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Decision-Making Rules and the Influence of Memory Data

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Abstract: The problems that decision-makers face can escalate under imbalances, turbulent development, risks, uncertainties, disasters, and other influences. The development of processes in technical and economic structures is generally considered complex and chaotic, and it usually expands into innumerable dynamic influences. The paper focuses on the evaluation of the decision criteria choice structure, such as the factual cause of the consequences (e.g., future threats, opportunities, chances, occasion). It offers a graphical vision of the future forecast. It draws attention to prevention and prophylaxis versus criterion-generated time–space (TS). The paper deals with the question: Is it possible to choose and recommend the right time and place of process activities? The paper formulates a positive answer and illustrates a range of consequences. Developed activities (investment, production, etc.) take place in a defined TS; over time, they create new time-series states and expand the space by defining processes as a time series of activities. In a broader context, the article deals with the issue of the lifecycle of decision rules (dynamic proposal of opportunities) as the first step of decision making, i.e., the decision about the existence of opportunity. On the one hand, it respects static applications based on equilibrium states, while on the other hand, it draws attention to the need for a dynamic view of turbulent, dynamic, chaotic, and nonlinear phenomena.

Keywords: decision making; decision rules; data memory; decision opportunity; utility; decision space; cellular automata

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1. Introduction

The modern debate about sustainability encompasses the application of commonly used complex simulations that are considered useful to predict tomorrow's world opportunities with good accuracy [1–4]. Some authors, such as [5], claimed that the economics and financial industry and risk management, in particular, need more advanced and detailed mathematical models [6–8]. Others, especially representatives of the Austrian School of Economics [9], have long denied the appropriateness of mathematical methods for economic and technical economic research. The approach has deep roots and, in part, good and simplistic arguments.

Tarasov aptly summarized the differences in opinion conducted in the 20th century and late 19th century by stating that modern economics was born in the marginal revolution and the Keynesian revolution. These revolutions led to the emergence of fundamental concepts and methods in economic theory, which allow for the use of differential and integral calculus to describe economic phenomena, effects, and processes. At present, the new revolution, which can be called memory revolution, is taking place in modern economics [10]. In this context, we are addressing the development of discipline dynamics economic [11–13] and the narrow profiles of the methods and theories used to date [1,14,15]. The works of Mandelbrot are still interesting contributions (see [16,17]). Statistical models challenged by the authors in [18] emphasize the need to capture unexpected events, and the authors proposed the use of fractals. Fractals are, in Mandelbrot´s approach, tied to the geometrical criteria. The approach presented here shapes decision rules (DRs) as a logical construct (based on verbal formulated premises and valid in time). Therefore, DRs form the decision space [19], and moreover, they concretize the admissibility of potential solutions (e.g., variants, alternatives). The DRs fulfil the dual role of being judges (for the past) on the imaginary time axis of the simulation calculus and being legislators (for the prospect of the future).

Fractal features appear in almost any economics data and any modern project realization. However, all real projects have strict dynamic behavior in time. Most calculations about such a project are based on deterministic equilibrium, such as in the design, financing, use and innovation. The need of a more sophisticated theoretical approach is desirable, and more practical experience is necessary. There are many different project realizations. The general theory and analysis are, therefore, extremely difficult. An economical dynamic of fractals is still developing. The path and direction of the theory may already be known, but an effective application tool is still missing.

The initial state is determined by assigning a starting position for a cell in time sequence $t = 1, 2, 3, \ldots, m$ as the utility value $u(t)$ [20]. Any new generation step is created by following a logical decision rule. The rule determines the new states of the current short-term ex post situation in its past neighborhood.

The idea of a 2D cellular automaton (CA) was originally discovered in the 1940s by Stanislaw Ulam and John von Neumann at the US Los Alamos National Laboratory and presented as Conway's Game of Life (see [21] for more). The idea was subject to significant academic debate, through which scholars developed discrete simulation models that are studied in mathematics, physics, and theoretical biology, which can be found in Wolfram's *Approaches to Complexity Engineering*, as seen in [22,23]. However, the idea's applications in an economic context have been fragmented into many professional fields [17,18]. Furthermore, the author in [10] classified, in his review, the development of dynamic models in economics into five stages: autoregressive fractionally integrated moving average (ARFIMA) [24], fractional Brownian motion, econophysics, deterministic chaos, and mathematical economics. The affinity of Turing-completeness (for simulating data manipulation sets) to CA was published by Wolfram in [25], who pointed out CA applications in many science areas.

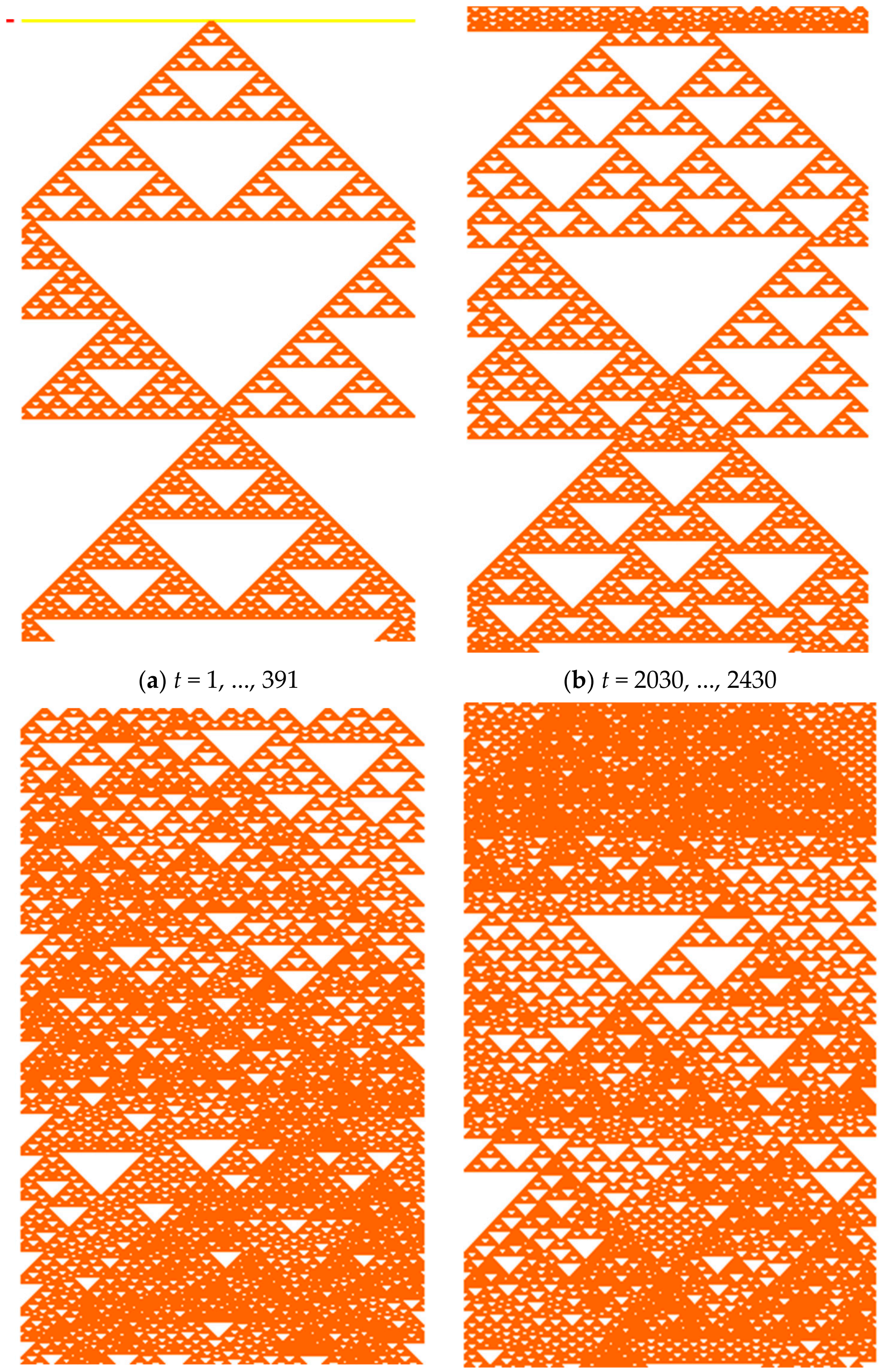
A decision rule in this paper is a deterministic function that generates a structure of opportunity actions over a problem-oriented time–space. The processing is devoted to aspects that are passable for IT deterministic simulation calculations.

This paper focuses on the interpretation of CA/fractals and on the economic context, that is, by means of combinational logic devoted to the formation of the decision space. The difference from Mandelbrot's approach lies in the formation of criteria. Mandelbrot forms criteria on the geometric basis. The general explanation about the decision process is given in the second subsection of Section 3 (i.e., Analysis).

The decision process and paper structure are based on three steps:

- Space for decision making (i.e., formation): Most of the arguments for fractal dynamic modelling are included in Section 2.2. Figure 1 shows the long-term application of a decision rule in the dynamic decision space $S_A = (\mathbf{A}, t)$, where $\mathbf{A}$ is a set of activities $A_{t,\,j}$ located in time t and their affiliation j to S_A for tracking opportunities as (0;1) or (N;Y), or expressed as colors. The choice or activity is determined by professional regulation (standards, laws, guideline, etc.);
- Decision making: Opportunities in S_A are calculated using decision rules $D_{opportunities} = \{(\mathbf{D}_{\text{rules}}, \mathbf{M}_{\text{data}}) \mid S_A\}$. A look at the calculation of opportunities is offered in Section 3. It deals with a basic decision rule grounded on a general, verbally formulated logical requirement. Comparing memory $\mathbf{M}_{\text{data}}$ dependence is the topic of the paper. This problem is treated as the productivity of the $\mathbf{D}_{\text{rule}}$ in Section 3 as well as Figures 2 and 3;
- Decision evaluation: Utility is the final step addressed in the article. Section 4 addresses the need for valuation opportunities. More implementation is offered in

Section 5. Both the space for decision making and the decision making itself create an introduction to practical application in the Appendix A example. The evaluation of the utility is also listed in the illustrative example as an evaluation of the frequency of maintenance and renewal interventions. The extensive construction of utility functions is, however, a separate problem in the design of decision rules.



(**a**) $t = 1, \ldots, 391$ (**b**) $t = 2030, \ldots, 2430$ (**c**) $t = 6630, \ldots, 7040$ (**d**) $t = 9630, \ldots, 10050$

Figure 1. Parts (**a**–**d**) of matrix U, where the blocks of time are noted below the figure as (**a**–**d**). The time unit is malleable and defined in the context of application (i.e., it could be a day, a week, etc.).

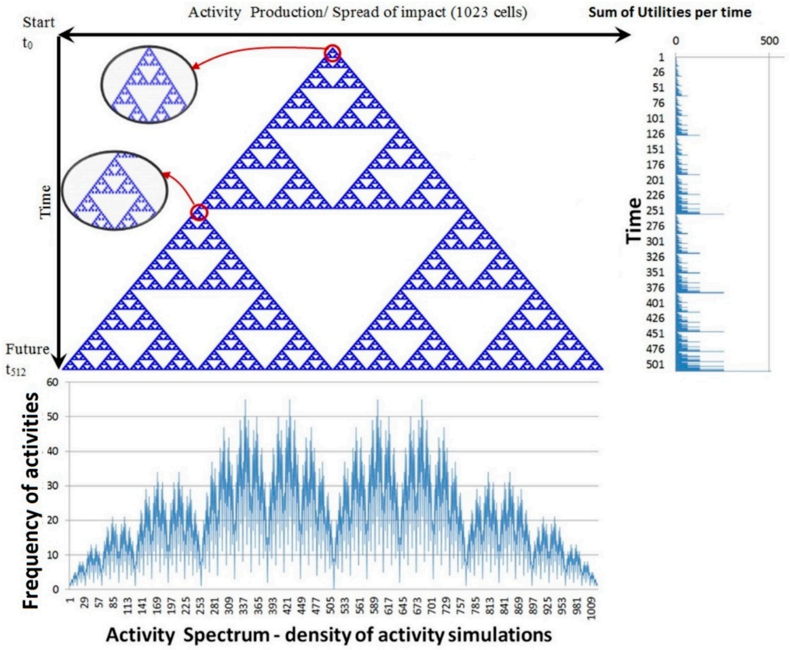


Figure 2. Simulation for decision rule D_α: *IF*$((A_{t-1,j-1} = A_{t-1,j+1})$ *then* 0 ⇒ "white", *else* 1 ⇒ "blue"). Calculation support found in [24]. Notes: Range of activities n = 1023. Number of time steps, m = 512. S□ = 1023 × 512 = 523,776, which is the total space that is available to be influenced by the horizon T = 512; SΔ = $\frac{1023 \times 512}{2}$ = 261,888 cells, is the decision space influenced by D_α; S▲ = $\sum u_{t,j}$, is the total utility potential, influenced by the decision rule $\boldsymbol{\alpha}$.

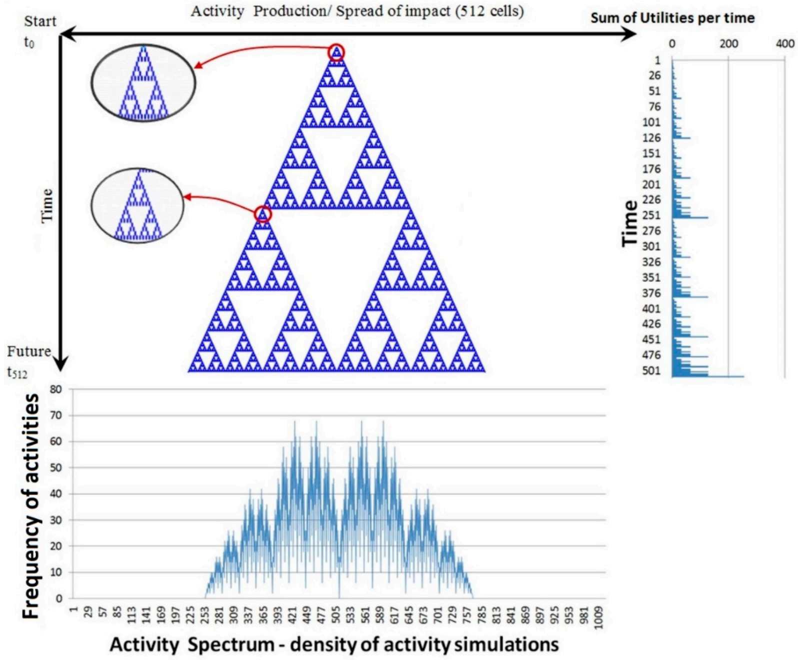


Figure 3. Process scheme period for the decision rule D_β, taking into account the data from the second most recent data $t - 2$ from the closest right and left neighbors, which in this case is the rule Scheme. Comments: Range of activities covered by the decision rule in t = 512. Number of time steps: S□ = 1023 × 512 = 523,773; total space available to be influenced, $S^\beta{}_\Delta$ = $\frac{512 \times 512}{2}$ = 131,072 cells, influenced space by the decision rule D_β, $S^\beta{}_\blacktriangle$ = $\sum u_{t,j}$, the total utility potential, influenced by the decision rule β, decision rule D_β as $u_{t,j}$ ⇔ *IF*$((A_{t-2,j-1} = A_{t-2,j+1})$ *then* $u_{t,j\,=}$ 0 ⇒ white, *else* $u_{t,\mathrm{j}\,=}$ *1 ⇒ blue).*

The decision process is constituted of (a) decision-making rule D [26], (b) a decision-making space S [23], (c) the decision data structure (M, memory data [27–29], which are data about past decisions), and (d) the evaluation of the decision space positions (U, utility set) [30].

2. Materials and Methods

2.1. The Decision Process—Decision Rule Horizon and Decision Rule Memory

The decision criterion (i.e., the decision rule) generates (or forms) the decision space. In the present paper, the economic applications are illustrated as a 2D space (t,j) for economic decision-making in time (sequence $t = 1, \ldots, m$) as one dimension and activities ($j = 1, \ldots, n$) as the second dimension of S. The calculation of the benefit of activity A_{tj} in the decision space is assessed in terms of potential utility u_{tj} (i.e., it is awarded utility, e.g., 0/functional award (value); or 0/1; false/true; impossible/possible; keep/sell; white/red; −100/+500). The assessment in decision-making requires calculation evaluation. The evaluation creates values $u(t,j)$, and these represent the economic interpretation [30,31]. The result is a matrix U, which covers the decision space, as presented in (1), that is, they are ex post values for the present state data. With advancing calculation time steps, the data of activities of the present A_{tj} become data about the past $A_{t\text{-}\tau,j}$, and are read as history and memory

$$U = \begin{bmatrix} u_{1,1} & \cdots & u_{1,j} & \cdots & u_{1,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ u_{t,1} & \cdots & u_{t,j} & \cdots & u_{t,n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ u_{m,1} & \cdots & u_{m,j} & \cdots & u_{m,n} \end{bmatrix} \tag{1}$$

where A_{tj} is an activity j adjusted (implemented) in time t.

The decision rules, D, have a broad interpretation in this article (see the formulation in (2) for more details). The D rules can be defined either on a geometric basis (e.g., [16]), as a mathematical function [25], or as a statistical dataset [17]. This paper prefers a logical structure, economic interpretation, rationality, or aesthetic principles, among other things. In most practical applications, the rudimentary decision criteria shape the space in time, which is described here as a dynamic development. In a multifaceted socioeconomic environment [32,33], which we perceive as a dynamic process [34], we recognize decision rules (criteria) as a dominant element. The paradox is that even if an economic decision criterion is held steady over time, this changes the opportunities inside the decision-making space (see Figure 1a and compare with Figure 1b–d).

For example, it is true that during different periods, the same set of rules can lead to significantly different results. Specifically, rules may lead to a certain outcome in the context of long-term decision making, represented here as D_{Long}. The time horizon T is significantly distanced from the initial t (i.e., $T >> t$), and shows various outcomes; this is seen in the context of short-term decision-making (D_{Short}), where $T = 1$ or 2, or decisions are typically developed without time horizons, that is, $T = 0$. In economics, decision rules typically serve the pragmatic purpose of selecting a goal, setting preferences, selecting options, identifying alternatives, representing regulatory requirements, and other acts of predominantly short-term execution. The substantive impact on decisions is the fuzzy time horizon (short-, medium-, long-term). Economic decision criteria are predominantly based on defined logical conditions and mostly relate to a general experience.

Figure 1 shows how different the decision-making situation is in the relatively short-term time horizon (see Figure 1a,b) and for the long-term horizon (see Figure 1c,d). The visual differences are already striking: the short-term and long-term time horizon T evokes the "experience" that the rule for long-term decision-making creates a more significant density of application (i.e., "opportunities").

One further important component is to be noted, namely, the memory of data [35]. In an economic context, a decision rule is related to ex post events, which are events with an evaluated history. In other words, the data memory is fixed to time $(t - \tau)$. If decision criteria use data from the past to develop an excluded decision rule with memory, the assumption is typically made to implement the decision rule D based on a general formula, such as

$$IF\ ((D \text{ is true})\ then\ (\text{evaluation } A_{tj} \text{ as } u_{tj} = 0)\ else\ (\text{evaluation } A_{tj} \text{ as } u_{tj} = 1)) \quad (2)$$

Every space element (t,j) may host activities A_{tj} or remain empty. The decision rule distinguishes the suitability of location (t, j) for activity A_{tj}. The utility can be evaluated as

$$u_{tj} = F(A_{tj} \mid N) \quad (3)$$

where N is the selected neighborhood of the cell (t, j) and F is an arbitrary functional specifying the utility value for the (t, j) location. The economic interpretation of the neighborhoods are logical rules or administrative rules, legislation, environmental requirements, etc. In this paper, the analysis will simply refer to the calculation based only on a set of ex post data.

If the analysis focuses on the current state $t = 0$ and a reflection on the last past outcome τ as -1, or the last and penultimate result -1 and -2, etc., then it is said to consider the decision rule memory. The delay τ defines the range of memory and specifies (or influences) the phenomenon of the actual economic opportunity. This is represented graphically in Figure 1a–d. More details are given in Figures 2 and 4. The short-term memory decision rule D_α is set as

$$IF((A_{t-1,j-1} = A_{t-1,j+1})\ then\ u_{t,j} = \mathbf{0} \Rightarrow \text{field fill } \mathbf{white}\ else\ u_{t,j} = \mathbf{1} \Rightarrow \text{field fill } \mathbf{red} \quad (4)$$

DECISION SPACE for $A_{t,j}$		**Activities**							
		1	2	…	j-1	j	j+1	…	n
Long term past …	…			…				…	
	…			…				…	
	…	…	…	…	…	…	…	…	…
	t-3			…				…	
	t-2			…				…	
Short term past …	t-1			…	(t-1, j-1)		(t+1, j+1)	…	
Actual time position …	t			…		(t, j)		…	
	t+1			…				…	
	t+2			…				…	
	…	…	…	…	…	…	…	…	…
Time horizont …	T			…				…	
	…			…				…	
Simulation limit …	m			…				…	

Figure 4. The description of decision space and labeling of particular activities, $A_{t,j}$, for different time points $t = 1, 2, \ldots, m$ and past and future time periods.

The color fill in Equation (4) simplifies the interpretation of Equation (2). The decision rule D_α is described as follows: When the last past period of both neighboring activities is successful, or when both activities are unsuccessful, no activity will be developed (white in the graph). The success of one of the neighboring activities in the past leads to the decision to implement one's own activity. Figures 1a and 2 have an identical initial simulation step. Further steps in Figure 1a–d show how long the horizon can change the structure of opportunities.

The illustration is given in Figure 1 (see Figures 3 and 4 for more) and verifies the demand for more comprehensive research of decision rules.

In a broader view, we touch on the sensitive issue of long-life dynamics decision-making. It is visible from practice that there is a gap in the methods used to date. This is mainly reflected in tenders and projects with a long lifecycle, innovative projects, etc. Some questions that emerge as follows: (a) to what extent D is bound to the space S_A; (b) to what degree are the benefits of U related to the functionality of D over time; (c) to which extent are the benefits of U are related to the link between D and historical (or memory) data; (d) to what extent is the lack of opportunities in S_A (opportunities = 0) an obstacle to the development of the investigated technical–economic projects.

2.2. Research Question

The essential question is to what extent the construction of the decision rules influences the decision space and how intensively the data memory might influence the decision space. If the data history significantly shapes the decision space, there is a need to give careful consideration to this information. Stored data about the past are relevant for the present and might shape the decision space for the future.

Real data sources typically represent a technical economic process. For this analysis, we substitute a real process with CA. Furthermore, we identify the process as a fractal created on the basis of decision rules, extending the interpretation of [17]. It is assumed that the decision rule is implemented in a process, according to Expression (2), whereby this expression is a schematic model that can be applied to most activities in a technical–economic context, as per Figure 2.

Typical examples of interpretations of this analytical approach are life-governing rules that factually form a dynamic process. The sequence (i.e., the dynamic) of time and space is often considered defined by increasing completeness and efficiency. Accordingly, the main focus of this paper is to shed light on this.

Specifically, the paper develops a response to the following questions:

- To what extent does the construction of the decision rules influence the decision space? This question reflects the considerations posed by [36];
- How intensively might the data memory influence the decision space?

We see a dispersion of activities (investment, knowledge, etc.) as a result of the technical–economic rules (built-in technicalities or prescribed functional properties). Section 3 further deals with the issue of evaluation of the decision criteria in detail. This concept is applicable in many areas of development, innovation, construction, and architecture. The present paper applies this concept to the development of technical–economic processes that represent regional development, investment, etc. [6,37]. An illustrative example is given in Appendix A, Figure A2, i.e., the horizontal form of the decision space S_A (read as $S_{\text{Bridge maintenance}}$), time steps 1–80.

The frequency of activities applied acts across time and per space of activities, as shown in Figure 2 and comparison to Figures 3–5. The spread of activities in time depends on the decision rule and the use of historical data. The rule states that if the former time period was successful for one of the next neighboring activities in the immediate past, the decision makers need to build on the success while in the current position (that is, initial innovation occurs, knowledge is gained, investment follows, along with further development [38]). It is believed that memory, information, and experience from the last time period are sufficient for making a decision in the current time period [29]. In other words, the memory of the recent time period is the primary enabler of decision-making for the current period.

The interpretation of memory data is frequently related to computer science and operation systems in practice [27]. In the context of the simulation described above, we could speak about dynamic time-dependent data and data memory that is constantly refreshed.

Business managers, engineers, and architects will face this dilemma at many points in time, and they make a decision based on the best information available to them at that point in time [39]. The decision maker has to evaluate whether the information is complete, unbiased, and authentic. The quantity and quality of data available for decision making become important for them to understand the quality of the decision made. Due to the uncertainty [40,41] of information, which is inherent to the dynamic situation of the business or other practical contexts, a decision maker has to evaluate an uncertain outcome, which is considered a random variable [42].

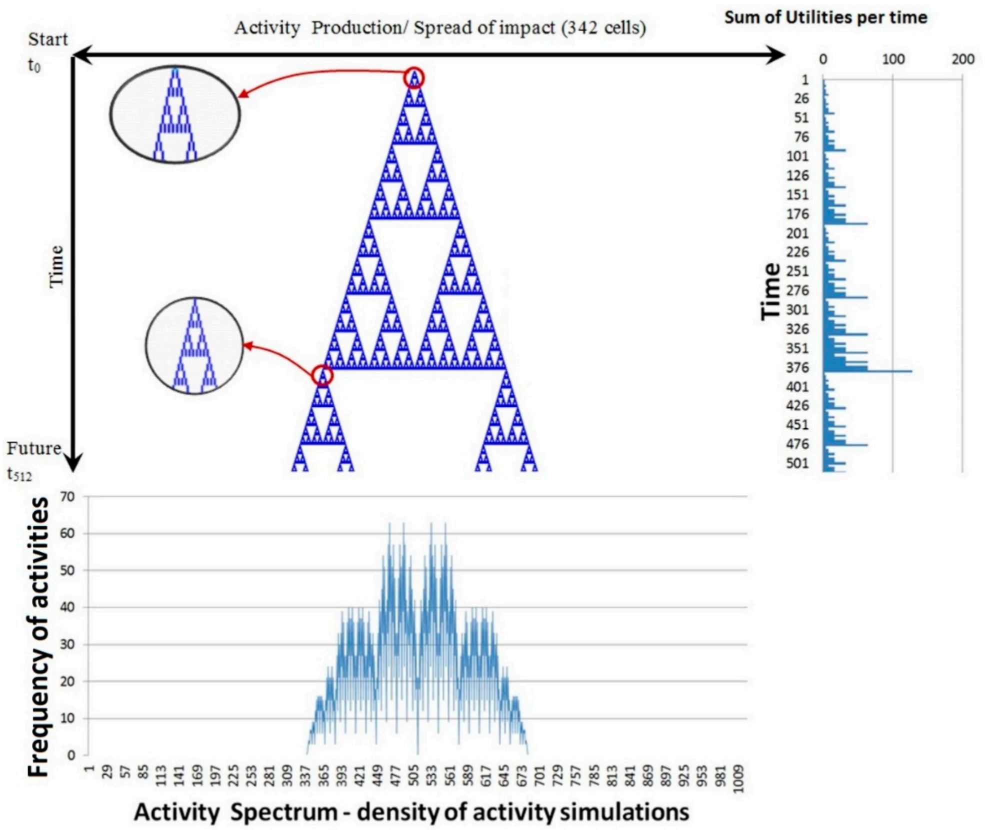


Figure 5. Process scheme for the decision rule D_γ, taking into account the third most recent historical data $t-3$, from the closest right and left neighbors. In this case, the rule starts to apply from the $t-3$ period. Comments: Range of activities covered by the decision rule in t = 512. Number of time periods: S☐ = 1023 × 512 = 523,773; the total space available to be influenced, $S^{\gamma}{}_{\Delta} = \frac{342 \times 512}{2}$ = 87,552 cells, influenced space by the decision rule γ, $S^{\gamma}{}_{\blacktriangle} = \sum u_{t,j}$, total utility potential. Decision rule D_γ as $u_{t,j} \Leftrightarrow IF((A_{t-3,j-1} = A_{t-3,j+1})$ *then* $u_{t,j} = 0 \Rightarrow$ white, *else* $u_{t,j} = 1 \Rightarrow$ blue).

2.3. Research Hypotheses

Three hypotheses can be developed to provide insight into the mechanism of decision making and data memory:

- Decision rules change the operational decision space for the decision maker depending on the time horizon of the memory used, e.g., $(t-1)$, $(t-2)$;
- The spread of activities in time is conditioned by the chosen DR. Comment: Industries and companies typically request stable regulation rules (in terms of regulatory, economic, ecological, or political rules, etc.), mostly without calculating the expected opportunities and their usefulness;
- The DR simulation utility characterizes the decision criterion both for (a) a schedule of opportunities and (b) individual activities over time.

3. Analysis

The previous chapter dealt with the broader context of the effects of the decision rule on the decision space and the potential of its use. An illustrative example is shown in Figure 3. To select suitable solutions (variants, alternatives of DR), it is necessary to quantify the use of created opportunities, their expiration, and intensification [43]. The DRC used for the simulation of opportunities in Figure 2 applies its basic graphical form, as given in Figure 3.

Using the decision rule, the structure of implementation opportunities to achieve reasonable utility outcomes needs indicators.

The opportunities created are unquestionable. These are the intensities of opportunities measured by the frequency of their occurrence in individual time periods and individual activities forming the monitored project. They are the main evaluation content in Figure 2 [44].

The sums of utilities in the time (applied activities in time sequence) are shown in the right part, by a vertical graph. They characterize the use of activities (individual utilities, in the sense of Equation (3), are evaluated as 1), introduced for simplicity: applied activity = utility = 1. Similarly, the frequency of application of individual activities in the simulation time sequence is characterized by the graph at the bottom of Figure 2. Fluctuation in the utilities is provided by individual activities.

The immediate characteristics that are offered for quantification are as follows:

(a) The total space S☐ for the involved activities in the project: S☐ = 1023 activities × 512 time horizon = 523,776 potential interactions;
(b) Total application space after D_α activation: SΔ = 1023/2 × 512 = 261,888 potential interactions;
(c) Total space after D_α simulation: S▲ = 19,171 proposed interactions according to D_α.

From the comparison of the defined space of activities SΔ and the total potential of concentrated activities S☐, we obtain (SΔ/S☐ = 261,888/523,776); we interpret this as the percentage of the potential of allocated activities. We see that for the limit purposes in D_α, it is possible to use only 50.00% of the total interaction space. The D_α reduces the available space for SΔ application in Figure 2 to 7.32%; the simulation of opportunities on the basis of D_α reduces these spaces further—fractal simulated opportunities use 3.66% of S☐ space.

The economic consequences and limits for different applications are significant in this context of dispersion. Application deals with opportunity and possibility. These are mostly investments, innovations, the application of inventions, patents, development strategies, and more [45]. The usability of opportunity at the level of percentage units is only associated with the usability of invested resources. In the current interpretation, we move on to the level of generalizing and reasoning. The effort to improve the productivity and efficiency of the decision-making rule through deeper binding to data obtained from the historical past has its limits.

In the following text, the article deals with the question of the consequences of connecting the criteria with historical data (as in statistical data, databases of knowledge, experience, evaluated experiments, and others).

The economic consequences and limits for different applications are significant in this context of dispersion. Application deals with opportunities and possibility. These are mostly investments, innovations, the application of inventions, patents, development strategies, and more. The usability of opportunity at the level of percentage units is only associated with the usability of invested resources. In the current interpretation, we move onto the level of generalizing and reasoning. The effort to improve the productivity and efficiency of the decision-making rule through a deeper binding to the data obtained from the historical past has its limits.

3.1. The Influence of Data Memory (i.e., Data History) on Decision Rules

The creation of the process structure in Figure 2 is based on the use of the most recently available ex post data ($t - 1$); the decision rule D_α, described earlier, is summarized in Table 1, and the simulation results shows and clarifies Figure 3. Similarly, Figure 4 is based on the ($t - 2$) memory scheme in Table 2, and the last ($t - 3$) of Table 3 results in Figure 5.

We need to distinguish the data forms of the past and current data used in the decision rule and the future (data ex ante). Suppose that Equation (2) represents a real process (such as a project of investment or innovation benefits or the spread of knowledge) that occurs in the market environment and causes the placement of quantitative parameters. This is interpreted as a market product and is presented in Figure 2 as blue-filled spaces.

Table 1. Decisions rule D_α on the basis of the most recent $(t-1)$ memory data.

Decision Rule D_α for $A_{t,j}$	Memory Data $(t-1)$			Decision on t about	Remarks
	$(j-1)$	j	$(j+1)$	$A_{t,j}$	
$u_{t,j} \Leftrightarrow$ IF$((A_{t-1,j-1} = A_{t-1,j+1})$ *then* 0 *else* 1)	0	~	0	0	Due to the negative result for $A_{t-1,j-1}$ and $A_{t-1,j+1}$ (*) in $t-1$, the rule D_α, states that the action is not recommended.
	1	~	1	0	Due to the full capacity for $A_{t-1,j-1}$ and $A_{t-1,j+1}$ (*) in $t-1$, the action is not recommended
	0	~	1	1	Due to the positive outcome in $t-1$ and the free capacity (**), the action is recommended
	1	~	0	1	Due to the positive outcome in $t-1$ and the free capacity (**), the action is recommended

Note: (*) can be interpreted as innovation, spread of knowledge, development, inventions, investment, etc. (**) for $t = 0, 1, 2, \ldots, m$ and for $j = 0, 1, 2, \ldots, m$.

Table 2. Decision rule D_β for the second most recent $(t-2)$ memory data.

Decision Rule D_β for $A_{t,j}$	Memory Data $(t-2)$			Decision on t about	Remarks
	$(j-1)$	j	$(j+1)$	$A_{t,j}$	
$u_{t,j} \Leftrightarrow$ IF$((A_{t-2,j-1} = A_{t-2,j+1})$ *then* $u_{t,j} = 0$ *else* $u_{t,j} = 1$)	0	~	0	0	Due to the lack of previous experience (*), the action is *not recommended*.
	1	~	1	0	Due to the fully exhausted capacity in the previous period (*), the action is *not recommended*.
	0	~	1	1	Due to the one positive outcome in $t-2$ and the free capacity (**), the action is *recommended*
	1	~	0	1	Due to the positive outcome in $t-2$ and the free capacity (**), the action is *recommended*

Note: (*) can be interpreted as innovation, spread of knowledge, development, inventions, investment, etc. (**) for $t = 0, 1, 2, \ldots, m$ and for $i = 0, 1, 2, \ldots, m$.

Table 3. Decision rule D_γ for the second most recent $(t-3)$ memory data.

Decision Rule D_γ for $A_{t,j}$	Memory Data $(t-3)$			Decision on t about	Remarks
	$(j-1)$	j	$(j+1)$	$A_{t,j}$	
$u_{t,j} \Leftrightarrow$ IF$((A_{t-3,j-1} = A_{t-3,j+1})$ *then* $u_{t,j} = 0$ *else* $u_{t,j} = 1$)	0	~	0	0	Due to the lack of previous experience (*), the action is not recommended.
	1	~	1	0	Due to the fully exhausted capacity in the previous period (*), the action is not recommended.
	0	~	1	1	Due to the one positive outcome in $t-3$ and the free capacity (**), the action is recommended
	1	~	0	1	Due to the positive outcome in $t-3$ and the free capacity (**), the action is recommended.

Note: (*) can be interpreted as innovation, spread of knowledge, development, inventions, investment, etc. (**) for $t = 0, 1, 2, \ldots, m$ and for $i = 0, 1, 2, \ldots, m$.

To have a practical interpretation in mind, it can be assumed that Figures 1–4 show the decision rules, which are the basis for a potentially successful strategic decision (e.g., investment or another economic asset). The calculation conditions are detailed in Tables 1–4. The purpose is to draw attention to the possibility of creating computational decision rules based on a text source or verbal formulation of decision rules. The formulations, which cannot be converted into a decision rule, most likely contain a formulation or interpretation defect. Simultaneously, with the transfer of the text to the decision rule, it is expedient to assess the question regarding to what extent it is desirable to use (incorporate) data with memory (statistical data, experience, and more) into the decision rules.

The usefulness or benefit created by means of the decision rules α, β, and γ as a tree structure is presented in particular time periods in Figures 2, 4 and 5. Details of the structure are offered in the frequency graphs on the right side and bottom of these figures. It is possible to distinguish the cycle and the excessive frequency values and volatility in some time periods for the decision rules α, β, and γ. The decision rules differ across time series and enable a comparison of decision rule implementation efficiency.

Table 4. Summary of findings according to the decision rules based by data memory, developed on extensions [44].

D Rule and Data Memory	Decision Rule Formula for 512 Periods of Simulations	Max Spread of A_j in t = 512	Total Space Available $S_{\square}$=a × b=1023×512 for $D_{\alpha\beta\gamma}$	Influenced Space Available $S_{\Delta}=\frac{a \times b}{2}$	Space Used $S_{\blacktriangle}$	Of S□ Used S_{Δ} as %	Of S_{Δ} Used $S_{\blacktriangle}$ as %
-	(a)	(b)	(c)	(d)	(e)	(f) = e/c	(g) = e/d
D_{α}: $(t-1)$	$u_{t,j} \Leftrightarrow IF((A_{t-1,j-1} = A_{t-1,j+1})$ *then* $u_{t,j=}$ 0 ⇒ white *else* $u_{t,j=}$ 1 ⇒ blue)	1023	523,776	261,888	19,683	3.76%	7.52%
D_{β}: $(t-2)$	$u_{t,j} \Leftrightarrow IF((A_{t-2,j-1} = A_{t-2,j+1})$ *then* $u_{t,j=}$ 0 ⇒ white *else* $u_{t,j=}$ 1 ⇒ blue)	512	523,776	131,072	13,123	2.51%	10.01%
D_{γ}: $(t-3)$	$u_{t,j} \Leftrightarrow IF((A_{t-3,j-1} = A_{t-3,j+1})$ *then* $u_{t,j=}$ 0 ⇒ white *else* $u_{t,j=}$ 1 ⇒ blue)	342	523,776	87,552	8493	1.62%	9.70%

Notes: Data in columns represent the influence of memory data: (a) Formulation of decision rules for 512 simulation steps results in Figures 2, 4 and 5. (b) S□ used for percentage comparison, both as the basis of the fraction and as an indication that it is still the same space with a range (t = 512, j = 1 to 1023 of potential activities). (c) The spread of activities based on memory-dependent decision rules D_{α}, D_{β}, and D_{γ}. (d) The total space available S□. (e) The potential space S_{Δ} required for simulation; D_{α}, D_{β}, and D_{γ} influenced and initiated decrease from 261,888 (=100%) to 131,072 = 50% and 87,552 = 33.4%. (f) The space $S_{\blacktriangle}$, used as application opportunities offered by memory-dependent decision rules D_{α}, D_{β}, and D_{γ}. (g) The space use S_{Δ} in %; S□ = 100%. (h) The space use $S_{\blacktriangle}$ in %; S_{Δ} = 100%.

It can be seen in Table 4 that, for example, productivity for $A_{t,j}$ is the highest for decision rule α.

3.2. Comparison of Decision Rule Productivity

The decision rules influence the outputs of the process under investigation and its utility (productivity). The decision rules based on ex post data $(t-1)$, $(t-2)$, $(t-3)$ explained in Tables 1–3 form the utility (productivity) of activities $A_{t,j}$. The shape of the decision outcome is evident from the dispersion of activities in Figures 2, 4 and 5.

The summarized results are concentrated in Table 4. The range $S_{\blacktriangle}$ points out the ability to spread the benefits of activities $A_{t,j}$ in simulating the opportunities in the defined space S_{Δ}. We see that the utilization dispersion indicates the volatility in (g) in the obtained data of Table 4.

The three research hypotheses formulated in "Section 2.3" find their confirmation in the simulations of Figures 2, 4 and 5. Dependencies are not linear; they have long-term dynamic development. The use of decision rules on memory leads to a narrowing of the application space (Table 4, columns of default data in columns (b), (d), and subsequent).

The usefulness or advantage created by the decision rules α, β, and γ as a tree structure is shown in time (simulation steps) in Figures 2, 4 and 5. Details of the structure are shown in the frequency graphs on the right and bottom of these figures. It is possible to distinguish high or low values of frequency and high volatility in some periods for decision rules α, β, and γ. Decision-making rules differ in time steps and allow for a comparison of the implementation effectiveness of the decision-making rules.

It can be seen, for example, that the spread of application opportunities (productivity) is the highest for the decision rule α. The volatility in simulation steps changes over time. Under such circumstances, significant follow-up economic and technical consequences may occur.

3.3. Productivity Efficiency of Decision Rules α, β, and γ

The summary of the results based on decision rules α, β, and γ is given in Table 4. Results are attained on the basis of long-term simulation. A number of observations can be made:

- Decisions based on the data memory narrow the range of information and space available for potential actions A_j (opportunity of economic response), as seen in Figures 2, 4 and 5;
- The inclusion of a backward (historical data) time $(t-1)$ into the decision criterion D_{α} $(t-1)$ in Figure 2 leads to a decrease in opportunities to 50.0% of the influenced space (261,888 units), compared to space available (523,776 units). The reduction of 131,072

of 523,776 units is 25% for decisions on the basis of two time periods ($t - 2$), as well as the reduction on the basis of data ($t - 3$) to 16.72% (523,773 units to 87,552) for D_γ on the basis of data ($t - 3$); details are in Table 4 and Figures 2, 4 and 5. The inclusion of one or more backward time steps into the decision criterion leads to narrowing the influenced implementation area ($S_\blacktriangle(t - 1)$ in Table 4, column (e) 19,683 as 100%) to 13,123; this is 68.4% for ($t - 2$), and 43.1% for ($t - 3$). Further comparisons are given in Table 4;
- The spread of the impact of decision rules of activities A_j slows down at a rate of tangent α; (indicator) is given as $tg(D_\alpha)$ = 0.999 to tg (D_β) = 0.500 and $tg(D_\gamma)$ = 0.334.

On this basis, conclusions can be drawn about the avoidance of risk [46,47] or conservative behavior. The economic consequences are a decrease in productivity for an ongoing process, as well as the reduction in opportunities as a future economic response.

4. Discussion

The above analysis provides the answer to fundamental questions regarding decision processes in real life. An example of a maintenance schedule is given in Appendix A. Some important questions are as follows:

- Does a decision process really act in a homogeneous decision space? The decision space is not homogenous for the rules modelling real-life situations, which were examined in this paper as decision rules (α, β, and γ). A time range of productive periods for $D_{\alpha\beta\gamma}$ exists, as do a range of unproductive time arrangements, which exhibit a lack of utility [48];
- Does the lack of homogeneity of the decision space affect the environment for implementing decisions? The decision space creates bubbles that cover substantial areas of the decision space. These bubbles indicate the absence of productive activity. Moreover, the decision space affects the utility (productivity) of subsequent time periods and parallel-running activities in the long term. To some extent, these are affected activities A_j, which can cause a long-term negative domino effect, that is, the productive space at various subsequent time steps is a sensitive structure of opportunities. The knowledge about decision consequences, which helps to choose an appropriate action in an appropriate time step, helps to generate a higher utility [49];
- To what extent is the decision in the decision layers ($t - 1$), ($t - 2$), ($t - 3$), etc., affected by decisions in previous layers (i.e., the decision history)? Binding $D_{\alpha\beta\gamma}$ to data with history causes significant changes in the decision space; it changes the range of effects that are available (see Figure 6). The pattern of productive activity is exhibited in Figures 3 and 4. This clarifies the significant modifications across the decision layers;
- To what extent is the decision in the decision layers ($t - 1$), ($t - 2$), ($t - 3$), etc., able to influence decisions in future layers?
- The pattern of active (productive) space in Figures 2, 4 and 5 shows a strong sensitivity of $D_{\alpha\beta\gamma}$ to ex ante development. Because of this, the $D_{\alpha\beta\gamma}$ exhibits similarity.

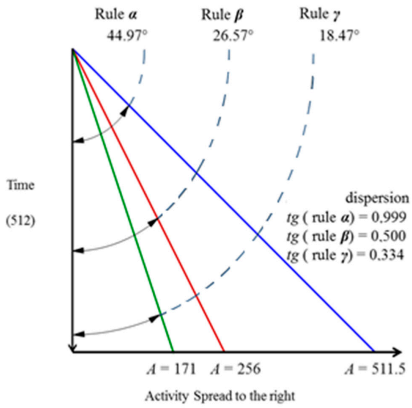


Figure 6. Change in the spread of activities, according to the decision rules α, β, and γ.

5. Conclusions

Decision making is one of the crucial tools supporting progress in any discipline. The paper argued that the prerequisite of successful development is to know the expected chances of future development. In this sense, we presented the idea about opportunities. This is an inverse to the concept of missed opportunities. Figure 1 illustrates the consequences, the long-time application of a relatively simple decision rule. The decision opportunity was introduced as $D_{opportunity} = \{(\mathbf{D}_{rule}; \mathbf{M}_{data}) \mid \mathbf{S}_A\ \}$ and is a surrogate for term prognosis, plans, prospect, etc.

A preview of the long-simulated simulation segment (a, b, c, d) suggests the following:

- A long-term life of activities included in the created decision-making space;
- The "thickening" opportunities over time;
- The existence of bubbles without application opportunities and their displacement during the lifecycle of the decision rule;
- Creating application cycles of opportunities.

Information on the consequences of the application of the decision criterion has a long-term strategic character. This creates an essential support of economic and technical disciplines.

Decision-making rules in common practice are based on the experience and knowledge created in the past. They compose the future based on the knowledge of the past. This issue was addressed in particular by the investigation of the examples in Sections 2 and 3 of this article.

We appreciate this point for situations in which transparent, mostly long-term valid databases dominate. Empirical confirmation is generally a necessary condition, but not without sufficient exceptions [50,51]. The values created by large national units should comply with the principles of evaluation, based on the idea of sustainable economic growth [52] and the need for return on investment, capital, and invention, to create long-term benefits [53,54]. However, many carefully selected activities do not achieve their goals [55,56]; for instance, the completion deadline was exceeded, the solution sustainability failed, or the externality costs unexpectedly rose [57].

Consequently, it is often misunderstood that the dynamics of the application opportunities for the intended economic and technical capital goods are dominant [58,59]. The calculation of opportunities is often overlooked, and it either did not occur or internal or external influences were excluded. The discussed fact is that future developments are difficult to detect. This argument is not full-fledged because of the current absence of tools for evaluating the decision criteria, and their application in processes depicting the space in which activities (such as large capital-intensive projects) are to be invested.

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Appendix A

An application example in a fractal environment is seen in the maintenance and renewal of a pedestrian–cyclist bridge. Engineering projects are known as investment-intensive. At the same time, they also have high costs for maintenance, renewal, and modernization during their lifetime. It is these issues that are neglected or ignored at the time of investment. The correct selection of construction variants or alternatives is essential. Figure A1a,b illustrates the two projects of the integrated column. The presented pedestrian and cyclist bridge case study follows the scheme in Figure 3, and it shapes the space for maintenance application. The individual activities of the bridge construction (i.e., structural elements) are the foundation (A1), pillars (A2), integral beams girder (A3), girder (A4), and deck (A5). The example brings the presentation closer to the usual time schedule scheme; the time axis is horizontal. In Figure A3, the information is divided into the past (information on the feasibility of individual structural elements), present (implementation), and future (forecast of the design of time intervals for the implementation of cyclic maintenance and renewal).

The study has its specific background in a number of bridge structures such as footbridges in the Czech Republic. There was also a reclassification of these structures to the category of "emergency". Photographs are shown in Figure A1a,b. They illustrate the new design solutions chosen for the new installations in Prague–Troja and Pisek.

(**a**)

(**b**)

Figure A1. Transverse profile of the pedestrian and cyclists' bridge. (**a**) Prague–Troja in 2020, (**b**) Pisek-Pleskot pedestrian and cyclist bridge in 2018. (**a**) Reproduced with permission from S. Vojtasek, *Construction journal*; published by KONSTRUKCE Media Plc., 2020.; (**b**) Reproduced with permission from H. Malik, Construction of the Year 2019; published by Triangl Corp., 2019.

Although the project of the new solution focuses on technical functionality and the authorial effect, it is in the public interest that the costs of the entire lifecycle are efficient per unit time. The economics of evaluating technical solutions, i.e., the applied rules of current decision practice, are mostly based on (empirical) cost budgets. This has to do with short-term horizons, as in the project is limited to the design and construction (the pedestrian bridge Prague–Troja m^2 cost ca. EUR 5859 and length meter ca. EUR 23,437; Pisek–Pleskot bridge, ca. 5809 EUR/m^2, and ca. 17,428 EUR/m).

The paper offers an alternative as a dynamic fractal life cycle. Figure A2 shows the spreadsheet segment and comments. The construction structure is simplified. Most EU national economies work with a standardized work breakdown structure (WBS). The reason for using a WBS is to eliminate the cost control risks of construction projects. A summary of the calculation using the decision rule α is shown in Figure A3.

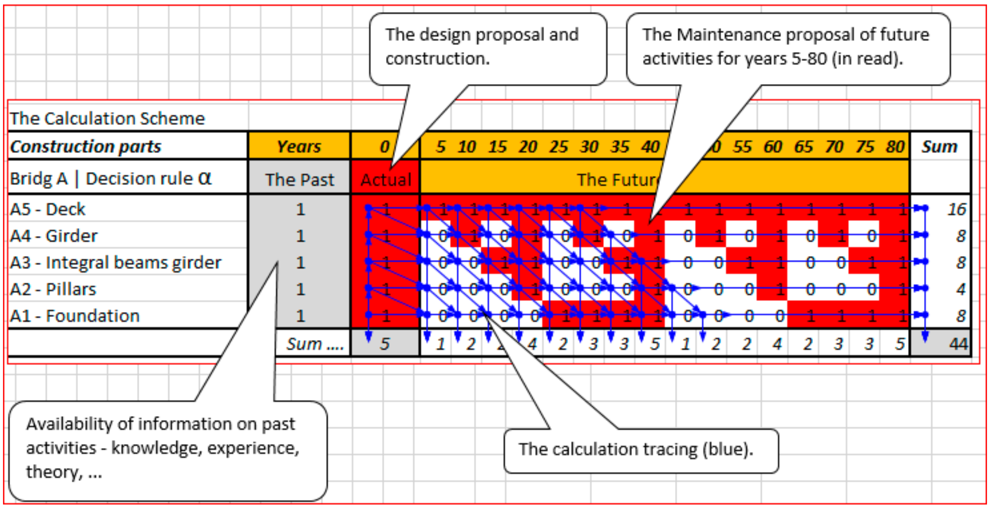

Figure A2. The calculation scheme—MS Excel; Bridge activities A1–A5 and time steps for t = 80.

The main result of the case study is the creation of a maintenance and renewal schedule based on the fractal structure created by the decision rule α. The individual maintenance cycles are a fractal structure (Figure A3). Moreover, activities A1–A5 create maintenance cycles with a fixed time and structure of the activities. They are visualized as bar graphs. A total of 44 maintenance interventions are included. The bridge deck in A5 requires 16 hits (as shown in the horizontal density graph, Figure A3 in the right part). Similar information about technical lifecycle is presented in a vertically oriented density graph for individual time periods of maintenance.

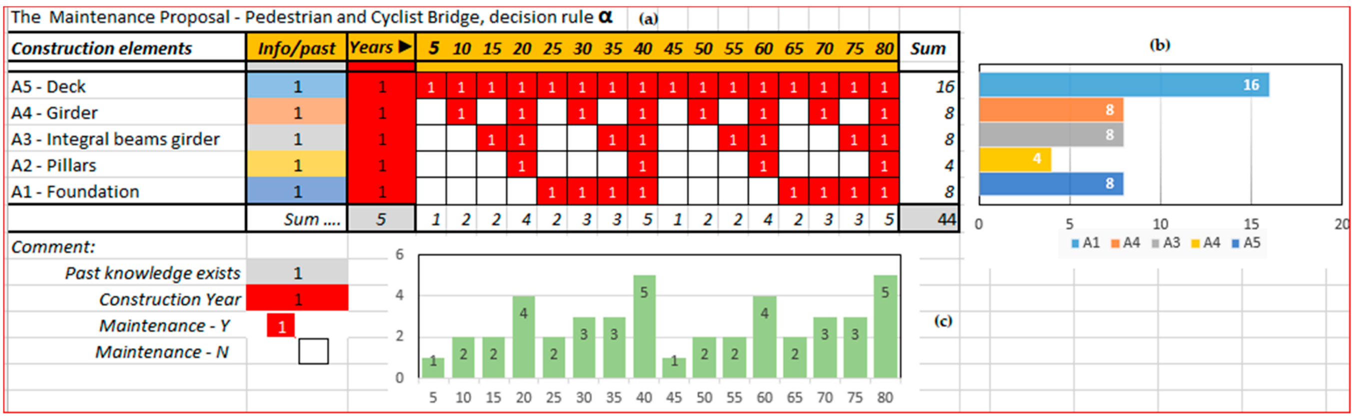

Figure A3. Calculation results: (**a**) maintenance time schedule, (**b**) construction elements maintenance density (service-life 80 years, in right), (**c**) for two technical life cycles, 2 × 40 years.

A clear segmentation of the individual maintenance cycles is shown separately in Figure A4. The time sequences are particularly noticeable.

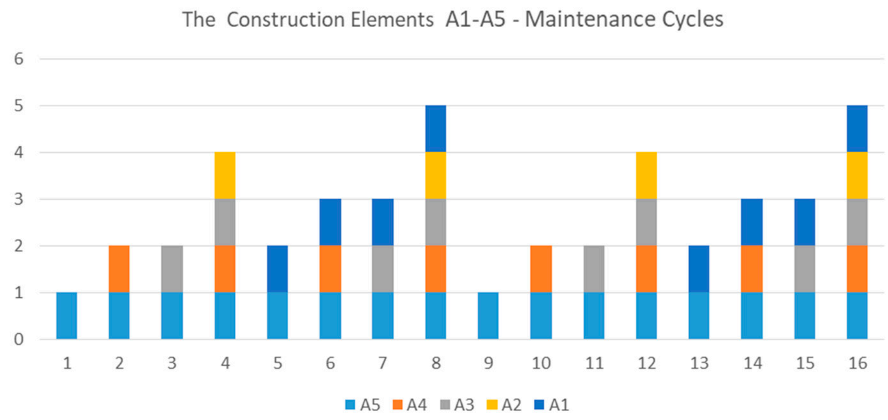

Figure A4. The construction elements A1–A5—Maintenance cycles; displayed as two technical life cycles (1–8), (8–16).

The issue of maintenance cycles is summarized in Table A1. Attention is given to the high concentration of maintenance needs for activities A5 and A4. For the first cycle of technical life, it is necessary to spend 36.36% + 18.18% of interventions. In this case, it is 54.54% of maintenance activities.

Table A1. The issue of maintenance cycles.

Activity	Cycle	Density *	Density **
Maintenance (a)	Years (b)	LC 80: % (c)	Σ 44: % (e)
A5—Deck	5	20.00%	36.36%
A4—Girder	10	10.00%	18.18%
A3—Integral beams girder	20	10.00%	18.18%
A2—Pillars	20	5.00%	9.09%
A1—Foundation	25	10.00%	18.18%

Note: * Maintenance density %; life cycle (LC) 80 years. ** Maintenance density %, construction elements. Applied (44 times) in LC 80 years.

However, the actual engineering standard of the project consists of innovations of elements A3, A2, and A1. Although disproportions can be further supported by cost analysis, the possibilities of project modifications can already be deduced from the assessment of the fractal structure.

References

1. Forrester, J. *World Dynamics*; Wright-Allen Press: Cambridge, MA, USA, 1971.
2. Lorenz, E.N. Deterministic nonperiodic flow. *J. Atmos. Sci.* **1962**, *20*, 130–141. [CrossRef]
3. Miao, J. *Economic Dynamics in Discrete Time*; MIT Press: Cambridge, MA, USA, 2014; p. 710. ISBN 978-0-262-02761-8.
4. Nguyen, M.H.; Nguyen, V.Q. *Dynamic Timing Decisions under Uncertainty: Essays on Invention, Innovation and Exploration in Resource Economics*; Springer: Heidelberg/Berlin, Germany, 2013; p. 198. ISBN 978-3-540-57649-5.
5. Trees of Life. *Philosophy of Biology*, 1st ed.; Griffiths, P.E., Ed.; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1992; pp. 1–13. ISBN 978-94-015-8038-0.
6. Schumpeter, J.A. *The Theory of Economic Development*; Harvard University Press: Cambridge, MA, USA, 1934.
7. Kuda, F.; Teichmann, M.; Proske, Z.; Szeligova, N. Modern approaches of facility management in the management and maintenance of underground services. In Proceedings of the 17th International Multidisciplinary Scientific Geoconference, SGEM 2017, Albena, Bulgaria, 29 June–5 July 2017; Volume 17, pp. 279–288. [CrossRef]
8. Schumpeter, J. *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*; McGraw-Hill Book Company, Inc.: New York, NY, USA, 1939.

9. Anderson, B.A. *Economics and The Public Welfare*; BoD—Books on Demand: Princeton, NJ, USA, 1949; p. 620. ISBN 9783846046814.
10. Taarasov, V.E. On history of mathematical economics: Application of fractional calculus. *Mathematics* **2019**, *6*, 509. [CrossRef]
11. Patten, S.N. *The Theory of Dynamic Economics*; University of Philadelphia: Pennsylvania, PA, USA, 1892; p. 153.
12. Harrod, F.R. An essay in dynamic theory. *Econ. J.* **1939**, *49*, 14–33. [CrossRef]
13. Hicks, J.R. *Methods of Dynamic Economics*; Oxford University Press: Manhattan, NY, USA, 1985; ISBN 978-0-19-877287-3.
14. Forrester, J. *Industrial Dynamics*; Pegasus Communications: Waltham, MA, USA, 1961.
15. Forrester, J. *Urban Dynamics*; Pegasus Communications: Waltham, MA, USA, 1969.
16. Mandelbrot, B.B. *The Fractal Geometry of Nature*; Henry Holt and Company: New York, NY, USA, 1982; p. 468.
17. Mandelbrot, B.B.; Hudson, R.L. *Misbehaviour of Markets*; Basic Books: New York, NY, USA, 2004.
18. Mandelbrot, B.B.; Taleb, N. Focus on the Exceptions that Prove the Rule. *Financial Times*. 24 March 2006. Available online: http://steveambler.uqam.ca/6080/articles/mandelbrot.taleb.2006.pdf. (accessed on 26 January 2021).
19. Jammer, M. *Concepts of Space: The History of Theories of Space in Physics*; Dover Publications: Cambridge, MA, USA, 1969.
20. Neumann, J.; Morgenstern, O. *Theory of Games and Economic Behaviour*; Princeton University Press: Princeton, TX, USA, 1944; p. 625.
21. Britannica, The Editors of Encyclopaedia. "Cellular automata". *Encyclopedia Britannica*. 30 May 2014. Available online: https://www.britannica.com/science/cellular-automata (accessed on 26 January 2021).
22. Wolfram, S. Cellular Automata and Compaxity: Collected Papers. In Proceedings of the International Conference of the Center for Nonlinear Studies, Los Alamos, New Mexico, 8 March 2018; p. 19.
23. Gardner, M. The fantastic combinations of John Conway's new solitaire game life. *Sci. Am.* **1970**, *223*, 120–123. [CrossRef]
24. Granger, C.W. *The Typical Spectral Shape of an Economic Variable*; Technical Report; Department of Statistics, Stanford University: Stanford, CA, USA, 1964.
25. Wolfram, S.A. *New Kind of Science*; Wolfram Media Inc.: Champaign, IL, USA, 2002.
26. Raiffa, H.; Schlaifer, R. *Applied Statistical Decision Theory*; Harvard University: Cambridge, MA, USA, 1961; p. 356.
27. Ball, P. Cellular memory hints at the origin of intelligence. *Nature* **2008**, *451*, 385. [CrossRef] [PubMed]
28. Batty, M.; Torrens, P.M. Modelling and prediction in a complex world. *Futures* **2005**, *10*, 745–766. [CrossRef]
29. Saenz, G.D.; Smith, S.M. Testing judgments of learning in new contexts to reduce confidence. *J. Appl. Res. Mem. Cogn.* **2018**, *12*, 540–551. [CrossRef]
30. Beran, V.; Dlask, P. Rational expectation in facility management. In Proceedings of the CESB 2010 Prague—Central Europe towards Sustainable Building from Theory to Practice, Prague, Austria, 30 June–2 July 2010; pp. 1–4, ISBN 978-802473624-2.
31. Burton, H.K. *Dynamic Economics*; Harvard, University Press: Cambridge, MA, USA, 1936; e-Book 9780674188679.
32. ISO. *ISO 27001. Information Technology, Security Techniques, Information Security Management Systems Requirements. ISO/IEC 27001*; ISO: Geneva, Switzerland, 2005.
33. ISO. *Environmental Management Systems. ISO 14001:2015*; ISO: Vernier, Geneva, Switzerland, 2019; Available online: https://www.iso.org/files/live/sites/isoorg/files/store/en/PUB100371.pdf (accessed on 26 January 2021).
34. Batty, M. Fifty years of urban modeling: Macro-statics to micro-dynamics. In *The Dynamics of Complex Urban Systems*; Albeverio, S., Andrey, D., Giordano, P., Vancheri, A., Eds.; Physica-Verlag HD: Berlin, Germany, 2008; pp. 1–20.
35. Kristoufek, L.; Lunacková, P. Long-term memory in electricity prices: Czech market evidence. *J. Econ. Financ.* **2013**, *63*, 407–424.
36. Aumann, R.J. Subjectivity and correlation in randomized strategies. *J. Math. Econ.* **1974**, *1*, 67–96. [CrossRef]
37. Nation, J.B.; Trofimova, I.; Rand, J.D.; Sulis, W. Formal Descriptions of Developing Systems. In *NATO Science Series II: Mathematics, Physics and Chemistry*; Nation, J.B., Trofimova, I., Rand, J.D., Sulis, W., Eds.; Springer Science & Business Media: Manoa, HI, USA, 2012; Volume 121, pp. 47–48, 306.
38. OECD. *Guidelines for Collecting and Reporting Data on Research and Experimental Development*; OECD: Paris, France, 2015.
39. Box, G.E.P.; Draper, N.R. *Empirical Model-Building and Response Surfaces*; Wiley & Sons: New York, NY, USA, 1987.
40. Kahneman, D.; Tversky, A. *Choices, Values, and Frames*; Cambridge University Press: Cambridge, UK; Russell Sage Foundation: New York, NY, USA, 2000; p. 840.
41. Cyert, M.; DeGroot, R. *Bayesian Analysis and Uncertainty in Economic Theory*; Springer Science & Business Media: Pittsburgh, PA, USA, 2012; p. 206. ISBN 978-94-010-7922-8.
42. Kahneman, D.; Slovic, S.P.; Tversky, A. *Judgment under Uncertainty: Heuristics and Biases*; Cambridge University Press: Cambridge, UK, 1982; p. 555. ISBN 0 521 28414 7.
43. Kuda, F.; Teichmann, M. Maintenance of infrastructural constructions with use of modern technologies. *J. Heat. Vent. Install.* **2017**, *26*, 18–21. Available online: https://www.researchgate.net/publication/316474681_Maintenance_of_infrastructural_constructions_with_use_of_modern_technologies (accessed on 26 January 2021).
44. Omar, G.R.S. Analysis of Delays in Construction Tasks. Ph.D. Thesis, Czech Technical University of Prague, Prague, Czech Republic, 2012.
45. Dlask, P.; Beran, V. Long-term infrastructure investment: A new approach to the economics of location. *E M Ekon. Manag.* **2016**, *19*, 40–56. [CrossRef]
46. Flyvbjerg, B.; Bruzelius, N.; Rothengatter, W. *Megaprojects and Risk—An Anatomy of Ambition*; Cambridge University Press: Cambridge, UK, 2003.
47. Grimscheid, G.; Thorsten, B. *Projektrisiko-Management in der Bauwirtschaft*; Bauwerk Verlag: Berlin, Germany, 2008. (In German)
48. Beran, V.; Teichmann, M.; Kuda, F.; Zdarilova, R. Dynamics of regional development in regional and municipal economy. *Sustainability* **2020**, *12*, 9234. [CrossRef]

49. Beran, V. Decision-making for long memory data in technical-economic design, fractals and decision area bubbles. *Appl. Math.* **2003**, *48*, 455–467. [CrossRef]
50. Flyvbjerg, B.; Steward, A. *Olympic Proportions: Cost and Cost Overrun at the Olympics 1960–2012*; Said Business School: Oxford, UK, 2012.
51. Segelod, E. *Project Cost Overrun: Causes, Consequences, and Investment Decisions*; Cambridge University Press: Cambridge, UK, 2018; ISBN 978-1-107-17304-0.
52. Madsen, J.B. The anatomy of growth in the OECD since 1870. *J. Monet. Econ.* **2010**, *57*, 753–767. [CrossRef]
53. World Bank Group. *Global Economic Prospects*; International Bank for Reconstruction and Development/The World Bank: Washington, DC, USA, 2020; ISBN 978-1-4648-1580-5.
54. World Bank Group. *Purchasing Power Parities and the Size of World Economies. Results from the 2017 International Comparison Program*; International Bank for Reconstruction and Development/The World Bank: Washington, DC, USA, 2020; ISBN 978-1-4648-1531-7.
55. Antuchevičiene, J.; Kou, G.; Maliene, V.; Vaidogas, E.R. Mathematical models for dealing with risk in engineering. *Math. Probl. Eng.* **2016**, 1–3. [CrossRef]
56. Flemming, C.; Netzker, M.; Schöttle, A. Probabilistische Berücksichtigung von Kosten- und Mengenrisiken in der Angebotskalkulation. *Bautechnik* **2011**, *88*, 94–101. (In German) [CrossRef]
57. Wilson, A. *Catastrophe Theory and Bifurcation: Applications to Urban and Regional Systems*; Routledge: Abingdon, UK, 1981; p. 331. ISBN 978-0-415-68782-9.
58. Piketty, T. *Capital in the Twenty-First Century*; Harvard University Press: Cambridge, UK, 2017.
59. Piketty, T. *Capital and Ideology*; The Belknap Press of Harvard University Press: Cambridge, MA, USA, 2020.