

Article

Supply Chain Replenishment Decision for Newsvendor Products with Multiple Periods and a Short Life Cycle

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Abstract: Traditionally, the newsvendor problem is a single-period model for a retailer and can be applied in the replenishment decision for a product with a short life cycle. However, many fashionable commodities are seasonal; not all of these products must be sold within a single period of a selling season, and they can be replenished once in each cycle. This study develops a novel multi-period model to determine multiple ordering replenishment decisions for a product over a short selling season. This study not only demonstrates the profit function for a retailer, but also provides those for both the manufacturer and the entire channel in a supply chain problem. The proposed multi-period ordering model provides explicit insights into how the ordering decisions of the retailer are affected in a specific period by considering unsold inventory or unsatisfied demand from a previous period. A numerical analysis and the simulation results illustrate the feasibility of the proposed model.

Keywords: supply chain management; inventory management; newsvendor; multiple period



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1. Introduction

The newsvendor model is a single-period, single-product model for a retailer and can be applied in replenishment decisions for a product with a short shelf or demand life [1]. Practical examples are retailers selling seasonal or fashionable goods, newsstands, and food retailers selling dairy products before their expiration dates. In the conventional newsvendor problem, the unit selling price, cost, salvage value and shortage penalty of an inventory item, and the density function of the item's stochastic demand are assumed to be known as prior knowledge [2,3]. The critical property of these products is a deadline after which selling must stop. The leftover stock becomes worthless if not sold by a specific deadline. The key issue is that of determining the optimal order quantity under a given optimization objective for maximizing the retailer's profit. That is, a retailer must decide how great of a quantity to order. Many studies have investigated this issue, including those of Pasternack [4], Lau and Lau [2], and Guo et al. [5].

Many fashionable commodities are seasonal, but not all of these products must be sold within a single period of a selling season. When a selling season consists of multiple ordering cycles, the commodities can be replenished once in each cycle. For example, many shops order computer, communication, consumer electronic, and fashion goods once a week. Then, determining the ordering quantity for each period in order to maximize the retailer's profit during a short selling season is a critical problem. Retailers cannot accurately forecast market demand and fluctuating product supply; then, either excessive inventory or product shortages occur, resulting in resource waste or loss of goodwill. Then, retailers must reduce such risk and increase profits by adopting novel contracts with suppliers.

However, retailers in a supply chain no longer make selling plans alone. They must manage with supply chain members and increase the synergy along an entire supply chain to enhance total channel profit. Supply chain management (SCM) has shifted the emphasis from internal structure to external processes and is dependent on the interactions between the organization and its external environment, with strong feedback linkages and collective learning [6–10]. Sustainable SCM has been defined as integrating system thinking, strategy, and action into supply chain management, including financial, ecological, and social performance [11]. Due to the rapid development of information and communication technology, supply chain members can quickly transfer messages and communicate with each other. Quick replenishment for products with short life cycles can be realized. A multi-period newsvendor model can be applied to solve multi-period replenishment decisions for short-life-cycle commodities. Retailers gain sustainable competitive advantage through improved supply chain relationships. The members of the supply chain can also cooperate closely to reduce the waste of excess inventories and increase the efficiency of resource use in order to achieve the sustainable development of the supply chain.

Considering cooperation between retailers and manufacturers, it is not enough to only consider the decision of the retailer. It is necessary to understand how one can affect the profits of all supply chain members and the entire channel through the replenishment decision in a multi-period newsvendor problem.

Because the classical newsvendor model was developed to deal with the single-period replenishment problem for a retailer, it did not consider how to solve the multi-period replenishment problem for short-lived products. The aim of this study is to investigate ordering decisions faced by a retailer and a manufacturer in a channel for a commodity in a short selling season with multiple ordering periods. A multi-period hierarchical inventory model will be developed based on the newsvendor problem with an ordering cycle with an identical length in order to examine replenishment decisions and maximize the retailer's expected profit. If there are remaining inventories during the previous period, some portion of the unsold commodities will be sold in next period. However, if unsatisfied demand exists and some customers are willing to wait and get the product in the next period, the retailer can request that the manufacturer offers additional goods to complement some portion of the unsatisfied demand in the next period. The ordering quantity of the next period must be determined in the model by considering these unsold inventories and the unsatisfied demand.

Although the proposed multi-period model is intended to maximize the retailer's profit similarly to the classical single-period newsvendor model, in order to understand the profits of the manufacturer and the entire channel, their expected profit models will also be simultaneously developed. The holding cost of the remaining inventories and the goodwill cost due to losing sales opportunities are integrated into the proposed models. The changes in the retailer's, manufacturer's, and whole channel's profits that are influenced by the ordering quantity decisions of the retailer can be observed.

This study extends the single-period newsvendor problem to a multi-period model for a supply chain. The extension has several contributions. First, this study considers the effect of the remaining inventories and backorders in the present period on the ordering quantity of the retailer. Because the product has a short life cycle or selling season and the retailer can only order the product once at the beginning of each period, the retailer would be eager to deal with the inventories or backorders within next period. The retailer will decrease or increase the ordering quantity based on the inventories or backorders in the previous period. As in the traditional newsvendor problem, the retailer has to bear the inventory holding cost or the goodwill cost. These cost factors should be considered in the multi-period inventory model. Second, this study expands the model into a two-tier supply chain model, which includes the expected profits of the retailer, the manufacturer, and the whole channel. In practice, the behavior of the manufacturer and the retailer is characterized by the Stackelberg game. The profit of one rises; the profit of the other is reduced. The coordination of the supply chain cannot be achieved if we only consider

the optimal decision of the retailer. The proposed model offers a good basis for future research on solving multi-period replenishment problems to maximize the profit of the whole channel and increase the profits of the retailer and manufacturer by using some channel coordination mechanisms.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. Section 3 presents the assumptions, notations, and primary elements used in this study. The multi-period model with the expected profits of the retailer, manufacturer, and entire channel is developed in Section 4. Section 5 uses numerical simulations to demonstrate the analytical results for the ordering quantity under the multi-period replenishment model. We discuss the results of the simulations in Section 6. In Section 6, we also compare the proposed model with those in other studies. In Section 7, we provide the concluding remarks and directions of future research.

2. Literature Review

Traditionally, a newsvendor problem considers a bilateral monopoly in which a manufacturer produces a commodity to sell to a retailer. We assume that the ordering model is a single-period and single-product model. In the model, the commodity produced has a relatively short shelf or demand life, and it is assumed that the retailer will only place one order with the manufacturer. If there are some remaining commodities after the demand life is exhausted, the retailer needs to bear the inventory holding cost. If the commodity is depleted, a goodwill cost associated with customers whose demands are unsatisfied will occur. The key issue is the determination of the optimal ordering quantity for maximizing the retailer's expected profit. Numerous studies have investigated this issue, such as those of Pasternack [4], Hwang and Hahn [12], Zhou and Yang [13], Rekik et al. [14], and Guo et al. [5].

The classical newsvendor model has been applied in decision making in the fashion, magazine, airline, and sporting industries at both the manufacturing and retail levels [15]. For example, Wang and Benaroch [16] adopted a newsvendor model to analyze the decisions of suppliers and buyers in the electronic market from the perspective of the supply chain when selling perishable products with random demand. Lee et al. [17] used a newsboy model to study a carbon emission cap-and-trade system. They formulated a non-cooperative game between the policymaker and companies and found an equilibrium for the policymaker's carbon pricing decisions and each company's production and pricing decisions. Shaban et al. [18] developed a multi-item newsvendor model to determine the extra-baggage price with reference to existing cargo prices. Jian et al. [19] considered a green closed-loop supply chain consisting of a manufacturer and a retailer. They designed a green closed-loop supply chain with profit-sharing contract coordination fairness.

Based on the traditional newsvendor model, researchers began to express some limitations and assumptions of the original model. For example, researchers showed that demand is dependent on price [2,20,21]. Some studies further developed single-retailer models to address multi-retailer problems [22,23]. Some researchers transformed single-product models into multi-product ordering models [24–27]. Some studies extended the single-period model to a two-period model [28–30] or a multi-period problem [31,32].

Many researchers have demonstrated that operating costs can be reduced and total profit can be increased through channel coordination and have suggested that good partnership is a supply chain's lifeblood, especially in a global economy with short life cycles and technology-intensive industries. Members in a supply chain no longer generate manufacturing or selling plans alone; that is, they must manage and coordinate with all supply chain members and increase the synergy via collaboration along an entire supply chain to enhance total channel profit. The best coordination of the supply chain is a win-win situation. That is, the total channel profit and the profit of each member in the supply chain can be increased. The newsvendor model has been integrated with numerous channel cooperation mechanisms and has been applied to many supply chain issues, such as cost-sharing or subsidy provisions for holding inventory [33], backordering to meet

unsatisfied demand, quantity discounts [34–36], return policies [4,28,37], consignment contracts [16,38], and revenue-sharing contracts [19,39–42]. These mechanisms are often designed as channel contracts to assure win–win situations or Pareto improvement. Vendors can use contract-based policies to urge retailers to stock and sell goods more aggressively, as well as to improve the service level and increase profits for each other.

3. Assumptions and Notations

This study considers a channel in a supply chain in which a manufacturer produces a specific commodity that is for sale by a retailer. The commodity produced has a short life cycle, and we assume that the retailer can sell it during a short selling season. A selling season H consists of n successive ordering periods. The retailer orders the commodity in each period and only places one order to the manufacturer in a period. Based on Pasternack [4], Khouja [43], Matsuyama [31], and Qin et al. [44], the basic newsvendor model usually assumes that the manufacturer has an unlimited capacity to supply goods. The quantity is delivered to the retailer at the beginning of each period. The probability density function of demand is a function of the selling price. The selling price is fixed, and demand is independent of the selling price. The probability density function of the demand may be known as prior knowledge, and random demand occurs in each period. To simplify the model, this study also follows these assumptions. No salvage value exists at the end of the selling season. Table 1 lists the circumstances and assumptions of this problem. The following notations are used to formulate the problem.

x_i = total amount of market demand during the i th period, $i = 1, 2, \dots, n$,
 $f(x_i)$ = probability density function of demand x_i ,
 $F(x_i)$ = cumulative probability density function of $f(x_i)$,
 q_i = ordering quantity by the retailer from the manufacturer at the beginning of the i th period,
 l_i = initial inventory level of the retailer at the beginning of the i th period,
 p = retailer selling price per unit,
 w = manufacturer wholesale price per unit,
 c = manufacturing cost per unit,
 h = inventory holding cost per unit item and per unit of time paid by the retailer,
 g = goodwill cost per unit item and per unit of time due to sellout by the retailer,
 CS = retailer setup cost,
 MS = manufacturer setup cost,
 $EP_R(q_i)$ = retailer's expected profit in the i th period,
 $EP_M(q_i)$ = manufacturer's expected profit in the i th period,
 $EP_T(q_i)$ = entire channel's expected profit in the i th period.

The relationships between the values are assumed to be:

$$c < w < p \quad (1)$$

Table 1. Circumstances and assumptions.

Items	Circumstances and Assumptions
Channel structure:	Two-tier supply chain, including a retailer and a manufacturer
Product characteristic:	One commodity with a short life cycle and short selling season
	Fixed selling price for the commodity
Objective function:	Retailer profit maximization in a multi-period newsvendor problem
Decision variables:	Replenishment quantity sent by the manufacturer to the retailer
Demand function:	Uncertain demand
Cost:	Manufacturing cost, setup cost, goodwill cost, and inventory holding cost
	Short selling season (limited planning horizon)
Time interval:	Multiple periods in the selling season
	Same and fixed duration for each period

4. The Model

This study considers a hierarchical situation in which a retailer determines the quantity to order from a manufacturer. The objective of the multi-period replenishment model is to determine an ordering plan such that the retailer can maximize the expected profit, as in the conventional single-period newsvendor model. In the model, the inventory or backorders at the end of a period are carried forward from the previous period. That is, the retailer decides the ordering quantity at the start of each period based on the real sales volume of the previous period. To give an insight into the change in the supply chain members' profit, the manufacturer's expected profits and those of the total channel are modeled simultaneously. In this study, we do not consider any channel coordination mechanisms to coordinate the retailer and manufacturer. The following ordering rules are utilized to transform the conventional single-period newsvendor model into the multi-period model.

1. If unsold commodities exist in a certain period i , demand x_i is less than the initial inventory level l_i and some portion of unsold commodities is stored by the retailer [31]. Since the retailer can determine the replenishment order for each period, the quantity ordered for the next period must be reduced to sell out the inventory. In this case, the retailer must bear the inventory holding cost. The ordering quantity for the next period becomes less than that when the retailer has no inventory.
2. If demand is unsatisfied in a certain period i , demand x_i is larger than the initial inventory level l_i , and the retailer then loses sale opportunities. The goodwill cost is borne by the retailer. If some customers are willing to wait and get their products in the next period, the retailer can request that the manufacturer offers additional goods to complement some portion of the unsatisfied demand. The manufacturer is also willing to satisfy the backordered quantity to enhance profits. Then, the quantity ordered for the next period exceeds that when the retailer has no inventory [31,45].

Based on the rules, the models can be formulated iteratively as follows.

When $i = 1$

Assume that the retailer has no initial inventory before the start of the selling season; the retailer orders q_1 from the manufacturer at the beginning of the selling season. Then, $q_1 = l_1$. The retailer's expected profit during the first period, $EP_R(q_1)$, is given as follows:

$$EP_R(q_1) = -wq_1 + \int_0^{l_1} [x_1 p] f(x_1) dx_1 + \int_{l_1}^{\infty} [pq_1 - g(x_1 - q_1)] f(x_1) dx_1 - CS \quad (2)$$

That is, if the demand x_1 is smaller than or equal to l_1 , the amount of the product that the retailer can sell is x_1 . If the demand x_1 is larger than l_1 , the maximum quantity that the retailer can sell is l_1 ; then, the retailer needs to bear the goodwill cost due to the loss of sales opportunity. In the traditional single-period newsvendor model, to obtain optimal ordering quantity q_1 to maximize the retailer's profit, $EP_R(q_1)$ is differentiated with respect to q_1 , and the computational result is set to 0. We get

$$F(q_1^*) = (p + g - w) / (p + g) \quad (3)$$

$$q_1^* = F[(p + g - w) / (p + g)]^{-1} \quad (4)$$

The retailer's expected profit, $EP_R(q_1)$, is at its maximum when the retailer orders quantity q_1^* in the classical single-period newsvendor model (see the Appendix A for details). On the other hand, the manufacturer's expected profit, $EP_M(q_1)$, can be expressed as

$$EP_M(q_1) = (w - c)q_1 - MS \quad (5)$$

To simplify the model, this study does not consider the capacity of the manufacturer. By combining Equations (2) and (5), the total profit for the entire channel is

$$EP_T(q_1) = -cq_1 + \int_0^{l_1} [x_1 p] f(x_1) dx_1 + \int_{l_1}^{\infty} [pl_1 - g(x_1 - l_1)] f(x_1) dx_1 - CS - MS \quad (6)$$

Similarly, in the conventional newsvendor model, by substituting q_1^* into Equations (5) and (6), the expected profits of the manufacturer and entire channel when the retailer's profit is maximal can be obtained.

When $i = 2, 3, \dots, n - 1, n$

According to rule 1, the single-period model can be expanded into a multi-period model. If unsold product exists from the previous period ($x_{i-1} \leq l_{i-1}$), some portion of the unsold quantity can be stored. The retailer can sell leftover product during period i . Although the retailer can order the product from the manufacturer at the beginning of each period, the retailer is unwilling to hold excessive inventory, as the product's life cycle is short. The quantity ordered for the current period is usually determined by calculating the unsold quantity left from the previous period. Assume that α ($0 \leq \alpha \leq 1$) is the ratio of the amount of stock to the amount unsold during the previous period. Then, the ordering quantity of the present period, q_i , equals the initial inventory of the present period, l_i , minus the remaining inventory from the previous period, l_{i-1} . The ordering quantity for the present period can be expressed as $q_i = l_i - \alpha(l_{i-1} - x_{i-1})$. Then, the initial inventory of the present period can be denoted as $l_i = \alpha(l_{i-1} - x_{i-1}) + q_i$. Additionally, the retailer needs to incur a holding cost for the inventory. h is the holding cost per unit item and per unit of time, which is paid by the retailer when the inventory is held at the end of a period. Therefore, the retailer's profit, $EP_R(q_i)$, manufacturer's profit, $EP_M(q_i)$, and channel's profit, $EP_T(q_i)$, are, respectively, modeled as follows.

$$EP_R(q_i) = -wq_i + \int_0^{l_i} [x_i p] f(x_i) dx_i + \int_{l_i}^{\infty} [pl_i - g(x_i - l_i)] f(x_i) dx_i - h\alpha(l_{i-1} - x_{i-1}) - CS \quad (7)$$

$$EP_M(q_i) = (w - c)q_i - MS \quad (8)$$

$$EP_T(q_i) = -cq_i + \int_0^{l_i} [x_i p] f(x_i) dx_i + \int_{l_i}^{\infty} [pl_i - g(x_i - l_i)] f(x_i) dx_i - h\alpha(l_{i-1} - x_{i-1}) - CS - MS \quad (9)$$

On the other hand, according to rule 2, when demand exceeds the initial inventory of the previous period ($x_{i-1} > l_{i-1}$), as the manufacturer can only offer the retailer products once within each period, the retailer will lack sufficient products to sell and loses sale opportunities; then, the retailer needs to bear the goodwill cost in period $i - 1$. If some customers are willing to wait, the manufacturer can offer the retailer additional products at the beginning of period i to satisfy the backordered quantity. The ordering quantity of period i is determined by the retailer to meet the unsatisfied demand from the previous period. Assume that β ($0 \leq \beta \leq 1$) is the ratio of the amount sold at the beginning of the current period to the amount of unsatisfied demand during the previous period. The ordering quantity of the present period, q_i , equals the initial inventory of the present period, l_i , plus the remaining backorders from the previous period. The ordering quantity for the present period can be modeled as $q_i = l_i + \beta(x_{i-1} - l_{i-1})$. Then, the initial inventory of the present period can be expressed as $l_i = q_i - \beta(x_{i-1} - l_{i-1})$. The expected profits for the retailer, $EP_R(q_i)$, manufacturer, $EP_M(q_i)$, and channel, $EP_T(q_i)$, are formulated as

$$EP_R(q_i) = -wq_i + \int_0^{l_i} [x_i p] f(x_i) dx_i + \int_{l_i}^{\infty} [pl_i - g(x_i - l_i)] f(x_i) dx_i + p\beta(x_{i-1} - l_{i-1}) - CS \quad (10)$$

$$EP_M(q_i) = (w - c)q_i - MS \quad (11)$$

$$EP_T(q_i) = -cq_i + \int_0^{l_i} [x_i p] f(x_i) dx_i + \int_{l_i}^{\infty} [pl_i - g(x_i - l_i)] f(x_i) dx_i + p\beta(x_{i-1} - l_{i-1}) - CS - MS \quad (12)$$

The retailer's expected profit can be maximized in the same manner as that modeled using the traditional single-period newsvendor model. Let EP_R be the cumulative profit expected by the retailer during the entire selling season, and it can be expressed as

$$EP_R = \sum_{i=1}^n EP_R(q_i) \quad (13)$$

By differentiating EP_R with respect to q_i ($i = 1, 2, \dots, n$) and setting the computational result to 0, the ordering quantity q_i^* for each period can be determined.

$$\frac{d}{dq_i} EP_R = \frac{d}{dq_i} \left(\sum_{i=1}^n EP_R(q_i) \right) = 0 \quad (14)$$

The concavity property of the retailer's total profit model can be verified using the following inequality:

$$\frac{d^2}{dq_i^2} EP_R = \frac{d^2}{dq_i^2} \sum_{i=1}^n EP_R(q_i) < 0 \quad (15)$$

Due to the high-power expression in Equation (14) and the inequality in Equation (15), determining the signs of the evaluation analytically is difficult. The numerical value of the ordering quantity for each period can be calculated using computer programs, and the concavity of the problem can be evaluated accurately.

Furthermore, the cumulative profits of the manufacturer and channel during the selling season can be, respectively, calculated as follows:

$$EP_M = \sum_{i=1}^n EP_M(q_i) \quad (16)$$

$$EP_T = \sum_{i=1}^n EP_T(q_i) \quad (17)$$

Substituting the quantity q_i^* of each period, determined by the retailer, into all formulas for the manufacturer and channel, the profits for each period and the cumulative profits during all selling seasons for the manufacturer and channel can be obtained.

5. Numerical Studies

Numerical illustrations are made to analyze the expected profits of the members and channel. The selling season of a product is 3 months, which consists of 12 periods. The retailer replenishes the stock once per week during the selling season. Table 2 shows the numbers of parameters in the numerical simulation. The price of the product is 50, and the per-unit manufacturing cost is 12. The unit wholesale price paid by the retailer is 30. If the retailer has inventory, the unit holding cost per period is 6, and the ratio of the amount of stock to the amount of unsold stock, α , is 0.9. However, if there is unsatisfied demand in a period, the ratio of the amount sold at the beginning of the current period to the amount of unsatisfied demand during the previous period, β , is 0.5, and the goodwill cost per unit is 14 due to the sellout, which is incurred by the retailer. In practice, the values of α and β are not constant and change from period to period. To simplify the simulation process and understand the influence of the two variables on the profits of the supply chain members, this study assumes that their values are fixed. It also assumes that the probability density functions of demand x_i are independent and follow normal distribution. The means of $f(x_i)$ decline as time goes on, $\mu_{x_i} = 500 - 20 \times (i - 1)$, $1 \leq i \leq 12$, and the standard deviation σ_{x_i} is 70. The complex procedures for determining the ordering quantity for each period and expected profits of the retailer, manufacturer, and channel were performed using Mathematica for the multi-period scenarios. Table 3 lists the simulation results for the first trial by using the proposed model (variable quantity obtained from the models). We also used a smaller fixed quantity ($q_i^s = 300$) and a bigger fixed quantity ($q_i^b = 450$) to simulate the replenishment decisions. The results are also listed in Table 3.

Under the same conditions as those in trial 1, ten trials were carried out with various demands. Table 4 shows the total ordering quantity and profits for the retailer, manufacturer, and channel with the variable quantities from the proposed models. Additionally, scenarios with a small quantity ($q_i^s = 300$) and a bigger fixed quantity ($q_i^b = 450$) were also simulated in ten trials. Table 4 presents the results.

Table 2. Numbers of parameters in the numerical simulation.

Items	Notation	Number
Retailer's selling price per unit	p	50
Wholesale price per unit	w	30
Manufacturing cost per unit	c	12
Inventory holding cost per unit per period paid by the retailer	h	6
Goodwill cost per unit paid by the retailer	g	14
Ratio of the amount of stock to the amount of unsold stock during the previous period	α	0.9
Ratio of the amount sold at the beginning of the current period to the amount of unsatisfied demand during the previous period	β	0.5

Table 3. Simulation results of trial 1.

Period (i)	Demand (x_i)	Variable Quantity Obtained from Models						Fixed Quantity								
		l_i	q_i	$EP_R(q_i)$	$EP_M(q_i)$	$EP_T(q_i)$	l_i^s	q_i^s	$EP_R(q_i^s)$	$EP_M(q_i^s)$	$EP_T(q_i^s)$	l_i^b	q_i^b	$EP_R(q_i^b)$	$EP_M(q_i^b)$	$EP_T(q_i^b)$
1	502	478	478	8824	8004	16,828	300	300	2772	4800	7572	450	450	7872	7500	15,372
2	493	457	470	8510	7860	16,370	199	300	1484	4800	6284	424	450	7634	7500	15,134
3	451	538	556	6345	9408	15,753	153	300	1428	4800	6228	416	450	8135	7500	15,635
4	427	519	440	7280	7320	14,600	151	300	1736	4800	6536	432	450	8325	7500	15,825
5	433	398	315	9069	5070	14,139	162	300	1806	4800	6606	455	450	7718	7500	15,218
6	398	478	496	5495	8328	13,823	165	300	2331	4800	7131	470	450	5881	7500	13,381
7	377	458	386	6438	6348	12,786	183	300	2859	4800	7659	514	450	5339	7500	12,839
8	351	439	365	5763	5970	11,733	203	300	3528	4800	8328	574	450	2910	7500	10,410
9	350	317	239	7412	3702	11,114	226	300	3864	4800	8664	650	450	2401	7500	9901
10	322	297	314	5544	5052	10,596	238	300	4174	4800	8974	720	450	580	7500	8080
11	295	278	290	5162	4620	9782	258	300	5082	4800	9882	809	450	−1299	7500	6201
12	266	285	294	4505	4692	9197	282	300	5849	4800	10,649	912	450	−3370	7500	4130
total			4643	80,347	76,374	156,721		3600	36,913	57,600	94,513		5400	52,126	90,000	142,126

Table 4. Simulation results of ten trials.

Trial	Variable Quantity Obtained from Models					Fixed Quantity						
	q_i	$EP_R(q_i)$	$EP_M(q_i)$	$EP_T(q_i)$	$q_i^s=300$	$EP_R(q_i^s)$	$EP_M(q_i^s)$	$EP_T(q_i^s)$	$q_i^b=450$	$EP_R(q_i^b)$	$EP_M(q_i^b)$	$EP_T(q_i^b)$
1	4643	80,847	76,374	156,721	3600	36,913	57,600	94,513	5400	52,126	90,000	142,126
2	5084	87,384	84,276	171,660	3600	18,807	57,600	76,407	5400	84,300	90,000	174,300
3	4655	81,818	76,590	158,408	3600	37,132	57,600	94,732	5400	53,917	90,000	143,917
4	4787	84,866	78,943	163,809	3600	31,323	57,600	88,923	5400	62,763	90,000	152,763
5	4590	73,927	75,420	149,347	3600	40,583	57,600	98,183	5400	49,899	90,000	139,899
6	4831	72,012	79,794	151,806	3600	24,341	57,600	81,941	5400	52,586	90,000	142,586
7	4801	70,205	79,218	14,9423	3600	23,112	57,600	80,712	5400	57,799	90,000	147,799
8	4625	70,119	76,032	146,151	3600	35,644	57,600	93,244	5400	52,276	90,000	142,276
9	5201	87,576	86,400	173,976	3600	14,710	57,600	72,310	5400	75,849	90,000	165,849
10	4705	76,541	77,490	154,031	3600	27,884	57,600	85,484	5400	31,968	90,000	121,968

6. Discussions

The columns of variable quantities in Table 3 show that the ordering quantity in each period is determined based on the amount of inventory or unsatisfied demand from the previous period according to real customer demand. For example, when the initial inventory of the retailer for Period 3 exceeds customer demand, the retailer's inventory increases. The quantity ordered for Period 4 is reduced to balance the inventory, and the retailer should bear the inventory holding cost. Conversely, when some demands cannot be satisfied in Period 2, the ordering quantity for Period 3 is increased to fulfill some unsatisfied demands from Period 2. Additionally, the retailer can earn some profits from the backordered quantity in Period 3, but the retailer has to burden the goodwill cost in Period 2. This study also illustrates the profits of the manufacturer and whole channel for the multi-period model.

To compare the results of the variable quantities obtained from the models with the results of the fixed-quantity scenarios, this study simulated the results of the retailer, manufacturer, and overall channel by using a smaller fixed quantity ($q_i^s = 300$) and a bigger fixed quantity ($q_i^b = 450$). The simulation results in Table 3 show that the total profit of the retailer in the selling season under the proposed variable quantity model is better than that generated in the fixed-quantity scenarios.

Similar simulations were carried out 10 times. In Table 4, similarly to trial 1, the expected profits of the retailer under the variable-quantity conditions are better than those under the fixed-quantity conditions in other trials. The retailer can reduce the risk associated with accumulating inventory or undesired shortages by using this model.

Comparing the proposed variable-quantity scenario with the fixed-quantity scenario, if the fixed replenishment quantity is small, the initial inventories of the retailer in many periods are less than the demand. The retailer does not have enough commodities to sell in the current period, and at the same time, they have to bear the loss of goodwill, resulting in a decrease in the profit of the retailer. On the other hand, if the fixed replenishment quantity is large, the retailer will have a lot of excess inventory and must bear higher inventory costs. Accumulating too much of an inventory seriously hurts the profit of the retailer.

However, because the newsvendor model only aims to maximize the profit of the retailer, under the variable-quantity model, the whole channel does not necessarily obtain greater profits than those under the fixed-quantity conditions, such as trial 2. Because of the lack of any mechanisms for channel cooperation, the profit of the manufacturer will inevitably increase under conditions of a large, fixed replenishment quantity. However, due to an excess of inventory, the retailer's profit will be greatly reduced. If we maximize the profit of the retailer, the profit of the manufacturer will decrease. Neither the retailer nor the manufacturer would be willing to accept less profit in the variable-quantity condition than they would achieve in the fixed-quantity scenario. From the view of supply chain coordination, it is necessary to offer some channel coordination policies, such as return policies [46], quantity discounts [35,47], revenue-sharing contracts [9,16,19,40,41,48,49], and a vendor-managed inventory (VMI) [8,38,50], in order to encourage the retailer and manufacturer to make active ordering decisions to sell as many products as possible during a short selling season. The profit of the overall channel can be enhanced, and both the retailer's and manufacturer's profits can be increased simultaneously. Yang et al. [30] studied a two-tier supply chain that consists of a retailer and a supplier. A two-period newsvendor model was developed to determine the ordering quantities. In the first sales period, the supplier sets the wholesale price and grants trade credits to the retailer. The retailer decides on the order quantity according to the wholesale price and the market demand. In the second sales period, the retailer can reorder when he pays the full loans. The supplier offers an incentive of a discounted wholesale price in the second order to encourage the retailer to select early payment. Compared with their study, the proposed model in this study extended the single-period newsvendor model into a multi-period model. In addition to providing the expected profits of the retailer and manufacturer, as in their research, this study also integrated the profit models of the two channel members into

the expected profit model of the overall channel. The study of Yang et al. used a flexible trade credit and a revenue-sharing contract to achieve the supply chain coordination. As discussed above, these supply chain cooperation mechanisms provide us with a good reference for future research.

Matsuyama [31] formulated a multi-period newsboy model. They mentioned that ordering quantity is always equal to the initial inventory quantity in a traditional single-period newsvendor model. Instead of deciding on the ordering quantity, they determined the optimal initial inventory level for maximizing the expected profit of the retailer. They also made 10 trials to simulate their model. Matsuyama's study only extended the classical single-period newsvendor model to a multi-period model of the retailer. Compared with his study, in addition to the profit of the retailer, our multi-period newsvendor model also established the expected profit models of the manufacturer and overall channel from the perspective of the supply chain. The random variable of demand x that they used followed a uniform distribution. However, in the proposed model, we used a random variable x with a normal distribution. The retailer's profits in our simulations are similar to those in the results of Matsuyama. However, the profit structure for the supply chain members and the overall channel in the proposed model can provide managers with better insights into the changes in the profits of supply chain members.

The basic single-period newsvendor model is used to determine the ordering quantity of a product that will maximize the expected profit of a retailer. Then, some researchers studied the minimization of the expected costs of overestimating and underestimating demand [43]. Kim et al. [32] proposed a multi-period newsvendor model to arrange the amount of inventory to minimize the total costs, including delivery, transshipment, holding, and shortage costs. The difference from their model is that the proposed multi-period model considered the factors that affect the expected profits of supply chain members from the perspective of outputs, such as unsold inventory resale and unsatisfied demand backordering. As mentioned above, the proposed model adds to the exploration of the expected profit structures of retailers, manufacturers, and entire supply chains.

7. Conclusions and Future Research

This study demonstrated a model for developing the profit of an overall channel and those of both a retailer and manufacturer in a multi-period newsvendor environment. The traditional single-period newsvendor model was effectively extended to a multi-period model to meet real challenges that are encountered when dealing with a short product life cycle. The proposed model can handle replenishment decisions for a product with a short selling season or life cycle and allow the retailer to place multiple orders and replenish stock in each period during the selling season. If there are some inventories, then some of the unsold commodities are stored, and the retailer will bear the inventory holding cost. The remaining inventory can be sold during the next period. On the other hand, if some unsatisfied demand exists, then the retailer should bear the goodwill cost due to the loss of sale opportunities. The retailer can request that the manufacturer offers additional goods to complement some portion of the unsatisfied demand if customers are willing to wait. The inventory holding cost and backordering to meet unsatisfied demand are integrated into the proposed models. We ran a series of numerical trials to simulate the proposed model. The results clearly indicate that the proposed multi-period model is effective in calculating the profits of the retailer and outperforms models in fixed-ordering-quantity conditions. The proposed multi-period model provides an opportunity to solve the replenishment decision for a short-life-cycle product with multiple periods and a fixed ordering duration.

This study assumes that the selling price is fixed and independent of demand. This is reasonable for simplifying the complexity of the initial model. The price and demand distributions are assumed to be fixed in the original newsvendor problem [2]. However, in practice, the demand distribution is a function of the selling price. In competitive business environments, enterprises have relied heavily upon pricing schemes to improve their profits. The price-demand curve often slopes downward from left to right. Thus, modeling

the selling price with demand can enhance the accuracy of the proposed model and increase the expected profits of the whole channel, retailer, and manufacturer in a multiple-period environment [20,21,51]. The existing literature usually uses two kinds of demand–price functions, namely, the linear demand model and nonlinear demand model [21]. The linear model denotes the demand function as $D(p) = a - bp$, $a > 0$, $b > 0$. The parameter b is the price-elasticity index of demand. The larger the value of b , the more sensitive the demand is to a change in price [52]. The nonlinear model is $D(p) = \tau p^{-b}$, $\tau > 0$, $b > 0$, where b is also the price-elasticity index of demand. Future extensions of this study will consider the selling price as a decision variable and will discuss the relationship of selling price and demand.

Additionally, the proposed multi-period newsvendor model contributes to a research base on the effective integration and cooperation of supply chain members. Numerous channel coordination policies, such as quantity discounts, wholesale price contracts, return policies, revenue-sharing contracts, consignment contracts, and VMI have been developed to maximize the profits of the overall channel and, simultaneously, to increase the profits of the retailer and manufacturer. Excellent and long-term cooperation between supply chain members can decrease operating costs, reduce the waste of excess inventories, and increase the efficiency of resource use in order to achieve the sustainable development of supply chains. In the future, we will consider some supply chain coordination policies in the multi-period newsvendor model to analyze the profits of the supply chain members.

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Appendix A

Derivation of Equation (2):

$$EP_R(q_1) = -wq_1 + \int_0^{l_1} [px_1]f(x_1)dx_1 + \int_{l_1}^{\infty} [pq_1 - g(x_1 - q_1)]f(x_1)dx_1 - CS \quad (A1)$$

Because the retailer has no initial inventory at the beginning of the selling season, $l_1 = q_1$.

$$\begin{aligned} EP_R(q_1) &= -wq_1 + \int_0^{q_1} px_1f(x_1)dx_1 + \int_{q_1}^{\infty} pq_1f(x_1)dx_1 - \int_{q_1}^{\infty} gx_1f(x_1)dx_1 + \int_{q_1}^{\infty} gq_1f(x_1)dx_1 - CS \\ &= -wq_1 + \int_0^{q_1} px_1f(x_1)dx_1 + pq_1(1 - F(q_1)) - \int_{q_1}^{\infty} gx_1f(x_1)dx_1 + gq_1(1 - F(q_1)) - CS \end{aligned} \quad (A2)$$

We differentiate $EP_R(q_1)$ with respect to q_1 and set the amount equal to 0.

$$\begin{aligned} \frac{d}{dq_1}EP_R(q_1) &= -w + (p - pF(q_1) - pq_1f(q_1)) + gq_1f(q_1) + (g - gF(q_1) - gq_1f(q_1)) \\ &= -w + p - pF(q_1) + g - gF(q_1) \\ &= -w + p + g - (p + g)F(q_1) \\ \text{Set } \frac{d}{dq_1}EP_R(q_1) &= 0 \\ F(q_1) &= \frac{p+g-w}{p+g}q_1^* = F\left(\frac{p+g-w}{p+g}\right)^{-1} \end{aligned} \quad (A3)$$

And,

$$\frac{d^2}{dq_1^2} EP_R(q_1) = -(p + g)f(q_1) < 0 \quad (A4)$$

The retailer's expected profit, $EP_R(q_1)$, is maximized, as the retailer's order quantity is q_1^* in the conventional single-period newsvendor model.

References

1. Peidro, D.; Mula, J.; Poler, R. Quantitative models for supply chain planning under uncertainty: A review. *Int. J. Adv. Manuf. Technol.* **2009**, *43*, 400–420. [\[CrossRef\]](#)
2. Lau, A.H.L.; Lau, H.S. The newsboy problem with price-dependent demand distribution. *IIE Trans.* **1988**, *20*, 168–175. [\[CrossRef\]](#)
3. Zhang, G.F.; Hu, Z.H.; Wang, W.Q. Self-Adaptive newsvendor model with information updated. *Procedia Comput. Sci.* **2019**, *154*, 89–95. [\[CrossRef\]](#)
4. Pasternack, B.A. Optimal pricing and return policies for perishable commodities. *Mark. Sci.* **1985**, *4*, 166–176. [\[CrossRef\]](#)
5. Guo, M.; Chen, Y.; Wang, H.; Yang, J.; Zhang, K. The single-period (newsvendor) problem under interval grade uncertainties. *Eur. J. Oper. Res.* **2019**, *273*, 198–216. [\[CrossRef\]](#)
6. Hsieh, T.P.; Weng, M.W.; Su, R.H.; Lai, K.K. An integrated vendor-buyer inventory model with defective items and trade credit under holding cost reduction. *Int. J. Inform. Manag. Sci.* **2018**, *29*, 145–196.
7. Azevedo, S.G.; Pimentel, C.M.O.; Alves, A.C.; Matias, J.C.O. Support of advanced technologies in supply chain processes and sustainability impact. *Appl. Sci.* **2021**, *11*, 3026. [\[CrossRef\]](#)
8. Hossain, M.I.; Parvez, M.S. Investigating the effect of extended vendor managed inventory in the supply chain of health care sector to enhance information exchange. *Int. J. Inf. Manag. Sci.* **2020**, *31*, 171–189.
9. Khorshidvand, B.; Soleimani, H.; Sibdari, S.; Esfahani, M.M.S. Revenue management in a multi-level multi-channel supply chain considering pricing, greening, and advertising decisions. *J. Retail. Consum. Serv.* **2020**, *24*, 102425. [\[CrossRef\]](#)
10. Xu, N.; Huang, Y.-F.; Weng, M.-W.; Do, M.-H. New retailing problem for an integrated food supply chain in the baking industry. *Appl. Sci.* **2021**, *11*, 946. [\[CrossRef\]](#)
11. Bratt, C.; Sroufe, R.; Broman, G. Implementing strategic sustainable supply chain management. *Sustainability* **2021**, *13*, 8132. [\[CrossRef\]](#)
12. Hwang, H.; Hahn, K.H. An optimal procurement policy for items with an inventory-level-dependent demand rate and fixed lifetime. *Eur. J. Oper. Res.* **2000**, *127*, 537–545. [\[CrossRef\]](#)
13. Zhou, Y.W.; Yang, S.L. An optimal replenishment policy for items with an inventory-level-dependent demand rate and fixed lifetime under the LIFO policy. *J. Oper. Res. Soc.* **2003**, *54*, 585–593. [\[CrossRef\]](#)
14. Rekik, Y.; Sahin, E.; Dallery, Y. A comprehensive analysis of the Newsvendor model with unreliable supply. *OR Spectr.* **2007**, *29*, 207–233. [\[CrossRef\]](#)
15. Gallego, G.; Moon, I. The distribution free newsboy problem: Review and extensions. *J. Oper. Res. Soc.* **1993**, *44*, 825–834. [\[CrossRef\]](#)
16. Wang, C.X.; Benaroch, M. Supply chain coordination in buyer centric B2B electronic markets. *Int. J. Prod. Econ.* **2004**, *92*, 113–124. [\[CrossRef\]](#)
17. Lee, J.; Lee, M.L.; Park, M.A. newsboy model with quick response under sustainable carbon cap-N-trade. *Sustainability* **2018**, *10*, 1410. [\[CrossRef\]](#)
18. Shaban, I.A.; Wang, Z.X.; Chan, F.T.S.; Chung, S.H.; Qu, T. An extra-baggage service price setting with reference to cargo prices using multi-item newsvendor model. *Comput. Ind. Eng.* **2019**, *128*, 877–885. [\[CrossRef\]](#)
19. Jian, J.; Guo, Y.; Jiang, L.; An, Y.; Su, J. A multi-objective optimization model for green supply chain considering environmental benefits. *Sustainability* **2019**, *11*, 5911. [\[CrossRef\]](#)
20. Federgruen, A.; Heching, A. Combined pricing and inventory control under uncertainty. *Oper. Res.* **1999**, *47*, 454–475. [\[CrossRef\]](#)
21. Petruzzzi, N.; Dada, M. Pricing and the newsvendor problem: A review with extensions. *Oper. Res.* **1999**, *47*, 183–194. [\[CrossRef\]](#)
22. Mantrala, M.K.; Raman, K. Demand uncertainty and supplier's return policies for a multi-store style-good retailer. *Eur. J. Oper. Res.* **1999**, *115*, 270–284. [\[CrossRef\]](#)
23. Lau, A.H.L.; Lau, H.S. Comparative normative optimal behavior in two-echelon multi-retailer distribution systems for a single-period product. *Eur. J. Oper. Res.* **2003**, *144*, 659–676.
24. Abdel-Malek, L.; Montanari, R.; Morales, L.C. Exact, approximate, and generic iterative models for the multi-product newsboy problem with budge constraint. *Int. J. Prod. Econ.* **2004**, *91*, 189–198. [\[CrossRef\]](#)
25. Abdel-Malek, L.; Montanari, R. An analysis of the multi-product newsboy problem with a budget constraint. *Int. J. Prod. Econ.* **2005**, *97*, 296–307. [\[CrossRef\]](#)
26. Zhang, B. Multi-tier binary solution method for multi-product newsvendor problem with multiple constraints. *Eur. J. Oper. Res.* **2012**, *218*, 426–434. [\[CrossRef\]](#)
27. Punia, S.; Singh, S.P.; Madaan, J.K. From predictive to prescriptive analytics: A data-driven multi-item newsvendor model. *Decis. Support Syst.* **2020**, *136*, 113340. [\[CrossRef\]](#)

28. Taylor, T.A. Channel coordination under price protection, midlife returns, and end-of-life returns in dynamic markets. *Manag. Sci.* **2001**, *47*, 1220–1234. [[CrossRef](#)]
29. Lee, C.H. Coordination on stocking and progressive pricing policies for a supply chain. *Int. J. Prod. Econ.* **2007**, *106*, 307–319. [[CrossRef](#)]
30. Yang, H.; Zhuo, W.; Zha, Y.; Wan, H. Two-period supply chain with flexible trade credit contract. *Expert Syst. Appl.* **2016**, *66*, 95–105. [[CrossRef](#)]
31. Matsuyama, K. The multi-period newsboy problem. *Eur. J. Oper. Res.* **2006**, *171*, 170–188. [[CrossRef](#)]
32. Kim, G.; Wu, K.; Huang, E. Optimal inventory control in a multi-period newsvendor problem with non-stationary demand. *Adv. Eng. Inf.* **2015**, *29*, 139–145. [[CrossRef](#)]
33. Wang, Y.; Gerchak, Y. Supply chain coordination when demand is shelf-space dependent. *Manuf. Serv. Oper. Manag.* **2001**, *3*, 82–87. [[CrossRef](#)]
34. Munson, C.L.; Rosenblatt, M.J. Coordinating a three-level supply chain with quantity discount. *IIE Trans.* **2001**, *33*, 371–384. [[CrossRef](#)]
35. Viswanathan, S.; Wang, Q. Discount pricing decisions in distribution channels with price-sensitive demand. *Eur. J. Oper. Res.* **2003**, *149*, 571–587. [[CrossRef](#)]
36. Sheen, G.J.; Tsao, Y.C. Channel coordination, trade credit and quantity discounts for freight cost. *Trans. Res. Part E* **2007**, *43*, 112–128. [[CrossRef](#)]
37. Lee, C.H.; Rhee, B.D. Channel coordination using product returns for a supply chain with stochastic salvage capacity. *Eur. J. Oper. Res.* **2007**, *177*, 214–238. [[CrossRef](#)]
38. Chen, L.T. Dynamic supply chain coordination under consignment and vendor-managed inventory in retailer-centric B2B electronic markets. *Ind. Mark. Manag.* **2013**, *42*, 518–531. [[CrossRef](#)]
39. Gerchak, Y.; Wang, Y. Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Prod. Oper. Manag.* **2004**, *13*, 23–33. [[CrossRef](#)]
40. Kocabiyikoglu, A.; Gogus, C.I.; Gonul, M.S. Revenue management vs. newsvendor decisions: Does behavioral response mirror normative equivalence? *Prod. Oper. Manag.* **2015**, *24*, 750–761. [[CrossRef](#)]
41. Su, J.; Li, C.; Zeng, Q.; Yang, J.; Zhang, J. A green closed-loop supply chain coordination mechanism based on third-party recycling. *Sustainability* **2019**, *11*, 5335. [[CrossRef](#)]
42. Jian, J.; Li, B.; Zhang, N.; Su, J. Decision-making and coordination of green closed-loop supply chain with fairness concern. *J. Clean. Prod.* **2021**, *298*, 126779. [[CrossRef](#)]
43. Khouja, M. The single-period (news-vendor) problem: Literature review and suggestions for future research. *Omega* **1999**, *27*, 537–553. [[CrossRef](#)]
44. Qin, Y.; Wang, R.; Vakharia, A.J.; Chen, Y.; Seref, M.M.H. The newsvendor problem: Review and directions for future research. *Eur. J. Oper. Res.* **2011**, *213*, 361–374. [[CrossRef](#)]
45. Pal, A.K.; Bhunia, A.K.; Mukherjee, R.N. Optimal lot size model for deteriorating items with demand rate dependent on displayed stock level (DSL) and partial backordering. *Eur. J. Oper. Res.* **2006**, *175*, 977–991. [[CrossRef](#)]
46. Hahn, K.H.; Hwang, H.; Shinn, S.W. A return policy for distribution channel coordination of perishable items. *Eur. J. Oper. Res.* **2004**, *152*, 770–780. [[CrossRef](#)]
47. Raju, J.; Zhang, Z.J. Channel coordination in the presence of a dominant retailer. *Mark. Sci.* **2005**, *24*, 254–262. [[CrossRef](#)]
48. Zhang, Y.; Yang, X. Online ordering policies for a two-product, multi-period stationary newsvendor problem. *Comput. Oper. Res.* **2016**, *74*, 143–151. [[CrossRef](#)]
49. Palsule-Desai, O.D. Supply chain coordination using revenue-dependent revenue sharing contracts. *Omega* **2013**, *41*, 780–796. [[CrossRef](#)]
50. Kiesmuller, G.P.; Broekmeulen, R.A.C.M. The benefit of VMI strategies in a stochastic multi-product serial two echelon system. *Comput. Oper. Res.* **2010**, *37*, 406–416. [[CrossRef](#)]
51. Chen, L.T.; Wei, C.C. Multi-period channel coordination in vendor-managed inventory for deteriorating goods. *Int. J. Prod. Res.* **2012**, *50*, 4396–4413. [[CrossRef](#)]
52. Wang, Y.; Jiang, L.; Shen, Z.J. Channel performance under consignment contract with revenue sharing. *Manag. Sci.* **2004**, *50*, 34–47. [[CrossRef](#)]