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# The Potential of Biophilic Fractal Designs to Promote Health and Performance: A Review of Experiments and Applications

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Abstract: Fractal objects are prevalent in natural scenery. Their repetition of patterns at increasingly fine magnifications creates a rich complexity. Fractals displaying mid-range complexity are the most common and include trees, clouds, and mountains. The "fractal fluency" model states that human vision has adapted to process these mid-range fractals with ease. I will first discuss fractal fluency and demonstrate how it enhances the observer's visual capabilities by focusing on experiments that have important practical consequences for improving the built environment. These enhanced capabilities generate an aesthetic experience and physiological stress reduction. I will discuss strategies for integrating fractals into building designs to induce positive impacts on the observer. Examples include fractal solar panels, fractal window shades, and fractal floor patterns. These applications of fractal fluency represent a fundamental and potentially impactful form of salutogenesis.

Keywords: aesthetics; biophilia; fractals; human-centered design; stress-reduction

### 1. Introduction

Nature's visual beauty is profound. Yet it is surprisingly under-utilized when building the environments in which we work and live. In 1975, architect Christopher Alexander published The Oregon Experiment [1] which described his famous approach to campus planning at the University of Oregon. Declaring that human aspirations and needs should be the primary driver when creating community spaces, The Oregon Experiment became a powerful demonstration of human-centered design. Although the term "biophilia" predates this project (being first used by psychologists in the 1960's [2]), the associated movement gained momentum in the 1980's when naturalist Edward Wilson promoted his Biophilia Hypothesis. Biophilia—nature-loving—recognizes the inherent need of humans to connect with nature [3]. Around this time, pioneering psychology experiments by Roger Ulrich and colleagues showed that exposure to nature's scenery induced positive changes in people, including significant stress reduction. This even accelerated the recovery of patients from major surgery [4–6]. In their book from the same era, The Experience of Nature: A Psychological Perspective [7], Rachel and Steven Kaplan introduced Attention Restoration Theory (ART) [8] which explores our inherent fascination for viewing nature. They proposed that the "soft" attention induced by nature differs from the "hard" attention required for unnatural tasks (like reading books and looking at artificial objects such as buildings) and restores depleted mental resources rather than exhausting them [8]. Consequently, nature's restorative power could reduce mental fatigue and refresh the ability to concentrate, and in doing so prevent occupational burn-out.

Over the past two decades, interdisciplinary teams have sought to confirm that the aesthetic qualities of fractals are inducing these striking effects. Fractals are patterns that repeat at increasingly fine sizes and so create shapes of rich visual complexity [9]. This hypothesis was inspired by the prevalence of fractal objects in nature, as catalogued in mathematician Benoit Mandelbrot's 1982 book The Fractal Geometry of Nature [9]. Common examples from our daily lives include clouds, trees, and mountains. Further emphasizing their visual impact, fractals have also permeated the artistic expression of cultures spanning many centuries [10,11] from Hellenic friezes (300 B.C.E) to Jackson



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Pollock's abstract paintings (mid-20th century) [12–14]. Whether natural or created, fractals represent a profound ingredient of our visual experiences. As Pollock famously declared "My concerns are with the rhythms of nature" and concluded "I am nature" [15].

I will review experiments suggesting that adaptation to nature's fractals influences many stages of the human visual system—from how the eye moves when acquiring the visual data of fractal patterns through to how the brain responds when processing their characteristics. Based on these findings, I will outline the "fractal fluency" model in which human vision has become fluent in the visual language of nature's fractals and can process their features efficiently. The model predicts that the increased performance of basic visual tasks during this "effortless looking" will create an aesthetic experience. This natural preference for fractals has special significance for the field of experimental aesthetics. In the 1930's, when mathematician George Birkhoff proposed Aesthetic Measure (the idea that there is relationship between measurable mathematical properties of images and their aesthetics) complexity was a critical property in his discussions [16].

I will show that the aesthetic experience induced by fractal complexity is accompanied by a decrease in the observer's physiological stress-levels. In addition to providing a deeper understanding of the visual system's relationship with nature's visual stimuli, fractal aesthetics studies therefore have the potential to improve the built environment. Applications of fractal fluency represent a fundamental form of salutogenesis—the medical approach of focusing on factors that promote well-being through the management of stress, health and coping [17]. Spending more time surrounded by nature's fractals will enhance well-being but people's work restraints often limit this possibility. Although more challenging, incorporating fractal patterns into the built environment will be more impactful. The call for fractal architecture is not new and today's advocates [18–23] are often inspired by Alexander's work. Yet the need for fractal designs—whether for the interior or exterior of individual buildings, or for the arrangement of multiple buildings—is escalating. The World Health Organization views stress to be the "Health epidemic of the 21st Century," with associated illnesses ranging from depression to schizophrenia [24]. As people increasingly find themselves surrounded by urban landscapes, they risk becoming disconnected from the relaxing qualities of nature's fractals. In response, designers and architects will need to rise to the inter-disciplinary challenges and rewards of creating fractal designs informed by the art and science of fractal aesthetics. I will use some recent approaches to incorporating fractals into the interior and exterior of buildings to highlight the advantages and also the challenges of various approaches. Focusing on the practicalities of implementation, the examples presented here impose fractal designs on conventional buildings (by introducing patterns through carpets, ceiling tiles, window blinds, solar panels etc.). The hope is that this current demonstration of salutogenesis will be extended to future buildings which will be shaped using fractal architecture.

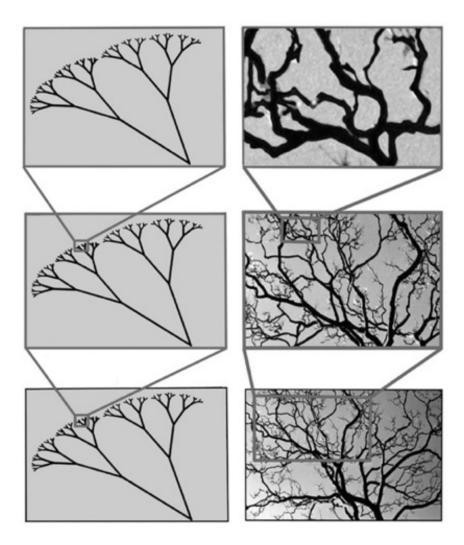
# 2. Fractal Dimension: The Visual Complexity of Fractals

Leading traditional studies of preference for nature's scenery overlooked its subtle complexity by adopting vague descriptions such as degree of "naturalness" [8] or through the use of simplified representations based on Euclidean shapes [16,25]. Although some theories considered optimal balances between order and disorder [25–27] or simplicity and complexity [28], they fell short of presenting a unified model that relates aesthetic response to the parameters of the scene's underlying structure. As has been stressed, "one must understand the nature of the environment before one can understand the nature of visual processing" [29]. Experiments that characterize natural environments in terms of their fractal characteristics therefore represent a major step forward in studies of human perception of nature.

In Figure 1, a prevalent form of fractal—a tree—is used to highlight their intrinsic visual properties. Fractals fall into two families—"exact" (left image) and "statistical" (right image). Exact fractals are assembled by repeating a pattern precisely at many scales. Randomness disrupts this repetition for statistical fractals and only the pattern's statistical

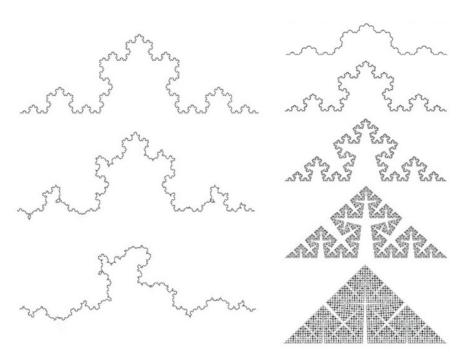
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qualities repeat. Statistical fractals therefore simply appear similar at different scales leading to the term "self-similarity". Exact fractals have been studied by mathematicians since the 1860s: famous examples were introduced by Weierstrass (1861), Cantor (1883), Peano (1890), Hillbert (1891), von Kock (1904), and Sierpinski (1915). In contrast, they are scarce in nature and a small degree of randomness inevitably creeps in. Consequently, natural examples of exact fractals, such as snowflakes and romanesco broccoli, lack the cleanliness of the mathematical versions. The large degree of randomness within statistical fractals provides the organic signature commonly on display in nature's scenery. The relative subtly of nature's version of fractality explains why it took a century for mathematicians to fully appreciate that nature shared the same underlying geometry as the early exact fractals. The left column of Figure 2 employs another common fractal—a coastline—to further demonstrate that introducing randomness morphs the cleanliness of the exact fractal into the subtle statistical version.



**Figure 1.** The branch patterns of an artificial tree repeat exactly at different magnifications (left column). In contrast, only the statistical qualities repeat for a real tree (right column).

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**Figure 2.** Left column: A computer-generated coastline based on exact fractals (top) is morphed into a statistical fractal coastline (bottom) by introducing randomness. For the top fractal, all of the headlands point upward. For the bottom fractal, half point downward and the positions of the up and down headlands are randomized. Note the D value (1.24) is preserved for all 3 patterns (top, middle, and bottom). Right column: The effect of increasing D is shown for 5 exact coastlines. Each of the coastlines is built using the same coarse scale pattern. Increasing the contributions of the fine scale patterns causes the coastlines to occupy more of the 2-dimensional plane, thus raising their D values: 1.1 (top), 1.3, 1.5, 1.7, and 1.9 (bottom).

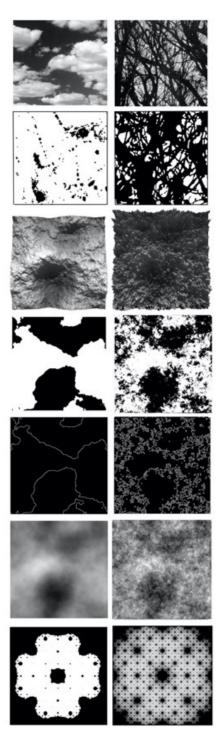
The ability to visualize fractals is central to their investigation. Around the time that Mandelbrot wrote The Fractal Geometry of Nature [9], he also helped to develop the most published image created by any mathematician—The Mandelbrot Set. Although similar equations had be around for fifty years, it required 1980's computing power to generate the associated images. Computer technology continues to radically expand the ability to explore fractal patterns. For example, it is now possible to generate 3-dimensional analogs of The Mandelbrot Set called Mandelbulbs. Although such mathematical objects join nature's fractals in 3-dimensional space, our retinas receive 2-dimensional projections of them.

Psychologists employ a parameter developed by the mathematicians to assess the visual intricacy resulting from the fractal pattern repetition. Fractal dimension D [9,30] quantifies how the patterns at different scales assemble into the fractal image projected on the retina. For simple (i.e., non-fractal) shapes, D matches what we would expect for dimension: a smooth line has a D value of 1 while a completely filled area has a value of 2. The repeating patterns embedded in a fractal line cause it to begin to occupy space. Accordingly, its D value lies between 1 and 2. When the contribution of fine structure to this fractal mix is increased, the line gradually fills in the 2-dimensional surface of the retina and the fractal's D value therefore approaches 2 (Figure 2, right column).

Figure 3 shows how the D value impacts the appearance of example fractal stimuli found in nature, art, and mathematics [31–34]. For each row, the left column image has a lower D value than the right image. The low content of fine structure within the low D fractals builds a very sparse and simple shape. However, as the D values move closer to 2, the increase in fine structure content creates a much more intricate, detailed shape. Because D charts the ratio of fine to coarse structure, it measures the visual complexity produced

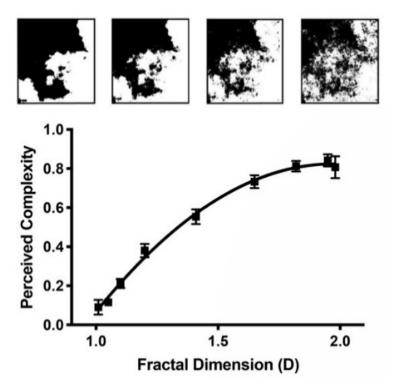
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by the repeating patterns. Behavioral research [35,36] confirms that people's perception of complexity increases with *D* (Figure 4).



**Figure 3.** Demonstrations of fractal complexity found in nature, art, and mathematics. The different rows summarize the variety of fractal images employed in my studies (see text for details). In each case, the left column shows examples of low D fractals and the right column show the equivalent high D fractals.

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**Figure 4.** Perceived complexity increases with the fractal's D value. Examples of computer-generated fractals quantified by D = 1.2, 1.4, 1.6, and 1.8 are shown above the graph.

The images of Figure 3 capture the variety of our previously studied images. The top row are photographs of natural objects (clouds and forests). The 2nd row are Pollock paintings from different periods [37,38]. The 3rd row shows mathematically-generated geographical terrains viewed from above. These terrains are used to generate the images below them. To create the 4th row images, a horizontal slice is taken through the terrain and then any terrain above this height is colored white. This is referred to as the coastline pattern (white being land) and is used to create the 5th row images by coloring the coastline edges white. The grayscale images shown in the 6th row are created by converting the terrain heights at each location into greyscale values. The bottom row shows mathematical examples of exact fractals.

Despite any superficial differences in the appearance of these 7 image types, their underlying fractal scaling generates a complexity that ramps up rapidly with *D* value. As an indication of its central role, this shared complexity is responsible for how fractals came to be named. Looking for a term to unite the exact and statistical patterns, Mandelbrot noted that their complexity created a fractured appearance and so took the Latin for fracture ("fractus") and altered it to the now famous name. This complexity is also responsible for 2 commonly experienced tell-tail signatures of fractals. While "fractal scaling" refers to the fact that fractal objects look similar when viewed near to and far away (picture a cloud), "fractal displacement" conveys their spatial uniformly (picture trees in a forest) [39]. Whereas these 2 signatures can be ascertained through conscious inspection, in the next section I will consider a set of "automatic" processes that unfold within the visual system well before conscious deliberations take hold.

#### 3. Fractal Fluency: Visual Processing of Fractals

Although there are examples of natural fractals with *D* values from 1.1 to 1.9, the most common lie in the narrower range between 1.3 to 1.5. As examples, many clouds and trees lie in this range. This informs the fluency model, which proposes that humans (and presumably animals) have adapted to efficiently process these mid-complexity patterns using a cascade of automatic (i.e., not consciously driven) processes evident at many levels

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of the visual system. To examine the first "entry" level, we used eye-motion studies to follow the observer's gaze when they look at fractal images displayed on a monitor [30,31]. As expected, the eye follows long saccade trajectories when jumping between points of interest and smaller micro-saccades during dwell periods. When we examined the saccade trajectories of a variety of people, we found that these trajectories traced out fractal patterns described by D = 1.4. This result held for all images observed even though they varied across the wider range from D = 1.1 to 1.9 [30,31]. We found that participants with and without neurological conditions revealed the same fractal gaze dynamics, suggesting that the fractal motion is intrinsic to eye-motion and is not modified by higher level processing in the visual system [40]. Subsequently, other groups exposed our fractal images to primates to investigate commonalities with animals [41].

We proposed that the eye searches through the scenery to confirm its fractal character [30]. If the gaze concentrates on just one location, the peripheral vision lacks the resolution to detect fine scale patterns in regions further away from the gaze's focus. The gaze therefore moves so that the eye's fovea can sample the fine patterns at many locations. The eye then experiences the full distribution of coarse and fine scale patterns necessary for confirming the scene's fractality. The answer to why the eye follows a fractal trajectory during this search can be found in foraging behavior. Animals benefit from fractal searches when exploring their natural terrains [42]. The mathematical efficiency of these multiscaled searches provides the likely explanation for why they are exploited both by animal searches for food and the eye's search for visual information [30]. The mid-*D* saccade is optimal during this fractal search because it matches the *D* values of prevalent fractal scenery. The saccades then match the fractal mix of coarse and fine structure found in the scenery, so facilitating an efficient sift through its visual information. We found that pupil dilation also varies in a fractal manner as the eye moves over the fractal images, suggesting further refinements to the search mechanism [43].

Evidence for the enhanced processing of mid-*D* fractals can also be found at later stages of the visual system. The brain's visual cortex can be modelled as a system of virtual "pathways" that process scenic information [44,45]. The number of pathways dedicated to processing objects of a particular size has been shown to be proportional to the relative number of objects of that size within in the scene. Through evolution, the distribution of pathways has therefore matched the *D* values that dominate the environment. Fractal processing also makes use of fractal images stored in our memories by utilizing simultaneous synthesis (an integration of current perceptual information with long-term memory) [46]. This would suggest that, as the eye searches efficiently through the image to confirm its fractal content, the brain is calling on fractal memories to help in this confirmation process. As to why this confirmation of fractality is so important, such a strategy would, for example, have allowed our ancestors to identify the non-fractal forms of animals within fractal scenery—so promoting their survival. Once a non-fractal element is detected then the fractal search is suspended and the "effortless looking" switches to focused attention on the element of interest.

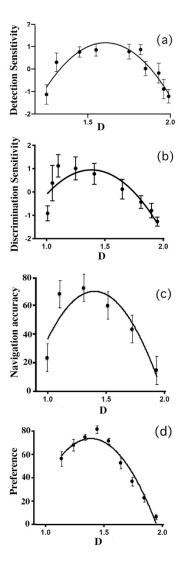
It is appealing to consider the role of fractal memory within the context of Jungian psychology and to propose that fractal imagery resides within the collective unconscious of all humans. However, modern neurophysiological techniques can provide a more focused and quantitative analysis of our brain activities. Employing quantitative EEG, peaks in "alpha waves" are associated with wakefully relaxed states while peaks in "beta waves" indicate heightened attention [47]. Strikingly, the D=1.3 fractals induced the largest changes in both alpha and beta responses [48,49]. This ability of stimuli to simultaneously relax and arouse is unusual and points to the unique role of nature's fractals for the visual system. Preliminary studies employing fMRI to examine which regions of the brain are being utilized also reveal D-dependent responses [31,50]. On-going fMRI studies will consider the role of the parahippocampal region (which is known to be involved in memory retrieval and scene recognition) and the default mode network (a large brain network

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associated with wakefully restful activities such as daydreaming and mind-wandering, and which features in modern versions of ART [51]).

#### 4. Fractal Aesthetics: The Visual Impact of Fractals

Taken together, the above experiments describe a sequence of automatic processes that enhance our capability to process the visual information of mid-D fractals. The peak in the qEEG beta response emphasizes that the viewer's attention is being engaged by mid-D fractals [48]. While engaged, fractal fluency improves the performance of visual tasks. For example, participants in behavioral studies exhibit increased sensitivity to mid-D fractals [52]. To demonstrate this, the pattern contrast of fractals shown on a monitor was gradually decreased until the monitor displayed uniform luminance. When observing mid-D images, participants could see these fractals under lower contrast conditions (Figure 5a) [52] and could distinguish their D values more accurately (Figure 5b) [45,46,52].

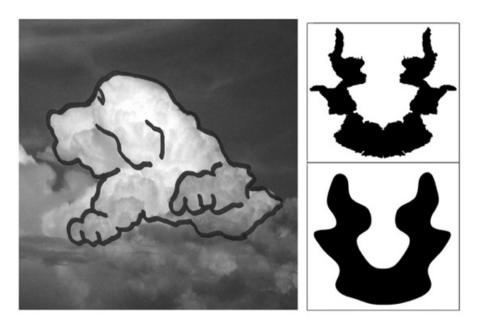


**Figure 5.** Performance tasks (detection (a), discrimination (b), and navigation (c)) and preference ratings (d) plotted against the fractal's *D* value. Refer to the individual studies discussed in the text for details of the measurements and the relevant y-axis scale.

Pattern recognition capabilities also heighten for mid-*D* fractals. For example, associated improvements in spatial awareness led to superior navigation through environments containing mid-*D* fractals [53]. When participants were instructed to navigate an avatar to find an object randomly placed within a virtual landscape, accuracy and completion speeds

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peaked for the mid- complexity landscapes predicted by the fluency model (Figure 5c). Imaginary objects induced by clouds serve as another example of heightened pattern recognition processes (Figure 6). The visual system becomes "trigger happy" when viewing these mid-D fractals and so we perceive objects that don't actually exist (pareidolia) [54]. Our research confirms that mid-D fractals induce large numbers of percepts [55] and that they activate the visual cortex's object perception and recognition regions [56]. This is supported by our studies of Rorschach ink blots. Perception of shapes in the fractal blots peaks in the lower D range [57] and declines when the fractal structure is electronically removed from the blot images (Figure 6).



**Figure 6.** Left: Fractal clouds are renowned for inducing perceived images (as an illustration, a perceived dog is drawn on the photograph of a cloud). Right: an example ink blot (top) which induces fewer percepts when these edges are smoothed (bottom).

All of these enhanced performances raise a crucial question for biophilic studies: because we find mid-*D* fractals so easy to subconsciously comprehend, is fractal fluency accompanied by a powerful aesthetic experience? Behavioral experiments show that ninety-five per cent of people prefer fractal images over ones which have had their fractal content reduced [58]. Fractal aesthetics experiments also confirm that preference for mid-*D* complexity occurs for a wide variety of fractal image types [31,59,60] and that this preference is already evident by the age of two [61]. Preference is robust to the method employed to measure aesthetics (for example, some experiments adopt the forced choice method in which observers chose the most preferred from pairs of displayed images. Other studies show images individually and observers rate them). Figure 5d shows example results for computer-generated fractals and analogous results hold for fractals found in art and nature [31].

Most studies of nature's fractals focus on individual objects (for example, those of Figure 3) even though the overall scene is expected to set fluency. Natural scenes typically feature a rich fractal content [44,62,63] originating from several factors: (1) the fractal shapes of a range of individual objects [9], (2) the fractal distribution of sizes of these objects [44,64], (3) the fractal luminance textures within the objects [65], and 4) the fractal shapes formed when neighboring objects combine visually to create "fractal composites" [66,67]. Our studies show that the skyline—the dominant fractal composite in many scenes—typically dictates preference, with mid-*D* skylines being the most preferred [66]. Notably, winter skylines have a higher *D* and are less preferred than their summer mid-*D* equivalents (Figure 7), perhaps contributing to seasonable affective disorder.



**Figure 7.** Left: A photograph of a natural scene (top) and the fractal composite formed by its skyline (bottom). Right: photographs taken in summer and winter featuring trees with leaves (low *D*) and without leaves (high *D*).

This "universal" peak preference for mid-*D* statistical fractals (revealed for artistic and mathematical creations, along with individual and combined natural objects) shifts to higher *D* values when viewing exact fractals [34]. This is expected from the fractal fluency model. When morphing from statistical fractals to their exact equivalents, the removal of randomness results in a lower complexity pattern (Figure 2). The preferred *D* value therefore has to rise to regain the optimal complexity set by exposure to nature's statistical fractals. In other words, the simplicity introduced by exact repetition increases the tolerance for higher fractal complexity.

Consistent with the alpha wave study, a NASA-funded project suggests that the aesthetic resonance induces a state of relaxation. The study examined the stress-levels of participants in a mock-up space laboratory [68]. While exposed to images, participants performed a sequence of stress-inducing mental tasks separated by recovery periods, thus creating a sequence of alternating high and low stress periods. The physiological response to the stress was recorded using the skin conductance method employed in Ulrich's original stress studies of nature [4]. The stress saw-tooth was found to dampen when participants viewed mid-*D* fractals, indicating a stress-reduction of 60% [68].

Building on these laboratory-based experiments, a computer server has sent fractal screen-savers to 5000 people's monitors in their homes who then voted electronically for their preferred images [69]. Through an iterative voting process, the fractal screensavers evolved with time towards the most aesthetic fractals. The results agreed with the preference for mid-D statistical fractals found in the laboratory. When fractal images are projected on the walls of a room rather than displayed on monitors, the preferred D shifts to higher values ( $D \sim 1.6-1.7$ ) [70]. The observer then sees the fractal surrounded by the blank surface of the wall. This introduction of Euclidean simplicity increases the tolerance for high fractal complexity [70]. This experiment serves as a warning for future fractal studies aimed at biophilia. Because practical applications will embed fractals within artificial environments, we will have to adapt them (bio-inspiration) rather than simply copy them (biomimicry). For example, when sat in a room surrounded by simple walls, the preferred *D* value will be higher than when the observer walks through a forest and is engulfed by other fractals. Experiments investigating the importance of matching city skylines to the backdrop of fractal mountains [71] further emphasize the importance of viewing context.

Building on the core principles of human-centered design, it is also important to acknowledge that one fractal will not fit all! Although the overall population prefers mid-*D* values when viewing fractals on a monitor, there are 3 sub-groups which exhibit distinct preferences. Whereas the majority's preference peaks at mid-*D*, just under 25 per cent of

observers are instead "sharpies" (preferring high *D*) and a similar number are "smoothies" (preferring low *D*) [35]. One recent study proposed that genetic factors might influence the fractal aesthetics of individuals [72]. It will also be intriguing to explore if there are underlying personality traits that characterize these subgroups. For example, it has been suggested that creative people might have a preference for higher *D* values [73]. Some studies show that urban versus rural living along with aging can shape fractal preference, indicating that adaptation during our lifetimes might also be a factor [74]. Clearly, as we shift from fundamental to applied research, the specifics of the individual spaces along with the needs of the individuals who occupy them will be crucial. Nevertheless, the basic requirements look favorable for applications. The amount of fractal repetition required to trigger the positive effects can easily be achieved. Set by typical magnification ranges of nature's fractals [32,75], the largest pattern needs to be just a factor of twenty-five larger than the smallest. Furthermore, participants typically took less than a few seconds to rate aesthetics (and much of this time was spent recording their judgment). Consistent with automatic processes, long exposure times are not necessary.

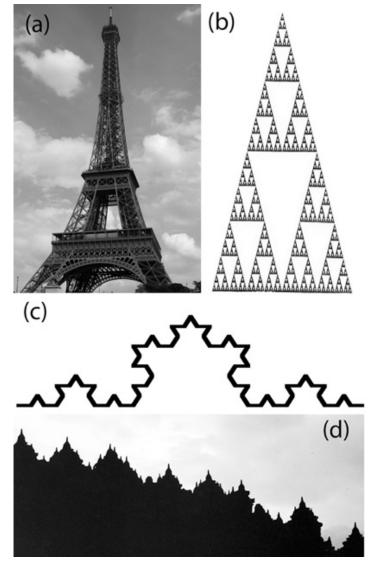
#### 5. Fractal Expressionism: The Creation of Fractals in Art, Design and Architecture

Fractal fluency pictures the visual system as an efficient fractal detector. This detection occurs well before conscious deliberations shape our aesthetic experiences. This might explain a recurring theme throughout the history of fractals—that intuitive artistic creation of fractals often pre-dates their conscious mathematical "discovery". For example, the repeating triangles found in von Koch's famous Koch Curve of 1904 were actually first used to illustrate waves in Hellenic friezes (300 B.C.E.). The Book of Kells (circa 800 C.E.) and sculpted arabesques in India's The Jain Dilwara Temple (1031 C.E.) also display remarkable examples of exact fractals. Repeating triangles appear in the 12th century pulpit of Italy's The Ravello Cathedral. Similarly, in the 13th century, triangles within Cosmati mosaics created a fractal shape that 7 centuries later became celebrated in mathematics as the Sierpinski Triangle.

The Ryoan-ji Rock Garden in Japan (15th century) along with the artistic works of Leonardo da Vinci (eg The Deluge, 1500), Katsushika Hokusai (eg The Great Wave, 1833), Salvador Dali (eg Visage of War, 1940) and Mauk Escher (eg Circle Limit III, 1959) serve as more recent examples. Fractals continue to hold fascination for artists, and we can learn from their creative processes. Take, for example, the Abstract Expressionists whose gestural techniques generated statistical fractals referred to as Fractal Expressionism [39]. The work of Willem De Kooning decreased in *D* value as he descended into Alzheimer's, highlighting the influence of neurological conditions on fractal fluency and therefore on fractal aesthetics [76]. In contrast, the *D* value of Pollock's paintings increased during his career. Pollock was aware that his drive towards higher complexity paintings would reduce the percepts: "I try to stay away from any recognizable image; if it creeps in, I try to do away with it. I don't let the image carry the painting. It's extra cargo—and unnecessary" [57]. Computer analysis of Pollock's fractal characteristics has been employed to distinguish authentic works from imitations and agrees with the judgments of the pre-eminent Pollock scholar Francis O'Connor—emphasizing O'Connor's highly trained fractal eye [77,78]!

In addition to fractal art, there have been stunning cases of architecture that incorporate repeating layers. The Borobodur temple constructed in Java during the 8th century is an early example (Figure 8). Built by the Holy Roman Emperor Frederick II in the 13th century, the layout of the Castle del Monte features several sizes of octagon. The Gothic cathedrals of Europe (12–16th century) also exploit fractal repetition of shapes (arches, windows, and spires) while the repetition of triangles in Frank Lloyd Wright's Palmer House in Ann Arbour (1950–1951) and the bubble patterns of the Beijing Olympics' Water Cube (2008) [79] add to their appeal. Moving beyond individual buildings, some African villages follow a fractal plan [80] and fractals appear in the skylines [71] and boundaries [81] of modern cities.

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**Figure 8.** The Eiffel Tower (**a**) compared to a Sierpinski Triangle (**b**) and a Kock Curve (**c**) compared to the Borobodur temple (**d**).

Gustav Eiffel's tower (1889) (Figure 8) enjoys some of the practical implications of fractal architecture. Eiffel employed the structural rigidity of a triangle at many different size scales—if the tower had instead been a solid pyramid, the extra iron would have added to the weight and cost without adding significantly to its strength. Fractals offer other practical advantages based on their large surface area to volume ratios, including large surfaces for solar panels and windows. The repeating structures can also dissipate the energy of impinging waves so fractal building designs can minimize noise from traffic and vibrations from earthquakes. By combining these physical advantages with their visual impact, artificial fractal environments should have a vibrant future.

To bring this vibrant future into reality, it will be necessary to create fractal designs that are informed by the science of the biophilic movement—and as such represent a natural and powerful approach to salutogenesis—and simultaneously meet the practical demands of the spaces that we live and work in. I will now discuss some of my own fractal design projects within the context of this balancing act. Placing art on a wall is the most obvious starting point. Given that the positive effects of fractal aesthetics have been triggered by an impressive variety of fractal stimuli, considerable artistic creativity can be applied to

the task of realizing a target *D* value. An Orwellian-like future of staring at the one perfect fractal image is therefore avoided!

The Buckley Tree of Figure 9 highlights the capacity to manipulate fractality for artistic effect. For the standard model of a fractal tree (introduced in Figure 1), each branch splits into two smaller versions and the *D* value is controlled through the rate of shrinkage of the branches (the lower shrinkage rate of high *D* fractals increases the contribution of the fine scale branches). Inspection of the Buckley Tree reveals that although the first branch splits into 2, this then splits into 3, which then splits into 4 branches etc., so offering a novel adjustment of the fine structure within the fractal mix. My simple invention would have been easy work for the likes of da Vinci and Escher, both of whom generated their fractals through an integration of artistic and scientific observation. Realizing that the invisibility of air turbulence hindered his development of air machines, da Vinci's meticulous studies of the equivalent fractal vortices in water currents resulting in The Deluge. Inspired by the Islamic tiles of Spain's Alhambra, Escher decided to replicate the tiling process at many size scales [82]. Making his patterns fit together required a helping hand from the mathematical work of Harold Coxeter who declared: "Escher got it absolutely right to the millimeter" [83].

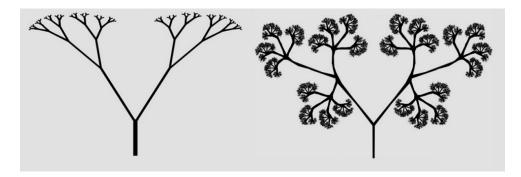
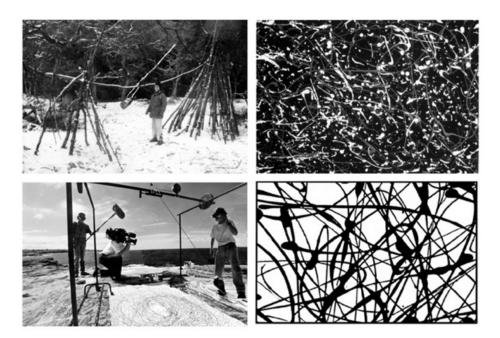


Figure 9. A comparison of a standard model of a tree (left) and the Buckley Tree (right).

However, to achieve the prevalence of fractal art necessary for promoting salutogenesis, we cannot rely on artists matching da Vinci and Escher's fluency in both art and science. An alternative approach is for artists to harness natural processes known to generate fractals. As an example, I used the pendulum shown in Figure 10 to translate a storm's fractal wind currents into a painting [37]. While a sail attached to one end of the pendulum was driven by the wind's motion, painting vessels at the other end poured paint onto a horizontal canvas stretched out across the floor below. A search for a more practical equivalent painting machine led to the Pollockizer (Figure 10) which uses electromagnets to knock the pendulum in a controlled manner and adjust its D value (the Pollockizer is so named because it replicates Pollock's patterns, which originated from the fractal motions of his body's balancing mechanisms) [31,39]. Although audiences are drawn to such demonstrations of fractal expressionism, inevitably even this approach is far from ideal for the mass production of fractal art. Emerging in the 1980s, computers have therefore become the most effective tool for generating fractal art. Ushered in with popular posters of the Mandelbrot Set, many early examples of computer fractal art demonstrated that mathematics wasn't sufficient—the most successful creations needed the artistic sensibilities offered by collaborative teams composed of both designers and scientists.

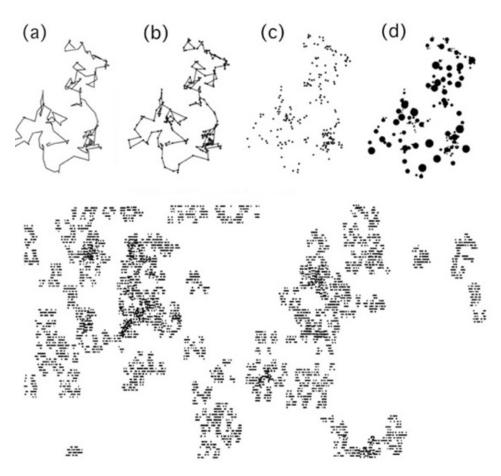


**Figure 10.** A photograph of the fractal wind machine (top left) and the resulting fractal art (top right) along with the Pollockizer (bottom left) and its fractal art (bottom right).

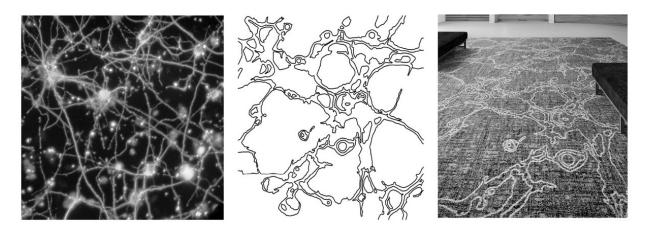
I formed The Science and Design Laboratory (SDL) in 2017 with Austrian designers Ana and Martin Lesjak to apply science-informed fractal designs to building interiors. Because floors occupy such a large part of our eye's visual field, an award-winning collection of carpet designs called Relaxing Floors was developed for the Mohawk Group, one of the world's largest carpet manufacturers [84]. As shown in Figure 11, the eye's fractal trajectories provided the basic lay-out for one of the carpet designs. A circular "seed" pattern was inserted at the locations between trajectories, and its size was scaled according to the length of the preceding trajectory. Then each circle was replaced by a fractal pattern. Fractals therefore contributed to the final carpet design in three ways: (1) the fractal trajectories determining the location of the seed patterns, (2) the fractal distribution of the seed sizes, and (3) the fractal shape of each seed. This approach was inspired by the rich fractal content of nature's scenery (Figure 7). Its D = 1.6 value was informed by the D = 1.6-1.7target range suggested by the aesthetics experiments that projected fractal images into rooms. Emphasizing the versatility of statistical fractals, cutting the pattern into tiles and randomly re-arranging them did not disrupt the fractal character nor significantly shift their D values. This has important consequences because many carpets in large spaces ranging from airports to hotels are installed as tiles rather than as continuous carpets.

A second flooring design employed images of nature's fractals—retinal neurons—as the starting point (Figure 12) [84]. These images were obtained as part of a research project which develops retinal implants to restore vision to patients with diseases such as macular degeneration [85]. Fluorescence microscopy was used to acquire detailed images of the retinal neurons in order to quantify parameters such as their D values (Figure 12 (left)). For the floor designs, the images were converted into grayscale versions and then contoured (Figure 10 (middle)). We initially expected to use software to manipulate the D values of the neuron contours but, fortuitously, the selected image's D value of 1.7 fell within the D = 1.6-1.7 target range.

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**Figure 11.** Top: The carpet's pattern generation process. (a) The eye's fractal trajectories provided the basic lay-out for the carpet design, (b) circular seed patterns were added to the "landing" locations between these trajectories, (c) the trajectories were removed, (d) the sizes of the circles were scaled based on the length of the previous trajectory. Bottom: the complete carpet pattern created by replacing the circular seeds by fractal seeds.



**Figure 12.** (left) Fluorescence image of retinal neurons, (middle) contours extracted from the greyscale image of the neurons, (right) an image of the installed carpet.

Figure 13 highlights an important key to success—the development of versatile designs that form the basis of multiple applications, in this case as carpet patterns for a university environment, as wall patterns used to disperse light throughout a chapel, and as computer screen savers (the latter are being made available for free personal use during the pandemic). The same form of design is also being used in a collaboration with *Fact Design* to install

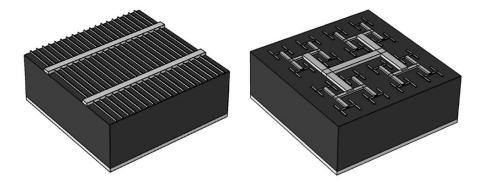
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patterned tiles on ceilings. This application demonstrates a second key strategy for success—that patterns should, if possible, provide simultaneous benefits. In this case, the patterns are embossed in the tiles, offering the potential to create an aesthetic impact coupled with the established noise-dampening capabilities of fractal surfaces [86]. We are currently determining if there is an optimal *D* value that maximize both functions.



**Figure 13.** The fractal pattern of Figure 11 employed as a floor design at the University of Oregon, USA (left), as wall patterns in the Fractal Chapel in the State Hospital in Graz, Austria (middle), and as a design for computer screen-savers (right).

Inspired by this strategy of combining aesthetics with other favorable functions, a psychology-engineering project recently incorporated fractal aesthetics into solar panels (Figure 14) [87]. Because the electrical power increases with the panels' surface area, the associated increase in visual impact will be critical for determining their success [88]. "Blended" panels have recently been introduced to neutralize the poor aesthetics of conventional designs. They cover the panels with camouflaging louvers which match the panels to their surroundings [89]. In contrast, rather than neutralizing their visual impact, our panels actively enhance the environmental aesthetics. This required the development of a hybrid electrode pattern which integrates a fractal design (based on the exact repetition of an H pattern shown in Figure 14 right) with the traditional solar panel design (the "bus-bar" shown in Figure 14 left). This novel hybrid electrode matches the electrical performance of bus-bars while promoting fractal aesthetics [87].

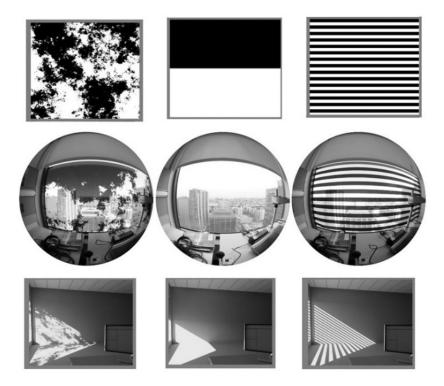


**Figure 14.** Left: The traditional bus-bar design of solar panels in which the electrode (light grey pattern) features large bars and finer perpendicular bars. Right: A fractal electrode design based on a repeating H pattern.

Fractal window blinds (Figure 15) offer further possibilities for multi-functionality [70,90]. The fractal pattern can be used to obscure an unattractive view, it can provide shade, and

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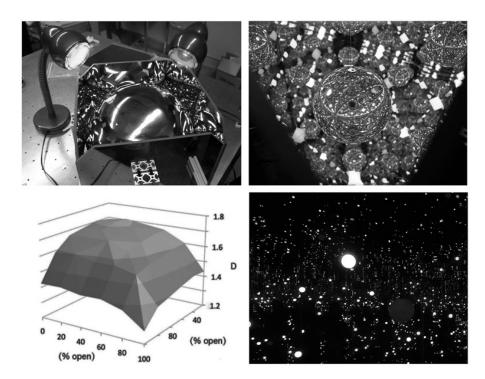
it can also cast a fractal shadow pattern across a room. For open windows, the shade can also generate a fractal breeze. In addition to the fractal variations in light, it can therefore provide analogous variations in heat and air currents for the room's occupants (I will return to the potential of experiencing fractals across multiple senses in the Conclusion). The shades also offer the advantage of impacting the building's interior and exterior appearance simultaneously. Most importantly, however, the shades can generate dynamic fractals, which are expected to maintain the observer's attention to a higher degree than their static equivalents (nature's dynamic fractals include moving ripples of water, tree branches swaying the breeze, flickering flames, and clouds moving across the sky). In this case, the shifting sun will move the fractal shadows across the room during the day and clouds will create extra variations on shorter time scales. The fractal blinds could be supplemented with water troughs located outside of the windows and their ripples could cast fractal light patterns into the room which vary with the wind. This idea of projecting nature into rooms is central to the biophilia movement [91].



**Figure 15.** A comparison of window shades with fractal (left), square (middle), and striped (right) shapes. The top row shows the basic patterns, the middle rows show them applied as shades, and the bottom row shows their shadows cast into a room.

Inspired by the prevalence of light patterns in nature, these ideas led to my investigations of rays of light reflected between multiple mirrors to create fractal light patterns (Figure 16). Our design is informed by the mathematical research of Yakov Sinai. Based on his studies of the game of billiards [92], he focused on a shape that became known as the Sinai billiard. He showed that if the walls of the table repeatedly reflect balls onto a circular wall placed at the billiard's center, then their trajectories map out fractal patterns. Sinai's billiard is celebrated for studying fractals in a controlled system and consequently he became an Abel Laureate in 2014, the mathematical equivalent of a Nobel Laurette. Sinai billiards have technological applications such as fractal transistors [93–95] but here we exploit them to cross into the art world by using mirrors to create tunable fractal light patterns [96].

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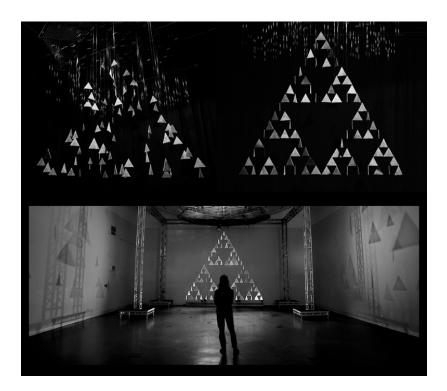
**Figure 16.** Top: the prototype Sinai mirror system featuring a spherical mirror within a cube (left) and the fractal light image created by the Sinai apparatus (right). Bottom: a graph showing the ability to tune the pattern's *D* value by varying the openings (left) and a room filled with fractal light patterns generated by spherical mirrors (right).

The photographs of the prototype apparatus (Figure 16) show a cube of mirrors and a central spherical mirror. By shining light through openings in the cube, we found that widening the openings reduces the chances of the light rays circulating around the cube and undergoing multiple reflections. This effect allows the fractal light pattern's D value to be adjusted and so allows us to accommodate the individual observer's aesthetic preferences [96]. These studies featured high fidelity mirrors to generate exact fractal reflections. Introducing random bumps into the sphere's surface will result in the equivalent statistical fractals. We plan to install smaller versions of this apparatus around light fixtures to cast fractal light patterns throughout rooms. The apparatus can also be hung in front of windows to cast natural light which varies as the apparatus sways in the breeze.

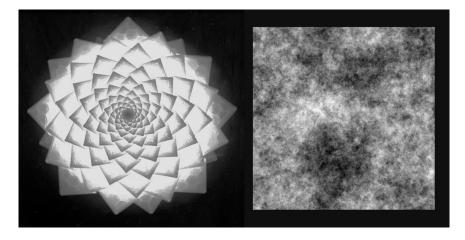
Finally, consider the use of dynamic patterns to create "virtual" fractals. Given that the repeating patterns can present a construction challenge, is it possible for the visual system to perceive fractals without them being physically assembled? Discussions with artist Richard Downing led to him creating *The Fractal Clock* (Figure 17). Based on the Sierpinski Triangle, Downing dangled a set of individual triangles (in one version they are made from glass, in another from slate) from cables attached to motors that allow the triangles to rotate at different rates. At a certain "resonance" time, all of the triangles align and the fractal is "assembled" in the observer's mind (even though in physical space the triangles remain a set of disconnected objects that a person can stroll between). In addition to being a stunning artwork, The Fractal Clock serves as a reminder that creative solutions can be used to overcome physical limitations.

Novatropes are less dramatic but more practical examples of virtual fractals. These are based on the principle of the zoetrope which was introduced in the 1860s. In the modern Novatrope built by Master Plan Industries (Figure 18), a 3-dimensional sculpture is rotated at high speed above a strobe light. The interplay of the strobe frequency with sculpture frequency creates virtual fractals that appear to move. If the surfaces shown in Figure 3 (3rd column) are used for the Novatrope sculptures then the observer can choose their *D* value. Furthermore, the strobe frequency can be used to tune the motion of the fractal image. Based

on the fractal fluency model, the optimal rate of change of dynamic fractals is expected to be set by the typical motions of nature's fractals. Given that this motion raises the stimulus complexity, the preferred D value of the dynamic fractal is expected to reduce. The optimal rate of change is a matter of current research [97]. However, given that the Novatrope can tune D and apparent motion, and that it is small (6 inches square) and portable, this device is ideal for studying the potential of dynamic fractals in real-life environments.



**Figure 17.** The Fractal Clock photographed at an angle that highlights the individual triangles (top left), close to "resonance" when the mind perceives the fractal (top right) and with an observer to give a sense of scale (bottom).



**Figure 18.** The top view of an exact fractal generated by the Novatrope (left) and the statistical fractal image that holds potential for future designs (right).

#### 6. Conclusions

Evolution has transformed the eye from its initial role as a simple motion detector to the remarkable system that we benefit from today. Vision is our dominant sense: the brain receives 2 billion pieces of information from our eyes every second (it receives

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only 1 billion pieces from the whole of the rest of the body) and consequently up to a 3rd of the brain's volume is dedicated to visual processing (compared to just 3 per cent for hearing). Under such data pressure, it is no surprise that the visual system has developed efficient strategies for processing what we see. In this review, I have described experiments showing that we have become fluent in the visual language of nature's fractal scenery. We are "wired" to look at fractals and not the Euclidean catalogue of circles, squares, and triangles that most buildings are composed of. The fractal patterns described here are increasingly referred to as biophilic fractals because they are likely responsible for biophilia's well-known effects, including the reductions in mental fatigue and stress observed in the pioneering psychology experiments that examined exposure to nature. Our own research has demonstrated significant increases in detection sensitivity, attention, visual performance (e.g., pattern recognition and navigation), aesthetic appeal and stress-reduction. Conversely, the lack of fractal aesthetics in unnatural (man-made) environments puts a strain on the visual system, inducing negative responses such as headaches [98].

Although far from being a comprehensive list of fractals utilized in today's buildings, the fractal applications discussed here capture the variety of possibilities along with their respective advantages and challenges. Future behavioral experiments will employ virtual environments to optimize the preferred D values of these fractals to allow for the specific visual characteristics of a room, including whether the fractal is covering a window, or a wall or the floor, and whether they are partially obscured by furniture etc. The function of the room will also be important. For example, a person running through an airport to catch a flight might require a different fractal experience than a patient recovering from surgery, and environments optimized for senior living will have different restraints than educational settings. As more examples are installed, these virtual experiments can then be supplemented by on-site studies. In addition to quantifying the behavioral aesthetics, future stress-reducing assessments could build on our previous skin conductance and EEG measurements to include, for example, cortisol tests. Previous research indicates that on-site experiences of natural stimuli are more impactful than when observing images displayed on monitors [99], suggesting that the range of effects that we have previously quantified in laboratory settings will be enhanced when studied in real-life settings.

The long-term goal is to generate patterns that will gain an occupant's attention in busy environments, and through this engagement will promote health through stress-reduction and improve diverse task performances through concentration restoration and enhanced pattern recognition capabilities. Adaptation with be a key consideration for on-site studies and whether people's aesthetic preferences evolve. This was a concern for the NASA experiments—would preference shift in a long journey to Mars? The same applies to staring at the same office wall day after day. As mentioned earlier, Pollock's paintings evolved towards higher *D* values over a decade. Was he adapting to his fractals? The ability to change fractals over a period of time could therefore be crucial. Dynamic fractals also maintain attention. As a test, we converted some Pollock paintings from his static originals to dynamic equivalents. Pollock didn't have the technology to do this but I'm sure he would have been impressed with their transformation.

Our behavioral and brain-mapping experiments might appear to be strange tools for judging aesthetics. Neuro-aesthetics in particular fuels a fundamental concern: to what degree is aesthetics driven by inevitable biological responses rather than cultured, intellectual deliberations? Both are important for fractal aesthetics. The initial aesthetic wave driven by the automatic processes outlined here does not exclude a second, conscious wave of aesthetic experience. Take, as an example, Escher's conscious integration of mathematics into art. It seems reasonable that the cleanliness of his exact fractals makes them more accessible to conscious inspection. In contrast, artists such as Pollock (who painted at speeds that excluded conscious intervention to generate his statistical fractals) were primarily driven by the first wave. As fractal art becomes more common, some forms of fractal art (the mid-*D* complexity enjoyed by the majority?) might become more socially acceptable than others. This acceptability might well arrive through conscious deliberation.

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The concept of the "complexity triangle" was introduced to explore this possibility [100]. Its 3 vertices are the fractal image's objective complexity (quantified through mathematical analysis), its subjective complexity (induced by the automatic processing associated with an individual's vision), and the social complexity dictated by peer interaction.

Here I have concentrated on the visual experiences of fractals. Based on synesthesia (when sensations are transferred between the senses), mid-complexity fractals could also impact audial and tactile experiences. Given that natural sounds ranging from bird song to running water have fractal qualities, and that fractal music is attracting growing attention [100], initial explorations of the aesthetics of fractals sounds [101] are expected to evolve into a vibrant research field. Along these lines, I am transcribing Pollock's paintings into music to compare people's responses to these equivalent visual and sonic fractals [100]. One of the pioneering examples of electronic fractal music could be employed in a similar fashion. Hugh McDowell (cellist in the Electric Light Orchestra) composed music using algorithms based on the Mandelbrot Set. His music therefore has the same objective complexity as the famous posters, but the subjective complexity might be different (with different preferred *D* values?) due to the distinctive processing functions of our visual and audial systems. In particular, the differing roles of time for fractal art and music have previously been considered [100].

In terms of tactile fractals, 3-dimensional printers allow computer-generated patterns to be printed ("contour-crafted") as physical objects and artists such as Daniel Della-Bosca have used them to construct fractal sculptures [102]. In discussions with Della-Bosca, we pictured rooms incorporating fractal surfaces for passers-by to touch. Mandelbrot had previously asked: "In order to understand geometric shapes, I believe you have to see them." Della-Bosca took this thought process one-step further by asking "what happens if you touch them too?" [102]. In on-going experiments, physical versions of the terrains shown in Figure 3 are being printed in order to compare their visual and tactile impacts [103]. In addition to fractal surfaces, we could also explore sensations created by fractal temperature variations and fractal air currents.

Imagine a future in which we immerse building occupants in synesthetic fractals—a "fractal atmosphere" of visual, sonic, thermal, and tactile experiences—inducing an emergent experience that we have all evolved to expect and appreciate. Based on the examples presented in this review, we already have the potential to walk into a room in which the fractal ceilings dampen the noise, the fractal window shades provide an optimal breeze, the fractal solar panels deliver efficient energy to the Sinai lighting, and all of their patterns combine to create a stress-reducing visual environment analogous to the complex scenes of nature. However, studies of on-site fractals are still in a preliminary phase, which is both surprising and disappointing given that The Biophilia Hypothesis is now almost 50 years old. The purpose of this review is therefore to highlight possibilities for future investigations of this promising approach to salutogenesis.

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