

Article

Determination of Optimal MR&R Strategy and Inspection Intervals to Support Infrastructure Maintenance Decision Making

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Abstract: In the commonly used approach to maintenance scheduling for infrastructure facilities, maintenance decisions are made under the assumptions that inspection frequency is periodical and fixed, and that the true state of a facility is revealed through inspections. This research addresses these limitations by proposing a decision-making approach for determining optimal maintenance, repair, and rehabilitation (MR&R) strategy and inspection intervals for infrastructure facilities that can explicitly take into account non-periodical inspections as well as previously considered periodical inspections. Four transition probabilities are proposed to represent four different MR&R strategies. Then, an optimization program is suggested to minimize MR&R and inspection costs of a bridge element network over a given time period, while keeping the condition states of the element network above a predetermined level. A case study was applied to illustrate the proposed approach. The results show that the proposal approach can support decision making in situations where non-periodical inspections and MR&R actions are incorporated into the model development. If employed properly, this may allow agencies to maintain their infrastructure more effectively, resulting in cost savings and reducing unnecessary waste of resources.

Keywords: maintenance, repair, and rehabilitation (MR&R); inspection; optimization; infrastructure; decision making



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1. Introduction

Under continuous wear by traffic, environment and weather, infrastructure facilities inevitably deteriorate and require effective maintenance, although most agencies lack sufficient funding and effective decision-making approaches for allocating limited resources [1,2]. Given available resources, highway agencies are faced with a number of choices: (1) How often should the inspection be done? (2) After an inspection, what type of maintenance actions should be performed? (3) How to determine the optimal inspection interval and maintenance strategy to minimize costs [3]. With regard to infrastructure issues, these choices are based on the consequences of possible maintenance, repair, and rehabilitation (MR&R) actions on the future condition of the infrastructure facilities. Since information about the future condition of infrastructure facilities is not available, performance prediction models are used. This framework is common in the current maintenance decision making of infrastructure facilities, although the actual formulation of the performance prediction and optimization models may differ [4].

The main difficulty faced by the current maintenance decision making for infrastructure facilities is the lack of empirical data related to the infrastructure facilities' historical behaviour, which to a large extent relies on the experience of managers and technical personnel [5,6]. Second, the currently used approaches and models have some limitations, which affect the effectiveness of maintenance decision making. Frangopol, et al. [7] reviewed the research related to models and modelling approaches of maintenance decision

making for infrastructure facilities. They concluded that no single model has yet proven to be generally applicable, and each model has its advantages and disadvantages. Therefore, several studies have suggested some advanced options to overcome these barriers. Third, infrastructure maintenance decisions are usually made by practitioners. Therefore, models in the context of infrastructure applications need to be interpreted in a more direct and simple manner, and should be easy to use in practice. However, the methods and models currently proposed for infrastructure maintenance decisions are usually quite theoretical in nature. It is evident that the difficulties faced by the current maintenance decision making arise from the data as well as the model. As mentioned by Mishalani and McCord [8], much more effort needs to be devoted to transferring the models and approaches into practice by better utilizing more pertinent data, and explicitly addressing limitations to practical implementation.

Therefore, the aim of this paper is to propose a decision-making approach for determining the optimal MR&R strategy and inspection intervals for infrastructure facilities that can explicitly take into account non-periodical inspections as well as previously considered periodical inspections, which mainly include two parts: (1) to identify the limitations of the current maintenance decision making used for infrastructure facilities based on a literature review; (2) to develop a decision-making approach that can address these identified limitations. The proposed approach is expected to extend current maintenance decision making by addressing non-periodical inspection issues, and hence enhance the capability and feasibility of the optimization module in current maintenance decision-making systems. The remainder of the paper is organized as follows. Section 2 reviews the current literature of maintenance decision making for infrastructure facilities, followed by a discussion of maintenance decision making for elements undergoing periodical inspection in Section 3. Section 4 develops an optimization model to support the maintenance decision making of elements undergoing non-periodical inspection. A general maintenance decision-making support architecture covering non-periodical as well as periodical inspections is proposed in Section 5. The last section concludes the paper by discussing the limitations and suggestions for future research.

2. Review of Maintenance Decision Making for Infrastructure Facilities

Based on the current state evaluation and future condition prediction of the infrastructure, the key of maintenance decision making for infrastructure facilities is to develop effective optimization models for programming maintenance and/or inspection schedules under a limited financial budget [9]. The current maintenance management of infrastructure facilities relies on information collected from periodical inspections, which can be used to assess the condition states and conduct maintenance optimization [10]. An important requirement when making maintenance decisions for infrastructure facilities is data availability [11]. However, resources to be invested in data collection and optimization analysis are usually limited in practice [12]. The Markov decision process (MDP) and Semi-Markov decision process (SMDP) are often used for maintenance decision analysis due to the following advantages: (1) they are state-based models suited to incorporate information from visual inspection [7]; (2) they are probabilistic models which can address both the uncertainties associated with deterioration patterns and the dynamic decisions of the model [5]; and (3) they can manipulate networks with a large number of facilities because of their computational efficiency and simplicity of use [13]. As a result, after an inspection, the decision maker can apply the MR&R activity specified by the optimal policy for that condition state of the facility [14,15].

In the MDP and SMDP, infrastructure maintenance decisions are made under the assumptions that inspections are performed at periodical intervals and that they reveal the true condition state of a facility, with no measurement error [14,16]. The assumptions raise several concerns which come from the simplification required to predict deterioration of the facilities and the uncertainty from the inspection [10,17]. The assumption that the performance prediction model depends on the state of a facility revealed through

periodical inspections is an example of the former, while the uncertainty that exists in generating transition probabilities for the MDP and SMDP is an example of the latter [18,19]. The assumptions ignore the effects, the cost, and the complexity of MR&R actions and inspections, thus limiting the effectiveness of the approach in many situations. Examples include the implementation of certain types of infrastructure facilities that are susceptible to accidental damage and need frequent MR&R actions during the life cycle, but have not been studied extensively, e.g., expansion joints, pipeline segments, and parapets.

The assumption of error-free inspections has been demonstrated to be incorrect in several empirical studies [10,20]. Additionally, there is a large measurement uncertainty in visual inspection, and this uncertainty will affect maintenance decisions. This is because measurement errors will affect the performance prediction of the facilities and ultimately lead to the selection of the “wrong” activities. The assumption of periodical and fixed intervals forces decision makers to schedule inspections at the beginning of planning, regardless of the cost and effectiveness of the inspection. Indeed, parts of this assumption may be valid for certain bridge elements, e.g., beam, pier, and girder, those where the deterioration is primarily governed by mechanical processes. However, it is unrealistic for some bridge components, e.g., expansion joints, bearing, and parapets, which are susceptible to accidental damage and need to be inspected non-periodically [21,22]. Madanat and Ben-Akiva [23] identified that increasing the frequency of inspections increases inspection costs but enhances the quality of information available to the decision maker.

A latent Markov decision process (LMDP) was proposed for maintenance decisions that accounts for the presence of non-fixed time intervals and measurement uncertainty [10,20]. Although relaxing the assumptions, LMDP research still manifests some limitations in determining the optimal inspection and MR&R strategy. First, the effects of MR&R actions between two adjacent inspections are still not properly incorporated into the model development. This means that the state derived from LMDP may still not reveal the true condition state of the bridge elements due to inconsistency in the bridge deterioration profile. This is especially critical for some infrastructure facilities, e.g., expansion joints, pipeline segments, and parapets, which are susceptible to accidental damage and need to be inspected non-periodically and treated in time in order to provide safe and good service quality for users [21]. Second, the approaches to date assume a presumed and constrained inspection frequency in MDP [5,14], or only address the decision of whether to inspect in a given year or not in LMDP [10,20,24]. Nazari, Noruzoliaee, Zou and Mohammadian [16] used LMDP to seek the optimal facility-specific inspection intervals, and the MR&R policies, but focused only on inspection error associated with technology. As discussed above, due to the effects of non-periodical inspections and subsequent MR&R actions not being incorporated into the model development, a “wrong” maintenance decision will be made, resulting in unnecessary waste of resources and materials. In fact, as suggested by Hu and Samer [24], the effects of MR&R actions must be considered and conducted to ensure the performance of infrastructure at safe and satisfactory levels [24]. Third, the current LMDP models for infrastructure still seem quite theoretical, and hence appear difficult to use in practice. It can be seen that the above limitations affect the effectiveness of maintenance decision making and the use of the model. Therefore, it is necessary to conduct research to address the above limitations.

Furthermore, the proposed maintenance scheduling should take into account the multiple constraints, including technical and economical considerations, as well as the balanced development imperatives [25,26]. If a decision maker uses a single objective algorithm to optimize the inspection intervals or maintenance scheduling separately, the conflict may result in an unsatisfied demand.

3. Maintenance Decision Making for Elements Undergoing Periodical Inspection

MDPs and SMDPs can be used for maintenance-decision analysis of the elements undergoing periodical inspection with the following underlying assumptions [14]:

1. The deterioration is represented by transition probabilities;

2. Perfect inspection is carried out to identify the condition state of the elements;
3. The process has a finite state space-maintenance action, and it is assumed that at every each period a set of maintenance actions is available;
4. The elements are inspected on a predetermined and fixed-time interval;
5. After the optimization analysis, the optimal periodical inspection interval and the optimal MR&R strategies can be determined.

MDPs have been extensively used for maintenance decision making for elements undergoing periodical inspections. The deterioration in this case is modelled as a Markov process. The inspection interval in most MDP-based MR&R optimization is pre-determined, usually 12 or 24 months. Some research has attempted to determine the optimal periodical inspection with MDP, for example, Kallen and Van Noortwijk [27] proposed a decision model to determine the optimal time between periodic inspections. Linear programming (LP) can be used for optimal MDP-based maintenance scheduling, which was first proposed by Golabi, et al. [28]. The decision variables of the optimization model are the fractions of the facilities in the network that are in various condition states, and to which different MR&R actions should be applied [29]. To improve computational efficiency, researchers have proposed evolutionary-based algorithms, e.g., the genetic algorithm (GA), for searching a near-optimum solution [30–32].

Compared with MDP, the generalized characteristics of SMDP make decision analysis more effective in choosing the optimal inspection interval. For example, Berenguer, et al. [33] used SMDP to derive a predictive maintenance policy which indicates, at each inspection and according to the observed value, whether a preventive maintenance is necessary and when the next inspection should be performed; Ge, Tomasevicz and Asgarpour [3] used SMDP to determine the optimal inspection rate and maintenance policy by maximizing equipment availability and minimizing the cost.

4. Maintenance Decision Making for Elements Undergoing Non-Periodical Inspection

4.1. Problem Description

Referring to the rating system of PONTIS [5], a bridge network is not considered as a set of individual bridges but as a combination of bridge elements that interact with each other in various forms and quantities. Thus, each bridge can be defined as a combination of its constituent elements. Now, if one unit of an element is considered, it is possible to define the condition state of a bridge element at any time. This makes it possible to specify the MR&R actions that can be applied to the specific state of each element. In reality, since one unit of a bridge element can usually be in one of four or five condition states at any given time, there are only a few MR&R actions available to correspond. So, at any time, the possible discrete states and available MR&R actions may be associated with one unit of the bridge element.

Thus, it is possible to use different types of MR&R actions to discretize the condition state of a bridge element, i.e., States 1,2,3, and 4 are distinguished by different MR&R actions ("Do nothing", "Preventive maintenance", "Corrective maintenance", and "Rehabilitation or Replacement"). More information about this rating system is described by Yang, Pam and Kumaraswamy [22]. According to the rating system, the number of possible condition states is determined for one unit of a bridge element at any given time. More pertinent data from past inspections and maintenance records are added to the maintenance decision analysis. The states of the bridge element take into account the impact of non-periodical inspections and MR&R actions, which can be seen as an extension of the approach from PONTIS. In earlier MDP and SMDP, the state is an integer representing the condition of the infrastructure facility, assuming there are no measurement errors [27]. However, in the model presented in this study, the state is represented by the different types of MR&R actions. Data from actual MR&R actions were used to supplement information from non-periodical inspections, relaxing the assumption of no measurement errors. These aspects include the time since the last inspection, as well as the most recent MR&R actions and the impact of these actions.

Markov and semi-Markov models are used for deterioration modelling by defining discrete condition states and accumulating the transition probabilities from one condition state to another, over multiple discrete time intervals, in which the deterioration process is represented by transition probabilities [34]. The transition probabilities are integrated in a transition probability matrix (TPM). Based on the rating system defined above, the following four TPMs are derived, denoted as M_1 , M_2 , M_3 , and M_4 . These four TPMs are proposed to represent four different MR&R strategies, which are expected to simplify MR&R decision making, i.e., strategy I takes all the MR&R actions ("Do nothing", "Preventive maintenance", "Corrective maintenance", and "Rehabilitation or Replacement"); strategy II takes three MR&R actions ("Do nothing", "Corrective maintenance", and "Rehabilitation or Replacement") apart from preventive maintenance, strategy III takes three MR&R actions ("Do nothing", "Preventive maintenance", and "Rehabilitation or Replacement") apart from corrective maintenance, while strategy IV only takes "do nothing" and "rehabilitation/replacement" actions.

$$M_1 = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

$$M_2 = \begin{bmatrix} \beta_{11} & \beta_{13} & \beta_{14} \\ \beta_{31} & \beta_{33} & \beta_{34} \\ 1 & 0 & 0 \end{bmatrix} \quad (2)$$

$$M_3 = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{24} \\ 1 & 0 & 0 \end{bmatrix} \quad (3)$$

$$M_4 = \begin{bmatrix} \beta_{11} & \beta_{14} \\ 1 & 0 \end{bmatrix} \quad (4)$$

4.2. Objective Function

Discrete-time SMDP can be used for maintenance-decision analysis of the elements undergoing non-periodical inspection with the following assumptions:

1. The deterioration is represented by transition probabilities;
2. Perfect inspection is carried out to identify the state of the elements;
3. The process has a finite state space—MR&R actions—and it is assumed that at every period a set of MR&R actions is available;
4. The elements are inspected at non-periodical intervals;
5. After each analysis, two main issues are determined: (1) the optimal inspection interval and (2) the optimal MR&R strategies.

Based on the four TPMs derived above, an optimization program is described below.

$$\text{Min} \cdot \sum_{i=0}^N \alpha^i [S_i \cdot \text{TPM}_{(i,i+1)} \cdot C_M + C_I] \quad (5)$$

$$s \cdot t \cdot S_0 = [1 \ 0 \ 0 \ 0] \quad (6)$$

$$S_{k+1} = S_k \cdot \text{TPM}_{(k,k+1)}, \forall k = 0, 1, 2 \dots i \quad (7)$$

$$S_i \leq S^{Thr} \quad (8)$$

where S_i = condition state vector (1×4) at the time point i ; $\text{TPM}_{(i,i+1)} = \text{TPM}$ (4×4) from time point i to $i + 1$; C_M = MR&R unit cost vector (4×1); C_I = inspection unit cost; α = discount rate; T = planning horizon; S_i^{Thr} = the threshold condition state vector (1×4).

5. Case Study

A bridge element (e.g., expansion joints) was used as an example to validate the approach, which was proposed to determine the optimal MR&R strategy and inspection intervals for a network of bridge element. The TPMs derived from Yang, et al. [35] were used as inputs to the optimization Equation (5). Searching for appropriate algorithms is the most important step factor in optimization. Network-level optimization problems of infrastructure are frequently formulated as MDPs, and the optimal MDP-based MR&R policy can be determined by short-term and/or long-term optimization. Long-term optimization is based on infinite planning horizons, while short-term optimization is based on a predetermined and finite planning horizon [14]. Considering the inspection frequency of the actual project, 6 months was used as a calculation step in this case. The planning period was set at 20 years. A common approach of solving an MDP-based model is by transforming it into a LP model. Therefore, a short-term LP optimization was considered in this case study, for which efficient algorithms exist (e.g., Golabi, Kulkarni and Way [28]; Smilowitz and Madanat [10,14]).

The objective function (5) minimizes the total expected cost of MR&R activities and inspection by selecting the optimal values of the decision variables over the planning horizon T . The required constraints for this minimization problem are listed in Equations (6)–(8). The first constraint (6) limits the initial variable of condition state vector. Constraint (7) shows the Chapman–Kolmogorov equation. Constraint (8) defines a predetermined threshold value to the condition states of the bridge element. More detailed calculation procedures can be seen in [14].

Referring to the real costs and the suggestions from the engineers, the inspection unit cost is assumed to be 10 (USD/m), and the ratio of MR&R action unit cost is assumed to be 0:10:100:1000 for Strategy I; 0: 100:1000 for Strategy II; and 0:10:1000 for Strategy III. The last value of the condition state vector is related to the structural safety and can be ensured by setting a critical value. According to Yang, Kumaraswamy, Pam and Xie [35], 1% can be used as a critical value for the last value of the condition state vector, i.e., if the last value of the condition state vector exceeds the critical value, a penalty in the form of an additional MR&R cost of USD 1000 per unit is applied. The above values can be adjusted according to the actual project.

As shown in Table 1, Strategy II is a good choice if the lowest total cost is considered, and Strategy I can be a good choice if good condition performance is considered. One possible explanation for the lowest total cost of Strategy II is that it does not use the preventive maintenance, reducing the frequency of inspections and associated costs that result from the preventive maintenance. In contrast, Strategy III has the worst conditional state and the highest total cost, making it an unsatisfactory choice.

Table 1. Output of analysis.

Strategy	Fractions of Condition State	Optimal Inspection Interval (Month)	Total Cost (\$)
I	[0.060; 0.092; 0.765; 0.083]	12–24–30–36–42–78–120–240	3.657×10^3
II	[0.073; 0.837; 0.090]	6–24–78–240	2.026×10^3
III	[0.438; 0.386; 0.078]	12–114–120–126–210–240	4.328×10^3

The results of the case study also indicate that relaxing the frequency of inspections may contribute to cost savings, due to the increased costs of frequent inspections. In particular, for higher levels of measurement error, the increased costs may be higher. The results suggest that the instrumentation could be used to improve the accuracy of infrastructure inspections in the future and the importance of jointly optimizing inspection intervals and MR&R policies. The results of the case study have some similarities to those of previous research (e.g., [10]).

6. Discussion

A case study was performed to illustrate the proposed approach to programming and formulating an optimal inspection interval and MR&R strategy for a network of bridge elements. The optimization problem is formulated as a short-term LP problem. The optimization objective is to minimize the MR&R and inspection cost of a bridge element network over a given time period, while keeping the element network condition above a predefined threshold level.

For a given time period (e.g., 20 years), the results of the case study comprise the inspection intervals, the total cost, and fractions of condition state for a given time period. Decisions are thus made by balancing these items. This application illustrated the feasibility, efficiency, and capability of the proposed approach. A general maintenance decision-making support architecture covering non-periodical as well as periodical inspections was thus proposed (see Figure 1). Four modules are included in the architecture, which are (1) data input, (2) optimization process, (3) output, and (4) making decisions. Just like the optimization results generated by the case study, engineers can choose one of them and make maintenance decisions. If the optimization outputs are not satisfied, the model can be run again with different parameters adjusted and entered, e.g., adjusting MR&R and inspection costs, and planning horizon.

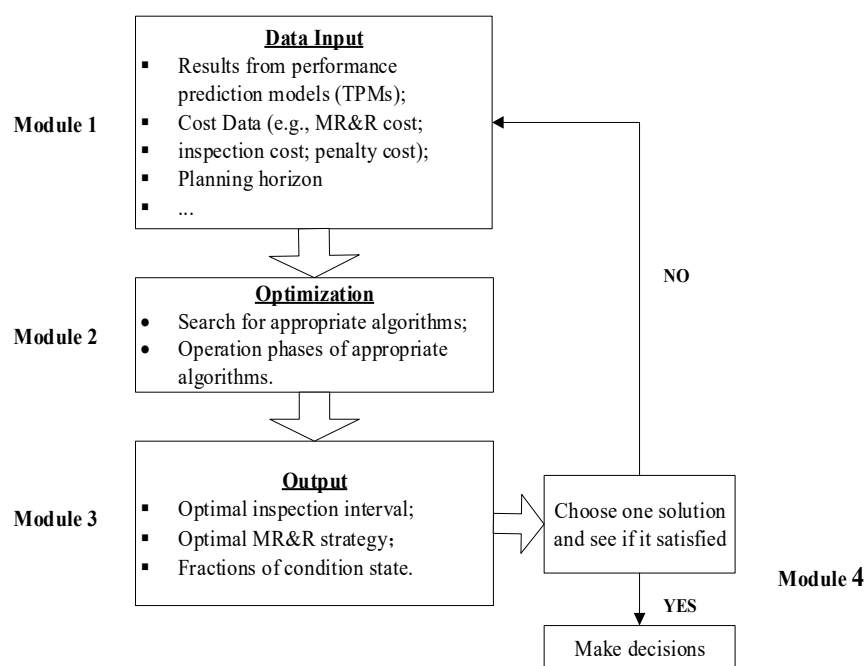


Figure 1. General maintenance decision-making support architecture.

7. Conclusions

Highway agencies are increasingly recognizing the need for an effective approach to allocate limited resources in a cost-effective and environmental-friendly way for infrastructure maintenance [2,36]. Much of the current research on sustainable infrastructure is focused on the design phase, encouraging more efficient use of natural resources and green design [37], and neglecting sustainable issues in the maintenance phase [38]. A literature review was used to identify the limitations of the current maintenance decision making used for infrastructure facilities. Based on this, a decision-making approach was proposed for determining optimal MR&R strategy and inspection intervals for infrastructure facilities that can explicitly take into account non-periodical inspections as well as previously considered periodical inspections. The optimization model enables highway agencies to determine the optimal maintenance policy for a given level of performance and minimum cost, while ensuring the security of critical infrastructures and reducing waste of resources.

After all, many materials and resources could be wasted, and infrastructure facilities could deteriorate more severely with improper MR&R policies. The outputs of the proposed decision-making support architecture comprise the inspection intervals, the total cost, and fractions of condition states for a given time period. Multi-criteria decisions can thus be made by optimizing the targeted balance between these items. This makes the proposed approach attractive, because it considers the engineering practices. It is impractical to make strong assumptions about performance prediction and maintenance optimization. For example, the existing approach to maintenance decision making for certain types of infrastructure elements, e.g., expansion joints, pipeline segments, and parapets, usually assumes that inspection is fixed, with no measurement error.

Through this approach, there is a great potential for highway agencies to make their infrastructure sustainable by saving costs and reducing unnecessary waste of resources. Compared with the optimization models presented in previous research (e.g., Memarzadeh and Pozzi [12]; Wu, Yuan, Kumfer and Liu [2]), the assumptions were addressed in this study; and the outputs include not only the optimal MR&R strategy but also the optimal inspection intervals.

The MR&R strategies vary greatly from one bridge element to another. The TPMs proposed in this study may not be applicable to another bridge element (e.g., bridge pier). Appropriate TPMs can be proposed for different types of bridge elements in future research. The inspection unit cost and the MR&R action unit cost are needed in the optimization model. Future studies will continue to collect cost data to ensure accurate and reasonable results. Additionally, the user cost (e.g., including traffic delay and resource use cost) and other social costs could be incorporated into the overall cost optimization. A comprehensive database could be developed to obtain such data. Assuming this is possible, it is also necessary to develop a cost model to incorporate all such pertinent cost data for overall optimization in future research. The proposed approach was only verified using the data from one important bridge element. More data from other bridge elements should be collected to verify the proposed approach in the future. Furthermore, superposing the additional variables and constraints will lead to an exponential increase in the number of variables, making it computationally expensive to reach an optimal solution. To improve computational efficiency, robust optimization techniques, e.g., the genetic algorithm (GA), could be tested in a future study.

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References

1. Chu, J.C.; Huang, K.H. Mathematical programming framework for modeling and comparing network-level pavement maintenance strategies. *Transp. Res. Part B Methodol.* **2018**, *109*, 1–25. [\[CrossRef\]](#)
2. Wu, D.; Yuan, C.; Kumfer, W.; Liu, H. A life-cycle optimization model using semi-markov process for highway bridge maintenance. *Appl. Math. Model.* **2016**, *43*, 45–60. [\[CrossRef\]](#)
3. Ge, H.; Tomasevicz, C.L.; Asgarpour, S. Optimum Maintenance Policy with Inspection by Semi-Markov Decision Processes. In Proceedings of the 39th North American Power Symposium, Las Cruces, NM, USA, 30 September–2 October 2007; pp. 541–546.
4. Robelin, C.-A.; Madanat, S.M. History-dependent bridge deck maintenance and replacement optimization with Markov decision processes. *J. Infrastruct. Syst.* **2007**, *13*, 195–201. [\[CrossRef\]](#)
5. Golabi, K.; Shepard, R. Pontis: A system for maintenance optimization and improvement of US bridge networks. *Interfaces* **1997**, *27*, 71–88. [\[CrossRef\]](#)
6. Madanat, S.; Smilowitz, K.; Lago, A. Optimal inspection and maintenance policies for infrastructure systems: Facility and network problems. *Transp. Res. Rec. J. Transp. Res. Board* **1999**, *1667*, 1–7. [\[CrossRef\]](#)
7. Frangopol, D.M.; Kallen, M.-J.; Van Noortwijk, J.M. Probabilistic Models for Life-Cycle Performance of Deteriorating Structures: Review and Future Directions. *Prog. Struct. Eng. Mater.* **2004**, *6*, 197–212. [\[CrossRef\]](#)
8. Mishalani, R.G.; McCord, M.R. Infrastructure condition assessment, deterioration modelling, and maintenance decision making: Methodological advances and practical considerations. *J. Infrastruct. Syst.* **2006**, *12*, 145–146. [\[CrossRef\]](#)
9. Fang, Y.; Sun, L. Developing A Semi-Markov Process Model for Bridge Deterioration Prediction in Shanghai. *Sustainability* **2019**, *11*, 5524. [\[CrossRef\]](#)
10. Smilowitz, K.; Madanat, S. Optimal inspection and maintenance policies for infrastructure networks. *Comput. Civ. Infrastruct. Eng.* **2000**, *15*, 5–13. [\[CrossRef\]](#)
11. Mandiartha, P.; Duffield, C.F.; Thompson, R.G.; Wigan, M.R. Measuring pavement maintenance effectiveness using markov chains analysis. *Struct. Infrastruct. Eng.* **2016**, *13*, 844–854. [\[CrossRef\]](#)
12. Memarzadeh, M.; Pozzi, M. Integrated inspection scheduling and maintenance planning for infrastructure systems. *Comput. Civ. Infrastruct. Eng.* **2016**, *31*, 403–415. [\[CrossRef\]](#)
13. Morcous, G. Performance prediction of bridge deck systems using markov chains. *J. Perform. Constr. Facil.* **2006**, *20*, 146–155. [\[CrossRef\]](#)
14. Madanat, S.M.; Park, S.; Kuhn, K. Adaptive Optimization and Systematic Probing of Infrastructure System Maintenance Policies under Model Uncertainty. *J. Infrastruct. Syst.* **2006**, *12*, 192–198. [\[CrossRef\]](#)
15. Durango-Cohen, P.L. Maintenance and repair decision making for infrastructure facilities without a deterioration model. *J. Infrastruct. Syst.* **2016**, *10*, 1–8. [\[CrossRef\]](#)
16. Nazari, F.; Noruzoliaee, M.; Zou, B.; Mohammadian, A. Optimal facility-specific inspection and maintenance decisions under measurement uncertainty: Unifying framework. *J. Infrastruct. Syst.* **2017**, *23*, 04017036. [\[CrossRef\]](#)
17. Kim, J.; Ahn, Y.; Yeo, H. A comparative study of time-based maintenance and condition-based maintenance for optimal choice of maintenance policy. *Struct. Infrastruct. Eng.* **2016**, *12*, 1525–1536. [\[CrossRef\]](#)
18. Hong, T.; Chae, M.J.; Kim, D.; Koo, C.; Lee, K.S.; Chin, K.H. Infrastructure asset management system for bridge projects in south korea. *KSCE J. Civ. Eng.* **2013**, *17*, 1551–1561. [\[CrossRef\]](#)
19. Moreira, A.V.; Fwa, T.F.; Oliveira, J.R.M.; Costa, L. Coordination of user and agency costs using two-level approach for pavement management optimization. *Transp. Res. Rec. J. Transp. Res. Board* **2017**, *2639*, 110–118. [\[CrossRef\]](#)
20. Durango-Cohen, P.L.; Madanat, S.M. Optimization of inspection and maintenance decisions for infrastructure facilities under performance model uncertainty: A quasi-bayes approach. *Transp. Res. Part A Policy Pr.* **2008**, *42*, 1074–1085. [\[CrossRef\]](#)
21. Mishalani, R.G.; Madanat, S.M. Computation of infrastructure transition probabilities using stochastic duration models. *J. Infrastruct. Syst.* **2002**, *8*, 139–148. [\[CrossRef\]](#)
22. Yang, Y.N.; Pam, H.J.; Kumaraswamy, M.M. Framework development of performance prediction models for concrete bridges. *J. Transp. Eng.* **2009**, *135*, 545–554. [\[CrossRef\]](#)
23. Madanat, S.M.; Ben-Akiva, M. Optimal inspection and repair policies for transportation facilities. *Transp. Sci.* **1994**, *28*, 55–62. [\[CrossRef\]](#)
24. Liang, W.Y.; Huang, C.C.; Lin, Y.C.; Chang, T.H.; Meng, H.S. The multi-objective label correcting algorithm for supply chain modeling. *Int. J. Prod. Econ.* **2013**, *142*, 172–178. [\[CrossRef\]](#)
25. Hu, X.; Samer, M. Determination of optimal mr&r policies for retaining life-cycle connectivity of bridge networks. *J. Infrastruct. Syst.* **2015**, *21*, 04014042.
26. Lee, S.Y.; Park, W.; Ok, S.Y.; Koh, H.M. Preference-based maintenance planning for deteriorating bridges under multi-objective optimisation framework. *Struct. Infrastruct. Eng.* **2011**, *7*, 633–644. [\[CrossRef\]](#)
27. Chen, L.; Henning, T.F.P.; Raith, A.; Shamseldin, A.Y. Multiobjective optimization for maintenance decision making in infrastructure asset management. *J. Manag. Eng.* **2015**, *31*, 04015015. [\[CrossRef\]](#)
28. Kallen, M.J.; Van Noortwijk, J.M. Optimal periodic inspection of a deterioration process with sequential condition states. *Int. J. Press. Vessel. Pip.* **2006**, *83*, 249–255. [\[CrossRef\]](#)
29. Golabi, K.; Kulkarni, R.B.; Way, G.B. A statewide pavement management system. *Interfaces* **1982**, *12*, 5–21. [\[CrossRef\]](#)

-
30. Yang, Y.N.; Kumaraswamy, M.M. Towards life-cycle focused infrastructure maintenance for concrete bridges. *Facilities* **2011**, *29*, 577–590. [[CrossRef](#)]
 31. Morcous, G.; Lounis, Z. Maintenance optimization of infrastructure networks using genetic algorithms. *Autom. Constr.* **2005**, *14*, 129–142. [[CrossRef](#)]
 32. Lee, C.K.; Kim, S.K. GA-based algorithm for selecting optimal repair and rehabilitation methods for reinforced concrete (RC) bridge decks. *Autom. Constr.* **2007**, *16*, 153–164. [[CrossRef](#)]
 33. Cheng, T.M.; Yan, R.Z. Integrating messy genetic algorithms and simulation to optimize resource utilization. *Comput. Civ. Infrastruct. Eng.* **2009**, *24*, 401–415. [[CrossRef](#)]
 34. Berenguer, C.; Chu, C.; Grall, A. Inspection and maintenance planning: An application of semi-Markov decision processes. *J. Intell. Manuf.* **1997**, *8*, 467–476. [[CrossRef](#)]
 35. Thomas, O.; Sobanjo, J. Semi-markov models for the deterioration of bridge elements. *J. Infrastruct. Syst.* **2016**, *22*, 04016010. [[CrossRef](#)]
 36. Yang, Y.N.; Kumaraswamy, M.M.; Pam, H.J.; Xie, H.M. Integrating semiparametric and parametric models in survival analysis of bridge element deterioration. *J. Infrastruct. Syst.* **2013**, *19*, 176–185. [[CrossRef](#)]
 37. Benitez, P.; Rocha, E.; Talukdar, S.; Varum, H.; Rodrigues, F. Efficiency analysis of optimal inspection management for reinforced concrete structures under carbonation-induced corrosion risk. *Constr. Build. Mater.* **2019**, *211*, 1000–1012. [[CrossRef](#)]
 38. Vicent, P.-P.; Tatiana, G.-S.; José, M.; Víctor, Y. An optimization-lca of a prestressed concrete precast bridge. *Sustainability* **2018**, *10*, 685.