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Volatility Spillovers between Equity and Green Bond Markets

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Abstract: This study examines the market for green bonds, which have been in the spotlight as an eco-friendly investment product. We analyze the volatility dynamics and spillovers between the equity and green bond markets. As the return dynamics of financial products typically exhibit asymmetric volatility, we check whether green bonds also share this property. Our analyses confirm that although green bonds do exhibit the asymmetric volatility phenomenon, their volatility, unlike that of equity, is also sensitive to positive return shocks. An analysis of the association between the green bond and equity markets confirms that although the two markets have some volatility spillover effects, neither responds significantly to negative shocks in the other market.

Keywords: asymmetric volatility; Baba–Engle–Kraft–Kroner model; dynamic conditional correlation-generalized autoregressive conditional heteroskedasticity; green bond; sustainable growth; G10; G11; Q56

1. Introduction

As interest and research in sustainable growth and corporate environmental responsibility (CER) have increased, investments in environment, social, and governance factors have also been growing. Corporate social responsibility (CSR) enhances firms' long-term performance through various channels, such as higher standard business practices and financial transparency (Popescu, 2019 [1]; Popescu and Popescu, 2019 [2]). CER is a subset of CSR, and it has been recognized as an important factor for firms' sustainability (Kim, Park, and Ryu, 2017a [3]). Indeed, investments in eco-friendly financial products are becoming more important as environmental pollution intensifies (Kim, Park, and Ryu, 2017b [4]). Accordingly, the issuances and trading volumes of green bonds, which are used to raise money for eco-friendly projects, are also increasing. Green bonds are issued mainly by international and local public organizations. These bonds are fixed-income securities such that the funds raised must be used only for eco-friendly and sustainable projects, such as renewable energy facilities. The green bond started to be issued in 2007 by the European Investment Bank for the development of technologies to improve energy efficiency or new renewable energy technologies (Banga, 2019 [5]; Hachenberg and Schiereck, 2018 [6]; Tang and Zhang, 2020 [7]).

Green bonds are considered stable and sustainable investments in terms of funding for long-term social projects. Eco-friendly infrastructure and sustainable projects, such as wind and solar energy facilities, are long-term projects that require vast amounts of funding. Thus, raising funds through a large bond market that can provide capital over a long period of time might be an appropriate choice. For investors, green bonds are stable investments that provide long-term income. Moreover, because green bonds are issued for social purposes, the political and social demands for the bonds are growing. Thus, green bonds provide an advantage for investors by allowing them to carry out their green initiatives without burdening their portfolios. Traditional eco-friendly financial products

are difficult to integrate into a portfolio owing to their variable interest payments, maturity adjustments, and liquidity constraints. In contrast, green bonds are relatively easy to integrate into portfolios because their issuance method is similar to that of traditional government bonds.

Because of these advantages, green bonds circulate actively in the market, and academic researchers continue to study them accordingly. Green bonds can be analyzed by considering the general characteristics of bonds, and a long-standing research interest related to the bond market is its association with the equity market. The mechanism by which volatility is transferred between the equity and bond markets plays an important role in portfolio composition and risk management. In portfolio composition, bonds are relatively stable assets compared to stocks and, thus, can offset the risk of stocks in a portfolio. Many previous studies, therefore, analyze the dynamic relationship between the equity and bond markets (Cappiello, Engle, and Sheppard, 2006 [8]; Christiansen, 2010 [9]; Chuliá and Torro, 2008 [10]; Dean, Faff, and Loudon, 2010 [11]; Fleming, Kirby, and Ostdiek, 1998 [12]; Steeley, 2006 [13]). Because green bonds are a type of bond, analyzing the relationship between the green bond and stock markets should help investors organize their portfolios. However, because green bonds just started to be actively traded recently, academic research on this topic is insufficient.

Thus, this study presents meaningful conclusions about the correlation between the green bond and stock markets and the characteristics of the green bond market by analyzing the volatilities of green bond and stock indices. Both bond and stock markets generally exhibit an asymmetric volatility that reacts more sensitively to negative shocks. Thus, we also test for the presence of asymmetric volatility in the green bond and stock markets. In addition, we utilize the Baba–Engle–Kraft–Kroner (BEKK) model and the dynamic conditional correlation-generalized autoregressive conditional heteroskedasticity (DCC-GARCH) model to analyze the volatility spillovers between the two markets. We consider that previous studies empirically and widely use the multivariate GARCH models (Kim and Ryu, 2014 [14]; Park, Ryu, and Song, 2017 [15]; Ryu and Shim, 2017 [16]; Sayed and Auret, 2020 [17]; Shim, Kim, and Ryu, 2017 [18]; Song, Park, and Ryu, 2018 [19], Vardar, Coskun, and Yelkenci, 2018 [20]).

The remainder of this paper is organized as follows. We review the literature related to the green bond market, the relationship between bond and equity markets, and the asymmetric volatility of financial products in Section 2. We then describe our data and conduct preliminary tests in Section 3. We provide the empirical results and interpretations regarding asymmetric volatility in the green bond market and volatility spillovers between the two markets in Section 4, and we draw conclusions in Section 5.

2. Literature Review

Because the green bond market has emerged recently, most prior studies on this market conduct basic financial analyses. Pham (2016) [21] uses the GARCH model to analyze the volatilities of the green and traditional bond markets using the S&P (Standard & Poor's: S&P Global Ratings, NY, USA) Green Bond Index, the S&P Green Project Bond Index, and the S&P U.S. Aggregate Bond Index to identify spillover effects. Reboredo (2018) [22] uses a threshold GARCH model to show that green bonds do not provide diversification effects to traditional bond market investors. He also argues that it is important to analyze the links between the markets for green bonds and other financial instruments based on green bonds' relationship with the stock and energy commodity markets.

The relationship between equity and bond markets has been studied for a long time. However, no unified relationship exists to describe the connection between the two markets. Campbell and Vuolteenaho (2004) [23] use the vector autoregressive model to demonstrate different changes in future cash flows and changes in discount rates in both markets. Steeley (2006) [13] analyzes the link between British short- and long-term bonds and stocks and shows that the returns in the two markets are negatively correlated. Christiansen (2010) [9] uses a bivariate GARCH model to analyze volatility spillover effects in the United States and England, and shows that each market's volatility is more affected by its own volatility than by that of the other market. These previous studies show somewhat inconsistent results depending on the region, financial products, and sample periods.

Volatility dynamics are an on-going hot research topic in the field of economics (Kim, Park, and Ryu, 2018 [24]; Kim and Ryu, 2015a, 2015b [25,26]; Chun, Cho, and Ryu, 2019, 2020 [27,28]; Lee and Ryu, 2014a, 2014b [29,30]). For example, some previous studies analyze the relationship between volatility movements and economic factors in financial markets (Shim, Kim, Kim, and Ryu, 2015 [31]; Lee and Ryu, 2018 [32]; Lee and Ryu, 2019 [33]). Other studies examine the return and volatility transmissions between related markets (Guo, Han, Liu, and Ryu, 2013 [34]; Kim, Ryu, Seo, 2015 [35]; Lee, Kang, and Ryu, 2015 [36]; Lee and Ryu, 2016 [37]; Lee, Lee, and Ryu, 2019 [38]). The more recent studies introduce the cross-market effects in explaining return dynamics, volatility dynamics, and volatility spillovers, aiming to broaden our knowledge on the volatility dynamics (Han, Kutan, and Ryu, 2015 [39]; Izadi and Hassan, 2018 [40]; Park, Kutan, and Ryu, 2019 [41]; Song, Ryu, and Webb, 2016, 2018 [42,43]; Yang, Kim, Kim, and Ryu, 2018 [44]).

In the field of finance and market microstructure, the studies on volatility have somewhat different focuses. For example, they examine the information contents of option-implied volatilities (Ryu, 2012 [45]; Ryu, Kang, and Suh, 2015 [46]), investigate whether trading volume and activities predict future volatilities (Ryu and Yang, 2020 [47]), and test which investors have an information advantage in volatility dynamics (Ryu and Yang, 2019 [48]; Ryu, Ryu, and Yang, 2020 [49]; Yang, Kutan, and Ryu, 2019 [50]).

In particular, existing studies in financial economics have continuously reported the asymmetric volatility phenomenon. In a pioneering study of asymmetric volatility, Black (1976) [51] and Christie (1982) [52] reveal that negative price shocks lead to higher volatility than positive shocks in the stock market. They argue that the leverage effect is the cause of asymmetric volatility. In contrast, Pindyck (1984) [53], French, Schwert, and Stambaugh (1987) [54], and Campbell and Hentschel (1992) [55] argue for the volatility feedback hypothesis. More recently, Hibbert, Daigler, and Dupoyet (2008) [56] and Han, Guo, Ryu, and Webb (2012) [57] examine the asymmetric volatility based on the behavioral explanation.

Following these studies, researchers have materially and empirically investigated asymmetric volatility as a feature of financial time series data. Understanding asymmetric volatility helps to increase the accuracy of predictions. Kroner and Ng (1998) [58] use the asymmetric bivariate GARCH model to show that the covariance matrix of returns to the stocks of large and small businesses is asymmetric about signals of return shocks. Koutmos (1999) [59] shows that positive returns are sustained longer than negative ones in an emerging Asian stock market. Cappiello, Engle, and Sheppard (2006) [8] argue that stock returns strongly exhibit asymmetry in their conditional variance, whereas little asymmetry is observed in bond returns. In addition, both stocks and bonds exhibit asymmetry in their conditional correlation. Stocks react more sensitively to negative shocks than bonds do. Chuliá and Torro (2008) [10] use an asymmetric multivariate GARCH model to consider volatility spillovers between European equity and bond markets and a dynamic trading strategy. Volatility is transferred in both directions. However, the conditional variance for stocks is asymmetric to shocks in both markets, whereas the conditional variance for bonds reacts asymmetrically only to stock market shocks. Dean, Faff, and Laudon (2010) [11] use a bivariate GARCH model to analyze the asymmetric volatility and spillover effects in daily returns to the Australian stock and bond markets. Lee and Ryu (2013) [60] suggest a vector autoregressive (VAR) framework to examine the asymmetric volatility. Based on these studies, we can expect the green bond market to exhibit asymmetric volatility as well.

3. Predictive Regression Models

3.1. Data Description

Because the purpose of this study is to analyze the stock and green bond markets, we carry out the analysis by converting the indices representing the two markets into return series. We use the S&P 500 Index to represent the stock market price and the S&P Green Bond Select Index, developed by Standard & Poor's, to reflect the green bond market. Standard & Poor's began publishing the S&P Green Bond

index as a general index of green bonds. More recently, the S&P Global Ratings started separately selecting the green bonds that are highly relevant to the stock market, and it now releases the S&P Green Bond Select index as a separate index. In this study, we use the S&P Green Bond Select index rather than the S&P Green Bond index because we analyze the correlation between the stock market and the green bond market. The data range from January 2010 to January 2020 and are converted to log returns, as shown in Equation (1).

$$R_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \quad (1)$$

We calculate the return rates of the S&P 500 Index and the S&P Green Bond Select Index, which we label as SPX and GBS, respectively. Table 1 shows the descriptive statistics for each dataset. As Table 1 shows, SPX and GBS both have asymmetrical structures with negative skewness. In addition, both series have leptokurtic distributions with thick tails, as they both have high kurtosis. We reject the null hypothesis of the Jarque–Bera test for both data series, meaning that neither follows a normal distribution. The null hypothesis of the augmented Dicky–Fuller (ADF) test is rejected, meaning that there is no unit root.

Table 1. Descriptive statistics.

	SPX	GBS
Mean	0.0002	0.0001
Max	0.0210	0.0118
Median	−0.2995	−0.0147
Min	0.0003	0.0003
Standard deviation	0.0040	0.0021
Skewness	−0.4980	−0.0971
Kurtosis	7.5970	7.1990
Jarque–Bera test	2336.8 ***	1866.3 ***
ADF test	−52.629 ***	−53.282 ***
Correlation	0.2301	

Note: This table summarizes the descriptive statistics of the returns to the S&P 500 Index (SPX) and the S&P Green Bond Select Index (GBS). The rows for the Jarque–Bera and Dicky–Fuller (ADF) tests report t-statistics. *** means that the test is significant at the 1% level.

3.2. Preliminary Test

This section tests whether SPX and GBS are suitable for the analysis in Section 4. First, we check the volatility clustering of SPX and GBS. Figure 1 shows a graphical representation of the two data series, and the volatility clusters of both series can be identified from these graphs.

Figure 1. Time series of SPX and GBS returns and squared returns. Note: This figure depicts the time series of S&P 500 Index (SPX) returns and squared returns and S&P Green Bond Select Index (GBS) returns and squared returns. Panels A and B show the time series of SPX and GBS returns, respectively, and Panels C and D show the time series of squared SPX and GBS returns, respectively.

In Panels A and B of Figure 1, both time series observations appear to follow a white noise process, but volatility clusters appear to exist. These patterns are more evident in Panels C and D, which show squared returns. Figure 2, which shows the autocorrelation between the two data series, reveals volatility clustering more clearly. Additionally, the Ljung–Box Q-statistic of each time lag is above the critical value for Panels C and D, demonstrating the autocorrelation in the squared returns and, thus, in the volatilities of SPX and GBS. This result means that both data series have an ARCH effect, meaning that the GARCH family of models is suitable for analyzing both indices. Table 2 shows the results of the Ljung–Box test for the returns and squared returns of SPX and GBS, and these results lead to the same conclusions as Figures 1 and 2.

Figure 2. Autocorrelation. Note: This figure depicts the autocorrelation (AC) in each index. SPX indicates S&P 500 Index returns, and GBS indicates S&P Green Bond Select Index returns. Panels A and B show the autocorrelation in SPX and GBS returns, respectively, and Panels C and D show the autocorrelation in SPX and GBS squared returns, respectively.

Table 2. Ljung–Box test.

	SPX	GBS
LB(10) returns	0.0298b ***	0.0191b **
LB(10) squared returns	1.1800b ***	0.8490b ***

Note: This table shows the results of the Ljung–Box test for the returns and squared returns of the S&P 500 Index (SPX) and the S&P Green Bond Select Index (GBS). The row labeled “LB(10) returns” shows Ljung–Box statistics with ten lags for the returns to each index, and the row labeled “LB(10) squared returns” shows Ljung–Box statistics with ten lags for the squared returns to each index. ** and *** indicate significance at the 5% and 1% level, respectively.

4. Empirical Analyses and Results

4.1. Asymmetric Volatility

The analysis described in this section tests for asymmetric volatility in the stock and green bond markets. As discussed in Section 2, financial data generally exhibit asymmetric volatility. Dean, Faff, and Loudon (2010) [11] find that the Australian stock and bond markets exhibit asymmetric volatility. Thus, in this study, we also check the volatilities of the stock and green bond markets for asymmetry. To this end, we implement the sign and size bias tests proposed by Engle and Ng (1993) [61]. These tests determine the significance of volatility changes when a negative or positive return shock occurs by

regressing the squared residual term on a dummy variable reflecting the sign of the residual term. Using $\epsilon_{i,t}$, $i = \text{SPX}$, GBS to indicate the residual of asset i at time t , we perform each test by estimating the following regressions.

$$\text{Signbiastest } \epsilon_{i,t}^2 = a + bS_{i,t-1}^- + u_{i,t}. \quad (2)$$

$$\text{Negativesizebiastest } \epsilon_{i,t}^2 = a + bS_{i,t-1}^- \epsilon_{i,t-1} + u_{i,t}. \quad (3)$$

$$\text{Positivesizebiastest } \epsilon_{i,t}^2 = a + bS_{i,t-1}^+ \epsilon_{i,t-1} + u_{i,t}. \quad (4)$$

$$\text{Jointbiastest } \epsilon_{i,t}^2 = a + b_1 S_{i,t-1}^- + b_2 S_{i,t-1}^- \epsilon_{i,t-1} + b_3 S_{i,t-1}^+ \epsilon_{i,t-1} + u_{i,t}. \quad (5)$$

$S_{i,t-1}^-$ and $S_{i,t-1}^+$ are dummy variables that take a value of one for negative and positive residuals of i , respectively. Table 3 presents the results.

Table 3. Bias test.

	SPX	GBS
Sign bias test	4.1875 ***	1.3341
Negative size bias test	−10.940 ***	−7.8589 ***
Positive size bias test	0.68606	2.272 **
Joint bias test	46.4 ***	30.4 ***

Note: This table shows the results of conducting the bias test proposed by Engle and Ng (1993) [61] for the residuals of S&P 500 Index (SPX) returns and S&P Green Bond Select Index (GBS) returns. The sign, negative size, and positive size bias tests are conducted using t -tests in Equations (2)–(4), and the joint bias test is conducted using an F -test in Equation (5). Each number in the table is a t -statistic or an F -statistic value. ** and *** indicate significance at the 5% and 1% level, respectively.

Table 3 shows that SPX follows the typical form of asymmetric volatility, as sensitivity to negative shocks is observed. However, for GBS, the coefficient of $S_{i,t-1}^-$ is not significant, whereas the coefficients and models for the other tests are all significant. Generally, financial instruments respond more sensitively to bad news than to good news, which seems to reflect some investors' tendency to hedge. In the case of GBS, however, volatility is also sensitive to the positive return shock. These results may be driven by the recent positive market response to eco-friendly financial instruments. In other words, as green bonds are experiencing rapid growth in recent years, investors view them hopefully and react strongly to minor pieces of good news, resulting in increased volatility. GBS also exhibits asymmetric volatility, as its volatility reacts differently to negative and positive shocks. However, given that Dean, Faff, and Loudon (2010) [11] find that the volatility in the Australian bond market is asymmetrical and sensitive to negative return shocks, green bonds have slightly different characteristics from those of conventional bonds.

4.2. Bivariate GARCH Estimation

In this section, we analyze volatility transfers between the two markets using a bivariate GARCH model. The earlier tests show that the volatilities of SPX and GBS exhibit ARCH effects and have asymmetrical characteristics. Thus, the correlation between the two markets indicates that the BEKK and DCC-GARCH models, which can reflect asymmetric volatility, are appropriate in this context. GARCH family models are represented by a conditional mean process and a conditional variance process. In this study, the conditional mean processes of models are defined as shown in Equations (6) and (7) because we focus on the analysis of the volatility spillover. In the subsequent analyses, we use $i = 1$ to mean SPX and $i = 2$ to mean GBS for convenience. ϵ_t is a residual vector following a multivariate normal distribution with $E_{t-1}(\epsilon_t) = 0$ and $E_{t-1}(\epsilon_t \epsilon_t') = H_t$.

$$r_{i,t} = \mu_{i,t} + \epsilon_{i,t}, \text{ for } i = 1, 2. \quad (6)$$

$$\epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix} \sim N(0, H_t). \quad (7)$$

The BEKK model presented by Engle and Kroner (1995) [62] has the conditional variance process given by Equations (8).

$$H_t = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B, \\ \text{where } H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}, C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}. \quad (8)$$

The BEKK model is intuitive and suitable for describing volatility spillovers. It can be transformed into the asymmetric BEKK model to reflect asymmetrical volatility (Kroner and Ng, 1998 [58]), as shown in Equations (9).

$$H_t = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B + G'\eta_{t-1}\eta'_{t-1}G, \\ \text{where } H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}, C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \\ G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, \eta_{t-1} = \begin{bmatrix} \max(0, -\epsilon_{1,t-1}) \\ \max(0, -\epsilon_{2,t-1}) \end{bmatrix} \quad (9)$$

A term for the negative return shock is incorporated. η_t is a vector of negative residuals. Thus, the product of the matrix G and η_t reflects the asymmetrical change in volatility response to the negative shock. Table 4 shows the estimates of the BEKK model for the stock and green bond markets.

Table 4. Baba–Engle–Kraft–Kroner (BEKK) estimation.

Panel A. Symmetric BEKK			Panel B. Asymmetric BEKK		
c_{11}	0.0010 ***	(9.1423)	c_{11}	−0.0008 ***	(−15.7276)
c_{12}	−0.0081 ***	(−12.5446)	c_{12}	−0.0018 ***	(−16.7175)
c_{22}	0.0001 ***	(7.8447)	c_{22}	−0.0027 ***	(−105.4916)
a_{11}	0.2523 ***	(12.2209)	a_{11}	0.1598 ***	(9.3944)
a_{12}	0.0065	(0.4422)	a_{12}	0.1449 ***	(3.3753)
a_{21}	0.0671 ***	(3.2148)	a_{21}	−0.0931 ***	(−6.4990)
a_{22}	0.2464 ***	(11.3875)	a_{22}	0.1593 ***	(20.7743)
b_{11}	0.9136 ***	(51.8925)	b_{11}	0.8875 ***	(6.7269)
b_{12}	−0.0680 ***	(−6.9604)	b_{12}	0.0056	(−0.3462)
b_{21}	0.1020 ***	(5.7324)	b_{21}	0.0062	(−0.8648)
b_{22}	0.8881 ***	(95.3600)	b_{22}	0.9270 ***	(4.3252)
			g_{11}	0.4044 ***	(67.5478)
			g_{12}	−0.0293	(1.3163)
			g_{21}	−0.0281	(0.4498)
			g_{22}	0.2068 ***	(500.9299)
<i>LogLike</i>	22828.923		<i>LogLike</i>	20344.498	
<i>AIC</i>	−45646.846		<i>AIC</i>	−40673.996	
<i>SIC</i>	−17772.846		<i>SIC</i>	−2663.996	

Note: This table estimates the parameters of the symmetric and asymmetric BEKK models for S&P 500 Index returns and S&P Green Bond Select Index returns. Panel A shows the estimation of Equation (8), and Panel B shows the estimation of Equation (9). The numbers in parentheses represent *t*-statistics. *** indicates significance at the 1% level. *LogLike* stands for log-likelihood value, and *AIC* and *SIC* stand for the Akaike information criterion and the Schwarz information criterion, respectively.

As Table 4 shows, the estimates of the BEKK model imply that volatility in the stock market affects volatility in the green bond market, and vice versa. However, g_{12} and g_{21} are not significant in Panel B, indicating that a negative shock in one market does not spread to the other market. That is, the bad news in the stock market does not create a major shock in the green bond market, and the reverse is also true. However, the BEKK model has the disadvantage of having too many parameters to estimate. Accordingly, Engle (2002) [63] proposes the DCC-GARCH model, which requires the estimation of

fewer parameters and can analyze a multivariate GARCH model. The conditional variance process in the DCC-GARCH model is given by Equations (10) and (11). Equation (10) shows a conditional covariance. Equation (11) presents a dynamic conditional correlation.

$$H_t = D_t R_t D_t, \text{ where } D_t = \text{diag}[\sqrt{h_{i,t}}] \text{ and } h_{i,t} = c_i + a_i \epsilon_{i,t-1}^2 + b_i h_{i,t-1}, \text{ for } i = 1, 2. \quad (10)$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \text{ where } Q_t^{*-1} = \text{diag}[\sqrt{Q_t}];$$

$$Q_t = (1 - \alpha - \beta) \bar{R} + \alpha z_{t-1} z_{t-1}' + \beta Q_{t-1}, \text{ where } z_{t-1} = \begin{bmatrix} \epsilon_{1,t-1} / \sqrt{h_{1,t-1}} \\ \epsilon_{2,t-1} / \sqrt{h_{2,t-1}} \end{bmatrix}, \bar{R} = E[z_{t-1} z_{t-1}']. \quad (11)$$

As with the BEKK model, the DCC-GARCH model cannot estimate asymmetrical effects. Accordingly, Cappiello, Engle, and Sheppard (2006) [8] extend the DCC-GARCH model to the asymmetric DCC-GARCH (ADCC-GARCH) model to reflect asymmetrical effects. Equations (12) and (13) present the conditional variance process of the ADCC-GARCH model. Equation (12) shows a conditional variance with the additional term $g_i \eta_{i,t-1}^2$, which reflects the asymmetry by the definition of $\eta_{i,t}$. Equation (13) shows a dynamic conditional correlation with additional terms $-\gamma^2 \bar{N}$ and $\gamma^2 \eta_{t-1} \eta_{t-1}'$. γ is a leverage parameter.

$$H_t = D_t R_t D_t, \text{ where } D_t = \text{diag}[\sqrt{h_{i,t}}];$$

$$h_{i,t} = c_i + a_i \epsilon_{i,t-1}^2 + b_i h_{i,t-1} + g_i \eta_{i,t-1}^2, \text{ for } i = 1, 2, \text{ where } \eta_{i,t-1} = \max(0, -\epsilon_{i,t-1}). \quad (12)$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \text{ where } Q_t^{*-1} = \text{diag}[\sqrt{Q_t}];$$

$$Q_t = (1 - \alpha^2 - \beta^2) \bar{R} - \gamma^2 \bar{N} + \alpha^2 z_{t-1} z_{t-1}' + \gamma^2 \eta_{t-1} \eta_{t-1}' + \beta^2 Q_{t-1},$$

$$\text{where } z_{t-1} = \begin{bmatrix} \epsilon_{1,t-1} / \sqrt{h_{1,t-1}} \\ \epsilon_{2,t-1} / \sqrt{h_{2,t-1}} \end{bmatrix}, \bar{R} = E[z_{t-1} z_{t-1}'], \eta_t = \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}, \bar{N} = E[\eta_t \eta_t']. \quad (13)$$

Thus, we estimated the DCC-GARCH model, as shown in Table 5.

Table 5. DCC-GARCH estimation.

Panel A. Symmetric DCC-GARCH			Panel B. Asymmetric DCC-GARCH		
c_1	0.0000 ***	(4.4884)	c_1	0.0000 ***	(4.4884)
c_2	0.0000	(1.5180)	c_2	0.0000 *	(1.8203)
a_1	0.1648 ***	(6.4289)	a_1	0.0000 **	(2.3922)
a_2	0.0319 ***	(4.5130)	a_2	0.0264 **	(2.5715)
b_1	0.7950 ***	(30.3176)	b_1	0.7950 ***	(30.3176)
b_2	0.9649 ***	(130.1763)	b_2	0.9666 ***	(142.6195)
			g_1	0.2794 ***	(7.1945)
			g_2	0.0076	(0.5427)
α	0.0202 *	(1.8399)	α	0.0193	(1.4769)
β	0.9764 ***	(69.4404)	β	0.9728 ***	(21.4186)
			γ	0.0092	(0.0852)
<i>LogLike</i>	23374.415		<i>LogLike</i>	23374.881	
<i>AIC</i>	−46739.830		<i>AIC</i>	−46736.762	
<i>SIC</i>	−23933.830		<i>SIC</i>	−13794.762	

Note: This table estimates the parameters of the symmetric and asymmetric DCC-GARCH models for S&P 500 Index returns and S&P Green Bond Select Index returns. Panel A shows the estimation of Equations (10) and (11), and Panel B shows the estimation of Equations (12) and (13). The numbers in parentheses are *t*-statistics. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively. *LogLike* stands for log-likelihood value, and *AIC* and *SIC* stand for the Akaike information criterion and the Schwarz information criterion, respectively.

The estimated results shown in Table 5 are similar to those shown in Table 4. Panels A and B both have significant coefficients, confirming the volatility transfers between the two markets. However, when we estimate the ADCC-GARCH model, we can conclude that the volatility transfer

between the two markets is not asymmetric, given that the coefficient γ is not significantly different from zero.

5. Conclusions

This study presents an econometric analysis of the green bond market. In particular, we analyze the relationship between the green bond and equity markets. We test whether green bonds exhibit asymmetric volatility and confirm that although green bonds do exhibit asymmetrical volatility, they have the unique characteristic that their volatility is sensitive to positive shocks, unlike other financial instruments. We also analyze the association between the green bond and equity markets and confirm that the two markets have some volatility spillover effects but that neither responds significantly to negative shocks in the other market. This analysis is expected to help investors and policymakers make decisions because it newly identifies some characteristics of green bonds.

However, this study only focuses on the volatilities of both markets, and it may be extended by considering different conditional mean processes. Through such analyses, finding evidence regarding the specific correlations between the returns of the two markets is expected to provide meaningful academic discussions on green bonds.

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