performance evaluation for economic fundamentals monthly data using the metrics MAPE and RMSE.

Country	Criteria	VAR/VECM			LS			SVR			RFR			AdaBoost			FSPSOSVR
		PPP	UIRP	MM	PPP	UIRP	MM	PPPP	UIRP	MM	PPP	UIRP	MM	PPP	UIRP	MM	Time-Series
Australia	MAPE (%)	4.030	4.788	3.649	5.155	14.232	6.667	3.366	**1.569**	3.121	2.580	2.927	3.152	1.955	23.038	9.196	3.410
	RMSE	0.097	0.113	0.088	0.117	0.358	0.137	0.084	0.041	0.079	0.065	0.076	0.076	0.051	0.495	0.206	**0.030**
Canada	MAPE (%)	5.427	5.060	4.838	15.461	5.308	9.017	5.910	5.417	4.699	3.563	3.674	3.387	10.067	2.977	2.906	**2.872**
	RMSE	0.073	0.068	0.065	0.173	0.067	0.108	0.079	0.071	0.064	0.058	0.052	0.050	0.132	0.056	0.058	**0.045**
China	MAPE (%)	2.597	2.626	2.611	15.305	15.139	5.983	24.371	11.377	2.698	6.237	5.228	4.430	9.582	3.695	3.353	**1.076**
	RMSE	0.205	0.194	0.222	1.012	1.003	0.435	1.598	0.779	0.2105	0.457	0.390	0.322	0.660	0.278	0.262	**0.092**
Euro	MAPE (%)	2.526	1.906	**1.891**	17.380	3.525	14.631	3.131	2.644	3.227	7.355	7.341	6.174	6.094	3.813	4.599	3.701
	RMSE	0.031	**0.027**	**0.027**	0.207	0.050	0.176	0.043	0.033	0.044	0.102	0.104	0.083	0.079	0.053	0.073	0.049
Japan	MAPE (%)	2.417	1.654	1.726	34.077	24.005	5.292	9.432	11.642	11.440	2.492	3.430	3.598	10.673	41.318	2.040	**1.622**
	RMSE	2.952	2.308	2.471	28.134	34.965	5.863	10.589	12.987	12.768	3.808	4.574	4.685	12.017	46.181	2.932	**2.095**
Taiwan	MAPE (%)	1.719	1.606	1.679	2.680	5.680	7.931	3.913	5.412	3.886	3.672	4.716	3.390	3.466	3.032	1.724	**1.551**
	RMSE	0.676	0.593	0.622	0.951	1.790	2.412	1.279	1.685	1.272	1.3224	1.637	1.176	1.159	1.045	0.625	**0.574**
UK	MAPE (%)	2.394	2.209	3.399	19.421	24.224	11.186	27.735	20.145	23.500	3.815	4.655	4.843	12.822	18.994	11.083	**1.837**
	RMSE	0.037	0.036	0.052	0.262	0.326	0.152	0.320	0.274	0.317	0.060	0.075	0.077	0.176	0.258	0.153	**0.026**
Avg	MAPE (%)	3.016	2.836	2.828	15.640	13.159	8.672	11.123	8.315	7.510	4.245	4.567	4.139	7.808	13.781	4.829	**2.296**
	RMSE	0.582	0.477	0.507	4.408	5.508	1.326	1.999	2.267	2.108	0.839	0.987	0.924	2.039	6.907	0.611	**0.416**

Abbreviations: VAR/VECM = Vector Autoregression (VAR) or Vector Error Correction Model (VECM), LS = Least Square Method, SVR = Support Vector Regression, RFR = Random Forest Regression, AdaBoost = Adaptive Boosting, PPP = Purchasing Power Parity, UIRP = Uncovered Interest Parity, MM = Monetary Model. Bold: the superior values

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