

Article



Remaining Useful Life Prediction-Based Maintenance Decision Model for Stochastic Deterioration Equipment under Data-Driven

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Abstract: Currently, the Remaining Useful Life (RUL) prediction accuracy of stochastic deterioration equipment is low. Existing researches did not consider the impact of imperfect maintenance on equipment degradation and maintenance decisions. Therefore, this paper proposed a remaining useful life prediction-based maintenance decision model under data-driven to extend equipment life, promoting sustainable development. The stochastic degradation model was established based on the nonlinear Wiener process. A combination of real-time update and offline estimation estimated the degradation model's parameters and deduced the equipment's RUL distribution. Based on the RUL prediction results, we established a maintenance decision model with the lowest long-term cost rate as the goal. Case analysis shows that the model proposed in this paper can improve the accuracy of RUL prediction and realize equipment sustainability.

Keywords: stochastic deterioration equipment; data-driven; nonlinear Wiener process; RUL prediction; maintenance decision; sustainability

1. Introduction

With the advancement of science and technology, the requirements for equipment reliability are getting higher and higher. The health management of equipment is an essential factor that determines the sustainable development of enterprises and improves production efficiency [1,2]. However, the complex working environment gradually reduces the equipment's Remaining Useful Life (RUL) and reliability, leading to serious safety accidents [3]. At present, preventive maintenance is still the primary maintenance method for stochastic deterioration equipment, and there is a phenomenon of over or under maintenance [4]. With the rapid development of Sensor technology [5], Data mining [6], and Intelligent algorithms [7], Prognostics and Health Management (PHM) has gradually become the main management methods for complex equipment systems [8], which has attracted the attention of many scholars at home and abroad [9–12].

PHM technology mainly includes RUL prediction and health management. At present, most researches focus on the RUL prediction. The main methods include degradation mechanism, [13–15], data-driven, and hybrid models [16,17]. The model method based on the equipment degradation mechanism requires an in-depth understanding of the physical principles of the equipment, and the actual complex working environment limits its development [18,19]. The data-driven method is widely used in RUL estimation due to its flexibility [20], mainly including neural network-based method, regression-based method [21], random filtering-based method [22,23], and random process-based method. Han, C. et al. [24] proposed a mobile convolutional neural network (TCNN) to learn domain invariant features. The trained TCNN can be used to predict RUL by feeding data. Chen, Z. et al. [25] proposed an attention-based deep learning framework to predict



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Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). the RUL of the equipment. Meng, M. et al. [26] proposed a convolution-based long shortterm memory (CLSTM) network to predict the RUL of rotating machinery. Unlike the literature [25], the proposed network performs convolution operations on both the input state transition and state transition of LSTM, which retains the advantages of LSTM and integrates time-frequency characteristics.

However, the above method requires a large amount of status monitoring data to model the degradation process. Furthermore, due to the complex operating environment of the equipment, the error in the measurement data and other factors cannot guarantee the accuracy of the results. Considering the uncertainty of life prediction, the modeling method based on a stochastic degradation process proposed by Nozer, D. [27] has been recognized as an effective method for equipment health management and remaining life prediction [28–30]. In addition, many scholars have established random degradation models based on the Gamma process [31,32], Gaussian process [33,34], Inverse-Gaussian process [35], and Wiener process. Both the Gamma process and the Gaussian process are only applicable to the monotonic degradation process. However, the Wiener process can describe the non-monotonic process of equipment degradation and has become a research hotspot [36,37]. Li, N. et al. [38] designed an RUL prediction method based on Wiener Process Model, which is based on the Maximum Likelihood Estimation (UMLE) algorithm to predict RUL. Man, J. et al. [39] established a random degraded signal model based on the Wiener process with drift and proposed a joint model and a Markov model of the Wiener process. Wang, H. et al. [40] proposed an improved Wiener process model for RUL prediction, in which drift, and diffusion parameters are obtained by real-time updates of monitoring data. Liu, D. et al. [41] proposed a reliability estimation and degradation modeling method based on the Wiener process and evidence variables. By applying evidence variables to describe the model parameters, the Wiener process is combined with evidence theory. The above studies are mainly based on the Wiener process to predict the remaining life of the equipment, and there are few studies on the maintenance decisions taken from the prediction results. Zhang, M. [42] established an imperfect maintenance model based on Wiener stochastic process and considered the impact of degradation rate on maintenance. Liu, B. et al. [43] developed an imperfect maintenance strategy for task-oriented systems, established a random equipment degradation model based on the Wiener degradation process. They adopted preventive replacement and corrective replacement strategies to establish a decision-making model. Based on the Wiener process, Pei, H. et al. [44] proposed a performance degradation and maintenance decision model that considers imperfect maintenance. They adopted a regular maintenance strategy and comprehensively consider the impact of equipment degradation and degradation rate. However, the above models describe the linear degradation process without considering the nonlinear degradation characteristics. The preventive threshold is specified in advance, resulting in inaccurate decision-making results. Wang, Z. et al. [45] used the nonlinear Wiener process and the homogeneous Poisson process to model the cumulative effect of equipment, thereby improving the accuracy of RUL prediction. However, this model does not study equipment maintenance decisions and assumes that the number of imperfect repairs is unlimited. Therefore, Chen, Y. et al. [46] used nonlinear Wiener process and nonhomogeneous Poisson process based on literature [45] to establish an imperfect decisionmaking model that satisfies the upper limit of the number of repairs. However, this model adopts the maximum likelihood method for parameter estimation and cannot update the parameters according to the equipment's real-time data, making the predicted result inaccurate and affects the accuracy of maintenance decision.

In summary, the current RUL prediction methods can hardly reflect the true degradation process of equipment. Researchers does not consider the impact of imperfect maintenance on equipment degradation and maintenance decisions. Therefore, this paper establishes the equipment maintenance decision model based on the RUL prediction under the data-driven method. First, we use the nonlinear Wiener process to establish the equipment degradation model, and the method of online parameter update is introduced. The offline estimation and online update are combined to estimate the parameters of the degradation model, and the RUL distribution of the equipment is derived. Then, based on the result of the RUL prediction, an imperfect maintenance strategy with limited maintenance times is adopted. The maintenance decision model is established with the lowest long-term cost rate as the goal. Finally, taking the bearing as an example, we compared the validity and accuracy of the model.

2. Problem Description and Model Assumptions

2.1. Problem Description

Maintenance includes perfect maintenance, imperfect maintenance and minor repairs. It is not easy to achieve perfect maintenance in the actual production process, especially in the face of complex equipment. Most projects use imperfect maintenance, and maintenance activities cannot restore the equipment to a brand new state. With the development of sensing technology, we can detect the operating data of the equipment, obtain the current degradation state, and provide a basis for maintenance. Based on the idea of imperfect maintenance, this paper adopts a restrictive maintenance strategy. When equipment degradation reaches a certain threshold, it will be maintained. However, it cannot be restored as new and equipment maintenance does not increase indefinitely. As the number of maintenance increases, equipment degradation increases until it completely fails. Figure 1 shows the equipment degradation process.



Figure 1. Equipment degradation process.

In Figure 1, the abscissa is the equipment operating time *t*, the ordinate is the equipment degradation X(t), ω is the failure threshold, and ω_p is the maintenance threshold.

2.2. Model Assumptions

The assumptions of this article mainly include the following points:

- (1) Preventive maintenance does not affect the degradation mechanism of the equipment itself and can slow down the process of equipment degradation. However, preventive maintenance activities are not unlimited. When the number of maintenance reaches the prescribed upper limit N, maintenance activities will no longer be carried out.
- (2) The status value of each period can be obtained through the detection method, and the replacement and repair time of the equipment under any maintenance means is ignored.
- (3) When equipment degradation is less than the preventive maintenance threshold ω_p , no maintenance, and the equipment usually operates.
- (4) When equipment degradation is greater than the preventive maintenance threshold ω_p , but the number of maintenance does not exceed N, we take preventive maintenance.

(5) When the overall degradation exceeds the failure threshold ω , the system will fail, and time-sensitive replacement will be adopted.

3. Model Building

3.1. Non-Linear Drift Wiener Degradation Model

The mathematical expression of the Wiener process [47] can be expressed as:

$$X(t) = X(0) + \mu t + \sigma B(t) \tag{1}$$

where, X(t) is the degradation of the equipment at time t. X(0) is the degradation of the equipment at time 0. μ is the drift coefficient. σ is the diffusion coefficient. B(t) is the standard Brownian motion.

The Wiener process expression of nonlinear drift can be written as [48]:

$$X(t) = X(0) + \alpha \int_0^t \mu(\tau, \beta) d\tau + \sigma B(t)$$
(2)

where, $\alpha \int_0^r \mu(\tau, \beta) d\tau$ represents the average cumulative effect of the equipment degradation process, which is the nonlinear drift of the degradation amount. α is a random parameter, β and σ are common parameters, $\alpha \sim N(\mu_0, \sigma_0^2)$. α and B(t) are statistically independent.

The Wiener degradation process based on imperfect maintenance can be expressed as [44]:

$$X(t) = X_i + \alpha \int_{T_i}^t \mu(\tau, \beta) d\tau + \sigma B(t - T_i)$$
(3)

where, X_i is the degradation amount of the equipment after *i* maintenance, $i(0 \le i \le N)$. *N* is the total number of maintenance. The probability density of X_i satisfies:

$$f(X_i) = \frac{a^{i-1}b}{1 - exp(a^{i-1}b\omega_p)} exp[\frac{a^{i-1}b(\omega_p - X_i)}{\omega_p}X'_i]$$
(4)

3.2. RUL Prediction Model

Based on the first arrival time, the equipment degradation life *T* represented by the Wiener process with nonlinear drift is defined as [49]:

$$T = inft, X(t) \ge \omega | X(0) < \omega$$
(5)

where, ω is the known failure threshold.

In order to obtain the probability density of the analytical form of the equipment life *T*, considering the randomness of the parameters, redefine the probability density function of the life *T* can be given as:

$$f_{T|\alpha}(t|\alpha) \simeq \frac{\omega - \alpha \int_{T_i}^t \mu(\tau, \beta) d\tau + \sigma B(t)}{\sigma t \sqrt{2\pi t}} exp\{\frac{[\omega - \alpha \int_{T_i}^t \mu(\tau, \beta) d\tau]^2}{2\sigma^2 t}\}$$
(6)

The unconditional total probability density function of life *T* is $f_T(t)$, and the total probability formula can be derived as:

$$f_T(t) = \int_{\Phi} f_{t|\alpha} f(\alpha) d\alpha \tag{7}$$

where, f_T and Φ are the probability density function and parameter space of random parameters.

Through the probability density of life *T*, the probability density of the RUL of equipment degradation can be obtained. Let t_n be the current time and l_n be the remaining life of the current time. If the equipment reaches the failure threshold for the first time at time *t*, the actual remaining life is $l_n = t - t_n$. Based on the Wiener process, we can get:

$$X(t) - X(t_n) = X(0) + \alpha \int_{t_n}^t \mu(\tau, \beta) d\tau + \sigma B(l_n)$$
(8)

From Equations (6)–(8), we know that the probability density function of the remaining life of the degraded equipment at time t_n can be written as:

$$f_{L_n}(l_n) \cong \frac{1}{\sqrt{2\pi l_n^2 [\sigma_{\alpha,n}^2(\alpha \int_{t_n}^t \mu(\tau,\beta)d\tau)^2 + \sigma^2 l_n]}} \\ \{\omega - x_n - [\alpha \int_{t_n}^t \mu(\tau,\beta)d\tau - \mu(l_n + t_n,\beta)l_n] \\ \frac{\sigma_{\alpha,n}^2(\omega - x_n \int_{t_n}^t \mu(\tau,\beta)d\tau) + \mu_{\alpha,n}\sigma^2 l_n}{\sigma_{\alpha,n}(\int_{t_n}^t \mu(\tau,\beta)d\tau)^2 + \sigma^2 l_n} \}$$

$$exp\{-\frac{[\omega - x_n - \mu_{\alpha,n} \int_{t_n}^t \mu(\tau,\beta)d\tau]^2}{2[\sigma_{\alpha,n}(\int_{t_n}^t \mu(\tau,\beta)d\tau)^2 + \sigma^2 l_n]} \}$$

$$(9)$$

where, $\mu_{\alpha,n}$ and $\sigma_{\alpha,n}^2$ are the mean value and variance of the random parameter α obtained after the update at time t_n , respectively. x_n is the degradation observation of the degraded equipment at time t_n .

3.3. Maintenance Decision Model

This paper takes the lowest long-term cost rate as the decision-making goal, and uses the maintenance threshold as the decision variable to establish a maintenance decisionmaking model. According to related theories [50], the average cost rate in the life cycle can be expressed as:

$$C^{\infty} = \lim_{t \to +\infty} C(t) = \frac{E[C]}{E[T]}$$
(10)

where, C^{∞} is the average cost rate. E[C] is the average cost, and E[T] is the average life cycle. The total cost includes inspection cost C_I , preventive maintenance cost C_M , preventive replacement cost C_P , failure replacement cost C_S , and $C_I < C_M < C_P < C_S$ The total cost can be expressed as:

$$C^{\infty} = \frac{C_I E[N_I] + C_M E[N_M] + C_P P_P + C_S P_S}{E[T]}$$
(11)

 N_I is the number of inspections. N_M is the number of maintenance. P_P is the probability of preventive replacement of the equipment. P_S is the probability of failure replacement of the equipment.

The equipment inspection time interval is Δt , when the amount of degradation after detection meets $0 \leq X(d_i\Delta t) < \omega_p$, and no maintenance measures are taken. When the amount of degradation after detection meets $\omega_p \leq X(d_i\Delta t) < \omega$, preventive maintenance is taken. When it is detected that the amount of equipment degradation meets $\omega_p \leq$ $X[(k+1)\Delta t] < \omega, (K \geq N)$ after completing *N* maintenance, preventive replacement is adopted. When the equipment has undergone *N* preventive maintenance, it is detected that the degradation amount satisfies $\omega \leq X[(k+1)\Delta t]$ in the interval $(k\Delta t, (k+1)\Delta t)$, and the failure replacement is adopted.

The maintenance process is shown in Figure 2.



Figure 2. Maintenance process.

3.3.1. Preventive Replacement

Assuming that the time of the *i*-th maintenance is $d_i\Delta t$, the probability of preventive maintenance of the equipment at time $d_i\Delta t$ can be expressed as [51]:

$$P(d_{i}\Delta t) = \int_{0}^{\omega_{p}} \{1 - F_{i-1,\omega_{p}}[(d_{i} - (d_{i} - 1))\Delta t | X_{i-1}] \\ \int_{X_{i-1}}^{\omega_{p}} f_{i-1}[x, (d_{i} - (d_{i} - 1) - 1)\Delta t](F_{i-1,\omega_{p}-x}(\Delta t, x | X_{i-1}) \\ - F_{i-1,\omega-x}(\Delta t, x | X_{i-1}))dx\}f(X_{i-1})d_{X_{i-1}}$$

$$(12)$$

where,

$$f_i(x) = \frac{1}{\sqrt{2\pi\sigma^2 t}} exp[-\frac{(x - X_i - \alpha \int_0^{T_i} \mu_i(\tau, \beta) d\tau)^2}{2\sigma^2 t}]$$
(13)

$$f_{i,\omega-x}(\delta|X_i) = \frac{\omega - x}{\sqrt{2\pi\sigma^2\delta^3}} exp\left[-\frac{\left[\omega - x - (\alpha\int_{T_i}^t \mu_i(\tau,\beta)d\tau)\delta\right]^2}{2\sigma^2\delta^3}\right]$$
(14)

When the number of maintenance is less than *N* and the equipment degradation satisfies $X((d_i - 1)\Delta t) < \omega_p \leq X(d_i\Delta t) < \omega$, preventive maintenance is taken. The probability at this time is expressed as:

$$P_{1}(d_{i}\Delta t) = P\omega_{p} \leq X(d_{i}\Delta t) < \omega \cap X((d_{i}-1)\Delta t) < \omega_{p} | i \leq N$$

$$= P\omega_{p} \leq X((d_{i}-1)\Delta t) < \omega_{p}$$

$$- PX((d_{i}-1)\Delta t) < X(d_{i}\Delta t) \geq \omega$$

$$= \omega_{p} \cap PX((d_{i}-1)\Delta t) < \omega \cap X(\Delta t) \geq \omega_{p} - X((d_{i}-1)\Delta t)$$

$$- PX((d_{i}-1)\Delta t) < X(d_{i}\Delta t) \geq \omega_{p} \cap \omega - X(d_{i}-1)\Delta t$$
(15)

The expenses incurred at this time can be expressed as:

$$C_1' = C_I N_I^1 + (i-1)C_M (16)$$

When the number of maintenance exceeds N and the equipment degradation satisfies $X(k\Delta t) < \omega_p \leq X((k+1)\Delta t) < \omega)$, preventive replacement is adopted. The probability at this time is expressed as:

$$P_{2}(k+1,N) = PX(k\Delta t) < \omega_{p} \leq X((k+1)\Delta t) < \omega|k \geq N$$

$$= \sum_{i=1}^{k} [\prod_{i=1}^{N} P_{1} \cdot PX(k\Delta t) < \omega_{p} \leq X((k+1)\Delta t) < \omega|k \geq N]$$

$$= \sum_{i=1}^{k} [\prod_{i=1}^{N} P_{1} \int_{0}^{\omega_{p}} \{1 - F_{N,\omega_{p}}[(k-d_{N}\Delta t)|X_{N} \int_{X_{N}}^{\omega_{p}} f_{N}(x,(k-d_{N})\Delta t) | (F_{N,\omega-x}(\Delta t,x)|X_{N}) - (F_{N,\omega_{p}-x}(\Delta t,x)|X_{N})]\} dx f(X_{N}) dX_{N}]$$

$$(17)$$

After *N* times of maintenance, the probability of preventive replacement of equipment can be given as:

$$P_P = \sum_{k=N}^{\infty} (k+1, N) P_2$$
(18)

The cost at this time can be expressed as:

$$C_1'' = C_I N_I^1 + N C_M + C_P (19)$$

The total cost of preventive replacement at the end of the life cycle can be written as:

$$C_I = C_1' + C_1'' \tag{20}$$

3.3.2. Failure Replacement

When the number of maintenance exceeds *N* and the equipment degradation satisfies $X(k\Delta t) < \omega_p - \omega \le X((k+1)\Delta t)$, the failure replacement is adopted, and the probability at this time can be expressed as:

$$P_{S}(k+1,N) = PX(k\Delta t) < \omega_{p} \land \omega X((k+1)\Delta t)|k \ge N$$

$$= \sum_{i=1}^{k} [\prod_{i=1}^{N} P_{1} \cdot PX(k\Delta t) < \omega_{p} \land \omega X((k+1)\Delta t)|k \ge N]$$

$$= \sum_{i=1}^{k} [\prod_{i=1}^{N} P_{1} \int_{0}^{\omega} \{1 - F_{N,\omega}[(k-d_{N}\Delta t)|X_{N} \int_{X_{N}}^{\omega} f_{N}(x,(k-d_{N})\Delta t) | (F_{N,\omega-x}(\Delta t,x)|X_{N}) - F_{N,\omega-x}(\Delta t,x)|X_{N}]\} dxf(X_{N})]$$
(21)

The cost C_2 incurred at this time can be given as:

$$C_2 = C_I N_I^2 + N C_M + C_p \tag{22}$$

The total cost of the entire maintenance process can be written as:

$$C = C_1 + C_2 \tag{23}$$

The life cycle expectations can be expressed as:

$$E(T) = \sum_{k=N}^{\infty} \sum_{i=0}^{N} [(k+1)\Delta t P_S] + \sum_{k=N}^{\infty} [(k+1)P_P]$$
(24)

The number of detections can be written as:

$$E(N_i = \frac{E(T)}{\Delta t}) \tag{25}$$

The number of maintenance can be expressed as:

$$E(N_M) = \sum_{k=N}^{\infty} \sum_{i=0}^{N} kP_S + \sum_{k=N}^{\infty} NP_P$$
(26)

The long-term minimum cost rate of the maintenance decision model can be expressed as:

$$\min C^{\infty} = \min \frac{E[C]}{E[T]}$$
(27)

3.4. Parameter Estimation

Traditional parameter estimation methods mainly use offline estimation, which has low accuracy. This paper introduces an online update method that combines offline estimation and online updates to estimate the parameters. Parameter estimation is divided into two steps: (1) Offline estimation: Estimate the standard parameters β and α based on the historical data of equipment degradation, and hyperparameters $\mu_{\alpha,0}$ and $\sigma_{\alpha,0}^2$ in the prior distribution of random parameter α . (2) Online update: at any time t_n use the actual monitored degradation data to update the parameters in the random parameter α distribution $f(\alpha)$.

3.4.1. Parameter Offline Estimation

Suppose there are degradation data of *M* devices of the same type, and the degradation data of the *j*-th device is M_j . Then the degradation amount of the *j*-th device at the *k*-th moment is $X(t_{j,k})$.

From Equation (2), we can get:

$$X_{j,k} = X(0) + \alpha_0 \int_0^{t_{j,k}} \mu(\tau, \beta) d\tau + \sigma B(t_{j,k})$$
(28)

where: α_0 is the prior value of the random parameter α , and its distribution is $\pi_0(\alpha) \sim N(\mu_{\alpha,0}, \sigma_{\alpha,0}^2)$.

Assume that the degradation data of different degraded equipment is irrelevant, while the data of the same degraded equipment is related. The *j*th equipment degradation data is $X_j = (X_{j,1}, X_{j,2}, X_{j,3}, \dots, X_{j,M})$, X_j obeys the multivariate Gaussian distribution, and the mean and covariance can be written as [52]:

$$\mu_i = \mu_{\alpha,0} I_j$$

$$\Sigma_j = \sigma_0^2 I_j I_j \prime + \sigma^2 K_j$$
(29)

$$I_i = \left(\int_0^{t_{j,1}} \mu(\tau,\beta) d\tau, \int_0^{t_{j,2}} \mu(\tau,\beta) d\tau, \cdots \int_0^{t_{j,M_j}} \mu(\tau,\beta) d\tau\right)$$
(30)

$$K_{j} = \begin{bmatrix} t_{j,1} & t_{j,1} & \cdots & t_{j,1} \\ t_{j,1} & t_{j,2} & \cdots & t_{j,1} \\ \vdots & \vdots & \ddots & \vdots \\ t_{j,1} & t_{j,2} & \cdots & t_{j,M_{j}} \end{bmatrix}$$
(31)

From Equation (11), the mean value of X_j is $\mu_j = \mu_{\alpha,0}I_j$, and the covariance matrix X_j can be expressed as:

$$\Sigma_{j} = \begin{bmatrix} D(X_{j,1}) & covD(X_{j,1}, X_{j,2}) & \cdots & covD(X_{j,1}, X_{j,M_{j}}) \\ & \ddots & covD(X_{j,1}, X_{j,M_{j}}) \\ & \ddots & \vdots \\ & & covD(X_{i}, X_{j,M_{j}}) \end{bmatrix}$$
(32)

The variance $D(X_{j,k})$ can be expressed as:

$$D(X_{j,k}) = D(\alpha_0 \int_0^{t_{j,k}} \mu(\tau, \beta) d\tau + \sigma B(t_{j,k}))$$

= $\sigma_{\alpha,0}^2 [\int_0^{t_{j,k}} \mu(\tau, \beta) d\tau]^2 + \sigma^2 t_{j,k}$ (33)

From Equation (12), the likelihood function formed by the degradation data of *M* degraded devices is:

$$\ell(\Theta|X_j) = -\frac{1}{2}\ln(2\pi)\Sigma_{j=1}^M M_j - \frac{1}{2}\Sigma_{j=1}^M \ln|\Sigma_j|$$

= $-\frac{1}{2}(X_j - \mu_j)\Sigma_{j=1}^M (X_j - \mu_j)\Sigma_j^{-1} (X_j - \mu_j)$ (34)

where: $\Theta = (\mu_{\alpha,0}, \sigma_{\alpha,0}, \beta, \sigma)$ is an unknown parameter, and $X_j = (X_1, X_2, \cdots, X_M)$ is the degradation data of *M* devices.

3.4.2. Real-Time Parameter Estimation

For a certain device, at any time t_k during its life, the random parameter α of the degradation model can be estimated from all the data previously observed by the device. Based on Bayesian theory, the posterior parameter distribution of the random parameters of the degradation model at time t_k is as follows [53]:

$$P(\alpha|x_{1,k}) \propto p(x_{1,k}|\alpha)\pi_0(\alpha) \tag{35}$$

where, $P(\alpha | x_{1,k})$ represents the likelihood function under given random parameters, and the prior distribution can be obtained by Equation (17).

Using the basic characteristics of Brownian motion, we can get:

$$P(\alpha|x_{1,k}) = \frac{1}{\prod_{q=1}^{k} \sqrt{2\pi\sigma^2(t_q - t_{q-1})}} exp\{-\sum_{q=1}^{k} \frac{[x_q - x_{q-1} - \int_{q-1}^{t_q} \mu(\tau, \beta) d\tau]^2}{2\sigma^2(t_q - t_{q-1})}\}$$
(36)

Since $P(\alpha|x_{1,k})$ and π_0 are both normally distributed, $P(x_{1,k}|\alpha)$ is also normally distributed. From Equations (18) and (19), it can be seen that the mean and variance of $P(x_{1,k}|\alpha)$ are expressed as:

$$\mu_{\alpha,k} = \frac{\frac{\mu_{\alpha,0}}{\sigma_{\alpha,0}^2} + \Sigma_{q-1}^k \{ \frac{(x_q - x_{q-1} - \int_{q-1}^{t_q} \mu(\tau,\beta)d\tau)^2}{\sigma^2(t_q - t_{q-1})} \}}{\Sigma_{q-1}^k \{ \frac{(\int_{q-1}^{t_q} \mu(\tau,\beta)d\tau)^2}{\sigma^2(t_q - t_{q-1})} \} + \frac{1}{\sigma_{\alpha,0}^2}}$$
(37)

$$\sigma_{\alpha,k}^{2} = \frac{1}{\sum_{q=1}^{k} \{\frac{(\int_{q=1}^{t_{q}} \mu(\tau,\beta)d\tau)^{2}}{\sigma^{2}(t_{q}-t_{q-1})}\} + \frac{1}{\sigma_{\alpha,0}^{2}}}$$
(38)

Incorporating Equations (34), (37) and (38) into Equation (9), the probability density function of the remaining life of the equipment at the moment can be obtained.

4. Case Analysis and Discussion

4.1. Case Analysis

Bearing is a critical component of mechanical equipment. Its primary function is to support the mechanical rotating body, reduce the friction coefficient during its movement, and ensure its rotation accuracy. At present, bearings are widely used in complex equipment such as aerospace, large machinery, and equipment. As a crucial part of significant equipment, bearings develop in high speed, high precision, high reliability and long life. However, the complex and harsh working environment has led to a relatively high failure rate of bearings, affecting the regular operation of the equipment and even causing severe safety accidents. In the actual operation process, people take regular replacement strategy for bearings, which does not extend the remaining service life of the bearings, which is not conducive to sustainable development.

This paper takes bearing as the research object, we use the bearing degradation data of PHM competition [54] as the original data for analysis and calculation. FEMTO-ST Research Institute provides the PHM data set, and the experiment is carried out on the laboratory experimental platform (PRONOSTIA). The sampling frequency is 25.6 kHz, and the original vibration signal changes of the three sets of data of the same bearing during the whole life cycle are shown in Figure 3.



Figure 3. Original signal of three sets of data.

The Root Mean Square (RMS) can reflect the vibration signal well, so we use the RMS as the characteristic signal data. The RMS value of the three sets of data is shown in Figure 4.

The bearing failure threshold after RMS processing is 6. It can be seen from Figure 4, the first set of data and the second set of data have failed. The third set of data has not failed. The first set of data has a longer life cycle. Therefore, this paper selects the first set of data as historical degradation data and the second set of data as test data to verify the effectiveness of the degradation and prediction method.

To further analyze the degradation process of the vibration signal of the first set of data, plot its Spectral Kurtosis. The changes of spectral kurtosis with time and frequency is shown in Figure 5.

It can be seen from Figure 5, as the frequency of the bearing signal increases, the greater the value of spectral kurtosis and the more pronounced the signal characteristics. With time, the health status of the bearing gradually decreases. As the bearing increases, the amplitude of the bearing vibration signal gradually increases, and the bearing gradually fails. The spectral kurtosis results show that the degradation process of the vibration signal of the first set of data satisfies the degradation law of the whole life cycle.



Figure 4. The degradation process of the three sets of data.



Figure 5. Spectral kurtosis of the first set of data.

4.2. RUL Prediction

The method proposed in this paper is denoted as M1. The method proposed in [44] is denoted as M2. The method in [45] is denoted as M3. By comparing the prediction results of the three prediction methods, the accuracy of the prediction model proposed in this paper is proved. Taking the bearing data of the first group as historical data, it is brought into the degradation model of this article, and the results of parameter estimation are $\mu_{\hat{\alpha},0} = 0.189$, $\sigma_{\hat{\alpha},0} = 0.015$, $\hat{\beta} = 1.135$, $\hat{\sigma} = 0.9871$. Bring the parameter estimation

results into Equation (9) to perform RUL prediction. The prediction degenerate path is shown in Figure 6.



Figure 6. Three methods to predict degradation path.

It can be seen from Figure 6 that the degradation result predicted by M1 is closer to the true value, followed by M3, and M2 has the lowest accuracy. With the running time increases, the degree of deviation between the predicted signal of M2 and M3 and the original signal becomes more and more apparent. Further analysis shows that the larger predicted value of M2 is because it does not consider the influence of nonlinearity on the Wiener process, which leads to the more immense predicted value. M3 considers the non-linear Wiener process but does not adopt the real-time update strategy, and the parameter estimation value cannot be updated in real-time, making the predicted value larger. Both M2 and M3 are pessimistic estimates, which may cause delays in replacement time and reduce equipment reliability.

In order to further prove the superiority of this method, we introduced Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) [55] to analyze the error between the predicted value and the true value of the three methods.

$$MAE = \frac{1}{N} \sum_{1}^{N} |l_{n}^{*} - l_{n}|$$
(39)

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{1}^{N} (l_n^* - l_n)^2}$$
(40)

where, l_n^* is the predicted value at the current moment, l_n is the true value at the current moment, and N is the number of samples. The smaller the value of MAE and RMAE, the higher the accuracy.

Table 1 shows the error between the predicted value of the three methods and the true value. The method M1 in this paper is minor than M2 and M3 in both MAE and RMSE. The values of MAE and RMSE of M2 are the lowest, followed by M3, which matches the prediction result of Figure 6. The MAE of M1 is 56.20% and 50.25% higher than M2 and

M3. Compared with M2 and M3, the RMSE of M1 has increased by 46.23% and 24.30%, respectively. Therefore, the method presented in this paper has higher accuracy in the prediction of residual life. It is worth noting that M3 is lower than M2 in both MAE and RMSE, which indicates that the introduction of a non-linear Wiener process will make the prediction model more effective.

Table 1. Bearing error under 3 methods.

Method	MAE	MAE Lift Rate	RMSE	RMSE Lift Rate
M1	0.240	-	0.378	-
M2	0.548	56.20%	0.703	46.23%
M3	0.485	50.52%	0.621	24.30%

In order to further prove the accuracy of the method in this paper, we introduce the relative accuracy (RA) criterion [56].

$$RA = 1 - |\frac{l_n^* - l_n}{l_n}|$$
(41)

The larger the RA, the higher the accuracy of the method. Figure 7 shows the accuracy analysis results of the three methods.

It can be seen from Figure 7a that the relative accuracy of M1 is greater than that of M2 and M3. It further shows that the M1 method proposed in this article is more accurate than M2 and M3, proving that the nonlinear Wiener degradation process is close to the actual degradation process of the equipment.

According to the literature [56], the confidence interval is 20%. It can be seen from Figure 7b that most of the remaining values predicted by M1 fall within the confidence interval of $\pm 20\%$ of the true life. The RUL predicted by M2 and M3 almost all fall outside the confidence interval, which further proves that the method in this paper can accurately predict the remaining life of the equipment.



Figure 7. Cont.



Figure 7. Accuracy analysis of three methods. (a) Comparison of three methods of RA. (b) Error Analysis.

4.3. Decision Analysis

According to the information of the remaining life prediction, the maintenance activities are arranged. The specific costs of the maintenance activities are shown in Table 2.

 Table 2. Maintenance fee schedule.

Parameter	C_I	C_M	C_P	Cs
Cost/\$	10	100	500	1000

Based on the above prediction information, the genetic algorithm is used to iterate the three models to solve the optimal long-term expense ratio. The iteration parameters are consistent. The number of iterations is 200. The iteration results are shown in Figure 8.

Figure 8 shows the process of solving the optimal average cost rate by two different methods. It can be seen from the figure that the long-term average cost rate of M1 is \$8.33, the average cost rate solved by M2 is \$13.26, and the average cost rate solved by M3 is \$10.58. Compared with M2 and M3, the lowest average cost of M1 is reduced by 37.17% and 21.26%, respectively. When $\omega_p < 5.2$ in the M1 solving process, the maintenance cost rate decreases monotonically with the increase of the maintenance threshold. When $\omega_p > 5.2$, the maintenance cost rate increases monotonically with the increase of the maintenance threshold. When $\omega_p > 5.2$, the maintenance cost rate increases monotonically with the increase of the maintenance threshold. When the preventive maintenance threshold $\omega_p = 5.2$, there is a minimum average cost rate of \$8.33. Similarly, when $\omega_p = 4.4$ in M2, there is a minimum average expense rate of \$13.26. When $\omega_p = 4.6$ in M3, there is a minimum average expense rate of \$10.58. It further illustrates that the method of this article can obtain better maintenance decision results. This is because the prediction results of M2 and M3 are conservative, which reduces the RUL value, increases the number of maintenance and maintenance costs. The prediction model of this article can accurately predict the RUL, reducing the loss caused by excessive maintenance.



Figure 8. Solving process of the three methods.

4.4. Discussion

It can be seen from the above results that model in this paper dramatically improves the accuracy of equipment life prediction and makes the maintenance decision results more reasonable. The maintenance decision model established based on the RUL prediction information eliminates the disadvantages of traditional preventive maintenance, reduces maintenance costs, and improves maintenance efficiency. The following further discusses the sensitivity of maintenance cost parameters in the maintenance decision model proposed in this paper. Based on the controlled variable method, the single maintenance cost for each change, other costs and conditions remain unchanged. Therefore, we studied the changes in the maintenance strategy with the changes of various costs. Since $C_I < C_M < C_P < C_S$, let $C_I \in [1,50]$, $C_M \in [1,500]$, $C_P \in [1,1000]$, $C_S \in [1,5000]$. The relationship between maintenance cost parameters and optimal maintenance strategy is shown in Figure 9.

Figure 9 a–d are the relationship between inspection costs C_I , preventive maintenance costs C_M , preventive replacement costs C_P , failure replacement costs C_S , and maintenance strategies, respectively. For the C^{∞} , the lowest long-term expense ratio increases approximately linearly with various expenses. With C_I from 1 to 50, the C^{∞} from 5.69/day to 16.58/day, with a growth rate of 0.22. The rate of increase in the C^{∞} caused by C_M , C_P and C_S was 0.018, 0.006 and 0.0014. It can be seen that the long-term average cost rate has the highest sensitivity to the inspection cost. This is because the inspection cost directly affects the total maintenance cost.

For the ω_p , as C_I , C_P , and C_S increase, ω_p decreases, and as C_M increases, ω_p increases. The main reason for the increase in ω_p is that when C_M is low, there is a lower ω_p , which can increase the maintenance frequency to ensure the stable operation of the equipment, and C_M continues to increase, a lower ω_p will generate many maintenance costs, so ω_p rises. C_I has a low impact on the ω_p , so ω_p does not change significantly with C_I . However, with the increase of C_P and C_S , it is necessary to increase the preventive frequency to extend preventive replacement and failure replacement and reduce the total cost. Therefore, the ω_p decreases.



Figure 9. The relationship between maintenance cost parameters and maintenance strategy. (**a**) The relationship between C_I and Optimal maintenance strategy. (**b**) The relationship between C_M and Optimal maintenance strategy. (**c**) The relationship between C_P and Optimal maintenance strategy. (**d**) The relationship between C_S and Optimal maintenance strategy.

For the Δt , as the various costs increase, Δt keeps increasing, and the total cost is reduced by increasing the Δt . The impact of the C_I pair is the most significant because when the C_I is low, the operation of the equipment can be learned in time through frequent inspections to ensure the reliable and safe operation of the equipment. When the C_I increases, frequent inspections will inevitably lead to increased maintenance costs, so increase t to reduce costs. In summary, C_I has the most significant impact on C^{∞} and Δt , C_M has the most significant impact on ω_p , and C_P and C_P have relatively low impacts on various indicators of the maintenance strategy. Therefore, the actual maintenance activities should focus on controlling C_M and C_M to reduce maintenance costs on the premise of ensuring the reliable operation of the equipment.

5. Conclusions

Based on the data-driven method, this paper established the RUL prediction and maintenance decision model of the deterioration equipment. We used the measured data to verify the model's accuracy. The conclusions are as follows:

(1) For stochastic deterioration equipment, the nonlinear Wiener process can represent the true degradation process of the device more accurately than the linear Wiener process, which conforms to objective laws and effectively improves the accuracy of RUL prediction.

- (2) The introduction of online parameter updates can improve the accuracy and reliability of RUL prediction.
- (3) Based on the more accurate RUL prediction results, a more realistic optimal maintenance strategy can be formulated, reducing maintenance costs and prolonging the RUL of the equipment.
- (4) Inspection costs and preventive maintenance costs have a significant impact on maintenance strategies. Therefore, it is necessary to focus on controlling inspection costs and preventive maintenance costs to promote sustainable development in actual maintenance activities.

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